

**Mathematics Socialization through Games: A Study of Bilingual Latinas/os in an
After-School Context**

BY

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DEDICATION

I dedicate this work to my mother, Vidosava, who did not live to see the completion of my studies.

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I. INTRODUCTION

Why is it that the connection between play and higher math escapes us?
(Piers, 1972, p. 170)

This study is inspired by my experience as a researcher and facilitator with the CEMELA *de Los Rayos* after-school mathematics club that brought together third grade Spanish/English bilingual students, their parents, graduate researchers, and undergraduate pre-service teachers. Educational activities were organized around play-based mathematics enrichment activities in a bilingual atmosphere using games, puzzles, projects in small groups and one-on-one interactions. Early into this program, the school principal expressed concern that the children participating in the *Los Rayos* after-school mathematics club were “just playing games” and not engaged in serious mathematics. In essence, the school principal was questioning the value of mathematical game play. The fact that researchers carefully chose games they believed contained fundamental as well as advanced mathematical concepts did not seem relevant. Instead, what the principal wanted was visible evidence that children were *doing mathematics*.

Game activities are known to be effective learning contexts for mathematics, as evidenced by the National Council of Teachers of Mathematics (NCTM) Standard 5.1 Communicating about Mathematics Using Games: “Mathematical games can foster mathematical communication as students explain and justify their moves to one another. In addition, games can motivate students and engage them in thinking about and applying concepts and skills”, (NCTM, 2000, p. 63). My focus on mathematical games is also inspired by the fact that games have a long (ancient) and distinctive history in human activity. Today, game theory is

a prominent field in mathematics in which several mathematicians received Nobel prizes. In computer science, a prominent paradigm used for creating interactive software entails posing problems as two-player game scenarios. In mathematics education, board games, dice games, card games, and computer games have been used successfully to help students learn mathematical concepts. These game activities also foster metacognition. My study is concerned with the value of games for helping students practice and appropriate mathematical *behaviors*. In other words, metacognitive skills associated with mathematical activity are my prime concern.

The purpose of this study is to document the types of actions participants use in the *process* of acquiring mathematical thinking as represented by the Common Core State Standards (CCSS) for Mathematical Practice (National Council of Supervisors of Mathematics, 2013, pp. 80-82). These eight standards are essentially metacognitive learning goals and represent the distillation of decades of research on the qualities of thinking processes most relevant to mathematical learning and development. What is most significant about these CCSS Mathematical Practice standards is that these mathematical practices, although simply and succinctly crafted, are not so simple to hand down to students in ways that they can effectively use. In short, they are not algorithms for memorization. Practices are actions that, according to our current understanding, require considerable amounts of social interaction so that novices can learn by observation, through various forms of participation, and engagement through discourse.

The type of study I have pursued is *descriptive*. My reason for choosing this type of approach stems from the school principal's concern I mentioned earlier. There is a need to uncover visible markers of valuable mathematical activity. Policy officials, school administrators, teachers, parents, and all other adults who are in positions of power over children

need to understand what is valuable about play activities that are focused on academic enrichment.

A. The Goals of *Los Rayos*

The after-school program was meant to mitigate the influences of language, racial, and cultural bias. It was a place where their home language, cultural heritage, and immigrant status was “normal”. Furthermore, their linguistic and cultural heritage was viewed as an asset rather than a deficit. Also, this context provided resources to children who did not have the extra resources available to those who are more privileged. Dr. Lena Liçon Khisty designed this program with several complimentary goals. These goals are; 1) eliminate linguistic/cultural bias, 2) expose pre-service teachers to children outside the classroom, 3) create space for informal mathematical development, and 4) provide contexts and activities for study and experimentation. Facilitators did not adopt practices aimed at teaching English. It was assumed that children could use Spanish or English as they chose and that children were not there to be remediated in mathematics or their language use. This was an environment where the focus was educational enrichment, not remediation.

The Mexican American third grade children in the *Los Rayos* after-school program were all bilingual Spanish/English speakers of immigrant parents. My interest in this group of children stems from my own status as a child of Serbian immigrant parents, having grown up in a bilingual home. I sympathize with the plight of bilingual children in the United States. Too often these children grow up in neighborhoods where the public education system has diminished resources and their linguistic and cultural heritage is often viewed as a deficit that must be overcome (Gándara & Contreras, 2009; Gutiérrez, 2008; Valenzuela, 1999). This is also true for

African American children and other children of color who experience diminished resources in their neighborhoods and public schools.

B. Statement of the Problem

In light of current understanding, mathematics has a significant influence on how individuals deal with key aspects of private, social, and civil life (e.g., Restivo & Van Bendegem, 1993). Conversely, research shows how social and cultural influences contribute to individuals' opportunities to learn mathematics and become empowered with mathematical fluency (Bishop, 1988a; Martin, 2000). Conspicuously absent from discussions on these influences is how play mediates these relationships. While there has been considerable effort to account for the influence of play in educational activities (e.g., Christie & Johnsen, 1983; Ginsburg, 2006; Kamii & Housman, 2000), there exist few studies of how play, mathematics learning, and language factors intersect to provide affordances for language minority students. A few noteworthy exceptions to this gap in the research are Hansen-Thomas (2009), Vásquez (2003), and Morales, Vomvoridi-Ivanović & Khisty (2011).

As educators move towards emphasizing discursive practices for teaching mathematics (Lerman, 2000; Walkerdine, 1990), language minority students may become even more excluded from effective practice unless their language issues are addressed (Bielenberg & Wong-Fillmore, December 2004/January 2005; Durán, 2008; Khisty, 1995; Moschkovich, 2007). There is much controversy over what type of language learning practices and theories are most appropriate (Gutiérrez, 2002, Téllez & Waxman, 2006). Language issues play an important role in understanding language minority students' mathematical achievement. Many states require testing of bilinguals in English after only one year of receiving specialized English language instruction (Rivera & Collum, 2006). Current research suggests bilinguals require 5 years or

more in order to master the academic language necessary for them to take full advantage of content area instruction in English (Cummins, 2009). When instruction is offered only in English, language minority students do not learn at the same rate as their English speaking peers. Under these circumstances, language minority students cannot effectively benefit from teachers' instructions nor can they fully understand assessment questions (Abedi, 2005; Abedi, Hofstetter, & Lord, 2004). Confounding this situation is the fact that there is no consensus among state education policies over how to assess and designate English language proficiency (Rivera & Collum, 2006).

This study links children's actions during game play with the Common Core State Standards for Mathematical Practice (2010). In my study I attempt to show that play-based activities, specifically mathematical games, engage children in mathematical thinking and learning practices outlined in the Common Core State Standards for Mathematical Practice (CCSS, 2010, pp. 6-8) listed below.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

There are three main reasons for my focus on mathematical practices as they are defined in the Common Core State Standards. First, there is a high demand for research that maps the Common Core State Standards to mathematics pedagogical practices and learning activities for classroom and out-of-school contexts. Second, the eight Common Core State Standards for Mathematical Practice succinctly summarize principles of mathematical thinking and practice found in mathematics education research. Third, there is a need to explore the mathematical

value of activities that counter traditional back-to-basics trends by uncovering alternate and effective routes to development and learning (Fisher, Hirsh-Pasek, & Golinkoff, 2012). An appropriate response to language minority students' underachievement is not the back-to-the-basics initiatives that currently dominate urban school policy (Gutiérrez, 2008). By mapping the eight Common Core State Standards for Mathematical Practice to play activities, I hope to contribute to the search for effective instructional pedagogies.

C. Definitions of Terms

1. Play and Games

Throughout my study I will refer to participants who are “playing” a game. In these instances I am using the common, non-academic version of “play” which I choose to mean “participants engaging in an activity” that is a “game”. My definition of mathematical games comes from Denzin (p. 463-4, 1975).

A [mathematical] game is defined as an interactional activity of a competitive or cooperative nature involving one or more players who play by a set of rules which define the content of the game. Skill and chance are the essential elements that are played over. Played for the amusement of the players and perhaps also for a set of spectators, games are focused around rules which determine the role that skill and chance will have. The objects of action are predetermined (checkers, chess players, dice). Furthermore, the elements of pretense are known beforehand and specified by the rules of conduct deemed appropriate to the game at hand. These rules specify the number of players, tell the players how to play, and determine how they will relate to one another as game players. Typically these rules hold constant outside game relationships and make each player a co-equal (subject to the variations of skill and chance) during the game itself. (However, some games deliberately build on inequality; they assign in sequence, or by chance, undesirable roles... [and]... winners and losers.

To the above I add that mathematical games are designed to highlight specific mathematical concepts embedded within the operations and rules of the game. It is also important to note that puzzles are subsumed under this definition and are considered to be a subset of mathematical games according to the definition above. Specifically, the clause that identifies mathematical

games as “involving *one or more players*” is crucial to my use of the word “games”. Even though folk definitions of games inspire a separation between games and puzzles, I use the word “games” interchangeably with “puzzles”. In other words, the card game solitaire is a “game” even though it involves only one player. Children in *Los Rayos* interact in small groups and “puzzles” become repurposed as either collaborative or competitive games. Finally, subsuming puzzles under the term “games” serves the additional purpose of maintaining an economy of words in my narrative.

2. Mathematics Ideology

Definitions of ideology vary according to the field of study (Gerring, 1997). For example, the disciplines of history, political science, psychology, and feminism all have different theoretical principles at work in their definition and use of ideology in their analytical frames. In critical discourse analysis, (Fairclough, 2003, p. 218),

Ideologies are representations of aspects of the world which contribute to establishing and maintaining relations of power, domination and exploitation. They may be enacted in ways of interacting (and therefore a *genre*) and inculcated in ways of being or identities (and therefore in *styles*). Analysis of texts (including perhaps especially *assumptions* in texts) is an important aspect of ideological analysis and critique, provided it is framed within a broader social analysis of events and social practices.

Following this reasoning, a *mathematical* ideology addresses how mathematics should be enacted, learned, discovered, created, and taught. In the context of this study, I am concerned with participants’ ideologies of teaching and learning mathematics. I expect that children and adults in the *Los Rayos* after-school club will display ideologies of teaching and learning mathematics in their actions and discourse. When I use the word ideology, I refer to an individual’s way of organizing activity and thinking. Mathematics ideologies of learning and instruction are centered on who is in control and who has power to direct activity.

3. Socialization

In mathematics education there is a need for new frameworks, perspectives, and methods to address both cognitive and social analyses of human thought and action (Biesta, 2009; Lester, 2010). A prominent movement to address this call comes through studies centered on socialization processes (Biesta, 2009). Examining mathematics socialization involves how students learn mathematical concepts but with a broader focus that includes how students “learn how to learn”. Such studies proceed from the assumption that mathematics learning is not only dependent on mastery of operations and applications, but also on meta-cognitive issues such as knowledge about one's thought processes and self-regulation during problem solving (Schoenfeld, 1992).

Socialization is “the way in which individuals are assisted in becoming members of one or more social groups”, (Grusec and Hastings, 2007, p. 1). Socialization research typically involves the examination of opportunities students have to participate in the process of becoming competent in mathematics as well as how these students accept or reject these opportunities. The notion of *socio-mathematical norms* (Yackel & Cobb, 1996) is instrumental in examining what these opportunities afford to students. In *Los Rayos*, prevailing norms carry the message that playing with mathematics is acceptable as is students’ use of Spanish, English, or hybrid forms of both.

There are several lines of research dealing with socialization processes. Cultural, emotional, gender, and class socialization are only a few examples of the lenses used in this field. Cognitive socialization (Gauvain & Perez, 2007) is the lens I use in this study. Schoenfeld (1992) points out in his often cited monograph on mathematical problem solving and metacognition that mathematical thinking practices are learned through indirect means, a

consensus shared by psychologists (Fletcher & Carruthers, 2012) and sociologists. Quoted in Schoenfeld (1992), Resnick summarizes this notion eloquently:

Several lines of cognitive theory and research point toward the hypothesis that we develop habits and skills of interpretation and meaning construction through a process more usefully conceived of as socialization rather than instruction. (Resnick, 1989, p. 39)

D. Research Questions

In the following paragraphs I state my research questions and briefly elaborate on each one in terms of what they are designed to uncover. The secondary questions are designed to inform the more generalized primary question with specific phenomena related to students' mathematical socialization through game activities in the *Los Rayos* after-school context.

Primary Question

1. How are bilingual students socialized in mathematics through play?

To answer this question I map the Standards for Mathematical Practice from the Common Core State Standards (CCSS) to participants' actions during game activities. In this study, actions refer to speech acts (discourse) and physical acts such as strategic manipulations of game pieces during game activities. For the purposes of this study, I view these mathematical practice standards as goals for mathematical socialization.

Secondary Questions

2. How do mathematical game activities in the after-school setting mediate affordances/limitations to mathematics socialization?

This question involves analyzing how participants interact with the rules of games and the participation styles suggested by the nature of particular games. For example, some games

are designed for competition while others are more collaborative. It was not uncommon for children in Los Rayos to change the participation style of a game to suit their own preferences.

3. How do mathematics ideologies mediate mathematics socialization for bilingual Latinas/os during play?

This question is aimed at finding the ideologies guiding participants' actions during the course of mathematical game activity. During game activities, participants' mathematical ideologies are visible through their game strategies and discourse. For example, when more experienced participants mark mathematical errors, they use a variety of communicative techniques to support and encourage children's understanding. The facilitators use suggestions, prompts, questions, hints, directives, praise, demonstration, and modeling. Children also use these communicative devices in the course of game play. This study will examine how these communication devices influence children's understanding, motivation, and persistence as well as how their use changes over the course of interactions in game play.

4. How do participants use language and cultural resources (Funds of Knowledge) as they negotiate systems of meaning in game play?

This question is aimed at documenting participants' choices of Spanish, English, or hybrid styles of language use. Children and adults in the *Los Rayos* after-school club are at various levels of bilingual proficiency which affects how they use their language resources in the context of game activities, varying their use of each language according to other participants' status, role, and preferences for interaction as well as for meaning making. Their cultural resources in this context may be reflected in participants' preferences for styles of participation. Children's home and family culture may also inform their prior knowledge when they provide

assistance to each other to understand rules, procedures, strategies, and mathematical reasoning during game activities. For example, a child in *Los Rayos* revealed a more complex set of rules for a dominoes game that she learned from adults during her family's game play activities.

E. Significance of the Study

This study provides research on the intersection of mathematics socialization with game activities that have not been sufficiently addressed. Although there is general agreement that children's mathematical content learning is enhanced through game activities, it is not clear how children engage with mathematical thinking and practice during game activities. This investigation provides a first step towards building an understanding of the possibilities for enhancing children's development of mathematical thinking and practice through game activities.

II. CONCEPTUAL FRAMEWORK

In this section I relate a general description of key concepts that will inform my analysis of how bilingual students are socialized in mathematics through games. The following concepts are implicit in the coding scheme I describe in my research methods.

A. Cultural Historical Activity Theory

Learning and socialization emerge as part of affordances being taken up and used for further action. This view of situated learning and socialization is different from the assumptions of the type of scientific research in which every input has an output, and every effect has an identifiable cause preceding it. Investigating learning from this perspective cannot be done effectively by using the methods of experimental science. New ways of doing scientific work are needed,” (van Lier, 2004, pp. 8-9).

Cultural historical activity theory (CHAT) represents the efforts of theorists to build a unified account of Vygotsky’s (1987a, 1987b) ideas on the nature and development of human behavior, which results from the integration of socially and culturally constructed forms of mediation into human activity. Leont’ev (1978), Luria (1957/1998), and Engeström (1999) are perhaps the most well known activity theorists who follow in Vygotsky’s tradition. Luria elaborated on the functional system resulting from the integration of artifacts into human psychological and social activity. He follows Vygotsky’s argument that the goal of psychology is to understand these functional systems (and their artifacts) and therefore must study their formation (history) and activity, not their structure. Applying this idea to mathematical game play, it follows that the game itself is not the sole affordance for mathematical socialization and learning. Affordances for mathematical socialization also depend on how the individuals negotiate social interactions, the extent of their prior mathematical knowledge, and the artifacts/conventions that are part of a particular activity context.

An important concept introduced by Vygotsky is mediation. People rely on tools, signs (symbolic tools), language, and activity to mediate and regulate relationships with others and to change the circumstances which govern their situations. Human culture creates physical and symbolic tools over time, making them available to succeeding generations that modify them during use or as they pass them on. The most important symbolic tools for consideration in the study of mathematical play are numbers, mathematical systems, and language. Humans use these tools to establish indirect, or mediated, relationships between themselves and the world. According to Vygotsky, the task of psychology is to understand how human social and mental activity is organized through culturally constructed artifacts. The term *semiotic mediation* has been expanded by Wertsch (1985) who introduced the phrase, *tool-mediated goal-directed action*.

Vygotsky used the metaphor, “zone of proximal development” (ZPD), to describe the site where social forms of mediation develop. He maintained that all higher mental abilities appear twice in the life of the individual. The process first appears on the intermental plane, distributed between the individual and cultural artifacts and/or some other person. Next, this faculty appears on the intramental plane, with the individual acting via psychological mediation. The ZPD is the difference between what a person can achieve when acting alone and what that same individual can accomplish when acting with support from someone else and cultural artifacts. According to Lantolf and Thorne (2006), the ZPD does not necessarily involve interaction between a novice and an expert, or more experienced other. In this sense, the ZPD is more of a collaborative construction of opportunities for individuals to develop their mental abilities. Novices do not merely copy the experts’ abilities but instead transform what is offered to them as they appropriate it. Imitation and collaboration are instrumental to this transformation. However,

imitation in the ZPD is not copying but rather a complex activity in which the novice is not viewed as a repeater but as a communicative being. In traditional school settings, the instructor frequently coerces the novice to produce an exact copy of what is offered in instruction. Play situations, on the other hand, open more opportunities for individuals to experiment with possibilities unique to the participants and the context.

The communicative acts formulated by more experienced participants as they attempt to level the novice's understanding with their own are *scaffolds*. However, in the spirit of CHAT, it is more germane to refer to the activity of *scaffolding* in order to emphasize the continual coconstruction of meaning between participants. In other words, scaffolding begins when planned pedagogical actions stops. A form of dialectic ensues and during that process, participants often engage in multimodal representations. These modes of communication are usually recognized as organized, regular, and socially specific means of representation (Jewitt, Kress, & Ogborn, 2001; 2000). Gestures, facial expressions, images drawn on paper, sound, and action are a few examples from a repertoire of communicational resources. In general, the concept of multimodality shifts the focus from language as the single source of meaning making. However, this should not diminish the importance of language in this process, and there is much to consider regarding language as it operates in the context of the playful, mathematical, and bilingual context of the *Los Rayos* after-school program.

Understanding the process of co-constructing meaning involves an analysis of the social process of learning and assumes that meaning does not reside solely in the mind of the learner. If learning is viewed as a situated activity, then the central defining characteristic of this process involves legitimate peripheral participation (Lave & Wenger, 1991). In this sense, learners participate in communities of practitioners and move toward full participation in the social

practices of the community. Using the concept of legitimate peripheral participation allows for discussions about the interactions between more and less experienced group members with respect to their activities, identities, knowledge, practices, and artifacts that are used or produced in the situation.

B. Discourse

References to “discourse” in this analysis is connected to a particular view of language that takes into account ‘texts’ occurring in print, as visual/photographic images, social events, and the structuring of these and other elements present in everyday life. These representations of text are all closely interconnected and further references to “language” should be understood as verbal language in the form of words and sentences, either aural or lexical.

Linguistic modality is concerned with expressions associated with notions of possibility and necessity. Textual analyses of modality concentrate not only on the use of a particular modal, but also on where the modal occurs in a sentence, the meaning of the sentence independent of the modal, the conversational context, and several other factors. For example, the word ‘should’ is a statement of inference or knowledge (epistemic modality) or a statement of how something ought to be (deontic modality). Such markers are seen as intermediate categories between speakers’ varying degrees of commitment to truth or necessity (Halliday, 1994; Fairclough, 2003). Modality is a key concept for interpretations of speakers’ social and personal identities as well as their ideologies. It is important to know how students’ styles of learning, communication, participation operate not just to change teaching style or to target “deficits” but also to use these observations to provide scaffolds to guide students in other forms of participation, thinking, language, and math problem solving strategies (Gutiérrez, 2002).

The study of socialization in mathematics necessarily addresses language use as well as the tools and social networks learners use as they negotiate mathematical norms of acting and thinking. Experimentation with strategy, attentive observation, symbolic reasoning, interpretation of physical events, translating between different systems of representation, acting on evidence, argumentation, and use of the mathematical (formal and informal) register are some of the most prominent markers of mathematically socialized behaviour. These markers are visible and indexed primarily through discourse, participants' physical actions as they manipulate tools and objects, and their production of physical artifacts such as writing, drawings, charts, graphs, pictorial representations, and models. Because mathematical socialization refers to a process, it is not necessary for participants to successfully learn mathematical concepts in order to become socialized into mathematical thinking and acting. Instead, the focus is on what happens on the way to either successful or unsuccessful problem solving.

III. REVIEW OF LITERATURE

A. Mathematics Socialization

The concept of mathematical socialization is based on the premise that mathematics is a cultural product. This view is supported by studies that embrace the notion of culture as a *community of practice* (Lave, 1988; Wenger, 1999). Bishop (1988a; 1988b) elaborates by stating that all cultural groups generate mathematics and that “Western” mathematics is one among many current and historical methods of performing mathematics. Ethnomathematics studies (e.g. Nunes, Schliemann, & Carraher, 1993; Saxe, 1994) further document these practices, providing evidence that mathematical competence takes many forms. Implications for dealing with bilingual/bicultural students point to the challenges of overcoming “hidden” values in teaching practices that devalue students’ social identities and linguistic and cultural capital. Culturally responsive schooling uses curricular material and methods to enforce connections between home and school, acknowledging the value of children’s knowledge base, and engages students in meaningful activities based on everyday mathematics to scaffold the recontextualization of school mathematics concepts (Brenner, 1998).

There exists a potential for educators to miss powerful opportunities for recontextualizing students’ cultural practices as demonstrated by Nasir and Hand (2008) in their study of African-American children’s mathematics as manifested in their basketball and dominoes games. Students could easily perform complex mental mathematics when mathematical questions were framed in terms of basketball related situations involving scoring percentages and associated statistics but could not duplicate such competence when asked school-based mathematics

questions with identical mathematical operations and concepts. Likewise, children's complex strategies for dominoes did not translate into successful problem-solving tasks recontextualized as school mathematics questions (Nasir, 2008). This corroborates the assertion by Pellegrino, Chudowsky, and Glaser (2001) that experts in a content domain have their knowledge "encoded in a way that closely links it with the contexts and conditions for its use", (p. 73).

Various studies have outlined norms and values associated with different mathematical communities of practice, namely, research/practicing mathematicians (e.g., Burton, 1999; Davis & Hersh, 1981), K-12 classrooms (e.g., Yackel & Cobb, 1996; Gorgorió & Planas, 2005) and everyday or "street" mathematics (e.g., Lave, 1988; Nunes, Schliemann, & Carraher, 1993). My study focuses on children's socialization in K-12 mathematics practices represented by the Common Core State Standards which share some of the characteristics of research/practicing mathematicians.

The various forms and contexts of mathematical practice need not be in contradiction with each other. Different contexts for learning mathematics can be complementary and productive for learners' overall mathematical development. However, there are also contexts of mathematics experience that limit and even negatively impact learners' trajectories. Jackson (2007) studied the mathematics learning trajectories of two middle school students and their parents with emphasis on the relationship between the social construction of mathematics and the social construction of learners. One of the two students in Jackson's study experienced a narrow and limiting mathematics instruction regimen and was further limited by his teacher's negative social construction of his mathematical identity and academic potential. The other student realized and sustained an identity of a successful mathematics student. She persevered in her academic pursuits with her mother's help in spite of difficult home conditions. There are few

studies that detail the complex trajectory of mathematical “success” under challenging circumstances, with Martin’s (2000) study being a notable exception. Jackson’s detailed study illustrates the complex interactions of numerous social factors (e.g., identity, ideology, power relations) involved in learners’ engagement with mathematics. My summary of this study is an essentialization of some of Jackson’s main findings but it is an important work to consider in light of the fact that mathematics socialization is a process that has not been fully explored.

B. Previous Studies on Games in the *Los Rayos* After-school Club

Three prior studies on the *Los Rayos de CEMELA* after-school club (Razfar, Khisty, Willey, & Radosavljević, 2008; Willey & Radosavljević, 2007; Morales, Vomvoridi-Ivanović, & Khisty, 2011) contributed significantly to the conceptualization of my proposed study. In examining these instances of game play, I found some of the factors relevant to mathematical socialization, leading me to extend these initial observations to a larger data set, namely all instances of game play in the after-school. I will note within these descriptions particular findings that have implications for the mathematics socialization of the students in my study.

The first study (Willey & Radosavljević, 2007) focuses on one of several game activities in the after-school project, the *Counters Game*, which is based on probability concepts. In terms of procedure, each player has equal status, an identical role, and identical operations to perform in order for the game to proceed. In addition to introducing students to probability concepts, the *Counters Game* provides a different type of drill and practice in basic arithmetic operations that is multimodal and rooted in social goals. Adding the sums of the numbers on the dice is necessary for every turn of game play. It is quite different from typical classroom exercises like worksheets that require students to solve multiple versions of similar arithmetic problems in the name of promoting automaticity in basic operations.

In this study, Willey and Radosavljević (2007) described three episodes of game play, each illustrating different implications for alternative mathematical learning activities. The first episode showed how Natalia, a third grader, became motivated to learn a mathematical strategy for winning after losing several rounds of the game. The second episode illustrated how a didactic, non-playful instructional pedagogy negatively impacted students' opportunities to advance their mathematical understanding of the *Counters Game*. Finally, the third episode revealed how creative tensions between players over the fairness of game rules led to a rule change that increased the complexity of the underlying mathematics principles.

Inspired by findings in the previous study, the authors of this second study (Razfar, Khisty, Willey, & Radosavljevic, 2008) analyzed all rounds of the Counters Game in the after-school in order to examine the role of problem solving in this game. We focused on *language ideologies* work (see Razfar, 2005) in order to understand the function and purposes of language use (i.e. code-switching) in relation to the values and beliefs of the participants as well as the larger sociopolitical context. As activity systems, the CEMELA after-school club and the *Counters Game* are both intentionally and explicitly designed on the basis of language ideologies that encourage the use of all available linguistic and cultural tools among the participants which in this case includes Spanish, English, and hybrid language practices.

We isolated approximately 177 minutes of video containing combinations of all levels of participants (intergenerational, expertise) engaged in playing the *Counters Game*. Focusing on mathematical problem solving as mediated by multiple modalities within the game, we developed codes that included 1) *Language Choice* 2) *Code-switching* 3) *Epistemic Stance* 4) *Assistance Strategies (with special emphasis on the use of questions, types of questions, and direction of questions)* 5) *Repair* 6) *Instances of disagreement between participants*.

Through our analysis of the participation of the *Counters Game* we found complex, multilayered, and multimodal discourse practices mediating the participation of children playing the *Counters Game*. With respect to language choice, we found participants switched between English and Spanish regularly and in purposeful ways. There were over 500 instances of Spanish use, 270 instances of English use, and nearly 100 instances of code-switching or hybrid practices. Code-switching occurred systematically for the purposes of assistance, making tasks more comprehensible, asking questions, joking, inclusion, and sometimes exclusion of central and peripheral participants. Children often indexed their awareness of speakers and non-speakers of a particular code by switching to accommodate understanding. The amount of Spanish use was substantially more than English (approximately 2:1) and this ratio increased over time as their game play progressed. We also examined numerous instances of ‘cognitive’ and ‘discursive’ tension (nearly 250). Sometimes the tension related to ‘fairness’ suggesting self-regulation of fairness within the game play and other times it coincided with participant attempts to repair or correct one another. The ensuing disagreements about the nature of the correction suggested how meaning making is never absolute and is contested terrain. In terms of mathematical socialization, this study provides evidence of how mathematical meaning making is mediated by influences that are not typically considered “mathematical”.

A third study (Morales, Vomvoridi-Ivanović, & Khisty, 2011) focused on one child, Rodrigo, who eventually became more confident and assumed more agency over time as he engaged with language and cultural resources mediated through his experiences in the *Los Rayos* after-school club. Rodrigo responded to the cultural and linguistic resources that were made available to him. At first, the facilitators thought that he was English dominant and very shy. After Rodrigo’s experience with an undergraduate facilitator who spent considerable effort and

time attending to questioning and other scaffolding techniques, Rodrigo began to assume more control over the direction of game and other play activities. Researchers and facilitators noticed that Rodrigo and the other children still displayed characteristics of English dominant speakers and were concerned that their intent to encourage students' use of Spanish as a resource for mathematical meaning making was frustrated. As a result, facilitators decided to expand the involvement of childrens' parents by inviting them to participate in playing the *Counters Game* which focused on probability concepts. The result was that children, especially Rodrigo, displayed their Spanish speaking skills and became noticeably more enthusiastic in their participation with *Los Rayos* activities. During the *Counters Game* activity with the parents, Rodrigo used Spanish exclusively and showed that he could play the game strategically. In an excerpt of Rodrigo's explanation in Spanish of the probability principles in the *Counters Game* to an undergraduate facilitator, Rodrigo observed that there is more probability of getting a 7 from a roll of dice because there are more possible combinations of 7 than other numbers. "In other words, Rodrigo's thinking was not only based on an experiential model or playing the game (experimental probability), but also demonstrated that he was able to think about the game theoretically (theoretical probability)" (Morales, Vomvoridi-Ivanović, & Khisty, 2011, p. 184). Rodrigo's trajectory towards assuming more agency in his activities eventually led him to create his own game using components of several different games he played in *Los Rayos* with assistance from his mother. Rodrigo's mother also shared her own past experience with a mathematical game she played with in her youth during the after-school sessions. Her son Rodrigo also incorporated a key principle from her game in his new game. This series of interactions showed that Rodrigo's transformation was mediated by several resources found in *Los Rayos*. Linguistic resources (use of Spanish) and cultural resources (Rodrigo's mother, the

undergraduate facilitator, and game activities) were important influences on Rodrigo's mathematical development.

C. Persistence and Motivation

Persistence and motivation to achieve mathematical competence is determined by many socializing agents found in children's family, school, and community environments. Learners, community members, and teachers are not neutral variables but active participants in the processes of learning. What the research literature fails to explain, according to Martin (2000), is how some African-American and other minority students manage to succeed despite their environmental circumstances. Further, he notes that reductive measures of mathematics achievement essentially characterize these students' failures as their own fault instead of focusing the blame on educational institutions and practices. Even more revealing is that Martin's interviewees related how their persistence and motivation to pursue mathematics was affected by both overt and tacit messages they received in school.

The socialization of cognition outside of school is relevant to the after-school context in my study because *Los Rayos* is structured to accentuate the roles of children's peers. Foremost in this perspective is that "interaction" alone is not enough for productive development. When activities with peers involve problem-solving tasks, it has been shown that participants are more likely to formulate advanced strategies and better meta-cognitive understanding of strategy (Fleming & Alexander, 2001). In another study of problem-solving tasks with peers, children performed better on later individual tasks when partners shared responsibilities during group interactions (Gauvain & Rogoff, 1989). However, social interaction does not promote learning when partners display directive and disapproving behavior (Gauvain, Fagot, Leve, & Kavanaugh, 2002).

Viewing CCSS Mathematical Practice Standard 1: Make sense of problems and *persevere* in solving them, it is apparent that the concept of persistence is important for learning mathematics and is a hallmark of mathematical thinking practice. Research mathematicians are accustomed to working on a single problem for a very long time - for months or even years yet this is difficult for children or even adults to comprehend if they are only familiar with school mathematics.

D. Play

Vygotsky's (1987a, 1987b) theories attempt to account for play as a developmental activity. In general, his findings on play and learning can be indexed with student's attempt to experiment with rules and concepts inherent in an educational setting. He stresses the role of imaginary play in the development of children's abstract thought and as the first manifestation of a child's emancipation from situational constraints. Behavior and thought become separated from the immediate perceptual field in imaginary play. He further concludes that the separation of meaning from objects and actions have different consequences. Operations with the meaning of objects lead to abstract thought while operations with the meaning of actions index development of the ability to make conscious choices. These conclusions are significant with respect to mathematics learning because play situations encourage abstract thinking and stimulate processes related to semiotic mediation in which symbols become tools for thought and conversely, physical tools take on symbolic values.

“Choice is often presented as a defining feature of play and it has been argued that a perception of choice contributes to the developmental potential of play” (King & Howard, 2012). Choice, or *perceived control* (Schunk & Pajares, 2005), is important in this study because the research on competence points to a strong link to motivation and *perceived self-efficacy*

(Bandura, 1986, 1993). Schunk and Pajares (2005) note that “...it is demoralizing to believe that one has the capabilities to succeed but that environmental barriers (e.g., discrimination) preclude one from doing so” (p. 91). This point is relevant to the *Los Rayos* context because, for example, children are not barred from their choice of language (Spanish, English, or hybrid combinations of the two), choice of activity, or their choice of peer and adult affiliations.

The following discussion attempts to catalogue some of the more recent developments in the field of play research with a focus on theories and studies that justify the role of play in mathematically related thinking and its role in culture.

E. Mathematical Play

The field of play research yields some rigorous descriptions and tentative definitions for types of play that resonate with characteristics of mathematical activity. Holton, Ahmed, Williams, and Hill (2001) identify types of play that are most relevant to students’ cognitive gains in mathematics learning by comparing commonalities among a number of studies related to mathematics play. Equating experimentation with object play, Holton et al (2001) maintain that scientists (and I include mathematicians in this category) engage in play behaviors in their actual practice. But, as Holton et al (2001) note, there is difficulty in arriving at a consensus on the relationship between play and experimentation. Experimentation occurs in a novel situation with the directional goal of determining the properties of an object. Vygotsky makes the distinction that the goal of experimentation in play is to determine ‘what can be done with this object’ as opposed to ‘what does this object do’.

Rubin (1982) offers the following ideas concerning the importance of play in children’s development in his review of mathematically relevant play studies before 1980:

- play allows the child to transform reality and thus to develop symbolic representations of his world;

- play aids in the development of creativity and aesthetic appreciation;
- play in childhood allows the practice and mastery of activities which are later useful for serious endeavors in adulthood;
- play serves a cathartic function in development.

He notes that mathematical play does not necessarily lead to a correct solution of a mathematics problem. Like professional mathematicians, the solver may explore many different strategies that produce incorrect solutions. This process is useful because mathematical play activities provide a non-threatening environment where wrong solutions are not stigmatized as mistakes but instead lead to a better understanding of the problem and helps the solver confront misconceptions. Bruner (1985) maintains that the teacher's role in these situations entails asking questions that make misconceptions clear while providing scaffolding questions or additional information that points the student in a more productive direction.

Ainley (1990) studied the Germany Mathematics Project, which used games designed to encourage participants to make conjectures. She asserts that in a regular classroom setting, children are not likely to make conjectures because of the stigma associated with making errors in front of the teacher and entire class. Also, she states that games provide a safe space where conjecturing is natural, thus providing opportunities for participants to talk explicitly about the process of conjecturing. In many games, it is not possible to learn all the best moves for all possible configurations. Therefore, Ainley maintains that it is natural for students to devise general strategies. While this is true for a number of mathematical games, there exist other games specifically designed to limit the degrees of freedom for strategizing so that the implicit mathematical learning goal may become apparent in a relatively short period of time.

F. Mathematical Play in After-School Sites

After-school projects inspired by the *Fifth Dimension* (Cole & The Distributed Literacy Consortium, 2006) at various sites in the United States have yielded the most promising results regarding mathematical problem-solving. One study at the Santa Barbara site sought to find whether Fifth Dimension students would show a greater gain in their ability to comprehend arithmetic word problems than matched children who had not attended the after-school club (Mayer, Schustack, & Blanton, 1997). Sixteen third- and fourth-grade students were individually matched for grade level, English language proficiency, schoolteacher, and gender with the majority speaking Spanish as their primary language. They participated in a word problem comprehension test designed to measure their skill in representing word problems, with a pre-test at the beginning of the academic year and a post-test at the conclusion of the school year. Computational solutions were not required. Instead, the test included three types of items: word problems which directed them to select one of four equations to match each problem; word problems requiring selection of numbers appropriate for problem solutions; and word problems asking for the selection of appropriate arithmetic operations. For all types of problems, the treatment group showed significantly greater gains than the control group and the difference was consistent with the results of a delayed post-test administered the following fall semester. These gains remained consistent when the treatment group was compared to all third- and fourth-grade students at the same school matched for pre-test scores. In addition, the study was replicated at an Appalachian State University site with comparable results.

In another study at the Fifth Dimension Escondido site, twenty-five children ages eight to twelve were classified into two groups; one with substantial experience in the Fifth Dimension and the other with minimal exposure. These children played a problem-solving computer

software game, “Puzzle Tanks”. The game requires players to pour juice into tanks of various sizes in order to obtain a particular amount of juice with rules specifying procedures for filling, emptying, and measuring. Measures of success in the game involve both the number of problems completed successfully as well as the level of sophistication in strategies employed. Also, the game included several types of the same general problem. Children with greater exposure to the Fifth Dimension were more successful according to quantitative measures. Using the same game at the North Carolina site, researchers analyzing videotapes of children’s game play found that student learning went beyond mastery of problem-solving strategies (Blanton, Simmons, & Warner, 2001). Students learned to make rapid and appropriate changes in tool use, invented new tools, used the same tool independently and simultaneously, and moved in both directions between levels of joint activity as they collaborated with peers while providing and receiving guidance.

IV. RESEARCH METHODS

A. Overview

The purpose of this chapter is to describe the procedures used to answer the question: How are children socialized in mathematical thinking and practice through games? Guided by CHAT theory, this study uses qualitative methods to analyze video data triangulated with field notes, student artifacts, and electronic messages between students and the fictional mathematics wizard, *El Maga*.

B. *Los Rayos* After-school Context

The data collection was conducted in a large, urban public school district in the Midwestern United States. In the middle of this large urban area is the Rivera Dual Language Academy, the site of the *Los Rayos de CEMELA* after-school club. Rivera is made up of approximately 425 students in Pre-Kindergarten through 6th grade. At the time of this study, the student population of Rivera was 97.4% Latina/o, approximately 1% White students, and had a relatively high mobility rating of 25.6%. Most of Rivera students (98.3%) were eligible for the government's free or reduced lunch program and 68% were categorized as English language learners (School District Data², 2007). As a "World Language Magnet Cluster School", Rivera offered a Dual Language Program in Spanish and English in which it was expected that all students would achieve academic language proficiency in both languages by the time they graduated (School District Data, 2007).

Los Rayos was a project loosely modelled after the work of *The Fifth Dimension* (Cole

² This citation does not appear in the Cited Literature section in order to protect the identities of the participants and their school.

and The Distributed Literacy Consortium, 2006) and *La Clase Mágica* (Vásquez, 2003). The main research focus of CEMELA was to explore the intersections between mathematics, language, and culture in order to show the importance of academic biliteracy in learning mathematics. In general, CEMELA goals were also focused on discovering research-based recommendations to address educational inequities impacting Latina/o bilingual mathematics learners.

CEMELA researchers in *Los Rayos* explored how to create mediating environments to empower students, parents, and university partners to bring together funds of knowledge in mathematics. Designed to give Latinas/os experiences with non-remedial mathematics in curriculum topics beyond the students' grade level, topics included probability, algebra concepts, geometry, combinatorics, and complex problem solving. Reading, writing, speaking, and listening skills were emphasized in both Spanish and English with a primary focus on students' home language, namely Spanish. Activities were designed to encourage students to play and have fun with mathematics in collaboration with their peers and adults. As a rule, facilitators promoted childrens' self direction in choosing activities and activity partners.

Los Rayos was an after-school setting that was not quite school but not quite free play. There were no assignments, tests, drills, lectures, assigned seats, punishments, or requirements for children to be silent. Children were free to pick who they wanted to be with, and to change their affiliation at any time, which they did frequently. When they did not feel like participating in a particular game or other activity, they moved to another group, sought out a friend, or simply said that they did not want to "play" or "work" on an activity anymore. To label all their actions as "play" is not appropriate. They were, after all, in an institutional setting and were not allowed to leave until the after-school session ended for the day. There was an asymmetric power

relationship between children and adult participants and this is true for many contexts in which Mathematics is often constructed with help from more experienced participants. However, the children received mathematics instruction in their regular classrooms and came to the after-school with their own mathematical knowledge and ideologies.

The children in *Los Rayos* forged relationships with the adult facilitators and were happy to be with them. In interviews with children at the end of their 2nd and third years of participation, many described their relationship with the undergraduate pre-service teachers (UGs) in terms of having a big sister or big brother. The purpose of populating *Los Rayos* with an abundance of adult participants was not to control children but rather to give them access to adults who would listen to them, interact in meaningful ways, and guide them in activities that fostered communication. It was also a goal to expose pre-service teachers to children outside the typical classroom setting in order to provide meaningful, individualized interactions between children and adults.

The *Los Rayos* context represented a major influence on the mathematical socialization of these participants. The emphasis on mathematics was an overtly stated goal in the recruitment of the participants and the activities were chosen by CEMELA facilitators to reflect this focus. Activity prompts were presented in both Spanish and English, allowing the children to choose and use either or both languages. They were frequently encouraged to read aloud and explain the activity instructions and game rules.

The after-school sessions occurred twice per week for 10 weeks of every school semester for 90 minutes from 2:30 to 4:00 p.m. in a classroom at the Rivera Dual Language Academy. A typical day at *Los Rayos* proceeded as follows:

A typical day in the after-school

- students arrived and chose partners (15 minutes)
- students examined and chose activities (5-10 minutes)
- students engaged in activities (45-60 minutes)
- students wrote to *El Maga* (10-15 minutes)

The students sat at desks with their UGs and their activity was recorded by stationary video cameras although sometimes cameras were operated by roving facilitators. Typically there would be 4 to 5 groups engaging in activities independently so that each one of four stationary cameras captured a separate group. For example, on October 25, 2006 there were four videos. Each video documents a separate group with different participants and different activities. On that day, two groups engaged in mathematical games and the other two groups engaged in a probability and a pattern-finding activity.

The norms of *Los Rayos* dictated that the choice of activities should be a product of negotiation between the facilitators and the children. The facilitators had prepared a collection of games and other mathematical activities for the children to choose from. These activities were written in both Spanish and English on sheets of paper bound in notebooks and included the necessary manipulatives, pictures, and diagrams, while other materials such as graph paper, colored paper, lined paper, pens, pencils, crayons, markers, glue, scissors, game pieces, calculators, protractors, compasses, etc. were always available to the students in the classroom reserved for *Los Rayos*. Students would arrive at the after-school, choose an UG (facilitator) and group they wanted to be with, and then examine the available materials to decide what type of activity to engage in. Frequently, one group of children would pick an activity and other groups would want to engage in the same activity. In this way, game activities occurred in several

groups simultaneously. On some occasions the facilitators steered the groups to pick certain activities.

1. Elementary school participants

The children in *Los Rayos* were all bilingual Spanish/English speakers with varying degrees of proficiency in each language. All the children attended the same school, the Rivera Dual Language Academy. The data corpus covers a time that spans the children's third grade and fourth grade academic years.

Children attended the after-school twice a week in 90-minute sessions during the course of the school year, 10 weeks from September to December in the fall semester and 10 weeks from January to May in the spring semester. Attendance was high at approximately 90%.

The school had several alternatives for after-school activity groups such as tutoring, sports, and musical instrument lessons. These alternatives represented a potential obstacle for recruitment to the *Los Rayos* mathematics club, especially since mathematics is not always considered a "fun" activity. On the other hand, it is possible that parents believed that *Los Rayos* represented a mathematics tutoring opportunity that would allow their students to become more academically proficient. Additionally, CEMELA researchers theorized that students and their parents might be discouraged from participating in *Los Rayos* because of the stigma attached to being recorded on videotape as part of an educational study.

Before the after-school program began, the targeted number of students was between 12 and 20. This number was considered ideal for the type of interactions planned. It was thought that students would engage with facilitators in groups of 2 to 4. The elementary school participants were recruited as follows. In the late fall of 2005, teachers read an announcement to third grade students. At a later date, CEMELA representatives came to the school to give a

presentation to the two combined third grade classrooms on the nature of the after-school club, *Los Rayos de CEMELA*, which was to begin operation for the spring semester of 2006. Parents were also invited to this session via a letter in English and Spanish previously sent home with students. All in attendance were given a handout describing the club activities and the goals of the research study. The handout also contained a consent form to be signed by both parent and student.

2. Undergraduates (UGs)

Undergraduate facilitators (UGs) were recruited with an email announcement and interviewed by CEMELA Fellows and Principal Investigators (PIs). The criteria for choosing the UGs was that they should be bilingual Spanish/English, preferably in the University pre-service teacher program, and willing to work for an hourly wage. They participated in weekly meetings with CEMELA Fellows and PIs who led them in activities that informed them of CEMELA goals. The undergraduates also planned activities, analyzed after-school data, and wrote responses to children's emails to El Maga, the fictional mathematics wizard who presided over the after-school club. Their role in weekly meeting with PIs and Fellows continued until spring 2007 when scheduling conflicts prohibited their participation. This scheduling conflict became an opportunity for one of the CEMELA Fellows to create a different meeting schedule that led to more debriefing about the UGs' experiences and to developing their proficiency in Spanish academic language in mathematics concepts.

The undergraduates' presence represented a significant portion of the socializing influences children experienced in *Los Rayos*. As far as the UGs' expertise with mathematics instruction, it must be said that they did not have any experience as professional teachers nor had they completed a significant amount of teacher education classes. *Los Rayos* was their first taste

of what it might be like to facilitate a group of children in an informal educational context. The UGs' participation in the after-school club was part of CEMELA goals to give pre-service teachers early field experiences in their program in order to acquaint them with real students' thinking and learning needs. It was, in effect, an attempt to socialize the undergraduates to their students' culture. Socialization, after all, is not a one-way interaction and teachers are socialized to adapt their teaching styles to children's thinking and learning.

The UG facilitators' attendance at *Los Rayos* was sporadic due to their university class schedules. However, the minimum number of UGs at a given session was never less than 4. Combined with the presence of graduate fellows, there were always enough adult facilitators to accommodate at least 4 groups of students and many one-on-one interactions. One UG occupied the role of data manager whose duties included setting up and monitoring video recorders as well as cataloging the recordings.

3. Fellows (Grads)

The Fellows were recruited by CEMELA's lead PI, Dr Lena Licón Khisty, and were graduate students in the sponsoring university's College of Education. The criteria for recruiting the Fellows was that they should have experience with K-12 teaching, be bilingual to some degree but not necessarily in Spanish, and committed to CEMELA goals revolving around the values of mathematics and bilingual education. Fellows received tuition waivers and a stipend and were expected to occupy lead roles in facilitating the after-school club which included planning, data collection, working with elementary school students and their parents, UGs, and the PIs.

4. *El Maga* the mathematics wizard

The fictional mathematics wizard, *El Maga*, existed for the students of *Los Rayos* in the form of email messages that were personalized for each student. In reality, UGs and Grads wrote these messages but kept the identity of *El Maga* secret. Even *El Maga's* gender was ambiguous since the construction of the wizard's name is gender neutral in Spanish. *El Maga* was all-knowing and an expert mathematician. Activity directions were written as a first-person form of address from *El Maga* and emails from the wizard frequently encouraged students to explain their role in the after-school activities, encouraging students to extend and amplify their mathematical understanding.

C. Data Corpus

The videos are characterized as follows:

- 35 meetings of Los Rayos between February 9, 2006 (first day) and Jan. 31, 2007
- 106 videos of independent groups during those 35 meetings
- 44 videos of game sessions that occurred on 23 separate days (i.e. 1 or more groups playing games on some days)

Additional Data from the same time span used for triangulation:

- written email communications between the *Los Rayos* students and the fictional mathematics “wizard” *El Maga*
- field notes from grad students and UGs
- writing & calculations, diagrams, digital stories, posters, and photographs produced by *Los Rayos* students

Although Los Rayos continued meeting until December 2009, the activities became more project-based and the activity books with games, puzzles, and worksheets were seldom used after

January 2007. Because my study is focused on game activities, I am excluding video data captured after January 31, 2007.

Descriptions of the games and mathematical content found in the data corpus are listed in Table I Descriptions of Games.

TABLE I
DESCRIPTIONS OF GAMES

| Games | Description | Mathematics Content |
|------------------|---|--|
| Blokus | Participants take turns placing their 21 pieces (different sizes and shapes) on a square board with a grid. Each piece is a combination of 1 to 5 squares arranged in different configurations. The winner is determined when a player uses up all their pieces or when no space remains on the game board and points are calculated based on each player's left over pieces. | spatial relations, geometry |
| Counters Game | Participants take turns rolling dice and removing game pieces strategically placed on the numbers 2-12. The winner is the person who removes all their pieces first. | probability, patterns, addition, data collection |
| Chess | Two players, each with 16 game pieces on a square checkered board, take turns moving their pieces to strategically trap their opponent's main game piece, the "king". | strategic knowledge, induction, deduction |
| Chinese Checkers | 2, 4, or 6 participants each with 10 game pieces take turns moving one piece at a time to relocate them to another section of the game board. The winner is the first to completely relocate all their pieces. | strategic knowledge, induction, deduction |
| Colored Dice | A pair of dice, each die a different color, is rolled repeatedly. Each roll is recorded on a 6 by 6 grid to illustrate sums | probability, patterns |

TABLE I (continued)

DESCRIPTIONS OF GAMES

| Games | Description | Mathematics Content |
|---------------|--|--|
| Dominoes | Participants use a set of 28 tiles, each with a unique combination of 2 numbers from 0 to 6. Each player begins with 7 tiles, selected at random, and the remaining pieces are used to replace each participant's stock of 7 pieces as the game progresses. Players take turns placing one tile at a time in configuration to match like numbers and attempt to strategically block opponents' moves. The winner has the highest score calculated when all tiles are used. | addition, strategic knowledge |
| El Maga's Hat | A fixed number of different colored cubes are place in a bag. Players take turns removing one cube at a time without looking in the bag and predict how many cubes of each color remain. The winner is one who predicts the closest approximation to the actual number of cubes of each color. | probability, prediction |
| Grocery Cart | Players use a spinner and move game pieces to different sections of the game board. Each section has instructions for a different grocery store transaction. The winner is the player who makes the best purchases, makes correct payment, and keeps accurate records. | arithmetic operations, estimation, approximation |
| Higher/Lower | A single deck of standard playing cards is used. Players guess whether the next card drawn from the deck will be higher or lower in value than the previous card. | probability, estimation |
| Mancala | A two player "count and capture" game. Players place a number of stones in pits on a game board arranged in rows and take turns with the object of capturing the game pieces. | arithmetic strategic knowledge |

TABLE I (continued)

DESCRIPTIONS OF GAMES

| Games | Description | Mathematics Content |
|-----------------------|--|---|
| Othello | Two players compete to turn each other's pieces over, each using a set of disks, light colored on one side and dark on the other, on a board with 8 rows and 8 columns. | patterns, prediction, strategy |
| Race to the Moon | Players toss dice and fill out a table cataloguing numbers from each roll of the dice. The object is to fill in enough numbers to reach "the moon" (the upper limit of a probability distribution). | data collection, probability, patterns |
| | | |
| Spinner Games | Students create various spinners (a movable pointer mounted in the center of a circle which is divided radially into equal fractions). Students spin the pointer and move pieces on a game board according to the location of the pointer on the circle. There are several versions of this game. | probability |
| Student Created Games | <i>Los Rayos</i> third graders are prompted to create their own mathematical games. | arithmetic operations, spatial relations |
| Tower of Hanoi | Participants take turns trying to find the least number of moves to complete a puzzle. 7 disks piled on top of each other in ascending order of size must be moved from one location to another, one at a time while preserving an order of ascending size. | recursion |
| Travel Math | Players use a spinner and move game pieces to different sections of the game board. Each section has instructions for a different travel-related mathematical task such as mileage calculations, cost of trip, and purchases for travel supplies. The winner is the player who makes the most economical trip, best purchases, makes correct payments, and keeps accurate records. | addition, subtraction, multiplication, division |

D. Methods

This study employed qualitative methods to examine the processes involved in mathematics socialization through game activities. A qualitative approach places emphasis on *process* and is highly dependent on context.

Traditionally, the collection of research video and materials is conducted according to a plan and a set of research questions determined beforehand (Barron & Engle, 2007). However, there have been occasions when previously collected and archived video and other materials must be analyzed (e.g., Leonard & Derry, 2006; Hiebert, Stigler, Jacobs, et al, 2005). The amount of accumulated recorded observations from the CEMELA project is unlike many previous studies in this field and stems from the exponential growth of technology which has made data collection more efficient in the last ten years.

Once I isolated all of the video data containing game play, my analysis consisted of two main phases. First, I viewed all of the video data several times using a constant comparative method (Glaser, 1965) and created summaries of the activities. Second, I chose three vignettes as case studies for closer analysis, transcription, coding for language tensions and, subsequently, coding for mathematical practices.

My rationale for choosing three vignettes containing the game Tower of Hanoi as a set of interrelated case studies is based on the fact that they represent a comprehensive description of mathematical socialization processes found throughout the data corpus. The Tower of Hanoi problem represents an advanced mathematical concept, recursion, typically not introduced to mathematics curricula until high school. Furthermore, there exist several studies on the Tower of Hanoi that assisted my analysis (Anderson, Albert, & Fincham, 2005; Anderson & Douglass, 2001; Anzai & Simon, 1979; Handley, Capon, Copp, & Harper, 2002; Jia-Jiunn, & Fu-Mei,

2012; Karakus, Inal, & Cagiltay, 2008; Welsh, 1991; Zook, Davalos, DeLosh, & Davis, 2004). Perhaps the most famous Tower of Hanoi study was conducted by Piaget (1976) who explored the association between formal operational thought and executive function. All of these studies were conducted in laboratory contexts with experimental controls and only one (Welsh, 1991) addresses the effects of assistance strategies on children playing the Tower of Hanoi. Even so, the study Welsh (1991) conducted does not match the informal and naturalistic context of *Los Rayos*. What these studies contribute to my understanding of children's mathematical play with the Tower of Hanoi is that the problem-solving strategies needed to solve the more complex versions of the game entail a demanding cognitive load.

The two main units of reference in my method are speech acts, or what participants say while engaged in game activities, and "game acts", the operations participants perform while manipulating the physical objects used in game play. Mathematical strategies are visible through participants' physical movements of game pieces while speech interactions index participants' intentions, values, and beliefs, and mathematical thinking.

I used NVivo 8 software for simultaneously viewing, summarizing significant events, and coding video. This program does not supplant the interpretive nature of coding but rather is a tool for enhancing the efficiency of data storage/retrieval, applying codes to the data, and recursive examination of data. The program is a tool for representing complex data in vivid and comprehensible patterns. During this process, I assigned codes for mathematical activity and language tensions. Language tensions serve as a heuristic that guides my interpretation of patterns found in interactions. The method for applying codes is interpretative but marking of discourse devices also serves as a link with established methodologies and theories. Although I do not employ an inter-rater reliability check with additional researchers, the explicit marking of

discursive practices from established theory, Systemic Functional Linguistics (Fairclough, 2003), makes analytical decisions visible to the reader. Razfar (2011) has examined the role of several types of discursive acts (tensions) such as questions and challenges used for eliciting knowledge as well as for their role in leading students to higher order thinking.

I used codes specific to the discursive tensions in playful mathematics activities and interactions found in the *Los Rayos* after-school program. Previous studies in this after-school setting have shown how tensions abound as learners adapt to new knowledge and ways of thinking (Razfar, et al, 2008; Razfar, Sutton, & Radosavljević, 2009). Sometimes a large number of tensions are correlated to unconventional or experimental solutions to problems. These tensions are indexed by language use and discourse practices, providing insight into learning processes. Likewise, these markers were used to index the significant aspects of children's mathematical socialization as they move from everyday modes of thinking and acting to more scientific, mathematical norms of behavior. These situations may also reveal student interests that can help motivate their adaptation to mathematical ways of knowing by scaffolding connections between formal learning goals and everyday activities.

The language tensions codes used in this study were adapted from previous studies on games in *Los Rayos* (Razfar, et al, 2008; Razfar, Sutton, & Radosavljević, 2009). During my analysis, I added a code for *Questions* because I found a large number of questions throughout the data corpus. Questioning practices seemed to play a large role in game activities as participants negotiated rules and explanations of strategy. Other codes used in previous *Los Rayos* studies were discarded. For example, I discarded the *Math* code because it was too general. In place of the *Math* code I decided to repurpose the CCSS Standards for Mathematical Practice as codes to mark participants' mathematical acts.

Table II Definitions and Examples of Discursive Codes shows my final version of codes used to identify discursive interactions as participants played mathematical games. Phrases in quotations are examples of participants' speech from the data corpus.

Table II
DEFINITIONS AND EXAMPLES OF DISCURSIVE CODES

| Language Tensions | Definitions | Examples |
|--------------------------|--|--|
| Challenge/Counterversion | Someone tells a narrative of events and another speaker offers either a counter or an alternative version of the same event | “I didn’t cheat!” “Yes you did! You moved that piece when we weren’t looking.” |
| Divergent Goals | Participants voice disagreement over activity goal(s) | “This is boring. Let’s do something else.” |
| Error | The marking of errors - when participants explicitly mark the error (of themselves or a peer) | Self-marking: Player moves her game piece then says, “I shouldn’t have done that.” Participants mark others’ error: “Mira, she messed them all up man.” |
| Expert or Novice | Students are constructed as 'experts' or 'novices' within a particular interaction in relation to mathematical content or ability to solve a problem | Expert construction: “Okay let’s see how many time it takes GR cuz she’s like an expert.” Novice construction: When one student in a group is constructed as an “expert”, the other students may be constructed as “novices”. |
| English | Talk in English | Talk in English (and Spanish) is coded with respect to time, not by number of utterances. |
| Mocking | Teasing | “Que te limpié las orejas si no oyes cochino marron.” (For you to clean your ears if you can’t hear disgusting pain.) |
| Negation | Also includes refusals, denials (this could pertain to any aspect of the activity; problem solving strategy, etc.; use of 'No') | “No” or statements that express disagreement. |

TABLE II (continued)

DEFINITIONS AND EXAMPLES OF DISCURSIVE CODES

| Language Tensions | Definitions | Examples |
|-------------------|--|--|
| No Uptake | Lack of uptake, minimal feedback (e.g. one participant makes a suggestion but the others ignore her/him) | "I think that (game piece) should go over there," but none of the participants acknowledge the suggestion. |
| Question | Participant asks a question | "How many moves did you make?" |
| Repair | Error correction | "That's seventeen, not twenty-three." |
| Silencing | The explicit act of silencing other participants (e.g. shhh!, be quiet, silence gestures or other non-verbal cues that are interpreted by participants to be silent. There are implicit ways in which this is done as well (i.e. lack of uptake; voice volume; non-verbal cues, interjections) | "Shhh!" (speaker raises her hand and moves it left/right, directed at another participant) |
| Spanish | Talk in Spanish | Talk in Spanish (and English) is coded with respect to time, not by number of utterances. |
| Strategy Shift | Change in strategy for playing game or solving problem | Shown by use of different procedure for moving game pieces |
| Uptake | Acknowledgement of a student contribution | "You got it. Awesome!" |

After coding for language tensions in the three Tower of Hanoi vignettes, I reviewed the video data and coded the activities for the Common Core State Standards for Mathematical Practice (CCSS, 2010, pp. 6-8) shown in Table III Mathematical Practice Codes. Each one of the Standards for Mathematical Practice has a series of elaborative statements detailing additional practices that are subsumed under the 8 standards. See Appendix A for the complete list of the National Council of Supervisors of Mathematics (NCSM) Bulleted Mathematical Practices (pp. 80-82).

Table III

MATHEMATICAL PRACTICE CODES

| | |
|---|--|
| 1 | Make sense of problems and persevere in solving them. |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics. |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning. |

The following list is a synopsis of the sequence of my analytic methods:

- iterative viewing of *Los Rayos* video data containing game activities
- write summaries of game activities
- examine other data sources (field notes, electronic messages to *El Maga*) for triangulation
- select a small number of game activities for case study

- code case study video data for language tensions
- transcribe case studies
- cross-check language tensions coding with transcripts
- code video data and transcripts for CCSS mathematical practices
- examine other data sources (field notes, electronic messages to *El Maga*) for triangulation

In the next chapter, I present summaries of the Tower of Hanoi vignettes with tables of language tensions coding and tables of CCSS mathematical practices.

E. Limitations

This study does not provide evidence of children's socialization into mathematical practice in the long term. I cannot speak to the possibility of transfer of the mathematical practices children used during their game activities. This study only identifies the actions of participants and their relative connections to mathematical thinking. As with other studies concerned with indirect processes of cognitive socialization, critics often question the value and significance of the social aspects of cognition. "Where's the math?" is a common retort in public discourse criticizing studies of this nature.

Another limitation of this study is that there are no inter-rater reliability measures. Decisions of coding and interpretation were made solely by one researcher. Validity is also in question because of the indirect relationships of participants' actions to formal mathematical concepts through their use of speech, mathematical notation, and calculation. Children's experiences and interactions in the game activities may not be replicable for all populations.

The lens of this study was focused on discursive tensions and repair practices and represents only one of many different approaches that could have been used to uncover

mathematics socialization in this after school context. Other methods may uncover important factors not found in this study. Finally, this study covers only one group of children and their cohort of adult facilitators. Possibilities for generalizations to larger populations are limited because of this small sample.

V. Findings

I present three vignettes of children and undergraduate facilitators playing the Tower of Hanoi (TOH) game. By focusing first on this trio of vignettes, I intend to illustrate a detailed version of my analytical approach to the entire data corpus. I have chosen these three versions of TOH because, taken together, they contain elements of several themes found across the entire data set. The TOH vignettes represent a set of events that show how various styles of participation with the same game yield various implications for mathematical socialization.

My interpretive choices for subdivisions of each vignette are based on multiple viewings and patterns in coding compared with transcripts and summaries of the data. Typically, I divide the game activities according to the duration of one participant's turn at solving the TOH puzzle and also when players start over and use a different number of disks to play the game. For example, I include separate summaries when the same participants play a simpler version of TOH with 5 disks instead of the full complement of 7 disks that are provided with the game materials.

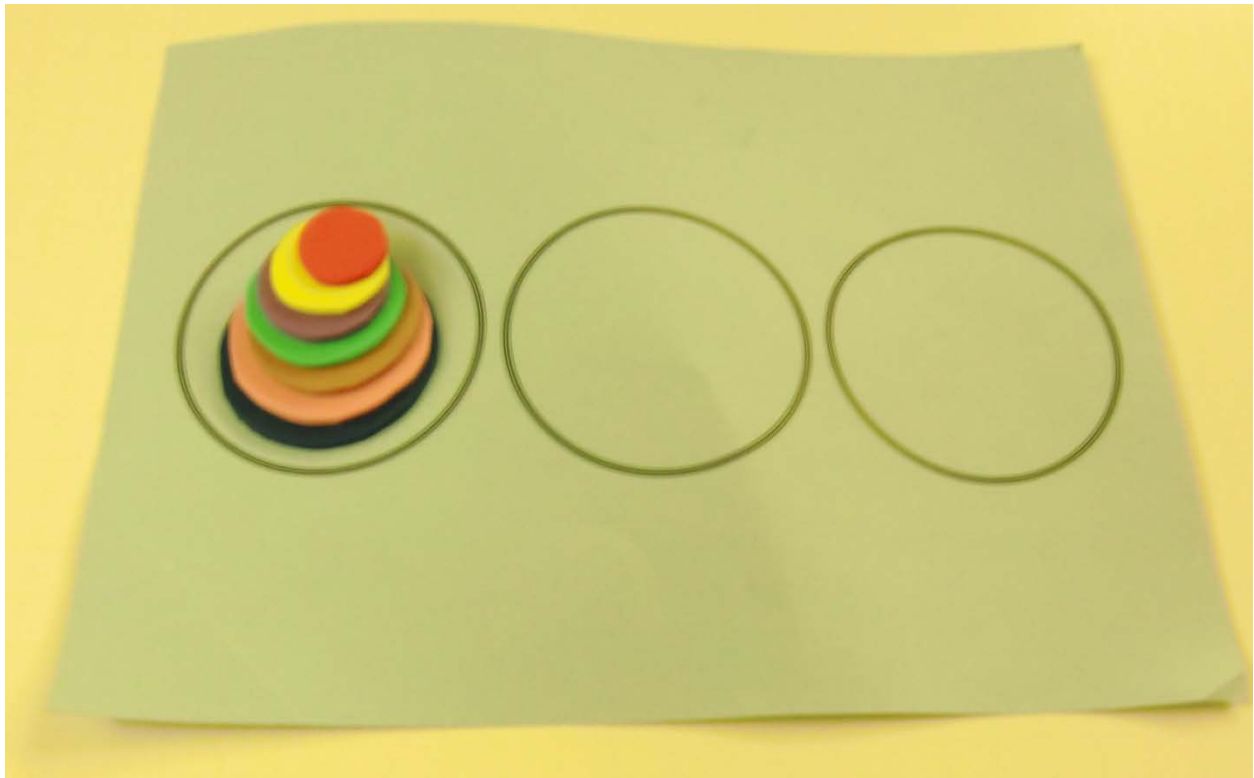
I refer to all participants by pseudonyms. To avoid confusion between adults and children, I preface all undergraduate facilitators' pseudonyms with "UG". Children in the after-school affectionately refer to these adults as "UGs" when they are talking about them, as in "Marisol's my UG", but the children always address them by name and not as "teacher", "maestra", Mr., or Ms.

A. Tower of Hanoi Introduction

Here it is useful to briefly describe the Tower of Hanoi. There are 7 disks piled on top of

each other in ascending order of size. The disks must be moved from one location to another so that the tower of disks shown below must be transferred to one of the other open circles on the game board. Disks are moved one at a time while preserving an order of ascending size so that a larger disk is never placed on top of a smaller disk. The least number of total moves required to solve TOH is $2^n - 1$, where n is the number of disks.

FIGURE 1
TOWER OF HANOI GAME MATERIALS



The solution strategies children used for the Tower of Hanoi were all recursive procedures. Mathematical recursion is fundamental to defining many functions, sets, and especially fractals. Recursive processes are also important in computer programming and mathematical induction.

In the CCSS Mathematics Content Standards, recursion first appears at the high school level when functions are introduced. The following definition comes from a high school college algebra text:

A recursive definition of a sequence is one in which each term after the first term or first few terms are defined as an expression involving the previous term or terms.
(Lial, Hornsby, Schneider, & Daniels, 2012)

Solving the Tower of Hanoi involves first determining a base case, where one figures out how to move a larger disk from underneath a smaller disk without violating the rule of never stacking a larger disk on top of a smaller disk. Beginning with the stack of disks on position A, one moves the smallest disk from the top of the stack to position C. Next one moves the second smallest disk to position B, followed by stacking the smallest disk from position A onto the larger disk on position B. This leaves position C open for the next set of moves. Each successive step involves invoking a similar procedure in order to end with the stack of disks on position C. This set of rules reduces all other successive steps towards the base case.

Understanding recursion and recursive processes exemplifies fully formalized mathematical thinking. Components of recursive thinking include iteration, induction, transitivity, generalization, embedding, and structural and functional dependence (Vitale, 1989). An important goal of mathematics education is to help students become proficient in recursive thinking. CCSS Mathematical Practice Standard 7 “Look for and make use of structure” and 8

“Look for and express regularity in repeated reasoning” represent important components in understanding and using recursive procedures.

When players in *Los Rayos* decided to play the game with 4 disks, the least number of moves required for a solution is 2^4-1 , or 15 moves. For 7 disks, the least total number of moves required for a solution is 2^7-1 , or 127 moves. Of course, it is possible to reach a solution with more moves, especially if players make mistakes and then correct their strategy. My point in reviewing the number of moves required for a solution is to suggest that the cognitive demand on an individual’s attention can be considerable and that the time it takes to execute a correct solution can take more than a few minutes. For example, the *One-on-one* vignette contains 41 minutes of game play with only 3 rounds of TOH. On the other hand, players in the *What’s the point?* vignette averaged 4 minutes for each player’s turn at playing TOH with 6 turns total over 31 minutes of game play. The number of discursive tensions I documented during the three vignettes is presented in Table IV below.

Table IV
SUMMARY OF DISCURSIVE CODING REFERENCES IN TOWER OF HANOI
VIGNETTES

| Tower of Hanoi Game Vignettes | | | |
|----------------------------------|--|--------------------------------|-----------------------------|
| Descriptive Title | Participants | Number of Coding References | Minutes of Game Activity |
| <i>One-on-one</i> | Lucinde & UG Jessinia | 158 | 44 |
| <i>Two at Once</i> | Lucinde, Marguerita, & UG Jessinia | 62 | 15 |
| <i>What's the point?</i> | Gabriela, Elicia, Maritza & UG Teresa | 151 | 32 |

In the discussion that follows, I describe participants' actions with respect to language tensions and how these discursive actions are connected with mathematical thinking and practice as outlined in my research methods.

There are a total of five third graders and two UGs, all female, interacting around the TOH game in these three vignettes. The first vignette, *One-on-one*, shows 10 year old Lucinde and UG Jessinia playing TOH in a collaborative style where players take turns moving one piece at a time. The second vignette, *Two at Once*, shows UG Jessinia with Lucinde and Elicia. The two third graders each have their own game board and work independently. UG Jessinia, who does not have a game board, interacts with Lucinde and Elicia as they play TOH. The third vignette, *What's the point?*, shows a different group of three third grade girls, Gabriela, Maritza, and Jocelyne with their UG Teresa playing the TOH game. Each player takes a turn solving the

puzzle while the group tracks the number of moves to determine who has the most efficient solution.

B. Tower of Hanoi Vignette #1: *One-on-one*

For this vignette, I present my summaries of activity rather than transcripts because participants' verbal interactions were sparse and interspersed with long periods of silence as they concentrated on the game. Also, the participants' actions were more revealing than their speech in this vignette. Specifically, their use of less complicated versions of the game was the most significant action in this vignette with respect to mathematical thinking and practice.

First I present a summary of interactions over the TOH game set-up where the two participants figure out how to play the game. Next I summarize game activity in sections defined by the times when players begin and end a round of the game, adding one more disk for each successive instance of game play. Players began with 3 disks, added another disk for the next round of game play, and so on until the last round where they used 7 disks. There are a total of 7 subsections in the following narrative, one for game set-up and one for each round of game play with successively larger numbers of disks. Lucinde and UG Jessinia sat facing each other for over 40 minutes while playing the game. All talk was in English.

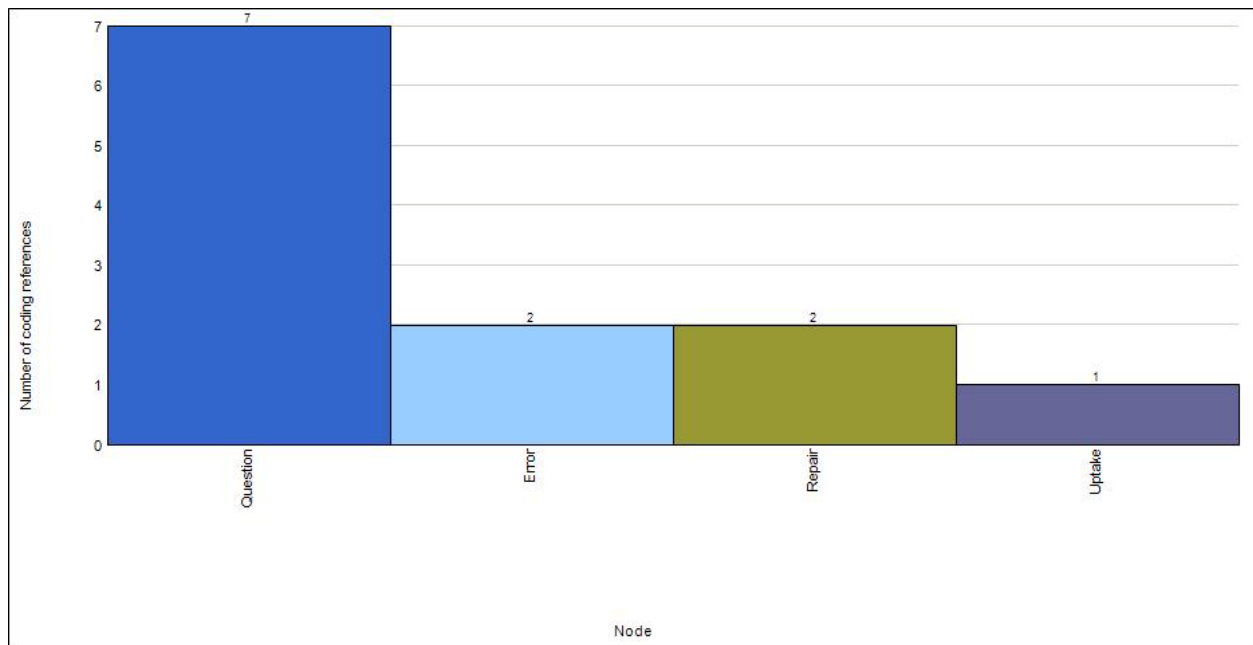
1. Game set-up – 4 minutes.

Lucinde read the TOH rules in English to UG Jessinia. There was one instance of error marking and repair when Lucinde paused in her reading and Jessinia read the next three words aloud. Lucinde repeated the phrase and continued reading the text. They set up the game using only 3 disks as suggested in the activity sheet. With 3 disks, the least possible number of moves for a solution is 7 moves. Together, they reached the solution in 7 moves but with several digressions. UG Jessinia demonstrated how the game works, talking through the TOH rules as

she moved game pieces. She explained to Lucinde why some moves are not allowed and why other arrangements of pieces fulfill the conditions of TOH rules of movement. There were 7 questions from Lucinde to UG Jessinia on game procedure and rules during these first 4 minutes and one sequence of error marking and repair directed at Lucinde. The one instance of uptake occurred when Lucinde moved a disk without assistance and UG Jessinia acknowledged her accomplishment.

FIGURE 2

SUMMARY OF DISCURSIVE CODING REFERENCES FOR *ONE-ON-ONE* GAME SET-UP

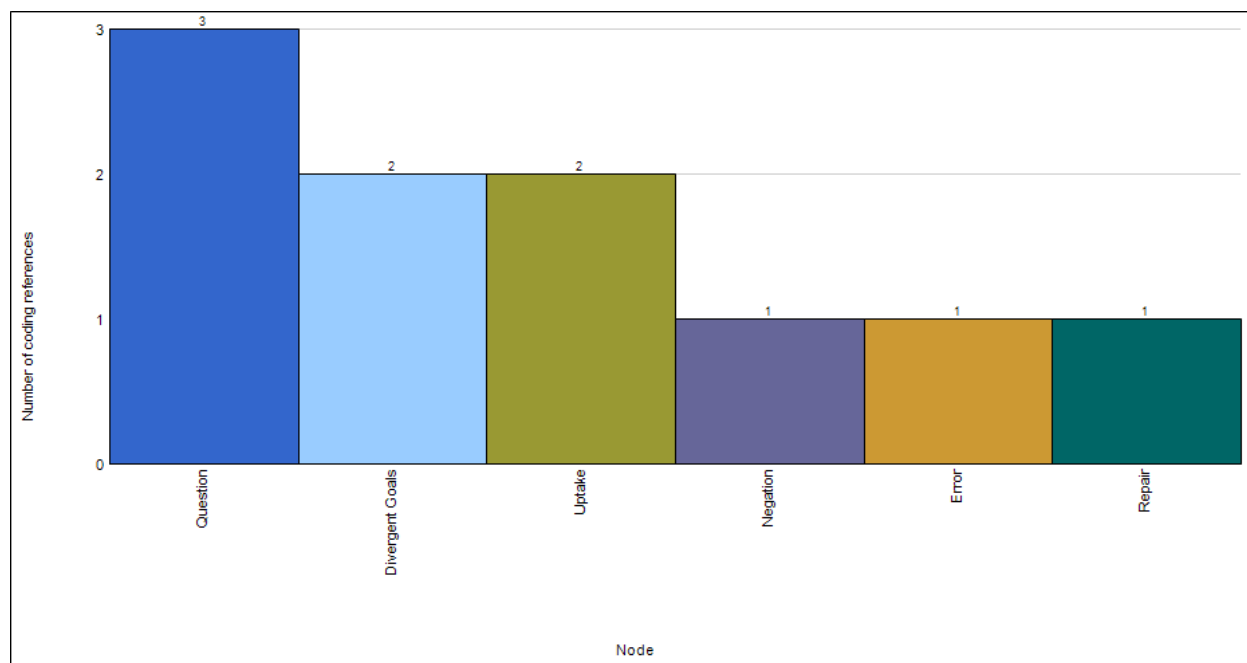


2. Game play – 4 disks - 2 minutes.

UG Jessinia suggested adding another disk for the next round, making the number of disks 4 with the least possible moves for a solution equal to 15 moves. They took turns at each move although the game was meant to be played by one person at a time while other players track and count moves. Lilynana asked three more questions and this round of the game resulted in a solution after 20 moves. There was one sequence error marking followed by repair directed from UG Jessinia to Lucinde. Figure 3 below summarizes discursive coding references for this 4 disk attempt.

FIGURE 3

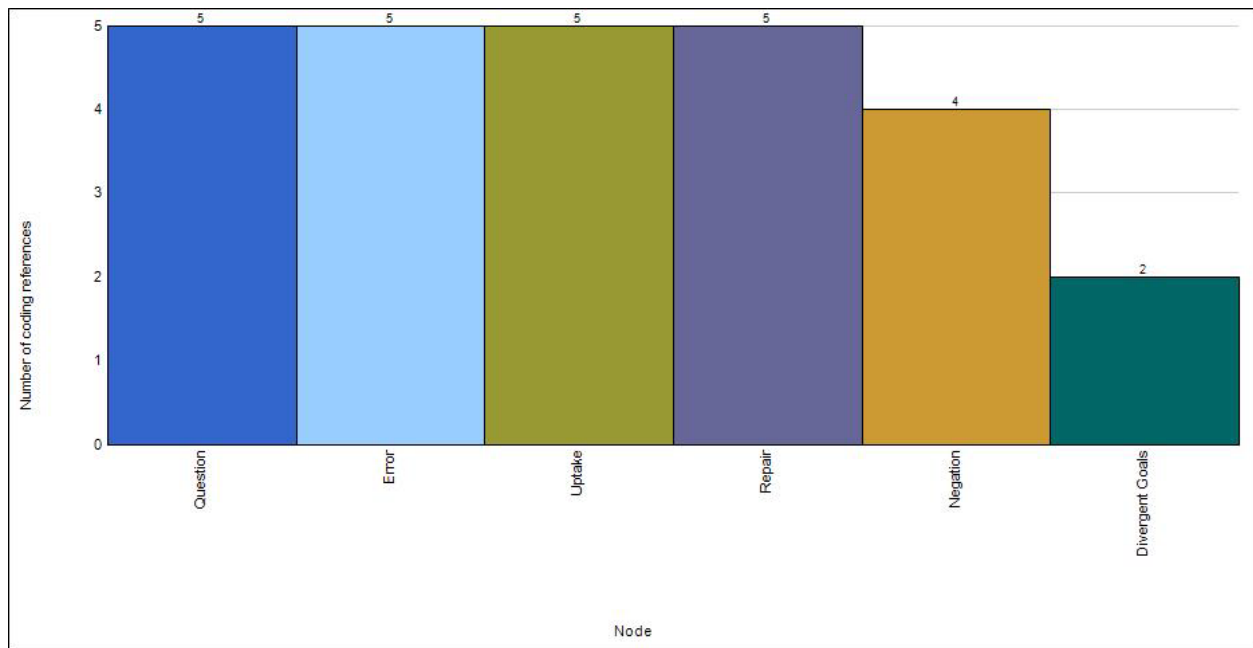
SUMMARY OF DISCURSIVE CODING REFERENCES FOR *ONE-ON-ONE* 4 DISK SOLUTION ATTEMPT



3. Game play – 5 disks - 3 minutes.

UG Jessinia added another disk and now, 5 disks require at least 31 moves for a successful result. At the beginning of this episode, UG Jessinia suggested, “Now you try it first, by yourself,” but Lucinde disagreed saying, “No way”. They continued playing with the 5 disks taking turns, Lucinde making one move followed by UG Jessinia taking the next move. After about 7 moves, UG Jessinia began directing Lucinde rather than making a move herself. Instead, she pointed to positions on the game board indicating how she wanted Lucinde to move each successive piece. They reached a solution in less than 40 moves. There were 5 instances of error marking followed by repair directed at Lucinde with UG Jessinia saying “no” four times and pointing at disks and positions on the game board. For one instance of error marking, Lucinde repaired her own mistake. All of the questions in this segment were from Lucinde to UG Jessinia. Figure 4 below summarizes discursive coding references for this 5 disk attempt.

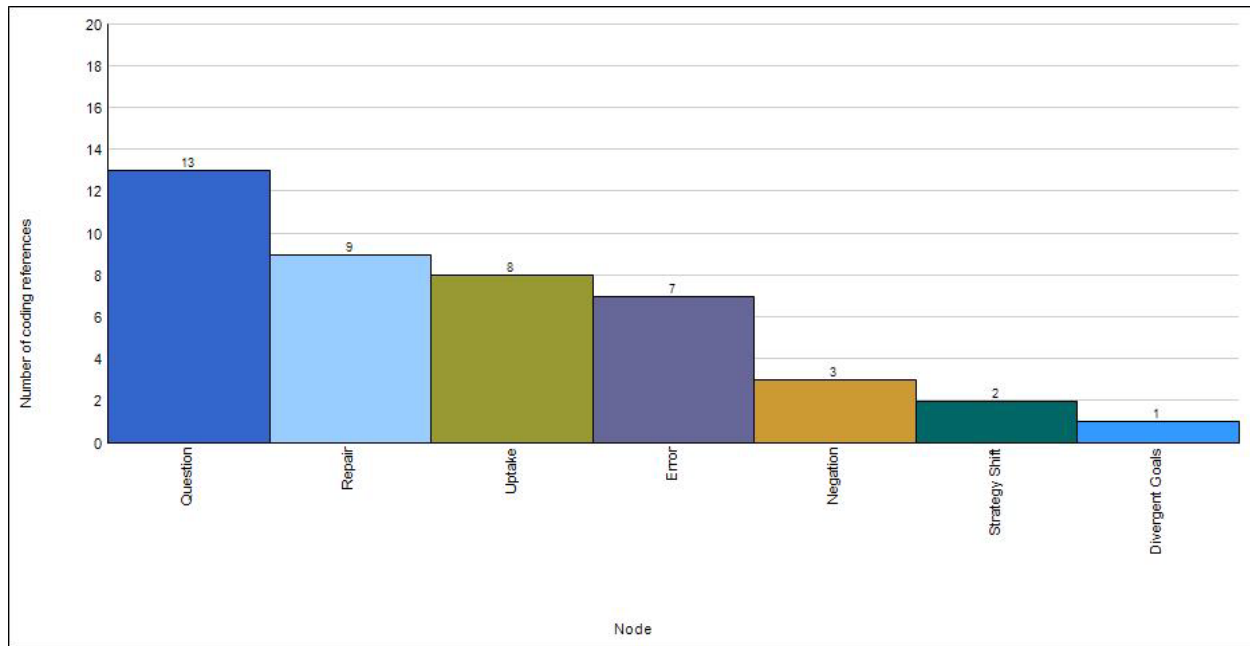
FIGURE 4
SUMMARY OF DISCURSIVE CODING REFERENCES FOR *ONE-ON-ONE* 5 DISK
SOLUTION ATTEMPT



4. Game play – 6 disks – 9 minutes.

The players used 6 disks in this round of game play. Lucinde and UG Jessinia continued taking turns moving disks. Instances of repair and questions related to procedure were less than the previous game with 5 disks. However, there is a segment in the middle of this section in which several social questions are posed and personal information not related to game play was exchanged, accounting for 5 out of the 7 questions posed. Figure 5 below shows a summary of discursive coding references for this 6 disk solution attempt.

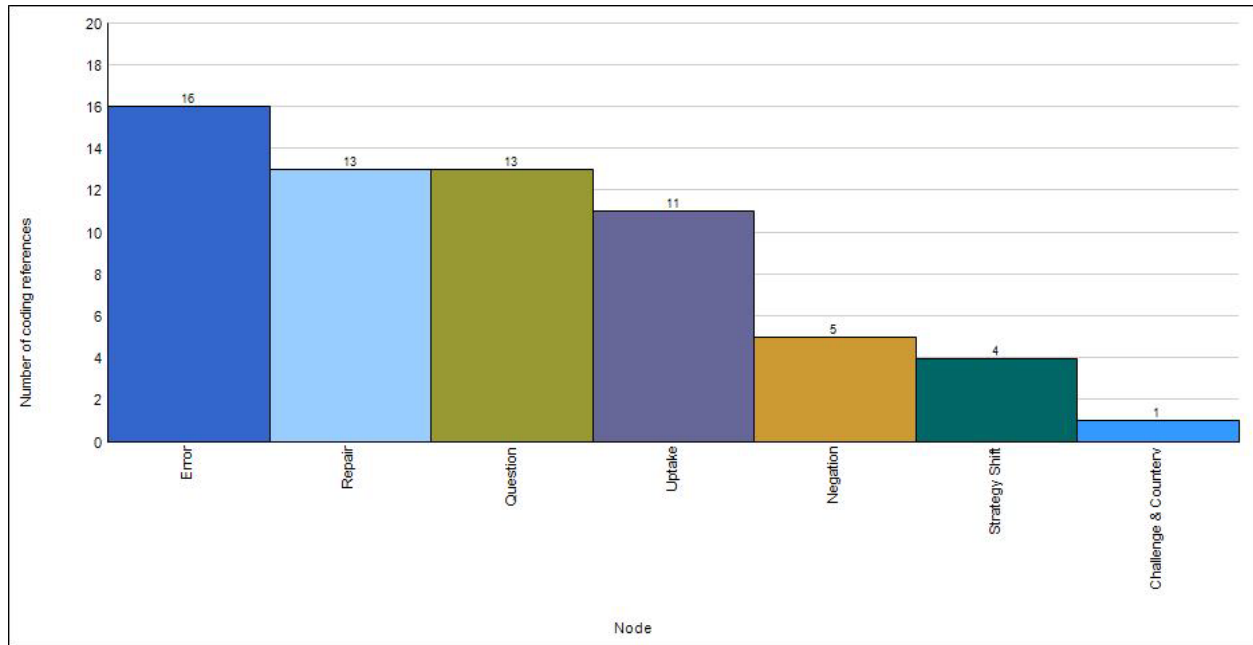
FIGURE 5
SUMMARY OF DISCURSIVE CODING REFERENCES FOR *ONE-ON-ONE* 6 DISK
SOLUTION ATTEMPT



5. Game play – 23 minutes - unsuccessful attempt with 7 disks.

Players started over and continued taking turns to solve the TOH. For the first time in this activity, Lucinde offered a counterversion of how to proceed to a solution and UG Jessinia accepted the alternative and conceded her error. UG Jessinia marked Lucinde's errors 16 times verbally and with gestures indicating correct moves. Lucinde was able to repair her own mistakes 3 times out of the 16 instances of error marking. UG Jessinia acknowledged Lucinde's corrections 11 times with brief statements such as "yes" and "good". However, they did not reach a solution during this round of game play.

FIGURE 6
SUMMARY OF DISCURSIVE CODING REFERENCES FOR *ONE-ON-ONE* 7 DISK
SOLUTION ATTEMPT



6. Game play – 3 minutes – Lucinde’s successful solo solution.

Lucinde asked if she could solve the puzzle on her own. UG Jessinia agreed and monitored her progress. There was very little talk during these minutes as Lucinde moved game pieces at more than twice the speed of previous game rounds. There was only one instance of repair directed from UG Jessinia to Lucinde. Jessinia moved her hand to the game board but before she could say anything, Lucinde quickly corrected her last move before UG Jessinia’s hand reached the game board. UG Jessinia offered assistance once more by pointing and saying, “You have to move this yellow one under this pile.” When Lucinde had successfully moved all 7 disks, UG Jessinia congratulated her and they exchanged a “high-five” hand slap (uptake).

Discursive coding references during this segment included only one each of error, repair, strategy shift, and uptake.

7. Language tensions during *One-on-one*.

Several language tensions are absent from this activity. There were no instances of *Silencing*, *No Uptake*, or *Mocking/Teasing*. In other words, Lucinde's contributions to the activity were all acknowledged by UG Jessinia and neither one of the participants engaged in teasing each other over game errors or personal issues. However, there were 32 instances where UG Jessinia marked Lucinde's errors and engaged in repair practices. Twenty-four error markings were not illegal moves according to game rules but were errors that would have led to a longer, less efficient solution. The types of repair Lucinde experienced were centered on her manipulation of game pieces with UG Jessinia pointing to game pieces and positions on the game board while referencing game rules. Lucinde made 7 distinct *Strategy Shifts* during game play, each following one of UG Jessinia's repair practices. *Strategy Shifts* in TOH indicate a more thorough understanding of how to manipulate the game pieces, showing a focus on long-term rather than intermediate results. With 7 disks, the back and forth movement of piles of disks becomes increasingly complex so that intermediate steps do not reflect the final solution in an obvious manner. The 12 instances of *Negation* come from 10 of UG Jessinia's repair practices, one from Lucinde's refusal to play TOH on her own, and one as Lucinde asserted her own version (coded as *counterversion*) of how to proceed in moving the game pieces. A summary of the total number and types of discursive tensions found in this vignette is given below in Table V.

Table V

SUMMARY OF DISCURSIVE CODING REFERENCES FOR *ONE-ON-ONE*

| Language Tensions | Game Set-up | 4 Disks | 5 Disks | 6 Disks | 7 Disks Unsuccessful Attempt | 7 Disks Successful Attempt | Totals |
|--------------------------|-------------|---------|---------|---------|------------------------------|----------------------------|--------|
| Challenge/Counterversion | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| Divergent Goals | 0 | 2 | 2 | 1 | 0 | 0 | 5 |
| Error | 2 | 1 | 5 | 7 | 16 | 1 | 32 |
| Expert or Novice | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Mocking/Teasing | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Negation | 0 | 1 | 4 | 3 | 5 | 0 | 13 |
| No Uptake | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Question | 7 | 3 | 5 | 13 | 13 | 0 | 41 |
| Repair | 2 | 1 | 5 | 9 | 13 | 1 | 31 |
| Silencing | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Strategy Shift | 0 | 0 | 0 | 2 | 4 | 1 | 7 |
| Uptake | 1 | 2 | 5 | 8 | 11 | 1 | 28 |
| | | | | | | | 158 |

The coding summary in Table V shows that as the players added more disks, making the game more complex, errors and repair practices increased until Lucinde's successful solution with 7 disks when there was only one instance of error marking. There were no coding references to expert or novice interactions even though it was clear that Lucinde was not an expert. My decision not to code for expert or novice is because UG Jessinia did not make any references to Lucinde's lack of skill. Instead, UG Jessinia often acknowledged Lucinde's contributions (uptake). The two players interacted collaboratively by taking turns in each round of the game. If UG Jessinia's marking of Lucinde's errors and subsequent repairs were to be interpreted as instances of expert/novice interactions, then each error marking and repair would count as an expert/novice interaction. Likewise, Lucinde's questions could be viewed as appeals

to authority and therefore expert/novice interactions. I chose not to code these questions as expert/novice interactions because the tenor (Fairclough, 2003) of their exchanges suggested children's descriptions of their relations with their UGs as "big sisters" or "big brothers", found in yearly interview data.

In Table VI below, I have summarized the most prominent examples of mathematical practice found in the *One-on-one* vignette.

Table VI

SUMMARY OF CODING REFERENCES FOR MATHEMATICAL PRACTICES

| <i>One-on-one</i> | |
|--|---|
| Observed Activity | CCSS Mathematical Practices |
| Lucinde reads aloud. | Try to communicate precisely to others. |
| Lucinde restates rules in her own words. | Try to communicate precisely to others. |
| Play game with 3 disks | Break problem into simpler version. |
| Play game with 4 disks | Break problem into simpler version. |
| Play game with 5 disks | Break problem into simpler version. |
| Play game with 6 disks | Break problem into simpler version. |

C. Tower of Hanoi Vignette #2: *Two at Once* (Lucinde Plays Again)

Lucinde played TOH again on the day after her first experience with the game. UG Jessinia, who also played TOH with Lucinde on the previous day, sat next to Lucinde. At the same time, UG Jessinia interacted with Elicia. There were almost no utterances exchanged between Lucinde and UG Jessinia. The majority of conversation occurred between Elicia and

UG Marina. This game activity lasted for only 15 minutes, less than half the time spent on TOH during the previous day when only Lucinde and UG Jessinia played together. All talk was in English.

1. Game play – Lucinde, Marguerita, & UG Jessinia play together – 4 minutes.

Lucinde & UG Jessinia had already set up TOH with 7 disks when Marguerita appeared and sat with them. Marguerita looked at the game and said, “This is easy”. Next, the three players took turns moving one disk at a time on the same game board. After a few moves, UG Jessinia asked Marguerita how she learned the game and Marguerita said, “Cynthia”, indicating another third grade member of *Los Rayos* after-school club but there is no record of her experience in the video data. However, her familiarity with the rules shows how the community of the after-school program yielded opportunities for children to learn from their peers.

Marguerita talked more than the other two participants. Most of the conversation was not focused on the game. Instead, talk focused on interpersonal relations with peers in the after-school, and classmates in the school who were not members of *Los Rayos*. Marguerita’s choices for positioning the disks showed that she was following the rule in TOH that specifies only smaller disks may be placed on larger disks. However, her strategy did not seem to lead to the correct solution. After four minutes of game play, Lucinde asked, “What are you doing?” Following this last comment, Lucinde and Marguerita talked and gestured simultaneously. Their talk overlapped with incomplete sentences as they discussed the strategic errors that occurred over the previous four minutes. Lucinde had now assumed a role of expert, evidenced by her intervention in Marguerita’s play. Next, Marguerita pulled the game board closer to her and announced that she wanted to play TOH first. I interpret Marguerita’s action as a *divergent goal*

because she initiated a departure from the collaborative turn-taking format of game play. UG Jessinia then found another TOH game package and gave it to Lucinde.

2. Game play – Lucinde & Marguerita play on separate game boards – 11 minutes.

During this interval, Lucinde and Marguerita played TOH, each with their own game materials. UG Jessinia did not participate in the games but remained in conversation with the players. The majority of talk between Marguerita and UG Jessinia was not focused on the game but on personal information exchanges such as make-up and jewelry, topics initiated by Marguerita. During the exchange of personal information between UG Jessinia and Marguerita, Lucinde silently continued to work through a 7 disk solution of TOH. Marguerita did not complete her solution and appeared to lose interest in spite of UG Jessinia's attempts to include her in the activity through conversation. UG Jessinia divided her attention between Lucinde and Marguerita, who did not cooperate with each other on the game but remained seated side by side, playing with TOH independently.

In Table VII below, I did not include a column for Game Set-up as I did in the other two vignettes because this aspect of game activity was not captured on video. I do not know how Lucinde and UG Jessinia began this game activity or who initiated Lucinde's involvement on this second day of her play with the Tower of Hanoi game. The lack of a Game Set-up sequence for the other participant, Marguerita, is due to the fact that she already knew how to play the game and began playing almost immediately after her arrival at the group's table.

TABLE VII
SUMMARY OF DISCURSIVE CODING REFERENCES FOR *TWO AT ONCE*

| Language Tensions | Game Play |
|--------------------------|-----------|
| Challenge/Counterversion | 1 |
| Divergent Goals | 2 |
| Error | 2 |
| Expert or Novice | 1 |
| Mocking/Teasing | 3 |
| Negation | 2 |
| No Uptake | 0 |
| Question | 40 |
| Repair | 3 |
| Silencing | 0 |
| Strategy Shift | 2 |
| Uptake | 6 |
| Total | 62 |

During *Two at Once*, the shortest of the three Tower of Hanoi vignettes (15 minutes versus 31 minutes and 41 minutes), there were 40 questions. Most of these questions were exchanged between Marguerita and UG Jessinia over social conversation and not over game related topics. Instances of uptake occurred twice, once for Lucinde's successful completion of the game and once for Marguerita's "success". Marguerita completed the Tower of Hanoi sequence by making an illegal move which was not noticed by any of the participants yet she received congratulations on her accomplishment when she announced her success. There were no instances of silencing and there were not any instances where participants did not acknowledge each other's contributions (no uptake).

In Table VIII below, I have recorded the mathematical practices found in the *Two at Once* vignette.

Table VIII
SUMMARY OF MATHEMATICAL PRACTICE CODING REFERENCES FOR *TWO AT ONCE*

| <i>Two at Once</i> | |
|--|--|
| Observed Activity | CCSS Mathematical Practices |
| Lucinde plays the game for the second time two days in a row. | Persistence in problem-solving. |
| Lucinde plays the game using the rules correctly without assistance. | Look for and express regularity in repeated reasoning. |
| Lucinde plays the game using the rules correctly without assistance. | Look for and make use of structure. |
| Lucinde questions Marguerita's game strategy. | Construct viable arguments and critique the reasoning of others. |

D. Tower of Hanoi Vignette #3: *What's the point?*

In this vignette, each player had her own turn at trying to solve TOH while the rest of the group members waited for their turns. However, stating that they “waited” for their turn is misleading because these players did not refrain from interrupting each other’s play with their own ideas about how to proceed to a solution. Often they reached onto the game board to point out their suggestions and even moved game pieces when it was not their turn. There were so many instances of overlapping talk, gesture, and contestation that it was difficult to capture all of the activity faithfully in the following narrative. I have attempted to distill the most important aspects as much as possible.

Maritza is the only child who remains at the group's table for the entire vignette. The other two, Gabriela and Elicia, leave for portions of this activity but return to witness Maritza's final successful solution of TOH.

1. Game Set-up – 8 minutes

Excerpt 1 shows the participants' interactions as they attempted to understand the game rules. This excerpt is significant in the way that it relates to problem-solving because these interactions detail how the participants reach a definition of the problem, a necessary first step to any problem-solving activity.

Excerpt 1

1. Gabriela: You have to empty this one and put it in another spot and when you put them all in sizes you win. (moves game pieces)
2. Maritza: I don't understand you.
3. UG Teresa: Okay, okay, okay. Otra vez. Okay otra vez. One more time. Okay, okay, okay. One more time. One more time. One more time.
4. Gabriela: You have to, um, switch this spot to one of these. So it can be there and it has to be like all from sizes. (pointing to various spots on the game board)
5. Maritza: And what's the point?
6. Gabriela: And you can't go like. Okay so I'm in this one, so I want to do this one here, these on top and this one like that. You can't grab it. You have to do it from where it's starting. (moves game pieces between positions on game board)
7. Maritza: But what's the point?
8. UG Teresa: Okay.
9. Gabriela: The smallest one always has to go on the top of all of them.
10. UG Teresa: Okay.
11. Maritza: But what's the point?

12. UG Teresa: How many people can play?
13. Gabriela: Uh.
14. UG Teresa: ¿Todos nosotras podemos jugar?
Can we all play?
15. Gabriela: Yeah.
16. UG Teresa: Okay mira.
Okay look.
17. Maritza: But what's the point?
18. Gabriela: Like it's...
19. UG Teresa: Mira I'm gonna go and then va decir Gabriela if I'm right.
Look I'm gonna go and then Gabriela is going to tell me if I'm right.
20. Gabriela: Uh huh.
21. UG Teresa: Okay. Okay do this?
22. Gabriela: Yeah you can take some off if you want to.
23. UG Teresa: I can go like this? I can go like this? (moves game pieces)
24. Gabriela: No, no, no. Only one at a time.
25. UG Teresa: So I can do this? (moves game pieces)
26. Gabriela: So you have, no. So you could only go like this one. And then you go like that. Put it on top. This one here. Wait. This one was there. And then no. But then you have to count whatever move you make. You have to count it. (moves game pieces)
27. Elicia: This is boring.
28. Maritza: Yeah it's boring.
29. Gabriela: If I have seventeen points, I need seventeen moves.
30. UG Teresa: Hold on. (to Maritza and Elicia)
31. Gabriela: So then I put it in the chart and then...

32. UG Teresa: What chart?
33. Gabriela: In a notebook paper we were doing it and if you have eighteen points and since she has fourteen and sixteen, whoever has the least points...
34. UG Teresa: Wins?
35. Gabriela: ...moves, uh moves, gets to win.
36. UG Teresa: So we have to try to reconstruct this.
37. Maritza: But what, but what's the point?
38. UG Teresa: We have to try to reconstruct this in a different circle.
39. Maritza: But what's the point?
40. UG Teresa: That. (points to game) Look I'm going to go like this (moves game pieces)
41. Gabriela: But you can't move all of them.
42. UG Teresa: And I can go like this? (moves all pieces at once)
43. Gabriela: But you can't move it all though.
44. UG Teresa: So...
45. Gabriela: So it has to be one at a time.
46. Maritza: I don't get that.
47. Gabriela: So pretend you already moved and you can't go like this (moves game pieces)
48. Elicia: I don't get it.
49. Gabriela: One. This one here. This one here. This one here. (moves game pieces)

Before Excerpt 1, Gabriela left the group and briefly appeared with the group in Vignette #2, *Two at Once*, whose members were engaged with the Tower of Hanoi. She interacted briefly and found more Tower of Hanoi game packets to bring back to her group. Seconds later Gabriela

returned to her group and said, “Let’s play this one, it’s fun.” UG Teresa responded, saying, “OK, you’re the expert.” Gabriela tried to explain the game rules saying, “You have to empty this one and put it in another spot and when you put them all in sizes you win” (Line 1) as she demonstrated by moving the game pieces. Her explanation did not convey the rules and concepts of the game for Maritza who challenged her 5 times in 2 minutes. Maritza stated “I don’t understand you” once (Line 2) and “What’s the point?” four times (Lines 5, 7, 11, and 17). Maritza’s challenges to Gabriela represented a demand for precision of explanation, one of the standards for mathematical thinking and practice used for coding these interactions.

UG Teresa scaffolded Gabriela’s attempt at explaining the game saying, “OK, let’s start again.” When Gabriela tried again, UG Teresa asked a question while modeling a counterexample, moving three game pieces at once (an illegal move) saying, “I can go like this?” (Line 23). Gabriela corrected her and elaborated further. In this instance, UG Teresa reacted to unresolved meaning making tensions surrounding the explanation of game rules and procedure. This situation speaks to the bi-directional nature of socialization processes (Kuczynski, Lollis, & Koguchi, 2003). Maritza’s demand for an explanation that she can understand, coupled with Gabriela’s inability to satisfy Maritza, prompted UG Teresa’s intervention through scaffolding techniques, namely, questioning while modeling counterexamples.

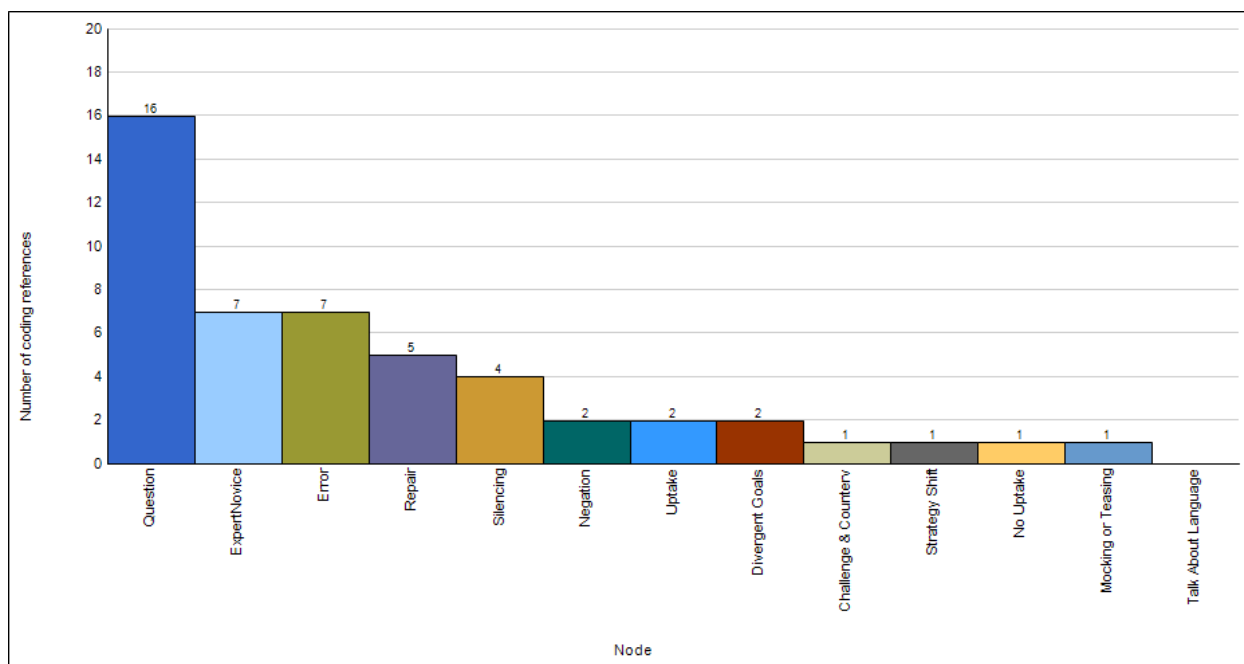
Maritza announced, “Let me practice” and she moved the game pieces while Gabriela interacted with her by giving directions verbally and by pointing to the disks and positions on the game board. Elicia simultaneously observed the practice session while drawing a chart for keeping track of each player’s moves. Gabriela corrected Maritza at key moments (“You could have put this one here”) but also congratulated her when she performed appropriate moves (“You’re doing great”). Their talk was in both Spanish and English in approximately equal

amounts. They used all 7 disks during this practice session but before they reached a complete solution, UG Teresa prompted them to start over and begin the game.

Figure 7 shows the language tension coding for the game set-up in *What's the point?*

FIGURE 7

SUMMARY OF DISCURSIVE CODING FOR *WHAT'S THE POINT?* GAME SET-UP



Questioning played a prominent role in the participants' interactions around understanding how to solve the Tower of Hanoi. The marking of errors and subsequent repairs between participants, partially represented in Excerpt 1, are characterized by an argumentative or

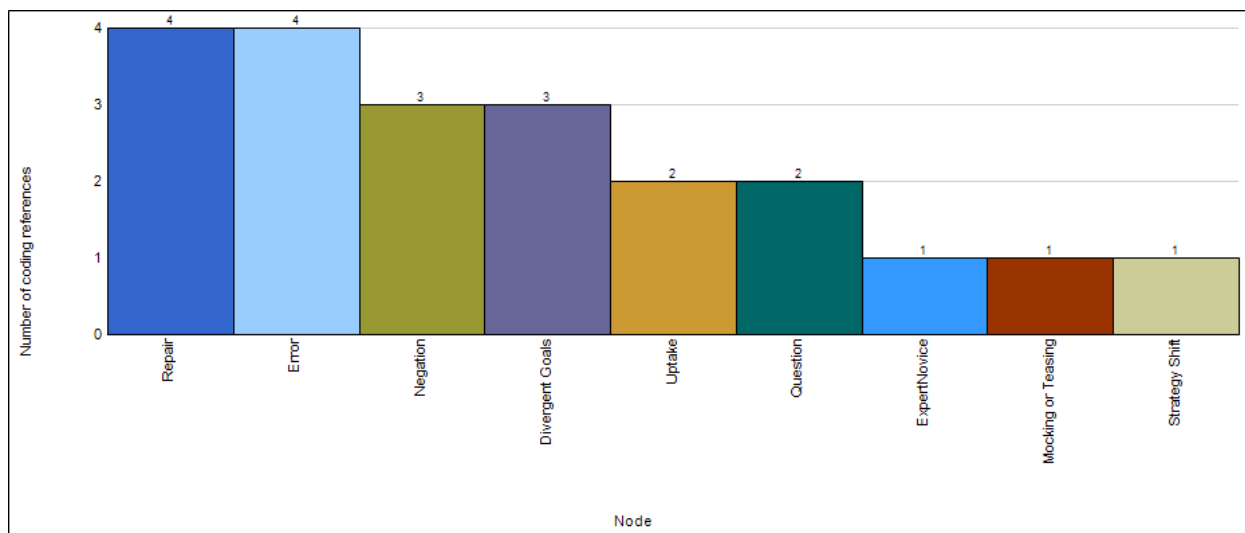
challenging tenor that included four instances of silencing, two instances of negation, and one teasing comment by UG Teresa as she silenced Maritza.

2. Elicia's turn – 3 minutes.

Elicia began her turn using 6 disks and the other participants called out the number of each move. For each move, Elicia said, “This one here?” and the others affirmed her correct move with a brief, “Uh huh”. After 9 moves, Elicia laughed, held her hands to her head and said, “This is hard.” At this point, Gabriela first and then Maritza intervened with suggestions and by reaching onto the game board to illustrate the next move. Maritza became so involved in Elicia's turn that UG Teresa had to tell her to back down with a teasing comment. Maritza moved away from the game board and announced that she would “do something else”. She opened a Mancala game board and started manipulating the game pieces but remained at the group's table. With Maritza's withdrawal, Gabriela moved closer to the game board and intervened in Elicia's turn even more than Maritza. Next there is a dispute about how many moves have taken place. Elicia announced that she wanted to start over but UG Teresa decided that starting over is not allowed and that the next player should take a turn. She picked Maritza and coerced her to join the game again.

Figure 8 below represents the discursive tensions found in this segment. All of the error markings and repairs were directed at Elicia from the other two children, Gabriela and Maritza.

FIGURE 8

SUMMARY OF DISCURSIVE CODING REFERENCES FOR ELICIA'S TURN IN *WHAT'S THE POINT?***3. Maritza's turn - 4 minutes.**

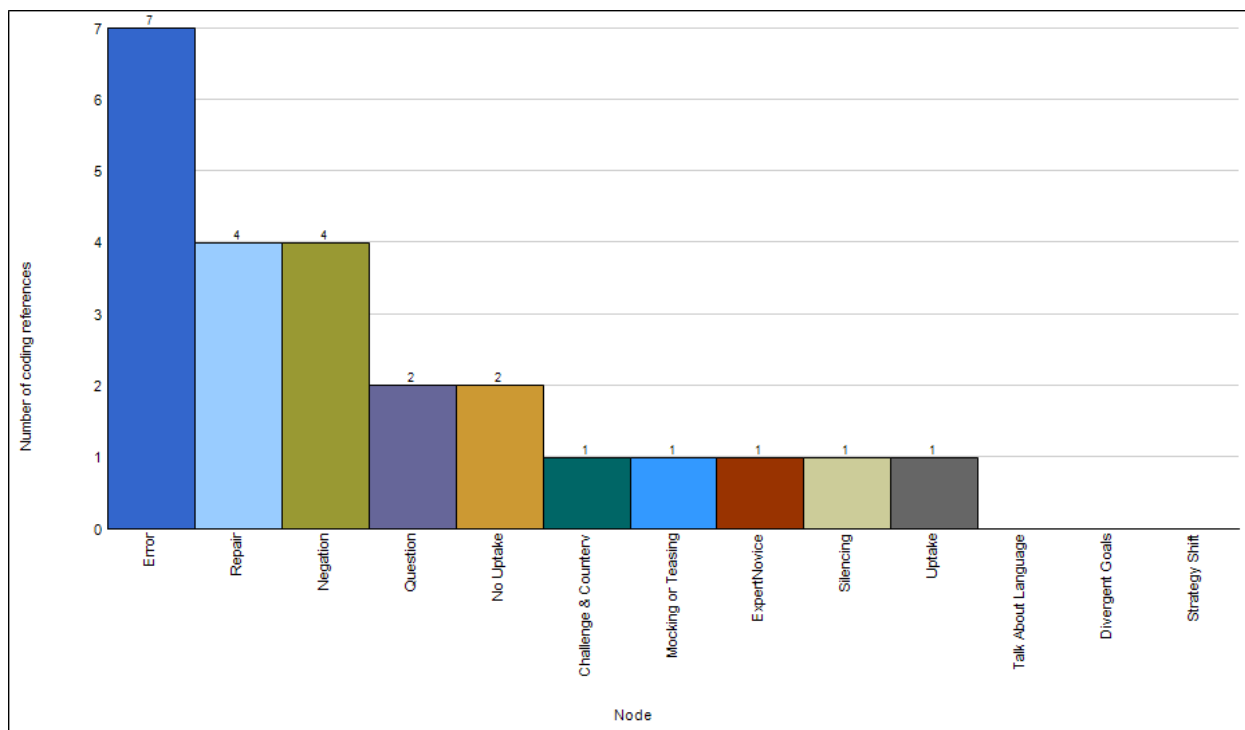
Gabriela produced two other TOH games and handed one each to Elicia and Maritza. After encouraging comments from the other participants, Maritza moved the Mancala game and began moving TOH game pieces while the others counted her moves. As she watched Maritza's game moves, Elicia suddenly shouted out, "I get it!" A few moves later, both Elicia and Gabriela commented on Maritza's progress and said, "If she had done it another way..." and "She was close!" UG Teresa next commented on how many moves Maritza made saying, "I'm guessing 20" but the others had stopped counting Maritza's moves. Elicia, for the third time in this episode announced, "This is boring!" Next, Gabriela and Elicia became distracted by another child walking across the room and continued talking about him. UG Teresa was still paying

attention to Maritza while listening to the others and commented on Maritza's progress, "You almost had it!" Two minutes later, having completed her "gossip" with Gabriela, Elicia shouted "No, no, no!" in reference to Maritza's moves, prompting Gabriela's focus back on the game. Soon after, Elicia, Gabriela, and UG Teresa debated how many moves Maritza had made and teased her with, "20!", "No, more like 50!" and "You messed it all up!". UG Teresa next announced that Maritza's turn was over and the next person should play. Maritza immediately returned to her solitary game of Mancala.

Figure 9 below shows the discursive tensions exchanged during Maritza's turn. Most of the talk during this episode was directed at Maritza from Gabriela and Elicia, taking the form of error markings, repairs, and negations.

FIGURE 9

SUMMARY OF DISCURSIVE CODING REFERENCES FOR MARITZA'S TURN IN
WHAT'S THE POINT?



4. Gabriela's turn - 5 minutes.

UG Teresa stated, "Let's see how many moves Gabriela takes because she's like an expert". Maritza played with a Mancala game by herself after saying, "I'm not playing anymore". Gabriela responded by saying, "OK Maritza, that's what you said the time before." As an aside to this activity, it is important to mention that Maritza has played Mancala on the previous day with a different group. She attempted to introduce Mancala to UG Teresa, Gabriela, and Elicia prior to their TOH play but could not explain how it should be played.

After about one minute, Elicia left the group. Seconds later, Gabriela left the group to check out some sort of verbal conflict that can be heard in the background but returns within 30 seconds. Gabriela continued her turn and soon Elicia returned with a piece of string used for playing Cat's Cradle, distracting UG Teresa with it. At this time, no one is paying attention to Gabriela as she continued to play TOH. Soon, Gabriela has stopped moving game pieces and has her gaze focused on UG Teresa and Elicia as they played with Cat's Cradle string. After Elicia left the group again, UG Teresa turned to Gabriela to inquire about the number of moves she had made. Gabriela responded with, "Oh, 30 or 40-something". Less than a minute later, UG Teresa said to Gabriela, "You started all over again" and "Let me see, I'm gonna try to do it". Gabriela handed her game board to UG Teresa but she had not completed a solution.

The amount of discursive tensions in this 5 minute episode are few: two questions, one instance of strategy shift, one instance of divergent goals, and one teasing comment. Gabriela played the game while her group members were engaged in other activities though in close proximity. Maritza played the Mancala game by herself while UG Teresa was occupied with Elicia's Cat's Cradle play.

5. Last round: Maritza tries again – 10 minutes.

Gabriela and Elicia have left the group and have gone somewhere else in the after-school classroom. Next, UG Teresa tried to solve TOH while Maritza continued playing Mancala. After a minute, UG Teresa announced, "I give up. Maritza, let's play this [other] game," and picked up another game packet. Maritza responded and said, "No, let me try" and then "I'll take this one off", referring to one of the disks, leaving 5 disks on the game board. After about a minute of silent activity, UG Teresa said "Oh, no", marking one of Maritza's strategic errors. Following

this moment, there were many contestations revolving around Maritza's attempts to complete a correct solution. First, UG Teresa interacted with Maritza's attempt to solve a 5 disk solution.

Excerpt 2

1. UG Teresa: How are you doing?
2. Maritza: Good.
3. UG Teresa: Ooh no. ¿Este es más grande no? Mira vez...
Ooh no. This one's bigger no? Look see...
4. Maritza: No it's cuz I'm gonna do something.
5. UG Teresa: Pero ya puedes mover el blue one. Acuerdate que debes que mover el azul para tratar de poner en otro lugar. ¿Me entiendes?
But now you can move the blue one. Remember that you must move the blue one to try to put it in another place. Do you understand?
6. Maritza: I could put it in the, I could put it in the, like right here. (points to position on game board)
7. UG Teresa: Where?
8. Maritza: In the middle.
9. UG Teresa: Where was that stack?
10. Maritza: Like this. (move game pieces)
11. UG Teresa: No. Este le puedes mover aca.
12. Maritza: No.
13. UG Teresa: Pero that's the point le debes que mover de aquí. Y ahorra debes que poner, ahorra debes que poner todo esos aquí arriba.
But that's the point you are supposed to move it here. And now you're supposed to put, now you're supposed to put all those up here.
14. Maritza: And.
15. UG Teresa: Pero un por uno.
Put one by one.

16. Maritza: Oh. No like that, like this, like this, like this, like this, like this, like this. (moves several game pieces) This one's bigger right?
17. UG Teresa: Yeah.
18. Maritza: Then I put this one here, this one here, and this one right here. Switch this one, then here, then I change, no. Right here, right here. Change this one right here. And this one here. Right here, right here, no right here, right there, right here.
19. UG Teresa: Nice. You got it, awesome.

This excerpt shows how UG Teresa provided assistance during Maritza's first successful attempt at solving a 5 disk version of the game. In Line 3, UG Teresa marked Maritza's error and provided an explanation of what went wrong. Maritza contested UG Teresa's repair "No it's cuz I'm gonna do something" (Line 4) and offered a counterversion "I could put it in the, I could put it in the, like right here" (Line 6). Next, they engaged in more contestations as UG Teresa explained a key goal in the game that Maritza had not understood. Although Maritza had mastered the procedure for moving disks, she did not understand that she had to move the tower of disks to a location *different* from the starting position. Maritza then showed that she understood UG Teresa's explanation and continued moving disks, verbalizing each move, "...like this, like this..." Maritza continued this process and in Line 20, she marked and corrected her own mistake shown by her negation and movement of disks, "...right here, *no* right here,..." Finally, Maritza completed a successful solution and UG Teresa showed uptake of Maritza's success, "You got it, awesome." During this excerpt, UG Teresa used Spanish (Lines 3, 5, 11, 13, and 15) but Maritza responded in English throughout this section. Neither one of them kept track of how many moves were made.

Maritza then urged UG Teresa to take a turn but within a few moves, Maritza began correcting UG Teresa's strategy by moving the disks and explaining her reasoning.

Excerpt 3

1. Maritza: Now you try.
2. UG Teresa: Okay.
3. Maritza: Try.
4. UG Teresa: A ver. Right? And then. (moves disks)

Let's see. Right? And then.
5. Maritza: No move this one right there. (pointing to game board)
6. UG Teresa: I don't know.
7. Maritza: No you had it, you had these ones right here. No you had this one right here right? (pointing to game board) So then you can move this one. Put this one right here, then put this one right here.
8. UG Teresa: I know that's what I did y luego ya ahorra este.
I know that's what I did and then now this one.
9. Maritza: And then put this one right here. And then you can move this one right here. (moves disks)
10. UG Teresa: But I have to move the blue one.
11. Maritza: That's why. Then right here. And then right here. And then right here. Then right here. And then this one right here. No. This one right here, this one right here. And then this one I remember right here. Right here. No right here. This one right here. This one, no this one right here and then this one right here. No right here. (moves disks)
12. UG Teresa: No este acá mira. Acá para que. Oh wait a minute, wait a minute, wait a minute.
No this one here see. Right here for what. Oh wait a minute, wait a minute, wait a minute.
13. Maritza: I was going to put it right here and then this one I could put it over here and then. No wait, right here. And then I could put this one right there, right here. No wait, right here. (7 sec) You could put this. (moves disk)
14. UG Teresa: Can't figure it out?
15. Maritza: I can't either. I can't remember how I did it.

16. UG Teresa: You did a good job. (8 sec) A ver let me see.

At Line 11, Maritza took over UG Teresa's turn and took control of the game for the rest of the activity. Beginning at Line 12, UG Teresa disputed Maritza's directions but Maritza did not back down from the challenge and offered her own strategy. Line 18 shows a 7 second pause where Maritza noticed that her strategy led to a configuration of disks that proved to be an impasse. UG Teresa congratulated her in spite of Maritza's inability to reach a solution.

Maritza began again and during her experimentation, she moved a disk illegally by pulling it out from underneath another disk. UG Teresa smiled and told her that she could not do that. At this time, Gabriela returned and stated that she knew how to reach a solution but Maritza refused to give up at first. Gabriela questioned UG Teresa about Maritza's strategy and then Gabriela began moving the disks while Maritza observed. After 30 seconds and before Gabriela reached a solution, Maritza claimed, "That's how I did it" and took over control of the game. Soon, Maritza completed her solution but UG Teresa and Gabriela disputed the solution. During the time that Gabriela and Maritza fought for control of the game, Maritza lost track of which position on the game board was supposed to be the final position of the tower of disks. Gabriela and UG Teresa explained the situation to Maritza who eventually acknowledged the mistake.

There was a commotion off camera in the room which signaled that it was time to write to *El Maga*. UG Teresa then told Gabriela to "put everything away" and as Gabriela reached for the game, Maritza protested, "Wait, wait, wait!" Gabriela backed away and Maritza completed the unfinished game correctly in less than 20 seconds and said, "Got it". UG Teresa smiled and said, "Nice".

Figure 10 below shows the language tensions that occurred during this 10 minute episode.

FIGURE 10

SUMMARY OF DISCURSIVE CODING REFERENCES FOR MARITZA TRIES AGAIN IN *WHAT'S THE POINT?*

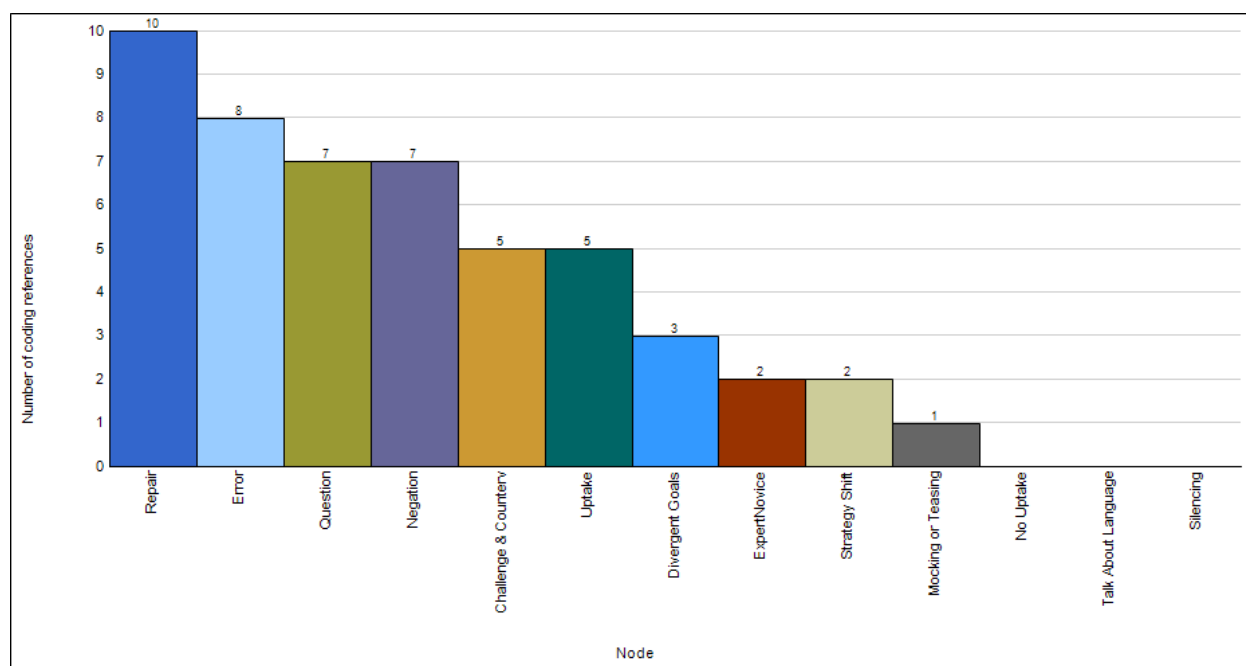


Table IX shows 8 instances of error marking directed at Maritza from UG Teresa and Gabriela. Usually, error marking is immediately followed by repair but in this segment, two of the repair practices were instances of self repair where Maritza corrected her own mistakes, evidenced by her hand movements with the disks. Questions in this segment came from Gabriela and UG Teresa when they asked Maritza about her moves and strategy. The 7 instances of negation were related to the 5 challenges and counterversions that occurred during exchanges between Gabriela and Maritza over game strategy and also between UG Teresa and Maritza over

disk placement. Maritza asserted her position on using her preferred strategy which also accounted for the three instances of divergent goals. Uptake of Maritza's assertions occurred four times with one instance of uptake related to Gabriela's observations of disk movements. The conflicts over disk moves were related to two of Maritza's strategy shifts when UG Teresa and Gabriela explained to Maritza how her errors would lead to an incorrect solution. The only instance of mocking/teasing came from UG Teresa's comment to Maritza during a dispute over strategy.

Compared to the other two Tower of Hanoi vignettes, the participants in *What's the Point?* spoke more Spanish. The ratio of Spanish to English measured over time was 1 to 5, with English used more than Spanish.

TABLE IX

SUMMARY OF DISCURSIVE CODING REFERENCES FOR *WHAT'S THE POINT?*

| Language Tensions | Game Set-up | Elicia's Turn | Maritza's Turn | Gabriela's Turn | Maritza Tries Again | Totals of Language Tensions |
|---------------------------------------|-------------|---------------|----------------|-----------------|---------------------|-----------------------------|
| Challenge/Counterversion | 1 | 0 | 1 | 0 | 7 | 9 |
| Divergent Goals | 2 | 3 | 0 | 1 | 3 | 9 |
| Error | 7 | 4 | 7 | 0 | 8 | 26 |
| Expert or Novice | 7 | 1 | 1 | 0 | 2 | 11 |
| Mocking/Teasing | 1 | 1 | 1 | 1 | 1 | 5 |
| Negation | 2 | 3 | 4 | 0 | 7 | 16 |
| No Uptake | 1 | 0 | 2 | 0 | 0 | 3 |
| Question | 16 | 2 | 2 | 2 | 7 | 29 |
| Repair | 5 | 4 | 4 | 0 | 10 | 23 |
| Silencing | 4 | 0 | 1 | 0 | 0 | 5 |
| Strategy Shift | 1 | 1 | 0 | 1 | 2 | 5 |
| Uptake | 2 | 2 | 1 | 0 | 5 | 10 |
| Total Language Tensions for Each Turn | 49 | 21 | 24 | 5 | 52 | 151 |

In Table VIII above, discursive tensions are shown for the various participants' turns playing the Tower of Hanoi game. The Game Set-up episode and the Maritza Tries Again episode had the most number of tensions compared with the others. During Gabriela's Turn which had the least amount of tensions, the other participants were not watching closely. Maritza was playing Mancala alone while Elicia showed UG Teresa her Cat's Cradle game. Combining the number of tensions during Maritza's turns shows that she was the recipient of the most repair practices and error markings. She also posed the most questions. If I had coded all of Maritza's questions during Game Set-up (e.g., "What's the point?") as challenges, the count for challenge/counterversion would be higher. However, I chose not to do so because she did not actually offer a counterversion during the course of her questioning.

Table X summarizes coding references for mathematical practices during the *What's the Point?* vignette.

Table X

SUMMARY OF MATHEMATICAL PRACTICE CODING REFERENCES FOR *WHAT'S THE POINT?*

| <i>What's the point?</i> | |
|--|---|
| Observed Activity | CCSS Mathematical Practices |
| UG labels student “expert” and asks her to explain the game. | Try to communicate precisely to others. |
| Student: “If she had done it another way...” | Evaluate peer solutions and reasoning. |
| Student challenges peer’s explanation 7 times saying “What’s the point?” & “I don’t understand”. | Evaluation of explanation. |
| Student restates explanation of game rules after being challenged. | Try to communicate precisely to others. |
| UG asks question using game pieces and models incorrect procedure. | Modeling how to use counterexamples. |
| Student asks question using game pieces, “Can I do it this way?” | Recognize & use counterexamples. |
| Group uses only 6 out of 7 game pieces. | Break problem into simpler version. |
| Students interrupt another student’s turn and move game pieces differently. | Evaluate peer solutions. |
| Student fails to solve problem 3 times but executes correct solution (31 minutes). | Persistence in problem solving. |
| Student asks “...you cannot move a pile like that?” while moving game pieces. | Ask useful questions and use counterexamples. |
| Student says “Let me practice!” & rehearses solution using game pieces. | Persistence in problem solving. |

In Table X above, I have coded mathematical practices found in the vignette, *What’s the Point?* This vignette contained the most variation of mathematical practices of all three vignettes.

E. Other Participants and Other Games

In this section, I present findings from my analysis of the rest of the games in the data corpus. The scope of this dissertation does not allow for the level of analysis used in the Tower of Hanoi vignettes since the number of language tensions I observed throughout the data corpus was too large. I found, however, that the types of discursive practices targeted in my research methods occur regularly during all of the game activities in *Los Rayos*.

Figure 11 below, documents students' participation in the various games found in the after-school. At a glance, one can see that some students played games more than others, and most students played the same game no more than twice.

In Figure 11 above, I have included only the full pseudonyms of children in the Tower of Hanoi vignettes while all of the other children are represented by initials. Rodrigo, Natalia, and Sara are the only other children's pseudonym included because they appear in other *Los Rayos* studies (Razfar et al, 2008; Willey & Radosavljević, 2007; Morales, Vomvoridi-Ivanović, & Khisty, 2011) covered in my literature review and also because I refer to them in the text that follows.

Figure 11 shows the number of times children appeared in video data that captured game play and which games they played. The largest number of game activities that one child participated in was 12 while the smallest number was one (a child who only attended *Los Rayos* for a few weeks). Many of the children created their own games after having engaged with the game activities in *Los Rayos*. Additionally, many children focused on their game creations at the end of the data corpus timeline when the *Los Rayos* club provided resources for them to create digital stories. The digital stories were created with assistance from the facilitators on laptop computers and consisted of sequences of digital photographs with audio commentary and music provided by the children. Out of the 21 children who attended *Los Rayos* during the year of my data corpus, 10 highlighted their game creations in their digital stories. Their choices for topics of their digital stories were left to them. Some who participated in a small number of games activities during this year used their own games in digital stories while others who participated in the largest number of games did not. Rodrigo, a child who showed a significant change in participation style and agency (Morales, Vomvoridi-Ivanović, & Khisty, 2011) played games in 9 videos from the data corpus and also created a digital story based on his own game creation. Lucinde, who appeared in two Tower of Hanoi vignettes, played only four games in the video data corpus but also created a digital story featuring a game she created.

Looking at the other children's participation in game activities shows the difficulty in predicting whether they would create their own games and if they would feature a game in their digital story. The mathematics children used in the creation of their own games was not advanced beyond their current level of elementary school arithmetic. In a study of a similar after-school mathematics enrichment program, Civil (2002) also found that the children in her study did not create games with mathematics more advanced than their current school arithmetic level. However, Civil noted that the students in her after-school study who previously were not interested in mathematical discussions became more engaged in this new environment. They noticed what their peers were doing, shared strategies with each other, and asked mathematically significant questions. I was not able to compare the *Los Rayos* children's prior engagement with mathematical discussions with their behavior in the after-school setting like Civil (2002) did in her study. Instead, I was able to document mathematically significant discursive tensions and mathematical thinking practices in the *Los Rayos* children's invented game activities. The most striking difference between *Los Rayos* children's participation with inventing and playing their own games and their participation with all of the other games was in the way questions were directed. During the invented game activities, the majority of questions were between children and adults. In contrast, the majority of questions were between children during all of the other games. Could it be that the adults who assisted the children during the creation of the games were trying to lead the children to more advanced mathematics like the adults in Civil's study? Or is it because children needed more assistance when their task involved the creation of mathematical games as opposed to playing a ready-made game? This situation raises more questions than I am able to explore in this current study.

VI. Discussion

In this chapter I discuss my findings from the three Tower of Hanoi vignettes. Next I present themes that cut across these three vignettes. These themes also represent aspects of the entire data corpus and inform the answers to my research questions. During my discussion of themes, I make specific references to details contained in the previous chapter on findings. After my discussion on themes, I restate my research questions and provide answers that result from my findings. Next, I discuss limitations and implications of my study. Finally, I conclude with a brief summary of findings.

A. Tower of Hanoi Vignettes

The Tower of Hanoi vignettes represent three different styles of participation in the same game activity. I have assigned descriptive titles for each one based on the characteristics of their participation styles. *One-on-one* is a dyadic interaction between a child and an undergraduate facilitator who take turns with each move of the Tower of Hanoi disks. *Two at Once* shows an undergraduate facilitator overseeing two children, each with their own game board playing independently side by side. In *What's the Point?* there are three children and one undergraduate facilitator. Each of the children takes their own turn with the game while the other participants count the player's moves. The children interact with each other in a highly contentious manner, marking each other's errors and repairing them both verbally and by reaching onto the game board to point at the game pieces and even interrupting each other's turns by moving the disks to positions they believe to be correct. In the following narratives, I discuss the events in the three

vignettes with respect to their significance to mathematical thinking and practice.

1. *One-on-one.*

Lucinde and UG Jessinia sat facing each other with the Tower of Hanoi game between them. During the game set-up, UG Jessinia explained the game rules to Lucinde who did not understand how to proceed. Lucinde asked a series of 7 questions within this four minute sequence which prompted UG Jessinia to respond with more detailed and precise explanations, moving the disks on the game board to show correct moves and counterexamples of incorrect moves. These interactions during the game set up were the beginning of a series of scaffolding events that contributed to Lucinde's developing mastery of the game. The next five rounds of the game began with a four disk simplified version and then became more difficult and complex as they played with 5 disks, 6 disks, and finally two rounds using 7 disks. Looking at the language tensions at each successive version of game play shows that Lucinde received more repair from UG Jessinia as complexity increased. Until the last round with 7 disks, Lucinde and UG Jessinia continued with their turn-taking style of game play, sharing responsibility during this joint problem-solving activity. Lucinde refused to give up this style of participation even though UG Jessinia suggested that she should try to solve the problem by herself. Finally, during the last round of game play when Lucinde attempted to solve the Tower of Hanoi by herself using 7 disks, she reached success with only one instance of error.

During this vignette, the players used simpler versions of the Tower of Hanoi on the way to more complex versions. These acts reflect the mathematical practice standard of breaking a problem into simpler versions in order to gain understanding of a more complex version. In the process of building complexity, Lucinde asked 41 questions over the course of this vignette, received 32 error corrections, and made 7 strategy shifts. During this time, she also received 28

instances of positive feedback or uptake from UG Jessinia. Lucinde's perseverance in achieving success during this 44 minute vignette was not diminished by the large number of repair practices she experienced. This vignette shows how the repair practices during a joint problem solving activity between an adult and child can function as a resource for mathematical socialization rather than an impediment. Although Lucinde participated in only four games found in the data corpus, she created her own game and featured her creation in a digital story she produced at the end of the year. Further, her email correspondence (below) with El Maga, the fictional mathematics wizard, shows her interest in the mathematical games found in *Los Rayos*.

Date: 10/31/07

Author: El Maga

HOLA AMIGA!! Thank you! oye why dont you do the activities with the other kids? I am the president of CEMELA and I want your opinion are the activities boring? What kind of activities do you want to do with math? Please help make the program better.

MAGA

Date: 11/01/07

Author: Lucinde

Re: Sorry

Dear El Maga,

Sorry it's just that I was busy. Besides I love the games we do in cemela by!

sincerlly,

Lucinde

2. Two at Once.

Lucinde and UG Jessinia appear again in this vignette but did not engage in the turn-taking style of participation found in the *One-on-one* vignette. Instead, Lucinde played Tower of Hanoi by herself while Marguerita also played the game independently. UG Jessinia sat with them and spent almost the whole time interacting with Marguerita, who already knew how to play the game. Almost all of the conversation between Marguerita and UG Jessinia was focused on women's make-up, jewelry, and social relations. Marguerita played Tower of Hanoi during the conversation and suspended her game play at one point in order to retrieve costume jewelry from her backpack, showing them to UG Jessinia. The only two instances of repair occurred when Lucinde took notice of Marguerita's progress on the Tower of Hanoi and questioned the validity of Marguerita's strategy. A short exchange between Marguerita and Lucinde ensued over correct strategy. Lucinde returned to her own game and Marguerita continued her progress. Both Lucinde and Marguerita reached solutions but Marguerita used an illegal move by picking up two disks simultaneously yet claimed success when another child, Gabriela, briefly inspected the group's games. Gabriela stated that their use of 7 disks was difficult and that was when both Lucinde and Marguerita announced their successful solutions. Marguerita's conversation with UG Jessinia during game play is characterized by her continuous appeals for UG Jessinia's approval. This behavior leads to my conclusion that Marguerita's perseverance and subsequent illegal game move was motivated by her desire to gain adult approval and affiliation. The fact that UG Jessinia did not provide scaffolding assistance for Marguerita's game play leads me to the following conclusion. UG Jessinia's attention to Marguerita's social appeals provided motivation for perseverance with the game but limited Marguerita's mathematical thinking practices. However, I do not fault UG Jessinia for this oversight since the attention necessary for

her to monitor the progress of two independent games of Tower of Hanoi is overwhelming. This situation shows that the participation structure used in this vignette limits the quality of scaffolding assistance. In this case, the absence of repair seems to limit progress.

3. What's the Point?

The title of this vignette comes from one child's repeated use of the phrase, "What's the point?" Maritza, the author of this phrase, was the only one of the three children who reached a successful solution. Although there was an abundance of assistance strategies directed at the child players from other child participants and the undergraduate facilitator, only Maritza persisted through several attempts to reach success. During the game set up sequence, Maritza demanded a more precise explanation of the game from Gabriela who introduced the game to the group. Gabriela's struggle to achieve a level of precision in her explanation was scaffolded by UG Tina who used questioning and counterexamples with the game pieces to assist. Gabriela picked up on this strategy and also used counterexamples as she attempted to appease Maritza's demand for a precise explanation.

The group played the game in the following manner. Each participant had their own turn at solving the Tower of Hanoi using 6 disks while the others kept track of the number of moves. The first player, Elicia, experienced repair and interruptions of her turn from Gabriela and Maritza after only a few moves. Within a few minutes, Elicia gave up and left the group, only to rejoin later on two occasions. On her first return, she demanded the full attention of UG Tina, appealing for her approval with a display of Cat's Cradle skill. Elicia's second return was at the end of the vignette as she participated as an observer of Maritza's final and successful attempt at Tower of Hanoi. Elicia did not weather the repair practices her peers directed at her, giving up soon after their intrusions.

Maritza's turn at the game immediately following Elizabeth was also filled with repair practices from her peers. After reaching an impasse in her strategy, she attempted to start over but was prevented by UG Tina who directed Gabriela to take her turn. Not satisfied with this turn of events, Maritza retreated to her corner of the table and played a game of Mancala by herself. While Gabriela took her turn, Elizabeth returned with her Cat's Cradle string and diverted UG Tina's attention. Gabriela was left without any observers and left the group with Elicia, not having completed her turn.

Left alone with UG Tina, Maritza continued with her Mancala game. UG Tina then took up the Tower of Hanoi, reached an impasse, and announced to Maritza that they should try another game. Instead, Maritza announced that she would try Tower of Hanoi again. During her turn, Gabriela and Elicia returned as observers and participated in a series of repair practices directed at Maritza as she persisted in reaching a solution.

This vignette shows how one child, Maritza, reached a successful solution in the face of a series of contentious repair practices. Elicia, on the other hand, did not respond to her peers' interventions in the same manner. Her other appearances in the data corpus of games shows that her other three game activities were all structured to allow for short turns and less contentious collaboration among the game players, suggesting that she preferred a different participation style. The fact that her participation in games was less than some of the other children corroborates this assertion.

Gabriela, who introduced Tower of Hanoi to the group and proclaimed it to be "fun", also did not participate in more than four games in the data corpus. However, she was motivated to create her own game, a complex synthesis of bingo, arithmetic rules, and dice. Her creative

process with the assistance of an undergraduate facilitator was captured in one of the videos from the data corpus.

Maritza, who demanded precise explanation from Gabriela and persisted in the face of contentious repair practices from her peers, appeared to thrive on challenge. Faced with occupying the role of passive observer, she retreated to her own choice of a different game and was content to play on her own. When she observed that the adult facilitator had given up on the challenging Tower of Hanoi game, she returned to the game and persevered to reach a successful solution while negotiating her peers' relentless repair practices.

In the following sections, I discuss various themes suggested by my findings in the Tower of Hanoi vignettes that also pertain to the rest of the game activities in the data corpus.

B. Explanation

Peer-mediated mathematical explanations have been identified as important learning goals in the mathematics education literature (Fuchs, Fuchs, Hamlett, Phillips, Karns, & Dutka, 1997). The children in *Los Rayos* were often and regularly encouraged to explain their reasoning to adults and peers. Likewise, the children were also prompted to take charge of their play activities by, for example, explaining game rules and procedures to their peers. The contribution of peer play explanatory talk has been linked to positive outcomes for second language learners' academically mediated language skills (Aukrust, 2004).

In the *What's the Point?* vignette, Excerpt 1, Gabriela and Maritza engaged in a tension filled exchange over the explanation of the Tower of Hanoi game rules. Maritza repeatedly demanded a more precise explanation of the procedures while Gabriela struggled to convey her meaning verbally while supplementing her explanation by moving game disks. This episode represents an important activity for practicing the type of precise language needed for

mathematical explanation. Throughout the data corpus, children regularly provided explanations of game rules, strategies, and corrective feedback to their peers in the course of game activities. According to Vygotsky (1987a), speech is essentially an analytic act because language demands that the speaker should translate internal thought into a precise sequence of words, an act that is qualitatively different from silent contemplation. In terms of mathematical explanations, Kazemi and Stipek (2001) provide helpful guidelines for analyzing the types of explanations found during game play in *Los Rayos*:

...a high press for conceptual thinking is characterized by the following sociomathematical norms: (a) an explanation consists of a mathematical argument, not simply a procedural description; (b) mathematical thinking involves understanding relations among multiple strategies; (c) errors provide opportunities to reconceptualize a problem, explore contradictions in solutions, and pursue alternative strategies; and (d) collaborative work involves individual accountability and reaching consensus through mathematical argumentation.

Children's explanations during *Los Rayos* game activities do not appear *mathematical* because they were focused on procedure, game rules, fairness, and strategies for winning. However, their explanations did involve understanding relations among multiple strategies. Children also marked errors in their peers' strategies, providing opportunities for reconceptualizing problems contained in the game structures. They actively explored contradictions in solutions and pursued alternative strategies. Their collaborative activities involved individual accountability and though they managed to reach consensus on many occasions, they did not reference formal mathematical concepts. Considering that *Los Rayos* was not created to be a remedial tutoring establishment or connected to their classroom mathematics curriculum, children's explanations during game activities were surprisingly *mathematical* when viewed with respect to the Common Core State Standards for Mathematical Thinking.

C. Persistence

Lucinde played TOH during two concurrent after-school sessions shown in the *One-on-one* and *Two at Once* vignettes. She experienced a great deal of repair from UG Jessinia and persisted in her attempts at mastering the game. In the *What's the point?* vignette, Maritza tried to solve TOH and failed several times yet persisted to reach the solution. She also experienced a great deal of error correction from both UG Teresa and her peers, Gabriela and Elicia. It seems that Maritza thrived on the challenges she faced.

In contrast to Lucinde (*One-on-one* and *Two at Once*) and Maritza (*What's the Point?*), Elicia (*What's the Point?*) never reached a solution and spent less than 10 minutes in her only attempt at solving TOH. She also experienced error marking and repair from her peers. One possible explanation for Elicia's lack of persistence could be that she was not comfortable with the type of repair practices she experienced from Maritza and Gabriela. When it became apparent that her solution attempt would not be successful and her peers had marked and tried to repair her errors, she laughed, put her hand to her face, and gave up trying to complete the task. This behavior suggests that Elicia has a performance orientation rather than a learning orientation. Further evidence of my claim comes from the fact that she immediately left the group when her turn was over. When she first returned, she claimed the full attention of UG Teresa, the adult and the person with most authority in the group. Elicia then proceeded to engage UG Teresa in a show of her mastery of Cat's Cradle, two loops of string wrapped around the fingers of both hands used to make patterns. Elicia's demand for attention and praise from UG Teresa is also a marker of performance orientation.

It is possible that Elicia would act differently if, for example, she participated in the style of collaboration that Lucinde seemed to prefer in the *One-on-one* vignette. Another possible

explanation is that she is not interested in the game. She said on several occasions “This is boring!” Whether Elicia is performance oriented, averse to repair practices, bored and disinterested in the game, or both bored and intimidated, this situation corroborates findings in the literature on play showing that it is difficult and ill-advised to force children to play with someone or something when they do not want to. On this note, I now move to the theme of *choice*.

D. Choice

Children in *Los Rayos* have the freedom to choose activities and the people with whom they interact. This situation is best characterized as *perceived control* because the adults in *Los Rayos* have provided the activities that are available and the children’s perception of choice is limited by what the adults provide. Also, children’s choice of who to participate with is limited by conditions beyond their control, namely by who is available on any given day. However, the fact that they are free to change activities and vary their peer, adult, and group affiliations mediates their sense of perceived self-efficacy. Bandura (1993) reviews four major processes influenced by students’ perceived self-efficacy; cognition, motivation, affect, and selection. He outlines how students’ belief in their self-efficacy to regulate their own learning determines their aspirations, level of motivation, and academic accomplishments. This may partially account for Maritza’s persistence in solving the Tower of Hanoi. Maritza seemingly lost interest in solving TOH during the latter part of the *What’s the point?* vignette, withdrawing from the game to play a different game by herself. When UG Teresa attempts to solve TOH, gives up, and invites Maritza to play a different game, Maritza drops the materials she has in her hands and announces, “Let me try!” Maritza not only rejects UG Teresa’s invitation but also chooses to persist in trying to successfully complete a solution to TOH. Maritza’s motivation to defy UG

Teresa's suggestion may also be explained by her sense of self-efficacy, supported by the *Los Rayos* environment which is responsive to children's needs and allows them to exercise their capabilities without restraint (Bandura, 1986).

In the *One-on-one* vignette, Lucinde exercised a different type of choice. She chose to play TOH in a turn-taking style with UG Jessinia, rejecting the competitive format suggested by the printed material that explains the rules. After several rounds of game play, UG Jessinia attempted to coerce Lucinde into trying a solution without assistance. Lucinde rejected the suggestion and did not attempt a solution on her own until the end of the vignette. Presumably Lucinde recognized that she needed more assistance and did not attempt independent action until she felt confident to do so. When Lucinde played TOH on the following day, she chose to play on her own without assistance but with UG Jessinia in close proximity.

E. Participation Styles

The styles of participation in *Los Rayos* are characteristic of indirect social processes of cognitive socialization (Gauvain & Perez, 2007). The processes of observational learning, intent community participation, guided participation, and legitimate peripheral participation are all present in the *Los Rayos* interactions. Observational learning (Bandura, 1986) has a long history in the psychological literature. In the context of observational learning it is important to note that imitation is not as simplistic as the conventional use of the term suggests. Observers use information to adapt to new problems and situations through encoding, retaining, retrieving, reproducing, and modifying information gleaned from their experiences. When UG Teresa used counterexamples (Excerpt 1, Lines 19, 23 and 25) in the *What's the point?* vignette, Gabriela adopted this strategy by moving game pieces and said, "So pretend you already moved and you can't go like this" (Excerpt 1, Line 47).

Intent community participation is another style found throughout the *Los Rayos* data corpus. Rogoff et al (2007) outline salient characteristics of intent community participation that match the children's actions as they wait for their turn during game play in the vignette, *What's the point?* Children waiting for their turn closely observed the player's actions in anticipation of their own turns. In this vignette, the observers took on the roles of advanced participants who are involved in the process through their guidance of the player. They took initiative through repair practices by calling out directions and pointing at game pieces and positions on the game board.

Guided participation, also known as guided repetition, is evident in Lucinde's first experience with TOH in the *One-on-one* vignette. Lucinde reacted favorably to UG Jessinia's frequent repair practices. Lucinde posed questions and refused to play without guidance until she felt comfortable proceeding on her own. On the next occasion of Lucinde's play with TOH, she did not solicit guidance from UG Jessinia and appeared content to play in relative silence.

While Lucinde plays side-by-side with Marguerita in the *Two at Once* vignette, the situation of two independent players with this complex game may be more than one adult facilitator can handle. The cognitive demand involved in playing one TOH versus two simultaneous TOH games could be too much for one person to monitor effectively. Although side-by-side independent play is not uncommon among children, this style of play seemed to limit UG Marguerita's ability to provide effective scaffolding for this game activity. In their discussion of side-by-side play, Paradise and Rogoff (2009) describe the importance of this participation style for supporting learning and socialization in both school and informal settings. Children learn by listening, observing, and taking purposeful initiative as they participate in everyday activities.

There are several qualities about the TOH vignette, *One-on-one*, that evoke comparison with the passage below by Marcus Du Sautoy (2008, p. 115). He describes his experience with face-to-face interaction between mathematicians collaborating on research and problem-solving.

It is a very fragile process. People often assume that we must all be doing mathematics by email and there is no need to meet. But our brand of collaboration could never be done electronically. For a start we often sit for hours, quietly thinking to ourselves, saying nothing, every now and again scribbling something down. But then a single word spoken can spark something in the mind of the other. Looking Fritz right in the eye, waving my hands and grunting is not something that can be replicated by email.

Lucinde and UG Jessinia were often silent as they moved the game pieces during the last 20 minutes of their TOH game activity when they used 7 disks, the most complicated TOH version they have experienced. Their utterances during their 7 disk attempt were short statements accompanied by gestures towards the game pieces and game board positions. The number of language tensions in this vignette was less than in the *What's the point?* vignette. In short, their interactions are like the mathematicians' hand-waving and scribbling interspersed with quiet thinking. The point I wish to emphasize is that the two situations in my comparison share the following qualities: 1) informal discourse focused on problem-solving, 2) a small number of utterances, 3) collaboration, and 4) silent contemplation during a joint goal-directed activity.

Even though the participants in the *What's the point?* vignette seem to be livelier during their TOH activity, Maritza has her own experience with the silent, contemplative version of problem-solving. This silent concentration lasts less than 4 minutes and she completes a 5 disk solution successfully with no verbal assistance or gestures (i.e. pointing to game pieces) from others in the group. UG Teresa and Gabriela are present with their gazes focused on the game board but they provide no comments or gestures that would count as direct assistance. However, they do provide support by their intent focus on the game activity. These observers were engaged

in legitimate peripheral participation (Lave & Wenger, 1991) through keen observation and listening in anticipation of correcting Maritza's game acts.

F. Repair Practices and Discursive Tensions

Marking of errors and repair practices from adults and peers are some of the main types of scaffolding assistance children experienced in *Los Rayos*. In the *Los Rayos* data, children received fewer repairs as they became able to complete tasks on their own. Children also engaged in repair practices with their peers, helping each other to understand game rules and strategies but sometimes also discouraged each other's persistence.

G. Bi-directional Socialization

The adult facilitators' scaffolding techniques are mediated by children's actions, indicating that children exercised agency when they asked questions related to game rules and operations. In the *One-on-one* vignette, Lucinde read the game instructions aloud with some assistance but she posed several questions that indicated she had not understood the game rules. Her questions prompted UG Jessinia to alter her strategy for explanation by providing visual mediation in her acts of moving the disks, illustrating correct and incorrect moves. Also, instead of a recitation of the game rules, UG Jessinia and Lucinde engage in several questions and answers. This type of interaction suggests "...a dialectical process in which human agents construct meanings out of each other's behavior and, thereby, produce transformational change," (Kuczynski & Parkin, 2007, p.262). There is dialectic of control in the relationship between Lucinde and UG Jessinia. When UG Jessinia encouraged Lucinde to solve the Tower of Hanoi by herself, Lucinde's refusal to do so until she was ready is an example of the control Lucinde exercised in their relationship. Likewise, Maritza refused to play a different game as UG Tina suggested and instead took up the game UG Tina wanted to abandon.

H. Research Questions Revisited

In this section I restate my research questions and discuss how they relate to my findings.

Primary Question

1. How are bilingual students socialized in mathematics through play?

Children in *Los Rayos* were socialized through a combination of several mediating influences. Interactions with adults and peers, repair practices they received from other participants and demands for following game rules were some of the most powerful mediating influences that encouraged mathematical thinking and practice. It appears certain that when children engaged in adding numbers on dice, recognizing patterns in game operations, and attending to precision in their explanations of rules, for example, they practiced thinking that is indirectly related to mathematical practice. However, some children did not appear to engage with games in the same way that other children practiced mathematical thinking. When children did engage with mathematical thinking practices, they displayed actions that map to the CCSS Mathematical Practice standards I outlined in my findings (see Table VI, Table VIII, and Table X). Specifically, children in Los Rayos engaged in the following practices:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Attend to precision.
- Use clear definitions in discussion with others and in their own reasoning.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

- Continually evaluate the reasonableness of intermediate results.
- Recognize and use counterexamples.
- Justify their conclusions, communicate them to others, and respond to the arguments of others.
- Construct arguments using objects, drawings, diagrams, and actions.
- Listen or read the arguments of others, decide whether they make sense, and ask useful questions.

Regarding formal mathematical concepts, I did not find, for example, children learning the formula for the least number of moves required to solve Tower of Hanoi, $2^n - 1$, where n is the number of disks used in the problem. I also did not expect to find children using formal mathematical discourse. However, my findings corroborate the following conclusions stated by Walkerdine (1988).

There are at least three ways in which the activities of the out-of-school practices differ from the mathematical activities of the classroom:

- The goals of the activities in the two settings differ radically
- Discourse patterns of the classroom do not mirror those of everyday practices
- Mathematical terminology and symbolism have a specificity that differs markedly from the useful qualities of ambiguity and indexicality (interpretation according to context) of terms in everyday conversation.

2. How do mathematical game activities in the after-school setting mediate affordances/limitations to mathematics socialization?

There were two major mediating influences on children's affordances and limitations for mathematical socialization in the Los Rayos context: 1) alteration of game participation structures and 2) the embedded mathematical concepts within each game.

Game structures mediated the affordances and limitations children had to engage with mathematical thinking and practice. In the TOH vignettes, two groups altered the game structure which resulted in different affordances and limitations for engaging with actions related to mathematical thinking.

In *One-on one*, Lucinde and UG Jessinia collaborated by taking turns, moving one disk at a time instead of competing with each other to find who can solve TOH in the least number of moves. This alteration of the game structure allowed Lucinde to receive assistance and feedback for almost every move she made in the game. At the same time, she was able to observe UG Jessinia's game moves. Further, Lucinde was able to suspend game activity and ask as many questions as she desired. This situation is a hybrid of two participation styles, namely, guided instruction and observational learning.

The *Two at Once* vignette showed Lucinde and her peer, Marguerita, playing with TOH independently side by side. The main difference between the participation styles in *One-on one* and *Two at Once* lies in how these structures afforded opportunities for assistance and scaffolding. With Marguerita and Lucinde playing independently, UG Jessinia's ability to monitor progress and provide scaffolding was limited by the complexity of the task. Also, this participation style of simultaneous independent activity did not yield enough cooperative interaction for the players to benefit from each other's thinking.

The participants in the *What's the Point?* vignette used a competitive participation style for playing TOH. This game structure, if followed in its intended form, would have allowed participants to observe each other's strategy when it was not their turn. As observers counted the player's moves, they would have been able to engage in observational learning and use this knowledge to improve their own strategies during their turns. However, the players in *What's the*

Point? did not follow a strict observational learning style. Instead, the observers interacted with the player through repair practices.

The mathematical concepts within each game determined the extent that children were able to engage with mathematical content. I expected this finding but the extent to which children were able to practice and receive scaffolding with these concepts was mediated by the style of participation chosen by each group as they played the games. In my research methods, Table 4.1 Descriptions of Games lists the mathematical concepts inherent to each game.

3. How do mathematics ideologies mediate mathematics socialization for bilingual Latinas/os during play?

Identifying students' mathematical ideologies was problematic. I found no instances of specific statements by children about how mathematics should be taught or learned. At the ages of 10 and 11, children have only a vague sense of self-image (Panaoura & Philippou, 2005). When I examined end-of-year interviews with the *Los Rayos* children, I found that their ideas about what constitutes mathematics and mathematical practice were also vague and restricted to statements about "numbers", "solving problems", and calculations. My expectations of uncovering specific ideological statements and actions proved to be unrealistic in this context. However, video data of game activities showed that *Los Rayos* children acted in ways that confirm their engagement with the following qualities:

- Valuation of strategy
- Repair practices
- Learning and playing styles characterized by collaboration as well as competition

An example of a child's valuation of strategy is shown in an episode of the *Counters Game* which was documented in detail in a previous *Los Rayos* study by Willey & Radosavljevic (2007). "It's only a game" was a statement by a student who subsequently changed her mind and showed that she valued a game winning strategy. In all of the games in *Los Rayos*, children's extensive use of repair practices indicated that they valued both correctness and fairness when playing games. Video data also showed that various styles of collaboration were valued by children as well as competitive formats.

When I searched for adult facilitators' mathematical ideologies, data showed that their styles of interaction with children were almost always sensitive to children's preferred modes of participation and non-didactic. Facilitators supported children's choices for activities and they rarely silenced children or ignored their contributions to activities. One notable exception, mentioned in Willey and Radosavljević (2007) described how an adult graduate facilitator, Paula, tried to explain the embedded mathematical concepts in the *Counters Game* to her group during the game set up. Her talk dominated the interactions and she used the pedagogical practice known as *Initiation-Response-Evaluation* extensively. Children responded to her questions with short utterances and she gave them short evaluative responses.

However, "...the practice of *repair* can serve as an index of cultural authority and power thus making it an ideological practice," (Razfar, 2005, p.405). In the *Los Rayos* after school "culture" where children are at play, these repair practices can be said to index struggles over power and authority among the children. Although the children may not recognize their games as inherently *mathematical*, they are exercising agency within their own play culture. In this sense, the children are displaying their ideologies of play when they engage in repair practices directed at their peers and at the adults.

4. *How do participants use language and cultural resources (Funds of Knowledge) as they negotiate systems of meaning in game play?*

Play is a significant part of children's funds of knowledge. Children organized themselves during game play by taking turns, enforcing fair play, and changing rules to suit their preference for different styles of participation. The participation styles included one-on-one turn taking, individual solo turns in *What's the point?* and side-by-side individual play in the *Two at Once* vignette.

Spanish was a linguistic resource for meaning making in the *Los Rayos* context. Hybrid language practices were used spontaneously. There was almost no talk about language use. Some notable exceptions include an UG's question to Maritza in a game not covered in my analysis of the three TOH vignettes. When asked at the beginning of the game, *El Maga's Hat*, "Which language do you want to use?" Maritza chose Spanish and the play group continued the entire activity using Spanish exclusively. However, Maritza used English exclusively during TOH. Gabriela used English in TOH but during the *Counters Game* she showed her Spanish speaking proficiency when mothers were part of the game. Elicia used Spanish in her side conversations with Gabriela during TOH. Her Spanish use during TOH was restricted to "gossip" with Gabriela and mocking of other children not present at their group. UG Teresa used Spanish/English hybrid constructions during TOH when she scaffolded children's strategic understanding of the game and also as she intervened in conflict and tension between the children. Lucinde spoke only English during both of her TOH episodes but like Gabriela, she used Spanish exclusively when her mother participated in the *Counters Game* episodes.

The list below summarizes my findings for this question.

- Participants used Spanish, English, and hybrid language practices
- Play is the children's culture
- Children identified with adult facilitators who shared common experiences with bilingualism, Mexican American and other immigrant home cultures.

I. Implications

Current discussions on after school programs focus mostly on remediation models (e.g., Lauer, et al, 2003) and mathematics oriented after school programs rarely involve enrichment activities. Notable exceptions include the Algebra Project (Moses, 2010) and the Fifth Dimension (Cole, & The Distributed Literacy Consortium, 2006). This study is aimed at uncovering the value of mathematics based games for encouraging children's mathematical thinking and practice, behaviors considered to be metacognitive in nature and not clearly defined in the literature in terms of how these practices can be taught. Socialization research suggests that these thinking styles are best taught through indirect means, namely through observation of and participation with more experienced participants in the course of activity. This study shows that the context of activity mediates possibilities for children's engagement with targeted mathematical thinking practices. Children in *Los Rayos* have opportunities to choose their activities, participants, and style of interaction. When children are allowed to follow their interests in this particular context, they spontaneously engage in the types of mathematical practices thought to be useful in the development of mathematical thinking.

Further implications are specifically related to the use of mathematical games in both after school and classroom contexts. First, children respond to games differently according to their preference for participation style, for example, collaborative versus competitive games and multi-player versus one-on-one structures. Second, when children are encouraged by their peers to

participate in an activity, they are likely to persist in that activity as long as the group members are not overly directive or inattentive.

For evaluations of after school enrichment programs, evaluators should be aware of the value of indirect cognitive socialization processes that target thinking practices and are not easily measured or observed in informal contexts where activities are spontaneous and child-directed. These types of activities do not resemble formal or traditional classroom models and can be easily dismissed as having no value. Instead, important components of social, emotional, and cognitive development are present in these informal adult-mediated contexts.

J. Concluding Remarks

I should mention that there still remains a question about the *degree of socialization* that occurred in the *Los Rayos* after-school context. I attempted to describe the process that leads to mathematical socialization during children's game play. But a descriptive study is, by some definitions, a study that explores *what happened* and not *how* or *why*. Although I ask "how are children socialized in mathematics through games", a more apt question is "*are* children socialized in mathematics through games" and if so, to what *degree*? This question was posed to me at the end of my study and I admit that I have not arrived at a satisfactory answer. In Martin's (2000) study on mathematics socialization, he was able to interview adults who could reflect back on their personal histories. This allowed Martin to arrive at convincing explanations about these individuals' character and degree of mathematics socialization, although I am not sure he would agree with my use of the word *degree*. If I could compare the amounts of discursive tensions and mathematical thinking practices found in *Los Rayos* with the same children's classroom experiences, I am not sure that type of study would help me answer the question of the

degree of mathematics socialization these children experienced. In this study, I had only hoped to find the qualities of a process of mathematics socialization in a very specific context.

Socialization occurs all the time and when children are in a context where mathematics is involved, they are being socialized into some kind of mathematics whether it is everyday mathematics, school mathematics, or mathematicians' mathematics. Caswell (2005) attempted to determine what children aged 9-12 thought about the value of play in learning mathematics by asking them after they had engaged in mathematical play activities. The children's responses were favorable. For example, one child stated, "It is much harder than just doing a sheet, [it] makes you think a lot more and use much more maths at once" (Caswell, 2005, p. 223). This begs the question, if children are told that what they are doing is mathematical, do they automatically find the value of that activity as it relates to mathematics? Are they being led to mathematize a situation that they otherwise would not have mathematized? The same situation exists in the *Los Rayos* afterschool context. Children were told that their afterschool club was about mathematics, so they had been primed to look for the mathematics in their play activities. But their school principal, who had also been informed that these activities were mathematical, was not convinced. On the other hand, the researchers facilitating the activities could see rich mathematics in these activities although often their realizations occurred after the fact while debriefing each other, writing field notes, and reviewing video data. Had I conceived of this study before the *Los Rayos* after-school began, perhaps I could have created survey and interview questions aimed at the question of the *degree* of socialization children experience in this context.

Returning to my assertion that people are always in the process of being socialized, I wanted to know what happened in the process of their game play that might be mathematical.

Since Schoenfeld (1992) and other prominent thinkers assert that mathematical thinking practices, *metacognitive* practices, are learned indirectly, what does this look like? How can we know when children are using mathematical thinking when they are not engaged in traditional mathematics problem solving? If teachers, parents, and other child care facilitators are going to learn how to encourage and expand children's mathematical thinking practices while they are engaged in play activities, then there must be some more economical way for them to recognize affordable moments to scaffold children's thinking practices. The only means they have in the moment has to be observable. This means that speech acts and children's physical manipulations of game pieces are the most accessible representation of children's thinking.

Next steps related to this study should consider how to structure after school academic enrichment programs to take advantage of the growing trend in teacher education programs that is moving towards a more field-based model for pre-service teachers. Pre-service teachers need more experience with children in contexts where they can observe and interact with children under conditions where children have more choice and agency in their activities. Without the presence of externally mandated testing and pressure from formal school assignments, children can freely express and act on their interests and participation styles. Ideally, pre-service teachers would learn different qualities about how children act and think compared to observing children in classroom contexts, providing insight into how to structure activities that can encourage children's motivation and persistence.

APPENDIX A

Common Core State Standards for Mathematics: Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1 Make sense of problems and persevere in solving them.

Mathematically proficient students:

- explain to themselves the meaning of a problem and looking for entry points to its solution.
- analyze givens, constraints, relationships, and goals.
- make conjectures about the form and meaning of the solution attempt.
- plan a solution pathway rather than simply jumping into a solution.
- consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.
- monitor and evaluate their progress and change course if necessary.
- transform algebraic expressions or change the viewing window on their graphing calculator to get information.
- explain correspondences between equations, verbal descriptions, tables, and graphs.
- draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- use concrete objects or pictures to help conceptualize and solve a problem.
- check their answers to problems using a different method.
- ask themselves, “Does this make sense?”
- understand the approaches of others to solving complex problems and identify correspondences between approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students:

- make sense of quantities and their relationships in problem situations.
- Bring two complementary abilities to bear on problems involving quantitative relationships:

- ✓ *decontextualize* (abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
- ✓ *contextualize* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).
- use quantitative reasoning that entails creating a coherent representation of the problem at hand, considering the units involved, and attending to the meaning of quantities, not just how to compute them
- know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context from which the data arose
- compare the effectiveness of plausible arguments
- distinguish correct logic or reasoning from that which is flawed and, if there is a flaw, explain what it is
 - ✓ elementary students construct arguments using concrete referents such as objects, drawings, diagrams, and actions..
 - ✓ later students learn to determine domains to which an argument applies.
- listen or read the arguments of others, decide whether they make sense, and ask useful question to clarify or improve arguments

4 Model with mathematics.

Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
 - ✓ In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
 - ✓ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
- identify important quantities in a practical situation
- map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.

- analyze those relationships mathematically to draw conclusions.
- interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students

- consider available tools when solving a mathematical problem. (These tools might include pencil and paper, concrete models, a ruler, protractor, calculator, spreadsheet, computer algebra system, a statistical package, or dynamic geometry software.)
- are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
 - ✓ High school students analyze graphs of functions and solutions generated using a graphing calculator
- detect possible errors by using estimations and other mathematical knowledge.
- know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
- identify relevant mathematical resources and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students:

- try to communicate precisely to others.
- try to use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.
 - ✓ In the elementary grades, students give carefully formulated explanations to each other.
 - ✓ In high school, students have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students:

- look closely to discern a pattern or structure.
 - ✓ Young students might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have.
 - ✓ Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for the distributive property.
 - ✓ In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
- step back for an overview and can shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or composed of several objects.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- notice if calculations are repeated
- look both for general methods and for shortcuts.
 - ✓ Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeated decimal.
 - ✓ Middle school students might abstract the equation $(y-2)/(x-1)=3$ by paying attention to the calculation of slope as they repeatedly check whether the points are on the line through (1,2) with a slope 3.
 - ✓ Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)(x^2+1)$ and $(x-1)(x^3+x^2+x+1)$ might lead high school students to the general formula for the sum of a geometric series.
- maintain oversight of the process of solving a problem while attending to the details.
- continually evaluate the reasonableness of intermediate results.

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Vita

Alexander Radosavljević

Education

Ph.D. Curriculum Studies/Mathematics Education 2014
University of Illinois at Chicago

M.A. English 1994
Purdue University at West Lafayette, Indiana

B. S. Secondary Education 1989
University of Illinois at Urbana-Champaign

B.A. Liberal Arts and Sciences 1986
English major and Mathematics minor
University of Illinois at Urbana-Champaign

Certification

Illinois Type 09 Secondary Teaching
Endorsements: Mathematics 9-12 and English 6-12

Education Research

Center for Mathematics Education for Latinas/os (CEMELA)
National Science Foundation Center for Learning and Teaching, grant number ESI-0424983
<http://math.arizona.edu/~cemela/english/>
Research Fellow (2005 – 2011)

University of Illinois at Chicago
PRAIRIE Group – University of Illinois at Chicago
Research Assistant (2011 – 2012)
Worked on formative evaluation of elementary school dual language programs, K-2 Summer Reading Programs, and high school summer enrichment programs in Chicago Public Schools

University of Illinois at Chicago
Learning, Instruction, and Teacher Development (LITD)
Research Assistant (2005 – 2007)
Mathematics Benchmark Assessment Pilot for Chicago Public Schools grades 3-8

University Teaching Experience

Governors State University

Adjunct Instructor (Summer & Fall 2013)

MATH 2137 Mathematical Foundations

ACM 6140 Methods of Teaching Math I

ACM 7240 Methods of Teaching Math II

Indiana University/Purdue University at Indianapolis (IUPUI)

Visiting Lecturer (2012-2013)

EDUC N102 Teaching & Learning Elementary Mathematics I

EDUC 343 Mathematics in the Elementary Schools

EDUC H440 Foundations of American Education Capstone Seminar

University of Illinois at Chicago

Instructor (2009-2010)

CI 507 Teaching & Learning Mathematics in the Elementary School

ED 194 Special Topics in Education – mathematics content for pre-service teachers

Purdue University - West Lafayette, Indiana

Instructor (1992-1993)

English Composition 101 & 102

Secondary Teaching Experience in Chicago Public Schools

Morgan Park High School (1995 – 2005)

Mathematics – Algebra I, Geometry, Algebra II/Trigonometry, College Algebra, Pre-Calculus

English – World Literature, English III, Topics in Literature

John F. Kennedy High School (1994-1995)

Special Education - Mathematics and English

George H. Corliss High School (1989-1991)

Mathematics – Pre-Algebra, Geometry, Algebra II/Trigonometry

English – English I with support, English II

Publications

Baldwin, J. T., Radosavljević, A., & Lee, H. S. (2010). P is not pizza: Variables from grade 3 to 13. In D. F. Berlin & A. L. White (Eds.), *Promising practices to meet global challenges in science and mathematics education* (pp. 115-130). Columbus, OH: International Consortium for Research in Science and Mathematics Education.

Thompson, A., & Radosavljevic, A. (in press). Fostering mathematical discourse with Socratic seminar. In M. Civil & E. Turner (Eds.), *The Common Core State Standards in mathematics for English language learner: Grades K-8*. Alexandria, VA: TESOL.

Conference Papers and Presentations

- Khisty, L. L., & Radosavljević, A. (2009, May). *Understanding teaching that supports ELLs in mathematics*. Powerpoint presentation at the Chicago Mathematics and Science Initiative Annual Conference.
- Khisty, L. L., & Radosavljević, A. (2010, April). *A descriptive analysis of Math Pathways and Pitfalls in a Latina/o bilingual classroom*. Paper presented at the American Educational Research Association Annual Meeting, Denver, CO.
- Khisty, L. L., Viego, G., & Radosavljević, A. (2007). *Mining Latino resources through play: Insights from an after-school project*. Powerpoint presentation at the National Council of Teachers of Mathematics Regional Conference, Kansas City, MO.
- Radosavljević, A. (2009, April). *Playing with mathematics*. In L. L. Khisty (Chair), *Lessons learned: Researching Latinos, language, and culture in mathematics*. Symposium conducted at the National Council of Teachers of Mathematics Annual Meeting, Washington, D.C.
- Radosavljević, A. (2012a, April 14). *Mathematics Socialization through games: Bilingual third graders after school*. Paper presented at the American Educational Research Association Annual Meeting, Vancouver, BC.
- Radosavljević, A. (2012b, June 1). *Mathematics Socialization through games: Bilingual third graders after school*. Paper presented at the CEMELA Dissemination Conference, Chapel Hill, NC.
- Radosavljević, A. & Thompson, A. (November, 2013). *Mathematical Socialization through Game Activity: Common Core Standards for Mathematical Practice in an After-school Club*. Paper presentation at the 35th Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA 2013), Chicago, IL.
- Razfar, A., Khisty, L. L., Willey, C., & Radosavljević, A. (2008). *Multimodal problem solving and probability: The counters game*. Poster session presented at the American Educational Research Association Annual Meeting, New York, NY.
- Razfar, A., Sutton, T., & Radosavljević, A. (2009, April). *Learning mathematics and language: Examining the tensions in two after-school clubs*. Presented in the symposium titled: *Tensions, contestations, and relations: Latina/o mathematical meaning making in hybrid spaces* conducted at the annual meeting of the American Educational Research Association, San Diego, CA.
- Thompson, A., Lopez Leiva, C., Radosavljević, A., & Willey, C. (2011, May 6). *Mathematics education, language, and Latinos/as: Prospects for the future*. Presentation at the Chicago Symposium Series, Chicago, IL.
- Thompson, A., & Radosavljević, A. (2012, March 23). *Your contributions to equity in STEM education*. Presentation at the Chicago Symposium Series, Northwestern University, Evanston, IL.
- Willey, C., & Radosavljević, A. (2007). *Play and building histories/identities of math competencies for Latinos/as*. Paper presented at the 2nd Socio-Cultural Theory in Education Conference: Theory, Identity, and Learning, Manchester, England.

Professional Organization Memberships

AERA - American Educational Researcher Association

CEMELA - Center for Mathematics Education for Latinas/os

NCTE - National Council of Teachers of English

NCTM - National Council of Teachers of Mathematics

TODOS: Mathematics for All

Awards and Honors

National Science Foundation Fellowship 2005-2011

Center for Mathematics Education for Latinas/os (CEMELA)

Eagle Scout 1981

Boy Scouts of America

Languages

Serbian: reading and conversation

Russian: reading