Experimental percolation studies of random networks
A. Feinerman and J. Weddell

Citation: Review of Scientific Instruments 88, 065114 (2017); doi: 10.1063/1.4989518
View online: http://dx.doi.org/10.1063/1.4989518
View Table of Contents: http://aip.scitation.org/toc/rsi/88/6
Published by the American Institute of Physics

Articles you may be interested in
Plasma fireball: A unique tool to fabricate patterned nanodots
Review of Scientific Instruments 88, 063507 (2017); 10.1063/1.4989701

Developments on a SEM-based X-ray tomography system: Stabilization scheme and performance evaluation
Review of Scientific Instruments 88, 063706 (2017); 10.1063/1.4989406

A novel sensor for two-degree-of-freedom motion measurement of linear nanopositioning stage using knife edge displacement sensing technique
Review of Scientific Instruments 88, 065110 (2017); 10.1063/1.4989517

Note: Fabrication of roughened tips for liquid metal ion sources
Review of Scientific Instruments 88, 066107 (2017); 10.1063/1.4985635

Optical derotator alignment using image-processing algorithm for tracking laser vibrometer measurements of rotating objects
Review of Scientific Instruments 88, 065111 (2017); 10.1063/1.4984125

Estimating two-point statistics from derivatives of a signal containing noise: Application to auto-correlation functions of turbulent Lagrangian tracks
Review of Scientific Instruments 88, 065113 (2017); 10.1063/1.4986467
Experimental percolation studies of random networks

A. Feinerman\(^{1,*}\) and J. Weddell\(^{2,\dagger}\)

\(^{1}\)Department of Electrical and Computer Engineering, University of Illinois at Chicago, Chicago, Illinois 60607, USA
\(^{2}\)Department of Bioengineering, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA

(Received 9 December 2016; accepted 9 June 2017; published online 28 June 2017)

This report establishes an experimental method of studying electrically percolating networks at a higher resolution than previously implemented. This method measures the current across a conductive sheet as a function of time as elliptical pores are cut into the sheet. This is done utilizing a Universal Laser System X2-600 100 W CO\(_2\) laser system with a 76 × 46 cm\(^2\) field and 394 dpc (dots/cm) resolution. This laser can cut a random system of elliptical pores into a conductive sheet with a potential voltage applied across it and measures the current versus time. This allows for experimental verification of a percolation threshold as a function of the ellipse’s aspect ratio (minor/major diameter). We show that as an ellipse’s aspect ratio approaches zero, the percolation threshold approaches one. The benefit of this method is that it can experimentally measure the effect of removing small pores, as well as pores with complex geometries, such as an asterisk from a conductive sheet. Published by AIP Publishing.

[http://dx.doi.org/10.1063/1.4989518]

I. INTRODUCTION

Electrical percolation in two dimensions has been numerically studied using theoretical probability models.\(^1\)\(^-\)\(^7\) While past research has shown how percolation can be studied experimentally,\(^8\) no method exists to make precise and arbitrary pores in the sheet. The method established in this report allows percolation to be simulated with control of the pore shape, size, location, orientation (random or otherwise), and number. We have studied elliptical pores cut into aluminum coated Mylar film. We measured how the percolation threshold was affected by the aspect ratio of the ellipse (minor/major diameter) while all other parameters are held constant with random orientation and distribution.

One of the biggest challenges in performing experimental percolation studies is the process of creating a random system of pores. In the past, some researchers would attempt to create a random system of circular porous cuts in a two-dimensional sheet by simply using a hand hole-puncher.\(^9\),\(^10\) While this method does allow for crude observations of how percolation affects electrical conductance, it does not accurately emulate a random network, and it is an improbable method for analyzing systems with hundreds to potentially hundreds of thousands of pores. The method established in this report aims to rectify these issues. We also compare our data to theoretically predicted results.\(^11\)

II. METHODS

Matlab randomizes the location, orientation, and removal order of all pores within the “percolation area” and stores this information into a script file that AutoCAD would then implement. AutoCAD communicates with a Universal Laser System X2-600, which sequentially cuts the pattern into the desired material. This is a 100 W CO\(_2\) laser system with a 76 × 46 cm\(^2\) field and 394 dpc (dots/cm) resolution. A 2 µm thick Mylar film that has been coated with an approximately 15 nm thick aluminum film was used as the material of study. The thickness of the aluminum film is estimated using Eq. (1) and the sheet resistance (R\(_{\text{Al}}\)) which is the resistance across one square of film.\(^12\) This estimate uses a bulk value for the aluminum resistivity (\(\rho_{\text{Al}}\)) of 2.7 µ\(\Omega\) cm,

\[
R_{\text{Al}} \approx \frac{\rho_{\text{Al}}}{t_{\text{Al}}} \cdot \frac{\text{length}}{\text{width}} = \frac{2.7 \, \mu\Omega \, \text{cm}}{15 \, \text{nm}} = 1.8 \, \Omega \, \text{square}^{-1}. \tag{1}
\]

This film is held on to a 0.64 cm thick aluminum plate by a sheet of double sided tape. A 100 mV potential was applied across the left and right ends of the defined rectangle sheet between electrodes attached to the aluminum coated Mylar film. This creates a current that can be measured, allowing the resistance to be determined. Elliptical pores would then be cut by the laser into the aluminum coating, while the current is measured as a function of time.

The area used in this study was a 6.9 × 7.1 cm\(^2\) rectangle. This region is then defined with a matrix of approximately 242 × 248 nodes. As shown in Fig. 1 on either side of the “percolation area” are unbroken 0.64 × 7.1 cm\(^2\) “contact areas” that smooth out any current variations between electrodes and the “percolation area.” The program will allow ellipses to be partially outside of the “percolation area,” but it clips off any portion of an ellipse that would enter the “contact area.” Ellipses will continue to be drawn until no spanning cluster is left, when no chain of adjacent nodes stretches from the left side to the right side of the “percolation area.” The program uses a Hoshen-Kopelman algorithm\(^13\) to evaluate if a cluster spans the “percolation area.”\(^14\)

In order to display the remaining area as ellipses are being removed, the computer program creates a second 6.9 × 7.1 cm\(^2\)
FIG. 1. Example of a random system of elliptical pores created by Matlab and displayed by AutoCAD. The aluminum/plastic film is between the dashed black lines and held on to an aluminum plate with double sided tape. Electrical contact to the film is made by pairs of 0.16 cm diameter brass rods (orange) clamped against the film with acrylic plates and nylon screws. Adjacent to the brass rods are contact regions to smooth out any contact point anomalies between the brass rods and the aluminum film. The magenta lines define the portions of the aluminum/plastic film that will be measured. The left sheet shows the “percolation area” and the right sheet displays the equivalent “remaining area” as it is reduced from the original “percolation area” by the ellipses. The red lines correspond to removed area, and the remaining white space is the final sheet area remaining. As ellipses are removed, the “remaining area” displayed is decreased whenever it has changed by 1% or more from last update. The cyan indicates the largest remaining continuous portion of the aluminum/plastic film.

Another 100 mV potential is applied across this second sheet so the current across it can also be measured. Thus, the two separate currents across the left and right sheets are measured simultaneously. The electrical measurements were made every 0.1 s by two Keithley 236 source measurement units (SMUs) using Metrics 2.1 software. The percolation threshold is determined when the current through the “percolation area” becomes “zero” after the last conductive path is removed by the laser. The SMUs will auto-range and “zero” is defined when no current is recorded even on the lowest SMU scale. The remaining area fraction is determined at this point by the ratio of final to initial current in the “remaining area.”

One of the difficulties incurred within this study was devising an experimental setup that had good electrical contacts between the electrodes and the conductive film. As the aluminum coated Mylar film is only 2 µm thick, electrodes had to be connected in a manner that would not wrinkle, or break the continuity of the film. We were able to obtain good electrical contact to the Mylar film using the experimental setup shown in Fig. 1.

First, double side adhesive tape is placed on a 20 cm square and 0.64 cm thick aluminum base plate. The base plate provides mechanical support for the film and electrodes. The Mylar film is stretched horizontally and placed on the tape with the ~15 nm thick aluminum coating facing up. The 100 W laser will cut through the film as well as the adhesive tape, but not the aluminum base plate. The magenta lines in Fig. 1 were laser cuts to define measurement regions in the Mylar film which was located between the dashed black lines. Three pairs of 0.16 cm diameter brass rods are placed vertically across the Mylar film and 0.32 cm apart, at the edges and center of the base plate. Acrylic pieces were cut and had grooves machined into them to match the brass rod diameter, which acted to clamp the brass rods against the Mylar film. Nylon screws secure the acrylic clamps to the base plate and prevented the base plate from shorting out the aluminum film. Alligator clips were connected to brass rods to perform electrical measurements.

III. RESULTS

Figure 2 shows a typical graph obtained from an experiment. The current through the percolation sheet follows a linear trend until it gets close to the percolation threshold, which
FIG. 3. Experimental results compared to theoretical results obtained by Xia and Thorpe. Each trial was performed so that there were 1000 ellipses, each with a single aspect ratio, for the experimental data for ten ellipse aspect ratios. Each aspect ratio was repeated three times.

TABLE I. Theoretical percolation threshold compared to the experimental percolation threshold of ellipses with different aspect ratios. The theoretical percolation thresholds are from computer simulations done by Xia and Thorpe. The experimental measurements are represented as mean ± standard deviation from three experimental runs.

<table>
<thead>
<tr>
<th>Aspect ratio</th>
<th>Theoretical threshold</th>
<th>Experimental threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000</td>
<td>0.33</td>
<td>0.35 ± 0.0035</td>
</tr>
<tr>
<td>0.7000</td>
<td>0.34</td>
<td>0.39 ± 0.0108</td>
</tr>
<tr>
<td>0.6000</td>
<td>0.35</td>
<td>0.41 ± 0.0147</td>
</tr>
<tr>
<td>0.4000</td>
<td>0.41</td>
<td>0.51 ± 0.0230</td>
</tr>
<tr>
<td>0.2500</td>
<td>0.50</td>
<td>0.59 ± 0.0314</td>
</tr>
<tr>
<td>0.1500</td>
<td>0.62</td>
<td>0.62 ± 0.0420</td>
</tr>
<tr>
<td>0.1000</td>
<td>0.70</td>
<td>0.72 ± 0.0503</td>
</tr>
<tr>
<td>0.0500</td>
<td>0.83</td>
<td>0.86 ± 0.0721</td>
</tr>
<tr>
<td>0.0250</td>
<td>0.91</td>
<td>0.91 ± 0.0815</td>
</tr>
<tr>
<td>0.0125</td>
<td>0.95</td>
<td>0.92 ± 0.0918</td>
</tr>
</tbody>
</table>

We compared our experimental results to the theoretical data of Xia and Thorpe. Their simulations were performed by creating a random network of 2000 ellipses, all with the same aspect ratio, in an approximately square sheet and calculating the percolation threshold. We used a random network of 1000 ellipses, all with the same aspect ratio in a nearly square sheet, ensuring that the percolation threshold depends only on the ellipse aspect ratio. Figure 3 shows a graphical representation of this comparison. Table I and Fig. 3 compare theoretical percolation threshold to experimental measurements after three runs at each aspect ratio.

A benefit of this experimental procedure is that the laser can cut virtually any shape desired. This allows percolation studies to be easily performed using pores with complex geometries that would be extremely difficult to determine theoretical percolation thresholds. We demonstrate this using asterisk shaped pores as shown in Fig. 4.

We determined the percolation threshold using asterisk shaped pores at three different aspect ratios and compared them to ellipse percolation thresholds at the same aspect ratio. Figure 5 shows this comparison graph of the asterisk percolation threshold to ellipse percolation threshold.

IV. DISCUSSION

As shown in Fig. 3 as the ellipse aspect ratio increases towards one, the percolation threshold decreases; more area

FIG. 4. (Left) Example of asterisk shaped pores to be used in experimental percolation studies. The percolation threshold of this system can be easily determined using experimental methods, but is extremely difficult to find using theoretical means. (Right) Picture of an asterisk cut into the aluminum coated Mylar with the laser at the edge of the defined percolation region. The asterisk is composed of four ellipses, each rotated 45°. The major ellipse diameter is 0.589 cm and the minor diameter is 0.147 cm and the laser kerf (width of the cut) on the majority of the elliptical path is ~0.034 cm.

FIG. 5. Comparison between percolation thresholds for asterisk pores (diamonds) and ellipse pores (squares) at identical aspect ratios. The error bars represent the standard deviation after three experiments with the same aspect ratio.
needs to be removed from the sheet before the current becomes zero. These results make sense physically; as the ellipse aspect ratio approaches zero, the ellipse becomes a straight line. A straight line cut into the film still prevents current from flowing, but takes up very little area. This is why at lower aspect ratios the percolation threshold is reached after removing less area.

The laser kerf (width of the cut) was initially estimated as 50 µm since we were using the laser’s HPDFOTM (High Power Density Focusing Optics) on a 5 cm focal length lens with a reported 25 µm diameter spot size.17 The Matlab program uses the kerf to calculate the area remaining in the “percolation area.” The kerf was bracketed with the following procedure. A voltage potential is applied across the Mylar sheet with a height H of 7.1 cm and a width W of 6.9 cm, and then horizontal lines are cut parallel to the flow of current. In each iteration, there are 2⁻ⁿ⁻¹ additional parallel lines cut across the conduction area subdividing the sheet from 2⁻ⁿ⁻¹ into 2ⁿ rectangles. The admittance of the conducting area is given by Eq. (2) for s ≥ 1, where Rsq is the resistance/square,

\[ G_s = \frac{H - (2^s - 1) \times \text{Kerf}}{R_{sq} \times W}. \]  

When the admittance goes to zero after dividing the sheet into 2ⁿ bins the kerf is bracketed between two values shown as follows:

\[ \frac{H}{2^{n-1}} > \text{Kerf} > \frac{H}{2^n}. \]  

The admittance went to zero after the 9th iteration, when H was 7.1 cm, putting the kerf between 280 and 140 µm.

This difference between the assumed and actual kerf leads to an error in measured percolation thresholds, where the error depends on the aspect ratio. Table I shows the experimental threshold compared to the theoretical threshold for several aspect ratios when the experiment assumed that the kerf was 180 µm. We conclude that this assumption is valid, as the experimental and theoretical results are in good agreement.

Figure 5 shows that the percolation thresholds obtained from asterisk shaped pores are very similar to those obtained from elliptical pores. This is expected, as an asterisk is comprised of several ellipses all with the same center but offset at different angles. This shows how the experimental method will give accurate results even for pores with a more complex geometry.

V. CONCLUSION

We have demonstrated a novel method for experimentally determining a system’s percolation threshold by cutting pores into a conductive sheet while measuring the current flow across the sheet. That the experimentally determined percolation thresholds are in good agreement with theoretical percolation thresholds for elliptical pores with different aspect ratios. This experimental procedure also allows for pores with complex geometries to be studied, including geometries that would be extremely difficult to conduct theoretical studies on. We have demonstrated this using asterisk shaped pores and shown how the percolation threshold of these asterisk pores is comparable to ellipse percolation thresholds of comparable aspect ratios.

It is also concluded that there is an inverse relationship between the aspect ratio of an ellipse and the percolation threshold. Using the information obtained in this study, future work can be performed using different pore shapes or on different conductive materials to determine different transport properties of interest.

ACKNOWLEDGMENTS

This research was made possible by funding from the National Science Foundation under EEC-NSF Grant No. 1062943.