Leptoquark induced rare decay amplitudes $h \to \tau^{\mp}\mu^{\pm}$ and $\tau \to \mu\gamma$

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Rare decay modes of the newly discovered standard-model-like Higgs boson h may test the flavor-changing couplings in the leptoquark sector through the process $h \to \tau^{\mp}\mu^{\pm}$. Motivated by the recently reported excess in LHC data from the CMS detector, we found that a predicted branching fraction Br $(h \to \tau^{\mp}\mu^{\pm})$ at the level of 1% is possible even though the coupling parameters are subjected to the stringent constraint from the null observation of $\tau \to \mu\gamma$, where the destructive cancellation among amplitudes is achievable by fine-tuning.

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I. INTRODUCTION

The newly discovered Higgs boson h at the mass 125 GeV is consistent with the Higgs boson predicted in the standard model (SM) [1]. The narrow decay width of a predicted size, about 4 MeV in the SM, provides hope that the unusual rare decay due to new physics (NP) can have a measurable branching fraction. Recently, the CMS Collaboration has reported [2] a possible excess in the decay process $h \to \tau^{\mp}\mu^{\pm}$ with a significance of 2.4σ in the search for lepton flavor violation (LFV). Assuming SM Higgs production, CMS obtained the best fit for the branching fraction summed over $\tau^-\mu^+$ and $\tau^+\mu^-$,

$$\mathrm{Br}(h \to \tau^{\mp}\mu^{\pm}) = 0.84^{+0.39}_{-0.37}\%. \tag{1}$$

We understand that it is too early to draw a positive inference until future analyses of higher statistics from both the CMS and ATLAS experiments are performed. However, the present sensitivity at the 1% level is interesting enough to call for possible NP to deliver such a detectable rate but satisfy other rare decay constraints such as $\tau \to \mu\gamma$,

$$\mathrm{Br}(\tau \to \mu\gamma) < 4.4 \times 10^{-8} \quad \text{at 90\% C.L.}, \tag{2}$$

from the *BABAR* experiment [3] at low energy. Indeed, there are many theoretical activities [4–22] along this line of investigation. There were also a number of studies on LFV Higgs boson decays in the literature [23]. We are particularly motivated by the leptoquark (LQ) associated with the third generation, which provides a large top quark mass insertion in the loop diagram. However, the LQ interactions also give rise to amplitudes for $\tau \to \mu\gamma$. We notice that the cancellation between two types of LQ contributions is possible for $\tau \to \mu\gamma$, leaving a large detectable decay rate for $h \to \tau^{\mp}\mu^{\pm}$. Each of these two types of LQs has been outlined in the literature, such as in Ref. [4], but the combined version necessary for the cancellation was overlooked.

The organization of this article is as follows. In the next section, we describe the LQ interactions associated with the top quark and tau lepton. In Secs. III and IV, we calculate the decays $\tau \to \mu\gamma$ and $h \to \tau^{\mp}\mu^{\pm}$, respectively. We give details on the numerical results in Sec. V and conclude in Sec. VI.

II. LEPTOQUARK INTERACTIONS ASSOCIATED WITH THE TOP QUARK

We associate the new LQs with the top quark of the third generation in order to avoid the very stringent constraints upon the flavor nonconservation among the first two generations. On the other hand, the mass insertion of the top quark can enhance the rate of the rare LFV Higgs decay mode among the second- and third-generation leptons. To satisfy the electroweak gauge symmetry, we can classify two types of LQs—the first one, $\chi^{\frac{1}{3}}$, is a weak $SU(2)$ singlet, and the other one is an $SU(2)$ doublet, i.e. $\Omega^T = (\Omega^{\frac{5}{3}}, \Omega^{\frac{2}{3}})$. The superscript denotes the electromagnetic charge number. LQs transform under the $SU(3)$ color group as 3 just like quarks. The relevant interactions are given by

$$\mathcal{L} \supset g^{\tau}_L \chi^{\frac{1}{3}} (Q_3)^T_L \epsilon L_{\tau,L} - g^{\tau}_R \chi^{\frac{1}{3}} t_R \tau_R + g'^{\tau}_L \Omega^T \epsilon \overline{t_R} L_{\tau,L} - g'^{\tau}_R \overline{Q_{3,L}} \tau_R \Omega + (\tau \leftrightarrow \mu) + \mathrm{H.c.} \tag{3}$$

The Feynman diagrams (for $\tau \to \mu\gamma$) that involve these LQ interactions are shown in Fig. 1. The shown symbol ϵ, basically $i\sigma_2$, links two $SU(2)$ doublets into a gauge invariant singlet. For brevity, we do not show other Levi-Civita symbols that contract Weyl spinor indexes.

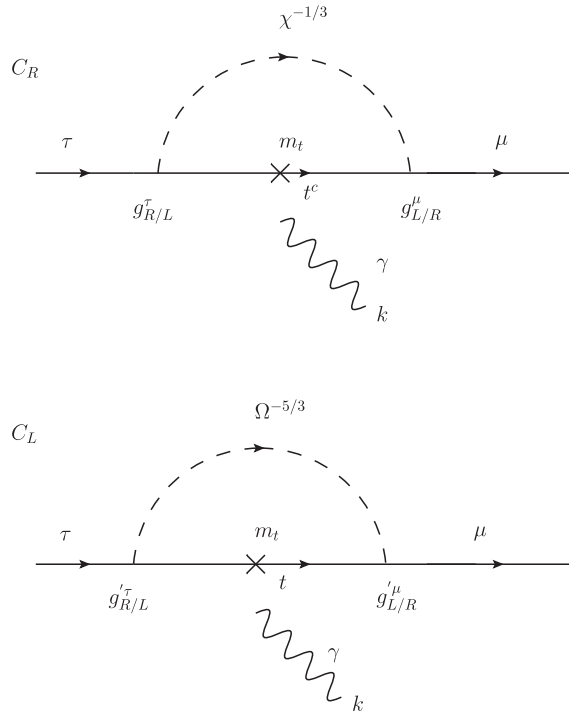

FIG. 1. Upper: Dipole transition of $\tau \to \mu\gamma$ via the singlet LQ χ. Lower: Dipole transition of $\tau \to \mu\gamma$ via the doublet LQ Ω.

Also, $(Q_3)^T_L = (t_L, b_L)$, $L_{\tau,L} = (\nu_{\tau,L}, \tau_L)^T$. The terms from exchanging $\tau \leftrightarrow \mu$ are needed to induce the LFV between the muon and the tau.

III. $\tau \to \mu\gamma$ AMPLITUDES INDUCED BY LEPTOQUARKS

We start with the contributions from the singlet χ. We define t^c to be the charge conjugated state of t. In this way, we can avoid the use of the unfamiliar Feynman rule for two fermions flowing into a vertex. Instead, one fermion flows in and the other out. For example, the incoming τ enters the first vertex and turns into a departing t^c plus a boson $\chi^{-\frac{1}{3}}$. The relevant vertices for the process $\tau \to \mu\gamma$ are

$$g^\tau_L(\chi^{-\frac{1}{3}})^\dagger(\overline{t^c}\tau_L) - g^\tau_R(\chi^{-\frac{1}{3}})^\dagger(\overline{t^c}\tau_R) + (\tau \leftrightarrow \mu) + \text{H.c.} \tag{4}$$

For the outgoing left-handed muon, the Feynman amplitude that the external photon line attaches to the t^c line is given by

$$i\mathcal{M}_1(\tau \to \mu\gamma) = -\frac{eq_{t^c} g^\tau_R g^\mu_L m_t}{16\pi^2} 3_c \int_0^1 \frac{(1-z)^2 dz}{zm_\chi^2 + (1-z)m_t^2} \sigma^{\mu\nu} k_\nu R, \tag{5}$$

where R stands for the right-handed chiral projection operator $(1+\gamma^5)/2$. It is understood that the external spinors $\overline{u(\mu)}$ and $u(\tau)$ sandwich the Dirac chain. We keep track of the color factor 3 by a subscript c. Another amplitude where the photon attaches to $\chi^{-\frac{1}{3}}$ is

$$i\mathcal{M}_2(\tau \to \mu\gamma) = \frac{eq_{\chi^{-\frac{1}{3}}} g^\tau_R g^\mu_L m_t}{16\pi^2} 3_c \int_0^1 \frac{(1-z)z dz}{zm_t^2 + (1-z)m_\chi^2} \sigma^{\mu\nu} k_\nu R. \tag{6}$$

We set charges $q_{t^c} = -\frac{2}{3}$ and $q_{\chi^{-\frac{1}{3}}} = -\frac{1}{3}$. Using $z \leftrightarrow (1-z)$ in $\mathcal{M}_1$, we obtain

$$i\mathcal{M}_{1+2} = \frac{eg^\tau_R g^\mu_L m_t}{16\pi^2 m_\chi^2} 3_c \int_0^1 \frac{\frac{2}{3}z^2 - \frac{1}{3}z(1-z)dz}{(1-z) + zm_t^2/m_\chi^2} \sigma^{\mu\nu} k_\nu R. \tag{7}$$

The numerator of the integral becomes $z^2 - \frac{z}{3}$. The overall result is

$$i\mathcal{M}_{1+2} = \frac{eg^\tau_R g^\mu_L m_t}{16\pi^2 m_\chi^2} 3_c \left(\xi_1(x_t) - \frac{1}{3}\xi_0(x_t)\right) \sigma^{\mu\nu} k_\nu R, \quad x_t = m_t^2/m_\chi^2, \tag{8}$$

$$\xi_n(x) \equiv \int_0^1 \frac{z^{n+1} \mathrm{d}z}{1+(x-1)z} = -\frac{\ln x + (1-x) + \cdots + \frac{(1-x)^{n+1}}{n+1}}{(1-x)^{n+2}},$$

$$\text{and} \quad \xi_{-1}(x) \equiv \int_0^1 \frac{\mathrm{d}z}{1+(x-1)z} = -\frac{\ln x}{1-x}. \tag{9}$$

So the amplitude is related to the integral function,

$$H_1(x) \equiv \xi_1(x) - \frac{1}{3}\xi_0(x) = -\frac{1}{6(1-x)^3}[7 - 8x + x^2 + 2(2+x)\ln(x)]. \tag{10}$$

Note that there is another chiral amplitude for the outgoing right-handed muon, using $g^\tau_L g^\mu_R$. These two amplitudes do not interfere in the zero muon mass limit.

Now we switch to the contributions from the LQ doublet Ω. The relevant vertices for the process $\tau \to \mu\gamma$ are

$$-g'^\tau_R(\Omega^{\frac{5}{3}})(\overline{t_L}\tau_R) + g'^\mu_L(\Omega^{\frac{5}{3}})(\overline{t_R}\mu_L) + (\tau \leftrightarrow \mu) + \text{H.c.} \tag{11}$$

For the outgoing left-handed muon,

$$i\mathcal{M}'_1(\tau \to \mu\gamma) = -\frac{eq_t g'^\tau_R g'^\mu_L m_t}{16\pi^2} 3_c \int_0^1 \frac{(1-z)^2 dz}{zm_\Omega^2 + (1-z)m_t^2} \sigma^{\mu\nu} k_\nu R. \tag{12}$$

This corresponds to the diagram that the external photon line attaches to the t line. Another amplitude where the photon attaches to $\Omega^{-\frac{5}{3}}$ is

$$i\mathcal{M}'_2(\tau\to\mu\gamma) = \frac{e q_{\Omega^{-\frac{5}{3}}} g'^{\tau}_R g'^{\mu}_L m_t}{16\pi^2} 3_c \int_0^1 \frac{(1-z)z dz}{z m_t^2 + (1-z) m_\Omega^2} \sigma^{\mu\nu} k_\nu R. \quad (13)$$

We set charges $q_t = \frac{2}{3}$ and $q_{\Omega^{-\frac{5}{3}}} = -\frac{5}{3}$. Using $z \leftrightarrow (1-z)$ in $\mathcal{M}_1$, we obtain

$$i\mathcal{M}'_{1+2} = \frac{e g'^{\tau}_R g'^{\mu}_L m_t}{16\pi^2 m_\Omega^2} 3_c \int_0^1 \frac{-\frac{2}{3}z^2 - \frac{5}{3}z(1-z) dz}{(1-z) + z m_t^2/m_\Omega^2} \sigma^{\mu\nu} k_\nu R. \quad (14)$$

The numerator of the integral becomes $z^2 - \frac{5z}{3}$. The overall result is

$$i\mathcal{M}'_{1+2} = \frac{e g'^{\tau}_R g'^{\mu}_L m_t}{16\pi^2 m_\Omega^2} 3_c \left(\xi_1(x'_t) - \frac{5}{3}\xi_0(x'_t)\right) \sigma^{\mu\nu} k_\nu R, \quad x'_t = m_t^2/m_\Omega^2. \quad (15)$$

So the amplitude is related to the integral function,

$$H_2(x) \equiv \xi_1(x) - \frac{5}{3}\xi_0(x) = -\frac{1}{6(1-x)^3}[-1 + 8x - 7x^2 - 2(2-5x)\ln(x)]. \quad (16)$$

Note that there is another chiral amplitude for the outgoing right-handed muon, using $g'^{\tau}_L g'^{\mu}_R$.

In general, the low-energy effective operators of dimension five are

$$\mathcal{L}_{\rm eff} \supset \frac{e}{m_t}[\bar{\mu}\sigma^{\alpha\beta}(C_L L + C_R R)\tau] F_{\alpha\beta} + \text{H.c.}, \quad (17)$$

$$C_R = \frac{3_c}{32\pi^2}(g^{\tau}_R g^{\mu}_L x_t H_1(x_t) + g'^{\tau}_R g'^{\mu}_L x'_t H_2(x'_t)), \quad (18)$$

$$C_L = \frac{3_c}{32\pi^2}(g^{\tau}_L g^{\mu}_R x_t H_1(x_t) + g'^{\tau}_L g'^{\mu}_R x'_t H_2(x'_t)). \quad (19)$$

The partial decay width of the process $\tau \to \mu\gamma$ is

$$\Gamma(\tau\to\mu\gamma) = \frac{e^2}{4\pi} m_\tau \left(\frac{m_\tau^2}{m_t^2}\right)(|C_L|^2 + |C_R|^2). \quad (20)$$

Our results in Eqs. (10), (16), and (20) for the rare decay $\tau \to \mu\gamma$ agree with the general loop formulas for the radiative transitions given in Ref. [24]. They are also compatible with those given in the study on the lepton-flavor violation by Hisano *et al.* in Ref. [25,26] if their corresponding supersymmetric couplings are replaced by the leptoquark couplings in the present context.

IV. $h \to \tau + \bar{\mu}$ VIA LEPTOQUARKS OF THE THIRD GENERATION

For the rare decay $h \to \tau\mu$, we start with the contribution from the $SU(2)$ singlet leptoquark $\chi^{-\frac{1}{3}}$ to the chiral amplitude of the outgoing right-handed τ. We take the zero mass limit for μ and τ. At the one-loop level, the Higgs coupling to $\tau(p_1)\bar{\mu}(p_2)$ is induced via a triangle diagram, which involves internal t^c, χ lines. First, we concentrate at the diagram where the external Higgs touches the internal t^c line, as shown in Fig. 2,

$$i\mathcal{M}^{\triangleleft}_{\chi,R} = (i)^6 3_c(-g^{\tau}_R g^{\mu}_L) \int \frac{d^4\ell}{(2\pi)^4} \frac{L(\not{\ell} + \not{p}_1 + m_t)(-\frac{m_t}{v})(\not{\ell} - \not{p}_2 + m_t)L}{((\ell+p_1)^2 - m_t^2)((\ell-p_2)^2 - m_t^2)(\ell^2 - m_\chi^2)}. \quad (21)$$

We use the Feynman parametrization trick to carry over the integration. The parameters α, β, γ are assigned to the denominator factors $(\ell+p_1)^2 - m_t^2$, $(\ell-p_2)^2 - m_t^2$, $\ell^2 - m_\chi^2$, respectively, under the constraint $\alpha+\beta+\gamma = 1$. Then we complete the square of the denominator as follows,

$$\begin{aligned}
&\alpha[(\ell+p_1)^2 - m_t^2] + \beta[(\ell-p_2)^2 - m_t^2] + \gamma[\ell^2 - m_\chi^2] \\
&= \ell^2 + 2\alpha p_1\cdot\ell - 2\beta p_2\cdot\ell + \alpha p_1^2 + \beta p_2^2 \\
&\quad - (\alpha+\beta)m_t^2 - \gamma m_\chi^2 \\
&= (\ell + \alpha p_1 - \beta p_2)^2 - m^2(\alpha,\beta), \\
&\text{where } m^2(\alpha,\beta) = m_\chi^2 - \alpha\beta s + (\alpha+\beta)(m_t^2 - m_\chi^2).
\end{aligned} \quad (22)$$

Shifting the loop momentum, we simplify the numerator of the Dirac matrices with the equation of motion,

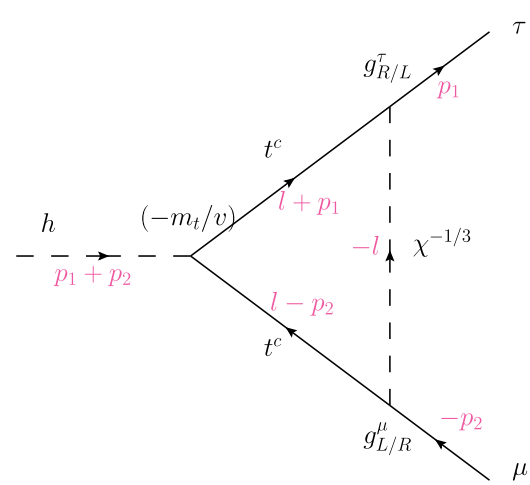

FIG. 2. $h \to \tau\bar{\mu}$ via the singlet LQ χ.

$$(\ell'^2 + m_t^2) \to \ell'^2 - 2\alpha\beta p_1 \times p_2 + m_t^2 \to \ell'^2 - \alpha\beta s + m_t^2.$$

Here the s variable is simply $2p_1 \cdot p_2 = m_h^2$. The amplitude becomes

$$i\mathcal{M}^{\triangleleft}_{\chi,R} = -3_c\left(g_R^\tau g_L^\mu \frac{m_t}{v}\right)\int_{\mathbb{L}} 2!d\alpha d\beta \times \int \frac{d^4\ell'}{(2\pi)^4}\frac{\ell'^2 - \alpha\beta s + m_t^2}{[\ell'^2 - m^2(\alpha,\beta)]^3}L. \tag{23}$$

The domain $\mathbb{L}$ covers positive α and β, as well as $\alpha + \beta \le 1$.

We perform the Wick's rotation by Euclideanizing $\ell'^0 \to iq_{E4}$, $\ell'^2 \to -q_E^2$, $d^4\ell' \to id^4q_E$, $d^4q_E \to d^3\boldsymbol{q}_E dq_{E4} = 4\pi|\boldsymbol{q}_E|^2 d|\boldsymbol{q}_E|dq_{E4} \to 4\pi(q_E^2\cos^2\phi)\frac{1}{2}dq_E^2 d\phi \to \pi^2 q_E^2 dq_E^2$. So

$$\begin{aligned}\mathcal{M}^{\triangleleft}_{\chi,R} &= -3_c\left(g_R^\tau g_L^\mu \frac{m_t}{v}\right)\int_{\mathbb{L}} 2!d\alpha d\beta \times \int \frac{-q_E^2 - \alpha\beta s + m_t^2}{-[q_E^2 + m^2(\alpha,\beta,s)]^3}\frac{q_E^2 dq_E^2}{16\pi^2}L,\\ &\to -3_c\frac{g_R^\tau g_L^\mu m_t}{16\pi^2 v}\int_{\mathbb{L}}\left(\log\frac{\Lambda^2}{m^2(\alpha,\beta,s)} - \frac{3}{2} + \frac{\alpha\beta s - m_t^2}{2m^2(\alpha,\beta,s)}\right)2!d\alpha d\beta L.\end{aligned} \tag{24}$$

The logarithmic divergence has to be canceled by the one-particle reducible (1PR) diagrams with bubbles in the external lepton lines. The h line is either attached directly to τ or μ, picking up, respectively, the mass couplings m_τ or m_μ, which are canceled by the propagators. It can be shown that the corresponding 1PR contribution becomes

$$\to +3_c\frac{g_R^\tau g_L^\mu m_t}{16\pi^2 v}\int_0^1\left(\log\frac{\Lambda^2}{\gamma m_\chi^2 + (1-\gamma)m_t^2} - 1\right)d\gamma L. \tag{25}$$

Overall, $\log\Lambda^2$ terms cancel. Therefore, the combined amplitude is

$$\mathcal{M}_\chi = -3_c\frac{1}{16\pi^2}\frac{m_t}{v}G_\chi(g_R^\tau g_L^\mu L + g_R^\mu g_L^\tau R), \tag{26}$$

$$\begin{aligned}G_\chi &= \int_{\mathbb{L}}\left(\log\frac{\Lambda^2}{m^2(\alpha,\beta,s)} - \frac{1}{2} + \frac{\alpha\beta s - m_t^2}{2m^2(\alpha,\beta,s)}\right)2!d\alpha d\beta\\ &\quad - \int_0^1 \log\frac{\Lambda^2}{\gamma m_\chi^2 + (1-\gamma)m_t^2}d\gamma.\end{aligned} \tag{27}$$

Note that in the intermediate step, we can choose an arbitrary Λ for the convenience of the calculation.

Alternatively, one can use the Passarino-Veltman (PV) [27] functions. The integral before the Feynman's parametrization as given in (21) is

$$\begin{aligned}&\int \frac{\frac{d^4\ell}{(2\pi)^4}(\ell^2 + m_t^2)}{((\ell+p_1)^2 - m_t^2)((\ell-p_2)^2 - m_t^2)(\ell^2 - m_\chi^2)}\\ &= \int \frac{[(\ell^2 - m_\chi^2) + m_\chi^2 + m_t^2]\frac{d^4\ell}{(2\pi)^4}}{((\ell+p_1)^2 - m_t^2)((\ell-p_2)^2 - m_t^2)(\ell^2 - m_\chi^2)}\\ &= \int \frac{d^4\ell}{(2\pi)^4}\left(\frac{m_\chi^2 + m_t^2}{((\ell+p_1)^2 - m_t^2)((\ell-p_2)^2 - m_t^2)(\ell^2 - m_\chi^2)} + \frac{1}{((\ell+p_1)^2 - m_t^2)((\ell-p_2)^2 - m_t^2)}\right).\end{aligned} \tag{28}$$

The first term simply gives the triangle function

$$\frac{i}{16\pi^2}(m_t^2 + m_\chi^2)C_0(0,0,s,m_t^2,m_\chi^2,m_t^2). \tag{29}$$

The second term, after shifting the loop momentum, gives the bubble function

$$\begin{aligned}\frac{i}{16\pi^2}B_0(s,m_t^2,m_t^2) &= \int \frac{d^4\ell/(2\pi)^4}{((\ell+p_1)^2 - m_t^2)((\ell-p_2)^2 - m_t^2)}\\ &= \int \frac{d^4\ell'/(2\pi)^4}{((\ell'+p_1+p_2)^2 - m_t^2)(\ell'^2 - m_t^2)}.\end{aligned} \tag{30}$$

The result including the 1PR bubbles is

$$\begin{aligned}G_\chi &= (m_\chi^2 + m_t^2)C_0(0,0,s,m_t^2,m_\chi^2,m_t^2) + B_0(s,m_t^2,m_t^2)\\ &\quad - B_0(0,m_t^2,m_\chi^2).\end{aligned} \tag{31}$$

We have cross-checked numerically that the G value from PV functions and from the Feynman parametrization method match each other.

For the Feynman diagram where the Higgs touches the leptoquark, the required vertex originates from the bosonic interaction of $-\lambda_\chi H^\dagger H\chi^\dagger\chi$. The G coefficient is updated with the new addition,

$$G_\chi \to G_\chi + \lambda_\chi v^2 C_0(0,0,s,m_\chi^2,m_t^2,m_\chi^2).$$

When we come to the contribution from the $SU(2)$ doublet leptoquark Ω, it is easy to see the simple translation,

$$\chi^{\frac{1}{3}} \leftrightarrow \Omega^{\frac{5}{3}}, \quad g_{L/R}^\ell \leftrightarrow g'^\ell_{L/R}, \quad \lambda_\chi \leftrightarrow \lambda_\Omega, \quad m_\chi \leftrightarrow m_\Omega, \quad \text{etc.}$$

Here m_Ω is the mass of the $\frac{5}{3}$ charged leptoquark. More explicitly,

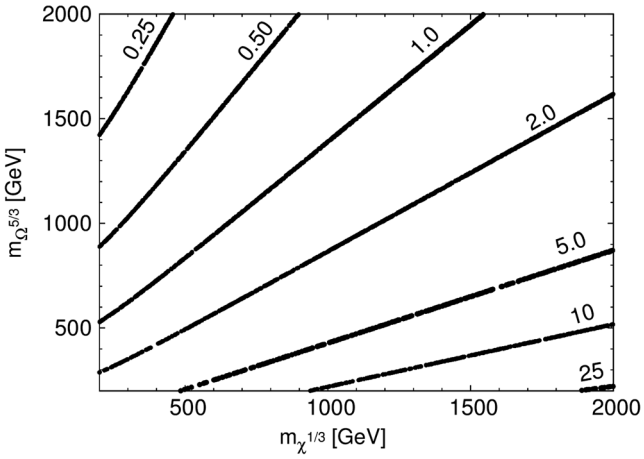

FIG. 3. Contour plot of the coupling ratio $g_R^\tau g_L^\mu/(g'^\tau_R g'^\mu_L)$ on the (m_χ, m_Ω) plane, satisfying the tuned cancellation in Eq. (35) in the amplitude $\tau \to \mu\gamma$.

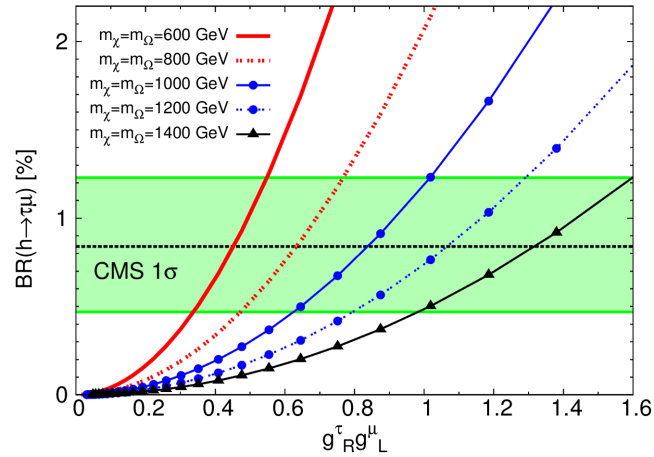

FIG. 4. The predicted numerical size of $\mathrm{Br}(h \to \tau^\mp\mu^\pm \text{ both})$ versus $g_R^\tau g_L^\mu$ for various LQ masses when the tuned cancellation is satisfied. The CMS 1σ range of Eq. (1) is also shown.

$$G_\chi = (m_\chi^2 + m_t^2)C_0(0,0,s,m_t^2,m_\chi^2,m_t^2) + B_0(s,m_t^2,m_t^2) - B_0(0,m_t^2,m_\chi^2) + \lambda_\chi v^2 C_0(0,0,s,m_\chi^2,m_t^2,m_\chi^2),$$
$$G_\Omega = (m_\Omega^2 + m_t^2)C_0(0,0,s,m_t^2,m_\Omega^2,m_t^2) + B_0(s,m_t^2,m_t^2) - B_0(0,m_t^2,m_\Omega^2) + \lambda_\Omega v^2 C_0(0,0,s,m_\Omega^2,m_t^2,m_\Omega^2), \tag{32}$$

$$\mathcal{M}^{\mathrm{ren}}(h \to \tau\bar{\mu}) = -3_c \frac{1}{16\pi^2}\frac{m_t}{v}[(G_\chi g_R^\tau g_L^\mu + G_\Omega g'^\tau_R g'^\mu_L)L + (G_\chi g_R^\mu g_L^\tau + G_\Omega g'^\mu_R g'^\tau_L)R]. \tag{33}$$

The partial decay width, summing both processes $h \to \tau^\mp\mu^\pm$, is[1]

$$\Gamma(h \to \tau^\mp\mu^\pm) = \frac{9}{2048\pi^5} m_h \left(\frac{m_t}{v}\right)^2 (|G_\chi g_R^\tau g_L^\mu + G_\Omega g'^\tau_R g'^\mu_L|^2 + |G_\chi g_R^\mu g_L^\tau + G_\Omega g'^\mu_R g'^\tau_L|^2). \tag{34}$$

V. PHYSICS POSSIBILITIES

To suppress the highly constrained $\tau \to \mu\gamma$, we tune the cancellation:

$$g_R^\tau g_L^\mu x_t H_1(x_t) + g'^\tau_R g'^\mu_L x'_t H_2(x'_t) \approx 0, \tag{35}$$

$$g_L^\tau g_R^\mu x_t H_1(x_t) + g'^\tau_L g'^\mu_R x'_t H_2(x'_t) \approx 0. \tag{36}$$

We choose a simplified scenario where only one chiral mode of the muon interactions is important, say, $g_L^\mu \gg g_R^\mu$ and $g'^\mu_L \gg g'^\mu_R$, then we only finely tune the corresponding one constraint, i.e. *the first of the two*. The ratio of the couplings $g_R^\tau g_L^\mu/(g'^\tau_R g'^\mu_L)$ is given in Fig. 3 for the tuned cancellation in $\tau \to \mu\gamma$. The contour plot demonstrates that a large parameter space remains available for the required fine-tuning.

Figure 4 shows the predicted numerical size of $\mathrm{Br}(h \to \tau^\mp\mu^\pm \text{ both})$ versus $g_R^\tau g_L^\mu$ for various LQ masses when the tuned cancellation is satisfied. We have set $\lambda_{\chi,\Omega} = 0$ in our numerical study. A desirable branching fraction at 1% level occurs for the coupling product $g_R^\tau g_L^\mu \simeq 0.3$–1 for the cases where $m_\Omega = m_\chi$ from 600 GeV to 1 TeV.

If we switch off either one of the canceling amplitudes in $\tau \to \mu\gamma$, the individual contribution to the $\mathrm{Br}(\tau \to \mu\gamma)$ is shown in Fig. 5. This demonstrates how much fine-tuning is required. The $\mathrm{Br}(\tau \to \mu\gamma)$ would be at about the 1% level for 500 GeV LQ and $g_R^\tau g_L^\mu$ about 0.3 to 0.8 if only using one of the two canceling amplitudes. To go down from 10^{-2} to

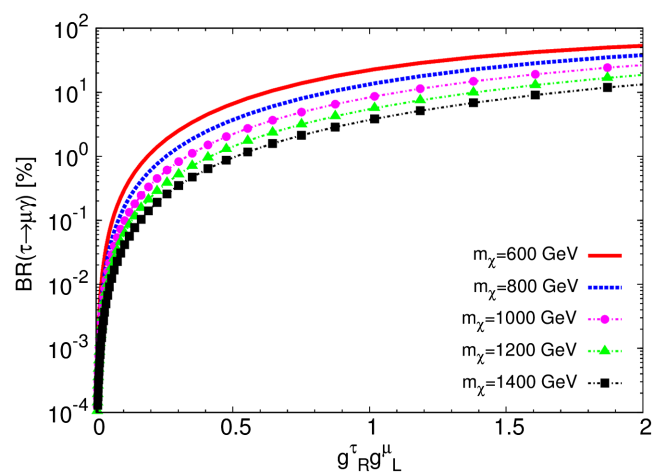

FIG. 5. Individual contribution to the $\mathrm{Br}(\tau \to \mu\gamma)$ if either one of the two canceling amplitudes is switched off.

[1]The rate given in Eq. (34) is a factor of 4 larger than that in Eq. (47) of Ref. [4].

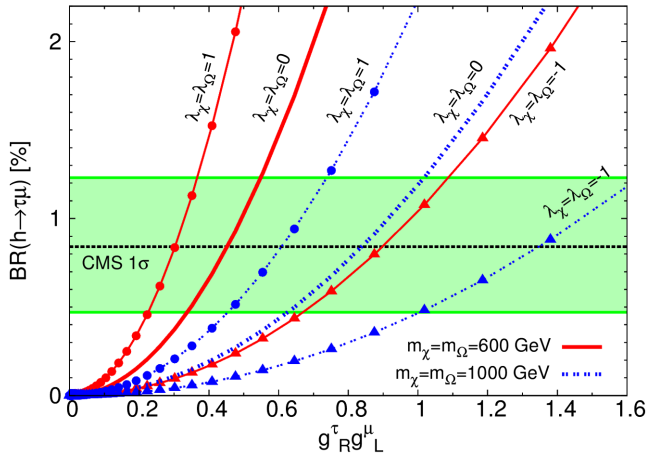

FIG. 6. The branching ratio $\mathrm{Br}(h \to \tau^{\mp}\mu^{\pm}\,\mathrm{both})$ versus $g^{\tau}_{R}g^{\mu}_{L}$ for various choices of $\lambda_\chi = \lambda_\Omega = -1, 0, 1$. The tuned cancellation is satisfied.

10^{-8} in the branching ratio, the two amplitudes are required to cancel each other by almost one part in 10^3.

Reference [4] also proposed a mechanism of cancellation in the amplitude of $\tau \to \mu\gamma$ with the help of an additional vectorial toplike quark. However, the detailed gauge quantum numbers of the added structure have not been shown to be feasible.

So far, we have set $\lambda_{\chi,\Omega} = 0$. We show in Fig. 6 the branching ratio $\mathrm{Br}(h \to \tau^{\mp}\mu^{\pm})$ for various choices of $\lambda_\chi = \lambda_\Omega = -1,\ 0,\ 1$. The tuned cancellation of Eq. (35) is satisfied. It gives additional freedom to achieve the desirable branching ratio for the rare Higgs decay.

VI. CONCLUDING REMARKS

The rare decay of $h \to \tau^{\mp}\mu^{\pm}$ can be at the current reachable sensitivity through the LFV LQ interactions; however, fine-tuning is needed to avoid the stringent constraint from the null observation of $\tau \to \mu\gamma$. Here we have invoked more than one of the LQs, which couple to the third-generation quarks, and the second- and third-generation leptons, in order to achieve a cancellation in $\tau \to \mu\gamma$ but sizable contributions to $h \to \tau^{\mp}\mu^{\pm}$.

There is another issue related to possible contributions of these LQs to the muon anomalous magnetic moments (aka $g-2$). It was shown a long time ago [28] and more recently [29] that if we choose, as we have chosen in the above analysis, the left-handed coupling to be much larger than the right-handed coupling for the muon, i.e. $g^{\mu}_{L} \gg g^{\mu}_{R}$ and $g'^{\mu}_{L} \gg g'^{\mu}_{R}$, the LQ contribution to $g-2$ is highly suppressed by m_μ/M_{LQ} and very small for the LQ mass range that we considered in this work.

The required leptoquarks χ and Ω can be strongly produced at the high-energy and high- luminosity hadron colliders. They have dominant decay channels into the top quark and the charged lepton τ or μ, which is a very identifiable signature. Both the ATLAS [30] and CMS [31,32] Collaborations have searched for the third-generation leptoquarks via pair production by strong interaction. The CMS have searched for the third-generation LQ with electric charged $-1/3$ (similar to the $\chi^{-1/3}$ of this work) decaying to a top quark and a tau lepton. They put a limit of 685 GeV at 95% C.L. on m_χ [31]. On the other hand, both ATLAS [30] and CMS [32] searched for the third-generation LQ with electric charge $-2/3$ decaying into a $\bar{b}$ antiquark and a tau lepton (similar to $\Omega^{-2/3}$ in this work) and put a limit of 534 and 740 GeV, respectively.

Therefore, there are still plenty of mass ranges for $\chi^{-1/3}$ beyond 685 GeV and for $\Omega^{-2/3,-5/3}$ beyond 740 GeV that one can directly search for a top or bottom quark with a tau or muon at run 2 of LHC-13.

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[1] See, for example, K. Cheung, J. S. Lee, and P. Y. Tseng, Phys. Rev. D **90**, 095009 (2014).
[2] V. Khachatryan *et al.* (CMS Collaboration), Phys. Lett. B **749**, 337 (2015).
[3] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **104**, 021802 (2010).
[4] I. Dorsner, S. Fajfer, A. Greljo, J. F. Kamenik, N. Kosnik, and I. Nisandzic, J. High Energy Phys. 06 (2015) 108.
[5] R. Harnik, J. Kopp, and J. Zupan, J. High Energy Phys. 03 (2013) 026.
[6] M. D. Campos, A. E. C. Hernndez, H. Ps, and E. Schumacher, Phys. Rev. D **91**, 116011 (2015).
[7] A. Celis, V. Cirigliano, and E. Passemar, arXiv: 1409.4439.
[8] D. Aristizabal Sierra and A. Vicente, Phys. Rev. D **90**, 115004 (2014).

[9] C. J. Lee and J. Tandean, J. High Energy Phys. 04 (2015) 174.
[10] J. Heeck, M. Holthausen, W. Rodejohann, and Y. Shimizu, Nucl. Phys. **B896**, 281 (2015).
[11] A. Crivellin, G. DAmbrosio, and J. Heeck, Phys. Rev. Lett. **114**, 151801 (2015).
[12] L. de Lima, C. S. Machado, R. D. Matheus, and L. A. F. do Prado, J. High Energy Phys. 11 (2015) 074.
[13] I. de Medeiros Varzielas and G. Hiller, J. High Energy Phys. 06 (2015) 072.
[14] D. Das and A. Kundu, Phys. Rev. D **92**, 015009 (2015).
[15] A. Crivellin, G. DAmbrosio, and J. Heeck, Phys. Rev. D **91**, 075006 (2015).
[16] B. Bhattacherjee, S. Chakraborty, and S. Mukherjee, arXiv:1505.02688.
[17] Y. n. Mao and S. h. Zhu, arXiv:1505.07668.
[18] B. Altunkaynak, W. S. Hou, C. Kao, M. Kohda, and B. McCoy, Phys. Lett. B **751**, 135 (2015).
[19] X. G. He, J. Tandean, and Y. J. Zheng, J. High Energy Phys. 09 (2015) 093.
[20] C. W. Chiang, H. Fukuda, M. Takeuchi, and T. T. Yanagida, J. High Energy Phys. 11 (2015) 057.
[21] W. Altmannshofer, S. Gori, A. L. Kagan, L. Silvestrini, and J. Zupan, arXiv:1507.07927.
[22] A. Crivellin, J. Heeck, and P. Stoffer, arXiv:1507.07567.
[23] A. Pilaftsis, Phys. Lett. B **285**, 68 (1992); J. G. Korner, A. Pilaftsis, and K. Schilcher, Phys. Rev. D **47**, 1080 (1993).
[24] L. Lavoura, Eur. Phys. J. C **29**, 191 (2003).
[25] J. Hisano, T. Moroi, K. Tobe, and M. Yamaguchi, Phys. Rev. D **53**, 2442 (1996).
[26] K. m. Cheung and O. C. W. Kong, Phys. Rev. D **64**, 095007 (2001).
[27] G. Passarino and M. J. G. Veltman, Nucl. Phys. **B160**, 151 (1979).
[28] K. m. Cheung, Phys. Rev. D **64**, 033001 (2001).
[29] F. S. Queiroz and W. Shepherd, Phys. Rev. D **89**, 095024 (2014).
[30] G. Aad *et al.* (ATLAS Collaboration), J. High Energy Phys. 06 (2013) 033.
[31] V. Khachatryan *et al.* (CMS Collaboration), J. High Energy Phys. 07 (2015) 042.
[32] V. Khachatryan *et al.* (CMS Collaboration), Phys. Lett. B **739**, 229 (2014).