

Quantum gravity and the nature of space and time

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Abstract

This is a non-technical overview of how various approaches to quantum gravity suggest modifications to the way we conceptualize space and time. A theory of quantum gravity is needed to reconcile quantum physics with general relativity, our best theory for gravity. The most popular approaches to quantum gravity are string theory and loop quantum gravity. So far, no approach has been empirically successful, and there is no commonly accepted theory. Thus, the conclusions presented here are tentative. Many approaches suggest that space and time - or perhaps, it is better to talk about spacetime - might not be fundamental. Various views on how to think about time in quantum gravity are briefly presented.

1. Introduction

This is a brief introduction to the changes to our understanding of space and time that a theory of quantum gravity might bring.¹ In a short overview such as this, it is impossible to give anything more than a highly selective presentation. Because the paper is intended for a general audience of philosophers, no technical background in physics is assumed; on the other hand, an acquaintance with basic philosophical concepts and terminology is expected of the reader.

There is no generally accepted and empirically successful theory of quantum gravity. Thus, the conclusions here are tentative but based on serious contemporary approaches to quantum gravity and what they suggest about the nature of space and time.² A theory of quantum gravity must solve the problem of combining and reconciling the already well-established theories of quantum physics and relativity in a consistent and coherent way.

Trying to figure out the nature of space and time is a project of a metaphysical character. Here - as is common in contemporary philosophy - any attempt to give a description of the nature of reality itself is considered to be metaphysical. With this broad characterization of metaphysics, scientific theories, if understood realistically, contain metaphysical claims. Some philosophers believe that metaphysical work can be done independently from science, but a common view - shared by the author - is that metaphysics worth taking seriously must take contemporary science into account.

Philosophers should be warned that using the term “metaphysics” might lead to misunderstandings when communicating with physicists since the expression can have rather negative connotations. This is probably due to lingering influences from logical positivism in the physics community. Nowadays most physicists have a more realist attitude towards science compared to logical positivists. This means that they have metaphysical views according to the terminology used here even if they would not describe their own work as metaphysical.

2. Why we need a theory of quantum gravity

In the 20th century, our physical worldview was radically altered by the theories of relativity and quantum physics. According to relativity, space and time are understood to form one cohesive whole, spacetime. The changes to our physical worldview due to quantum physics are generally considered to be even more dramatic but have so far not led to further modifications to our conception of space and time. To formulate a theory of quantum gravity it seems that we need to apply the lessons from quantum physics to spacetime itself. It is expected that this will require new modifications to the way we think about space and time. Effects from quantum gravity become important at the so-called Planck scale. By combining the speed of light c , the gravitational constant G and Planck's constant h in different ways, values of length, time and energy can be calculated. To give just one example, the Planck length is of the order of magnitude 10^{-35} m. At this length scale, effects from quantum gravity must be taken into account and we can no longer trust our current account of spacetime to give a good description of the world.

The debate about how to understand quantum physics and what it tells us about reality is still going on. There are also conceptual issues that can be raised regarding the theories of relativity. Although there are difficulties in understanding exactly what these already well-established theories tell us about reality, it is not in question that the theories must be taken very seriously. They are strongly empirically supported and have been relied on for a plethora of technological applications.

According to contemporary physics, there are four fundamental forces: gravity, electromagnetism, the weak force and the strong force. With the exception of gravity - described by Einstein's general theory of relativity (GR) - the other forces are described in terms of the formalism of quantum field theory. GR is not quantum and does not conceptually fit with the other forces in the standard model of particle physics. Because there are situations where one must take both quantum physics and strong gravitational effects into account - for example, to fully understand black holes or the early stages of the big bang - we need a new theoretical framework where quantum physics and relativity can be coherently combined.³ Unfortunately, there is a lack of relevant empirical data to guide us in formulating the theory and it is very difficult to experimentally test theories of quantum gravity. Our best strategy is to take our established theories seriously as a starting point and try to find a way of combining quantum physics with gravity in a coherent and consistent way. The success of the well-established theories provides us with constraints on how to formulate a theory of quantum gravity. It is not easy to come up with anything that does not immediately come into conflict with what we know from quantum physics and relativity. So, although there are a number of different approaches, we cannot just dream up anything. Hopefully a theory will eventually be formulated that can successfully be confronted with new empirical data.

3. How different approaches to quantum gravity modify our understanding of space and time

How could a quantum theory of gravity alter the way we think about the nature of space and time? To answer this question one needs to take a look at the various approaches to quantum gravity that have been developed. Different approaches suggest different modifications to our conception of space and time but there are some commonalities. Regardless of which approach

one follows, a necessary requirement for the theory to be successful is that one is able to explain how space and time as we experience them arise and thus account for the success of our current theories. In particular, at least some models of any theory of quantum gravity must be able to approximate the spacetime picture we are familiar with from GR. It is still an open question exactly how a more fundamental theory is supposed to approximate GR.

3.1 Spacetime, fundamental or not?

A common conclusion shared by many different approaches to quantum gravity is that spacetime might be “emergent” in some sense.⁴ When physicists and philosophers of physics discuss emergence in this context, they typically do not claim something as strong as when the term “emergent” is used in some other parts of philosophy. Here, saying that spacetime is emergent means that it is not part of a more fundamental theory of the world but derivable from this more fundamental theory. This way of talking about emergence is supposed to be compatible with reductionism.⁵

A rather generic idea suggested by many approaches is that we have to modify our view on space and time by revising our description of spacetime as a continuum. Maybe spacetime is better described in terms of some sort of discrete structure. One approach of this kind is causal set theory (CST). CST is much less developed compared to loop quantum gravity and string theory; so far, no complete quantum version of CST has been formulated.⁶ In CST, one explicitly starts out with a discrete collection of primitive elements forming a partially ordered set with respect to a primitive binary relation. The spatiotemporal properties of spacetime are supposed to be derivable from this fundamental network of discrete elements. Some causal set networks form structures that in a satisfying way approximate the spacetime descriptions we are familiar with from GR. In GR, the description of a spacetime encodes which points in spacetime that can causally be connected to each other; in the sense that signals or objects can be sent from the earlier event to the later. When the causal set approximates a GR spacetime, the binary relation used to form the partial order can be interpreted to be a causal relation indicating which events can causally influence others.

Another discrete picture appears in loop quantum gravity (LQG).⁷ Here, the discreteness is not put in from the start but is derived as a consequence of other assumptions. Quantum states of the theory can be graphically described forming a so-called “spin network”. It is important to understand that the network - or more precisely, a quantum superposition of such networks - is not supposed to lie in space, but to make up space. As a result of the formalism of LQG, areas and volumes will be quantized and take on discrete values. Not all solutions of LQG would give us something similar to a traditional spacetime though. For LQG, a challenge has been to find solutions that recover traditional spacetime pictures; it has for instance not yet been shown that LQG has solutions that would correspond to Minkowski spacetime, that is. the basic flat spacetime.

The most popular approach to quantum gravity among physicists is string theory.⁸ Prima facie, it looks like string theory has a more conservative picture with respect to spacetime, albeit with more dimensions than the four-dimensional spacetime we are used to; supersymmetric string theories require 10 dimensions.

A common complaint by opponents to string theory is that the theory is “background dependent” since it requires a fixed background spacetime to be put in from the start. In response to this, string theorists often claim that the theory is background independent because it can be shown that the geometry of the background must satisfy Einstein’s field equations. For a derivation of this result see for instance Polchinski (1998).⁹ The conflicting assessments derive from conflicting requirements for what is needed to judge something to be truly background dependent.¹⁰ Still, it is fair to say that string theory would be background independent in a weaker and less explicit sense compared to LQG, which is manifestly background independent.

The basic idea of string theory is to suggest that what we have previously believed to be point particles are actually one-dimensional extended entities, that is, strings. When the strings move through spacetime, they form two-dimensional surfaces called *world-sheets*. When we explore the consequences of this assumption in a quantum theory it seems that this makes it possible to combine and unify all forces including gravity in one unified picture. Different particles can be seen to correspond to different vibrational states of a quantum string, and one such vibrational state can be understood to correspond to the graviton, the quantum particle mediating gravitational interactions. The properties of the particles depend on the background in which the strings are moving. An often-used analogy is to say that one can think of different particles as different tones of the string and the background can be thought of as the resonance box deciding which tones that can be played. A flat background gives only massless particles and particles way too massive to correspond to the particles of the standard model.

I have so far - as is commonly done - considered the background manifolds to represent spacetime, this is however not unproblematic. This view is called into question by “dualities”. In physics, these are physically equivalent descriptions, which seem to describe very different situations. In string theory, dual pictures may have background “spacetimes” with very different geometries or topologies.¹¹ But according to physicists, dual pictures describe a common reality; this view undermines a direct interpretation of the background manifolds as accurate representations of spacetime. Huggett (forthcoming) argues that we cannot identify the background manifold with spacetime. The argument is based on considering T-dualities in string theory where two different background manifolds constitute a dual pair; in this case they have circular compact dimensions forming circles of different radii. In T-duality, we can figure out what the effective or phenomenal spacetime will be; this has to be the same in the dual pictures because they describe physically equivalent situations. It turns out that the larger of the two radii of the target manifolds is the same as the radius of the effective or phenomenal spacetime. The basic point that we cannot directly identify the background manifolds with spacetime can be generalized to more complicated dualities where even the topology differ between the manifolds in the dual pair of backgrounds.

So, although string theory superficially seems to have a more traditional treatment of spacetime, albeit with extra dimensions, dualities complicate the picture. One way of understanding how spacetime is supposed to appear in string theory is to emphasize that in a sense the two-dimensional world-sheets of the strings are more fundamental than the higher dimensional spacetime.¹² A quantum treatment of the string is based on quantum fields on the world-sheets. The metrical properties of spacetime are supposed to appear or emerge from the quantum description of a large number of strings. In practice, we have to introduce a classical background when trying to do calculations. In general it can be a challenge to figure out what the effective or

phenomenal spacetime would be. In the absence of a dual picture the background could give a good description of the effective spacetime. However, if two backgrounds upon quantization turn out to form a dual pair we have two physically equivalent dual pictures, so the relevant effective spacetime must in principle be constructible only from the shared physical content of the dual pictures. Thus there is a kind of emergence of spacetime also in string theory; here, the main point is however not primarily a contrast between continuous and discrete.

Another duality in string theory that has been used to argue for emergence of spacetime is the Anti de Sitter / Conformal Field Theory correspondence (AdS/CFT). This duality has not been rigorously proved but is believed to be true. Here, a quantum field theory in a four dimensional background is found to be equivalent to a 10-dimensional string theory. For an introduction to AdS/CFT, in which the view is adopted that this means that six dimensions of spacetime is emergent see de Mello Koch and Murugan (2012). Philosophers of physics have pointed out that if dual theories are truly physically equivalent, it does not make sense to see one of the theories as emerging from the other, see Rickles (2013) and Teh (2013). In Dieks, van Dongen and de Haro (2015) and de Haro (forthcoming), the argument is made that emergence from one picture to the other could work if the duality is only approximate. Because the duality has not been strictly proved, this is a live possibility.

3.2 Time in quantum gravity

When attempting to formulate a theory of quantum gravity, the so-called “problem of time “ shows up. The problem primarily appears in canonical approaches such as LQG. To use the canonical formalism, one has to divide up spacetime in some way into space and time and introduce a time variable with respect to which the space evolves. In the classical non-quantum case, the development of space can be taken to form a spacetime. However, in GR, there is no universal or fixed time, so one can choose very different slices through spacetime to start this process with correspondingly different time variables. At the classical level the spacetime we end up with will be the same regardless of which slice through spacetime we start with.

When attempting to quantize gravity in the canonical formalism, this freedom of redefining the time variable has the consequence that it turns out that it seems as if time plays no role at all. It leads to a timeless formulation where physical observables we would expect to change over time are fixed and do not allow for any evolution.¹³ This is in clear conflict with our experience of how things change. The problem of time has elicited a number of responses. Could it be that the conclusions we should draw about space and time in a future theory of quantum gravity are such that the same attitude should not be taken with respect to both space and time? Since the early days of relativity, we have learned to think of space and time as intimately connected making up just one spacetime. Hence, it seems plausible to think that space and time should be treated in the same fashion in a theory of quantum gravity. However some researchers have suggested that in quantum gravity space and time should not be treated in the same way thus abandoning what can be considered to be one of the major insights from the theories of relativity.

For instance Barbour thinks we need to eliminate time in a radical sense from the fundamental ontology.¹⁴ According to Barbour, reality is a collection of three-dimensional spaces; they can be seen as individual moments. They do not form a spacetime at all; they are separate and not connected in such a way that they form a cohesive whole. Some moments are called “time

capsules” and contain in them data that give the illusion of a history. It is from these moments that the appearance of time is derived, but time itself is not at all part of the fundamental ontology.

In contrast to Barbour’s extreme eliminativism with respect to time, Smolin has instead argued that time should be taken to be the one thing that is truly fundamental, even if space might not be.¹⁵ Smolin’s view is presentist in the sense used by metaphysicians, thus only the present exist, but the present changes and evolves as a consequence of a real and truly fundamental time. A necessary but not sufficient requirement for a presentist view is a privileged foliation of spacetime and relativity implies that no foliation can be empirically distinguished. Since Smolin’s view is presentist, he thinks that despite this there may exist an objective time according to which: for any pair of points in spacetime, it is an objective fact whether or not they are truly simultaneous according to the objective time and if they are not simultaneous in this sense, there is an objective fact which of the points are the earlier one.¹⁶

The above suggestions are quite radical and abandon a central idea from the theories of relativity, namely, the relativity of simultaneity. They both treat space and time very differently from each other distinguishing their views from the way in which space and time are combined to form a spacetime in GR. It is quite startling that they come to very different conclusions and where Barbour thinks space to be real and fundamental and time just an illusion, Smolin thinks that time is what is truly fundamental.

Others do not want to abandon such an important part of the theories of relativity. For instance Rovelli’s response to the problem of time is quite different and he thinks that the relativity of simultaneity is one of the discoveries that will not be abandoned.¹⁷ He thinks that we should not accept or expect there to be an objective time; this is already a lesson we should have learned from the theories of relativity. Neither time nor space is fundamental according to Rovelli’s account, but both are supposed to be emergent in the same way. Rovelli points out that to be able to measure how things change we always do this by comparing different observables, not by using an external fixed time, we need to understand the quantum measurements in this relational fashion. The predicted measurements are between relevant correlations between observables.¹⁸

I mention these various suggestions just to show the range of responses to the problem of time. Here I will not evaluate the various views or enter into arguments about which view is the best one. Given the current state of quantum gravity where we do not have an established theory it is probably good if a number of different ideas are tried out and pushed as far as possible.

4. Final comments

With this short presentation, I hope to have given an easily accessible overview of various ways in which modern research in quantum gravity suggests new and radical changes to our understanding of space and time. I would be very happy if this text inspires at least a few more philosophers to decide to further study and start working on the philosophical issues connected with the nature of space and time in quantum gravity.

Some have contemplated that solving the measurement problem and give a correct interpretation or view of quantum physics could be connected to the problem of formulating a quantum theory

of gravity; this may be the case but it does not have to be so. However, I think it is at least not unreasonable or out of the question that there is a connection. For this reason, I think it could be interesting to keep this option in mind. Perhaps one should even specifically think about how or if different views or interpretations are more easily compatible with or makes more sense in different approaches to quantum gravity.

Another interesting possibility to keep in mind is that various approaches may converge towards something. Perhaps different approaches each have found different parts of the puzzle and the right way forward is to combine a number of ideas from different approaches in a novel way.

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Extra Resources

For more on this topic the interested reader is advised to go to beyondspacetime.net. On this webpage, a large number of videos of talks about quantum gravity given by physicists and philosophers can be found.

Author biography

Keizo Matsubara received a PhD in theoretical physics from Uppsala University in 2004. After this, he studied philosophy and became a researcher in a project on philosophical aspects of string theory. He received a second PhD from Uppsala University in 2013, this time in philosophy. He held a two-year postdoctoral position at the Rotman Institute of Philosophy, Western University. He is currently a postdoctoral researcher at the University of Illinois at Chicago working on the project Space and Time after Quantum Gravity.

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Endnotes

¹ For more on quantum gravity written for philosophers of physics, see for instance: Callender and Huggett (2001),

² Because all approaches to quantum gravity are work in progress without empirical support, some are very skeptical towards the whole field of research. In this paper, the epistemic and scientific status of quantum gravity research will not be discussed.

³ Do we have to quantize GR to formulate a theory of quantum gravity? That might be the case but according to some approaches, this is not required. Here, all approaches attempting to reconcile quantum physics and gravity will be considered to be theories of quantum gravity. For more on whether or not a theory of quantum gravity requires GR to be quantized see Wüthrich (2005).

⁴ For more on the idea that spacetime might be emergent, see Huggett and Wüthrich (2013). The article also addresses some critique that can be leveled against theories where spacetime is emergent.

⁵ For an extensive discussion of a way of understanding emergence so that it is compatible with reduction, see Butterfield (2011a) and Butterfield (2011b).

⁶ For an introduction to causal set theory, see for instance Henson (2012). For a philosophical discussion of the approach, see Wüthrich (2012). Causal set theory is an approach towards quantum gravity that is much less developed compared to loop quantum gravity and string theory; there is for instance not yet a fully developed description of the quantum dynamics of the theory.

⁷ Two textbooks in loop quantum gravity are Gambini and Pullin (2011) and Rovelli (2004).

⁸ Zwiebach (2009) is a good introductory textbook in string theory. For more advanced treatments, see Polchinski (1998) or Becker, Becker and Schwarz (2006).

⁹ For more philosophical discussion on how GR is derived from string theory, see Huggett and Vistarini (2015).

¹⁰ Belot (2011) is an example of a recent article addressing the debate on the concept of background independence. He argues that whether or not a theory is background independent is not something that can be read directly from the formalism; it also requires a physical interpretation. He also argues that there can be different degrees of background independence.

¹¹ Dualities are discussed at different levels of technicality in the standard textbooks Polchinski (1998), Becker, Becker and Schwarz (2006) and Zwiebach (2009). For a philosophical discussion, see for instance Dawid (2006), Rickles (2011) and Matsubara (2013).

¹² See for instance Witten (1996); this article is reprinted in Callender and Huggett (2001).

¹³ There are obviously a number of technicalities and subtleties that cannot be addressed here. A discussion at a more technical level addressing the philosophical and metaphysical issues is Rickles (2006). See also Huggett, Vistarini and Wüthrich (2013).

¹⁴ For a popular account of Barbour's view, see Barbour (2000).

¹⁵ See Smolin (2013) for a popular treatment.

¹⁶ For a discussion and critique of other suggestions that quantum gravity might be hospitable to presentism, see Wüthrich (2010).

¹⁷ See Rovelli (2004) p. 1306.

¹⁸ There are obviously many subtleties involved here about how to construe the relevant observables and how this is consistent with our experience of the passage of time. For more details see Rovelli (2004) and Rovelli (2007).

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