



Bilingualism and symbolic abstraction: Implications for algebra learning

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ABSTRACT

Much of the research on bilingualism and math learning focuses on the potential challenges that bilinguals and language learners may face. The current line of research took an alternative approach and explored whether a bilingual advantage may emerge for a novel algebraic problem solving task that requires symbolic thought, the Symbol Math task. No differences were seen between bilingual and monolingual samples on basic math or executive control tasks; however, a bilingual advantage was seen in performance on the Symbol Math task across two experiments. The results suggest that bilingualism may improve the ability to engage in more abstract or symbolic thought processes, which may have important implications for algebra learning.

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1. Introduction

1.1. Algebra and symbolic abstraction

Although algebra is foundational for higher-level mathematics and considered to be a gatekeeper subject for careers in STEM fields, the shift from arithmetic to algebra is notoriously challenging for students (Herscovics & Linchevski, 1994; Humberstone & Reeve, 2008). Algebraic understanding requires moving beyond calculation of exact values to consideration of relationships among quantities and operations involving unknown values and variables. Thus, symbolic abstraction is an important component of algebraic understanding (Arcavi, 2005). One particularly important algebraic topic is functions, and many researchers have advocated teaching other algebraic topics, such as solving equations and manipulating expressions, within the context of functions (Chazan & Yerushalmy, 2003; Kieran, 2007). Students typically treat functions as recipes for obtaining an answer, and struggle with understanding functions as expressing a relationship between variables (Kalchman & Koedinger, 2005). The present research tests whether bilinguals demonstrate a performance advantage on a task that presents algebraic functions in a novel way, as part of a Symbol Math task.

1.2. Bilingualism and symbolic abstraction

Research on bilingualism and math learning has focused on the ways that bilinguals and language learners can be disadvantaged in traditional academic environments (Campbell, Davis, & Adams,

2007). For instance, certain features of items found on common mathematical assessments, such as the number of words in an item and grammatical features common to academic language, are associated with differential item functioning (DIF) with a bias against language learners (Haag, Heppt, Stanat, Kuhl, & Pant, 2013). Even in bilingual immersion programs, where learning in multiple languages is encouraged and supported, there may be costs associated with switching languages while learning mathematics. Saalbach, Eckstein, Andri, Hobi, and Grabner (2013) demonstrated that, despite the assumption that mathematics is a language-independent subject, the mismatches between the language of instruction and the language of testing can impact performance on mathematical tasks. High school students enrolled in a bilingual education program were trained to complete subtraction and multiplication problems either in their L1 (German) or L2 (French). Each participant was then tested on trained and untrained problems in both L1 and L2. Saalbach et al. (2013) found a switching effect (lower accuracy and higher response time) when the testing language differed from the training language, and this effect was greater when participants were trained in L1 and tested in L2. These results suggest that mathematics performance can be language dependent, and that educators in bilingual immersion programs should be mindful of the potential costs associated with teaching and testing in different languages.

Although understanding the ways that bilinguals can be disadvantaged in traditional learning environments is an important endeavor, a full understanding of the bilingual experience should consider not only the potential costs but also potential benefits that come with the bilingual experience with the goal of understanding how these costs and benefits are related (Cummins, 1976; Kempert, Saalbach, & Hardy, 2011). Planas (2014) argues that, contrary to viewing bilingualism as a disadvantage in math learning, bilingualism can actually create opportunities for learners to engage more deeply with mathe-

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mathematical concepts. Planas (2014) observed a small sample of Catalan language learners interacting with native Catalan speakers while solving algebra problems in groups. Because the Catalan learners lacked specific mathematical terminology to describe the problems, they attempted different problem solving strategies (e.g., using a geometric approach to understand an algebraic expression). The language learners also focused more on the meaning of mathematical terms than their native-speaker group members did because they were unfamiliar with the requisite terminology.

Recently, more research has taken the approach of exploring potential cognitive benefits associated with bilingualism. Several theories have been developed that are consistent with the idea that prolonged experience managing more than one language may place unique demands on bilingual minds, resulting in cognitive advantages (Adesope, Lavin, Thompson, & Ungerleider, 2010). The exact nature of these advantages, however, has been difficult to pin down. Some researchers have explored the idea that bilinguals' need to suppress competition from one language when using another may lead to improved inhibitory control (Bialystok, Craik, & Luk, 2012), more efficient allocation of executive control resources in the face of conflict, or an advantage in overall response time on tasks that feature competition (Costa, Hernández, Costa-Faidella, & Sebastián-Gallés, 2009; Hilchey & Klein, 2011). However, others have failed to find evidence of a bilingual advantage in executive control (de Bruin, Treccani, & Della Sala, 2015; Paap & Greenberg, 2013; von Bastian et al., 2016).

Despite increasing evidence that bilingual young adults do not outperform their monolingual counterparts on executive control tasks, it is still possible that bilingual experiences could confer other cognitive benefits. Recent research has demonstrated that bilinguals may learn novel rules more efficiently than monolinguals (Stocco & Prat, 2014). The bilingual experience may also impact the development of metalinguistic awareness (Adesope et al., 2010; Bialystok, 1997; Galambos & Hakuta, 1988), which could have implications for algebra learning (MacGregor & Price, 1999). Even Vygotsky believed that bilingualism could have positive consequences on the flexibility and sophistication of human thought (Cummins, 1976; Vygotsky, 1962). He argued that being able to express the same thought in different languages enables one to see that any particular language is just one system among many, to separate labels from their referents, to understand the symbolic function of words, and to view words in more abstract, semantic, and general terms. The present research takes up this suggestion and builds on the intuition that bilingualism may potentiate the ability to engage in more abstract or symbolic thought processes, which play a crucial role in algebra learning.

1.3. The present study

In order to test for a bilingual advantage in symbolic abstraction, we developed a new experimental task: the Symbol Math task. In the Symbol Math task algebraic functions are presented in an unfamiliar way using a novel symbol to represent a particular sequence of basic mathematical operations. Task items are of the form:

$$x\Theta y = xy + x - y \quad \text{What is } 5\Theta 3?$$

The task requires symbolic abstraction because participants must understand that the novel symbol denotes a relationship between variables—a relationship defined by a set of mathematical operations. Further, participants must manipulate letters representing unknown quantities in order to obtain a solution for several items, and several items require using the output of one function as the input for another function. This task was developed to test for symbolic abstraction in a way that does not depend on previous experience with algebraic func-

tions. The present experiments sought to address the question of whether bilinguals would outperform monolinguals on the Symbol Math task. If bilingualism confers specific advantages in symbolic abstraction, a bilingual advantage may emerge in performance on the Symbol Math task.

2. Experiment 1

In addition to the Symbol Math task, a basic math task was also included in this experiment in order to control for the impact of general mathematical ability. No differences were expected between monolinguals and bilinguals in basic math performance.

2.1. Method

2.1.1. Participants

Sixty-one undergraduates at an urban college in the northeastern U.S. (40 female) between the ages of 18 and 35 participated in this experiment for course credit in introductory psychology. Participants were classified as bilingual ($N = 29$) if they reported prolonged exposure to more than one language before the age of 7 (with prolonged exposure defined as both parents speaking a language other than English or attending school taught in a language other than English), or monolingual ($N = 32$) if they were native English speakers with no early prolonged exposure to another language. The decision to test for effects using only early bilinguals was based on previous literature exploring cognitive advantages associated with early bilingualism (see Luk, De Sa, & Bialystok, 2011 for a review) as well as prior work in creative problem solving that has identified benefits specifically among early (but not late) bilinguals (Cushen & Wiley, 2011).

For the bilingual sample, 90% reported English as their dominant language, and 31% reported English as the first language acquired (L1). Other dominant languages reported were Chinese (3%), Haitian Creole (3%), and Urdu (3%). Bilingual participants reported 13 different non-dominant languages: Russian (14%), Spanish (14%), Arabic (10%), Haitian Creole (10%), English (10%), Urdu (10%), Bengali (7%), Chinese (7%), French (3%), Hebrew (3%), Hindi (3%), Polish (3%), and Tagalog (3%). Bilingual participants reported using their dominant language 69.48% of the time. Participants indicated their speaking and comprehension proficiency for L1 and second language (L2) on a 0–10 scale (0 = none, 10 = perfect). Bilinguals reported similar speaking proficiency for L1 ($M = 8.34$, $SD = 1.59$) and L2 ($M = 8.69$, $SD = 1.14$), $t < 1$. Bilinguals also reported similar comprehension proficiency for L1 ($M = 8.76$, $SD = 1.27$) and L2 ($M = 8.93$, $SD = 1.07$), $t < 1$.

All monolingual participants reported English as their dominant language. The mean age of reported exposure to a second language was 14.13 ($SD = 2.54$). In contrast to the bilingual participants, monolinguals reported higher speaking proficiency for English ($M = 9.41$, $SD = 0.98$) than for their second language ($M = 2.41$, $SD = 1.76$), $t(31) = 22.74$, $p < 0.01$. Monolinguals also reported higher comprehension proficiency for English ($M = 9.55$, $SD = 0.77$) than for their second language ($M = 2.97$, $SD = 1.89$), $t(30) = 19.72$, $p < 0.01$.

2.1.2. Materials

2.1.2.1. Symbol math task

The task featured a training session with four sample problems of increasing difficulty administered on paper (see Fig. 1). The first sample problem was a worked example. After completing each of the remaining three sample problems, participants were informed if they had correctly answered the problem, or given the correct response.

Sample Problems		
Instructions: Here are some sample symbol problems. The first one is done out for you and the answers are provided for the remaining problems.		
1. $x \ominus y = xy + x - y$	What is $5 \ominus 3$?	$(5)(3) + 5 - 3$ $15 + 5 - 3$ Answer: 17
2. $x \odot y = x/y - x + 4$	What is $8 \odot 2$?	Answer: 0
3. $x \nabla y = 3x - 2y$	What is $5 \nabla (2 \nabla 1)$?	Answer: 7
4. $x \phi y = (3x - 2y) + 2$	What is $(2 \phi 1) \phi 3$?	Answer: 14
Easy Items		
1. $x \otimes y = (x + y) + (x - y)$	What is $5 \otimes 3$?	
2. $x \oplus y = (x + y)(xy)$	What is $2 \oplus 4$?	
3. $x \Theta y = \frac{x+y}{x-y}^2$	What is $4 \Theta 2$?	
4. $x \sim y = x^3 + 2xy - y^2$	What is $2 \sim 5$?	
Medium Items		
5. $x \odot y = (x + y)(xy)$	What is $(1 \odot 2) \odot 1$?	
6. $x \# y = y^2 + xy$	What is $(1 \# 2) \# 3$?	
7. $x \Rightarrow y = xy$	$x \Leftarrow y = x/y$	What is $(3 \Rightarrow 6) \Leftarrow 2$?
8. $\{x\} = x^2/2$	$\llbracket y \rrbracket = y + 3$	What is $\{6\} / \llbracket 6 \rrbracket$?
Hard Items		
9. $\langle x \rangle = 2x - 1$	$3 - \langle x \rangle = x$	What is x ?
10. $x \{y\} = \frac{10x}{y}$	$z \{7\} = 200$	What is z ?
11. $x \$ y = \frac{x - 2y}{y}$	$4 \$ 5 = z \$ 10$	What is z ?
12. $[x] = x^2 - x$	$[y] = [y - 2]$	What is y ?

Fig. 1. Symbol math task items.

Following the sample problems, participants completed 12 problems divided into three difficulty levels (easy, medium, and hard). Difficulty level was determined by the number of novel symbols in the problem as well as the number of operations required to solve the problem. Two examples were given of easy items, which featured a single novel symbol that appeared once in the item. Two examples were also given of medium items, which featured two instances of a symbol in the item. No examples were given of hard items, which required solving for an unknown by using the novel symbols. Problems were presented in order of increasing difficulty, and all participants completed the same problems in the same order. The items were presented one at a time on a computer and participants recorded their answers in an answer booklet. Participants were prompted to do as much work as possible in their heads, but they were provided with paper for calculations. Participants completed the task at their own pace, and received one point for each correct item. Proportion scores were calculated for items at each level of difficulty (Cronbach's $\alpha = 0.62$).

Fig. 1. Although the reliabilities of both the Symbol Math task and the basic math task are somewhat low as computed by Cronbach's α , this may be due to the small number of items and not a lack of reliability per se, as the measures do show reliable correlations with each other ($r = 0.55, p < 0.01$).

2.1.2.2. Basic math task

The Woodcock-Johnson® III Normative Update Tests of Achievement (Woodcock, McGrew, Mather, & Schrank, 2001) subtest 10 (applied problems) was administered using standard procedures. Items ranged in difficulty from basic arithmetic to algebra and geometry, and were often word problems such as: "If 75 pounds of flour lasts a bakery $2 \frac{1}{2}$ days, how much flour is used per day?" Items were presented on a computer and participants were asked to provide their solutions verbally (pencil and paper were provided for calculation). The subtest was designed for administration at different levels of development. It is comprised of 63 items, however standard administration for college-level students begins at item 35. Standard administration procedure includes basal and ceiling criteria. In order to meet the basal criteria, three consecutive items must be answered correctly on the first page (beginning with item 35) in order to proceed to the next page. If the first three items are not answered correctly, the administrator turns to the preceding page (beginning with item 30) and this process continues until the participant has answered the first three questions on a page correctly. Test administration stops after the participant has answered 3 or more consecutive items incorrectly on a page. Standard administration procedure results in different total numbers of problems administered for each participant. Proportion

scores were calculated for each participant based on the number of problems they completed (Cronbach's $\alpha = 0.56$).

2.1.3. Procedure

Once participants gave consent, they were asked to complete a language history questionnaire. Then the participants completed the Basic Math and the Symbol Math tasks.

2.2. Results

There were no significant differences between monolinguals ($M = 0.59$, $SD = 0.10$) and bilinguals ($M = 0.61$, $SD = 0.19$) in proportion scores on the basic math task; $t(59) = 0.40$, *ns*. Monolinguals and bilinguals also did not differ in their total scores or the number of items attempted on the basic math task ($ts < 1$), or in overall completion time for the Symbol Math task, $t < 1$.

Proportion scores for monolinguals and bilinguals for each difficulty level of the Symbol Math items can be found in Fig. 2 (solution rates for all items by language group are included in the Appendix). An analysis of covariance was performed with performance on the basic math task as a covariate in order to determine whether there were differences between monolinguals and bilinguals at each level of difficulty on the Symbol Math task. The analysis revealed a main effect for difficulty, $F(1,58) = 71.4$, $p < .0001$, $\eta_p^2 = 0.55$. No main effect was found for bilingualism, $F < 1$, but there was a significant

interaction, $F(1,58) = 6.83$, $p < .01$, $\eta_p^2 = 0.11$. Follow-up tests for each level of difficulty revealed a bilingualism effect only on the hard items, $F(1,58) = 4.87$, $p = 0.03$, $\eta_p^2 = 0.08$ (see Fig. 3).

2.3. Discussion

In this experiment, a bilingual advantage was observed in performance on the hard items in the Symbol Math task, and this advantage could be due to bilingual participants having an advantage in symbolic abstraction. This empirical finding is similar to another recent study by Kempert et al. (2011) who also found a bilingual advantage particularly on the more difficult problems in their set, which consisted of word problems with distracters.

However, the findings from Experiment 1 could also potentially be explained in terms of group differences in executive control or socioeconomic status (SES). Several researchers have raised the concern that some previously-reported differences between monolinguals and bilinguals might actually have been the result of differences in SES between the samples (Morton & Harper, 2007; Paap & Greenberg, 2013). Since SES is known to be related to a variety of cognitive skills (Sackett, Kuncel, Arneson, Cooper, & Waters, 2009; Von Stumm & Plomin, 2015) it is important to ensure that monolingual and bilingual samples are matched on level of parental income or other important characteristics of the household (such as level of parental education). However, this was not done in Experiment 1, so an alternative explanation based on differences between the samples cannot be ruled out.

Similarly, it is important to show that monolingual and bilingual samples do not differ in executive control. Previous studies have found executive control advantages for bilinguals (Adesope et al., 2010; Bialystok et al., 2012; Costa et al., 2009; Hilchey & Klein, 2011). Because executive control is thought to play a role in succeeding at novel tasks (Miyake et al., 2000; Wiley, Jarosz, Cushen, & Colflesh, 2011), a bilingual advantage in executive control could explain the results obtained in Experiment 1. Since the Symbol Math task required participants to remember the results of previous calculations in order to use these results in subsequent calculations, performance on the task may specifically depend on working memory capacity. To address these possible alternative explanations for the findings in Experiment 1, Experiment 2 included several additional measures.

3. Experiment 2

Experiment 2 was conducted in order to rule out alternative explanations for the bilingual advantage on the hard symbol math items as being due to group differences in executive control or SES. Experiment 2 served as a test of whether the results seen in Experiment 1 would replicate, but also included several measures of executive control: inhibitory control was measured by a Number Stroop task, set shifting was measured by a Plus-Minus task, and working memory capacity was measured by a Letter-Number Sequencing task. Another improvement from Experiment 1 was using only a subset of problems from the Woodcock Johnson Applied Problems subtest. This was changed so that all participants received the exact same problem set, in contrast to leaving this task free to vary in terms of how many and which problems each individual received, which is the case under standard administration. In addition, self-reported measures of SES (years of parental education and parental income levels), student math background, and student math confidence were collected for each participant.

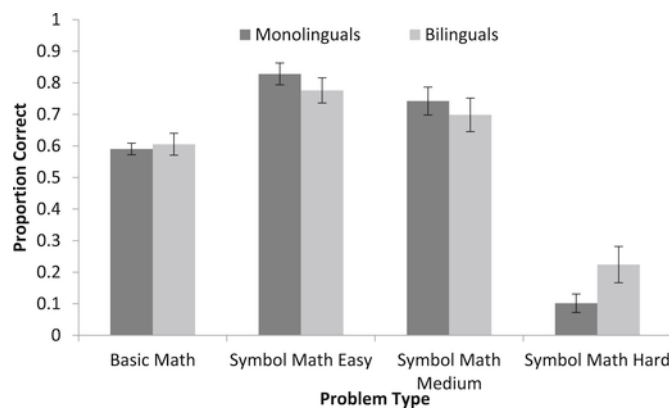


Fig. 2. Performance of Monolinguals and Bilinguals on Basic Math and Symbol Math Problems. Note: Error bars represent standard error.

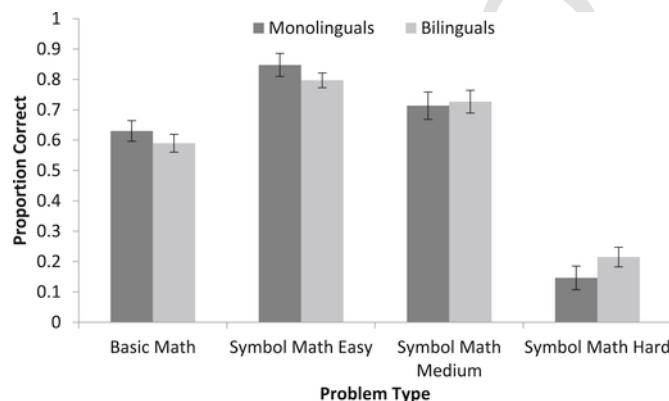


Fig. 3. Performance of Monolinguals and Bilinguals on Basic Math and Symbol Math Problems. Note: Error bars represent standard error.

3.1. Method

3.1.1. Participants

A sample of 105 undergraduates at an urban university in the mid-western U.S. (60 female) between the ages of 18 and 26 participated in this experiment for course credit in introductory psychology. The increase in sample was based on a power analysis using the effect size obtained in Experiment 1. Using the same criteria as Experiment 1, 64 participants were classified as bilingual and 41 were classified as monolingual.

For the bilingual sample, 91% reported English as their dominant language, and 55% reported English as their L1. Other dominant languages included Chinese (3%), Spanish (3%), Albanian (1%), and Telugu (1%). Bilingual participants reported 27 different non-dominant languages: Spanish (23%), English (9%), Arabic (6%), Korean (6%), Tagalog (6%), Polish (5%), Bulgarian (3%), Chinese (3%), Greek (3%), Gujarati (3%), Hindi (3%), Yoruba (3%), Assyrian (2%), Bengali (2%), Dagomba (2%), Esan (2%), French (2%), German (2%), Italian (2%), Laotian (2%), Malaysian (2%), Marathi (2%), Russian (2%), Thai (2%), Ukrainian (2%), Urdu (2%), and Vietnamese (2%). Bilingual participants reported using their dominant language 71.22% of the time. Participants indicated their speaking and comprehension proficiency for L1 and L2 on a 1–7 scale (1 = almost none, 7 = native speaker). In this sample, bilinguals reported higher speaking proficiency for L1 ($M = 6.57$, $SD = 0.64$) than for L2 ($M = 4.84$, $SD = 1.70$), $t(62) = 7.64$, $p < 0.01$. Bilinguals also reported higher comprehension proficiency for L1 ($M = 6.32$, $SD = 0.88$) than for L2 ($M = 5.08$, $SD = 1.57$), $t(62) = 6.05$, $p < 0.01$.

Only 11 monolingual participants reported exposure to a language besides English. The mean age of reported exposure to L2 was 13.15 ($SD = 3.65$). Monolinguals reported higher speaking proficiency for L1 ($M = 7.00$, $SD = 0.00$) than for L2 ($M = 2.73$, $SD = 1.56$), $t(10) = 9.11$, $p < 0.01$. Monolinguals also reported higher comprehension proficiency for L1 ($M = 7.00$, $SD = 0.00$) than for L2 ($M = 2.82$, $SD = 1.54$), $t(10) = 9.02$, $p < 0.01$.

Participants reported each of their parent's level of education using an 8 point scale (1 = less than high school, 2 = high school, 3 = professional training, 4 = some college, 5 = college, 6 = some graduate school, 7 = Masters, 8 = Ph.D./M.D./J.D.). A Mann-Whitney U test revealed no differences between groups for paternal education level, $U = 1219$, $p = 0.78$ (monolingual $M = 4.51$, $SD = 1.52$; bilingual $M = 3.95$, $SD = 1.94$), or maternal education level, $U = 1085$, $p = 0.13$ (monolingual $M = 4.00$, $SD = 1.95$; bilingual $M = 4.13$, $SD = 2.21$). Participants also reported their parental household income level using a 4 point scale (1 = Under \$45,000, 2 = \$45,000–\$50,000, 3 = \$50,000–\$60,000, 4 = Over \$60,000). This resulted in a significant Chi-square, $\chi^2(3, N = 105) = 10.81$, $p < 0.02$ with 57.5% of monolinguals reporting the highest level of income for their parents, but 53.4% of bilinguals reporting parental income under \$50,000. Although the monolingual and bilingual samples differed in parental income level, monolinguals reported higher levels of parental income, which is not consistent with a bilingual advantage in SES. However, of these three SES measures, only paternal level of education predicted performance on the Symbol Math task ($r_s(103) = 0.20$, $p < 0.05$), and the two samples did not differ on this measure. No reports were obtained from parents.

Monolinguals ($M = 1.25$, $SD = 1.32$) and bilinguals ($M = 1.16$, $SD = 1.16$) did not differ in the number of math courses taken since beginning college, $t < 1$, or in self-reported math confidence ratings using a 1–6 scale, $t < 1$ (with higher scores indicating higher confidence; monolinguals: $M = 4.08$, $SD = 1.25$; bilinguals: $M = 3.91$,

$SD = 1.27$). Out of the total sample, 31% of participants (34% of monolinguals and 30% of bilinguals) reported some previous exposure to problems similar to those in the Symbol Math task, however previous exposure was not associated with an advantage in performance on the task, $t < 1$.

3.1.2. Materials

3.1.2.1. Symbol math task

The administration of the Symbol Math task was the same as Experiment 1 and proportion scores were again calculated for each participant (Cronbach's $\alpha = 0.66$). A table showing simple correlations among measures is included in the Appendix. As in Experiment 1, there was a reliable correlation between scores on the two math tests ($r = 0.69$, $p < 0.01$).

3.1.2.2. Basic math task

In order to simplify the administration procedure of the basic math task, and to ensure that all participants completed the same items, we administered a subset of 12 items (items 37, 40, 41, 42, 45, 47, 49, 52, 55, 58, 60, and 63) from the Applied Problems subtest of the Woodcock-Johnson® III Normative Update Tests of Achievement. Items were selected to represent a range of difficulty (based on solution rates obtained from Experiment 1). Four items were selected for three levels of difficulty (easy, medium, and hard). The solution rates for the Applied Problems task were 87% for the easy items, 65% for the medium items, and 21% for the hard items. Cronbach's α for this set of items was 0.77, which indicates acceptable internal consistency. Items were presented on a computer screen, and participants were asked to indicate their solutions using paper and pencil. Participants completed the task at their own pace, and proportion scores were calculated for each participant.

3.1.2.3. Executive control measures

3.1.2.3.1. Inhibitory control

A Number Stroop task based on Hernández, Costa, Fuentes, Vivas, and Sebastián-Gallés (2010) was administered. Stimuli were presented on a computer screen, and participants were instructed to enter the number of items that appeared on the screen as quickly as possible. Possible answers were 1, 2, or 3, and participants indicated their response by pressing the corresponding number key on the numeric key pad using their right hand. Neutral stimuli items (25%) were strings of letters (Z, MM, GGG), congruent stimuli items (25%) featured numerals that corresponded to the number of items on the screen (1, 22, 333), and incongruent items (50%) featured numerals that did not correspond to the number of items on the screen (2, 3, 11, 33, 111, 222). Stimuli items were presented in random order, and all 12 items appeared 7 times during each experimental block. For each trial a fixation cross appeared on the screen for 1000 ms followed by the stimulus which appeared in the center of the screen until the participant entered a response. Stimulus items were presented in black, 18 point, Courier New font. Prior to beginning the task participants completed a practice block of 24 items and received feedback on their performance for each item. The experimenter observed the participant during the practice block and provided verbal instructions if the participant was not answering accurately. Then participants completed two blocks of 84 trials with a break in between. Items were scored as either correct or incorrect, and reaction time data was collected. Two measures of inhibitory control were computed. Following Hernández et al. (2010), interference effects were computed as the difference in reaction time between incongruent and neutral trials.

Following Costa et al. (2009), mean reaction time for all correct items was used as an indication of efficiency on a task that involves competition.

3.1.2.3.2. Working memory updating

The Letter-Number Sequencing subtest from the WAIS-III (Wechsler, 1997) was administered. The task requires participants to listen to a string of numbers and letters and repeat them back in a specific order (numbers first in ascending order followed by letters in alphabetical order). The stimuli increase in difficulty beginning with 2 alphanumeric characters per string and concluding with 8 characters per string. There are 3 items for each string length for a total of 21 items. Stimuli items were read by the experimenter at approximately 1 s per character. Participants received one point for each fully correct response for a maximum possible score of 21.

3.1.2.3.3. Set shifting

A Plus-Minus task based on the one used by Miyake et al. (2000) was adapted for administration on a computer. Participants were shown numbers ranging from 10 to 99 on a computer screen, presented in 3 blocks with an instruction screen preceding each block. Numbers were presented one at a time in the center of the screen, and were in random order with no number repeated during the task. For the first block participants were instructed to add 3 to each number, for the second block they were instructed to subtract 3 from each number, and for the last block they were instructed to alternate between adding and subtracting 3 from the number on the screen. During the alternating block there was a reminder at the top of the screen indicating whether the participant should add or subtract. During each block the number remained on the screen until the participant entered a response using the numeric keypad. Participants were instructed to complete each block as quickly and accurately as possible, and reaction time for each item was recorded. Typically shift cost, the difference between the reaction time for correct trials on the alternating block and the mean of correct trials on the addition and subtraction blocks, is used as a set-shifting measure (Miyake et al., 2000). To parallel the Number Stroop analyses, mean reaction time for all correct items on the alternating block was used as an indication of efficiency on a task that involves set shifting.

3.1.3. Procedure

Once participants gave consent, each participant completed the Basic Math and Symbol Math tasks, followed by the executive control tasks. A final survey assessed participants' language background as in Experiment 1, plus self-reports of parental education, parental income, number of mathematics courses taken in college, and mathematics confidence ratings.

3.2. Results

3.2.1. Data cleaning

For reaction time (RT) data from the Number Stroop and Plus-Minus tasks, means and standard deviations were calculated at both the subject and the task level for correct trials only. Overall accuracy on the Number Stroop task was 95.4% ($SD = 0.03$) and 97.0% ($SD = 0.02$) on the Plus-Minus task. As shown in Table 1, no differences in overall mean accuracy were seen for monolinguals and bilinguals on the Number Stroop task or on the Plus-Minus Task.

For the Number Stroop task, means were calculated separately for neutral, congruent, and incongruent items and for the Plus-Minus task means were calculated separately for the addition, subtraction, and alternating blocks. At the subject level we followed Paap and Greenberg's (2013) upper bound criteria of 2.5 SDs. RTs exceeding

Table 1

Means and standard deviations for all tasks in experiment 2.

Task	Monolinguals	Bilinguals	<i>t</i> -test (df)
	<i>M</i> (<i>SD</i>)	<i>M</i> (<i>SD</i>)	
Number Stroop Accuracy	0.96 (0.03)	0.95 (0.03)	1.43 (100)
Plus-Minus Accuracy	0.98 (0.02)	0.97 (0.03)	0.26 (98)
LNS Score (out of 21)	11.07 (2.28)	10.73 (2.31)	0.74 (103)
Number Stroop Interference Effect (ms)	35.21 (24.65)	44.86 (27.74)	-1.78 (100)
Number Stroop Overall Mean RT (ms)	551.61 (62.94)	577.42 (67.83)	-1.92 [†] (100)
Plus-Minus Shift Cost (ms)	301.20 (362.64)	400.10 (316.07)	-1.46 (98)
Plus-Minus Mean RT Alternating Block (ms)	2934.79 (769.85)	2905.55 (786.08)	0.18 (98)
Basic Math Completion Time (min)	16.24 (5.10)	17.82 (6.87)	-1.27 (102)
Symbol Math Completion Time (min)	10.00 (3.14)	10.53 (4.15)	-0.69 (102)

Note: [†] $p < 0.06$.

2.5 SDs above the mean were replaced with 2.5 SD times the mean (2.7% of trials for numerical Stroop and 3.2% of trials for Plus-Minus). Following the method used by Costa et al. (2009) to analyze Number Stroop RT data, RTs lower than 200 ms were replaced with 200 ms (fewer than 0.01% of trials). There were no correct trials with RTs below 2.5 SD times the mean in the Plus-Minus task. After replacing outliers at the subject level, subjects whose mean RTs exceeded 2.5 SDs above the mean at the task level were eliminated from the analysis. For the Number Stroop task analysis, 2 monolinguals and 1 bilingual were eliminated and for the Plus-Minus task analysis 3 bilinguals and 1 monolingual were eliminated. In addition, one bilingual participant did not complete the entire Plus-Minus task, so data from that participant was not included.

3.2.2. Executive control analyses

The means and standard deviations for performance on the three executive control tasks are presented in Table 1. Contrary to the prediction of a bilingual advantage in monitoring (Costa et al., 2009; Hilchey & Klein, 2011), bilingual participants had marginally *higher* overall mean RTs relative to monolinguals in the Number Stroop task. The difference between groups in the interference effect (incongruent – neutral RTs) was not significant, but also tended to show a monolingual advantage. The same analysis using the difference between RTs for incongruent and congruent trials also revealed no group differences, $t(100) = -1.09$, *ns*. Similarly, for the Plus-Minus task differences in shift cost and mean RTs on alternating trials were not significantly different, and again tended to show a monolingual advantage. No other group differences were found in performance on the executive control tasks.

3.2.3. Problem solving analyses

As shown in Table 1, no differences were seen in completion time for either the Basic Math or Symbol Math task. As in Experiment 1, an analysis of covariance was performed with performance on the Basic Math task as a covariate in order to determine whether there were differences between monolinguals and bilinguals at each level of difficulty on the Symbol Math task (solution rates for all items by language group are included in the Appendix). The analysis revealed a main effect of difficulty, $F(1,102) = 157.50$, $p < 0.01$, $\eta_p^2 = 0.61$. There was no main effect for bilingualism, $F(1,102) = 1.47$, $p < 0.23$, but there was a significant interaction, $F(1,102) = 6.63$, $p < 0.01$, $\eta_p^2 = 0.06$. Follow-up tests for each level of difficulty revealed a

bilingualism effect only on the hard items, $F(1,102) = 6.49$, $p < 0.01$, $\eta_p^2 = 0.06$.

3.3. Discussion

The results of Experiment 2 replicated those of Experiment 1, namely, bilinguals outperformed monolinguals on hard items in the Symbol Math task. This bilingual advantage in performance cannot be attributed to differences in SES, because the bilingual and monolingual samples were matched on education levels for each parent, and the bilinguals came from lower income households. The differences in performance on the hard items in the Symbol Math task also cannot be attributed to a bilingual advantage in executive control, because no group differences were found on the three executive control measures. These results also preclude a related alternate explanation of the results of Experiment 1, namely possible group differences in working memory capacity because no differences were observed between monolinguals and bilinguals on the Letter-Number Sequencing task, which is a measure of working memory capacity.

4. General discussion

The goal of this research was to explore a bilingual advantage on a novel algebraic problem solving task within young adult samples. An advantage in performance on hardest items of the Symbol Math task, over and above performance on a basic math task, was found across two experiments. Indeed, even though the bilingual samples came from different institutions located in different geographic areas across the two experiments, and the bilinguals in Experiment 1 were more balanced than the bilinguals in Experiment 2, this did not seem to influence the results. The primary result of a bilingual advantage was replicated across both experiments despite differences in the populations. Bilinguals and monolinguals in the current studies did not differ in performance on the basic math task, which fails to support the idea that bilinguals are generally better at math (Planas, 2014). Instead, as seen in Kempert et al. (2011), the bilinguals in these studies performed better specifically on the most difficult problems.

Further, the results of the second experiment failed to demonstrate a bilingual advantage in any of the three basic measures of executive control. These null results are not surprising in light of similar recent reports by Paap and Greenberg (2013); de Bruin et al. (2015); and von Bastian et al. (2016). Similarly, these results are consistent with a recent longitudinal study that found that a spending a year in an immersive bilingual kindergarten resulted in benefits in performance on the Raven's Coloured Matrices, a task that involves abstract reasoning and is generally considered as a measure of fluid intelligence, but led to no change in performance on executive control tasks (Woumans, Surmont, Struys, & Duyck, 2016).

The advantages seen here are also consistent with prior work suggesting that bilinguals may be faster to adapt to the demands of novel mathematical tasks. Since participants did not receive any training on the hard items in the Symbol Math task, these can be considered transfer items. Better performance on these items suggests that bilinguals may have a specific advantage in transferring their understanding of novel symbolic rules to a new context. Stocco and Prat (2014) examined monolingual and bilingual performance on a task where participants needed to execute a set of mathematical operations (e.g. multiply x by two, subtract one from y , and multiply the result) for each trial. Stocco and Prat (2014) found no differences between monolinguals and bilinguals on trials where a set of operations was repeated from a previous trial, but that bilinguals responded faster

than monolinguals on novel trials where the set of operations was new to the participants. These results suggest that bilinguals may have superior skills at the flexible selection and application of novel procedures. To the extent that succeeding at novel tasks requires fluid intelligence, these results can also be seen as consistent with the idea that bilingualism may confer advantages in fluid reasoning abilities (Woumans et al., 2016). Thus, several prior results seem consistent with the possibility that experience with symbolic abstraction and the ability to succeed at novel tasks may explain the bilingual advantage seen on the hardest symbol math items in these experiments.

Since the Symbol Math task essentially presents algebraic functions to participants in a novel form, the results of this study suggest that the bilingual experience may have specific implications for learning about algebraic functions, which is an important topic in algebra. In particular, bilingualism may be associated with an ability to separate symbols from their referents, which can be seen as a form of metalinguistic awareness. Several researchers have drawn parallels between algebra and language (Koedinger & McLaughlin, 2010; MacGregor & Price, 1999; Planas, 2014). Language and mathematics are both syntactic rule-based systems, and they may even engage similar processing mechanisms (Scheepers & Sturt, 2014; Van de Cavey & Hartsuiker, 2016). Because early bilingual children are confronted with the experience of a single concept having more than one name, they may be in a better position to develop metalinguistic awareness than monolinguals (Adesope et al., 2010). For example, Galambos and Hakuta (1988) found that bilingualism played a role in grammatical error detection, which they argued was consistent with a bilingual advantage in metalinguistic awareness. Similarly, Bialystok (1997) demonstrated that bilingual children have a better understanding of the symbolic nature of written language than their monolingual peers. In this study, bilingual children were more likely than monolingual children to recognize that a printed label still referred to the same object even if it was "accidentally" placed in front of a new object. Given that symbolic abstraction is an essential part of algebra, there is reason to suspect that advantages in metalinguistic awareness could also extend to algebraic tasks. MacGregor and Price (1999) discuss components of metalinguistic awareness and how these components have mathematical analogs that are applicable to algebraic problem solving. Word awareness involves the ability to treat words as variables with general properties, which is analogous to symbol awareness in algebraic problem solving. Syntax awareness pertains to recognizing the structure of syntactical arrangements and understanding the role of syntactical structure in the meaning of linguistic or algebraic expressions. Further, MacGregor and Price (1999) found that metalinguistic awareness was correlated with algebraic problem solving performance. If metalinguistic awareness predicts algebraic problem solving performance, and if bilinguals have an advantage in metalinguistic awareness, this could also explain the bilingual advantage on the hard items of the Symbol Math task that was found in the current studies. The results are consistent with the suggestion that bilinguals may be better able to attend to the meaning of the symbols than monolinguals.

Although the challenges that bilinguals and language learners experience during math learning have been well documented (e.g., Campbell et al., 2007; Haag et al., 2013; Saalbach et al., 2013), bilingualism may also confer certain advantages in symbolic abstraction, as the current studies demonstrate. Symbolic abstraction is a crucial part of algebra, thus bilinguals may also experience some advantages in algebra learning specifically, which is consistent with the findings of Planas (2014). This work adds to the growing body of research from a variety of disciplines that paints a complex picture of the effects of the bilingual experience. In order to make informed instruc-

tional choices it is important for educators to be aware of students' weaknesses as well as their strengths.

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Appendix.

Table A1 Correlations among Executive Control Measures, Basic Math Task, and Symbol Math Task for Monolinguals in Experiment 2.

Task	1	2	3	4	5	6	Symbol Math
1. Basic Math	–	0.55**	-0.04	-0.24	0.04	-0.39*	0.64**
2. L-N Sequencing	–	–	-0.06	-0.29†	0.12	-0.15	0.45**
3. Number Stroop Interference Effect	–	–	–	0.31*	0.26	0.30†	0.00
4. Number Stroop Overall Mean RT	–	–	–	–	-0.14	0.25	-0.13
5. Plus-Minus Shift Cost	–	–	–	–	–	0.63**	0.02
6. Plus-Minus RT Alternating Block	–	–	–	–	–	–	-0.31*

Note: * $p < 0.05$ ** $p < 0.01$ † $p < 0.07$.

Table A2 Correlations among Executive Control Measures, Basic Math Task, and Symbol Math Task for Bilinguals in Experiment 2.

Task	1	2	3	4	5	6	Symbol Math
1. Basic Math	–	0.26*	-0.03	-0.14	-0.33**	-0.51**	0.71**
2. L-N Sequencing	–	–	0.12	-0.05	-0.30*	-0.30*	0.36**
3. Number Stroop Interference Effect	–	–	–	0.62**	0.01	0.21	-0.00
4. Number Stroop Overall Mean RT	–	–	–	–	0.13	0.34**	-0.29*
5. Plus-Minus Shift Cost	–	–	–	–	–	0.55**	-0.27*
6. Plus-Minus RT Alternating Block	–	–	–	–	–	–	-0.40**

Note: * $p < 0.05$ ** $p < 0.01$.

Table A3 Solution Rates for All Items in the Symbol Math Task for Monolinguals and Bilinguals in Experiment 1.

	Item	Monolinguals <i>M (SD)</i>	Bilinguals <i>M (SD)</i>
Easy	1	0.97 (0.18)	0.79 (0.41)
	2	0.84 (0.37)	0.97 (0.19)
	3	0.72 (0.46)	0.59 (0.50)
	4	0.78 (0.42)	0.76 (0.44)
Medium	5	0.75 (0.44)	0.83 (0.38)
	6	0.72 (0.46)	0.62 (0.49)
	7	0.78 (0.42)	0.72 (0.46)
	8	0.72 (0.46)	0.62 (0.49)
Difficult	9	0.06 (0.25)	0.21 (0.41)
	10	0.13 (0.34)	0.31 (0.47)
	11	0.22 (0.42)	0.31 (0.47)
	12	0.00 (0.00)	0.07 (0.26)

Table A4 Solution Rates for All Items in the Symbol Math Task for Monolinguals and Bilinguals in Experiment 2.

	Item	Monolinguals <i>M (SD)</i>	Bilinguals <i>M (SD)</i>
Easy	1	0.88 (0.33)	0.88 (0.33)
	2	0.81 (0.40)	0.88 (0.33)
	3	0.85 (0.36)	0.66 (0.48)
	4	0.85 (0.36)	0.78 (0.42)
Medium	5	0.67 (0.48)	0.77 (0.43)
	6	0.67 (0.48)	0.70 (0.46)
	7	0.76 (0.43)	0.83 (0.38)
	8	0.78 (0.42)	0.61 (0.49)
Difficult	9	0.12 (0.33)	0.27 (0.45)
	10	0.17 (0.38)	0.23 (0.43)
	11	0.27 (0.44)	0.36 (0.48)
	12	0.02 (0.16)	0.00 (0.00)

Uncited reference

Bradley and Corwyn, 2002.

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