A $\gamma\gamma$ enhancement of the 125 GeV Higgs boson signal relative to the Standard Model (SM) expectation has been reported by the ATLAS and CMS collaborations, and can be obtained if it is identified principally with the neutral $H_0^0$ of the two Higgs doublets of minimal supersymmetry. We focus on sparticles and the pseudoscalar Higgs $A$ with TeV masses. The off-diagonal element of the $(H_0^0, H_0^0)$ mass matrix in the flavor basis must be suppressed, and this requires a large Higgsino mass parameter, $\mu \sim$ TeV, and large $\tan \beta$. A minimal supersymmetric Standard Model sum rule derived that relates $\gamma\gamma$ and $b\bar{b}$ rates, and a $\gamma\gamma$ enhancement relative to the SM predicts $b\bar{b}$ reduction. On the contrary, natural supersymmetry requires $|\mu| < \sim\!0.5$ TeV, for which $\gamma\gamma$ is reduced and $b\bar{b}$ is enhanced. This conclusion is independent of the $m_A$ value and the supersymmetry quantum correction $\Delta_{b\bar{b}}$. Relative $\tau\tau$ to $b\bar{b}$ rates are sensitive to $\Delta_{b\bar{b}}$.

The ratios of the $h$ and $H$ couplings to those of the SM Higgs $h_{SM}$, denoted as $r_{pp}^{hH} (= g_{hHpp}/g_{hSMpp})$, are given by

$$
\begin{align*}
& r_{VV}^h = s_\beta - a, & r_{tt}^h = c_a/c_\beta, & r_{\tau\tau}^h = -s_a/c_\beta, \\
& r_{bb}^{H} = \frac{-s_a}{c_\beta} \left[ 1 - \frac{\Delta_b}{1 + \Delta_b} \left( 1 + \frac{1}{\tan \beta} \right) \right], \\
& r_{VV}^H = c_\beta - a, & r_{tt}^H = s_a/c_\beta, & r_{\tau\tau}^H = c_a/c_\beta, \\
& r_{bb}^{H} = \frac{c_a}{c_\beta} \left[ 1 - \frac{\Delta_b}{1 + \Delta_b} \left( 1 - \frac{1}{\tan \beta} \right) \right]
\end{align*}
$$

where we include the 1-loop contribution $\Delta_{b\bar{b}}$ to the $b\bar{b}$ coupling. $\Delta_{b\bar{b}}$ is the $b$-quark mass correction factor [11,12], which may be sizable, especially if both $\mu$ and $\tan \beta$ are large.

$$
\Delta_{b\bar{b}} = -\mu \tan \beta \left[ 2a_s \left\{ \frac{2}{3} m_b I(m_{b}, m_{b}^2, m_{m}^2, m_{m}^2) + \frac{h_0^2}{16\pi^2} a_t I(m_{b}, m_{b}^2, m_{m}^2, m_{m}^2) \right\} \right],
$$

$$
I(x, y, z) = - xy \ln y + yz \ln y + zx \ln z / (x - y)(y - z)(z - x),
$$

$$
I(x, y, z = y) = - \left[ x - y + x \log \frac{y}{x} \right]/(x - y)^2,
$$

$$
I(x, x, x) = \frac{1}{2x}.
$$

The first (second) term of $\Delta_{b\bar{b}}$ is due to the sbottom-gluino (stop quark—chargino) loop. We nominally take $M_{susy} = 1$ TeV and express sparticle masses $\tilde{m}$ in units of $M_{susy}$. The top Yukawa coupling is $h_t = \tilde{m}_t/v_u = \tilde{m}_t/(v_u \nu_u)$ and $t_\alpha = \tan \alpha$. Our interest is in large $\tan \beta$, $\tan \beta \gtrsim 20$, and in the decoupling regime with large $m_A$ for which $\alpha \approx \beta = -\frac{\pi}{2}$.
\[ m_t = m_t(m_t) = 163.5 \text{ GeV} \] is the running top quark mass \[ [13]. \] We consider \[ m_Q = m_U = m_D = M_{\text{asy}} \] for the squark masses in the third generation.

The off-diagonal element of the stop quark squared mass matrix \( \tilde{m}_t = \tilde{m}_t(m_t) \) is \( \tilde{m}_t X_t \) where the stop quark mixing parameter \( X_t \) is given by \( X_t = A_t - \mu / \tan \beta. \) The quantities \( A_t, \mu \) and \( X_t \) are also defined in units of \( M_{\text{asy}} \) as \( a_t = A_t / M_{\text{asy}}, \mu = \mu / M_{\text{asy}}, \) and \( x_t = X_t / M_{\text{asy}} = a_t - \mu / \tan \beta. \) Our sign convention for \( \mu \) and \( A_t \) is the same as \[ [14], \] opposite to the sign convention of Ref. \[ [15]. \] We fix \( \tilde{m}_t = 2, \) well above the current LHC reach, \( \tilde{m}_{p_1} = \tilde{m}_{p_2} = 1, \) and \( \tilde{m}_{i_1} = 0.8, \) \( \tilde{m}_{i_2} = 1.2. \) A stop quark mass difference \( \tilde{m}_{i_1} - \tilde{m}_{i_2} \approx 0.4 \text{ TeV} \) is chosen in accord with the natural SUSY prediction \[ [16. \] Then \( \Delta_b \) is well approximated numerically by

\[
\Delta_b \approx \tilde{\Delta}_b \approx \frac{\tilde{\Delta}_b}{20} \left[ 0.26 + \left( \frac{0.09}{|\tilde{\mu}|} + 0.6 - 0.003 |a_t| \right) \right], \tag{5}
\]

where the first and the second terms in the square bracket are the values of the gluino and the chargino contributions, respectively.

The chargino and neutralino masses have no special role except possibly in \( b \to s \gamma \) decay, but consistency with natural SUSY has been found there \[ [17]. \] Large \( m_{\tilde{t}} \) implies a large charged Higgs \( H^\pm \) mass and this suppresses the \( H^\pm \) loop contribution to \( b \to s \gamma \).

The \( gg, \gamma \gamma \) coupling ratios \( r_{gg, \gamma \gamma}^{\phi} \) for \( \phi = h, H, A \) relative to those of \( h_{\text{SM}} \) are \[ [18], \]

\[
r_{gg}^{\phi} = I_{gg}^{\phi} \frac{r_{V}^{h} + r_{bb}^{h}}{r_{V}^{h} + r_{bb}^{h}},
\]

\[
r_{\gamma \gamma}^{\phi} = \frac{7}{4} I_{WW}^{\phi} r_{V}^{h} - \frac{4}{9} r_{V}^{h} - \frac{1}{9} I_{bb}^{\phi} r_{bb}^{h}.
\]

where \( I_{WW,bb}^{\phi} \) represent the triangle-loop contributions to the amplitudes normalized to the \( m_t \to 0 \) limit \[ [19–21]. \]

The \( XX \to h \to PP \) cross section ratios \[ [18] \] relative to \( h_{\text{SM}} \) are obtained from

\[
\sigma_{\phi}^{\text{SM}} = \frac{\sigma_{XX-\phi PP}}{\sigma_{XX-\phi PP}} = \frac{|r_{XX}^{h} r_{PP}^{h}|^2}{R^{h}}, \tag{7}
\]

\[
R^{h} = \frac{\Gamma_{h}^{\text{SM}}}{\Gamma_{h}^{\text{SM}}}, \tag{8}
\]

where \( R^{h} \) is the ratio of the \( h \) total width to that of \( h_{\text{SM}}. \)

\[
\Gamma_{h}^{\text{SM}} = 4.14 \text{ MeV} \ [22] \text{ for } m_h = 125.5 \text{ GeV}. \]

The coefficients in Eq. \[ (8) \] are the SM Higgs branching fractions. Here we have assumed no appreciable \( h \) decays to dark matter.

**Sum rule of cross-section ratios**.—In the large \( m_A \) region close to the decoupling limit, \( \alpha \) takes a value

\[
\alpha = \beta - \frac{\pi}{2} + \epsilon \tag{9}
\]

with \( |\epsilon| < \frac{\pi}{2} - \beta. \) Then, the \( r_{XX}^{h} \) of Eq. \[ (2) \] are well approximated by

\[
r_{VV}^{h} = 1, \quad r_{cc}^{h} = 1 + \epsilon / t_{\beta}, \quad r_{tt}^{h} \approx 1 - \epsilon t_{\beta}, \quad r_{bb}^{h} \approx 1 - \frac{1}{1 + \Delta_{b}^{\text{eff}}} \epsilon t_{\beta} \tag{10}
\]

through first order in \( \epsilon. \) The \( r_{cc}^{h} \) are close to unity because those deviations from SM are \( t_{\beta} \) suppressed. Thus,

\[
r_{gg}^{h} \approx r_{\gamma \gamma}^{h} \approx 1, \tag{11}
\]

since the bottom triangle loop function \( r_{bb}^{h} \) is negligible in Eq. \[ (6). \] Only \( r_{bb}^{h}, r_{tt}^{h} \) can deviate sizably from unity for large \( m_A \) and large \( \tan \beta. \) Following Eqs. \[ (7) \] and \[ \epsilon, \]

\[
\sigma_{\gamma} = \sigma_{W} = \sigma_{Z} = \frac{1}{0.6(r_{bb}^{h})^2 + 0.4}, \tag{12}
\]

and \[ 0.4 \sigma_{\gamma} + 0.6 \sigma_{h} = 1, \tag{13} \]

where the SM \( b \bar{b} \) branching fraction \[ [23] \] is approximated as 60%. Equation \[ (12) \] holds independently of the production process. Enhanced \( \sigma_{\gamma} \) implies reduced \( \sigma_{bb}, \) as well as enhanced \( \sigma_{W} \) and \( \sigma_{Z}. \)

**Flavor-tuning of mixing angle \( \alpha. \)**—Note that \( r_{bb}^{h}, r_{tt}^{h} = 1 \) in the exact decoupling limit \( m_A \to \infty \) for which \( \epsilon = 0. \) Flavor-tuning of \( \epsilon \) to be small but nonzero is necessary to obtain a significant variation of \( r_{bb}^{h} \) from unity. Positive (negative) \( \epsilon \) gives \( bb \) reduction (enhancement).

The mixing angle \( \alpha \) is obtained by diagonalizing the squared-mass matrix of the neutral Higgs in the \( u, d \) basis. Their elements at tree level are

\[
(M_{ij}^{\text{tree}})^2 = M_{ZZ}^2 s_\beta^2 + m_A^2 c_\beta^2, \quad M_{Z2}^2 c_\beta^2 + M_{\chi 2}^2 c_\beta^2, \quad M_{\chi 2}^2 s_\beta^2,
\]

\[
- (M_{Z2}^2 + m_\chi^2 s_\beta c_\beta, \tag{14}
\]

for \( ij = 11; 22; 12 \) respectively, which gives \( \epsilon < 0 \) in all regions of \( m_A. \) Thus, in order to get \( bb \) reduction, it is necessary to cancel \( (M_{ij}^{\text{tree}})^2 \) by higher order terms \( \Delta M_{ij}^{\text{tree}}. \)

In the 2-loop leading-log (LL) approximation the \( \Delta M_{ij}^{\text{tree}} \) are given \[ [14,24] \] by

\[
M_{ij}^{2} = (M_{ij}^{\text{tree}})^2 + \Delta M_{ij}^{2}, \tag{15}
\]

where
\[
\Delta M_{11}^2 = F_3 \frac{3m_t^4}{4\pi^2 v^2 s_\beta} \left[ t(1 - G_2 t) + a_i x_i \left( 1 - \frac{a_i x_i}{12} \right) (1 - 2G_2 t) \right] - M_Z^2 s_\beta^2 (1 - F_3),
\]
\[
\Delta M_{22}^2 = -F_2 \frac{m_t^4}{16\pi^2 v^2 s_\beta} \left[ (1 - 2G_2 t) (x_i \mu) \right]^2,
\]
\[
\Delta M_{12}^2 = -F_2 \frac{3m_t^4}{8\pi^2 v^2 s_\beta} \left[ (1 - 2G_2 t) (x_i \mu) \left( 1 - \frac{a_i X_i}{6} \right) \right] + M_Z^2 s_\beta c_\beta (1 - F_3),
\]
(16)

where \( F_i = 1/(1 + l_i h_i/t) \) with \( l = 3, \frac{3}{2}, \frac{9}{4} \) and \( G_i = -1/(\pi h_i/2) \), and \( h_i \) are due to the wave function renormalization of the Higgs field. The index \( l \) is the numerical value of \( h_i \) fields in the effective potential of the two Higgs doublet model. \( F_3 \) is due to the wave function renormalization of the Higgs field and the index \( l \) is the numerical value of \( h_i \) fields in the effective potential of the two Higgs doublet model. \( F_3 \xi^4 \approx F_3 \xi^4 \approx F_3 \xi^2 \approx 1 \) where \( \xi \) is defined by \( H_a(M_a) = H_a(m_t) \xi \) and \( \xi = F^{-1} \). The formulas of Refs. [14,24] are based on the expansion \( F_1 = 1 - t \xi \) but our formula of \( F_1 \) is more exact and has better approximation at large \( t \). The parameter \( tan \beta = v_u/v_d \) is defined in terms of the Higgs vacuum expectation values \( v_u = (H^0_u/M_u) \) at the minimum of the 1-loop effective potential at the weak scale \( \mu = m_t \) and \( v = \sqrt{v_u^2 + v_d^2} \approx 174 \) GeV, while \( a_i, x_i \), \( \mu = M_{susy} \). The relation \( cot \beta(m_t) = cot \beta(M_{susy}) \xi^{-1} \) will be used in the following calculation.

Numerically \( \alpha_i = \alpha_i(m_t) = 0.109 \) giving \( -32\pi a_s = -10.9 \). Also, \( h_i = m_t/v = 0.939 \), \( G_{1/2} = 0.0274 \), \( 0.0442 \) and \( t = \log(15\sqrt{v})^2 = 3.62 \); thus, \( G_{2/2} = 0.099 \), \( 2G_2 = 0.320 \), and \( F_2 = 0.892 \).

In the large \( m_A \) limit, the \( m_h^2 \) expression is

\[
m_h^2 = M_2^2 c_\beta^2 \]
\[
+ M_2^2 \frac{3m_t^4}{4\pi^2 v^2 s_\beta} \left[ t(1 - G_2 t) + \left( 1 - 2G_2 t \right) \left( x_i - \frac{x_i^4}{12} \right) \right] - M_Z^2 s_\beta^2 (1 - F_3) - \frac{3m_t^4}{8\pi^2 v^2 s_\beta} \left( x_i \mu \right) \left( 1 - \frac{a_i X_i}{6} \right)
\]
(17)

where the Higgs wave function renormalization factor \( \xi \) is retained in the denominator of \( F_3 \). In the usual expansion of the \( F_3 \) denominator \( G_2 \) and \( G_2 \) are replaced by \( G_2^2 \) and \( G_2^4 \) and \( G_2^4 \) are replaced by \( G_2^2 \); \( m_h^2 = M_2^2 c_\beta^2 + \frac{3m_t^4}{4\pi^2 v^2 s_\beta} \left( t(1 - G_2 t) + (1 - 2G_2 t) \left( x_i^3 - \frac{x_i^4}{12} \right) - M_Z^2 s_\beta^2 X_i \right) \). However, numerically Eq. (17) significantly increases \( m_h \) at large \( M_{susy} \) as shown in Fig. 1. Equation (17) gives increasing \( m_h \) as \( M_{susy} \) increases up to \( \sim 7 \) TeV, while the usual formula with the expansion approximation of \( F_3 \) gives decreasing \( m_h \) when \( M_{susy} > 1.3 \) TeV and it is not applicable at large \( M_{susy} \).

The experimental \( m_h \) determinations from the LHC experiments are [1,2]

\( m_h = 125.3 \pm 0.4 \pm 0.4, \quad 126.0 \pm 0.4 \pm 0.4 \) GeV. (18)

It seems unlikely that the central \( m_h \) determination will change much with larger statistics because of the excellent mass resolution in the \( \gamma \gamma \) channel. The experimental \( m_h \) value is near the maximum possible value of \( m_h \) in Eq. (17) and this constraint the value of \( x_i \) to \( \left| x_i \right| \approx \sqrt{6} \), to maximize the term \( x_i^2 - \frac{x_i}{12} \). This is known as “maximal-mixing” in the stop quark mass matrix [16]. In Eq. (17) we require \( m_h \gtrsim 124 \) GeV. This implies

\( 1.95(= x_{min}) < \left| x_i \right| < 2.86(= x_{max}) \), (19)

where we should note that the positive \( x_i \) branch is favored by the SUSY renormalization group prediction [16].

By using Eq. (15) the Higgs mixing angle \( \alpha \) is determined from

\[
\tan 2\alpha = \frac{2M_{12}^2}{M_{22}^2 - M_{11}^2} \approx \frac{(m_h^2 + M_Z^2) s_2 \delta - 2\Delta M_{12}^2}{(M_A^2 - M_Z^2) c_2 \delta + (\Delta M_{11}^2 - \Delta M_{22}^2)},
\]

(20)

\[
\Delta M_{12}^2 \approx - \frac{\mu}{s_\beta} x_i \left( 1 - \frac{x_i^4}{6} \right) 558 \text{ GeV}^2 + 24 \cdot \frac{20}{\tan \beta} \text{ GeV}^2.
\]

(21)

Defining \( \zeta(= M_{22}^2/m_h^2) \), \( \delta(= \Delta M_{12}^2/m_h^2) \), and \( \eta(= \frac{1}{2} \times (\Delta M_{11}^2 - \Delta M_{22}^2)/m_h^2) \), \( \epsilon \) is simply given in the first order of \( \zeta, \delta, \) and \( \eta \) by

\[
\epsilon = -2 \frac{\zeta + \eta}{\tan \beta} + \delta.
\]

(22)

We note that \( r_{hb}^2 \) is related to \( \epsilon \) through Eq. (10). With the \( x_i \) constraint in Eq. (19), we can derive the allowed
FIG. 2 (color online). $\bar{\mu}$ dependence of $\sigma_\gamma = \sigma_{\gamma}/\sigma_{\text{SM}}$ (upper panels), $\sigma_b = \sigma_{b}/\sigma_{\text{SM}}$ (middle panels), and $\sigma_{\tau} = \sigma_{\tau}/\sigma_{\text{SM}}$ (lower panels) for $m_A = 500$ GeV. Their allowed values are between the solid red curve (corresponding to $|\chi| = x_{\text{max}}$) and the dashed blue curve (corresponding to $|\chi| = x_{\text{min}}$). Left (right) panels show negative (positive) $\chi$ region. Deviations from unity are enlarged for a large negative $\bar{\mu}$, but there the perturbative calculation is unreliable due to a large quantum correction.
region of $r_{bb}^h$ for each $\mu$ value. Correspondingly, the allowed regions of $\sigma_s(=\sigma_{\gamma\gamma}/\sigma_{\text{SM}} = \frac{1}{\alpha_{\gamma\gamma}^2 + \alpha_{\text{SM}}^2})$, $\sigma_b(=\sigma_{bb}/\sigma_{\text{SM}} = \frac{(\mu_{\text{SM}}^2)^2}{0.6(\mu_{\text{SM}}^2)+0.4})$ and $\sigma_c(=\sigma_{\tau\tau}/\sigma_{\text{SM}} = \frac{(\mu_{\text{SM}}^2)^2}{0.6(\mu_{\text{SM}}^2)+0.4})$ are given respectively by the two curves in Fig. 2 where we take $\tan\beta = 50$.

The condition $r_{bb}^h = 1$, or equivalently $\epsilon = 0$, $t_{2\alpha} = t_{2\beta}$, defines the boundary that separates $\gamma\gamma$ enhancement and suppression in the parameter space:

$$r_{bb}^h = 1 \Leftrightarrow \epsilon = 0 \Leftrightarrow \Delta M_{12}^2 = M_{Zs}^2 - \frac{\Delta M_{11}^2 - \Delta M_{22}^2}{2} t_{2\beta}. \quad (23)$$

where $M_{\text{asy}} = 1$ TeV and $m_A = 0.5$ TeV. The relevant sparticle masses are taken commonly with the values given above Eq. (5). The $m_h$ value is predicted by Eq. (17). We also note that the predicted values of $B_{s} \to \mu^+ \mu^-$ branching fraction of these benchmark points are consistent with the experimental measurement [26], $(3.2^{+1.4}_{-1.2}\mu_{\text{SM}}) \times 10^{-9}$ within 2$\sigma$.

**Natural SUSY predictions.**—Natural SUSY always predicts $b\bar{b}$ enhancement and $\gamma\gamma$ reduction [27].

$$m_A \approx 500 \text{ GeV} \begin{array}{c} 0.82 \sim 0.91 \ 1.06 \sim 1.12 \ 1.04 \sim 1.08 \end{array} m_A \approx 1000 \text{ GeV} \begin{array}{c} 0.95 \sim 0.98 \ 1.01 \sim 1.03 \ 1.01 \sim 1.02 \end{array} \quad (25)$$

Here we have taken $|\mu| \approx 500$ GeV and the other parameters are fixed with the values given above Eq. (5).

**Concluding remarks.**—We have explored the $\gamma\gamma$, $b\bar{b}$ and $\tau\tau$ signals in the MSSM, relative to SM, and also in natural SUSY. In MSSM an enhancement in the diphoton signal of the 125 GeV Higgs boson relative to the SM Higgs can be obtained in a flavor-tuned model with $h = H^0_2$ provided that $|\mu|$ is large (TeV) and $\mu$ is negative. A $\gamma\gamma$ enhancement is principally due to the reduction of the $b\bar{b}$ decay width compared to $h_{\text{SM}}$. The ratios of $WW^*$ and $ZZ^*$ to their SM values are predicted to be the same as that of $\gamma\gamma$. There is also a corresponding reduction of the $h$ to $\tau\tau$ signal. The Tevatron evidence of a Higgs to $b\bar{b}$ signal in $W +$ Higgs production [29] does not favor much $b\bar{b}$ reduction. The flavor-tuning of the neutral Higgs mixing angle $\alpha$ requires a large $\mu \sim \text{TeV}$ and large $\tan\beta$. For small $|\mu| < 0.5$ TeV of natural SUSY, $\gamma\gamma$ suppression relative to the SM is predicted. Thus, precision LHC measurements of the $\gamma\gamma$, $WW^*$, $ZZ^*$ and $b\bar{b}$ signals of the 125 GeV Higgs boson can test MSSM models.

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[27] Here we note that as \( \mu = 0 \), then \( r_{bb}^{\prime} = 1 + 2M_{Z}^{2}/m_{A}^{2} \rightarrow \sigma_{\gamma\gamma}/\sigma_{SM} = 1 - 2.4M_{Z}^{2}/m_{A}^{2} \), independently of \( \tan \beta \). The smaller \( m_{A} \) gives the larger suppression of \( \sigma_{\gamma\gamma} \) [28]. Then, from the \( \gamma\gamma \) deviation from unity, the \( CP \)-odd state mass \( m_{A} \) could be estimated.


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