Okun’s Law:
A Non-linear Threshold Modeling Approach

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THESIS
Submitted as partial fulfillment of the requirements
For the degree of Doctor of Philosophy in Economics
in the Graduate College of the
University of Illinois at Chicago, 2015

Chicago, Illinois

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DEDICATION

This thesis is dedicated to my wife and children.

To Annette:
Thank you for allowing me to express my “midlife crisis” as an extended quest for a PhD. We made it! Without your unwavering support and strength within the home, it could not have been done.

To Reed, Mallory, and Caleb:
Learning is a lifelong process! It is never too late to start something different; do something new; master a new skill. Intelligence and inquisitiveness can get you started, but only effort and discipline will allow you to finish.
ACKNOWLEDGEMENTS

I would like to thank Professor Houston Stokes for advising me both on this thesis and throughout my years at UIC. Professor Stokes has always been able to connect the realms of theory and practice in econometrics, for which I am grateful.

I would also like to thank the members of my committee: Professors George Karras, Gilbert Bassett, Jr., Paul Pieper, and Jin Man Lee. Time is valuable, and I thank you for sharing yours with me through your thoughtful review and suggestions for this thesis.

The guidance of Marcus Casey was invaluable to the completion of this work. Professor Casey was able to describe a solid framework to structure the paper, and to get me started on the right track. He highlighted the importance of telling a story that will motivate economists and non-economists alike.

Finally I would like to thank Professors Steven Rivkin and Robert Kaestner and the entire Economics Department at UIC. Under their leadership and guidance, UIC Econ will continue to improve in both its rank and rigor. I am proud to be a newly minted PhD in Economics from the University of Illinois at Chicago.
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SUMMARY

The relationship between the Gross National Product of the United States and the Unemployment Rate was first estimated by Arthur Okun using a linear specification in 1962. This relationship became known as Okun’s Law.

Since the Great Recession of 2008, policy makers and advisors such as Janet Yellen, Chair of the Federal Reserve Board of Governors, and Christina Romer, former chief economic advisor to the President, among others have questioned the continued validity of Okun’s Law.

This paper uses the General Additive Models (GAM) approach from Hastie and Tibshirani to formally test Okun’s Law for nonlinearity.

This paper also uses the Multivariate Adaptive Regression Splines (MARS) estimation technique to identify whether threshold levels on the change in GDP exist that would better describe Okun’s Law.

A framework for testing a MARS estimation for a structural break in the estimated coefficients is suggested. The model is tested for evidence of a structural break between the pre oil shock period from 1948 until 1973 against the post oil shock driven inflationary period beginning in 1982 and ending in 2012. The model is tested a second time for evidence of a structural break, but now the partitioning of the data follows the theory
of the Great Moderation with a before and after period divided at first quarter of 1984.

Throughout this paper I investigate both threshold effects and structural breaks. Threshold effects are defined as changes in the value of an estimated coefficient due to a change in the level of the independent variable. A structural break is defined as a change in an estimated coefficient over time; in other words, a change in the estimated coefficient due to a change in the estimation period.

The findings support nonlinearity and threshold response. The results regarding the existence of a structural break are mixed. When partitioning the with respect to the oil shocks of the 1970’s, structural break is rejected. In contrast, while partitioning the data with respect to the Great Moderation a structural break is supported.

In addition, the MARS estimation suggests that for GDP growth within a threshold around zero there is a zone of no effect of GDP growth on the change in the unemployment rate. Within this zone of no effect, Okun’s law does not statistically exist.
I. INTRODUCTION

Arthur Okun estimated the relationship between percentage changes in the Gross National Product of the United States and changes in the unemployment rate using a linear specification in 1962. This relationship has become known as Okun’s Law.

At the December 2012 meeting of the Federal Open Market Committee, the Federal Reserve made history by explicitly linking their interest rate policy, as expressed through the Federal Funds Rate, to threshold levels in both the unemployment rate and the expected rate of inflation. The Federal Reserve made this historic move in order to continue an expansionary monetary policy in order to meet their dual mandate of promoting maximum employment and stable prices. This aggressive expansionary monetary policy follows the great contraction of GDP caused by the financial crisis of 2008.

Okun’s Law remains relevant today as it is the link between the Phillips Curve (inflation / unemployment relationship) and the short run Aggregate Supply Curve (price / output relationship). An accurate empirical relationship for Okun’s Law is essential for guiding sound decision making for both monetary and fiscal policy.

Mankiw provides a derivation of the Aggregate Supply, Okun’s Law, Phillips Curve relationship, which I include below.
\[ P = P^e + \left( \frac{1}{\alpha} \right) (GDP - GDP^{bar}) + \nu \]  

**Aggregate Supply** (I.1) with exogenous shock

where:

- \( GDP \) = Log of Gross Domestic Product
- \( GDP^{bar} \) = Log of Potential GDP
- \( UNEMP \) = Unemployment Rate
- \( UNEMP^n \) = Natural Rate of Unemployment
- \( \pi \) = \((P_t - P_{t-1})\) inflation
- \( \pi^e \) = \((P^e_t - P_{t-1})\) expected inflation
- \( P \) = log of Price level
- \( P^e \) = log of Expected Price level

By substitution we put the aggregate supply curve in terms of inflation.

\[ \pi = \pi^e + \left( \frac{1}{\alpha} \right) (GDP - GDP^{bar}) + \nu \]  

**Aggregate Supply** (I.2) in terms of inflation

\[ -\beta (UNEMP - UNEMP^n) = \left( \frac{1}{\alpha} \right) (GDP - GDP^{bar}) \]  

**Okun’s Law** (I.3) 

By substituting Okun’s Law into Aggregate Supply we derive the Philips Curve.

\[ \pi = \pi^e - \beta (UNEMP - UNEMP^n) + \nu \]  

**Phillips Curve** (I.4)  

(Mankiw, 1997)

The rule of thumb relationship that Okun established of a 3 percentage point increase in Gross Domestic Product (GDP) leading to a 1 percentage point reduction in the Unemployment
Rate (UNEMP) has been at the heart of both public policy decisions as well as economic forecasting.

Throughout this paper I investigate both threshold effects and structural breaks. Threshold effects are defined as changes in the value of an estimated coefficient due to a change in the level of the independent variable. A structural break is defined as a change in an estimated coefficient over time; in other words, a change in the estimated coefficient due to a change in the estimation period.

In this paper, I use the Multivariate Adaptive Regression Splines (MARS) modeling technique of Friedman as implemented in the open source version of R and the econometric software system, B34S, to identify whether threshold levels on the change in GDP exist that would better describe Okun’s Law. (Friedman, 1991) Once a non-linear form is identified, this model is then tested for evidence of a structural break between the pre oil shock period from 1948 until 1973 against the post oil shock driven inflationary period beginning in 1982 and ending in 2012. The findings support the hypothesis of asymmetric response, but reject the hypothesis of the existence of a structural break. In addition, the MARS estimation suggests that a zone of no effect, of GDP growth on the change in the unemployment rate, exists for a threshold on GDP growth around zero. Within this zone of no effect, Okun’s law does not statistically exist.
Chapter II reviews the literature that focuses on asymmetric response as well as the existence of a structural break in Okun’ Law. Chapter III reviews the data used within the model. Chapter IV presents the empirical methodology of the thesis. Chapter V tests the hypothesis of linearity in the Okun model across varied estimation windows using Generalized Additive Models (GAM). Chapter VI presents the MARS estimation of the original Okun model. Chapter VII estimates an alternative model proposed by Prachowny using MARS. Chapter VIII uses MARS to estimate a distributed lag model proposed by Silvapulle. Chapter IX replaces the changes in the unemployment rate with a recent indicator created by the Federal Reserve as a broad measure of labor market activity, the Labor Market Conditions Index. Chapter X uses the Geweke procedure to decompose the variance between unemployment and GDP by frequency. Chapter XI test the MARS estimation for structural break in the estimated coefficients. Chapter XII partitions the data according to dates related to the Great Moderation. Chapter XIII interprets the results and concludes the thesis.
II. LITERATURE REVIEW

Due to the importance that Okun’s law holds within macroeconomics, the relationship has been revisited many times in the academic literature. Authors, such as Hugh Courtney (Courtney, 1991), have suggested that Okun’s linear specification of the unemployment / output relationship is incorrect. These authors suggest that the relationship would be better specified as a non-linear function due to an asymmetric response of unemployment to GDP’s positive or negative changes. This literature review will focus on the authors that have made significant contributions to the investigation of asymmetry in Okun’s Law. This list in chronological order includes: Courtney (1991), Prachowny (1993), Lee (2000), Harris and Silverstone (2001), Viren (2001), Silvapulle, Moosa, and Silvapulle (2004), Huang (2006), Lin (2008).

Hugh Courtney’s PhD Thesis at MIT investigates both the Beveridge Curve and Okun’s Law. Courtney is one of the first to suggest an asymmetric response in the unemployment / output relationship. Building on work by Summers and Delong that recognized nonlinearity in the business cycle, Courtney suggests that Okun’s Law is best described as a discontinuous, nonlinear relationship. Courtney uses a production function approach to the model with

\[ Y = G K^\alpha E^\beta H^\gamma \]  

(II.1)
where:

\[ Y - \text{output} \]
\[ G - \text{multi-factor productivity} \]
\[ K - \text{capital} \]
\[ E - \text{employment} \]
\[ H - \text{hours worked} \]

In log first differences the estimating equation becomes

\[ e = (y - g - \alpha k - \gamma h)^{\beta - 1} \] (II.2)

Courtney states that the nonlinearity in Okun’s Law may be caused by factor substitution and fluctuation in productivity growth throughout the business cycle. His paper relies on the Burns Mitchell approach from 1946 to describe the business cycle. One assumption of the paper is that output follows a symmetric growth cycle.

Courtney partitions the output data, \( y \), into positive and negative values (\( y > 0 \) or \( y < 0 \)) and estimates equation II.2. Although Courtney does not reject the hypothesis of equality of the coefficient on \( y \) (Okun coefficient) or find evidence of factor substitution, Courtney does identify asymmetry within the four regions of the business cycle. His results show that the response of employment is strongest in expansionary peaks and contractionary troughs. Most importantly Courtney opens the
door for further research into nonlinear and production function based investigation of Okun’s Law. (Courtney, 1991)

Martin Prachowny, writing in 1993, suggests that Arthur Okun should have taken a first principles approach to modeling the relationship between output and unemployment. Okun suggested that other factors in the relationship would change pari passu, or by moving together. Okun identified these other factors as average hours in the work week, labor force participation, and man-hour productivity. Okun believed that the unemployment rate could serve as a proxy for all of these factors. Prachowny, in his first principles approach, specifies the estimating equation with additional variables derived from a production function approach. Prachowny estimates the following equation using linear estimation methods which he feels are statistically superior to the original Okun gap model.

\[
\Delta(y-y^*) = a_1\Delta(c-c^*) + a_2\Delta(l-l^*) - a_3\Delta(u-u^*) + a_4\Delta(h-h^*) + e
\]

(II.3)

Where:

\(y-y^*) = \text{output gap}\)

\(c-c^*) = \text{utilization gap}\)

\(l-l^*) = \text{natural log of the supply of workers gap}\)

\(u-u^*) = \text{deviation from the natural rate of unemployment}\)

\(h-h^*) = \text{hours worked gap}\)
When estimated with this specification the relation of a change in unemployment on output changes significantly. For a 1.5% decrease in unemployment, a 1% increase in output is implied. This in contrast to Okun’s gap results where a 1% increase in output implies a 0.37% decrease in unemployment. Prachowny feels that the results show that capacity utilization and hours worked have influence on output. From this he suggests that the Okun coefficient is a complicated weighted sum of all other effects. (Prachowny, 1993)

In 2000, Jim Lee focuses on the robustness of Okun’s Law. To do so, Lee uses postwar data from 16 OECD countries. Lee’s paper presents three major points. First, when using a gap model, that is a model based on the differences between potential values and actual values, the estimated coefficients are highly sensitive to the method used to calculate the potential levels. Lee uses three different methods to generate the potential levels: the HP filter, the Beveridge-Nelson decomposition procedure, and the Kalman filter based on the Non-accelerating Inflation Rate of Unemployment (NAIRU). Each of these filtering methods remove the low frequency information in the data. Lee finds that the choice of filter significantly affects the estimated coefficients within his model. Second, Lee tests the Okun coefficient for stability through time. He
finds evidence of a structural break in most countries as early as 1963 in Italy and as late as 1980 in Austria and the Netherlands. Third, Lee tests for asymmetry in Okun’s Law. Lee estimates the equation for the gap model as

\[(y_t - y_t^*) = \beta_0 - [\beta_1 I_{2t^+} (u_t - u_t^*) + \beta_1 I_{2t^-} (u_t - u_t^*)] + \epsilon_t \] (II.4)

where:
\[y = \text{log output (GDP)}\]
\[u = \text{unemployment rate}\]

\[I_{2t^+} = 1 \text{ if } (u_t - u_t^*) \geq 0 \text{ and } 0 \text{ if } (u_t - u_t^*) < 0\]
\[I_{2t^-} = 1 \text{ if } (u_t - u_t^*) < 0 \text{ and } 0 \text{ if } (u_t - u_t^*) \leq 0\]

In this way Lee investigates asymmetry by forcing a partition of the data based on positive and negative values on the unemployment gap.

Lee finds that there are great disparities in the estimate of the Okun coefficient across countries and significant differences in the estimate of the Okun coefficient for any single country when each of the three different methods (HP filter, Beveridge Nelson decomposition, Kalman Filter) for calculating the potential levels of output and unemployment for use in the gap model are compared. Lee states that “these results essentially highlight the difficulties of distinguishing between long-term trends and cyclical fluctuations in economic time series. From this perspective, inferences based on a
single model specification, as in many previous studies, should be interpreted with caution. On the other hand, our empirical results also complement the existing literature which reveals sizable distortionary effects of certain data detrending methods on evaluating macroeconomic models." (Lee, 2000)

Lee finds strong evidence of structural breaks that are likely associated with the years of the oil crisis.

Lee finds mixed evidence of asymmetry in the Okun coefficient across countries. The results for the US on asymmetry are inconclusive as they depend on which model and filter is estimated.

Matti Viren investigates non-linearity in the Okun Curve. Viren approaches the estimation by forming an error correction model. Unemployment is related to the working-age population and a second order time trend that is a proxy for labor market structure.

\[
\begin{align*}
  u_t &= \alpha_0 + \alpha_1 n_t + \alpha_3 t + \alpha_4 t^2 + \mu_t \\
  \Delta u_t &= \beta_0 + \beta_1 EC_{t-1} + \beta_2 \Delta n_t + \beta_3 \Delta y_{t-1}^+ + \beta_4 \Delta y_{t-1}^- + \varepsilon_t
\end{align*}
\]  

(II.5) (II.6)

where:

\begin{align*}
  \Delta &= \text{the first difference operator} \\
  u &= \log \text{ of the number of people unemployed} \\
  EC &= \text{the error correction term derived from II.5 and } y \text{ output} \\
  n &= \log \text{ of working age population} \\
  y &= \log \text{ of GDP} \\
  t &= \text{the time trend}
\end{align*}
Viren partitions the data by fixing the threshold value for $\Delta y$ at zero, $\Delta y > 0$ or $\Delta y < 0$. Viren compares 20 OECD countries with data from 1960 through 1997. Viren rejects symmetry in favor of an asymmetric Okun curve around the fixed threshold in all countries with the exception of Iceland and Finland. (Viren, 2001)

Richard Harris and Brian Silverstone approach Okun’s law asymmetry using a Threshold Autoregressive model. In the first step, Harris and Silverstone estimate the following equation.

$$u_t = \beta_0 + \beta_1 y_t + \beta_2 t + \varepsilon_t \quad (II.7)$$

They collect the residuals from the estimation and then estimate a second equation.

$$\Delta \hat{\varepsilon}_t = I_t \rho_1 \hat{\varepsilon}_{t-1} + (1-I_t) \rho_2 \hat{\varepsilon}_{t-1} + \nu_t \quad (II.8)$$

Once again zero is used as the fixed threshold to partition the data, but in this case the residuals are being partitioned.

$$I_t = \begin{cases} 
1 & \text{if } \hat{\varepsilon}_{t-1} > 0 \\
0 & \text{if } \hat{\varepsilon}_{t-1} < 0 
\end{cases} \quad (II.9)$$

Harris and Silverstone find that $y_t$ is weakly exogenous. They strongly reject symmetry in Okun’s Law. They conclude by stating that their results, “suggest that using an asymmetric model should become the standard approach to estimating what is a well-established empirical link between the labour market and output in the macroeconomy.” (Harris and Silverstone, 2001)
Paramsothy Silvapulle, Imad Moosa, and Mervyn Silvapulle investigate asymmetry using data from the US for the period 1947 to 1999. The Harvey structural time series model is used to generate the potential values of output. Silvapulle et al. formulate a distributed lag form of the unemployment / output relationship.

\[
\begin{align*}
\text{\(u_t^c = \sum_{j=1}^{p} \alpha_j u_{t-j}^c + \sum_{j=1}^{q} \beta_j y_{t-j}^c + \varepsilon_t\)}
\end{align*}
\]  
(II.10)

where:

\[
\begin{align*}
\text{\(u_t^c = u_t - u_t^n\)} \\
\text{\(u_t^c = \text{cyclical unemployment rate}\)} \\
\text{\(u_t = \text{observed unemployment rate}\)} \\
\text{\(u_t^n = \text{natural rate of unemployment}\)} \\
\text{\(y_t^c = y_t - y_t^n\)} \\
\text{\(y_t^c = \log \text{cyclical output} \)} \\
\text{\(y_t = \log \text{output} \)} \\
\text{\(y_t^n = \log \text{potential output} \)}
\end{align*}
\]

Once again a fixed partition around zero in \(y\) is formed.

\[
\begin{align*}
\text{\(u_t^c = \sum_{j=1}^{p} \alpha_j u_{t-j}^c + \sum_{j=1}^{q} \theta_j y_{t-j}^{c+} + \sum_{j=1}^{q} \delta_j y_{t-j}^{c-} + \varepsilon_t\)}
\end{align*}
\]  
(II.11)

Silvapulle et al. used the Akaike Information Criterion to select the optimal lag length. For \(u\), three lags are included.
in the model \((p=3)\). For \(y\), two lags were included in the model 
\((q=2)\).

Silvapulle et al. find strong evidence of asymmetry based on their estimation and hypothesis testing. (Silvapulle et al., 2004)

Ho-Chuan Huang and Shu-Chin Lin use the flexible nonlinear inference approach by Hamilton to investigate asymmetry in Okun’s Law. Huang and Lin take a first step in “let the data determine whether or not the relation is nonlinear and see how the nonlinearity looks like.”

Huang and Lin use the Harvey structural time series model to extract the trend and cyclical components to formulate the data from 1948 through 2004 for the gap model. “Overwhelming evidence is found in support of nonlinearity”. (Huang and Lin, 2006)

Ho-Chuan Huang and Shu-Chin Lin in a second paper use the smooth-time-varying-parameter approach using the Gibbs sampler to investigate asymmetry in Okun’s Law. They extend their data set from the prior paper to begin in 1948 and end in 2006. “We find that the mean estimates of (time-varying) Okun’s coefficients exhibit large variations for the entire analysis period and fluctuate around the fixed-Okun’s coefficient obtained from the conventional linear parametric model using differenced data.” This result supports Lee’s assertion of
instability in the Okun coefficient which he tested in the linear model using the Quandt likelihood function. Huang and Lin plot the time-varying coefficient over the range of the data. At all points the coefficient maintains the negative relationship of Okun’s original paper.

Huang and Lin believe the reason for the variation could be attributed to changes in labor productivity over the period 1948 to 2006. Huang and Lin find that the smooth-time-varying Okun’s coefficients are closely and positively related to productivity trends. (Lin, 2008)
III. DATA

The two variables of interest in estimating Okun’s law are the US Gross Domestic Product (GDP) and the US unemployment rate (UNEMP). The Gross Domestic Product is calculated by the US Bureau of Economic Analysis (BEA) on a quarterly basis. The data series is available from 1948. Originally Okun did his analysis using Gross National Product, but in 1991 the Gross Domestic Product replaced GNP as the accepted best indicator of national output.

The US unemployment rate is published monthly by the US Bureau of Labor Statistics (BLS). The unemployment rate is the percentage of people unemployed in the labor force. This indicator is estimated based upon the BLS household survey.

Okun suggested two models in his original paper, 1) the first differences model and 2) the trial gaps model. I focus on the first differences model in this paper in order to remove a possible confounder. Calculation of potential GDP and the non-accelerating inflation rate of unemployment, NAIRU, rely upon choosing a method for detrending the raw data. Lee has shown that the estimated coefficients in his model vary widely based upon which of the three detrending methods (HP filter, Beveridge Nelson decomposition, and Kalman Filter) that he used. (Lee, 2000) Silvapulle, Moosa, and Silvapulle (Silvapulle et al., 2004) use four different methods to calculate the potential
level of GDP. These methods include the linear trend, the Hodrick Prescott filter, the Beveridge Nelson decomposition, and the Kalman filter. The appropriate choice for modeling these potential levels is open to strong controversy, thus the focus on the first differences model in this paper.

The dependent variable in the first differences model is generated as the first difference of the unemployment rate on a quarterly basis.

\[
\text{UNEMP}_t = \text{quarterly change in the unemployment rate in percentage points} \\
= \text{US Unemployment Rate}_t - \text{US Unemployment Rate}_{t-1}
\]

The independent variable is the Quarter on Quarter percentage change in US GDP.

\[
\text{GDP}_t = \text{quarterly percent change in real GDP} \\
= (\text{real GDP}_t - \text{real GDP}_{t-1}) / \text{real GDP}_{t-1}
\]

I gather data from both the BEA and the BLS. The data series run from 1948 Q1 to 2012 Q2. Figure III.1 plots the raw data series. Figure III.2 plots the variables transformed for use in the first differences model.
FIGURE III.1
Real GDP and the Unemployment Rate since 1947

Raw data:

FIGURE III.2
First differences of real GDP and the Unemployment Rate since 1947

First Differences model data:
The appropriate transformation of data, represented as quarterly percentage changes for GDP and quarterly differencing for the unemployment rate in this paper, is essential to time series analysis. Over differencing, while resulting in a stationary series, would create unnecessary autocorrelation to the data series.

I plot both the spectrum and the autocorrelation function for the data used in the Okun model. The series are raw GDP, GDP quarter on quarter % change, the raw unemployment rate, and the quarterly difference in the unemployment rate.

Inspection of the plots in Figures III.3 through III.6 of both the spectrum and the ACF of both transformed series suggest the correct order of transformation has been used.
FIGURE III.3

Raw GDP: Spectrum and ACF
Spectral weights (1 2 3 2 1)

Spectrum GDP 2005 chain weighted
FIGURE III.4

GDP quarterly percent change: Spectrum and ACF
Spectral weights (1 2 3 2 1)

Spectrum GDP QonQ % change

ACF Plot for GDP ALL
FIGURE III.5

Unemployment Rate: Spectrum and ACF
Spectral weights (1 2 3 2 1)

Spectrum Unemployment Rate raw

ACF Plot for UNEMPRAW
FIGURE III.6

Unemployment Rate quarterly differences: Spectrum and ACF
Spectral weights (1 2 3 2 1)

Spectrum UNEMP QonQ diff

ACF Plot for UNEMPALL
The first differences model estimated by Arthur Okun presented the following results. (Okun, 1962)

\[ \text{UNEMP}_t = 0.30 - 0.30 \times \text{GDP}_t \quad r = 0.79 \] (III.1)

where:
\( \text{UNEMP}_t \) is the quarter on quarter change in the unemployment rate
\( \text{GDP}_t \) is the quarter on quarter percentage change in GDP

This model implies that a 3.3% increase in GDP from one quarter to the next would be associated with a 1 percentage point decrease in the unemployment rate.

The data used here differs from Okun’s in a couple of ways. The data begins in second quarter of 1948 and runs through the second quarter of 2012. Okun originally used GNP within his model, while this paper uses GDP in 2005 chained dollars. In 1991 the Bureau of Economic Analysis (BEA) changed the focus indicator for national income from Gross National Product (GNP) to Gross Domestic Product (GDP). Gross National Product measures the value of all goods and services produced by U.S. residents whether they are located within in the country or residing abroad. Gross Domestic Product measures the value of all goods and services produced within the boundaries of the United States. By 1991 most other OECD countries were focusing on GDP as their primary measure of production in economic accounting. The move by the BEA to feature GDP was consider a better measure of production and allows for more accurate cross
country comparisons. In 1991 the difference in the real notional value of GNP and GDP was less than 1%. (Fox, 1991)

The first step in modeling is to run an OLS estimation on the closest possible data set to that originally used by Okun, 1948-Q2 through 1960-Q4. The results below show a reasonable replication of Okun’s parameters. I report both the OLS standard errors in single parenthesis as well as the Newey-West standard errors in double parenthesis.

**Replication of Okun’s original estimation period:**

*1948 through 1960*

OLS

\[
\text{UNEMP}_n = 0.3201 - 0.3142 * \text{GDP}_n \quad \text{Adjusted } R^2 = 0.58238
\]

\[
(0.0374)
\]

\[
((0.0313))
\]

\[
\text{RSS} = 7.0425 \quad ((\text{Newey West SEs}))
\]

In addition to using the quarterly difference in the unemployment rate as in Okun’s original model, this paper will attempt to replace the unemployment rate with the Labor Market Conditions Index (LMCI) as defined and published by the Federal Reserve.

The US labor market is a complex system that may not be best described by a single rate such as the unemployment rate. In her speech at the Federal Reserve meeting in Jackson Hole in August 2014, Fed Chair Yellen pointed out two specific economic indicators that can affect the unemployment rate. Those
indicators are the labor market participation rate and involuntary part-time employment. The LMCI is an attempt to combine a broad base of economic indicators on the labor market into a single index value.

The methodology used by Chung, Fallick, Nekarda, and Ratner of the Division of Research & Statistics and Monetary Affairs, Federal Reserve Board, Washington, D.C. to create the LMCI is included below.

We instead estimate a dynamic factor model, along the lines laid out by Geweke (1977) and Sargent and Sims (1977). In the dynamic factor model, as in the principal components framework, the observable vector, $Y_t$, is a linear combination of a small number of “common” factors, $F_t$, and an “idiosyncratic” component $\omega_t$: $Y_t = HF_t + \omega_t$. In turn, the law of motion for the common factors is assumed to follow a vector autoregression (VAR). In our case, the persistence implied by the VAR dynamics substantially ameliorates the instability of factor estimates with missing end-of-sample data. As shown by Stock and Watson (1998), as the number of indicators in the panel grows large, principal component and factor analysis are equivalent.

Given our objective of capturing overall labor market conditions, we wish to focus on the common variation which accounts for the largest share of the variance of the indicators. Accordingly, we construct the LMCI as the first principal component of the projection of the indicators onto the common factors. Specifically, let $\theta$ be the eigenvector of $H \text{var}(F)H'$ associated with the largest eigenvalue. Then $\text{LMCI}_t = \theta H F_t$. (Chung, 2014)

The LMCI is a dynamic factor model created from nineteen economic indicators. In October, 2014 due to heightened interest by top policy makers regarding the LMCI the Federal
Reserve has formalized the release cycle for the LMCI. The monthly net change in the Labor Market Conditions Index will be released each month by 10:00am ET on the Monday following the release of the Bureau of Labor Statistics employment situation which includes non-farm payrolls and the unemployment rate. This BLS release is usually scheduled for the first Friday of each month. The nineteen included indicators sorted by their absolute value of correlation with the monthly change in LMCI are shown in Table III.1. The Spectrum and ACF for the average monthly change in the LMCI each quarter is included in Figure III.7.

In this paper in section IX, I will use MARS to estimate a model of LMCI ~ GDP. This model is an innovation beyond Okun’s original specification. We will investigate whether threshold levels hold in a model that includes the more broad-based indicator of labor market conditions the LMCI.
### TABLE III.1

Economic Indicators included in the Labor Market Conditions Index

<table>
<thead>
<tr>
<th>#</th>
<th>Economic Indicator</th>
<th>Correlation with change in LMCI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Private employment</td>
<td>0.83</td>
</tr>
<tr>
<td>2</td>
<td>Insured unemployment rate</td>
<td>-0.74</td>
</tr>
<tr>
<td>3</td>
<td>Temporary help services employment</td>
<td>0.70</td>
</tr>
<tr>
<td>4</td>
<td>Unemployment rate</td>
<td>-0.64</td>
</tr>
<tr>
<td>5</td>
<td>Job availability</td>
<td>0.53</td>
</tr>
<tr>
<td>6</td>
<td>Composite help-wanted index</td>
<td>0.47</td>
</tr>
<tr>
<td>7</td>
<td>Involuntary part-time employment</td>
<td>-0.39</td>
</tr>
<tr>
<td>8</td>
<td>Job losers unemployed less than 5 weeks</td>
<td>-0.28</td>
</tr>
<tr>
<td>9</td>
<td>Jobs hard to fill</td>
<td>0.20</td>
</tr>
<tr>
<td>10</td>
<td>Average weekly hours of production workers</td>
<td>0.20</td>
</tr>
<tr>
<td>11</td>
<td>Quit rate</td>
<td>0.17</td>
</tr>
<tr>
<td>12</td>
<td>Average weekly hours of persons at work</td>
<td>0.17</td>
</tr>
<tr>
<td>13</td>
<td>Net hiring plans</td>
<td>0.15</td>
</tr>
<tr>
<td>14</td>
<td>Average hourly earnings of production workers</td>
<td>0.14</td>
</tr>
<tr>
<td>15</td>
<td>Hiring rate</td>
<td>0.13</td>
</tr>
<tr>
<td>16</td>
<td>Transition rate from unemployment to employment</td>
<td>0.12</td>
</tr>
<tr>
<td>17</td>
<td>Labor force participation rate</td>
<td>0.06</td>
</tr>
<tr>
<td>18</td>
<td>Job leavers unemployed less than 5 weeks</td>
<td>0.06</td>
</tr>
<tr>
<td>19</td>
<td>Government employment</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

(Chung, 2014)
FIGURE III.7

LMCI average monthly change per quarter: Spectrum and ACF

Spectral weights (1 2 3 2 1)

Spectrum LMCI ave change per qtr

ACF Plot for LMCI Ave
IV. EMPIRICAL METHODOLOGY

Each of the authors cited in the literature prior to Huang and Lin in 2006 shares a common similarity in the modeling approach. In their models, the right hand side variable is decomposed into positive and negative changes, thus zero is forced as the threshold across which asymmetry is tested. This is a reasonable starting point for an assumption about asymmetry, but newer threshold modeling methods allow us to take a step forward in estimating the output / unemployment relationship with fewer prior assumptions.

I will make use of Friedman’s Multivariate Adaptive Regression Splines (MARS) model. This nonparametric model is a natural fit for identifying asymmetry. “MARS is an adaptive procedure for regression and can be thought of as a generalization of stepwise linear regression or a modified approach to Classification and Regression Tree (CART) modeling.” (Friedman, 1991) I will now describe the MARS algorithm relying heavily upon material in The Elements of Statistical Learning by Hastie, Tibshirani, and Friedman.

MARS uses expansions in piecewise linear basis functions of the form $\text{MAX}(X - \tau, 0)$ and $\text{MAX}(\tau - X, 0)$. Each function is linear, with a knot at the value $\tau$. The value $\tau$ is also referred to as a threshold level. These two functions with a shared
value for \( \tau \) are called a reflected pair. The algorithm forms knots at each observed value of X. At each stage of the algorithm, a new basis function pair is considered and the term that causes the largest reduction in the training error is included in the model. This is done up to M times; the limit of the number of knots specified by the user. The user also controls the highest order of variable interactions.

MARS has the following form.

\[
f(x) = \beta_0 + \sum_{m=1}^{M} \beta_m h_m(X)
\]

(IV.1)

Where:

\( h_m(X) \) are the basis functions

\( \beta_m \) are the coefficients estimated by minimizing the sum of squared residuals.

A large model that overfits the data is typically formed in the forward process. MARS then applies a backward deletion procedure to remove terms that cause the smallest increase to the residual sum of squares. Generalized cross-validation is used as the criteria for backward pruning. The MARS algorithm builds a regression surface parsimoniously including non-zero basis functions only when they are needed. The MARS algorithm is not bound by a priori assumptions about the appropriate
values of $\tau$, but instead searches for the threshold values that best fit the data. (Hastie et al., 2005)

For further detail on the MARS algorithm than can be provided in this paper, please refer to the original article by Friedman (Friedman, 1991) or the invaluable text by Hastie, Tibshirani, and Friedman (Hastie et al., 2005).

This paper makes extensive use of a second nonlinear modeling method, the Generalized Additive Model (GAM) from Trevor Hastie and Robert Tibshirani. GAM has the following form.

$$E(y | x_1, x_2, ..., x_k) = s_0 + \sum_{j=1}^{k} s_j(x_j) + e$$

(IV.2)

Where:
s_j(\cdot) are the smooth functions standardized so that $E[s_j(X_j)] = 0$.
The smooth functions are estimated one at a time using forward stepwise estimation using a scatterplot smoother. (Stokes, 1997)

Hastie and Tibshirani write,

We introduce the class of generalized additive models which replaces the linear form $\sum \beta_j X_j$ by a sum of smooth functions $\sum s_j(X_j)$. The $s_j(\cdot)$’s are unspecified functions that are estimated using a scatterplot smoother, in an iterative procedure we
call the local scoring algorithm. The technique is applicable to any likelihood-based regression model: the class of generalized linear models contains many of these. In this case the linear predictor $\eta = \sum \beta_j X_j$ is replaced by the additive predictor $\sum s_j(X_j)$; hence, the name generalized additive models. [T]he method proves to be useful in uncovering nonlinear covariate effects. It has the advantage of being completely automatic, i.e., no "detective work" is needed on the part of the statistician. As a theoretical underpinning, the technique is viewed as an empirical method of maximizing the expected log likelihood, of equivalently, of minimizing the Kullback-Lieber distance to the true model. (Hastie, Tibshirani, 1986)

GAM is a powerful estimation technique when nonlinearity is suspected. When using the statistical package, B34S, leverage plots of each predictor variable can be drawn to visualize the form of the nonlinearity. An important aspect of GAM models is the ability to formally test each predictor variable for nonlinearity. The test statistic for nonlinearity is equal to $(\text{RSS}_R - \text{RSS}_U) / \sigma^2$, where $\text{RSS}_U$ is the residual sum of squares for the GAM model and $\text{RSS}_R$ is the residual sum of squares for the restricted linear model. This test statistic is distributed $\chi^2$ with the degree of freedom equal to the degree of the polynomial used to smooth the variable being tested. (Stokes, 1997)
V. TESTING THE ORIGINAL OKUN MODEL FOR NONLINEARITY

GENERAL ADDITIVE MODEL (GAM)

The common question that has been posed by the papers cited in this thesis is whether Okun’s law is misspecified as a linear relationship and better specified in a nonlinear way. The common threshold for asymmetric hypothesis testing is overwhelmingly zero for some choice variable: change in unemployment, change in GDP, change in the error correction term.

I focus in this chapter on how Okun’s law has become nonlinear over time. Okun did not misspecify the relationship during the original time period used by Okun, 1948-Q2 through 1960-Q4. I will use the test statistic within the GAM estimation procedure in B34S to show that the relationship between unemployment and output has become nonlinear over time.

Using data in the first differences format, I run the GAM estimation with the degree of smoothing set equal to 2. GAM will first impose the restriction of a linear model or degree of smoothing equal to 1. The linear estimation results are shown.

\[ \text{UNEMP}_t = 0.320 - 0.314 \times \text{GDP}_t \]  \hspace{1cm} (V.1)

\[ (0.0620) (0.0374) \]

where:

\( \text{UNEMP}_t \) = the quarter on quarter change in the unemployment rate
\( \text{GDP}_t \) = the quarter on quarter percentage change in GDP
GAM then estimates the model with the degree of smoothing set equal to 3 allowing for a locally estimated nonlinear surface. The sum of squared residuals from the restricted and unrestricted model are used to form the hypothesis test with the null of linearity. The test statistic is distributed $\chi^2$ with degrees of freedom equal to the number of parameters (k) in the model. The results are shown below. The probability of nonlinearity is 20.89%. We cannot reject the null hypothesis of linearity, supporting Okun’s original specification of the model.

**TABLE V.1**

GAM Results for original OKUN model, 1948-Q2 through 1960-Q4

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>z score</th>
<th>Nonlinear pval</th>
<th>RSS w/ Linear Restriction</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>-0.3142</td>
<td>-8.3640</td>
<td>0.2089</td>
<td>7.042</td>
<td>2.0</td>
</tr>
<tr>
<td>RSS</td>
<td>6.9745</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R squared</td>
<td>0.5947</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Visual inspection of the surface estimated by GAM, with confidence intervals plotted, further reinforces a linear relationship. This graph is provided in figure V.1.
In order to test the hypothesis of a change from a linear to a nonlinear relationship between unemployment and output, I follow an identical testing procedure using GAM as that above, but using data beginning in 1961-Q1 through 2012-Q2.

During the new data period the test statistic strongly rejects the hypothesis of linearity; in other words Okun’s law became
nonlinear. The results of the GAM estimation are shown below with the probability of nonlinearity at 99.96%.

**TABLE V.2**

GAM Results for OKUN model, 1961-Q1 through 2012-Q2

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>z score</th>
<th>Nonlinear pval</th>
<th>RSS w/ Linear Restriction</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDPNEW</td>
<td>-0.2549</td>
<td>-12.5400</td>
<td>0.9996</td>
<td>13.720</td>
<td>2.0</td>
</tr>
<tr>
<td>RSS</td>
<td>12.7315</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R squared</td>
<td>0.4599</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Visual inspection of the surface estimated by GAM with, confidence intervals plotted, supports the finding of a nonlinear relationship. Further inspection suggests that the inflection point of the change in slope is not at zero on the axis of GDP. This raises the question as to whether threshold testing based on partitions of the data across the zero boundary is appropriate. This graph is provided in figure V.2.
Formal hypothesis testing for nonlinearity, using the methods of Hastie and Tibshirani (Hastie and Tibshirani, 1986), in a smoothed General Additive Model, shows us that the relationship between unemployment and output which was once linear, from 1948 to 1960, has become nonlinear when the data from 1961 to 2012 is considered.
VI. MARS ESTIMATION OF OKUN’S FIRST DIFFERENCES MODEL

To being the investigation of Okun’s law with the MARS estimation, I run the model using MARS on the same period originally used by Okun, 1948-Q2 through 1960-Q4. I show the results of both the OLS and MARS estimations below. Standard errors are shown in parenthesis below each coefficient. MARS was estimated using the B34S MARSPLINE procedure, as well as the mda package in R. The estimated coefficients and thresholds matched exactly between the two packages when the maximum number of knots was set equal to 5.

**Estimation Period: 1948-Q2 through 1960-Q4**

**OLS**

\[
\text{UNEMP}_t = 0.3201 - 0.3142 \times \text{GDP}_t \\
(0.0374) \\
((0.0313)) \tag{VI.1} \]  

\[
\text{RSS} = 7.0425 
\]

**MARS**

\[
\text{UNEMP}_t = 0.2130 + 0.5005 \times \text{MAX}( -0.2387 - \text{GDP}_t , 0) - 0.3556 \times \text{MAX}(\text{GDP}_t - 0.6153, 0) \\
(0.1239) \quad (0.0602) \\
((0.0565)) \quad ((0.0528)) \tag{VI.2} 
\]

\[
\text{RSS} = 6.9600 \\
# \text{Non-Zero} \quad 51 \quad 11 \\
\]

Within the MARS estimation, asymmetry is certainly supported, as negative changes in GDP effect unemployment with a -0.5005 coefficient while positive changes in GDP effect unemployment with a -0.3556 coefficient. It is interesting that
we jump between the 2x and the 3x rule of thumb, depending on the severity of the negative change in GDP. Even more interesting is where these changes occur. The MARS results suggest the there is a zone of no effect within the GDP / unemployment relationship where Okun’s Law breaks down and is not statistically significant. The threshold levels are -0.2387 and 0.6153. Within this range of changes in quarter on quarter (QonQ) GDP, there is no effect of GDP on unemployment. It may help to illustrate this point with a contribution chart for the MARS estimation.

**Figure VI.1**

![MARS estimation of Okun's Law 1948 through 1960](image-url)
Figure VI.1 shows the fitted values of the change in the unemployment rate for both the MARS and the OLS estimates. The solid blue line representing the MARS estimation is completely flat between the threshold values of GDP, -0.2387 to 0.6153.

One familiar problem with a nonparametric approach to modeling is that small changes in the data could cause additional basis functions or interaction variables to enter the estimated equation. Since Okun’s original first differences model includes only a single right hand side variable, we do not have to worry about interactions in this case. Friedman’s MARS uses Generalized Cross Validation to prune the model in the backward phase to better fit out-of-sample data. In the forward phase of the algorithm, the model is often over fit, but forecasting is improved by going through the pruning process. Given these issues one wonders how well MARS can model Okun’s law when applied to the data from the end of the original sample period up to our most recent data ending with the second quarter of 2012. The model was estimated over this period. The OLS and MARS results are shown below.
**Estimation Period**: 1961-Q1 through 2012-Q2

**OLS**

\[
\text{UNEMP}_t = 0.2060 - 0.2548 \times \text{GDP}_t
\]

\[
(0.00211)
\]

\[\text{(Newey West SEs)}\]

\[
\text{RSS} = 13.7178
\]

\[(VI.3)\]

**MARS**

\[
\text{UNEMP}_t = 0.0423 + 0.4467 \times \text{MAX}(0.2617 - \text{GDP}_t, 0) - 0.2475 \times \text{MAX}(\text{GDP}_t - 0.5787, 0) + 0.3191 \times \text{MAX}(\text{GDP}_t - 1.9504, 0)
\]

\[
(0.0455) \quad (0.0417) \quad (0.1332)
\]

\[\text{(Newey West SEs)}\]

\[
# \text{Non-Zero} \quad 206 \quad 41 \quad 129
\]

\[
\text{RSS} = 12.3158
\]

\[(VI.4)\]

The contribution chart again provides a good way to visualize the MARS equation. Once again, we see evidence of the zone of no effect of GDP growth on the change in the unemployment rate. This zone has shifted to the right and narrowed to between 0.2617 and 0.5787, but still reflects an area in which Okun’s law is no longer significant.

As feared in a nonparametric approach, the inclusion of five extreme observations of GDP QonQ percentage changes in 1971-Q1, 1972-Q2, 1973-Q1, 1976-Q1 and 1978-Q1 lead the MARS algorithm to add an additional basis function with a threshold above 1.9504 that carries a positive coefficient, contrary to theory. This contribution chart is displayed in Figure VI.2.
The MARS estimation was run on the full data set from 1948 through 2012. In this case, there is no evidence of a zone of no effect of GDP growth on the change in the unemployment rate. Instead we see the same structure as the within Okun’s sample period, that of two basis functions left in the model. The difference is that the threshold levels overlap with the lower threshold ending at 0.0131 and the upper threshold beginning at -0.2157. This overlapping basis function has more of an effect of a vertical jump implying asymmetrically differing intercepts along with differing slope coefficients. The OLS and MARS
results are shown below. The contribution chart for this estimation is included as Figure VI.3.

**Estimation Period: 1948 through 2012**

**OLS**

\[ \text{UNEMP}_t = 0.2367 - 0.2780 \times \text{GDP}_t \]

\[ (0.0180) \]

\[ ((0.0254)) \]

\[ \text{RSS} = 21.1501 \]

(VI.6)

**MARS**

\[ \text{UNEMP}_t = 0.2078 + 0.4112 \times \text{MAX}(0.0131 - \text{GDP}_t,0) - 0.2157 \times \text{MAX}(\text{GDP}_t-0.2387,0) \]

\[ (0.0509) \]

\[ ((0.0441)) \]

\[ \text{RSS} = 20.0264 \]

(VI.7)

**Figure VI.3**

MARS estimation of Okun's Law 1948 through 2012
VII. MARS ESTIMATION OF PRACHOWNY MODEL

Recall from the literature review that Prachowny, in his first principles approach, specifies the estimating equation with additional variables derived from a production function approach. Prachowny estimates the following equation using linear estimation methods which he feels are statistically superior to the original Okun gap model.

\[ \Delta(y-y^*) = a_1\Delta(c-c^*) + a_2\Delta(l-l^*) - a_3\Delta(u-u^*) + a_4\Delta(h-h^*) + e \]

(VII.1)

Where:

\( (y-y^*) = \) output gap
\( (c-c^*) = \) utilization gap
\( (l-l^*) = \) natural log of the supply of workers gap
\( (u-u^*) = \) deviation from the natural rate of unemployment
\( (h-h^*) = \) hours worked gap

When estimated with this specification the relation of a change in unemployment on output changes significantly. For a 1.5% decrease in unemployment, a 1% increase in output is implied.

Although Prachowny takes a step toward identifying the marginal effect of the individual factors for production such as capacity utilization, the size of the labor force, and weekly
hours worked, the estimation is based on the assumption of linearity. To investigate whether nonlinearity and threshold effects are mitigated by the production function approach to Okun’s Law I begin by extending the data set from the original period estimated by Prachowny of 1967 Q1 to 1988 Q4 to include data through 2012 Q2.

Prachowny originally used the potential levels of GDP and the Natural Rate of Unemployment from Gordon’s data set. Prachowny created the potential levels of Labor Force ($l^*$) and Hours Worked ($h^*$) by fitting a second degree polynomial regression of $x = c_0 + c_1T + c_2T^2$. In order to extend the data set I replaced Gordon data with the Congressional Budget Office values from January of 2012 for potential GDP and the Natural Rate of Unemployment. The data set begins in 1949 Q1 and forecasts potential levels out to 2022 Q4.

To confirm that the substituted potential levels do not significantly affect the Prachowny estimated coefficients, I replicate the estimation for Prachowny’s original estimation period, 1967 Q1 to 1988 Q4. In addition I run the regression on the period 1967 Q1 to 2012 Q2 and the period 1989 Q1 to 2012 Q2. The results of these estimations are shown in Tables VI.1, VI.2 and VI.2 below.

Within the B34S program the following variables were used:

$YY = output\ gap$
CC = utilization gap  
LL = natural log of the supply of workers gap  
UU = deviation from the natural rate of unemployment  
HH = hours worked gap

**Table VII.1**  
Prachowny estimation: 1967 Q1 to 1988 Q4

<table>
<thead>
<tr>
<th>Coefficients using CBO potential levels</th>
<th>Coefficients from (Prachowny, 1993)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967 Q1 to 1988 Q4</td>
<td></td>
</tr>
<tr>
<td>Estimate</td>
<td>Std.Error</td>
</tr>
<tr>
<td>(c-c*)</td>
<td>0.27039</td>
</tr>
<tr>
<td>(l-l*)</td>
<td>-0.14157</td>
</tr>
<tr>
<td>(u-u*)</td>
<td>-0.63204</td>
</tr>
<tr>
<td>(h-h*)</td>
<td>0.61775</td>
</tr>
</tbody>
</table>

**Table VII.2**  
Prachowny estimation: 1967 Q1 to 2012 Q2

<table>
<thead>
<tr>
<th>Coefficients using CBO potential levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967 Q1 to 2012 Q2</td>
</tr>
<tr>
<td>Estimate</td>
</tr>
<tr>
<td>(c-c*)</td>
</tr>
<tr>
<td>(l-l*)</td>
</tr>
<tr>
<td>(u-u*)</td>
</tr>
<tr>
<td>(h-h*)</td>
</tr>
</tbody>
</table>
As a diagnostic test for nonlinearity, I run GAM on the Prachowny model from 1967 Q1 to 1998 Q2. All variables with the exception of Hours Worked (HH) reject the null hypothesis of linearity. Table VII.2 reports the probability of nonlinearity for each estimated coefficient. Figure VII.1 displays GAM surface plots for the variables CC, LL, UU, HH.

**Table VII.3**

Prachowny estimation: 1989 Q1 to 2012 Q2

<table>
<thead>
<tr>
<th>Coefficients using CBO potential levels</th>
<th>1989 Q1 to 2012 Q2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
</tr>
<tr>
<td>(c-c*)</td>
<td>-0.0556</td>
</tr>
<tr>
<td>(l-l*)</td>
<td>-0.0232</td>
</tr>
<tr>
<td>(u-u*)</td>
<td>-1.7396</td>
</tr>
<tr>
<td>(h-h*)</td>
<td>0.2815</td>
</tr>
</tbody>
</table>

**Table VII.4**

GAM Results for Prachowny model
1967-Q1 through 1998-Q2

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>z score</th>
<th>Nonlinear pval</th>
<th>RSS w/ Linear Restriction</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c-c*)</td>
<td>0.2797</td>
<td>5.2610</td>
<td>0.9986</td>
<td>41.680</td>
<td>3.0</td>
</tr>
<tr>
<td>(l-l*)</td>
<td>-0.2809</td>
<td>-5.8000</td>
<td>0.9985</td>
<td>41.580</td>
<td>3.0</td>
</tr>
<tr>
<td>(u-u*)</td>
<td>-0.4786</td>
<td>-3.1870</td>
<td>0.9956</td>
<td>40.530</td>
<td>3.0</td>
</tr>
<tr>
<td>RSS</td>
<td>34.5076</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R squared</td>
<td>0.9452</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure VII.1

GAM surface plots of CC, LL

Surface CC

Surface LL
Figure VII.2

GAM surface plots of

UU, HH
The next step is to investigate threshold effects by estimating the Prachowny model with MARS.

In the MARS estimation, the primary choice variables are the number of knots, or basis functions, allowed in the model and the maximum degree at which basis functions can interact. We begin with the simplest form of the MARS estimation, setting $\text{max interaction} = 1$.

The leverage chart is the most intuitive way to understand the marginal effect of each variable on the output gap, $YY$. The OLS model and the GAM model both show us that in general the marginal effect on $YY$ of $CC$ is positive, $LL$ is negative, $UU$ is negative (supporting Okun’s Law), $HH$ is positive.

The MARS estimation identifies thresholds where the magnitude of the marginal effect changes. In the case of the supply of workers, $LL$, the marginal effect actually changes signs from negative to positive above the 5.046 threshold. The leverage plots for each variable with all other variables held at their median are shown in Figures VII.3 through Figures VII.6. The results of the MARS estimation with both coefficients and basis functions that denote the threshold values are shown in Table VII.5.
Figure VII.3

Leverage Chart of the effect of Utilization Gap on Output Gap (CC on YY)
Max Knots = 30, Max Interaction = 1

Prediction Leverage of CC [lag = 0, int = 1, o = Medians]
Figure VII.4

Leverage Chart of the effect of Supply of Workers on Output Gap (LL on YY)
Max Knots = 30, Max Interaction = 1
Figure VII.5

Leverage Chart of the effect of Unemployment Gap, UU on Output Gap, YY
Max Knots = 30, Max Interaction = 1

Prediction Leverage of UU [lag= 0 , int= 1, o=M edians]
Figure VII.6

Leverage Chart of the effect of
Hours Worked on Output Gap (HH on YY)
Max Knots = 30, Max Interaction = 1

Prediction Leverage of HH
[ lag= 0 , int= 1, o=Medians]
### TABLE VII.5

MARS estimation of the Prachowny Model

Max knots = 30, Max Interaction = 1

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Basis Function</th>
<th>t</th>
<th>% Non-zero</th>
</tr>
</thead>
<tbody>
<tr>
<td>YY = 0.66469432</td>
<td></td>
<td>1.01</td>
<td>100.000</td>
</tr>
<tr>
<td>+ 0.20957296</td>
<td>* max( CC - -3.587600, 0)</td>
<td>3.30</td>
<td>79.545</td>
</tr>
<tr>
<td>+ 1.3683559</td>
<td>* max( LL - 5.0460000, 0)</td>
<td>5.30</td>
<td>18.182</td>
</tr>
<tr>
<td>+ 0.44120042</td>
<td>* max( CC - 1.0727000, 0)</td>
<td>4.59</td>
<td>45.455</td>
</tr>
<tr>
<td>- 0.68988420</td>
<td>* max( LL - 1.5739000, 0)</td>
<td>-9.67</td>
<td>65.909</td>
</tr>
<tr>
<td>- 3.0080866</td>
<td>* max( UU - 1.0000000, 0)</td>
<td>-6.68</td>
<td>36.364</td>
</tr>
<tr>
<td>+ 1.4709398</td>
<td>* max( UU - 0.4000000, 0)</td>
<td>3.54</td>
<td>44.318</td>
</tr>
<tr>
<td>+ 3.4531264</td>
<td>* max( HH - 0.1336000, 0)</td>
<td>3.29</td>
<td>18.182</td>
</tr>
<tr>
<td>- 1.7768474</td>
<td>* max( 0.1336000 - HH, 0)</td>
<td>-3.28</td>
<td>80.682</td>
</tr>
<tr>
<td>- 2.0565409</td>
<td>* max( HH - -0.792500, 0)</td>
<td>-2.94</td>
<td>79.545</td>
</tr>
</tbody>
</table>

Where:

YY = (y-y*) = output gap  
CC = (c-c*) = utilization gap  
LL = (l-l*) = natural log of the supply of workers gap  
UU = (u-u*) = deviation from the natural rate of unemployment  
HH = (h-h*) = hours worked gap

In addition, the MARS estimation was run with the maximum degree of interaction at both 2 and 3. As the degree of interaction was increased the resulting models became more complex and improved the overall fit. A comparison is shown in table VII.6.
**Table VII.6**

Comparison of Residual Sum of Squares (RSS) and Generalized Cross Validation (GCV) for differing degrees of Interaction

<table>
<thead>
<tr>
<th>Max Degree of Interactions</th>
<th># Coefficients</th>
<th>RSS</th>
<th>GCV</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS without constant</td>
<td>4</td>
<td>55.5090</td>
<td>0.6923</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>26.4992</td>
<td>0.4898</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>15.5957</td>
<td>0.3688</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
<td>10.8189</td>
<td>0.3147</td>
</tr>
</tbody>
</table>
VIII. MARS estimation of the Silvapulle Distributed Lag Model

Recall from the literature review that Silvapulle et al. formulate a distributed lag form of the unemployment / output relationship.

\[
\begin{align*}
    u_t^c &= \sum_{j=1}^{p} \alpha_j u_{t-j}^c + \sum_{j=1}^{q} \beta_j y_{t-j}^c + \epsilon_t \\
    (VIII.1)
\end{align*}
\]

where:

\[
\begin{align*}
    u_t^c &= u_t - u_t^n \\
    u_t^c &= \text{cyclical unemployment rate} \\
    u_t &= \text{observed unemployment rate} \\
    u_t^n &= \text{natural rate of unemployment} \\
    y_t^c &= y_t - y_t^n \\
    y_t^c &= \text{logarithm of cyclical output} \\
    y_t &= \text{logarithm of output} \\
    y_t^n &= \text{logarithm of potential output}
\end{align*}
\]

Once again a fixed partition around zero in \( y \) is formed.

\[
\begin{align*}
    u_t^c &= \sum_{j=1}^{p} \alpha_j u_{t-j}^c + \sum_{j=1}^{q} \theta_j y_{t-j}^{c+} + \sum_{j=1}^{q} \delta_j y_{t-j}^{c-} + \epsilon_t \\
    (VIII.2)
\end{align*}
\]

Silvapulle et al. used the Akaike Information Criterion to select the optimal lag length. For \( u \), three lags are included in the model (\( p=3 \)). For \( y \), two lags were included in the model (\( q=2 \)).
To investigate this model, the OLS regression is run on

\[ UNEMP_t = \beta_0 + \beta_1 L^1UNEMP_t + \beta_2 L^2UNEMP_t + \beta_3 L^3UNEMP_t + \beta_4 L^1GDP_t + \beta_5 L^2GDP_t + \epsilon_t \]

(VIII.3)

The third lag of unemployment (L3UNEMP) is insignificant in the OLS model. Additionally, the MARS estimation does not select this variable in its MGCV guided stepwise process. Therefore L3UNEMP was removed from the model. The OLS form of the main estimating equation in this section becomes

\[ UNEMP_t = \beta_0 + \beta_1 L^1UNEMP_t + \beta_2 L^2UNEMP_t + \beta_4 L^1GDP_t + \beta_5 L^2GDP_t + \epsilon_t \]

(VIII.4)

Silvapulle et al. used the Akaike Information Criterion to select the optimal lag length (Silvapulle et al., 2004). Another method for identifying the appropriate lag length for the model is to use the btiden command in B34S and the estvar sentence to estimate the unconstrained VAR model. All lags of UNEMP and GDP are included in the unconstrained model. Initially the model was attempted with 6 lags. The appropriate lag length is the last significant lag in the stepwise autoregression as measured by the M test which is distributed \( \chi^2 \). As seen in table VIII.1 the appropriate lag length is 2.
Table VIII.1

M test for appropriate lag length (Stokes, 1997)

<table>
<thead>
<tr>
<th>Lag</th>
<th>( \chi^2 ) distributed</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>160.7</td>
<td>100.00%</td>
</tr>
<tr>
<td>2</td>
<td>25.8</td>
<td>100.00%</td>
</tr>
<tr>
<td>3</td>
<td>2.57</td>
<td>36.87%</td>
</tr>
<tr>
<td>4</td>
<td>5.46</td>
<td>75.70%</td>
</tr>
<tr>
<td>5</td>
<td>6.84</td>
<td>85.52%</td>
</tr>
<tr>
<td>6</td>
<td>5.37</td>
<td>74.89%</td>
</tr>
</tbody>
</table>

Next, I use MARS to estimate the distributed lag model (p=2, q=2), with max knots = 3 and max interaction = 1. The leverage plots are shown in figures VIII.1 through VIII.4. The results of the MARS estimation are shown in Table VIII.4. The OLS estimation is shown in table VIII.2. The comparison between MARS estimations with increasing maximum degrees of interaction is shown in table VIII.3.

Table VIII.2

Silvapulle et al. estimation: 1949 Q1 to 2012 Q2

|            | Estimate | Std.Error | t.value | Pr(>|t|) |
|------------|----------|-----------|---------|---------|
| Constant   | 0.17400  | 0.034407  | 5.057   | 0.0000  |
| L1UNEMP    | 0.50764  | 0.078018  | 6.507   | 0.0000  |
| L2UNEMP    | -0.30179 | 0.066739  | -4.522  | 0.0000  |
| L1GDP      | -0.11631 | 0.024634  | -4.721  | 0.0000  |
| L2GDP      | -0.08984 | 0.025577  | -3.513  | 0.0005  |
Table VIII.3

Comparison of Residual Sum of Squares (RSS) and Generalized Cross Validation (GCV) for differing degrees of Interaction

<table>
<thead>
<tr>
<th>Max Degree of Interaction</th>
<th># Coefficients</th>
<th>RSS</th>
<th>GCV</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>5</td>
<td>19.0326</td>
<td>0.0783</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>17.4481</td>
<td>0.0766</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>13.7909</td>
<td>0.0695</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>13.1151</td>
<td>0.0698</td>
</tr>
</tbody>
</table>

Figure VIII.1

Leverage Chart of the effect of L1UNEMP on UNEMP
Max Knots = 30, Max Interaction = 1

Prediction Leverage of L1UNEMP  [lag= 0, int= 1, o=Medians]
Figure VIII.2

Leverage Chart of the effect of L2UNEMP on UNEMP
Max Knots = 30, Max Interaction = 1

Prediction Leverage of L2UNEMP  [lag= 0 , int= 1, o=Medians]
Figure VIII.3

Leverage Chart of the effect of L1GDP on UNEMP
Max Knots = 30, Max Interaction = 1

Prediction Leverage of L1GDP [lag = 0, int = 1, o = Medians]
Figure VIII.4

Leverage Chart of the effect of L2GDP on UNEMP
Max Knots = 30, Max Interaction = 1

Prediction Leverage of L2GDP [lag = 0, int = 1, o = Medians]
**Table VIII.4**

MARS estimation of the Silvapulle Model

Max knots = 30, Max Interaction = 1

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Basis Function</th>
<th>t</th>
<th>% Non-zero</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNEMPALL</td>
<td></td>
<td>8.14</td>
<td>100.000</td>
</tr>
<tr>
<td>+ 0.47126839* max( L1UNEMP - 0.000000, 0)</td>
<td>5.12</td>
<td>35.178</td>
<td></td>
</tr>
<tr>
<td>- 0.49483982* max( 0.000000 - L1UNEMP, 0)</td>
<td>-4.09</td>
<td>50.198</td>
<td></td>
</tr>
<tr>
<td>- 0.12176564* max( L2GDP - -0.88000000, 0)</td>
<td>-4.77</td>
<td>93.281</td>
<td></td>
</tr>
<tr>
<td>- 0.32782713* max( L2UNEMP - -1.000000, 0)</td>
<td>-5.17</td>
<td>99.605</td>
<td></td>
</tr>
<tr>
<td>- 0.30479593* max( L1GDP - -0.88000000, 0)</td>
<td>-5.93</td>
<td>93.281</td>
<td></td>
</tr>
<tr>
<td>+ 0.29264970* max( L1GDP - 0.64000000, 0)</td>
<td>4.24</td>
<td>57.708</td>
<td></td>
</tr>
</tbody>
</table>
IX. A MARS ESTIMATION OF THE LABOR MARKET CONDITIONS INDEX

LMCI ~ GDP

In September 2014, Janet Yellen, Chair of the Federal Reserve Board of Governors, highlighted an indicator of the labor market that is considered to be a more broad-based indicator of the labor market, the Labor Market Conditions Index (LMIC). This index is the first principal component of nineteen labor market indicators. In this section I will substitute the LMCI for the Unemployment rate in the Okun relationship in order to answer the question as to whether the LMCI can remove the nonlinearity that has been evidenced in the Unemployment / GDP relationship.

The MARS estimation will treat the average monthly change over each quarterly period from 1976-Q4 though 2014-Q4 as the dependent variable and the quarter-on-quarter percent change in GDP as the independent variable.

It is important to highlight that the cyclical changes in the unemployment rate have a high negative correlation with the changes in the LMCI. The correlation to the change in LMCI is -0.64 and shown in Table III.1. We therefore expect a positive sign on the estimated coefficient for GDP in both the OLS and the MARS estimation. This is in contrast to the negative coefficient that results from the original Okun estimation of UNEMP ~ GDP.
The results of the MARS estimation of the average monthly change in LMCI over each quarterly period, expressed as the variable LMCIAVE, versus GDP is shown below.

**Estimation Period: 1976-Q4 through 2014-Q4**

**OLS**

\[ \text{LMCIAVE}_t = -5.2602 + 8.3643 \times \text{GDP}_t \]

\[ (0.6985) \]

\[ ((1.283)) \]

\[ \text{RSS} = 6946.7 \]

**MARS**

\[ \text{LMCIAVE}_t = -1.064 + -16.01\times\max(0.020 - \text{GDP}_t,0) + 5.862\times\max(\text{GDP}_t - 0.318,0) \]

\[ (1.538) \]

\[ (0.978) \]

\[ ((0.0565)) \]

\[ ((0.0528)) \]

\[ \text{RSS} = 6001.2 \]

\[ (\text{IX.2}) \]
The LMCI may be a better indicator of the overall condition of the labor market, but it does not correct the nonlinearity in the relationship between GDP and LMCI.

The MARS estimation of LMCI ~ GDP and the contribution chart supports the previous MARS results of the Okun model. There is a zone of no effect of GDP growth on the change in the
unemployment rate on the range of GDP growth between 0.02% and 0.318%. Above this range, GDP has a lower marginal effect of 5.862, while below this range GDP has a higher marginal effect of 16.01.
X. SPECTRAL DECOMPOSITION OF UNEMP AND GDP

THE GEWEKE PROCEDURE

This paper considers the relationship between quarterly differences in the unemployment rate and the quarterly growth rate of GDP. The assumption of linearity and continuity are relaxed. Threshold effects are explored in models with unemployment specified as the dependent variable and GDP specified as the independent variable in the Okun’s original specification as well as the models proposed by Viren, Harris and Silverstone, Silvapulle, and Lin. In the models specified by Courtney, Prachowny, and Lee, this relationship is reversed and the models are estimated with GDP as the dependent variable and unemployment as the independent variable.

It is worth a brief diversion into a linear method to understand the interaction between unemployment and GDP. For this I use the Geweke Procedure for frequency decomposition of a VAR model. The basic goal of the Geweke procedure is to decompose a VAR model in the frequency domain to ascertain how two series are related and at what frequency this relation is strongest. (Geweke, 1982) The linear feedback could be from unemployment to GDP, from GDP to unemployment, or instantaneously between unemployment and GDP. The feedback may occur at low frequency, middle frequency, high frequency or there may be no feedback at all. (Stokes, 1997)
I used the statistical package B34S to run the VAR model with Geweke procedure on the quarterly difference in the unemployment rate (UNEMP) and the quarter-on-quarter percent change in GDP (GDP) from 1948-Q2 through 2012-Q2. The Geweke frequency decomposition results are shown in Table X.1.

Inspection of Table X.1 shows that there is an overall mapping of GDP to UNEMP at 10.9%, while in the opposite direction UNEMP maps to GDP at 8% of overall variance. The instantaneous effect of UNEMP and GDP is 37.7%.

It is interesting to note that when breaking variance mappings down by frequency, GDP maps to UNEMP at low frequency with a total of 59.9% at periods 5, 6.667, and 10. On the other hand, UNEMP maps to GDP most strongly at higher frequency with 44% of variance at periods 2 and 2.222.
Table X.1

Geweke Frequency Decomposition of UNEMP and GDP
1948-Q2 through 2012-Q2
Lags = 8, Replications = 1000

<table>
<thead>
<tr>
<th>F ( UNEMP to GDP )</th>
<th>Adjusted Estimate</th>
<th>Adjusted %</th>
<th>25%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.083</td>
<td>8.0%</td>
<td>0.065</td>
<td>0.101</td>
</tr>
<tr>
<td>F ( GDP to UNEMP )</td>
<td>0.115</td>
<td>10.9%</td>
<td>0.092</td>
<td>0.136</td>
</tr>
<tr>
<td>F ( GDP.UNEMP )</td>
<td>0.473</td>
<td>37.7%</td>
<td>0.418</td>
<td>0.524</td>
</tr>
</tbody>
</table>

\[
f ( \text{UNEMP to GDP })
\]

<table>
<thead>
<tr>
<th>Period</th>
<th>Adjusted Estimate</th>
<th>Adjusted %</th>
<th>25%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.000</td>
<td>0.322</td>
<td>27.6%</td>
<td>0.182</td>
<td>0.419</td>
</tr>
<tr>
<td>2.222</td>
<td>0.179</td>
<td>16.4%</td>
<td>0.103</td>
<td>0.232</td>
</tr>
<tr>
<td>2.500</td>
<td>0.026</td>
<td>2.5%</td>
<td>0.009</td>
<td>0.035</td>
</tr>
<tr>
<td>2.857</td>
<td>0.014</td>
<td>1.4%</td>
<td>0.005</td>
<td>0.020</td>
</tr>
<tr>
<td>3.333</td>
<td>0.011</td>
<td>1.1%</td>
<td>0.004</td>
<td>0.015</td>
</tr>
<tr>
<td>4.000</td>
<td>0.025</td>
<td>2.5%</td>
<td>0.011</td>
<td>0.034</td>
</tr>
<tr>
<td>5.000</td>
<td>0.157</td>
<td>14.5%</td>
<td>0.090</td>
<td>0.210</td>
</tr>
<tr>
<td>6.667</td>
<td>0.115</td>
<td>10.8%</td>
<td>0.062</td>
<td>0.152</td>
</tr>
<tr>
<td>10.000</td>
<td>0.040</td>
<td>3.9%</td>
<td>0.021</td>
<td>0.055</td>
</tr>
<tr>
<td>20.000</td>
<td>0.073</td>
<td>7.0%</td>
<td>0.035</td>
<td>0.100</td>
</tr>
<tr>
<td>Infinite</td>
<td>0.055</td>
<td>5.4%</td>
<td>0.011</td>
<td>0.080</td>
</tr>
</tbody>
</table>

\[
f ( \text{GDP to UNEMP })
\]

<table>
<thead>
<tr>
<th>Period</th>
<th>Adjusted Estimate</th>
<th>Adjusted %</th>
<th>25%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.000</td>
<td>0.000</td>
<td>0.0%</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2.222</td>
<td>0.024</td>
<td>2.4%</td>
<td>0.010</td>
<td>0.033</td>
</tr>
<tr>
<td>2.500</td>
<td>0.076</td>
<td>7.3%</td>
<td>0.041</td>
<td>0.101</td>
</tr>
<tr>
<td>2.857</td>
<td>0.055</td>
<td>5.4%</td>
<td>0.023</td>
<td>0.077</td>
</tr>
<tr>
<td>3.333</td>
<td>0.007</td>
<td>0.6%</td>
<td>0.002</td>
<td>0.009</td>
</tr>
<tr>
<td>4.000</td>
<td>0.097</td>
<td>9.3%</td>
<td>0.049</td>
<td>0.133</td>
</tr>
<tr>
<td>5.000</td>
<td>0.258</td>
<td>22.7%</td>
<td>0.169</td>
<td>0.330</td>
</tr>
<tr>
<td>6.667</td>
<td>0.240</td>
<td>21.3%</td>
<td>0.154</td>
<td>0.308</td>
</tr>
<tr>
<td>10.000</td>
<td>0.173</td>
<td>15.9%</td>
<td>0.107</td>
<td>0.220</td>
</tr>
<tr>
<td>20.000</td>
<td>0.150</td>
<td>13.9%</td>
<td>0.091</td>
<td>0.190</td>
</tr>
<tr>
<td>Infinite</td>
<td>0.211</td>
<td>19.0%</td>
<td>0.080</td>
<td>0.295</td>
</tr>
</tbody>
</table>
XI. TESTING THE HYPOTHESIS OF STRUCTURAL BREAK IN THE MARS ESTIMATION

A. Defining the Pre and Post structural break periods

Lee found limited evidence of asymmetry in Okun’s Law. (Lee, 2000) A main finding in Lee’s work is evidence of a structural break in the coefficients of Okun’s Law. A structural break implies that the estimated coefficients of the linear model change over time, a violation of the classical assumptions of the OLS model. If the point in time of the break is known, the Chow test is an appropriate and powerful test to employ. Stokes writes that if the point of the break is unknown, the Quandt Likelihood function, a test based on the recursive residuals method, can be used to locate the suspected break. (Stokes, 1997) Lee uses the Quandt Likelihood function in order to identify a structural break in the coefficients in Okun’s Law. Lee identified differing dates at which the structural break occurred depending upon which of the Okun models, as well as, which form of filter was used to generate the cyclical component of GDP and unemployment. Lee focused on the break occurring in 1973, but had evidence, particularly when the Kalman filter was used, of a structural break as late as 1980.

Triggered by an OPEC embargo, the United States experienced its first oil shock in 1973. This oil shock led to a highly
volatile period of inflation as measured by the US Consumer Price Index (CPI). A second oil shock occurred in 1979 following the Iranian revolution and US embassy hostage crisis.

Paul Volker, as chairman of the Federal Reserve, waged a battle against inflation in the early 1980’s. To lower inflation, Volker aggressively raised short term interest rates through open market operations. After rising to an annual rate of 14.8% in 1980, the CPI finally fell below its five year moving average in the second quarter of 1982. Figure XI.1 plots the monthly level of inflation on an annualized basis.

Figure XI.1

![Graph](image-url)
The fundamental concept of structural break is an attempt to identify a point in time where a relationship changes such that the estimated coefficients significantly differ before and after the breakpoint. In order to test the hypothesis of structural break for the MARS estimation, I estimate the model on data from 1948-Q1 up to 1973-Q2. I then separately estimate the model on the data beginning in 1982-Q1 through 2012-Q2. Identifying a structural break within a MARS estimation is not as straightforward as with an OLS model due to the ever present possibility that the final functional form based on the included basis functions could change.
The results of the PRE and POST break estimations are shown below, with Figures XI.2 and XI.3 displaying the contribution charts of the respective estimations.

**Estimation Period: PRE Oil Shock, 1948-Q1 through 1973-Q2**

**OLS**
\[
\text{UNEMP}_t = 0.2894 - 0.2783 \times \text{GDP}_t
\]
\[
(0.0265) \quad (0.0296)
\]
RSS = 9.7560 \quad (XI.1)

**MARS**
\[
\text{UNEMP}_t = 0.2267 + 0.4821 \times \max(-0.2387 - \text{GDP}_t, 0) - 0.2605 \times \max(\text{GDP}_t - 0.1555, 0)
\]
\[
(0.0964) \quad (0.0436) \quad (0.0350) \quad ((\text{Newey West SEs}))
\]
# Non-Zero 101 \quad 13 \quad 80
RSS = 9.5439 \quad (XI.2)

**Estimation Period: POST Oil Shock, 1982-Q1 through 2012-Q2**

**OLS**
\[
\text{UNEMP}_t = 0.2153 - 0.3170 \times \text{GDP}_t
\]
\[
(0.0313) \quad (0.0471)
\]
RSS = 6.8176 \quad (XI.3)

**MARS**
\[
\text{UNEMP}_t = 0.0305 + 0.3818 \times \max(0.5167 - \text{GDP}_t, 0) - 0.2156 \times \max(\text{GDP}_t - 0.3598, 0)
\]
\[
(0.0535) \quad (0.0577) \quad (0.0514) \quad ((\text{Newey West SEs}))
\]
# Non-Zero 122 \quad 41 \quad 94
RSS = 6.5793 \quad (XI.4)
Figure XI.2

MARS estimation of Okun’s Law 1948 through 1973
Pre Oil Shock

Unemployment rate change in Percentage Points
QonQ

-3 -2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 2

-3 -2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 2 2.5 3 3.5 4

GDP percentage change Quarter on Quarter

MARS
OLS

Figure XI.3

MARS estimation of Okun’s Law 1982 through 2012
Post Oil Shock

Unemployment rate change in Percentage Points
QonQ

-3 -2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 2

-3 -2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 2 2.5 3 3.5 4

GDP percentage change Quarter on Quarter

MARS
OLS

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The contribution charts may best show the striking similarity between the PRE and POST MARS estimations. Both consist of two basis functions and an intercept with the coefficients and the thresholds changing. The next section considers methods for identifying a structural break in the context of a MARS estimation.

B. **Hypothesis testing for structural break in the MARS estimation**

In order to test the hypothesis of structural break in the MARS estimation, one approach is to test for the equivalence of the coefficients on the lower basis function in the PRE and POST models as well as equivalence in the coefficients in the upper basis function in the PRE and POST models. If the null of equivalence of the $\beta$’s is not reject in both cases, then the prior findings of a structural break become questionable. For this I employ the Z test for coefficient equivalence as in Paternoster (Paternoster et al., 1998).

\[
Z = \frac{\beta_1 - \beta_2}{\sqrt{SE_{\beta_1}^2 + SE_{\beta_2}^2}}
\]  

(XI.5)
Here we are reminded of the Pre and Post MARS estimations.

**MARS, PRE 1973**
\[
\text{UNEMP}_t = 0.2267 + 0.4821 \times \max(-0.2387 - \text{GDP}_t, 0) - 0.2605 \times \max(\text{GDP}_t - 0.1555, 0)
\]
\[(0.0964) \quad (0.0350) \]  
(XI.6)

**MARS, POST 1982**
\[
\text{UNEMP}_t = 0.0305 + 0.3818 \times \max(0.5167 - \text{GDP}_t, 0) - 0.2156 \times \max(\text{GDP}_t - 0.3598, 0)
\]
\[(0.0535) \quad (0.0514) \]  
(XI.7)

Table XI.1 shows the results of the Z test on both the lower and upper basis functions in the Pre and Post estimations.

**Table XI.1**

Pre/Post Oil Shock
Testing for Structural Break

<table>
<thead>
<tr>
<th></th>
<th>coefficients below the lower threshold</th>
<th>coefficients above the upper threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta_{1\text{PRE}} ) = -0.4821</td>
<td>( \beta_{2\text{PRE}} ) = -0.2605</td>
</tr>
<tr>
<td>PRE 1973</td>
<td>SE( \beta_{\text{1PRE}} ) = 0.0964</td>
<td>SE( \beta_{\text{2PRE}} ) = 0.0350</td>
</tr>
<tr>
<td></td>
<td>( t \text{-} -4.9990 )</td>
<td>( t \text{-} -7.4412 )</td>
</tr>
<tr>
<td>POST 1982</td>
<td>( \beta_{1\text{POST}} ) = -0.3818</td>
<td>( \beta_{2\text{POST}} ) = -0.2156</td>
</tr>
<tr>
<td></td>
<td>SE( \beta_{\text{1POST}} ) = 0.0535</td>
<td>SE( \beta_{\text{2POST}} ) = 0.0514</td>
</tr>
<tr>
<td></td>
<td>( t \text{-} -7.1410 )</td>
<td>( t \text{-} -4.1966 )</td>
</tr>
<tr>
<td>test coef diff</td>
<td>( Z \text{= -0.9096} )</td>
<td>( Z \text{= -0.7215} )</td>
</tr>
<tr>
<td></td>
<td>( p \text{= 0.637} )</td>
<td>( p \text{= 0.529} )</td>
</tr>
</tbody>
</table>

**H0**: \( \beta_{1\text{PRE}} = \beta_{1\text{POST}} \)
**H1**: reject H0

**H0**: \( \beta_{2\text{PRE}} = \beta_{2\text{POST}} \)
**H1**: reject H0
\( \beta_1 \)'s reflect the estimated value of the coefficient on the lower threshold basis function. \( \beta_2 \)'s reflect the estimated value of the coefficient on the upper threshold basis function before 1973 and after 1982. The \( Z \) tests show that we cannot reject the equivalence of \( \beta_{1\text{PRE}} \) and \( \beta_{1\text{POST}} \) nor can we reject the equivalence \( \beta_{2\text{PRE}} \) and \( \beta_{2\text{POST}} \). Under the MARS specification, the existence of a structural break, in the sense of differing coefficients on the upper and lower threshold areas is called into question.

In the MARS estimation the changes that are apparent are the shifts in the threshold values toward more positive values. This will be discussed further in the interpretation section to follow.

The hypothesis of asymmetry of the relationship between output and unemployment is supported in each of the MARS estimations that have been reported in this paper. The threshold levels may change, but negative changes in GDP elicit a large response in unemployment in every case. This finding is robust to each of the time periods examined as well as across a differing number of allowed maximum knots in the MARS algorithm. As previously mentioned, these results were robust across two econometric packages as well: B34S and R.
XII. A MARS ESTIMATION OF OKUN’S LAW AND THE EFFECT OF THE GREAT MODERATION

Ben Bernanke in a speech to the Eastern Economic Association on February 20, 2004 discussed The Great Moderation. “One of the most striking features of the economic landscape over the past twenty years or so has been a substantial decline in macroeconomic volatility. In a recent article, Olivier Blanchard and John Simon (2001) documented that the variability of quarterly growth in real output (as measured by its standard deviation) has declined by half since the mid-1980’s, while the variability of quarterly inflation has declined by about two thirds.” (Bernanke, 2004)

As a policy making member of the Board of Governors of the Federal Reserve, Governor Bernanke was intently interested in this changing dynamic. “Lower volatility of output tends to imply more stable employment and a reduction in the extent of economic uncertainty confronting households and firms. The reduction in the volatility of output is also closely associated with the fact that recessions have become less frequent and less severe.” Little did he know that within four years he would be the setting the course of monetary policy as the Chairman of the Federal Reserve Board of Governors in the midst of The Great
Recession of 2008, the most severe recession since the Great Depression of the 1930’s.

In the paper by Olivier Blanchard and John Simon, the authors plot a twenty quarter rolling standard deviation of the quarterly output growth of GDP. Blanchard and Simon use data for GDP from 1952 through 2000. Many economists have presented possible explanations for the Great Moderation. Bernanke summarizes these explanations as 1) structural change in technology, business practices and institutions 2) improved macroeconomic policies with a focus on improved monetary policy and 3) good luck. (Bernanke, 2004)

Benjamin M. Friedman comments on the work of Blanchard and Simon to say, “Blanchard and Simon’s basic point is that output has become less volatile, and this is surely true. But there is no way to duck the fact that, in their estimated autoregression, the absence of recessions is very much associated with the absence of large negative shocks.” (Blanchard and Simon, 2001)

To investigate the theory of the Great Moderation, I begin by extending the data set used beginning in 1950 but including the most recent data that contains the Great Recession of 2008 with data through 2014-Q4.

In Figure XII.1, I plot the rolling standard deviation of quarterly output growth. I show three differing rolling
periods: twelve, twenty, and forty quarter rolling standard deviations. 

Inspection of these plots, especially the slow (40 quarterly periods) plot, suggests that The Great Moderation is still intact following the most severe output shock in 79 years.

In order to quantify this visual inspection, I use the kmeans cluster analysis in R. Both the 20 period and 40 period rolling volatilities are separated into two groups in the cluster analysis. The means of the high and low volatility groups are shown in Table XII.1.

**Table XII.1**

The Great Moderation: Cluster Analysis

<table>
<thead>
<tr>
<th>cluster</th>
<th>Description</th>
<th>Mean 20 period vol</th>
<th>Mean 40 period vol</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>High Volatility</td>
<td>1.2110</td>
<td>1.1650</td>
</tr>
<tr>
<td>1</td>
<td>Low Volatility</td>
<td>0.6044</td>
<td>0.5982</td>
</tr>
</tbody>
</table>

Each data point within the 20 period rolling volatility data set is then identified as to which group it belongs. This group membership is then plotted on Figure XII.1 where the high volatility group has a value equal to 2 and the low volatility group has a value equal to 1. The outliers to the theory are the observations from 1964 through 1972 that we may expect to be included in the high volatility group, but instead are included
with the low volatility group. All observations after 1987 are included in the low volatility group, providing quantitative support for the conclusion that the Great Moderation is intact following the Great Recession of 2008.

**Figure XII.1**

Volatility of Output Growth  
1950-Q1 through 2014-Q4

Since improved monetary policy has been identified as a possible explanation of the Great Moderation, in Figure XII.2, I plot the twenty quarter rolling standard deviation of output.
growth and the effective Federal Funds rate. Please note the extended period of near zero interest rates beginning in 2008.

**Figure XII.2**

Volatility of Output Growth and the Effective Federal Funds Rate 1950-Q1 through 2014-Q4

Considering the prominence that the concept of The Great Moderation holds with both economists and policy makers alike, this would be a logical point in time to partition the data and investigate structural change. Rather than exclude all of the
rich data associated with the oil shocks of the 1970’s, I will divide the data set into two parts: before the Great Moderation (1948-Q2 through 1983-Q4) and after the Great Moderation (1984-Q4 through 2012-Q4). Using these before and after data sets, the MARS estimation of quarterly difference in the unemployment rate (UNEMP) versus the quarterly percent change in US Gross Domestic Product (GDP) will be estimated. The results of the MARS estimations are shown below. The contribution charts for the before and after period are shown in Figure XII.3.

**Estimation Period: BEFORE The Great Moderation, 1948-Q2 through 1983-Q4**

OLS
UNEMP<sub>t</sub> = 0.2808 - 0.2806 * GDP<sub>t</sub>  
(0.0225)  
((0.0243))  
((Newey West SEs))

RSS = 14.8639

(MII.1)

MARS
UNEMP<sub>t</sub> = 0.3598 + 0.6208 * MAX( -0.7983 - GDP<sub>t</sub>,0) - 0.2619 * MAX( GDP<sub>t</sub> - 0.3943,0)  
(0.1251)  
((0.0896))  
((Newey West SEs))

# Non-Zero 143 13 122
RSS = 14.4012

(MII.2)

**Estimation Period: After the Great Moderation, 1984-Q1 through 2012-Q2**

OLS
UNEMP<sub>t</sub> = 0.1947 - 0.2932 * GDP<sub>t</sub>  
(0.0337)  
((0.0469))  
((Newey West SEs))

RSS = 5.7143

(MII.3)

MARS
UNEMP<sub>t</sub> = -0.0175 + 0.4414 * MAX( 0.4866 - GDP<sub>t</sub>,0) - 0.1266 * MAX( GDP<sub>t</sub> - 0.4019,0)  
(0.0552)  
((0.0864))  
((Newey West SEs))

# Non-Zero 114 34 86
RSS = 5.1213

(MII.4)
Figure XII.3

MARS estimation of Okun’s Law
Before and After the Great Moderation

MARS estimation of Okun’s Law 1948-Q2 through 1983-Q4
BEFORE the Great Moderation

MARS estimation of Okun’s Law 1984-Q1 through 2012-Q4
AFTER the Great Moderation
Table XII.2

The Great Moderation
Testing for Structural Break

<table>
<thead>
<tr>
<th></th>
<th>coefficients below the lower threshold</th>
<th>coefficients above the upper threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BEFORE 1973</td>
<td>BEFORE 1973</td>
</tr>
<tr>
<td>( \beta_1^{\text{BEFORE}} )</td>
<td>-0.6208</td>
<td>( \beta_2^{\text{BEFORE}} )</td>
</tr>
<tr>
<td>( \text{SE}_1^{\text{BEFORE}} )</td>
<td>0.1252</td>
<td>( \text{SE}_2^{\text{BEFORE}} )</td>
</tr>
<tr>
<td>( t )</td>
<td>-4.96</td>
<td>( t )</td>
</tr>
<tr>
<td>( \text{test coef diff} )</td>
<td>-1.311</td>
<td>( \text{test coef diff} )</td>
</tr>
<tr>
<td>( Z )</td>
<td></td>
<td>( Z )</td>
</tr>
<tr>
<td>( p = )</td>
<td></td>
<td>( p = )</td>
</tr>
</tbody>
</table>

|                      | POST 1982                              | POST 1982                              |
| \( \beta_1^{\text{AFTER}} \) | -0.4414                                | \( \beta_2^{\text{AFTER}} \) | -0.1266 |
| \( \text{SE}_1^{\text{AFTER}} \) | 0.0552                                 | \( \text{SE}_2^{\text{AFTER}} \) | 0.0544 |
| \( t \)              | -8                                     | \( t \)                               | -2.32  |

**H0:** \( \beta_1^{\text{BEFORE}} = \beta_1^{\text{AFTER}} \)

**H1:** reject H0

**H0:** \( \beta_2^{\text{BEFORE}} = \beta_2^{\text{AFTER}} \)

**H1:** reject H0

We can test for structural break before and after the Great Moderation. As in section XI, to test the hypothesis of structural break in the MARS estimation, one approach is to test for the equivalence of the coefficients on the lower basis function in the BEFORE and AFTER estimation periods as well as equivalence in the coefficients in the upper basis function in the BEFORE and AFTER estimation periods. If the null of equivalence of the \( \beta \)'s is not reject in both cases, then the prior findings of a structural break become questionable. For
this I employ the Z test for coefficient equivalence as in Paternoster (Paternoster et al., 1998).

In contrast to section XI, when partitioning the data with respect to the Great Moderation, the null of equivalence in the estimated coefficients on the upper basis function is rejected. This suggests that the absolute magnitude of the response of unemployment for changes in GDP greater than 0.48% per quarter has been reduced by 57%. For changes in GDP greater than 0.48%, the model implies that it would take a 7.89% increase in GDP from one quarter to the next would be associated with a 1 percentage point decrease in the unemployment rate.

In the lower basis function equivalence cannot be rejected at conventional levels with a z score implying rejection at only an 81% confidence level. The standard set forth in this paper for acceptance of a structural break is high.

Consideration for the magnitude of the estimated coefficients is warranted at this point. In the lower basis function in the BEFORE period, the value of the estimated coefficient is -0.6208. In the lower basis function in the AFTER period, the value of the estimated coefficient is -0.4414. Although we cannot formally reject equivalence, these estimates imply that the absolute magnitude of the response of unemployment to GDP has risen dramatically for levels of small or negative GDP growth as compared with Okun’s original
estimate. The lower basis function implies that it would take only a 0.8054% to 2.26% increase in GDP from one quarter to the next to be associated with a 1 percentage point decrease in the unemployment rate.

The most important result related to the MARS estimation partitioned by the Great Moderation, may be the large shift in the threshold level between the lower and upper basis functions. The higher absolute magnitude effect has shifted to much higher levels of positive GDP growth. The AFTER estimation indicates that the effect is strong up to +0.4019% quarterly GDP growth; the effect becomes even stronger between +0.4019% and +0.4866% quarterly GDP growth; and then moderates significantly above +0.4866% quarterly GDP growth.
XIII. INTERPRETATION AND CONCLUSION

The proceeding work shows us first that Okun’s Law is as relevant today as it was in 1962 when originally proposed. The Law is relevant in the sense that new threshold modeling techniques allow us to estimate a more detailed description of the relationship between GDP and unemployment. The MARS estimation strongly supports the asymmetry hypothesis regarding Okun’s Law. Changes in unemployment are more sensitive to a quarter on quarter drop in GDP of 0.2387 in the Pre 1973 period. In the Post 1982 period, this increased sensitivity is seen shifting to a higher level of QonQ GDP changes at the positive 0.5167% level. When partitioning the data according to the Great Moderation, the hypothesis of asymmetry is strongly supported.

Lee held to the finding of structural change. (Lee, 2000) Under a MARS specification, I refute the existence of a structural break in the traditional definitional sense. Once properly specified, we cannot reject the equivalence of the lower and upper basis threshold coefficients between the Pre 1973 and Post 1982 periods.

When the data is partitioned based upon the Great Moderation, equivalence of the estimated coefficients on the upper basis function is rejected in favor of a structural break. Equivalence of the lower basis functions, however, cannot be
reject and by the standard established in this paper, a model wide structural break is rejected.

The MARS estimation allows us to consider other dynamics of the model in this new framework. The changes to consider are the shifting threshold values associated with the basis functions when estimating with differing partitions of the data. The Pre 1973 period illustrates many of the opposing theoretical influences on the output / unemployment relationship. Silvapulle, Moosa, and Silvapulle (Silvapulle et al., 2004) describe these opposing theoretical effects well.

Another explanation for asymmetry that leads to the prediction that unemployment responds more strongly to output growth on the downswing than on the upswing. This explanation is based on the assumption of pessimism on the part of the employers, in the sense that bad news is believed more quickly than good news. Assuming no restrictions on hiring and firing employees, employers respond very quickly by laying off workers when the economy goes into a downturn. On the other hand, when the economy recovers, firms would be reluctant to hire workers for fear of the possibility that the recovery may not last long. In that case, unemployment responds faster to output on the downswing than on the upswing.

However, there are two more explanations that lead to the conclusion that unemployment responds more strongly to output growth on the upswing than on the downswing. In the first explanation, asymmetry is attributed to the rigidity of the labour market as represented by institutional restrictions on the ability
of employers to lay off workers. If there are restrictions on the ability of employers to dismiss but not so much to hire workers, then, when the economy slows down, the response of unemployment would be slow. When the economy recovers, firms would hire more workers, and since institutional restrictions on hiring workers are not strong, the response of unemployment to growth would be fast. The second is based on the observation that firms invest heavily in the training on their staff. If this is the case, then they would be reluctant to lay off employees on the downswing. On the upswing, however, they would hire more employees, since the training process takes time.

[The empirical results show us that] When the crunch comes and profitability is hit hard, the urge to preserve the trained labour force will be relegated to secondary importance, particularly if the employers feel that they can always go back to the labour market when the conditions improve. (Silvapulle et al., 2004)

The MARS estimation for the Pre 1973 period illustrates all of these conflicting theories well. In the zone of no effect that occurs when QonQ changes in GDP are between −0.2387% and +0.1555%, the unemployment rate does not react to changes in GDP. Firms may make idiosyncratic decisions based on their specific circumstances but Pre 1973, Okun’s law was insignificant within this range. When GDP was below −0.2387% firms set all concern about training costs aside and reacted more strongly with their employment decision than they otherwise
would if convinced of a healthy, growing economy with GDP growth above +0.1555.

The Post 1982 MARS estimation provides much material for further exploration. The estimation shows us that although the reaction of unemployment to GDP has not significantly changed, the threshold levels at which this asymmetry is experienced does change rather significantly. These threshold levels move to more positive values. The heightened sensitivity of unemployment to GDP is triggered at +0.5167. This implies annual real growth rates of approximately 2%. Have firms become more conservative in their employment decisions as soon as growth falls below what many consider to be normal levels? How has the reduction of the percentage of unionized labor within the overall labor force, and the increase in temporary workers, as well as, the prevalence of outsourcing effected these thresholds? Has the increase in technology and its effects on the just-in-time supply chain and inventory methods allowed firms to make more efficient hiring decisions?

Fed Chair Yellen and the other Federal Reserve Board of Governors are tasked with the dual mandate of maintaining the growth of GDP at potential in order to maximize employment while maintaining stable prices. In the recovery to the Great Recession of 2008, the unemployment rate has fallen more rapidly than would be expected given the growth of GDP actually observed
and Okun’s original coefficient. As Fed officials consider the timing and the pace at which to raise the Federal Funds rate from its current near zero lower bound, they would be well served to investigate the MARS estimation of Okun’s Law. When the data is partitioned by the Great Moderation, the estimation suggests that unemployment can fall 1% with GDP growth as low as 1.61%. The estimation of the relationship in the period after the Great Moderation suggest that this higher sensitivity of the change in unemployment to the quarterly percentage change in GDP applies for annual levels of GDP growth at 1.946%.

The MARS estimation confirms the importance of a tested and robust relationship within macroeconomics that became “Law”. MARS estimation of Okun’s Law opens many avenues for further investigation into the underlying fundamentals that drive the changing threshold values of the output / unemployment relationship.
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