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Charged elliptic flow at zero charge asymmetry

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The difference between the flow ellipticities of oppositely charged pions $\Delta v_2 \equiv v_2[\pi^-] - v_2[\pi^+]$, measured recently by the STAR Collaboration at the Brookhaven National Laboratory Relativistic Heavy Ion Collider (RHIC) shows a linear dependence on the event charge asymmetry $A_\pm \equiv (N_+ - N_-)/(N_+ + N_-)$: $\Delta v_2(A_\pm) = \Delta v_2(0) + r A_\pm$ with a slope $r > 0$ and a nonzero intercept $\Delta v_2(0) > 0$ on the order of 10^{-4}. We discuss two novel mechanisms, which could explain the nonzero value of the charged elliptic flow Δv_2 at *zero* charge asymmetry $A_\pm = 0$, i.e., the nonzero positive intercept $\Delta v_2(0)$. Both effects are due to the electric fields created by the colliding ions. These fields have quadrupole asymmetry of the magnitude and the sign needed to account for the nonzero intercept $\Delta v_2(0) > 0$ in the RHIC data. One of the mechanisms also involves the chiral magnetic effect. This mechanism, although negligible at RHIC energies, may become important at the CERN Large Hadron Collider energies.

I. INTRODUCTION

The recent analysis [1,2] of the STAR event-by-event measurements of the difference between the elliptic flows of positive and negative pions $\Delta v_2 \equiv v_2[\pi^-] - v_2[\pi^+]$ finds an interesting behavior of this quantity as a function of the event charge asymmetry $A_\pm \equiv (N_+ - N_-)/(N_+ + N_-)$. As seen in Fig. 1, Δv_2 depends linearly on $A_\pm$, i.e.,

$$\Delta v_2(A_\pm) = \Delta v_2(0) + r A_\pm, \tag{1}$$

with a slope $r > 0$ and an intercept $\Delta v_2(0) > 0$. This observed positive slope agrees with the prediction made in Refs. [3,4] based on the chiral magnetic wave (CMW) [5,6]. The CMW is a gapless soundlike mode of chiral charges (left handed or right handed) that propagates along the direction of the magnetic field with a dispersion relation,

$$\omega = \pm v_\chi k - i D_L k^2 + \cdots, \tag{2}$$

where the sign and, hence, the direction of the propagation with respect to the magnetic field depend on the chirality of the charge fluctuations. The velocity v_χ is given by [5]

$$v_\chi = \frac{N_c e B}{4\pi^2} \alpha, \tag{3}$$

where α is the inverse susceptibility of the chiral charge and D_L is the longitudinal diffusion constant which depends on the microscopic transport properties of a given system. The mechanism described in Refs. [3,4] is based on the observation (also made in Ref. [7]) that the initial *net* electric charge Q is a sum of the net charge carried by left-handed (Q_L) and right-handed (Q_R) chiral carriers (quarks),

$$Q = Q_L + Q_R, \tag{4}$$

where, on average, $Q_R = Q_L$. In off-central heavy ion collisions at the Brookhaven National Laboratory Relativistic

Heavy Ion Collider (RHIC) energies, the heavy ions can create a transient magnetic field of strength as large as $B \sim 4m_\pi^2/e \sim 10^{19}$ G in the direction perpendicular to the reaction plane. The CMW moves chiral charges Q_L and Q_R along the axis of this magnetic field according to Eq. (2). Since Q_L moves in the direction opposite to that of Q_R, the net result will be an electric charge excess in the poles of the fireball and a depletion in the central region. See Fig. 2 for a schematic of the mechanism. Such a charge transport via CMW will result in an electric quadrupole moment whose magnitude is naturally proportional to the charge asymmetry. Combined with the subsequent radial flow, this leads to the charged elliptic flow of pions Δv_2, linear in $A_\pm$ [3,4]. A semirealistic theoretical simulation performed in Refs. [3,4] qualitatively and semiquantitatively agrees with the STAR data, which include a nontrivial dependence on the impact parameter (centrality).

Although the CMW mechanism described above accounts for the nonzero positive slope in Fig. 1, the origin of nonzero positive intercept $\Delta v_2(0)$ so far has not been understood to the same extent. An argument based on a simple quark coalescence model in Ref. [8] suggests that a positive contribution to the intercept may appear due to initial isospin asymmetry. In this paper, we will identify two different mechanisms which are more closely related to the CMW effect responsible for the nonzero slope. These mechanisms account for the sign as the well as the magnitude of the intercept.

One way to look at the problem is to note that the nonzero intercept suggests the existence of an additional source of electric quadrupole moment, approximately independent of the charge asymmetry $A_\pm$ and present even in the neutral plasma, i.e., at $A_\pm = 0$. We discuss two sources of such a net positive (out-of-reaction-plane) electric quadrupole moment. Both involve electric fields created by the heavy ions which are similar in strength to the magnetic fields responsible for the CMW. One of the sources is the quadrupole component of the electric field, which has been pointed out in Ref. [9], and we will show that it has the right magnitude and sign to
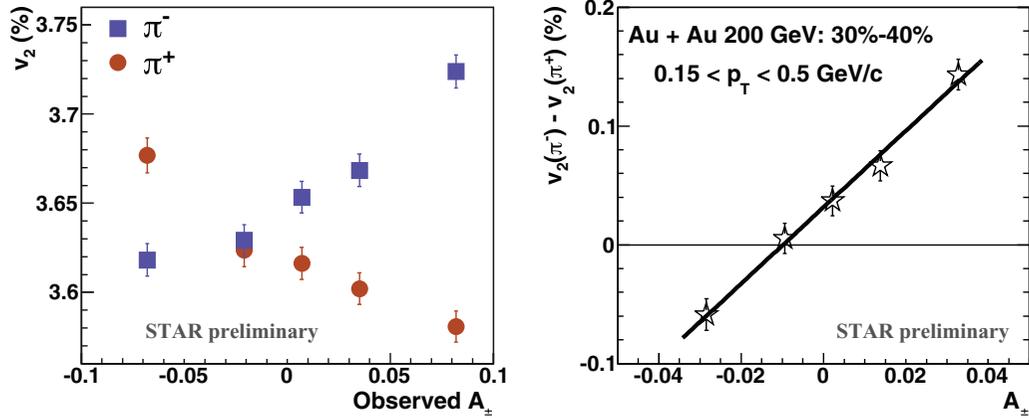
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FIG. 1. (Color online) The STAR data [1] on the elliptic flow difference between the positive and the negative pions that show a linear dependence on the charge asymmetry $A_\pm \equiv (N_+ - N_-)/(N_+ + N_-)$ with a finite positive intercept.

account for the intercept $\Delta v_2(0)$ observed at the RHIC. The other source involves chiral magnetic effect. We describe the two mechanisms in the subsequent sections and make an order-of-magnitude estimate of their effect on the charged elliptic flow $\Delta v_2(0)$.

II. SOURCE 1: ELECTRIC-FIELD QUADRUPOLE

A highly relativistic heavy ion carries around itself a Lorentz contracted "pancake" of the electromagnetic field perpendicular to its velocity. The magnetic-field lines are circular around the velocity direction, and the heavy ions that collide at a finite impact parameter create a net magnetic field in the overlapping region along the direction perpendicular to the reaction plane. This field is responsible for the CMW.

On the other hand, the electric field from a single heavy ion is radially directed, perpendicular to the magnetic field at all points in space-time. The superposition of the electric fields from the two colliding ions in the overlapping region is described schematically in Fig. 3 (left). The key feature of the effect we describe is that this electric field has a quadrupole component: The field pointing outward *perpendicular to* the reaction plane is stronger than the field pointing outward *in* the reaction plane due to partial cancellation of the in-plane components. Since the plasma (or preequilibrium state, such as glasma, at such early times) is conducting, there naturally arise

net currents moving charges outward from the reaction plane: $J \sim E$. One easily sees that this charge transport induces a net positive electric quadrupole moment along the out-of-plane axis (see Fig. 3) which, combined with subsequent radial collective flow, gives rise to $\Delta v_2 > 0$ as in the CMW.

Since the conductivity is nonzero also in a *neutral* plasma, the effect is present even when $A_\pm = 0$. It is also clear from charge-conjugation symmetry that the conductivity can only depend quadratically on the net charge density. Thus, the dependence of the contribution of the quadrupole electric-field effect to Δv_2 in Eq. (1) on $A_\pm$ is given by $\Delta v_2(0) + O(A_\pm^2)$. The term quadratic in $A_\pm$ is negligible for small enough $A_\pm$ ($A_\pm$ in Fig. 1 is $\leqslant 0.04$).

We emphasize again that the magnitude of the electric field is similar to that of the magnetic field, which indicates that the order of magnitude of the induced quadrupole moment should be comparable to the one induced by the magnetic field via the CMW. This aspect and other similarities make this mechanism (and the mechanism described in the following section) a natural possible explanation of the nonzero intercept in Fig. 1 if the nonzero slope is due to the CMW.

Let us make an order-of-magnitude estimate of the effect. The elliptic flow difference between positively and negatively charged particles is on the order of $\Delta v_2 \sim \Delta Q / N_{tot}$ where ΔQ is the total charge (in units of e) transported by this mechanism and N_{tot} is the total multiplicity of charged particles. The value of ΔQ can be estimated as $\Delta Q \approx J A \tau_J$, where J is the (units
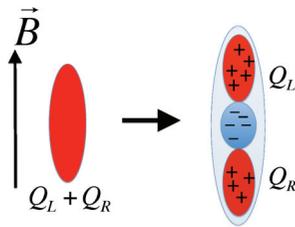

FIG. 2. (Color online) The CMW evolution of the initial charge asymmetry (taken to be positive in this example) produces a net electric quadrupole moment responsible for a nonzero $\Delta v_2 \equiv v_2[\pi^-] - v_2[\pi^+]$ (positive in this case). The collision plane is horizontal in this sketch.
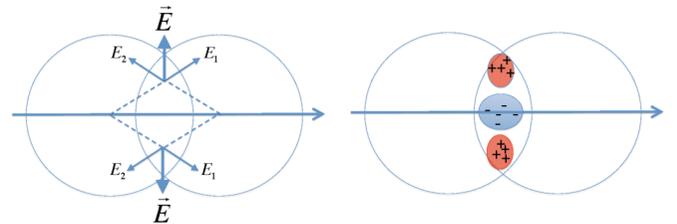

FIG. 3. (Color online) The net electric-field profile from the two heavy ions (*left*), which leads to a nonzero electric quadrupole moment (*right*) whose magnitude is independent of the charge asymmetry $A_\pm$ for small enough $A_\pm$. The reaction plane is horizontal.

of e) current density induced by the electric field, τ_J is the lifetime of the current, and A is the typical area of the fireball transverse to current during the lifetime of the electric field τ_E. The area transverse to the current at time τ_E is given by the product of the transverse size, which, for 30%–40% central events, we take to be on the order of the radius of the nucleus $R \sim 7$ fm and of the longitudinal size, which is on the order of τ_E: $A = R\tau_E$.

To estimate the current and its lifetime we need to consider two cases that depend on whether the lifetime τ_E of the electric field is shorter or longer than the typical (mean-free) time between collisions τ_{free}. Although the mechanisms of the charge transfer are different in the two cases, the relevant product $J\tau_J$ turns out to be similar. As we will see below, for the relatively long-lived electric field $\tau_E \gg \tau_{\text{free}}$, the magnitude of the current is determined by conductivity and $J \sim \tau_{\text{free}}$, whereas, its lifetime is as long as that of the electric field $\tau_J \approx \tau_E$. On the other hand, for the short-lived electric field $\tau_E \ll \tau_{\text{free}}$, the magnitude of the current is determined by the lifetime of the field $J \sim \tau_E$, whereas, its lifetime is limited by the mean-free time $\tau_J \approx \tau_{\text{free}}$. In either case the relevant product is $J\tau_J \sim \tau_E\tau_{\text{free}}$.

Let us now estimate the current J. First consider the case when the lifetime of the electric field τ_E is much longer than the time between collisions τ_{free}. This is the hydrodynamic regime, and thus, the current is given by Ohm's law $J = \sigma E$ where for the conductivity σ, we could use a lattice QCD result from Ref. [10],

$$\sigma = 0.4eT \sum_{F=u,d,s} q_F^2 \approx 0.27eT. \qquad (5)$$

When $\tau_E \ll \tau_{\text{free}}$ we cannot use Ohm's law. But in this case we could estimate the current in the collisionless approximation. The contribution of a each quark or antiquark species f to the current is given by $J_f = q_f n_f v_f$ (where $f = u, d, s, \bar{u}, \bar{d}, \bar{s}$), where v_f is the drift velocity induced by the electric field by increasing momentum density of this component of the plasma by $\pi_f = \tau_E e E q_f n_f$. By Lorentz invariance, $v_f = \pi_f/w_f$, where w_f is the momentum "susceptibility" (given by enthalpy $\epsilon + p$ in equilibrium). Putting this together and taking into account that, at $A_\pm = 0$, quark and antiquark of a given flavor contribute equally to the net current, i.e., $J_F = J_f + J_{\bar{f}} = 2J_f$, we find: $J = \sum_{F=u,d,s} J_F = 2eE\tau_E \sum_{F=u,d,s} q_F^2 n_F^2/w_F$, where the factor 2 in the last equation counts the equal contributions from quarks and antiquarks of a given flavor F. We can estimate n_F and w_F by using equilibrium momentum distribution for massless quarks: $n_F = 9\zeta(3)T^3/\pi^2$ and $w_F = 7\pi^2T^4/15$. Thus, we find

$$J \approx 0.26\tau_E e E T^2 \sum_{F=u,d,s} q_F^2. \qquad (6)$$

By comparing to Eq. (5) we can write $J \approx 0.65(\tau_E T)\sigma E$. Clearly, when $\tau_E \approx \tau_{\text{free}}$, the two equations should match from which we could infer a rough estimate of the mean-free time $\tau_{\text{free}} \approx 1.5/T$ implicit in the lattice result Eq. (5). By using this as an estimate of τ_{free}, we can write the result in a form

independent of which regime we consider

$$J\tau_J \approx 0.17eET^2\tau_E\tau_{\text{free}}. \qquad (7)$$

Thus,

$$\Delta v_2 \approx \frac{\Delta Q}{N_{\text{tot}}} \approx \frac{J\tau_J A}{N_{\text{tot}}} \approx 0.17\frac{eET^2\tau_E^2\tau_{\text{free}}R}{N_{\text{tot}}}. \qquad (8)$$

The electric-field strength can be approximated as $eE \approx \gamma Ze^2/(4\pi R^2)$, where Z is the atomic number of the heavy nucleus, R is its radius, and $\gamma = \sqrt{s}/(2 \text{ GeV})$ is the Lorentz contraction factor, which we express in terms of the center-of-mass energy per colliding nucleon pair. The lifetime of the electric field is roughly given by the longitudinally contracted thickness of the colliding nucleus, which is $\tau_E \approx 2R/\gamma$, whereas, $\tau_{\text{free}} \approx 1.5/T$ as inferred from the lattice result (5). By putting this together and by using $Z \approx 80$ and $R \approx 7$ fm, we find

$$\Delta v_2 \approx 1 \times \frac{Z\alpha RT}{N_{\text{tot}}\gamma} \approx 10^{-4}\left(\frac{T}{400 \text{ MeV}}\right)\left(\frac{10^3}{N_{\text{tot}}}\right)$$
$$\times \left(\frac{200 \text{ GeV}}{\sqrt{s}}\right), \qquad (9)$$

which has the right order of magnitude to account for the intercept $\Delta v_2(0) \approx 3 \times 10^{-4}$ in Fig. 1.

III. SOURCE 2: ELECTRIC QUADRUPOLE VIA CHIRAL MAGNETIC EFFECT

The other source of a net positive electric quadrupole moment arises through a combination of both $\boldsymbol{E}$ and $\boldsymbol{B}$. Consider a P- and CP-odd quantity $\boldsymbol{E}\cdot\boldsymbol{B}$. From the electric-field pattern in Fig. 3 as well as the magnetic field pointing upward perpendicular to the reaction plane, one easily recognizes that $\boldsymbol{E}\cdot\boldsymbol{B}$ is positive in the upper half region, whereas, it is negative in the lower half region as shown in Fig. 4 (left). Via the triangle anomaly relation for the axial current J_A^μ,

$$\partial_\mu J_A^\mu = \frac{N_c e^2}{2\pi^2}\left(\sum_F q_F^2\right)\boldsymbol{E}\cdot\boldsymbol{B}, \qquad (10)$$

this implies a net positive axial charge created in the upper region and net negative in the lower region. In the presence of the vertical magnetic field, the chiral magnetic effect [11–13] that acts on these axial charges induces an electric charge
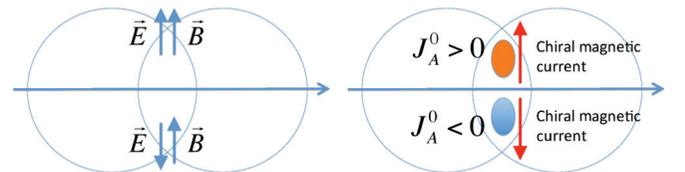

FIG. 4. (Color online) The profile of $\boldsymbol{E}$ and $\boldsymbol{B}$ that leads to a creation of axial charges via a triangle anomaly (*left*). The chiral magnetic effect (CME) that acts on these axial charges induces a net electric quadrupole moment (*right*).

current (in units of e),

$$\boldsymbol{J} = \frac{N_c e}{2\pi^2} \left(\sum_F q_F^2 \right) \mu_A \boldsymbol{B}, \tag{11}$$

from which it is easily seen that the net effect is a development of a positive electric quadrupole moment. See Fig. 4 (right) for a schematic. To some extent, the mechanism is similar to that of the CMW in Refs. [3,4] with the main difference that the mechanism is independent of the initial charge asymmetry $A_\pm$.

To estimate the contribution of this mechanism to Δv_2, we start from the amount of axial charge density created by the nonzero $\boldsymbol{E} \cdot \boldsymbol{B}$ during the lifetime τ_E of the electromagnetic field,

$$J_A^0 \approx \frac{N_c e^2}{2\pi^2} \left(\sum_F q_F^2 \right) E B \tau_E \approx 0.1(eE)(eB)\tau_E. \tag{12}$$

This corresponds to an axial chemical potential on the order of $\mu_A \approx j_A^0/\chi_A$ with the axial charge susceptibility χ_A whose magnitude we can estimate by assuming it is similar to the vector charge susceptibility χ known from the lattice QCD [14] to be about $\chi \approx 1.0\, T^2$. The CME in Eq. (11) then produces a current

$$J = \frac{N_c}{2\pi^2} \left(\sum_F q_F^2 \right) \mu_A(eB) \approx 10^{-2}\, T^{-2}(eE)(eB)^2\tau_E, \tag{13}$$

which would result in charge separation $\Delta Q \approx J A \tau_J$ with $A \approx R\tau_E$ as before. We could also expect that the duration of the chiral magnetic current τ_J is approximately given by the duration of the magnetic field $\tau_J \approx \tau_B$. Putting this together and estimating the electric and magnetic fields again as $eE \approx eB \approx \gamma Z\alpha/R^2$, one finds for $\Delta v_2 \approx \Delta Q/N_{\text{tot}}$,

$$\Delta v_2 \approx 10^{-2} \frac{(eE)(eB)^2 \tau_E^2 \tau_B R}{N_{\text{tot}} T^2} \approx 10^{-2} \frac{(Z\alpha)^3 \tau_E^2 \tau_B \gamma^3}{N_{\text{tot}} R^5 T^2}$$

$$\approx 10^{-7} \left(\frac{400\ \text{MeV}}{T} \right)^2 \left(\frac{10^3}{N_{\text{tot}}} \right), \tag{14}$$

where we used $\tau_E \approx \tau_B \approx 2R/\gamma$ and $R \approx 7$ fm as before. We see that the magnetically induced electric quadrupole effect is negligible at RHIC energies compared to the direct electric quadrupole effect in Eq. (9). However, the direct effect in Eq. (9) decreases with energy $\sqrt{s}$, whereas, the magnetically induced effect in Eq. (14) is energy independent.

In addition, the magnetically induced electric quadrupole is sensitive to the lifetime of the magnetic field τ_B. For example, if this time turns out to be much greater than our estimate $\tau_B = 2R/\gamma$ (due to the conductivity of the medium as in Ref. [15]), that effect may be significantly larger than our estimate in Eq. (14) and could possibly compete with the direct electric dipole effect in Eq. (9) at sufficiently high $\sqrt{s}$,

$$\Delta v_2 \approx 10^{-5} \left(\frac{\tau_B}{2\ \text{fm}} \right) \left(\frac{eB}{1\ \text{fm}} \right)^{-2} \left(\frac{400\ \text{MeV}}{T} \right)^2 \left(\frac{10^3}{N_{\text{tot}}} \right). \tag{15}$$

By comparing this with the estimate of the direct electric quadrupole effect (9), we see that, whereas, at the top RHIC energy, the direct electric quadrupole dominates, the

two effects could possibly become comparable in magnitude (10^{-5}) at the CERN Large Hadron Collider (LHC) energies due to different $\sqrt{s}$ dependences.

IV. DISCRETE SYMMETRIES

We end this paper by discussing two discrete symmetries, the charge conjugation C and the $180°$ rotation (or reflection) in the transverse plane $R_\perp$, useful for classifying possible mechanisms contributing to the observables: the slope r and the intercept $\Delta v_2(0)$. Under C,

$$\Delta v_2(0) \to -\Delta v_2(0), \qquad A_\pm \to -A_\pm, \tag{16}$$

therefore, r is C even, whereas, $\Delta v_2(0)$ is C odd.

Since QCD is C invariant and the source of C violation is in the initial condition, one concludes that the physics of $\Delta v_2(0)$ must have its origin in the initial charge asymmetry, such as the electromagnetic charge of the heavy ions (or the isospin asymmetry as in Ref. [8]). On the other hand, slope r could possibly receive contributions from other effects unrelated to the charge asymmetry of the initial conditions (as, e.g., in Ref. [16]).

Under the $R_\perp$ rotation (reflection) in the transverse plane, both r and $\Delta v_2(0)$ are even since v_2 is $R_\perp$ even. The pattern of the electric field is $R_\perp$ even, whereas, the magnetic-field pattern is $R_\perp$ odd. In conjunction with the C-parity discussion above, this tells us that intercept $\Delta v_2(0)$ can be linearly proportional to the electric field but not the magnetic field. Indeed, our first source can be viewed as being linear in electric field E, whereas, the second source should be considered as an $(EB)B \sim EB^2$ effect.

V. SUMMARY AND CONCLUSION

We identified two possible sources of the charged elliptic flow $\Delta v_2(A_\pm)$ at zero charge asymmetry, i.e., $\Delta v_2(0)$. The most straightforward source is the quadrupole pattern of the electric fields created by the relativistic heavy ions in the central overlap region. Although these fields last a short time, their magnitude is extremely large due to the well-known Lorentz contraction. Our order-of-magnitude estimates suggest that the contribution of this effect at the top RHIC energy is comparable with the observed value of $\Delta v_2(0)$ and that it should be inversely proportional to the collision energy $\sqrt{s}$, which drops by an order of magnitude at LHC energies.

The simultaneous presence of electric and magnetic fields in the overlap region via the chiral magnetic effect provides another source of quadrupole moment at zero charge asymmetry. The magnitude of this effect is less sensitive to the collision energy. It depends, however, on the lifetime of the magnetic fields, which could be longer than that of the electric fields. We estimate that this effect is negligible at RHIC energies but may become comparable with the direct electric quadrupole at the LHC.

We stress that the two mechanisms we discuss in this paper give rise to the same (positive out-of-plane) sign of the

electric quadrupole moment, which makes the prediction for the sign of the effect robust. However, a reliable quantitative prediction of the magnitude would clearly require a realistic simulation that includes spatial variations and fluctuations in the electromagnetic fields [9,17,18].

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