Business Intelligence and Smart Pricing in Uncertain Competitive Environment

BY

FERDI ERUYSAL
B.S., Middle East Technical University, 2000
M.S., Bahcesehir University, 2003

THESIS

Submitted as partial fulfillments of the requirements
for the degree of Doctor of Philosophy in Management Information Systems
in the Graduate College of the
University of Illinois at Chicago, 2014

Chicago, Illinois

Defense Committee:

Aris M. Ouksel, Chair and Advisor
Siddhartha Bhattacharyya
Stanley Sclove
Ali Tafti
Kunpeng Zhang
Canan Uckun, Argonne National Laboratory

ACKNOWLEDGMENTS

This study would not be possible without the help, guidance, and encouragement of my advisors, colleagues, friends, and family. I would like to give special thanks to my advisor, Prof. Aris M. Ouksel, for his inspiring mentorship as well as encouraging me to find my own research path. Unfortunately due to life's challenges, my dissertation could not always be the number one priority. If it were not for his patience and extraordinary guidance, I would not be writing this now. He has constantly pushed me to be a better researcher and significantly broadened the scope of my abilities. I will always be thankful to him.

Thank you to my dissertation committee including Siddhartha Bhattacharyya, Stanley Sclove, Ali Tafti, Kunpeng Zhang and Canan Uckun, who have all always been exemplary mentors and scholars. I also owe a great deal of gratitude to my other friends and former colleagues in Information and Decision Sciences department at University of Illinois, Chicago.

I would like to extend my deepest appreciation to my parents, Faruk and Nermin Eruysal. Their dedication to my academic success has been truly remarkable. I would also like to express gratitude to my brother, Fatih. His faith in me has been truly inspiring.

Part of this dissertation has already been published in Ouksel and Eruysal (2011). Authorization to reuse the material in this dissertation is provided in appendix I.

CONTRIBUTIONS OF AUTHORS

This dissertation consists of a collection of three original papers. One paper has already been published, another one has been submitted for publication and third one will be submitted shortly. The research presented in this thesis was carried out by me under the supervision of Prof. Aris M. Ouksel. All game theoretical models, proofs, analysis, and discussion were performed by me with close supervision of Prof. Ouksel. Prof. Aris M. Ouksel supervised the study, helped in building game-theoretical models, aided in the interpretation of results, and assisted in the proofreading, editing final revisions of the thesis. Chapter 1 is a literature review that places my dissertation question in the context of price discrimination and business intelligence. Chapter 2 represents a basic price discrimination model. Chapter 3 is a published manuscript (Ouksel and Eruysal (2011)) for which I was co-author and a major driver of the research. Prof. Aris Ouksel assisted and supervised in building the model, proofs and analysis. He also contributed to editing and proofreading of the manuscript. Chapter 4 and 5 is a series of unpublished papers directed at answering the question of how product cost, information quality and privacy affects the market equilibrium. I anticipate that this line of research will be continued in my future career and this work will ultimately be published as part of a co-authored manuscript.

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

LIST OF TABLES

LIST OF FIGURES

SUMMARY

Business intelligence tools have enabled novel and relatively low-cost capabilities to collect and analyze vast amounts of customer information. Accumulation of customer specific information along with transactional data empowers firms to categorize customers into segments and offer customized prices. The impact of smart pricing on competition, consumer purchase behavior and social welfare in a game-theoretic model with two asymmetric firms is studied.

Three issues that were neglected in the smart pricing literature are identified. First, segment granularity has not been analyzed in detail. Second, product costs were assumed to be uniform. Third, information quality regarding customer preferences and customer privacy were not analyzed in depth. Those three issues are analyzed in detail.

First, a basic game-theoretical model where customers differ in brand loyalty and respond differently to firms' products and promotions is introduced. Firms estimate individual customer price tolerance using business intelligence technology. Then, the basic model was extended to study segment granularity, product cost, privacy-conscious customers and classification errors.

Contrary to previous findings, analysis shows that firms do not necessarily become worse off when firms are able to categorize customers into multiple segments. Firms have strong incentive to acquire business intelligence technology to increase their profits and improve market share. The firm dominating the industry is likely to improve its profits at the expense of the rival firm, and consumer welfare will increase with business intelligence. Two fundamental parameters, market dominance and the technology cost to industry dominance ratio, are introduced to study business intelligence adoption decisions, as a basis for our analytical approach. High technology cost coupled with high dominance indicator results in a situation in which only very dominant

firm acquires business intelligence. Smart pricing and business intelligence leads to lower prices for price-sensitive customers, but higher prices to loyal customers.

Product cost has been ignored in past smart pricing studies as either negligible or equal for all firms. Previous literature implicitly assumes that product cost differential can be aggregated into the price tolerance, and thus product cost can be assumed away. However, product cost and price tolerance are in fact independent of each other, and thus must be examined separately. By using a game-theoretical model with two asymmetric firms, unequal product costs, and variable customers' price tolerance, this study proves that a change in product cost does not translate into a correlated adjustment of the price tolerance. Moreover, a whole spectrum of subcases parameterized on product cost differential and price tolerance is uncovered. Four distinct classes whose equilibrium conditions are significantly different in terms of market share, prices and profits has been identified. The case where product cost is considered to be negligible or equal across firms is shown to be but one special case.

Privacy-conscious customers could inhibit firms' data collection initiatives, resulting therefore in incorrect segmentation. Moreover, firms even could make classification errors due to imperfect business intelligence systems. The impact of information quality and size of privacy-conscious customers are also analyzed by using a duopolistic non-cooperative price discrimination model. When only one firm has sole access to business intelligence, the firm becomes more competitive, retains its loyal customers and even attracts its rival's loyal customers due to flexibility in pricing, The firm with the business intelligence capability improves its market share and profit at the expense of other firm. An improvement in information quality increases profitability of the firm with business intelligence capability as expected. Surprisingly, overall prices in the market go up as the firm with business intelligence capability indirectly affects the pricing strategies of the other firm.

When both firms have business intelligence, lower prices are offered to customers. This result is counter-intuitive. Even though, smart pricing is not considered as fair by customers, it drastically improves the level of competition which leads to more affordable prices. As both firms have access to business intelligence, competition confines to individual segments. Most important contribution of this study is that an increase in information quality lower firms' profits as this makes them more competitive.

I. INTRODUCTION

1.1 Introduction

Firms have devoted considerable human and technical resources to build business intelligence capabilities to collect and analyze vast amount of customer information. Gartner Group, a leading technology-related research firm, reported that BI was the top priority among companies in 2006, and the trend has since accelerated. Firms infer customers' price sensitivity and product preferences by analyzing customer information such as purchasing history, age, income and demographics. For example, for several years Capital One has used customer demographics to tailor credit cards to its customers (Kumar and Reinartz, 2005). By analyzing its huge database, Capital One personalizes credit cards taking into account individual needs like credit limit, credit card design, rebate program, and annual fees. Firms also use BI for market segmentation, the practice of partitioning customers into different groups within which customers have similar characteristics, needs and wants. Segments form homogeneous sub-markets, whose customers respond similarly to firms' product offerings (Dickson and Ginter, 1987).

When customers make a purchase, the transaction is recorded in digital format and a digital trail is left in firms' databases. Companies often use transactional data along with information from a number of other sources to segment the customers. An example is Swipely, a social network for sharing information about credit card purchases (Prior, 2010). By analyzing the current trends and purchasing patterns, Swipely helps firms to implement loyalty programs and special offers for a targeted set of customers. Another example is Cardstar, a startup that introduced a loyalty-card mobile phone application to help firms to attract customers and reward their loyal customers. Customers register their loyalty cards through a mobile application and then start getting customized promotions. Cardstar keeps track of customer purchases for a multitude of firms through a credit card or mobile application payments (Lapowsky, 2010). It is able therefore to share a customer's historical purchase behavior across these firms. Clearly, Cardstar is able to provide purchase data of a competitor's customers. It even goes so far as to provide data when consumers shop at a competitors' store.

Firms have a keen interest in collecting customer information such as age, income and purchasing history. Automated profiling, combining customer information from a number of sources to figure out consumer's purchase behavior, is reported to have been common as early as in 1993 (Clarke, 1993). An average American is estimated to be profiled at least 25 times in electronic databases (Shapiro and Varian, 1998). The Direct Marketing Association estimated the number of consumer mailing lists to be 15,000 in 1996, containing 2 billion names (including duplicates) (Crawford, 1996). Declining cost of hardware and software for the collection and analysis of customer information, and the concomitant increase in competition, has fueled the interest further.

Business intelligence, which can be defined as technologies, applications, and practices for the collection, integration, analysis, and presentation of business information to better understand firm's businesses and customers, has been steadily gaining in popularity. A recent study conducted by the Aberdeen group revealed that 76% of retailers currently use or have budgeted plans for business intelligence (Aberdeen Group, 2006). Another study conducted by Gartner Group has recently shown business intelligence applications to be the topmost technology priority for most firms (Pettey and Goasduff, 2009). Despite the popularity of business intelligence tools in industry and the continuous improvement in their sophistication, there has been a lack of rigorous research on the economic implications of business intelligence on prices, customers and firms. Worse, current studies show conflicting results.

Firms can proactively tailor products and services to individual customers based on their personal preferences. For example, Principal Financial Group delivers personalized investment advice and customer service to a fragmented customer base and sells retirement plans, supplementary mutual funds and insurance by collecting high-quality data about demographics, life milestones, and benefits-enrollment habits of its customers (Ricadela, 2006). Amazon.com suggests similar products to customers by analyzing customers' purchase patterns. Customers definitely profit from personalized offers and products. However, business intelligence tools can also harm some customers financially. Firms can offer higher prices to price-insensitive customers as business intelligence offers the capability of identifying them. By analyzing competitive effects, seasonal factors, product switching, customer loyalty, preference and habits, firms can boost their profits by optimizing their pricing policies.

Price discrimination is thus the practice of charging different consumers different prices for similar products or services when price difference in the market is not proportional to difference in marginal costs (Stigler, 1987). Price discrimination has been studied by researchers in economics, marketing, and information systems. Price discrimination terminology slightly varies from one discipline to the next. In fact, price discrimination, price differentiation and dynamic pricing are generally used interchangeably. We use the term smart pricing and price discrimination interchangeably in this study. Several firms practice smart pricing. Amazon, for example, extensively used a direct form of price discrimination in 2000, although it had to retreat somewhat as soon as there was a public outcry (Streitfeld, 2000; Krugman, 2000). Amazon and other online retailers still resort to smart pricing by implementing reward/loyalty programs and coupons.

Smart pricing is not a new pricing strategy. Railroad companies practiced it more than a century ago, and airlines pursue various forms of it. Advances in information technology are enabling novel ways to collect and analyze vast amount of customer information, and in turn, to exploit this information for the classification of customers into price discriminated categories. For example, more than 2 million customers -- who supply an abundance of transactional data to businesses to tailor their products and services -- have downloaded CardStar's mobile application. Firms are increasingly better informed about their customers' purchasing behaviors, and have available more sophisticated tools for price discrimination analysis. However, it is important to reiterate that, despite advances in the development of the business intelligence tools, their economical implications have not been adequately studied in the literature.

Firms cannot thus engage in smart pricing without being able to predict customers' price sensitivity accurately, which in turn depends on their ability to collect voluminous data regarding seasonal factors, customer loyalty, and preference information. But data collection efforts may run against the resistance of privacy-conscious consumers, who may be wary of its misuse. Indeed, consumers' privacy valuations play a greater role in determining the success of firms' price discrimination attempts. This study will examine the gap in the literature about the impact of privacy-conscious customers on effective price discrimination.

While there have been extensive developments in business intelligence, its impact on firm profitability, competition, and market and consumer welfare have received comparatively little attention. Most studies in this area have been done in economics. They do suffer from three common important weaknesses. First, previous smart pricing studies did not explore segment granularity in detail. Second, product costs were assumed to be uniform. This ruled out the possibility that a low-cost firm might have gained advantage through the use of business intelligence tools. Third, privacy-conscious customers could inhibit firms' data collection initiatives, resulting therefore in incorrect segmentation. Besides, firms could make wrong inferences about customers even though they have the best tools in the market.

Our research will first analyze the above drawbacks in detail, and propose game theoretic models, which are more likely to capture the characteristics of practical scenarios, including privacy concerns. We analyze segment granularity in chapter 3. Product cost is studied in chapter 4. Finally, the impact of privacy-conscious customers and wrong predictions are explored in chapter 5.

1.2. Literature Review

1.2.1 Information Privacy

Privacy is a complicated concept that is difficult to define at a theoretical level in a single and logically-consistent manner. Post (2001) describes the complexity of the concept as "*Privacy is a value so complex, so entangled in competing and contradictory dimensions, so engorged with various and distinct meanings, that I sometimes despair whether it can be usefully addressed at all*." Privacy is a multi-disciplinary concept encompassing law, philosophy, sociology, political science and economics. This dissertation focuses on economic aspects of privacy. Specifically, the study incorporates the quality of the information collected into smart pricing models and examines its implications on individuals, firms and markets. Recently, White House released a report to set up a series of policy regulations to protect consumers' privacy (Franceschi-Bicchierai, 2014). Podesta et al. (2014) in White House report wrote that "*the ability to segment the population and to stratify consumer experiences so seamlessly as to be almost undetectable* *demands greater review, especially when it comes to the practice of differential pricing and other potentially discriminatory practices*". As public is more informed about privacy issues, we expect to have more laws passed to regulate how information about customers are collected, analyzed and used.

Information privacy is defined as the individual's ability to control the collection and use of personal information (Stigler, 1980; Westin 1967). Consumers differ in how much they care about privacy and the amount of information they provide to firms. Current studies regarding an individual's approach to privacy revealed that customers are willing to provide demographics information in exchange for convenience and monetary reward. Pew Internet and the American Life Project found that 54% of survey respondents provide personal information to use a website (Fox et al., 2000). On the other hand, 27% of hard-core privacy protectors declared that they would never provide their personal information to a website (Fox et al., 2000). Actually, consumers differ a lot in their privacy concern. The perception of invasion of privacy differs from one customer to next (Bernett, 2000). Firms need to collect voluminous data about customer preferences to segment the customers into groups of similar needs, wants and characteristics. Customers' privacy concerns may inhibit data collection, and thus make it impossible to categorize customers into correct segments. The fact of a customers' being aware of data collection may distort the segmentation. Acquisti and Varian (2005) studied the case where some customers are aware of the fact that firms use their historical purchase data. Price discrimination yields higher profits when the majority of customers are unaware, i.e., short-sighted, and don't consider their purchase decisions' impact on future prices offered by firms. Similar to Acquisti and Varian(2005), we build a game theoretical model where privacy-conscious customers inhibit firm's data collection efforts. Firms cannot collect information about these customers and offer customized prices. On the other hand, those customers which share their personal information can get tailored prices.

Consumers face a tradeoff between the extent of revealed purchasing information and pricing. Privacy conscious consumers are very unlikely to reveal their information, making it thus harder for firms to estimate customers' tolerance for price increases. Clearly, customers' privacy plays an important role in determining market prices. This study does not address the issue of how customers arrive at a valuation of their own privacy, expressed as the extent of revealed information. Rather, the focus is on the impact of consumers' privacy valuations on market prices.

Public concern regarding privacy intensified with the development of advanced information technology tools, which can be used to gather, link and exploit personal data. The next section examines the recent technological advances that are exacerbating consumers' privacy concerns. Firms are investing heavily to build business intelligence infrastructures, which include technology, systems, processes, and people. Even though the cost of software and hardware has been declining, dramatically over the past few years, the total cost of investment in business intelligence has been increasing. This is mainly due to high demand for skilled experts in data mining, analysis and development. In addition, the volume of information requiring analysis has been increasing rapidly. Even though investment in business intelligence makes firms competitive, it also has an adverse effect on profitability. There is a trade-off between business intelligence investment and profitability. Information technology cost's impact on market prices haven't been studied extensively yet. This study incorporates technology costs into our game-theoretical models.

1.2.2 Information Technology

In recent years, there have been advances first in data storage technology, which in turn has lead to increases in the amount of customer information collected and stored, and second, in software-facilitated data extraction and analysis. We discuss below in more details these advances.

1.2.2.1 Hardware

Moore's law states that computing power doubles every 18 months at a reasonable price. Computing power is thus no longer a limiting factor in processing information. Interestingly, data storage capacity has been rising at a rate that has far-outpaced the rate of increase in computing power. Some studies show that it doubles on average every 12 months (Grochowski and Halern, 2003). Due to these improvements in data processing and storage, it becomes economically feasible to collect and store dramatically increasing volumes of personal data.

1.2.2.2 Software

Improvements in software, especially in the area of data mining and business intelligence pose a remarkable threat to privacy. Data mining is defined as "*the process of discovering meaningful correlations, patterns, and trends by sifting through large amounts of data stored in repositories, using pattern recognition technologies as well as statistical and mathematical techniques.*" (Gartner Group, 2004) Implicit information within data is often brought to light with data mining software.

Data mining has become important in everyday applications. Banks utilize it to identify the most affluent customers (Hoffman, 1998). Firms can use data mining to discern purchasing habits of targeted customers. With data mining, it is now possible to segment people into groups based on their preferences, and habits.

These improvements in both hardware and software provide the capability to efficiently collect, store, process, and analyze data at unprecedented scale.

1.2.2.3 Social Networks

Social networks have become very popular in recent years. They make it easier for firms to collect and analyze customer preferences. As mentioned before, Swipely is a social network for customers to save money and earn rewards. Customers share their credit card purchases to earn rewards, and this in turn enables firms to understand customer purchase behavior. Firms can now attract new customers, build loyalty and understand its customers shopping patterns better.

In the next section, we discuss how firms exploit collected data to gain insights into consumers' purchasing habits and preferences.

1.2.2.4 Profiling

As mentioned previously, profiling, which combines customer information from multiple sources to determine customers' purchase behavior, can be beneficial to customers. For example, Amazon uses profiling to suggest similar products by analyzing customers' purchase patterns. Profiling can also be used to personalize products and services. Personalization is the ability to proactively tailor products and services to individual customers based upon their personal preferences (Chellappa and Sin, 2005).

Profiling however suffers from essentially two drawbacks. First, some customers may be charged marginally higher prices, as firms use their customer databases to price discriminate. Second, some customers may feel their privacy violated as their personal information is used without their consent. Further, a firm may collect consumer information, and then turn around to sell it for extra revenue. Customers generally punish firms' misuse of personal information. This is the case of America Online, which stopped selling its subscribers' phone numbers to telemarketers (Litman, 2000). DoubleClick, an internet advertising firm that tracks consumer behavior to customize banner ads, postponed its plan to merge online and offline data due to the public's outcry (Hansen, 2000).

Acquisti and Varian (2005) incorporated privacy into their model by introducing "myopic" customers. Myopic customers are those unaware that firms are collecting data about them to potentially charge them higher prices in the future. The model, however, does not incorporate privacy concern and the impact of technology cost. Obviously, as privacy-conscious customers refuse to provide their data, firms need to allocate more resources to derive meaningful patterns hidden in the available data. Firms may not be able to track privacy-conscious customers. As a result, information about customers may lack accuracy, and firms may be forced to increase their investment in business intelligence for customer categorization, and ultimately, for price discrimination.

1.2.3 Smart Pricing

Krugman and Obstfeld(2003) stated that "*price discrimination exists when sales of identical goods or services are transacted at different prices from the same provider*". Similar smart pricing definitions can be found in many textbooks. These definitions generally focus on different prices in the market, and neglect the cost differences in producing identical products. The issue of cost difference stems from the concept of commodity, which Lancaster (1979) defines as a group of products possessing a particular set of characteristics. Product differentiation is "the practice of creating variations with distinct characteristics within the same product group" (Lancaster, 1979). Absolute homogeneity in the commodity is almost impossible (Lancaster, 1979). Dickson and Ginter (1987) define product differentiation as a marketplace state in which products are not a commodity and differ from each other in terms of physical and nonphysical characteristics. Firms in fact have to produce different products customized to different customer needs within the same product group and incur different costs in producing each variation. In other words, different prices for similar products made by the same provider are partly due to different cost structures. Net price is the price adjusted to take into account the cost associated with product differentiation. For Phlips (1983), price discrimination occurs when two varieties of the same commodity are sold by the same provider at different *net* prices. Steiner (1957) and Demsetz (1973) offered similar definitions. Thus, different prices for similar products in the market do not necessarily mean price discrimination.

Stigler (1987) stated that price discrimination exists when differences in prices are not proportional to differences in marginal costs. Stigler's definition is more realistic in that it incorporates cost differences. When adjacent seats in an airline are sold at different prices, there is obviously price discrimination. Airlines also charge a premium for business class services. Smart pricing also exists in this case as the premium charged for business class service far exceeds the cost to provide those extra services.

Note that smart pricing should not be confused with price dispersion. These are two different concepts. Price dispersion is variation in prices *across* sellers of the same item. On the other hand, price discrimination exists when a *single* seller charges different net prices. This study focuses solely on smart pricing.

Market segmentation and price discrimination terms are also often used interchangeably. While the concepts are very close, there are significant differences. Smith (1956) wrote that "*market segmentation involves viewing a heterogeneous market as a number of smaller homogeneous markets, in response to differing preferences, attributable to the desires of* *consumers for more precise satisfaction with their varying wants*". Market segmentation is basically a market condition in which customers can be categorized into distinct groups according to their preferences. In fact, market segmentation is a necessary condition for price discrimination (Carlton and Perloff, 1994). If customers cannot be categorized into segments via observable or unobservable characteristics, firms cannot charge different net prices. Firms engage in market segmentation for smart pricing.

Price discrimination is feasible when firms have some market power (the ability to change market price of a product or service), can segment the customers into distinct categories, and can prevent re-selling of the product or service (Carlton and Perloff, 1994). Similar to previous price discrimination studies, the research presented here makes the same assumptions.

Mohammed (2005) identified seven techniques to segment the customers. They are customer characteristics (Age, Gender, organizational affiliation…), hurdles (coupon, sales, membership), time (high prices for customers that want the product immediately, and low prices for customers which prefer to wait), quantity (discounts for customers which purchase multiple items), distribution (different prices for customers depending on where they make purchase), mixed bundling (offering several products for sale as one combined product), and negotiation. This study focuses on the economic implications of market segmentation and smart pricing and bypasses the question of how firms segment the market.

A price discrimination typology was first proposed by Pigou (1920). It basically classified price discrimination into three categories. First-degree price discrimination occurs when a customer faces a different price based on customer's reservation price -- the maximum price that the customer is willing to pay. First-degree price discrimination is considered to be hypothetical as it requires sellers to know the maximum price the consumer is willing to pay. Even though

Pigou stated that first-degree price discrimination is of academic interest only, recent advances in business intelligence have enabled firms to approximate closely customer’s reservation prices. Personalized marketing, or one-to-one marketing, is in fact a strategy used to make a customized product for each customer to charge a reservation price. Firms extract all consumer surplus, the difference between a customer's reservation price and the cost, in first-degree price discrimination. This is why first-degree price discrimination is sometimes referred as perfect price discrimination.

Second-degree price discrimination occurs when firms offer a set of products with different prices and customers self-select the product matching their needs. All customers choosing the same product are considered to be in the same group and customers in the same group pay exactly the same price. Price discrimination exists across different customer groups. For instance, the previously mentioned airline example in which customers are offered multiple classes of seats (namely, business class or economy class) is second-degree price discrimination.

Third-degree price discrimination occurs when firms categorize customers into different groups based on observable characteristics (past purchases, age, gender, geographical location and so on). Similar to second-degree price discrimination, customers within the same group pay exactly the same price and price discrimination exists among different customer groups. The main distinction with second-degree price discrimination is that customers cannot select their own group in third-degree price discrimination. This study focuses on third-degree price discrimination.

Another alternative classification is that of Armstrong and Vickers’ (2001). It is based on interpersonal price discrimination, which occurs when customers are offered different net prices for the same product. In other words, prices vary across customers in interpersonal price discrimination. For example, a firm may charge different prices for loyal and first-time customers. Intrapersonal price discrimination, on the other hand, occurs when different net prices are offered for a set of products. In other words, intrapersonal price discrimination is the case in which prices vary across different products for the same customer. For example, bundling, where several products are offered for sale as one combined product, is a form of intrapersonal price discrimination as customers could pay a different price if they purchase the bundled products separately.

Stole (2007) proposed an alternative classification based on whether price discrimination is based on observable or unobservable customer characteristics. Direct price discrimination exists when prices differ in the market based on observable characteristics (age, gender, organizational affiliation, location). On the other hand, indirect price discrimination is present when customers are heterogeneous in some unobservable characteristics. Actually, Stole's classification method is similar to the difference between second-degree and third-degree price discrimination in Pigou's typology.

Finally, Shapiro and Varian (1999) proposed an alternative typology for price discrimination, a modern version of Pigou's (1920). Personalized pricing refers to the case in which each customer is offered a different price. This corresponds to first-degree price discrimination in Pigou's typology. Versioning is the strategy to offer a range of products and let customers self-select the version of the product. Versioning is in fact second-degree price discrimination. Group pricing is the strategy of charging different prices for different groups of customers. Group pricing corresponds to third-degree price discrimination.

Even though Shapiro and Varian's typology appeals to business users, we shall adopt Pigou's typology throughout this study to conform to the traditional economics literature.

1.2.3.1 Monopolistic Price Discrimination

Villas-Boas (2004), Kennan (2001) and Battaglini (2005) are the most recent studies on monopolistic price discrimination. A monopolist can price discriminate by analyzing customers' purchase history to extract more consumer surplus. A price discriminating strategy is at least as profitable as a non-discriminating pricing policy for a monopoly (Phlips, 1988). Competition effects, which will be shown to have a great impact on prices, market share and social welfare, are not considered in monopolistic price discrimination studies. While the effect of surplus extraction improves firms' profitability, competition impacts work in the opposite direction. Monopolistic price discrimination studies rule out the possibility that competition effects may outweigh surplus extraction effects. In the end, a monopolist becomes more profitable. However, monopolistic industries are rare in practice. Even though competitive industries are more common, research on price discrimination in competitive markets is limited. This study focuses on price discrimination in competitive markets.

1.2.3.2 Duopolistic Price Discrimination

Unlike monopolitistic scenarios, firms in a duopoly must consider the effect of competition on price discrimination. The capability to track customers' previous purchases and reservation prices enables a firm to tailor different prices and products to customers. Duopolistic price discrimination research can be divided into two main categories. The first stream assumes that firms have no a-priori information about their customer preferences, and only learn it gradually as purchases are made, whereas the second assumes firms have the necessary information to derive customer preferences, and reservation prices. Both perspectives are analyzed further in the next two sections.

1.2.3.2.1 Firms have no information a-priori about customer preferences

In some industries, customers incur switching costs if they change their suppliers. Switching costs could be due to subscription fees, or to training on the use of a new product. The products of various firms are assumed to be homogeneous (i.e., customers have no preference towards one product) at the beginning, but the customers become partially locked in once they make a purchase. Firms discriminate between their own and their competitors' customers. They generally offer price incentives to induce their competitors' customers to switch. Chen (1997) considers a duopolistic two-stage game where customers are indifferent among firms but become locked-in with their initial purchase. A customer will switch to a rival firm in the second stage to take advantage of lower prices if the price differential compensates for the switching cost. The strategy of the firms is to deflate their prices initially to attract new customers and thereby expand market share, only to increase them sufficiently in a second stage to recover the losses in revenue in the first stage. Both firms are worse off due to price deflation; yet, customers are not necessarily rewarded as they incur switching costs. Switching intensifies competition and leads to the prisoner's dilemma. Taylor (2003) proposed a multi-period subscription-based model where multiple firms compete. Similar to Chen (1997), Taylor showed that price discrimination lowers firms' profitability.

Esteves (2010) studied a two-period duopolistic price discrimination model with customer recognition where customers' preferences are discrete. All customers are myopic and their loyalty level towards their preferred firm is the same. The results show that firms are worse off when they have the necessary information for price discrimination. Chen and Zhang (2009) studied a duopolistic symmetrical price discrimination model where customers are forward-looking, i.e., they can forgo a purchase in the first period to get a better price in the second period. Each firm has a set of loyal customers of equal size, and there exists a third set of switchers. No firm can recognize customer types in the first period. Firms can price discriminate in the second period. Price discrimination becomes more profitable for firms even though customers behave strategically. Chen and Zhang attributed this finding to the fact that firms can screen out price-sensitive customers and limit price competition.

1.2.3.2.2 Firms possess information about customer preferences

Hotelling (1929) proposed a location-based model where customers are identical and uniformly distributed along a fixed-length line. Customers incur a transportation cost based on the distance between customer and firm's location. Firms decide on their prices based on geographical location. Thisse and Vives (1988) extended the Hotelling model to explore perfect price discrimination where each customer incurs a linear transportation cost proportional to the distance between a customer and a firm. Firms know each customer's location on a linear market and set prices taking into account customer's location. They found that price discrimination intensifies competition and customers benefit from lower prices. Thisse and Vives (1988) then analyze whether firms can avoid price discrimination. Even though both firms can be better off with uniform pricing, price discrimination is dominant strategy. Therefore, the game is the prisoner's dilemma. Shaffer and Zhang (1995) reach the same conclusion when promotional discount coupons are introduced into a competition of symmetric firms to attract new customers.

Shaffer and Zhang (2000) explored a duopolistic model where each firm offers promotions either to its own customers or to its competitor's. If the demand is symmetric, a firm's best strategy is to offer discounts to their competitor's customers, but the game is the prisoner's dilemma. On the other hand, the game is not necessarily the prisoner's dilemma when the demand is asymmetric. One of the firms or both firms can be better off depending on their initial market share and the intensity of their customer's loyalty. Shaffer and Zhang (2002) extended their study to examine the impact of one-to-one promotions on market share and competition in a duopolistic perfect price discrimination model where asymmetric firms incur a targeting cost to deliver promotions. Contrary to earlier price discrimination findings above (Chen 1997, Shaffer and Zhang 1995), they found that their game model is not necessarily the prisoner's dilemma. The firm with avid loyal customers and greater market share is likely to be better off. Shaffer and Zhang (2002) attributed the prisoner's dilemma conclusion of earlier results to the fact that firms are assumed to be symmetric.

Liu and Serfes (2004) extended Shaffer and Zhang (2000)'s model and incorporated multiple customer segments for each firms' loyal customers. The analysis showed that the case is the prisoner's dilemma. Liu and Serfes (2005) later examined the effect of segmentation in a vertically differentiated market, where one of two firms offers higher quality product. Customers are inclined to purchase the high-quality product, but the premium a customer is willing to pay differs. The high-quality firm is found to have sufficient upward price flexibility to increase its market share at the expense of the low-quality firm through segmentation. The low quality-firm has no incentive to segment the market.

Ba et al (2007) study a two-dimensional price discrimination model in a duopoly where firms are differentiated in terms of service level and customer recognition. Note that service level corresponds to product quality and brand loyalty in previous studies. They assume that improvements in customer service levels are easier to accomplish than brand recognition. In the first period, firms decide the investment level on customer recognition; in the second, the service level is determined; and in the third, the price competition begins. Obviously, the firm with the higher service level is more profitable. Interestingly, they found that both firms are worse off as the low-service-level firm tries to improve its service level.

Esteves (2009) analyzes firms' price discrimination in the case of partial information about customers. Customers are assumed to be uniformly distributed in a two-dimensional Hotelling model, and have different preferences for firms' brand name and products. Customers' product preferences do not necessarily match their brand preferences. The main finding is that price discrimination does not necessarily lead to the prisoner's dilemma.

II. BASIC MODEL

First, we briefly describe the basic model. Then, we will discuss the challenging research extensions and modify the model in the next sections.

First, customers differ in brand loyalty and respond differently to firms' products and promotions. The model will assume that customers have varying degrees of brand loyalty, which measures the extent to which a customer is impervious to pricing enticements by rival firms. Second, firms collect information about customers by exploiting internal and external data sources. Shaffer and Zhang (1995) stated "data on past purchasing behavior has given firms information that allows them to discriminate in price according to customer heterogeneity in brand loyalty". Third, firms generally send customized coupons to attract new customers and retain old ones. Oftentimes, a promotional discount amount is determined by customers' purchase history and personal preferences. Firms in our model offer customized prices to each segment.

A customer's loyalty is influenced by past purchases. A loyal customer is one who allocates a majority of purchases to a preferred firm. Even though a firm has less information about rival firms' loyal customers, it can still obtain enough information from internal and external data sources. In this study, the question of how firms built customer loyalty is bypassed, and focus is on the implications of customer loyalty on firms' strategies.

Consider two firms, labeled A and B, which produce a similar product with zero marginal cost. We'll relax zero marginal cost in section 4. Customers purchase one unit of the product regardless of the price and a customer prefers one firm over another. Price tolerance is defined as the minimum price differential necessary to induce a customer to purchase from the less preferred firm. Customers are heterogeneous in price tolerance that a customer is loyal to either firm A or B, but not both.

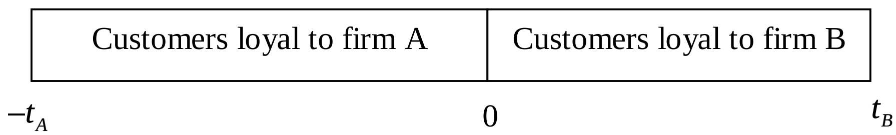


FIGURE 1: Customers' price tolerance

Figure 1 shows the price tolerance spectrum in the market. Loyalty parameters t_A and t_B $(t_A, t_B > 0)$ denote the maximum price tolerance towards firm A and B, respectively. Customers between $-t_A$ and 0 prefer firm A and those between 0 and t_B prefer firm B. The customer at point 0 is indifferent and has no brand loyalty. Let t denote a customer's price tolerance. A loyal customer to firm A results in $0 < t \leq t_A$; otherwise $0 < t \leq t_B$. Let p_A and p_B be the prices offered by firms A and B to a customer loyal to firm B. A customer purchases from the preferred firm as long as the premium charged by the preferred firm doesn't exceed the customer's price tolerance. For example, a customer loyal to firm B will only tolerate premiums up to t above firm A's price, i.e., $p_B \leq p_A + t$. Otherwise the customer will switch to firm A.

In Chen et al. (2001) and Narasimhan (1988), price tolerance is binary, i.e., a customer is either loyal to one firm or a switcher. In our case, price tolerance is modeled as a linear function. As a result, it is now possible to analyze a firm's pricing policies for various price tolerance levels, and as we shall show this has significant implications.

Let a and b represent the customers who would buy from firm A and B, respectively, as long as both firms offer the same price. That is, a and b customers prefer firm A and B respectively. Assuming customers are uniformly distributed along a closed interval [0, 1], let the fraction of a customers be denoted as q and the fraction of b customers as $1-q$.

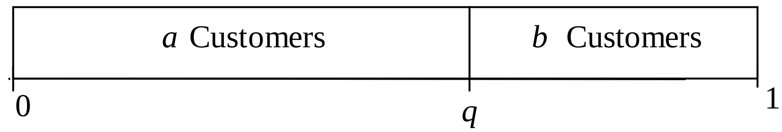


FIGURE 2: Firms' market share in case of equal prices

Figure 2 shows firm's market share when both firms offer the same price. Firms are asymmetric when $q \neq .5$ and symmetric if $q = .5$.

Let $f_i^j(p_A, p_B)$ denote the fraction of i customers purchasing from firm j given that firms A and B offer prices p_A and p_B where $i \in \{a, b\}$ and $j \in \{A, B\}$. Obviously, $f_i^A(p_A, p_B) + f_i^B(p_A, p_B) = 1$. $f_a^B(p_A, p_B)$ can be expressed as:

$$f_a^B(p_A, p_B) = \begin{cases} 1 & \text{if } t_A < p_A - p_B \\ \dfrac{p_A - p_B}{t_A} & \text{if } 0 \leq p_A - p_B \leq t_A \\ 0 & \text{if } p_A - p_B < 0 \end{cases}$$

When firm A offers a higher price than firm B, a customers are obviously inclined to switch to firm B. When the price difference exceeds maximum loyalty (i.e., $p_A - p_B > t_A$), a customers purchase from firm B. If the price differential is moderate (i.e., $0 < p_A - p_B \leq t_A$), customers with low loyalty switch to firm B and very loyal a customers still purchase from firm A. As long as firm A's price is lower than firm B's, firm A is able to keep all a customers.

Note that the Hotelling (1929) model is a special case of ours. Price tolearance (t_A and t_B) plays an equivalent role to the transportation cost (t) in the Hotelling model. If firms are symmetric, $t_A = t_B = t$ and $q = .5$, then a firm A's demand becomes $(t + p_B - p_A)/2t$ and firm

B's demand is $(t + p_A - p_B)/2t$ as in the Hotelling model. Our model is extended to incorporate asymmetric demand by changing values of the parameters t_A, t_B and q .

III. SEGMENT GRANULARITY

3.1 Motivation

Market segmentation today can be carried out to an extent never envisioned before. The granularity of segments is only limited by the amount of data available, the sophistication of the technological tools, and cost. In their seminal paper, Dickson and Ginter (1987) state that segments form homogeneous sub-markets, where customers within the same segment respond similarly to firms' product offerings. Wedel and Kamakura (2000) extensively reviewed the marketing segmentation literature, compiling a list of methods and identifying the variables used to assign customers to segments. Frank et al. (1972) categorized segmentation research into two main streams: microeconomic theory and behavioral sciences. The focus in this section will be on the economical implications of market segmentation rather than on the segmentation methods and variables.

In general, when firms decide to charge different prices to different segments, three important questions arise: What should be the price for a particular segment? What will be the impact of segmentation and price discrimination on market share? How would consumer welfare change with segmentation and price discrimination? In this section, we address these issues and the impact of business intelligence cost in a duopolistic price discrimination model, and specifically investigate the impact of segment granularity on price, market share and social welfare.

3.2 The Game

The game unfolds as follows. Firms independently and simultaneously determine their business intelligence adaptation and pricing strategies. If a firm acquires business intelligence technology, it can offer different prices to each segment through targeted coupons or special offers. Otherwise, a firm offers single price to whole market. Let's assume that business intelligence technology is capable of partitioning a customers into n segments and b customers into m segments. Parameters n and m represent the granularity possible under the implemented segmentation technology. A sophisticated business intelligence technology provides more information about customer preferences, and minimizes the error in estimating the individual customer's price tolerance level. The case $n = m = 1$corresponds to the case in Shaffer and Zhang's (2000) model where firms do not segment their loyal customer base.

In Shaffer and Zhang's (2000) model, firms know the fraction of customers switching to a rival firm given the price difference between firms, but they have no way of knowing the individual customer's price tolerance. In our model, firms estimate individual customer price tolerance using business intelligence technology. The precision achieved depends on the sophistication of business intelligence technology. The case of $n = m = 1$corresponds to perfect price discrimination where firms can tailor their prices to each individual customer. At first, business intelligence technology is considered to be free. Later, we relax this assumption.

3.3 Analysis and Strategies

The overall game between the two asymmetric firms in a duopoly consists in deciding whether it is profitable to acquire business intelligence technology. Thus, the analysis below examines first the pricing and market share strategies when the two firms make their business intelligence acquisition decision simultaneously. Given a business intelligence acquisition strategy, the goal is to determine whether the pricing behavior of the firms that leads to an equilibrium. As a result, it is necessary to consider three possible subgames, namely:

- No firm acquires business intelligence

- Only one firm acquires business intelligence (there are two symmetric cases)
- Both firms acquire business intelligence

Note that a subgame is any part of a game that can be itself played as a game. Our goal is to determine whether there exist dominant strategies in a duopoly. In other words, we seek to find out whether there is a best pricing strategy for both firms across all business intelligence acquisition strategies.

3.3.1. Pricing and market share strategies

3.3.1.1 Subgame 1: No firm acquires business intelligence

Each firm offers two prices,one to their own loyal customers, and one to rival firm's loyal customers. Each firm profits more from its loyal customer base, as these customers are willing to pay a premium for their preferred firm so long as it is not excessive. This result, established by Shaffer and Zhang (2000) restated below using our own notation.

Proposition 1: *Subgame 1 has a unique Nash Equilibrium. At equilibrium, firms A and B charge* $\frac{2}{3}t_A$ *and* $\frac{1}{3}t_A$ *to* a *customers and retain* $\frac{2}{3}$ *and* $\frac{1}{3}$ *of* a *customers, respectively. They charge* $\frac{1}{3}t_B$ *and* $\frac{2}{3}t_B$ *to* b *customers and acquire* $\frac{1}{3}$ *and* $\frac{2}{3}$ *of* b *customers respectively.*

We include this proposition simply to provide as a benchmark for comparisons with our own models, where firms may segment the market into finer segments. More precisely, subgame 1 raises the following question: why would firm A allow firm B to make inroads into its loyal customers base? After all, as the preferred brand for a customers firm A has always the flexibility to undercut firm B to retain all of its loyal customers. Yet, our results show that firm A is willing to loose some of its loyal customers. It can actually improve its profit simply by charging a higher premium to its loyal customers. As a consequence, some of its least loyal customers may find the premium so onerous that they switch to the rival firm. This scenario illustrates the classical tradeoff between market share and profitability: the premium may be increased only to the point where loss in market share starts having a negative impact on profitability.

A firm however cannot remain inward looking, lest it loses excessive market share to its competition by standing still. It must constantly pursue a strategy to make inroads into its rival firm's customer base by undercutting its prices. A reduction in price makes its products more attractive to its rival firm's loyal customers, and encourages their switching. All switchers are however not the same. Some may be able to pay higher prices than others. As the firm reduces its price to attract its rival firm's customers, its profitability continues to increase but only to a point. Gains in market share are not sufficient to offset the loss due to the decrease in prices. As a result, the firm faces again a trade-off between market share and profit.

3.3.1.2 Subgame 2: Only one firm acquires business intelligence

Firm A has business intelligence technology while firm B does not. Firm A exploits the technology to partition its a customers into *n* equal-sized segments, $\frac{q}{n}$. Let a_i denote the ith segment. The segments are ordered such that a_1 represents the most loyal customers and a_n is the least loyal. Firm A also segments b customers into *m* segments of equal size, $\frac{1-q}{m}$. Let b_j be the j-th segment, where b_1 represents firm B's least loyal customers and b_m the most loyal. Figure 3 illustrates how firm A segments the market.

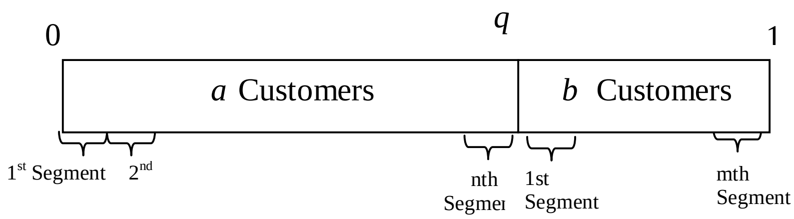


FIGURE 3: Firm A's market segmentation plan

Let $Min(a_i)$ and $Max(a_i)$ denote the minimum and maximum price tolerance in segment a_i. They can be expressed as follows:

$$Min\ (a_i) = t_A - \frac{i}{n} t_A$$
$$Max\ (a_i) = t_A - \frac{i-1}{n} t_A$$

Let t such that ($Min\ (a_i) < t \leq Max\ (a_i)$) denote the price tolerance level of an arbitrary customer in segment a_i. The customer purchases from firm A as long as $p_A \leq p_B + t$. If the price difference exceeds $Max(a_i)$, all customers in segment a_i switch to firm B. On the other hand, if the price differential is moderate (i.e., $Min(a_i) \leq p_A - p_B \leq Max(a_i)$), low-loyalty customers switch to firm B while high-loyalty customers continue to purchase from firm A.

Let $p_A(a_i)$ denote the price offered by firm A to segment a_i, and $p_B(a)$ the price offered by firm B to a customers. Let $f_{a_i}^{B}(p_A(a_i), p_B(a))$ be the fraction of customers in segment a_i who purchase from firm B. Then, $f_{a_i}^{B}(p_A(a_i), p_B(a))$ can be expressed as:

$$f_{a_i}^{B}(p_A(a_i), p_B(a)) = \begin{cases} 0 & \text{if } p_A(a_i) - p_B(a) < \mathrm{Min}(a_i) \\ \dfrac{p_A(a_i) - p_B(a) - \mathrm{Min}(a_i)}{t_A/n} & \text{if } Min(a_i) \le p_A(a_i) - p_B(a) \le Max(a_i) \\ 1 & \text{if } p_A(a_i) - p_B(a) > Max(a_i) \end{cases}$$

Proposition 2: *Subgame 2 has a unique Nash equilibrium. Assuming that firm A is the one that has access to business intelligence technology, then firm A: 1) retains all of its customers in segments* a_1 *to* a_{n-1} *and loses 1/3 of segment* a_n *to firm B at equilibrium; and 2) captures segments* b_1 *to* $b_{\lfloor \frac{m}{2} \rfloor - 1}$*, while firm B retains its customers in segments* $b_{\lfloor \frac{m}{2} \rfloor + 2}$ *to* b_m*. The two firms share segments* $b_{\lfloor \frac{m}{2} \rfloor}$ *and* $b_{\lfloor \frac{m}{2} \rfloor + 1}$*. [A detailed proof is provided in appendix A].*

When firm A has sole access to business intelligence technology, it is able to retain almost all its loyal customers, except for segment a_n. This is markedly different from the previous case where firm A's strategy led to a larger loss of its customer base. Access to business intelligence technology provides firm A pricing flexibility, and thereby, the ability to retain a larger percentage of its market share while increasing its profit. The tradeoff between market share and premium still exists, but firm A's scale of competition is now confined to specific segments.

Firm A increases its profit by charging a higher price within segments. Price increases are beneficial only to the point where customer switching starts offsetting gains in profitability. Should firm A charge the highest tolerable price that all customers within a segment can afford?

It turns out that in some segments, market share loss may not be the best strategy at all for firm A. This is the case for segments a_1 to a_{n-1}, where firm A does not offer highest tolerable prices in order not to lose customers to the rival firm. Segment a_n, which contains firm A's least loyal customers, is the only segment where the most profitable strategy for firm A is to loose its very least loyal customers to firm B and to offset this loss of revenue by charging the most loyal customers a higher but tolerable price.

Firm B has a loyalty advantage over firm A as far as b customers are concerned. Nevertheless, firm A can capture part of this market by exploiting its segmentation technology to tailor a judicious price discrimination policy for each segment. When firm A segments the whole market, it is able to capture almost half of firm B's customers as firm B is penalized by its inability to price discriminate. Obviously, firm B can block firm A's penetration into its own loyal customer base by setting its price to zero, which is obviously an unrealistic strategy. To boost profitability, firm B will opt instead to let go its very least loyal customers and recoup the loss by charging its most loyal customers a higher price. As firm B pursues a high price strategy, limiting thus its pricing flexibility, firm A's responds by setting its price lower to make inroads into its rival firm's least loyal customers.

3.3.1.3 Subgame 3: Both firms acquire business intelligence

Both firms have access to business intelligence technology and therefore can tailor their prices to individual segments. While a customers are partitioned into "n" segments, b customers are split into "m" segments. Let $p_B(a_i)$ denote the price firm B charges segments a_i. The fraction of customers in segment a_i purchasing from firm B can be expressed as:

$$f_{a_i}^{B}(p_A(a_i), p_B(a_i)) = \begin{cases} 0 & if\ p_A(a_i) - p_B(a_i) < \mathrm{Min}(a_i) \\ \dfrac{p_A(a_i) - p(a_i) - \mathrm{Min}(a_i)}{t_A/n} & if\ Min(a_i) \le p_A(a_i) - p_B(a_i) \le Max(a_i) \\ 1 & if\ p_A(a_i) - p_B(a_i) > Max(a_i) \end{cases}$$

Proposition 3: *Subgame 3 has a unique Nash equilibrium. Firm A retains segments a_1 to a_{n-1} and loses 1/3 of its market share in segment a_n at equilibrium. Firm A captures 1/3 of segment b_1 and firm B retains all its market share in segments b_2 to b_m. [A detailed proof is provided in appendix B].*

When both firms have business intelligence technology, the competition shifts to individual segments. Both firms are able to block the penetration of a rival firm into their loyal customer base. Firm A has a loyalty advantage over firm B on a customers. For segments a_1 to a_{n-1}, firm A blocks the penetration of firm B. Firm A sets in these segments the highest price tolerable to its least loyal customers. For segment a_n, firm A maximizes its profit by trading off loss of its least loyal customers for the benefit of charging a high premium to 2/3 of its most loyal customers in the segment.

With access to business intelligence technology, firm B behaves symmetrically to firm A. It is able to keep its market share in all segments, except the one with its least loyal customers, where it trades off 1/3 of its market share for the benefit of charging a high premium to 2/3 of its most loyal customers within the segment.

3.3.2 Business Intelligence Acquisition Strategies

The firms' payoffs are summarized in Table 1 below.

Firm A \ Firm B		No Segmentation	Segmentation
No Segmentation	Firm A's payoff	$\frac{4}{9}t_A q + \frac{1}{9}t_A(1-q)$ **(I)**	$(\frac{t_A}{4}+\frac{t_A}{4n}+\frac{t_A}{16n^2})q + \frac{t_B}{9m^2}(1-q)$ **(II)**
	Firm B's payoff	$\frac{1}{9}t_A q + \frac{4}{9}t_B(1-q)$	$(\frac{t_A}{8}-\frac{t_A}{8n}+\frac{5t_A}{32n^2})q$ $+(\frac{t_B}{2}-\frac{t_B}{6m}+\frac{t_B}{9m^2})(1-q)$
Segmentation	Firm A's payoff	$(\frac{t_A}{2}-\frac{t_A}{6n}+\frac{t_A}{9n^2})q$ $+(\frac{t_B}{8}-\frac{t_B}{8m}+\frac{5t_B}{32m^2})(1-q)$ **(III)**	$(\frac{t_A}{2}-\frac{t_A}{2n}+\frac{4t_A}{9n^2})q + \frac{t_B}{9m^2}(1-q)$ **(IV)**
	Firm B's payoff	$\frac{t_A}{9n^2}q + (\frac{t_B}{4}+\frac{t_B}{4m}+\frac{t_B}{16m^2})(1-q)$	$\frac{t_A}{9n^2}q + (\frac{t_B}{2}-\frac{t_B}{2m}+\frac{4t_B}{9m^2})(1-q)$

TABLE 1 : Firms' payoffs when they can categorize customers into multiple segments

Theorem: *Each firm's (strictly) dominant strategy is to acquire segmentation technology.*

Proof: Table 1 summarizes the results of the propositions. Firm A's profit in quadrant I is always less than profit in quadrant III. That is, firm A's best strategy is to acquire business intelligence technology to segment the market when firm B doesn't. Similarly, firm A's profit in quadrant II is always less than its profit in quadrant IV. Firm A's best strategy again is to acquire business intelligence when firm B does. To sum up, firm A's dominant strategy is to acquire business intelligence technology regardless of its rival's strategy. The same analysis could be made for firm B. In the end, both firms acquire business intelligence technology to segment the market. We have assumed that segmentation technology cost is nil. This assumption is relaxed later to enable the evaluation of segmentation technology cost impacts on firms' strategies.

It is not possible to determine whether the firms are better off or the game is prisoner's dilemma by comparing firms' profits in quadrant (I) and (IV) at this point. The change in profit is a function of the sizes of the firms' loyal customer bases and the magnitudes of price tolerance. In the next section, we discuss the conditions under which a firm becomes more profitable and show that the game is not necessarily prisoner's dilemma.

3.4. Implications

The results obtained in the previous section are examined to assess business intelligence's impact on firms' profits, prices, and consumer welfare.

3.4.1 Implications for firms

First, firms have a strong incentive to acquire business intelligence technology as supported by the findings above. When no firm segments the market, each firm retains 2/3 of its loyal customer base and captures 1/3 of its rival's. On the other hand, a monopoly on business intelligence technology over its competitor enables a firm to retain almost all of its loyal customers and to capture roughly 1/2 of its competitor's loyal customers. The best strategy for a firm is therefore to acquire business intelligence technology. Consequently, this finding provides the rationale for firms to invest in business intelligence tools for market segmentation in a competitive environment.

Second, the profits of a firm are directly correlated with the strength of its customers' price tolerance. The higher is price tolerance, the greater are the profits. Firms are thus compelled to seek strategies that elicit and improve customers' price tolerance. This finding explains in part why companies engage in loyalty improvement initiatives like reward programs, club cards and so on.

Third, and most importantly, the study shows that price discrimination coupled with segmentation does not necessarily lead to prisoner's dilemma, contrary to the result obtained by Liu and Serfes (2004) in the case of a symmetric model. Our model of asymmetric firms is not only more general, but also it is prisoner's dilemma only when no firm is dominant. We shall give a formal definition of dominance below.

Profits are impacted by several factors including, the size of a firm's base of loyal customers, the strength of its customers' price tolerance, as well as the number of segments. The behavior of a firm's profits, assuming segmentation is best analyzed in the case of perfect price discrimination, where the number of segments is infinite, and thus the price can be customized to each single customer. However, the findings still hold when the number of segments is finite. Table 2 summarizes the firms' profits.

Firms	No Firm segments the market	Both firms segment the market
A	$\frac{4}{9}t_A q + \frac{1}{9}t_B(1-q)$	$\frac{1}{2}t_A q$
B	$\frac{1}{9}t_A q + \frac{4}{9}t_B(1-q)$	$\frac{1}{2}t_B(1-q)$

TABLE 2: Firm's profits when number of segments is infinite

The game is prisoner's dilemma if both firms are worse off by segmenting the market. This situation is captured when the following conditions are simultaneously satisfied:

$$\frac{4}{9}t_A q + \frac{1}{9}t_B(1-q) \geq \frac{1}{2}t_A q$$
$$\frac{1}{9}t_A q + \frac{4}{9}t_B(1-q) \geq \frac{1}{2}t_B(1-q)$$

Simplifying the inequalities yields:

$$2t_B(1-q) \geq t_A q \quad (1)$$
$$2t_A q \geq t_B(1-q) \quad (2)$$

Recall that customer's price tolerance and the size of a firm's loyal customer base are denoted by t and q. Let $D_A = t_A q$ and $D_B = t_B(1-q)$ denote firm A's and firm B's dominance, respectively. We define ***dominance*** as a measure of a firm's influence in the market. It follows from the definition that a firm with a large market share of intense loyal customers is the most dominant. Conversely, a firm with a small market share of not so loyal customers is the least dominant. Substituting D_A and D_B into inequalities (1) and (2) yields:

$$2D_B \geq D_A \quad (3)$$
$$2D_A \geq D_B \quad (4)$$

We define $d_A = \dfrac{D_A}{D_A + D_B}$ as firm A's ***dominance indicator***, which is firm A's degree of dominance in the market. Firm B's dominance factor can be defined the same way. Let d_B be firm B's dominance indicator, defined in a similar way as for A. Obviously, $d_A + d_B = 1$. Substituting d_A and d_B into inequalities (3) and (4) and rearranging the terms leads to the following:

$$\frac{1}{3} \leq d_A \leq \frac{2}{3} \quad (5)$$

Figure 4 illustrates precisely the case when the game is prisoner's dilemma as a function of firm A's dominance indicator.

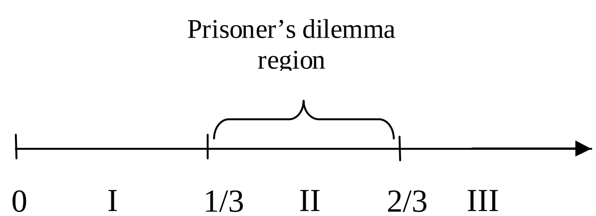


FIGURE 4: Relationship between prisoner's dilemma and dominance indicator

Inequality (5) is only satisfied in region II. Therefore, the game is prisoner's dilemma when firms' dominance indicators differ by no more than 1/3. This is exactly what happens in the symmetric model studied by Liu and Serfes (2004), where $t_A = t_B$ and $q = 0.5$, implying a dominance factor $d_A = 0.5$ for firm A, the mid-point of region II. As a result, their game-theoretic model is thus prisoner's dilemma.

The firms' profits in regions I and III in Figure 4 are a function of their dominance indicators. Let $\Delta\prod_A$ denote the marginal profit due to segmentation for firm A. Then $\Delta\prod_A$ can be expressed as:

$$\Delta\prod_A = \frac{d_A - 2d_B}{18} \quad (6)$$

Substituting $d_A = 1 - d_B$ leads to $\Delta\prod_A = \frac{3d_A - 2}{18}$. Firm A's profit is positively correlated with its dominance indicator. Marginal profit due to segmentation is positive, i.e., $\Delta\prod_A > 0$, only when the dominance indicator is greater than 2/3, which corresponds to region III in Figure 4. Firm B benefits from segmentation in region I, where firm A's dominance indicator is less than 1/3, but firm's B dominance indicator is higher that 2/3.

When no firm acquires business intelligence technology, the firms lose their least loyal customers because of high prices, which only their most loyal customers can afford. With segmentation, a firm will be able to keep some of the lesser loyal customers who would have otherwise switched if there were no segmentation and lose only its very least loyal customers. Competitors are likely to follow the same segmentation strategy. As a result, the net effect of segmentation is an exchange of the very least loyal customers. Very importantly, a dominant firm may become more profitable because of its ability to extract a greater surplus from its own customers. Because of the symmetric demand assumption, Liu and Serfes (2004) ruled out the possibility that a dominant firm can be better off.

Our model also explains how niche firms can coexist with large firms. To compete in the marketplace, a niche firm has to keep its dominance indicator strong. A niche firm may have a smaller market share, but it is able to compete by keeping its customers more loyal, which in turn keeps its dominance indicator high.

3.4.2 Implications for Customers

Customers always purchase one unit of a product regardless of the price. We analyze the prices paid by a customers. A similar analysis can be done for b customers. Table 3 shows the market prices for both a and b customers.

	a **Customers**		b **Customers**	
	Firm A's price	Firm B's price	Firm A's price	Firm B's price
No firm segments the market	$\frac{2t_A}{3}$	$\frac{t_A}{3}$	$\frac{t_B}{3}$	$\frac{2t_B}{3}$
Both firms segment the market	$p_{\mathrm{A}}(a_i) = t_A - \frac{i}{n} t_A$ where $i = 1,2...,n-1$ $p_{\mathrm{A}}(a_n) = \frac{2t_A}{3n}$	$p_{\mathrm{B}}(a_i) = 0$ where $i = 1,2...,n-1$ $p_{\mathrm{B}}(a_n) = \frac{t_A}{3n}$	$p_{\mathrm{A}}(b_J) = 0$ where j = $1,2...,m-1$ $p_{\mathrm{A}}(b_m) = \frac{t_B}{3m}$	$p_{\mathrm{B}}(b_j) = \frac{j-1}{m} t_B$ where i = $1,2...,m-1$ $p_{\mathrm{B}}(b_1) = \frac{t_B}{3m}$

TABLE 3: Prices in the market when they can categorize customers into multiple segments

Segments 1 to $\left\lfloor \frac{n}{3} \right\rfloor$ represent customers with the highest price tolerance levels. These customers can be charged very high prices with segmentation, but this causes the least loyal customers of firm A to become vulnerable to competitors. The ensuing intense competition puts a cap on the prices that firm A can charge, and thus on its profits. In fact, firm B is able to limit firm A's profits by setting a negligible price or simply zero.

Segments $\left\lceil \frac{n}{3} \right\rceil$ to $\left\lfloor \frac{2n}{3} \right\rfloor$ are also subject to competition. Firm B firm sets its price to zero to gain market share. Firm A counters by reducing its price, and is able to successfully defend its market share while experiencing a decline in profit. Firm A also captures almost all the least loyal customers in segments $\left\lceil \frac{2n}{3} \right\rceil$ to n. Despite firm B's attempts to make inroads into these segments by setting its price to zero, the customers choose firm A as the premium is now affordable.

To sum up, business intelligence leads to higher prices for very loyal customers. However, moderately and least loyal customers benefit from business intelligence by paying lower prices. Thisse and Vives (1988) have shown that all customers pay less when firms discriminate on prices. Borenstein (1985) and Holmes (1989) claimed that price discrimination leads to higher prices. The conclusions of our model are on the other hand more nuanced. Very loyal customers will pay more, but moderately and least loyal customers benefit from intensified competition by paying less.

Our results differ from those of Borenstein (1985), Holmes (1989) and Thisse and Vives (1988), in that our model deals with asymmetric firms. Prior results rule out the possibility that a dominant firm could gain market share with business intelligence. Our study, on the other hand, shows that in some cases, the dominant firm improves its profitability at the expense of the other firm by lowering prices for the least loyal customers. Moreover, a firm could charge higher premium to its very loyal customers without losing its revenue from least loyal customers. While very loyal customers face higher prices, the premium tolerated never exceeds a threshold, which will otherwise force them to switch to the competing firm.

3.4.3 Implications for consumer welfare

An important economic question is how business intelligence and price discrimination impacts consumer welfare. Consumer surplus is the difference between what a consumer is willing to pay and the actual price. We consider aggregate consumer surplus as the measure of consumer welfare. Assume that a customer obtains benefit v by purchasing a product (i.e., $v >> p$ where p is price) from a firm. An individual customer's surplus is therefore $v - p$. Let V and $\prod$ be the aggregate customer benefit and the firms' total revenue, respectively. The aggregate customer surplus is therefore $V - \prod$. Table 4 summarizes model's consumer welfare implications.

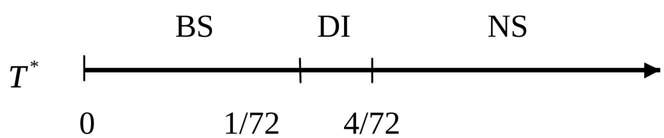
	No Segmentation	Segmentation
Consumer Welfare	$V - \frac{5}{9}t_A q - \frac{5}{9}t_B(1-q)$	$V - (\frac{t_A}{2} - \frac{t_A}{2n} + \frac{5t_A}{9n^2})q - (\frac{t_B}{2} - \frac{t_B}{2m} + \frac{5t_B}{9m^2})(1-q)$

TABLE 4: Consumer Welfare when firms can categorize customers into multiple segments

Clearly, consumer welfare improves with segmentation. The average market price decreases as segmentation becomes more refined.

3.5 Impact of Business Intelligence Technology Cost

So far, the cost of business intelligence technology is assumed to be zero so far. This is obviously unrealistic. We analyze the impact of business intelligence technology's cost on firms' strategy. Let T denote the cost of business intelligence technology. A firm has to take into account the cost of the business intelligence technology and its competitor's decision to segment or not. As in section 4.1, we examine the perfect price discrimination case. Let $T^* = \frac{T}{D_A + D_B}$ be the "technology cost/industry dominance" ratio. Denominator $D_A + D_B$ is the sum of the aggregate dominance of the firms in the industry, which, as we have seen earlier, is directly correlated to the aggregate profits of the firms. Technology cost / industry dominance is thus a proxy for the importance and the attractiveness of the technology to a particular industry. A lower technology cost / industry dominance ratio implies a higher industry dominance, which in turn means the cost of segmentation technology adoption is relatively cheap with respect to the industry. Firms in an industry with high revenues likely to be early adopters of business intelligence technology as their technology cost / industry dominance ratio is low. Figure 5 shows the cut-off ratios to support technology investment decisions. [Detailed derivations of these cut-offs ratios are provided in appendix C].



BS: Both firms acquire segmentation technology
DI: Firm’s decision is dependent on dominance indicator
NS: No firms acquires the segmentation

FIGURE 5: Relationship between cost/earnings ration and firm’s technology acquisition decision

A firm will not acquire business intelligence technology if the associated cost is not offset by accrued marginal profit. Technology is costly in interval NS in Figure 5, and therefore both firms decide not to acquire business intelligence. On the other hand, in interval BS, the technology is affordable, and thus both firms acquire it. In interval DI, a firm’s strategy, i.e., the decision to segment or not, is dependent on the dominance indicator

Figure 6 illustrates the firms’ strategies for interval DI. In region IV, no firm acquires business intelligence technology, as it is not affordable. In region II, the dominant firm acquires technology and its competitor cannot afford the cost of technology. This finding is noteworthy: high technology cost coupled with high dominance indicator leads to a situation where only very dominant firm segments the market. Even though the rival firm's profitability decreases, it will choose not to counteract the dominant firm's strategy to segment. , Both firms will segment the market in regions III and I.

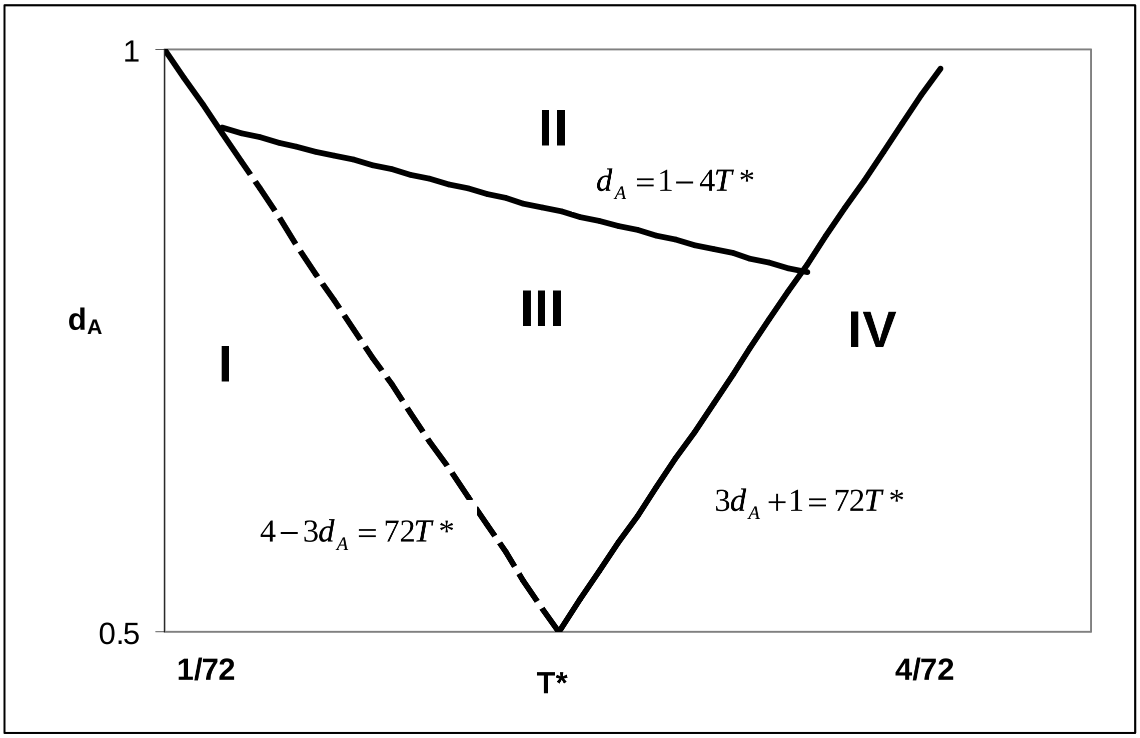


FIGURE 6: Relationship between cost/earnings ratio and firm's dominance indicator in region DI

3.6 Sensitivity Analysis

Segmentation granularity, i.e., the number of segments, has an impact on firms' profitability. Increase in granularity enables a company to customize prices to ever-smaller segments. Figure 7 shows the relation between a firm's profit and the number of segments that it can split its loyal following.

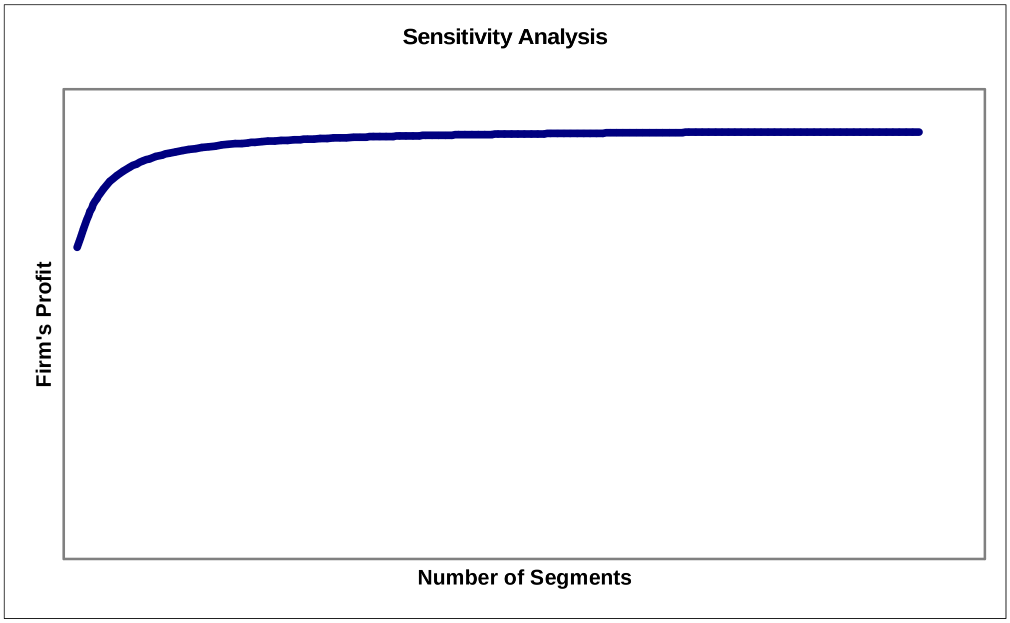


FIGURE 7: Sensitivity analysis showing how number of segments affects firms' profitability

Firm A's profit with respect to n and m are provided below. Due to symmetrical nature of the problem, we skipped firm B.

$$\frac{\partial \Pi_A}{\partial n} = \left(\frac{t_A}{2n^2} - \frac{8t_A}{9n^3}\right)q$$

$$\frac{\partial \Pi_A}{\partial m} = -\frac{t_B}{18m^3}(1-q)$$

A firm's profit is positively correlated with number of segments it can split its loyal following. As firms can customize their prices to smaller segments, they can extract higher consumer surplus. The maximum profitability is reached as the number of segments converges to infinity. This is predictable as perfect information means that prices can be tailored for each individual customer. Even so, the decision to acquire or not the technology should not be based solely on profitability. The cost of business intelligence technology must be taken into account, as finer segmentation requires larger investments in business intelligence. The acquisition decision must be based on the cut-off points determined in the previous section.

A striking result is that a firm's profit from rival's loyal customers is negatively related with number of segments. As the number of segments increases, a firm's market share from its rival's loyal customer diminishes.

Contrary to firm's profits, aggregate customer surplus $V - \prod$ is negatively correlated with number of segments. Even though aggregate consumer surplus goes down with improvements in segmentation, it is always higher than non-segmentation regime. A public policy that allows segmentation but limits the number of segments is most beneficial for society.

3.7. Conclusion

We have examined price discrimination and business intelligence's effects on competition and consumer's purchase behavior in a duopolistic game-theoretic model. Firms compete for customers with varying price tolerance. A firm acquiring business intelligence technology is able to partition the population of customers into categories of similar price tolerance and to tailor a different price to each segment. A firm that forgoes the acquisition of business intelligence technology, on the other hand, offers only two prices, one to their own customers, and the other to the rival firm's customers. The main results of the study can be summarized as follows:

1. Firms have a strong incentive to acquire business intelligence technology to increase profits and to gain market share. Their profits are proportional to customers' price tolerance. This finding explains in part the proliferation of loyalty enhancing programs.
2. The game is not necessarily prisoner's dilemma when the magnitude of the firms' base and the degree of price tolerance are considered. The dominant firm is likely to improve its profits at the expense of its competitor. The game is prisoner's dilemma only when firms are very similar in the degree of customer loyalty.

3. Price discrimination and business intelligence lead to lower prices for price-sensitive customers, but higher prices to loyal customers. Segmenting the market for not-so-loyal customers is detrimental for firms due to intense price competition. For example, long distance phone service turned out to be a low-profit business due to specialized discounts offered to rival firms' customers. AT&T, MCI and Sprint engaged in price wars in late 90's. This resulted in long-distance service becoming a low-margin business. AT&T's chairman, C Michael Armstrong, was reported to complain at the time about declining profits (Blumenstein, 1999).
4. Business intelligence improves consumer welfare. While the top loyal customers pay higher prices, the majority of the other customers enjoy lower prices. The premiums paid by the top loyal customers is largely offset by the actual savings achieved by less loyal customers. Thus, overall consumer welfare is improved. Importantly, the premiums incurred by loyal customers never exceed the threshold that will otherwise push them to switch to the competitor.
5. High technology cost coupled with high dominance indicator leads to a situation in which only very dominant firm segments the market. Even though the rival firm will see its profits decline as a consequence, its strategy is not to match a dominant firm's segmentation decision.

Our main contribution to price discrimination research is to characterize the conditions under which competition in a duopoly is prisoner's dilemma. When firms are similar in market share and loyalty, the game is clearly prisoner's dilemma. This is not true for asymmetric firms, as the outcome depends on the dominance indicator. The second contribution is that business intelligence and price discrimination improve consumer welfare, where welfare is defined as aggregate consumer surplus. Thus, while some (a few) individual customers may complain that a firm's price discrimination efforts as unfair, policy makers on the other hand may viewed positively since the general public benefits from the overall strategy. Third, a firm's dominant strategy is to segment the market. Thus, managers should make every effort to acquire segmentation technology as long as the "technology cost/industry dominance" ratio is low.

IV. PRODUCT COST

4.1 Motivation

Several attempts have been made to incorporate product cost into price discrimination definitions (Stigler, 1987; Krugman and Obstfeld, 2003), but the impact of product cost in a duopoly has been ignored. Product cost has been assumed to be either negligible or equal for all firms. Product cost differential is implicitly assumed to be incorporated in the price tolerance. We show that this is a gross oversimplification, which hides the complexity of possible competitive price discrimination games with variable customers' price tolerance. We show that the impact of product cost on equilibrium cannot be assumed away in price discrimination. We construct a duopolistic game theoretical model where firms are asymmetric, incur different product costs, and have heterogeneous customers' price tolerance. A strong submarket for a firm is one where customers incur a cost to purchase from a rival firm. We uncover a variety of duopolistic price discrimination subcases, which hereto have never been investigated, and determine the equilibrium conditions under which product cost differential makes a significant impact on competition. Specifically, if the product cost differential is greater than the maximum price tolerance of a low-cost firm, then the low-cost firm is able to prevent penetration of a rival into its submarket. If product cost differential is greater than twice the size of the high-cost firm's maximum price tolerance, then the low-cost firm is able to drive its rival out of its strongest submarket.

Let c_A and c_B denote respectively the cost incurred by two firms A and B, respectively. Table 5 summarizes the various ways product cost was incorporated in past price discrimination studies.

Cost	Explanation	Papers
$c_A = c_B = c$	Firms incur the same cost.	Fudenberg and Tirole(2000), Shaffer and Zhang (2002)
$c_A = c_B = 0$	Firms don't incur a cost. Actually, this is a subcategory of previous research stream where cost is normalized to zero.	Chen et al. (2001), Liu and Serfes(2004),Chen and Iyer(2002)
$c = F(q)$	Cost is a function of the quality. As quality improves, products costs increase. This research streams aims to find the quality level that maximizes firms' profitability.	Johnson and Myatt (2003), Choudhary et al.(2005)
$c = ax$	This research stream is based on Salop Model and cost is in fact the price tolerance a customer can bear.	Dewan et al.(2003), Liu and Serfes(2005)

Table 5: Product Cost in Price Discrimination Studies

Chen (1997), Shaffer and Zhang (2000), Chen et al (2001), Thisse and Vives (1998), Liu and Serfes (2004), Chen and Iyer (2002), Ouksel and Eruysal (2011) and Borenstein (1985) investigated price discrimination for firms that incur the same product cost. Prices are directly equated with profit margins, and product costs are ignored. Our model, in this section, assumes that product costs are not necessarily the same, and show that product cost differential has a significant impact on firms' profitability and market share in a competitive environment, and product cost is independent of profit margin differential.

Johnson and Myatt (2003), and Chaoudry et al. (2005) study the impact of product quality on market prices, where the firm offering higher quality products incurs higher cost. Product cost differential between two firms is correlated with quality differential between the products offered. All customers in product-quality models are alike, i.e. they all prefer high-quality firm.

If both firms offer the same quality level, their product costs would be the same. The cost in these models is not actually product cost. In our model, firms compete on cost-difference and they can offer the product to different segments at different prices.

4.2. The Game

The competition game between the two firms unfolds as follows. Firms independently and simultaneously determine their pricing strategies. Firms can offer different prices to each segment through targeted coupons or special offers. For example, LL Bean inserts into their catalogs "special offers" that vary across households (Shapiro and Varian, 1998). Both firms incur production costs which are not necessarily the same.

4.3. Analysis and Strategies

We first show that product cost and price tolerance are two independent dimensions. Then, we analyze the impact of product cost.

4.3.1 Product Cost and Price Tolerance are two independent dimensions

One may erroneously argue that product cost differential can be aggregated into price tolerance. We show that a change in product cost does not translate into a commensurate adjustment of the price tolerance. Let the model illustrated in Figure 1 be referred to as the basic model. We define a modified model where product cost and price tolerance are integrated together by subtracting product cost differential from price tolerance. We shall refer to this model as the cost-adjusted model. We show below that the two models are not equivalent.

Without loss of generality, assume $c_A > c_B$. Both firms may be viewed as having the same cost c_B, but price tolerance for firm A's customers is reduced by product cost differential

$(c_A - c_B)$ in the cost-adjusted model. Note that since price tolerance cannot be negative, firm A's cost-adjusted maximum price tolerance can be expressed as:

$$t_a^* = \begin{cases} t_A - (c_A - c_B) & \text{if } t_A \geq (c_A - c_B) \\ 0 & \text{otherwise} \end{cases}$$

If product cost differential is less than maximum price tolerance, then firm A retains some of customers in its strongest submarket, as illustrated in Figure 8a. Customers between 0 and $q^* = (1 - \frac{c_A - c_B}{t_A})q$ continue to prefer firm A while those between q and 1 continue to prefer firm B. On the other hand, customer between q^* and q are now indifferent between firms.

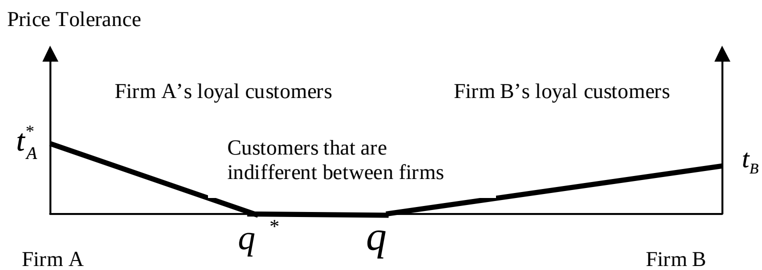


FIGURE 8a: Price Tolerance for the cost-adjusted model where price tolerance is greater than cost differential ($t_A \geq (c_A - c_B)$)

When product cost differential exceeds maximum price tolerance, cost-adjusted price tolerance is Zero for customers between 0 and q. The customers that used to prefer firm A are now indifferent between the firms. Figure 8b illustrates this case.

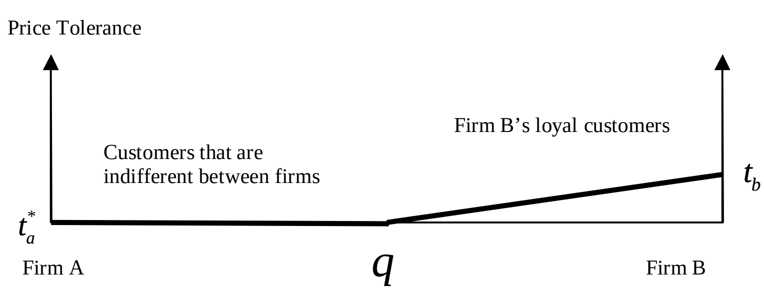


FIGURE 8b: Tolerance for the cost-adjusted model where price tolerance is less than cost differential ($t_a < (c_a - c_b)$)

Proposition 4: The basic model is not equivalent to the cost-adjusted model.

Proof: [By contradiction.] Assume that both models are equivalent. Therefore, customer responses to firms' prices must be the same, in that a customer will select to purchase from the same firm in both models. We will show that this will not actually occur by analyzing the two cases illustrated in Figures 8a and 8b:

Case 1: $t_A \geq (c_A - c_B)$ as illustrated in Figure 8a.

Consider the customers that prefer firm A with switching cost in the range (0, $t_A - (c_A - c_B)$) in the basic model. These are the exact customers between q^* and q in the cost-adjusted model. They buy from firm A so long as both firms offer the same price in the basic model. But these same customers ,are indifferent to the two firms in the cost-adjusted model. Contradiction.

Case 2: $t_A < (c_A - c_B)$, as illustrated in Figure 8b.

Consider customers in range $(0, q)$. They will buy from firm A so long as both firms offer the same price in the basic model. The same set of customers become indifferent to either firm in the cost-adjusted model. Contradiction.

<u>4.3.2 Pricing Strategies</u>

Let us assume that firm A incurs equal or higher product cost, i.e., $c_A - c_B \geq 0$. Due to symmetrical nature of the argument, the case where $c_A - c_B < 0$ will not be examined. Equilibrium conditions are determined for each of the following subcases:

i. $c_A - c_B \leq 2t_A$ and $c_A - c_B \leq t_b$

ii. $c_A - c_B > 2t_A$ and $c_A - c_B < t_B$

iii. $c_A - c_B < 2t_A$ and $c_A - c_B > t_B$

iv. $c_A - c_B > 2t_A$ and $c_A - c_B > t_B$

Lemma 1: *There exists a unique Nash equilibrium if* $c_A - c_B \leq 2t_A$ and $c_A - c_B \leq t_B$, *where the equilibrium prices are given by: [**proof in Appendix D**]:*

$$p_A^a = \frac{2t_A}{3} + \frac{2c_A}{3} + \frac{c_B}{3} \text{ and } p_A^b = \frac{t_B}{3} + \frac{2c_A}{3} + \frac{c_B}{3}$$
$$p_B^a = \frac{t_A}{3} + \frac{c_A}{3} + \frac{2c_B}{3} \text{ and } p_B^b = \frac{2t_B}{3} + \frac{c_A}{3} + \frac{2c_B}{3}$$

. **Lemma 2**: *There exists a unique Nash equilibrium if* $c_A - c_B > 2t_A$ and $c_A - c_B < t_B$ *where the equilibrium prices are given below: [**proof in Appendix E**]:*

$$p_A^a = c_A \text{ and } p_A^b = \frac{t_B}{3} + \frac{2c_A}{3} + \frac{c_B}{3}$$
$$p_B^a = c_A - t_A \text{ and } p_B^b = \frac{2t_B}{3} + \frac{c_A}{3} + \frac{2c_B}{3}$$

Lemma 3: *There exists a unique Nash equilibrium if* $c_A - c_B < 2t_A$ and $c_A - c_B > t_B$. *The unique equilibrium prices are given below: [**proof in Appendix F**]:*

$$p_A^a = \frac{2t_A}{3} + \frac{2c_A}{3} + \frac{c_B}{3} \text{ and } p_A^b = c_A$$
$$p_B^a = \frac{t_A}{3} + \frac{c_A}{3} + \frac{2c_B}{3} \text{ and } p_b^b = c_A - t_B$$

Lemma 4: *There exists a unique Nash equilibrium if* $c_A - c_B > 2t_A$ and $c_A - c_B > t_B$ *where the equilibrium prices are given below: [**proof in Appendix G**]:*

$$p_A^a = c_A \text{ and } p_A^b = c_A$$
$$p_B^a = c_A - t_A \text{ and } p_B^b = c_A - t_B$$

4.4 Discussion

We now analyze how variable product costs affect equilibrium. The results show that the game is in equilibrium regardless of cost differential. Figure 9 shows the equilibrium conditions when both firms can price discriminate.

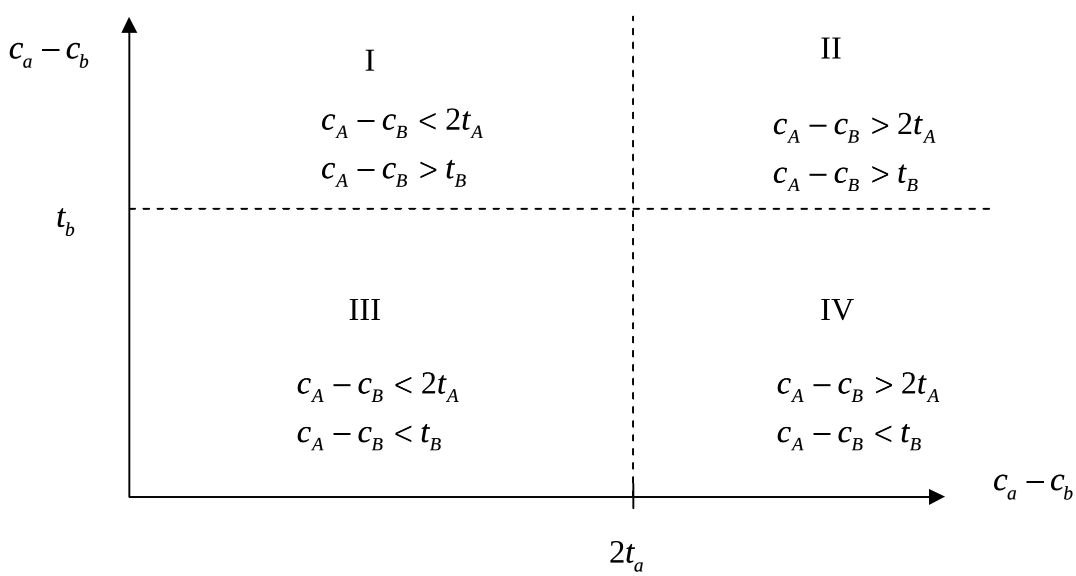


Figure 9: Equilibrium conditions when firms have different product costs

Both x and y axis are $c_a - c_b$.

The figure illustrates all the possible product cost differential cases. Previous findings dealt only with the case represented by the origin in this graph, i.e., the case where product cost differential is zero. Market equilibrium is sharply different in each quadrant. We briefly explain the each quadrant below:

- Quadrant I is the case where low-cost firm doesn't let its rival make inroads into its strongest submarket.
- Quadrant II is the case where low-cost firm drives its rival out of business.
- Quadrant III is the case where product cost differential is moderate and both firms are able to sell in some of each others' submarkets. It includes the case investigated in previous price discrimination studies, where product cost differential is zero.
- Quadrant IV is the case where the low-cost firm makes inroads into its rival's strongest submarket.

The findings of the previous studies still hold within quadrant III. On the other hand, quadrants I, II and IV have never been investigated.

4.5 Conclusion

Our results show product cost differential cannot be ignored in price discrimination. We build a game-theoretic model with two asymmetric firms, where firms incur variable product costs, and customers incur variable price tolerance. Incorporating product cost into the model enabled us to analyze how variable product cost differential impacts market equilibrium. We have shown that product cost and price tolerance impact competition independently. Then, we exhibit a variety of cases, which hereto have not been investigated previously. The assumption that product cost is negligible or equal for competing firms is a gross oversimplification. Our results show that product cost differential changes market structure.

V. INFORMATION QUALITY AND PRIVACY

5.1 Motivation

When firms split customers into segments under imperfect information quality, questions arise about how the uncertainty affects prices, market share, and competition. Using a duopolistic non-cooperative price discrimination model, we investigate the following research questions:

- How does uncertainty about customer preferences affect prices, market share and competition?
- Does more knowledge about customers lead to intensified competition?
- Is it more profitable to price discriminate even if information quality is low?
- How does consumer welfare change with varying degrees of information quality?

Privacy consciousness is a major factor affecting information quality. The collection and use of customers' information without their consent remains a material privacy concern (Awad and Krishnan 2006; Hui et al. 2007; Kasanoff 2001). A well-known example was that of DoubleClick -- an internet advertising company that tracks user behavior to better target banner ads – which postponed its plan to merge online and offline data due to a public outcry (Hansen, 2000). Likewise, firms that used customer information to price discriminate also faced a public outcry. Target used business intelligence to predict their pregnant customers to send them customized coupons (Hill, 2012).

Privacy-conscious consumers are unlikely to reveal their information, thus making it harder for firms to predict their preferences. Although sophisticated BI tools make it very easy to store and analyze customer purchase behavior, customers still have some control over their own personal information. Customers can take actions to avoid being tracked: blocking browser cookies, using multiple online identities, using temporary e-mail addresses, and purchasing with virtual credit cards. Not surprisingly, privacy concerns negatively affect consumers' willingness to disclose personal information (Culnan and Armstrong 1999; Dinev and Hart 2006; Malhotra et al. 2004; Stewart and Segars 2002). Clearly, privacy concerns play an important role in information quality.

The second factor affecting information quality is imperfect business intelligence systems. Firms might classify customers into incorrect segments due to gaps in data analysis. Firms may also make classification mistakes due to inaccurate and/or incomplete information. We shall refer to accuracy and completeness of customer preferences as information quality. A firm with low information quality is more likely classify customers into wrong segments. This may direct customers to products not matching their needs, in turn impacting the firm's profit through lost sales opportunities. A firm's business intelligence capability is a function of information quality and customers' privacy concerns. We study how varying degrees of information quality and privacy concern affect prices, profits and consumer welfare in this section.

When firms have access to business intelligence technology, customers are offered lower prices. This outcome is counter-intuitive. Even though, price discrimination is not considered as fair by customers, it significantly improves the competition in the market which leads to more affordable prices. As firms have business intelligence capability, competition confines to individual segments. Most important contribution of this study is that an increase in information quality lower firms' profits as this makes them more competitive.

This study differs from past price discrimination literature in two major ways. First, we assume inaccurate and incomplete information about customer preferences. Business intelligence does not offer perfect precision and accuracy in predicting customers' preferences. In practice, business intelligence does not deliver perfect results and firms regularly improve their business intelligence systems. For example, Netflix awarded $1 million to improve its movie recommendation system (Lohr, 2009). Second, firms may have no information about some extremely privacy-conscious customers who actively block firms' data collection efforts. We assume two different customer types in the market. One is myopic customers which reveal their personal preferences. The other one is privacy-conscious customers about which firms have no information. Firms can customize prices to myopic customers considering their purchase behavior and personal preferences. However, firms can't tailor their prices to privacy-conscious customers. Our novel approach in modeling privacy-conscious customers enables us to analyze how varying degrees of privacy consciousness in the market will impact competition, profits, and market structure.

This section connects several interrelated areas of marketing and economics. The primary contribution applies competitive price discrimination in a duopoly, especially the literature on behavior-based price discrimination. This paper extends the behavior-based price discrimination literature by incorporating information quality about customer preferences and privacy concern of customers into a game-theoretical model. In this research stream, firms tailor only prices (e.g., Chen and Iyer 2002; Chen et al. 2001; Choudhary et al. 2005; Shaffer and Zhang 1995, 2002, Thisse and Vives 1988, Ouksel and Eruysal 2011) or both products and prices (e.g., Dewan et al., 2003; Syam and Kumar 2006; Ghose and Huang 2009). Major finding from early studies in this research stream is that price discrimination leads to prisoner's dilemma where firms become worse-off (Chen et al. 2001; Choudhary et al. 2005; Shaffer and Zhang 1995; Thisse and Vives 1988). Firms in those studies attempt to lure rival firms' price sensitive customers by lowering their prices. Intensified competition in turn makes firms less profitable. However, more recent studies showed that symmetry between firms lead to prisoner's dilemma. Shaffer and Zhang

(2002) show that a firm might improve its market share at the expense of its rival. When improvements in market share outweigh the losses due to competition, a firm could be better off with price discrimination.

This section is also closely related with customer recognition studies where customers are indifferent among firms initially but become locked-in with their purchase. A customer has to incur switching cost to purchase from the other firm in subsequent term. A customer's purchase in the first period is a revelation of a preference for one firm's product. Firms use customers' purchase history and switching cost to price discriminate in the second period. Chen (1997) studies a two-period duopoly model where customers incur varying degrees of switching costs. The strategy of the firms is to lower their prices initially to build a loyal customer base and thereby expand market share, only to increase them sufficiently for locked-in customers in the second stage. Firms are worse off due to intensified competition for price sensitive customers; yet, customers are not necessarily rewarded as they incur switching costs. Taylor (2003) extends Chen's model by having more than two firms. Nevertheless, the findings are similar. The modeling approach in this research stream favors offering lower prices to rival firms' locked-in customers. Switching cost in these studies has similar impact as price tolerance in our model.

Another closely related study is Chen et al. (2001) where firms do not have perfect information about customers and can mistakenly classify customers into wrong segments. Both firms have their own loyal customer base and switchers are indifferent between firms. Firms compete for loyal customers and switchers. Firms prefer to offer higher prices for their own loyal customers as those customer can tolerate a premium. Due to imperfect information quality, a firm perceives some of its loyal customers as switchers, and thus those customers are likely to make a purchase from rival firm. The main finding of the study is that improved information quality is not always detrimental to rival firm. As a firm gets better in identifying its own loyal customers, it charges a higher price to its perceived loyal customers. Therefore, price competition gets softened as targetability improves.

Lee et al. (2011) classified customers into three main groups with respect to their privacy consciousness. They are privacy unconcerned, privacy pragmatists, and privacy fundamentalists. We argue that firms can target privacy unconcerned and pragmatists' customers as they are likely to reveal some of their preferences. Firms could make classification errors for privacy unconcerned and privacy pragmatists due to insufficient data or imperfect business intelligence systems. On the other hand, privacy fundamentalist prefer not to disclose any personal information and firms cannot offer customized prices due to lack of information. Our model takes into consideration these nuances.

5.2. The Game

Similar to Acquisti and Varian (2005), a customer in our model can be ***myopic*** or ***sophisticated*** with regards to privacy. Myopic customers reveal their preferences and firms can analyze their past purchases to predict their price tolerance. On the other hand, sophisticated customers adopt privacy-enhancing technologies not to establish a purchase history. We assume that sophisticated customers develop mechanisms to avoid being tracked and firms have no way of knowing their product preferences. A customer's price tolerance and privacy preferences are two separate dimensions. We assume that myopic customers are uniformly distributed within the interval [0, 1]. Let l be the fraction of myopic customers. Table 6 shows the privacy-consciousness and price tolerance matrix.

		Price Tolerance	
		a customers	b customers
Privacy	Myopic	lq	$l(1-q)$
	Sophisticated	$(1-l)q$	$(1-l)(1-q)$

Table 6 Price Tolerance and Privacy-Consciousness in the market

Parameters l (proportion of myopic customers) and q (proportion of a customers) are known by firms. This market-level information can easily be obtained from industry reports, survey results, aggregated sales data and census results (Kotler, 1997). An individual customer's product preference can be predicted by analyzing internal and external data sources. Firms in our model predict whether an individual customer is loyal to them. Privacy-conscious customers reveal no personal product preference; therefore, firms cannot predict price tolerance. On the other hand, myopic customers reveal their product preferences with their past purchases and firms can classify them into a and b segments. Let q_i denote the probability of categorizing i customers correctly where $i \in (a,b)$. If information quality is perfect, then firms can categorize all myopic customers into correct segments. This is the case where $q_a = q_b = 1$. If firms have no information about customers, they can still randomly categorize myopic customers based on market-level information. Therefore, $q_a \geq q$ and $q_b \geq 1-q$. We assume that both firms have exactly the same information quality. Therefore, they both make the same misclassification errors.

The competition game between the two firms unfolds as follows. Firms independently and simultaneously determine their pricing strategies. If a firm has business intelligence capability to segment the market, it can offer different prices to each segment through targeted coupons or special offers. In other words, business intelligence technology provides capability to customize prices to different segments. A firm without business intelligence cannot address individual customer and it has to offer the same price to all customers in the market.

5.3 Analysis and Strategies

Our aim is to find out how varying degrees of information quality and size of privacy-conscious customer segment affect the overall prices, market share, profitability and social welfare. We will first derive equilibrium conditions for the case where firms have business intelligence. Then, we will compare the results with the case where no firm has business intelligence.

5.3.1 Subgame 1: No firm has business intelligence

Firms cannot divide the market into segments, as they cannot figure out individual customers' loyalty preference. Each firm offers one price to the market. Customers compare firms' prices and make a purchasing decision considering their loyalty preferences. This subgame has been studied by Shaffer and Zhang (2000). Their main finding is restated below using our own notation.

Proposition 5: *Subgame 1 has a unique Nash Equilibrium. A pure-strategy Nash equilibrium to the game in which no firm can price-discriminate exists if and only if* $q, t_A,$ and t_B *are such that:*

$$\frac{9(1-q)}{(1+q)^2} \le \frac{t_A}{t_B} \le \frac{9q^3}{(1-q)(2-q)^2}$$

At equilibrium, firms A and B:

- *charge* $\frac{1+q}{3q}t_A$ *and* $\frac{2-q}{3q}t_A$*, respectively and,*
- *retain* $\frac{1+q}{3}$ *and* $\frac{2-q}{3}$ *customers, respectively.*

We include this proposition in order to provide a benchmark for comparisons with the other subgame, where firms acquire business intelligence to split the market into finer segments.

5.3.2 Subgame 2: Both firms have business intelligence

Firms have access to business intelligence and thus can categorize customers into three segments: perceived loyal customers, rival firms' perceived loyal customers and privacy-conscious customers. Figure 10 shows firms' perception of the market.

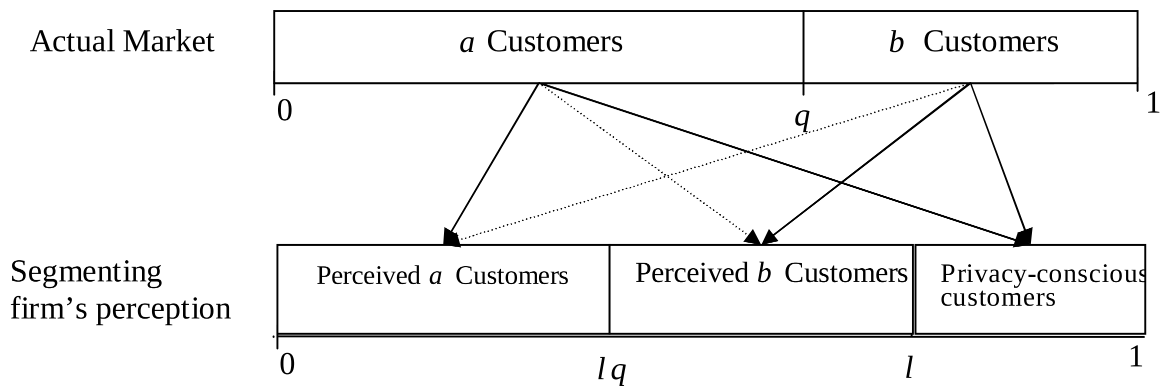


Figure 10: Firms' perception of the market

As a result of imperfect information quality, firms may categorize myopic customers inaccurately. Solid lines in figure 10 shows correctly classified customers. On the other hand, dashed lines show incorrectly classified customers. Size of perceived a customers is lq where l is the proportion of customers that firms have information about and q is firm A's loyal customer base. Note that perceived a customers segment has some incorrectly classified customers which are in fact b customers. The size of incorrectly classified a customers is $lq(1-q_a)$ where q_a is the probability of correctly categorizing a customers. Similar to perceived a customers, perceived b customers segment has incorrectly classified consumers.

The size of incorrectly classified customers is $l(1-q)(1-q_b)$ where $1-q$ is the proportion of customers that prefer firm B and q_a is the probability of correctly categorizing b customers.

A firm is strong in the submarket where its loyal customers are likely to buy its products. On the other hand, it is at a disadvantaged in the submarket where customers have a tendency to buy its rival's product. Contrary to other submarkets where all customers within the segment have exactly the same brand preference, the segment composed of privacy-conscious customers does not have a loyalty preference for only one firm. Some privacy-conscious customers prefer firm A; others favor rival firm. Note that a privacy-conscious customer is actually of type a or b; however, firms cannot predict individual privacy-conscious customers' loyalty preference due to lack of customer-profile related information. We assume firms are able to recognize privacy-conscious customers in the market even though they cannot predict their loyal preference.

Each firm offers three prices, one to its perceived loyal customers; one to its rival's perceived loyal following, one to privacy-conscious customers.

Proposition 6: *There is a unique Nash equilibrium when both firms in a duopoly, have access to business intelligence. At equilibrium, firm A charges* $\frac{1+q_a}{3q_a}t_A$, $\frac{2-q_b}{3q_b}t_B$, *and* $\frac{1+q}{3q}t_A$ *to its perceived loyal customers, its rival's perceived loyal following, and privacy-conscious customers respectively. On the other hand, firm B charges* $\frac{2-q_a}{3q_a}t_A$, $\frac{1+q_b}{3q_b}t_B$, *and* $\frac{2-q}{3q}t_A$ *to its rival's perceived loyal customers, its perceived loyal following, and privacy-conscious customers respectively. Both firms are able to hold most of its loyal customer base. [See* Proof is in appendix H].

The striking difference from the previous cases is that customers are offered lower prices in this subgame. This result is counter-intuitive. After all, price discrimination is considered to be unfair by customers. However, price discrimination in fact improves the competition in the market which results in lower prices.

When both firms have business intelligence capability, the level of competition confines to individual segments. Both firms allow its rival to make inroads into its loyal customer base. However, they are both able to limit the penetration of the rival firm.

Firm A has a loyalty edge over firm B on perceived a customers. Firm A can in fact block the penetration of firm B. However, it maximizes its profit by trading off market share with premium. Firm A prefers to lose some of its least loyal customers to charge a high premium to other perceived loyal customers in perceived a customers segment.

Firm B has a loyalty edge over firm A on perceived b customers. Due to symmetrical nature of the problem, it follows the same strategy of charging a premium to its perceived loyal customers. Thus, firm B loses some of its least loyal customers to maximize its profit from perceived b customers.

Privacy-conscious customers' segment resembles the subgame 1 where firms cannot predict customer's price tolerance. Thus, firm A charges a higher price than its rival to maximize its profit.

5.4 Implications

We analyze previous section's findings to obtain implications for consumers, firms and policy-makers. Table 7 and 8 shows firms' overall profits and prices in the market for each subcase. We cannot evaluate whether firms are better off or their profits decline by comparing firms' profits at this stage. The change in profit is a function of firms' initial loyal customer base size, price tolerance, information quality and percentage of privacy-conscious customers in the market. In the following section, we analyze the prices for each subgame and explore the conditions under which a firm becomes more profitable. Specifically, we analyze the impact of information quality and size of privacy-conscious customers on market share, prices and profitability.

	Profit Functions	
	Firm A	Firm B
No Business Intelligence	$\frac{(1+q)^2}{9q} t_A$	$\frac{(2-q)^2}{9q} t_A$
With Business Intelligence	$l q \frac{(1+q_a)^2}{9q_a} t_A + l(1-q)\frac{(2-q_b)^2}{9q_b} t_B + (1-l)\frac{(1+q)^2}{9q} t_A$	$l q \frac{(2-q_a)^2}{9q_a} t_A + l(1-q)\frac{(1+q_b)^2}{9q_b} t_B + (1-l)\frac{(2-q)^2}{9q} t_A$

Table 7: Firms' Profits when information quality and privacy is considered

Prices in the market						
	Alpha		Beta		No Info	
	firm A	firm B	firm A	firm B	firm A	firm B
No Business Intelligence	$\frac{1+q}{3q} t_A$	$\frac{2-q}{3q} t_A$	$\frac{1+q}{3q} t_A$	$\frac{2-q}{3q} t_A$	$\frac{1+q}{3q} t_A$	$\frac{2-q}{3q} t_A$
With Business Intelligence	$\frac{1+q_a}{3q_a} t_A$	$\frac{2-q_a}{3q_a} t_A$	$\frac{2-q_b}{3q_b} t_B$	$\frac{1+q_b}{3q_b} t_B$	$\frac{1+q}{3q} t_A$	$\frac{2-q}{3q} t_A$

Table 8: Prices in the market when information quality and privacy is considered

5.4.1 Implications for Customers

Lemma 5: Prices for perceived a customers are negatively correlated with information quality.

Proof: Remember q_a is the percentage of perceived a customers that are correctly classified. q_a indicates the information quality in the model. Firm A and B charge $\frac{(1+q_a)}{3q_a}t_A$ and $\frac{(2-q_a)}{3q_a}t_A$ to perceived a customers when both firms have access to business intelligence. As q_a increases, both firms lower their price for a customers. On the other hand, both firms increase their price if q_a goes down.

This result is counter-intuitive. Even though more knowledge about customers is expected to make prices higher, it substantially increases the level of competition in the market which leads to cheaper prices. In fact, uncertainty about customer information in fact keeps the prices high.

Lemma 6: Prices for perceived a customers go down when both firms have access to business intelligence.

Proof: a customers are charged $\frac{1+q}{3q}t_A$ and $\frac{2-q}{3q}t_A$ by firms A and B respectively when firms have no business intelligence. Firm A and B charge $\frac{(1+q_a)}{3q_a}t_A$ and $\frac{(2-q_a)}{3q_a}t_A$ to perceived a customers when both firms have access to business intelligence. Note that $q_a \geq q$. Therefore, $\frac{1+q}{3q}t_A \geq \frac{(1+q_a)}{3q_a}t_A$ and $\frac{2-q}{3q}t_A \geq \frac{(2-q_a)}{3q_a}t_A$. Due to intensified competition, prices for a customers goes down.

Lemma 7: Prices for perceived b customers are negatively correlated with information quality.

Proof: Firm A and B charge $\frac{2-q_b}{3q_b}t_B$ and $\frac{1+q_b}{3q_b}t_B$ to perceived b customers when both firms have access to business intelligence. As q_b increases, both firms lower their price for b customers. On the other hand, both firms increase their price if q_b goes down.

The impact of business intelligence on b customers are more nuanced. We analyze pricing changes for b customers below.

Lemma 8: Firm A charges a lower price for perceived b customers if the following condition is held.

$$\frac{t_A}{t_B} > \frac{(2-q_b)}{q_b}\frac{q}{1+q}$$

Proof: Firm A's price is $\frac{1+q}{3q}t_A$ when firms have no business intelligence. Firm A's price is changed to $\frac{2-q_b}{3q_b}t_B$ for perceived b customers when firms acquire business intelligence technology. Firm A's price is reduced if $\frac{1+q}{3q}t_A > \frac{2-q_b}{3q_b}t_B$.

Rearranging the terms yields $\frac{t_A}{t_B} \leq \frac{(2-q_b)}{q_b}\frac{q}{1+q}$.

Lemma 9: Firm B charges a lower price for perceived b customers if the following condition is held.

$$\frac{t_A}{t_B} > \frac{(1+q_b)}{q_b}\frac{q}{2-q}$$

Proof: Firm A's price is $\frac{2-q}{3q}t_A$ when firms have no business intelligence. Firm B's price is changed to $\frac{1+q_b}{3q_b}t_B$ for perceived b customers when firms acquire business intelligence technology. Firm B's price is reduced if $\frac{2-q}{3q}t_A > \frac{1+q_b}{3q_b}t_B$. Rearranging the terms yields $\frac{t_A}{t_B} > \frac{(1+q_b)}{q_b}\frac{q}{2-q}$.

Lemma 10: If firm B lowers its price for perceived b customers, then firm A is guaranteed to lower its price for perceived b customers.

Proof: If firm B lowers it price, then following condition is held.

$$\frac{t_A}{t_B} > \frac{(1+q_b)}{q_b}\frac{q}{2-q}$$

Note that $\frac{(2-q_b)}{q_b}\frac{q}{1+q} \leq \frac{(1+q_b)}{q_b}\frac{q}{2-q}$ as $q_b, q > \frac{1}{2}$. Therefore, $\frac{t_A}{t_B} > \frac{(2-q_b)}{q_b}\frac{q}{1+q}$ must hold.

Lemma 11: Price for privacy-conscious customers is not affected by business intelligence.

As firms have no information about privacy-conscious customers, they are offered exactly the same price when no firm price discriminates.

5.4.2 Implications for firms

We analyze how firms' profitability changes with business intellugence in this subsection. Specifically, we find the conditions that make firms more profitable. Then, we analyze the impact of l, the fraction of myopic customers, on firms' profits.

Lemma 12: Firm A becomes more profitable with business intelligence if the following condition is held:

$$\frac{t_A}{t_B} < \frac{(1-q\,q_a)^2 q\,q_a}{(q_a(1+q)^2 - q^2(1+q_a)^2)\,(1-2q+q\,q_a)}$$

Proof: Firm A's profit is $\frac{(1+q)^2}{9q}t_A$ when firms have no business intelligence. Its profit can be expressed as $l\,q\,\frac{(1+q_a)^2}{9q_a}t_A + l\,(1-q)\frac{(2-q_b)^2}{9q_b}t_B + (1-l)\frac{(1+q)^2}{9q}t_A$ when firms have business intelligence. Firm A's profit increase if the following condition is held:

$$l\,q\,\frac{(1+q_a)^2}{9q_a}t_A + l\,(1-q)\,\frac{(2-q_b)^2}{9q_b}t_B + (1-l\,)\frac{(1+q)^2}{9q}t_A > \frac{(1+q)^2}{9q}t_A$$

Rearranging the terms yields:

$$\frac{t_A}{t_B} < \frac{(1-q\,q_a)^2 q\,q_a}{(q_a(1+q)^2 - q^2(1+q_a)^2)\ (1-2q+q\,q_a)}$$

Remember firm A offers a lower price to its loyal customer base with business intelligence. Firm A could increase its profitability if and only if its losses from its loyal customer base is offset by its gains from b customers. This is possible if firm B has customers with extremely high price tolerance levels.

Lemma 13: Firm B becomes more profitable with business intelligence if the following condition is held:

$$\frac{t_A}{t_B} < \frac{(2-3q+q\,q_a)^2 q\,q_a}{(q_a(2-q)^2 - q^2(2-q_a)^2)\ (1-2q+q\,q_a)}$$

Remember firm B offers a lower price to its rival's loyal customer base with business intelligence. Firm B could increase its profitability if and only if its loss from its rival's loyal customer base is offset by its gains from its own loyal customers. This is possible if firm B has customers with extremely high price tolerance levels.

We are ready to analyze the impact of l, the fraction of myopic customers, now. We will do the analysis for firm A. Similar analysis can be done for firm B; however, we skip it as the impact of l is the same for both firms.

Remember firm A's profit is $l\,q\,\frac{(1+q_a)^2}{9q_a}t_A + l\,(1-q)\,\frac{(2-q_b)^2}{9q_b}t_B + (1-l\,)\frac{(1+q)^2}{9q}t_A$ when firms have access to business intelligence technology. The first term is firm A's profit from its loyal customer base, the second one is its profit from rival firm's loyal customers and third term is its profit from privacy-conscious customers. Note that an increase in l improves the first two terms; however, it has negative impact on the third term.

$l = 0$ corresponds to the case where all customers are privacy conscious. Note that firm A's profit becomes $\frac{(1+q)^2}{9q} t_A$ when $l = 0$. This is exactly the same profit function when firms have no business intelligence.

$l = 1$ corresponds to the case where no customer is privacy-conscious. Firm A's profit is $q \frac{(1+q_a)^2}{9q_a} t_A + (1-q) \frac{(2-q_b)^2}{9q_b} t_B$ when $l = 1$.

$l = 0$ and $l = 1$ are in fact extreme conditions and firm A's profit is always between $\frac{(1+q)^2}{9q} t_A$ and $q \frac{(1+q_a)^2}{9q_a} t_A + (1-q) \frac{(2-q_b)^2}{9q_b} t_B$.

If firm A becomes more profitable with business intelligence, then $q \frac{(1+q_a)^2}{9q_a} t_A + (1-q) \frac{(2-q_b)^2}{9q_b} t_B > \frac{(1+q)^2}{9q} t_A$ is satisfied. Therefore, l has a positive impact on overall profit. In other words, the size of privacy-conscious customers $(1-l)$ has a negative impact on profitability if firm A becomes more profitable with business intelligence.

If firm A incurs losses with business intelligence, then $q \frac{(1+q_a)^2}{9q_a} t_A + (1-q) \frac{(2-q_b)^2}{9q_b} t_B < \frac{(1+q)^2}{9q} t_A$ is satisfied. Therefore, l has a negative impact on overall profit in this subcase. In other words, the size of privacy-conscious customers ($1-l$) has a positive impact on profitability if firm A becomes less profitable with business intelligence.

To sum up, the impact of privacy-conscious customers' impact on profitability is nuanced. If a firm's profit increase with business intelligence, the size of privacy-conscious customers have negative impact. On the other hand, it has positive impact if firms profit decrease with business intelligence.

5.4.3 Implications for Policy Makers

An important question is how business intelligence impacts consumer welfare when firms make classification errors and some of the customer are privacy-conscious. We already defined consumer surplus as the difference between what a consumer is willing to pay and the actual price in section 3. We again consider aggregate consumer surplus as the measure of consumer welfare. Assume that a customer obtains benefit v by purchasing a product (i.e., $v >> p$ where p is price) from a firm. An individual customer's surplus is therefore $v-p$. Let V and $\prod$ be the aggregate customer benefit and the firms' total revenue, respectively. The aggregate customer surplus is therefore $V-\prod$. Table 10 summarizes model's consumer welfare implications.

Consumer Welfare	No Business Intelligence	$V-\frac{5-2q+2q^2}{9q}t_A$
	Both firms have business intelligence	$V- lq\frac{5-2q_a+2q_a{}^2}{9q_a}t_A - l(1-q)\frac{5-2q_b+2q_b{}^2}{9q_b}t_B-(1-l)\frac{5-2q+2q^2}{9q}t_A$

TABLE 10: Consumer Welfare when both firms have business intelligence

Lemma 14: Consumer welfare improves with business intelligence if the following condition is held:

$$\frac{t_A}{t_B} < \frac{(5+q(-12+2q_a)+q^2(9-6q_a\,2+2q_a^2)q\,q_a}{(5q-2q\,q_a+4q^3q_a-5q^2-2q^2q_a^2)\;(1-2q+q\,q_a)(1-q)}$$

Proof: Consumer welfare improves if the following condition is held:

$$V-\;l\,q\,\frac{5-2q_a+2q_a{}^2}{9q_a}t_A-l\,(1-q)\frac{5-2q_b+2q_b{}^2}{9q_b}t_B-(1-l)\frac{5-2q+2q^2}{9q}t_A>V-\frac{5-2q+2q^2}{9q}t_A$$

Rearranging the terms yields:

$$\frac{t_A}{t_B} < \frac{(5+q(-12+2q_a)+q^2(9-6q_a\,2+2q_a^2)q\,q_a}{(5q-2q\,q_a+4q^3q_a-5q^2-2q^2q_a^2)\;(1-2q+q\,q_a)(1-q)}$$

5.5 Conclusion

We have examined the impact of price discrimination on competition and consumer's purchase behavior in a duopolistic game-theoretic model. Firms compete for customers with varying price tolerance and privacy-consciousness. A firm acquiring business intelligence is able to partition the population of customers into categories of similar brand preference and to tailor a different price to each segment. A firm may suffer from low information quality and make errors in classifying customers into correct segment. The main results of the study can be summarized as follows:

- The game is not necessarily prisoner's dilemma when the magnitude of the firms' initial customer base, the degree of price tolerance, information quality and fraction of privacy-conscious customers are considered.

- As information quality about customer preferences improves, firms profit decline. This result is not intuitive. More knowledge about customer preferences intensifies competition and firms lower their prices to attract customers.
- When both companies have access to business intelligence, the customers that prefer the firm with higher initial market share are guaranteed to be offered lower prices. On the other hand, the customers that prefer the firm with small initial market share might be charged higher price.
- The impact of business intelligence on consumer welfare is nuanced. Initial customer base, the degree of price tolerance, information quality affects consumer welfare.

Our main contribution to price discrimination research is to show the relationship between information quality and firms profits. More knowledge about customer preferences have negative impact on profits.

There are several limitations in this research. First, we assume that business intelligence provides exactly the same level of information quality for both firms. In reality, a firm needs to collect customer-specific data to accurately estimate individual customer's price tolerance and firms could have different information quality levels. We plan to investigate the case where firms have different level of information quality.

Second, customer's price tolerance is assumed to decline linearly to simplify mathematical derivations. A U-shaped loyalty curve would be more realistic, as most of the population is price-sensitive and their loyalty is moderately low.

Third, customer’s brand loyalty never changes in our model. Purchasing from a rival firm can change a customer’s loyalty level in practice. A model incorporating the change in brand loyalty over time is more realistic.

APPENDIX A – Proof of Proposition 2

It is assumed that a firm possessing business intelligence technology and history of past purchases knows individual customers price tolerance. We assume perfect information about customers' purchase behavior. The outline of the proof is as follows:

1. Determine pricing strategies and prices for a customers. We show that the overlap of firms A and B occurs only in the n-th segment of a customers.
2. Determine pricing strategies and prices for b customers. We first show that firms A and B overlap in at most two overlapping segments. Then, this result is refined further to show firms A and B actually overlap in exactly one or two segments over the b customers. Finally, we show that the two-overlapping-segment case is the dominant strategy. Even though it is only the dominant strategy for firm B, the two-overlapping-segment equilibrium is imposed, as firm B is the pricing leader over b customers. The steps in proof are outlined below:
   - First, firm A and B overlap in at most two segments (Lemma A.1).
   - Second, zero-overlapping-segment case is not feasible (Lemma A.2).
   - Third, one-overlapping-segment case is a feasible solution when β customers are partitioned into m>2 segments. The overlapping segment is neither the first nor the last of the m segments (Lemma A.3).
   - Fourth, two-overlapping-segment case is an equilibrium, for any m. Lemma A.4 shows that it is the only one for m=2.
   - Finally, two-overlapping-segment is the dominant strategy (Theorem). It is dominant for both firms when the number of segments for β customers is even. When m is odd, one-overlapping-segment is the dominant strategy for firm A,

and two-overlapping-segment is the dominant strategy for firm B. Since firm B is the pricing leader for b customers, its dominant strategy is imposed.

i. Determine pricing strategies and prices for a customers

Let $\prod_A(a_i)$ and $\prod_B(a_i)$ denote firms' A's and B's profits in segment a_i. Then:

$$\prod_A(a_i) = \frac{q}{n}\, p_A(a_i)\ \left(1 - \frac{p_A(a_i) - p_B(a) - (t_A - t_A \frac{i}{n})}{\frac{t_A}{n}}\right)$$

At equilibrium, firm A cannot improve its profits by deviating from its pricing. Therefore:

$$\frac{\partial \prod_A(a_i)}{\partial p_A(a_i)} = \frac{q}{n}\left(1 - \frac{p_A(a_i) - p_B(a) - (t_A - t_A \frac{i}{n})}{\frac{t_A}{n}}\right) - \frac{q}{n}\frac{p_A(a_i)}{\frac{t_A}{n}} = 0 \Rightarrow$$

$$p_A(a_i) = \frac{p_B(a) + t_A + \frac{t_A}{n} - t_A \frac{i}{n}}{2} \quad (1)$$

The proportion of customers purchasing from firm B in the ***i***-th segment is bounded as:

$$0 \le \frac{p_A(a_i) - p_B(a) - (t_A - t_A \frac{i}{n})}{\frac{t_A}{n}} \le 1 \quad (2)$$

Substitution (1) into (2) yields:

$$0 \le \frac{-\frac{p_B(a)}{2} + \frac{t_A}{2n}(1 + i - n)}{\frac{t_A}{n}} \quad (3)$$

Since $p_B(a)$ is nonnegative, the numerator in (3) can be nonnegative only when i = n. Clearly, it does not hold for segments a_1 to a_{n-1}. Therefore, **firms A and B overlap only in segment** a_n. Firm B's profit function over a customers can be expressed as:

$$\prod_B(a_i) = \frac{q}{n} p_B(a) \left(\frac{p_A(a_n) - p_B(a)}{\frac{t_A}{n}} \right)$$

At equilibrium, we have:

$$\frac{\partial \prod_B(a)}{\partial p_B(a)} = \frac{q}{n} \left(\frac{p_A(a_n) - p_B(a)}{\frac{t_A}{n}} - \frac{p_B(a)}{\frac{t_A}{n}} \right) = 0 \qquad (4)$$

Solving (1) and (4) simultaneously results in:

$$p_A(a_n) = \frac{2t_A}{3n} \quad \text{and } p_B(a) = \frac{t_A}{3n}$$

For segments a_1 to a_{n-1}, firm A charges the maximum possible price that its loyal customers can tolerate. Therefore, firm A's prices for these segments are:

$$p_A(a_i) = p_B(a) + t_A - t_A \frac{i}{n} \; = \frac{t_A}{3n} + t_A - t_A \frac{i}{n} \qquad \textit{where } i = 1, 2, ..., n\text{-}1$$

Let $\prod_A(a)$ and $\prod_B(a)$ denote firms A's and B's profits from a customers, assuming that both overlap only in the n-th segment. They can be expressed as follows:

$$\prod_A(a) = \sum_{i=1}^{n} \prod_A(a_i) = \sum_{i=1}^{n-1} p_A(a_i)\frac{q}{n} + \; p_A(a_n)\left(1 - \frac{p_A(a_n) - p_B(a)}{\frac{t_A}{n}}\right)\frac{q}{n} \; = \left(\frac{t_A}{2} - \frac{t_A}{6n} + \frac{t_A}{9n^2}\right)q$$

$$\prod_B(a) = \sum_{i=1}^{n} \prod_B(a_i) = \sum_{i=1}^{n-1} 0 + \; p_B(a)\left(\frac{p_A(a_n) - p_B(a)}{\frac{t_A}{n}}\right) \frac{q}{n} \; = \; \frac{t_A}{9n^2}q$$

ii. Determine pricing strategies and prices for b customers

We now explore pricing strategies for b customers:

Lemma A.1: *Firm A and firm B overlap in at most two segments over the b customers.*

Proof: Since firm A possesses business intelligence technology, it can tailor its prices to each segment. To prevent firm A from cannibalizing its customer base, firm B's strategy would be to lower its price. If it sets its price to zero for example, it will indeed successfully keep its entire market share. But this is not a viable strategy for a profit-driven firm. Therefore, firm B must set its price to maximize its profits. But, in so doing, it must be willing to loose market share over its least loyal customers. Assume that firm A captures segments from b_1 to b_{q-1}, both firms overlap in segments b_q to b_r, and firm B controls alone segments from b_{r+1} to b_m, such that $1 \le q \le r \le m$.

Firm A must undercut its rival's price to capture segments b_1 to b_{q-1} since customers in these segments are loyal to firm B. The maximum price firm A could charge in segments b_1 to b_{q-1}, and yet undercut firm B is given below:

$$p_A(b_j) = p_B(b) - t_B \frac{j}{m} \quad \text{where j} = 1, 2, ..., q-1$$

Let $\prod_A(b)$ denote firm A's profit from b customers. It is given by:

$$\prod_A(b) = \sum_{j=1}^{q-1}(p_B(b) - t_b \frac{j}{m})\frac{1-q}{m} + \sum_{j=q}^{r} p_A(b_j)(\frac{p_B(b) - p_A(b_j) - t_B \frac{j-1}{m}}{\frac{t_B}{m}})\frac{1-q}{m}$$

where the first term is firm A's profits from segment b_1 to b_{q-1}, and the second its profits from the overlapping segments. Since firm A will not deviate from its equilibrium strategy, its price for overlapping segments is deduced as follows:

$$\frac{\partial \prod_A(b)}{\partial p_A(b_j)} = \frac{p_B(b) - p_A(b_j) - t_B \frac{j-1}{m}}{\frac{t_B}{m}} - \frac{p_A(b_j)}{\frac{t_B}{m}} = 0 \quad \text{where j} = q,\ q+1, \ldots, \text{r} \quad \Rightarrow$$

$$p_A(b_j) = \frac{p_B(b)}{2} - t_B \frac{j-1}{2m} \quad (5)$$

The proportion of customers purchasing from firm A in overlapping segments satisfies:

$$0 < \frac{p_B(b) - p_A(b_j) - t_B \frac{j-1}{m}}{\frac{t_B}{m}} < 1 \qquad (6)$$

Substituting (5) into (6) yields:

$$0 < \frac{p_B(b)}{2} - t_B \frac{j-1}{2m} < \frac{t_B}{m} \qquad (7)$$

Thus (7) must be satisfied for all overlapping segments.

$$j = q \quad \Rightarrow \quad 0 < \frac{p_B(b)}{2} - t_B \frac{q-1}{2m} < \frac{t_B}{m} \quad (8)$$

$$j = q+1 \quad \Rightarrow \quad 0 < \frac{p_B(b)}{2} - t_B \frac{q}{2m} < \frac{t_B}{m}$$

$$j = q+2 \quad \Rightarrow \quad 0 < \frac{p_B(b)}{2} - t_B \frac{q+1}{2m} < \frac{t_B}{m} \quad \Rightarrow \quad -\frac{t_B}{m} < \frac{p_B(b)}{2} - t_B \frac{q-1}{2m} < 0 \quad (9)$$

Observe that (8) and (9) cannot be satisfied simultaneously, whereas (7) is valid for only q and q+1. Thus, lemma A.1 is proved.

Lemma A.2: *Firms A and B have overlap in at least one segment over b customers.*

Proof: Since we have already shown there are at most two overlapping segments, it suffices now to only prove that zero-overlapping-segment case is not feasible. Assume the contrary, i.e., the zero-overlapping-segment case is feasible. Then, firm A sells to customers in segments *1,2, …q* and firm B sells to the customers in segments *q+1,q+2…m*. Firm B's price must be $t_B \frac{q}{m}$, minimum price tolearance level in segment *q+1*. If it were higher than $t_B \frac{q}{m}$, firm A will be able to undercut it, and thus, capture some customers from firm B in this segment. As a result, firms A and B would overlap in segment q+1, contradicting our initial assumption.

If, on the other hand, firm B lowers its price below $t_B \frac{q}{m}$, it will be able undercut firm A in segment q, and thereby capture customers from firm A in this segment. As a result, firms A and B will be overlapping in segment q. Thus, firm B's price must be exactly $t_B \frac{q}{m}$. In this case, firm A's profit from segment q is:

$$\Pi_A(b_q) = \frac{1-q}{m}\, p_A(b_q) \frac{p_B(b) - p_A(b_q) - t_B \frac{q-1}{m}}{t_B/m}$$

where the first term is the number of customers in segment q, the second is firm A's price, and the third is firm A's market share in segment q. The zero-overlapping-segment is feasible only if firm A's market share in segment q is 100%. In other words:

$$\frac{p_B(b) - p_A(b_q) - t_B \frac{q-1}{m}}{t_B/m} = 1 \Rightarrow p_A(b_q) = 0$$

Firm A will have to set is price to zero in segment q to make zero-overlapping-segment feasible, resulting in zero profit from segment q. This is obviously unrealistic, as firm A is a profit-maximizing agent. Thus, zero-overlapping-segment is infeasible.

Lemma A.3: *If m>2, there exists one one-overlapping-segment solution over b customers. The overlapping segment can neither be the first nor the last of the m segments.*

Proof: For completeness, the proof requires showing the following: (1) First segment can't be the only overlapping one; (2) Segment *m* can't be the only overlapping one; and (3) Exactly one of the segments 2,3, …*m*-1 can be the overlapping segment.

***I. First segment cannot be the only overlapping segment*:**

Assume the first segment is the only overlapping segment. The firms' profits are:

$$\Pi_A(b) = \frac{1-q}{m} p_A(b_1) \frac{p_B(b) - p_A(b_1)}{t_B / m}$$

$$\Pi_B(b) = \frac{1-q}{m}\left(1 - \frac{p_B(b) - p_A(b_1)}{t_B / m}\right) p_B(b) + \sum_{j=2}^{m} p_B(b) \frac{1-q}{m}$$

$\dfrac{p_B(b) - p_A(b_1)}{t_B / m}$ is firm A's market share in the first segment and $\dfrac{1-q}{m}$ is the number of customers in each segment. Thus, we can write:

$$\frac{\partial \Pi_A(b)}{\partial p_A(b_1)} = \frac{1-q}{m} \frac{p_B(b) - p_A(b_1)}{t_B / m} - \frac{1-q}{m} \frac{p_A(b_1)}{t_B / m} \Rightarrow p_A(b_1) = \frac{p_B(b)}{2}$$

$$\frac{\partial \Pi_B(b)}{\partial p_B(b)} = \frac{1-q}{m}\left(1 - \frac{p_B(b) - p_A(b_1)}{t_B / m} - \frac{p_B(b)}{t_B / m}\right) + (m-1)\frac{1-q}{m} = 0 \Rightarrow 2 p_B(b) - p_A(b_1) = t_B$$

The optimum prices are thus $p_A(b_1) = \frac{t_B}{3}$ and $p_B(b) = \frac{2t_B}{3}$. In the second segment, the minimum price tolerance is $\frac{t_B}{m}$ and firm B's price is $\frac{2t_B}{3}$. Obviously, $\frac{2t_B}{3} > \frac{t_B}{m}$, and thus firm A can undercut firm B’s price and capture some customers in segment 2. Therefore, the first segment cannot be the only overlapping segment. Contradiction.

***II. The m-th segment cannot be the only overlapping segment*:**

Similarly, assume the m-th segment is the only overlapping segment. Firms’ profits are:

$$\Pi_A(b) = \left(\sum_{j=1}^{m-1} p_A(b_j)\frac{1-q}{m}\right) + \frac{p_B(b) - p_A(b_m) - t_B\frac{m-1}{m}}{t_B/m}\, p_A(b_m)\frac{1-q}{m}$$

$$\Pi_B(b) = \frac{1-q}{m}\left(1 - \frac{p_B(b) - p_A(b_m) - t_B\frac{m-1}{m}}{t_B/m}\right) p_B(b)$$

$\dfrac{p_B(b) - p_A(b_m) - t_B\frac{m-1}{m}}{t_B/m}$ is firm A’s market share in segment m and $\dfrac{1-q}{m}$ is the number of customers in each segment. Thus:

$$\frac{\partial \Pi_A(b)}{\partial p_A(b_m)} = \frac{p_B(b) - p_A(b_m) - t_B\frac{m-1}{m}}{t_B/m}\frac{1-q}{m} - \frac{p_A(b_m)}{t_B/m}\frac{1-q}{m} = 0 \Rightarrow p_{\cdot}(b_m) = 2p_A(b_m) + t_B\frac{m-1}{m}$$

$$\frac{\partial \Pi_B(b)}{\partial p_B(b)} = \frac{1-q}{m}\left(1 - \frac{p_B(b) - p_A(b_m) - t_B\frac{m-1}{m}}{t_B/m}\right) - \frac{1-q}{m}\frac{p_B(b)}{t_B/m} = 0 \Rightarrow 2p_B(b) - p_A(b_m) = t_B$$

So the firms’ prices are: $p_A(b_1) = \frac{t_B}{3m}$(-m+2) and $p_B(b) = \frac{t_B}{3m}(m+1)$. Observe that firm A’s price is always negative for m>2. Thus, the solution is not feasible. Contradiction.

III. There exists exactly one one-overlapping-segment

Let q be the only segment shared by the two firms. Observe that (7) holds for only j=q.

$$j = q \quad \Rightarrow \quad 0 < \frac{p_B(b)}{2} - t_B\frac{q-1}{2m} < \frac{t_B}{m} \ \Rightarrow \ \frac{t_B}{m}(q-1) < p_B(b) < \frac{t_B}{m}(q+1) \text{ where } 1<q<m \ (10)$$

Consequently, the following inequalities do not hold:

$$j = q+1 \quad \Rightarrow \quad 0 < \frac{p_B(b)}{2} - t_B\frac{q}{2m} < \frac{t_B}{m} \ \Rightarrow \frac{t_B}{m}q < p_B(b) < \frac{t_B}{m}(q+2) \ (11)$$
$$j = q-1 \quad \Rightarrow \quad 0 < \frac{p_B(b)}{2} - t_B\frac{q-2}{2m} < \frac{t_B}{m} \ \Rightarrow \frac{t_B}{m}(q-2) < p_B(b) < \frac{t_B}{m}(q) \ (12)$$

Inequality (10) is true if one of the following holds:

i. $\frac{t_B}{m}(q-1) < p_B(b) < \frac{t_B}{m}q$

ii. $p_B(b) = \frac{t_B}{m}q$ (13)

iii. $\frac{t_B}{m}q < p_B(b) < \frac{t_B}{m}(q+1)$

If inequality (i) holds, then (12) is satisfied. Contradiction. Similarly, if inequality (iii) holds, then (11) is true. Contradiction. Thus, one-overlapping-segment exists if and only if the following equation is satisfied:

$$p_B(b) = \frac{t_B}{m}q \quad (13)$$

Firm A's price for the overlapping segment can be found by substituting (13) into (5).

$$p_A(b_j) = \frac{p_B(b)}{2} - t_B\frac{j-1}{2m} \Rightarrow \ p_A(b_q) = \frac{t_B}{2m}$$

Then, firm A's profit function can be expressed as:

$$\Pi_A(b) = \sum_{j=1}^{q-1}(\frac{t_B}{m}q - t_b\frac{j}{m})\frac{1-q}{m} + \frac{t_B}{2m}(\frac{\frac{t_B}{m}q - \frac{t_B}{2m} - t_B\frac{q-1}{m}}{\frac{t_B}{m}})\frac{1-q}{m} = \frac{t_B}{2m^2}(1-q)(q^2-q+\frac{1}{2}) \ (14)$$

where the first term is firm A's profit from segments 1 to q-1, and the second is its profit from the overlapping segment. Firm B's profit can be expressed as:

$$\prod_B(b) = \frac{t_B}{m} q\left(1 - \frac{\frac{t_B}{m} q - \frac{t_B}{2m} - t_B \frac{q-1}{m}}{\frac{t_B}{m}}\right)\frac{1-q}{m} + \sum_{j=q+1}^{m} \frac{t_B}{m} q \frac{1-q}{m} \quad = \frac{t_B}{m^2}(1-q)(mq - q^2 + \frac{q}{2}) \quad (15)$$

where the first term is firm B's profit from the overlapping segment, and the second are its profits from segments *q+1* to m.

Firm A's profitability increases with q, as $\frac{\partial \prod_A(b)}{\partial q} > 0$. However, firm B's profitability goes up only to a fixed *q*, at which time it starts to decline as $\frac{\partial \prod_B(b)}{\partial q} = \frac{l_B}{m^2}(1-q)(m - 2q + \frac{1}{2})$ is concave down.

Since firm A has business intelligence technology, it can customize its prices to each segment to undercut firm B's prices. Higher *q* values represent the situation in which firm A makes deeper inroads into firm B's loyal customer base. To prevent firm A from cannibalizing its customer base, firm B's strategy may be tempted to lower its price. For example, if it sets its price to zero, it will surely successfully protect its market share. But, this will not be a viable strategy for a profit-driven firm. Therefore, firm B must set a strictly positive price that maximizes its profit, but at the expense of loosing its least loyal customers. At the same time, firm A can't make inroads beyond the *q*-th segment, unless it offers negative prices to attract those customers. Firm B is the leader in setting the value of *q*. It will do so in such a way as to maximize its profits, and yet, to avoid being undercut by firm A. This *q* is given by:

$$\frac{\partial \Pi_B(b)}{\partial q} = \frac{t_B}{m^2}(1-q)(m-2q+\frac{1}{2}) = 0 \Rightarrow q = \frac{m}{2}+\frac{1}{4} \text{ as } \frac{\partial^2 \Pi_B(b)}{\partial q^2} < 0$$

Observe that $q = \frac{m}{2}+\frac{1}{4}$ is not integer for any m. Thus, firm B's profit is maximized if either $q = \left\lfloor \frac{m}{2}+\frac{1}{4} \right\rfloor$ or $q = \left\lfloor \frac{m}{2}+\frac{1}{4} \right\rfloor + 1$. If m is even, then $q = \frac{m}{2}$ or $q = \frac{m}{2}+1$. If m is odd, then $q = \frac{m-1}{2}$ or $q = \frac{m+1}{2}$. One can easily show that firm B's profits are maximized for $q = \frac{m}{2}$ when m is even, and for $q = \frac{m+1}{2}$ if m is odd. The two firms' optimum prices and profits are given below:

When m is even	When m is odd
$q = \frac{m}{2}$	$q = \frac{m+1}{2}$
$p_A(b_j) = \frac{t_B}{2} - t_B\frac{j}{m}$ where $j = 1, 2, ..., \frac{m}{2}-1$	$p_A(b_j) = \frac{t_B}{2} + \frac{t_B}{2m} - t_B\frac{j}{m}$ where $j = 1, 2, ..., \frac{m-1}{2}$
$p_A(b_{\frac{m}{2}}) = \frac{t_B}{2m}$	$p_A(b_{\frac{m+1}{2}}) = \frac{t_B}{2m}$
$p_A(b_j) = 0$ where $j = \frac{m}{2}+1, \frac{m}{2}+2, .., m$	$p_A(b_j) = 0$ where $j = \frac{m+3}{2}, \frac{m+5}{2}, .., m$
$\Pi_A(b) = (\frac{t_B}{8} - \frac{t_B}{4m} + \frac{t_B}{4m^2})(1-q)$	$\Pi_A(b) = (\frac{t_B}{8} + \frac{t_B}{8m^2})(1-q)$
$p_B(b) = \frac{t_B}{2}$	$p_B(b) = \frac{t_B}{2} + \frac{t_B}{2m}$
$\Pi_B(b) = (\frac{t_B}{4} + \frac{t_B}{4m})(1-q)$	$\Pi_B(b) = (\frac{t_B}{4} + \frac{t_B}{4m})(1-q)$

***Lemma A.4**: If m=2, the one-overlapping-segment is not feasible.*

Proof: The proof consists of two steps: (i) first segment cannot be the only one overlapping; and (ii) second segment cannot be the only one overlapping.

I. The first segment cannot be singly overlapping, when m=2.

Assume that the first segment is the only one overlapping, and the second is entirely captured by firm B. Then, firm A's overall profits are:

$$\Pi_A(b) = \frac{1-q}{2} p_A(b_1) \frac{p_B(b) - p_A(b_1)}{t_B/2}$$

The first term is the total number of customers in the first segment, the second is firm A's price for the first segment, and the third is its market share. Thus, as the second derivative is negative, firm A's maximum profits are obtained as follows:

$$\frac{\partial \Pi_A(b)}{\partial p_A(b_1)} = \frac{p_B(b) - 2p_A(b_1)}{t_B/2} \frac{1-q}{2} p = 0 \Rightarrow p_A(b_1) = \frac{p_B(b)}{2}$$

Firm B's profits are:

$$\Pi_B(b) = (1 - \frac{p_B(b) - p_A(b_1)}{t_B/2}) \frac{1-q}{2} p_B(b) + \frac{1-q}{2} p_B(b)$$

The first term are firm B's profits from the first segment, the second are its profits from the second segment, $\frac{1-q}{2}$ is the number of customers in each segment, and $1 - \frac{p_B(b) - p_A(b_1)}{t_B/2}$ is firm B's market share in the first segment. Since the second segment is completely captured by firm B, its market share in this segment is obviously 100%. As the second derivative is negative, firm B's maximum profits are given by:

$$\frac{\partial \prod_B(b)}{\partial p_B(b)} = (1 - \frac{p_B(b) - p_A(b_1)}{t_B/2})\frac{1-q}{2} - \frac{p_B(b)}{t_B/2}\frac{1-q}{2} + \frac{1-q}{2} = 0 \Rightarrow 2p_B(b) - p_A(b_1) = t_B$$

Thus, the two firms' prices are: $p_B(b) = \frac{2t_B}{3}$ and $p_A(b_1) = \frac{t_B}{3}$. Firm A's market share from the second segment must be zero since the first segment is the only one overlapping. Since firm B's price for the second segment is $\frac{2t_B}{3}$ and its minimum loyalty level in the second segment is $\frac{t_B}{2}$, this segment is vulnerable to firm A's market penetration as firm B's price is higher than minimum loyalty level. Therefore, the second segment is also overlapping. Contradiction.

II. The second segment cannot be singly overlapping, when m=2.

We omit the proof, as the argument is symmetrical to above.

Lemma A.5: *The two-overlapping-segment is an equilibrium point, for any m.*

Proof: Let the two overlapping segments be q and $q + 1$. Firm A's profits are then:

$$\prod_A(b) = \sum_{j=1}^{q-1}(p_B(b) - t_B\frac{j}{m})\frac{1-q}{m} + \sum_{j=q}^{q+1} p_A(b_j)(\frac{p_B(b) - p_A(b_j) - t_B\frac{j-1}{m}}{\frac{t_B}{m}})\frac{1-q}{m}$$

where the first term is firm A's profits from segments b_1 to b_{q-1}, and the second is its profits from the overlapping segments. Let $\prod_B(b)$ be firm B's profit function from b customers. It is given by:

$$\Pi_B(b)=\sum_{j=q}^{q+1} p_B(b)\left(1-\frac{p_B(b)-p_A(b_j)-t_B\dfrac{j-1}{m}}{\dfrac{t_B}{m}}\right)\frac{1-q}{m}+\sum_{j=q+2}^{m} p_B(b)\frac{1-q}{m} \quad (16)$$

where the first term is firm B's profits from the overlapping segments, and the second is firm B's profit from segments b_{q+2} and b_m. Substituting (5) into (16) yields:

$$\Pi_B(b)=\sum_{j=q}^{q+1} p_B(b)\left(1-\frac{\dfrac{p_B(b)}{2}-t_B\dfrac{j-1}{2m}}{\dfrac{t_B}{m}}\right)\frac{1-q}{m}+(m-q-1)p_B(b)\frac{1-q}{m}$$

Firm B's profits are maximized when its price is as follows:

$$\frac{\partial \Pi_B(b)}{\partial p_B(b)}=\left(1-\frac{\dfrac{p_B(b)}{2}-t_B\dfrac{q-1}{2m}}{\dfrac{t_B}{m}}-\frac{\dfrac{p_B(b)}{2}}{\dfrac{t_B}{m}}+1-\frac{\dfrac{p_B(b)}{2}-t_B\dfrac{q}{2m}}{\dfrac{t_B}{m}}-\frac{\dfrac{p_B(b)}{2}}{\dfrac{t_B}{m}}\right)\frac{1-q}{m}$$

$$+(m-q-1)p_B(b)\frac{1-q}{m}=0 \quad \Rightarrow p_B(b)=\frac{t_B}{2}-\frac{t_B}{4m} \quad (17)$$

Substituting (17) into (7) yields:

$$0\le\frac{p_B(b)}{2}-t_B\frac{j-1}{2m}\le\frac{t_B}{m} \quad \text{where } j = q,\ q+1$$

$$0\le\frac{t_B}{4}+\frac{t_B}{8m}-t_B\frac{j-1}{2m}\le\frac{t_B}{m} \quad \Rightarrow$$

$$\frac{m}{2}-\frac{3}{4}\le j\le\frac{m}{2}+\frac{5}{4} \quad \textit{where } j = q,\ q+1$$

$$j=q \ \Rightarrow\ \frac{m}{2}-\frac{3}{4}\le q\le\frac{m}{2}+\frac{5}{4} \quad (18)$$

$$j=q+1 \ \Rightarrow\ \frac{m}{2}-\frac{7}{4}\le q\le\frac{m}{2}+\frac{1}{4} \quad (19)$$

From (18) and (19), one can deduce that

$$\frac{m}{2}-\frac{3}{4}\le q\le\frac{m}{2}+\frac{1}{4} \quad .$$

Thus, $q = \left\lfloor \frac{m}{2} \right\rfloor$. Firm B's price is: $p_B(b) = \frac{t_B}{2} + \frac{t_B}{4m}$

if m is even, then	if m is odd, then
$\mathrm{p_A}(b_q) = \frac{5t_B}{8m}$	$\mathrm{p_A}(b_q) = \frac{7t_B}{8m}$
$\mathrm{p_A}(b_{q+1}) = \frac{t_B}{8m}$	$\mathrm{p_A}(b_{q+1}) = \frac{3t_B}{8m}$
$q = \frac{m}{2}$	$q = \frac{m-1}{2}$
$\prod_A(b) = (\frac{t_B}{8} - \frac{t_B}{8m} + \frac{t_B}{4m^2})(1-q)$	$\prod_A(b) = (\frac{t_B}{8} - \frac{t_B}{8m} + \frac{5t_B}{32m^2})(1-q)$
$\prod_B(b) = (\frac{t_B}{4} + \frac{t_B}{4m} + \frac{t_B}{16m^2})(1-q)$	$\prod_B(b) = (\frac{t_B}{4} + \frac{t_B}{4m} + \frac{t_B}{16m^2})(1-q)$

Theorem: The two-overlapping-segment is the dominant strategy

Proof: The results of Lemma A.1,A.2,A.3, and A.4 are summarized in the following table.

		One-overlapping-segment	Two–overlapping-segment
m is even	Firm A's payoff	$(\frac{t_B}{8} - \frac{t_B}{4m} + \frac{t_B}{4m^2})(1-q)$	$(\frac{t_B}{8} - \frac{t_B}{8m} + \frac{5}{32m^2})(1-q)$
	Firm B's payoff	$(\frac{t_B}{4} + \frac{t_B}{4m})(1-q)$	$(\frac{t_B}{4} + \frac{t_B}{4m} + \frac{t_B}{16m^2})(1-q)$
m is odd	Firm A's payoff	$(\frac{t_B}{8} + \frac{t_B}{8m^2})(1-q)$	$(\frac{t_B}{8} - \frac{t_B}{8m} + \frac{5}{32m^2})(1-q)$
	Firm B's payoff	$(\frac{t_B}{4} + \frac{t_B}{4m})(1-q)$	$(\frac{t_B}{4} + \frac{t_B}{4m} + \frac{t_B}{16m^2})(1-q)$

Obviously, firm B is better off with two overlapping segments regardless of firm A's pricing policy. Firm A's strategy is more nuanced. Firm A is more profitable with one overlapping segment if \$m\$ is odd and its dominant strategy is two-overlapping-segment if m is even. Firm A can customize its prices to each segment as it possesses segmentation technology. However, as both firms are competing for firm B's loyal customers, firm A will have to undercut firm B's price substantially to gain a larger market share. Firm B can lower its prices to prevent firm A from cannibalizing its loyal customer base. In fact, it can completely prevent firm A's incursions into its loyal customer base if it sets its price to zero. Because firm B is a profit-driven firm, it will have to set a strictly positive price that maximizes its profit. For that, it must be willing to loose market share of its least loyal customers. Obviously, firm A's segmentation technology pays off, as it provides flexibility to undercut firm B in some segments and, thereby, capture greater market share. However, firm A can't compete against firm B over highly loyal b customers. Firm B has enough price flexibility to lower its price to a level that can't be matched by firm A. It is also the lead price setter over b customers, and thus, it determines the market share to relinquish to firm A. Since the two-overlapping-segment strategy is its dominant for firm B, it can therefore force firm A to accept it. Therefore, the firms profit from b customers are given by:

$$\Pi_A(b) = \left(\frac{t_B}{8} - \frac{t_B}{8m} + \frac{5t_B}{32m^2}\right)(1-q)$$

$$\Pi_B(b) = \left(\frac{t_B}{4} + \frac{t_B}{4m} + \frac{t_B}{16m^2}\right)(1-q)$$

APPENDIX B - Proof of Proposition 3

For segment a_i, firms profit functions can be expressed as:

$$\Pi_A(a_i) = p_A(a_i)\ \left(1 - \frac{p_A(a_i) - p_B(a_i) - \ (t_A - \frac{i}{n} t_A)}{(\frac{t_A}{n})}\right)\ \frac{q}{n}$$

$$\Pi_B(a_i) = p_B(a_i) \frac{p_A(a_i) - p_B(a_i) - \ (t_A - \frac{i}{n} t_A)}{(\frac{t_A}{n})} \frac{q}{n}$$

The unique Nash equilibrium is obtained if following conditions are satisfied (1) the first derivative of profit functions at equilibrium prices is zero; (2) the profit function's second derivative is always negative.

$$\frac{\partial \Pi_A(a_i)}{\partial p_A(a_i)} = \left(1 - \frac{p_A(a_i) - p_B(a_i) - \ (t_A - \frac{i}{n} t_A)}{(\frac{t_A}{n})}\right) \frac{q}{n} - \frac{p_A(a_i)}{\frac{t_A}{n}} \frac{q}{n} = 0 \quad (20)$$

$$\frac{\partial^2 \Pi_A(a_i)}{\partial p_A^2(a_i)} = -2 \frac{p_A(a_i)}{\frac{t_A}{n}} q < 0 \quad (21)$$

$$\frac{\partial \Pi_B(a_i)}{\partial p_B(a_i)} = \frac{p_A(a_i) - p_B(a_i)\ -\ (t_A - \frac{i}{n} l_A)}{(\frac{t_A}{n})} \frac{q}{n} - \frac{p_B(a_i)}{\frac{t_A}{n}} \frac{q}{n} = 0 \quad (22)$$

$$\frac{\partial^2 \Pi_B(a_i)}{\partial p_B^2(a_i)} = -2 \frac{p_B(a_i)}{\frac{t_A}{n}} q < 0 \quad (23)$$

Inequalities (21) and (23) show that the second derivatives of the profit functions are always negative and thus the profit functions are concave down. The solutions to (20) and (22) yield Nash equilibrium prices. Solving (20) and (22) simultaneously results in:

$$p_A(a_i) = \frac{t_A}{3} - i\frac{t_A}{3n} + \frac{2t_A}{3n} \quad (24)$$

$$p_B(a_i) = -\frac{t_A}{3} + i\frac{t_A}{3n} + \frac{t_A}{3n} \quad (25)$$

Observe that the $p_B(a_i)$ are not positive for segments $a_1, a_2,..., a_{\text{n-1}}$. Thus, if firm B offers these prices, it will incur loss. But, firms must offer prices nonnegative prices to make a profit. Therefore, firm B has only one option for segments $a_1, a_2,..., a_{\text{n-1}}$. It has to set its price to zero. Setting $p_B(a_i) = 0$, solving (20) yields: $p_A(a_i) = \frac{t_A}{2} \; - i\frac{t_A}{2n} + \frac{t_A}{2n}$ where $i = 1, 2,..,n-1$. However, firm A cannot set its price to $p_A(a_i) = \frac{t_A}{2} \; - i\frac{t_A}{2n} + \frac{t_A}{2n}$, as its market share, given by $1 - \dfrac{p_A(a_i) - p_B(a_i) - \; (t_A - \frac{i}{n}t_A)}{(\frac{t_A}{n})}$, will become greater than 1, which is not feasible. Firm A has one option, its price has to be minimum loyalty level in the segment, i.e. $t_A - \frac{i}{n}t_A$ to be most profitable. The game is in equilibrium for segments $a_1, a_2,..., a_{\text{n-1}}$ when $p_{\text{A}}(a_i) = \text{t}_A - \frac{i}{n}t_A$ and $p_{\text{B}}(a_i) = 0$.

For segment a_{n}, prices offered by firms in equation (24) and (25) become positive. Solving equation (24) and (25) simultaneously when $i = n$ results in Nash equilibrium prices.

$$p_A(a_n) = \frac{2t_A}{3n} \text{ and } p_B(a_n) = \frac{t_A}{3n}$$

Similarly, one can deduce equilibrium prices for segment b_j:

$$p_A(b_j) = \frac{t_B}{3m} \text{ and } p_B(b_j) = \frac{2t_B}{3m} \quad \text{for } j = 1$$

$$p_A(b_j) = 0 \text{ and } p_B(b_j) = \frac{j-1}{m} t_B \quad \text{for } j = 2, 3, .., m$$

Firm A' profit function is given below:

$$\Pi_A = \sum_{i=1}^{n-1} p_A(a_i)\frac{q}{n} + p_A(a_n)\Big(1 - \frac{p_A(a_n) - p_B(a_n)}{\frac{t_A}{n}}\Big)\frac{q}{n} + \frac{1-q}{m} p_A(b_1)\Big(\frac{p_B(b_1) - p_A(b_1)}{\frac{t_B}{m}}\Big)$$

where the first term is firm A's profits from segments $a_1, a_2, ..., a_{n-1}$, the second represents the profits from the a_n-th segment, and the last is firm A's profits from b_1.

Firm B's profit function is given by:

$$\Pi_B = \frac{q}{n} p_B(a_n)\Big(\frac{p_A(a_n) - p_B(a_n)}{\frac{t_A}{n}}\Big) + \frac{1-q}{m} p_B(b_1)\Big(1 - \frac{p_B(b_1) - p_A(b_1)}{\frac{t_B}{m}}\Big) + \sum_{j=2}^{m} \frac{1-q}{m} p_B(b_j)$$

where the first is firm B's profits from segment a_n, the second is firm B's profit from segment b_1, and the last is the profits from segments $b_2, b_3, ..., b_m$. In summary:

$$\Pi_A = \Big(\frac{t_A}{2} - \frac{t_A}{2n} + \frac{4t_A}{9n^2}\Big)q + \frac{t_B}{9m^2}(1-q)$$

$$\Pi_B = \frac{t_A}{9n^2}q + \Big(\frac{t_B}{2} - \frac{t_B}{2m} + \frac{4t_B}{9m^2}\Big)(1-q)$$

APPENDIX C – Impact of Technology Cost

Assume that firm A is dominant $(i.e.\ d_A > d_B)$. Its marginal profit due to segmentation can be expressed as follows:

$$\Delta\prod_A = \frac{1}{2}t_A q + \frac{1}{8}t_B(1-q) - \left(\frac{4}{9}t_A q + \frac{1}{9}t_B(1-q)\right) = \frac{1}{18}t_A q + \frac{1}{72}t_B(1-q)$$

Firm A's marginal gain from its investment in technology must surpass the cost of acquiring the segmentation technology. Thus:

$$\begin{array}{ll}
\Delta\prod_A \geq T & \Rightarrow \quad \frac{1}{18}t_A q + \frac{1}{72}t_B(1-q) \geq T \\
\frac{1}{18}D_A + \frac{1}{72}D_B \geq T & \Rightarrow \quad \frac{D_A}{18(D_A+D_B)} + \frac{D_B}{72(D_A+D_B)} \geq \frac{T}{D_A+D_B} \\
\frac{1}{18}d_A + \frac{1}{72}d_B \geq T^* & \Rightarrow \quad 4d_A + d_B \geq 72T^* \\
d_B = 1-d_A & \Rightarrow \quad 3d_A + 1 \geq 72T^* \quad (26)
\end{array}$$

Let $\Delta\prod_B$ be firm B's marginal profit from segmenting. It can be expressed as follows:

$$\Delta\prod_B = \frac{1}{8}t_A q + \frac{1}{2}t_B(1-q) + -\left(\frac{1}{9}t_A q + \frac{4}{9}t_B(1-q)\right) = \frac{1}{72}t_A q + \frac{1}{18}t_B(1-q)$$

Technology acquisition will occur only marginal gains due to technology exceed the investment cost. Thus, we must have:

$$\begin{array}{ll}
\Delta\prod_B \geq T & \Rightarrow \quad \frac{1}{72}t_A q + \frac{1}{18}t_B(1-q) \geq T \\
d_A + 4d_B \geq 72T^* & \Rightarrow \quad 3d_B + 1 \geq 72T^* \quad (27)
\end{array}$$

Firm B will segment the market only if inequality (27) is satisfied. On the other hand, if inequality (26) is satisfied, then firm A will be the only firm acquiring segmentation technology. In this case, firm B looses almost half of its loyal customers, as firm A is the only one segmenting. This great loss leads firm B to reconsider its segmentation decision. Firm B's marginal profits by segmenting, $\Delta\prod_B$, is expressed as follows:

$$\Delta\prod_B = \frac{1}{2}t_B(1-q) - \frac{1}{4}t_B(1-q) = \frac{1}{4}t_B(1-q)$$

Firm B will segment the market if and only if its marginal profits from segmentation exceed the investment cost.

$$\begin{aligned} &\Delta\prod_B \geq T && \Rightarrow \frac{1}{4}t_B(1-q) \geq T \\ &\frac{1}{4}D_B \geq T && \Rightarrow \frac{1}{4}\frac{D_B}{D_A+D_B} \geq \frac{T}{D_A+D_B} \\ &d_B \geq 4T^* \end{aligned}$$

In summary, firm B segments the market if one of the following conditions is satisfied.

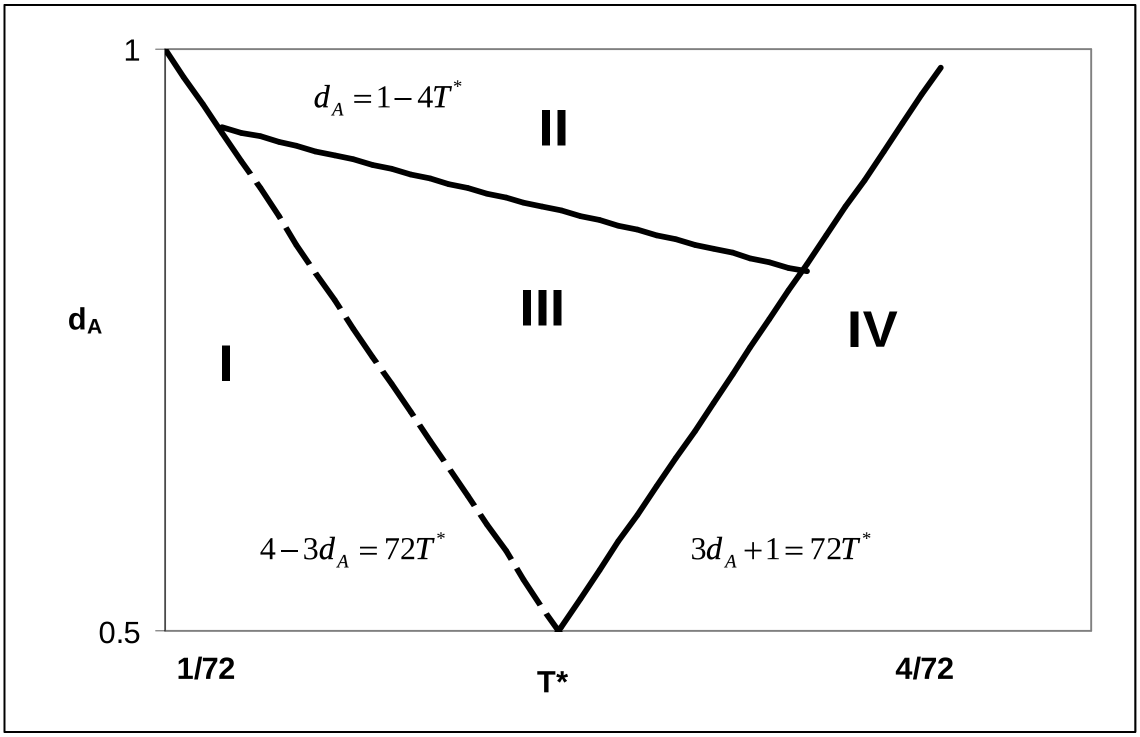
i) $3d_B + 1 \geq 72T^*$
ii) $3d_A + 1 \geq 72T^*$ and $3d_B + 1 < 72T^*$ and $d_B \geq 4T^*$ (28)

We know that $1 < 3d_B + 1 < 3d_A + 1 < 4$ as $0 < d_B < 0.5 < d_A < 1$. Therefore, if $72T^* \leq 1$, both firms segment the market, and if $72T^* > 4$, none of the firms segments the market. Inequalities (27) and (28) can be rewritten as follows:

$$3d_B + 1 = 3(1-d_A) + 1 = 4 - 3d_A \geq 72T^* \quad (29)$$

$$d_B = 1 - d_A \geq 4T^* \quad (30)$$

The figure below illustrates the firms' segmentation strategy when $1 < 72T^* \leq 4$



- Region 1: Inequalities (28) and (29) are satisfied, both firms segment the market.
- Region 2: Inequality (28) is satisfied, but neither (29) nor (30) are. Therefore, only the dominant firm segments the market.
- Region 3: Inequalities (28) and (30) are satisfied, but (29) is not satisfied. As a result, both firms segment the market.
- Region 4: Neither (28) nor (29) is satisfied, so no firm segments the market.

APPENDIX D – Proof of Lemma 1 in section 4

p_i^j : Firm i's price for customers in segment j where i$\in$ (A,B) and j$\in$ (a,b)

The outline of the proof is as follows:

1. We show that there exist a local optimal solution set $(p_A^a, p_B^a, p_A^b, p_B^b)$ that maximizes both firms' profit.
2. We show that the solution found in step 1 is global optimal if the following conditions are met. $c_A - c_B \le 2t_A$ and $c_A - c_B \le t_B$

Proof: Firms' profits can be expressed as:

$$\Pi_A = q(p_A^a - c_A)\frac{t_A - p_A^a + p_B^a}{t_B} + (1-q)(p_A^b - c_A)\frac{p_B^b - p_A^b}{t_B}$$
$$\Pi_B = q(p_B^a - c_B)\frac{p_A^a - p_B^a}{t_A} + (1-q)(p_B^b - c_B)\frac{t_B - p_B^b + p_A^b}{t_B}$$

As long as the following inequalities are held:

$$p_A^a - c_A \ge 0$$
$$p_A^b - c_A \ge 0$$
$$p_B^a - c_B \ge 0$$
$$p_B^b - c_B \ge 0$$
$$0 \le p_A^a - p_B^a \le t_A$$
$$0 \le p_B^b - p_A^b \le t_B$$

As $\frac{\partial^2 \Pi_A}{\partial p_A^{a\,2}} < 0, \frac{\partial^2 \Pi_A}{\partial p_A^{b\,2}} < 0, \frac{\partial^2 \Pi_B}{\partial p_B^{a\,2}}$ *and* $\frac{\partial^2 \Pi_B}{\partial p_B^{b\,2}} < 0$, maximum profit can be found by setting first derivative of profits to zero.

$$\frac{\partial \Pi_A}{\partial p_a^a} = q\,\frac{t_A - p_A^a + p_B^a}{t_A} - q\,\frac{p_A^a - c_A}{t_A} = 0$$
$$\frac{\partial \Pi_A}{\partial p_a^b} = (1-q)\frac{p_B^b - p_A^b}{t_B} - (1-q)\frac{p_A^b - c_A}{t_B} = 0$$
$$\frac{\partial \Pi_B}{\partial p_b^a} = q\,\frac{p_A^a - p_B^a}{t_A} - q\,\frac{p_B^a - c_B}{t_A} = 0$$
$$\frac{\partial \Pi_B}{\partial p_b^b} = (1-q)\frac{t_B - p_B^b + p_B^b}{t_B} - (1-q)\frac{p_B^b - c_B}{t_B} = 0$$

Firms' prices can be shown as:

$$p_A^a = \frac{2t_A}{3} + \frac{2c_A}{3} + \frac{c_B}{3} \text{ and } p_A^b = \frac{t_B}{3} + \frac{2c_A}{3} + \frac{c_B}{3}$$
$$p_B^a = \frac{t_A}{3} + \frac{c_A}{3} + \frac{2c_B}{3} \text{ and } p_B^b = \frac{2t_B}{3} + \frac{c_A}{3} + \frac{2c_B}{3}$$

Firms overall profit can be expressed as:

$$\Pi_A = q\,\frac{(\frac{2t_A}{3} - \frac{c_A}{3} + \frac{c_B}{3})^2}{t_A} + (1-q)\frac{(\frac{t_B}{3} - \frac{c_A}{3} + \frac{c_B}{3})^2}{t_B}$$
$$\Pi_B = q\,\frac{(\frac{t_A}{3} + \frac{c_A}{3} - \frac{c_B}{3})^2}{t_A} + (1-q)\frac{(\frac{2t_B}{3} + \frac{c_A}{3} - \frac{c_B}{3})^2}{t_B}$$

The following inequalities must be held for the solution to be feasible

$$p_A^a - c_A \ge 0 \Rightarrow \frac{2t_A}{3} + \frac{2c_A}{3} + \frac{c_B}{3} - c_A \ge 0 \Rightarrow c_A - c_B \le 2t_A$$
$$p_A^b - c_A \ge 0 \Rightarrow \frac{t_B}{3} + \frac{2c_A}{3} + \frac{c_B}{3} - c_A \ge 0 \Rightarrow c_A - c_B \le t_B$$
$$p_B^a - c_B \ge 0 \Rightarrow \frac{t_A}{3} + \frac{c_A}{3} + \frac{2c_B}{3} - c_B \ge 0 \Rightarrow c_A - c_B \ge -t_A$$
$$p_B^b - c_B \ge 0 \Rightarrow \frac{2t_B}{3} + \frac{c_A}{3} + \frac{2c_B}{3} - c_B \ge 0 \Rightarrow c_A - c_B \ge -2t_B$$
$$0 \le p_A^a - p_B^a \le t_A \Rightarrow 0 \le \frac{2t_A}{3} + \frac{2c_A}{3} + \frac{c_B}{3} - (\frac{t_A}{3} + \frac{c_A}{3} + \frac{2c_B}{3}) \le t_A \Rightarrow -t_A \le c_A - c_B \le 2t_A$$
$$0 \le p_B^b - p_A^b \le t_B \Rightarrow 0 \le \frac{2t_B}{3} + \frac{c_A}{3} + \frac{2c_B}{3} - (\frac{t_B}{3} + \frac{2c_A}{3} + \frac{c_B}{3}) \le t_B \Rightarrow -2t_B \le c_A - c_B \le t_B$$

The above solution is feasible as long as $c_a - c_b \le 2t_a$ and $c_a - c_b \le t_b$.

APPENDIX E – Proof of Lemma 2 in section 4

The solution found in appendix D does not apply to this case as the following inequality is not satisfied: $p_A^a - c_A \geq 0$.

As firm A can't charge a price less than its cost, the best firm A can do is to set its price to its cost c_A for a customers. Then, firm B's profit for a customers can be expressed as:

$$\Pi_B^a = q(p_B^a - c_B)\frac{p_A^a - p_B^a}{t_A} \text{ where } p_A^a = c_A$$

As $\frac{\partial^2 \Pi_B^a}{\partial p_B^{a\,2}} < 0$, the price maximizing firm B's profit from a customers can be found by setting $\frac{\partial \Pi_B^a}{\partial p_B^a} = 0$.

$$\frac{\partial \Pi_B^a}{\partial p_B^a} = q\,\frac{p_A^a - p_B^a}{t_A} - q\,\frac{p_B^a - c_B}{t_A} = 0 \Rightarrow p_B^b = \frac{c_A + c_B}{2}$$

Note that $0 \leq p_B^a - p_A^a \leq t_A$ needs to be held so that equilibrium is maintained. However,

$p_A^a - p_B^a = \frac{c_A + c_B}{2} - c_A = \frac{-c_A + c_B}{2} < 0$. Therefore, firm B can't set its price to $\frac{c_A + c_B}{2}$.

The best firm B can do is to set its price to $c_A - t_A$ for a customers.

As inequalities for b customers are satisfied, there is no change in firm A and B's price for b customers. Firm A and B's prices can be shown as:

$$p_A^a = c_A \text{ and } p_A^b = \frac{t_B}{3} + \frac{2c_A}{3} + \frac{c_B}{3}$$

$$p_B^a = c_A - t_A \text{ and } p_b^b = \frac{2t_B}{3} + \frac{c_A}{3} + \frac{2c_B}{3}$$

APPENDIX F – Proof of Lemma 3 in section 4

The solution found in appendix D does not apply to this case as the following inequality is not satisfied: $p_A^b - c_A \geq 0$.

As firm A can't charge a price less than its cost, the best firm A can do is to set its price to its cost c_A for b customers. Then, firm B's profit for b customers can be expressed as:

$$\Pi_B^b = (1-q)(p_B^b - c_B)\frac{t_B - p_B^b + p_A^b}{t_B} \text{ where } p_A^b = c_A$$

As $\frac{\partial^2 \Pi_B^b}{\partial p_B^{b\,2}} < 0$, the price maximizing firm B's profit from b customers can be found by setting $\frac{\partial \Pi_B^b}{\partial p_B^b} = 0$.

$$\frac{\partial \Pi_B^b}{\partial p_B^b} = (1-q)\left(\frac{t_B - p_B^b + c_A}{t_B} - \frac{p_B^b - c_B}{t_B}\right) = 0 \Rightarrow p_B^b = \frac{t_B + c_A + c_B}{2}$$

Note that $0 \leq p_B^b - p_A^b \leq t_B$ needs to be held so that equilibrium is maintained. However, $p_B^b - p_A^b = \frac{t_B + c_A + c_B}{2} - c_A = \frac{t_B - c_A + c_B}{2} < 0$. Therefore, firm B can't set its price to $\frac{t_b + c_A + c_b}{2}$. The best firm B can do is to set its price to $c_A - t_B$ for b customers.

As inequalities for a customers are satisfied, there is no change in firm A and B's price for a customers. Firm A and B's prices can be shown as:

$$p_A^a = \frac{2t_A}{3} + \frac{2c_A}{3} + \frac{c_B}{3} \text{ and } p_A^b = c_A$$

$$p_B^a = \frac{t_A}{3} + \frac{c_A}{3} + \frac{2c_B}{3} \text{ and } p_B^b = c_A - t_B$$

APPENDIX G – Proof of Lemma 3 in section 4

The solution found in appendix D does not apply to this case as the following inequalities are not satisfied: $p_A^a - c_A \geq 0$ and $p_A^b - c_A \geq 0$.

Findings for a customers in appendix B and b customers in appendix C apply here.

Therefore, firms' prices can be expressed as:

$p_A^a = c_A$ and $p_A^b = c_A$
$p_B^a = c_A - t_A$ and $p_B^b = c_A - t_B$

APPENDIX H – Proof of proposition 6

Each firm offers three prices, one for a customers, one for b customers and one for g customers. We will show the equilibrium conditions for perceived a customers. Similar analysis can be done for perceived b customers. Due to symmetrical nature of the problem, we skipped the proof for b customers. As g customers cannot not be addressed by firms, the results obtained in chapter 3 also apply to privacy-conscious customers.

The outline of the proof is as follows:

1. We show that there exist a local optimal solution set (p_A^a, p_B^a) that maximizes both firms' profit.

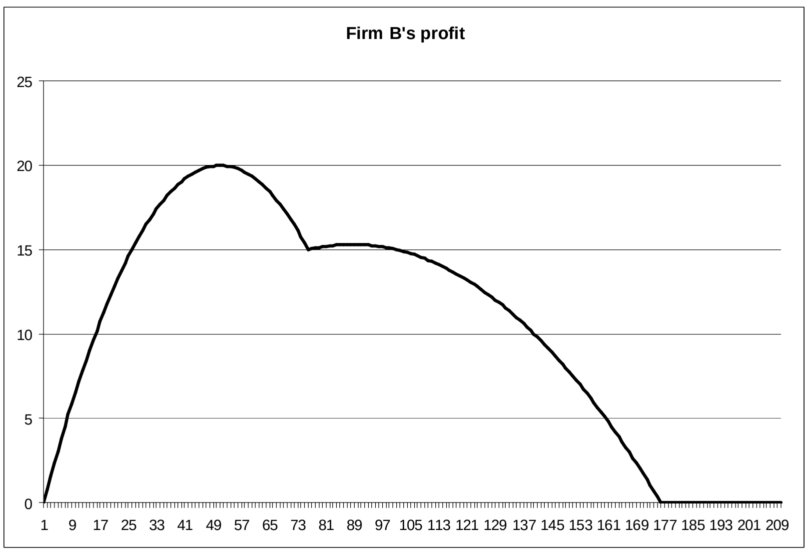
2. We show that the solution found above is in fact global optimal for both firms.

Firms' profit is a function of market share which is dependent on the price difference. Therefore, profit functions for a customers can be stated as:

$$\Pi_a^a = \begin{cases} 0 & \text{if } p_A^a \geq p_B^a + t_A \\ l\,q\,q_a\, p_A^a(1 - \dfrac{p_A^a - p_B^a}{t_A}) & \text{if } p_B^a \leq p_A^a < p_B^a + t_A \\ l\,q(q_a + \left(1 - q_a\right)\dfrac{p_B^a - p_A^a}{t_B})p_a^a & \text{if } p_B^a - t_B \leq p_A^a < p_B^a \\ l\,q\,p_A^a & \text{if } p_A^a < p_B^a - t_B \end{cases}$$

$$\Pi_b^a = \begin{cases} l\,q\,p_B^a & \text{if } p_A^a \geq p_B^a + t_A \\ l\,q\,(q_a \dfrac{p_A^a - p_B^a}{t_A} + \left(1 - q_a\right))\,p_B^a & \text{if } p_B^a \leq p_A^a < p_B^a + t_a \\ l\,q\,(1 - q_a)(1 - \dfrac{p_B^a - p_A^a}{t_B})\,p_B^a & \text{if } p_B^a - t_B \leq p_A^a < p_B^a \\ 0 & \text{if } p_A^a < p_B^a - t_B \end{cases}$$

Firms have two piecewise continuous profit functions. The following graph shows firm's profit function.



X axis is firm's price and Y axis is firm's profit. We will show that concave down two-piecewise continuous functions have important implications later.

Lemma I:1: There exist a local optimal solution set (p_A^a, p_B^a) that maximizes both firms' profit.

Consider the case $p_A^a \geq p_B^a$. Due to symmetrical nature of the problem, we skipped the case $p_A^a < p_B^a$.

Firm A can't penetrate into firm B's loyal customer base as $p_A^a \geq p_B^a$. However, firm B is able to capture some of its rival's customers by undercutting its price. Therefore, firms' profit functions can be expressed as:

$$\Pi_A^a = l\,q\,q_a\,p_A^a(1-\frac{p_A^a - p_B^a}{t_A})$$

$$\Pi_B^a = l\,q(q_a\,\frac{p_A^a - p_B^a}{t_A} + (1-q_a))p_B^a$$

As $\frac{\partial^2 \Pi_A^a}{\partial p_A^{a\,2}} < 0$ and $\frac{\partial^2 \Pi_A^a}{\partial p_A^{a\,2}} < 0$, maximum profit can be found by setting first derivative of profits to zero.

$$\frac{\partial \Pi_A^a}{\partial p_A^a} = l\,q\,q_a(1-\frac{p_A^a - p_B^a}{t_A} - \frac{p_A^a}{t_A}) = 0 \;\Rightarrow t_A = 2p_A^a - p_B^a \quad (1)$$

$$\frac{\partial \Pi_B^a}{\partial p_B^a} = l\,q(q_a\,\frac{p_A^a - p_B^a}{t_A} + (1-q_a)) - l\,q\,q_a\,\frac{p_B^a}{t_A} = 0 \;\Rightarrow -\frac{1-q_a}{q_a}t_A = p_A^a - 2p_B^a \quad (2)$$

From (1) and (2), one can show that

$$p_A^a = \frac{1+q_a}{3q_a}t_A \text{ and } p_B^a = \frac{2-q_a}{3q_a}t_A$$

The above solution is feasible if the following conditions are held:

$i.\; p_A^a \geq p_B^a \quad \Rightarrow \frac{2q_a - 1}{3q_a}t_A \geq 0 \Rightarrow q_a \geq \frac{1}{2}$

$ii.\; p_A^a - p_B^a \leq t_a \quad \Rightarrow \quad \frac{2q_a - 1}{3q_a}t_A \leq t_A$ $\Rightarrow$ always satisfied.

$iii.\; p_A^a \geq 0 \;\Rightarrow \frac{1+q_a}{3q_a}t_A \geq 0 \Rightarrow$ always satisfied.

$iv.\; p_B^a \geq 0 \Rightarrow \frac{2-q_a}{3q_a}t_A \geq 0 \Rightarrow$ always satisfied.

Firms actually have two piecewise continuous profit functions mentioned before. The solution given above is for one of the continuous functions. Maximum profit from the other piecewise-continuous function may be even higher than the solution found above. It must be less profitable for firms to deviate from their pricing strategy so that the game is in equilibrium. Specifically, firm A should not decrease its price and firm B should not raise its price. We analyze each possibility below:

Lemma I.2: Firm B cannot deviate from its pricing strategy if the following condition is held:

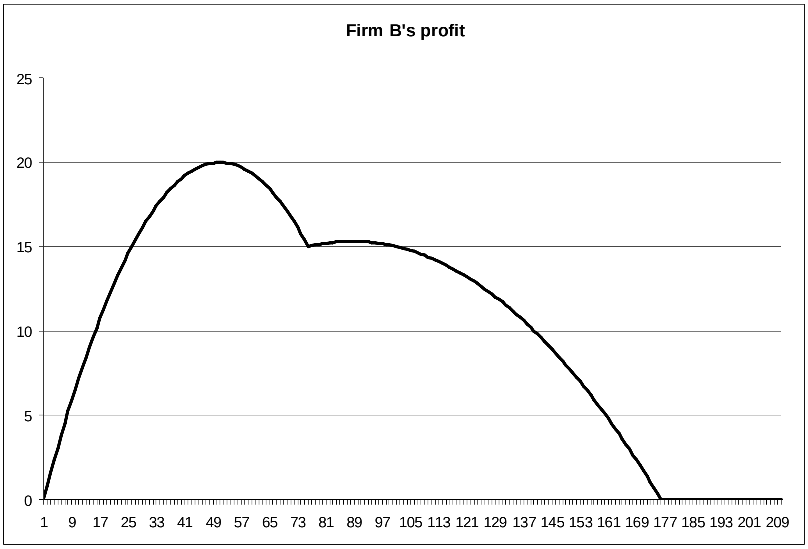
$$\min\left(\frac{3}{2}t_B - \frac{1+q_a}{2q_a}t_A, \frac{3}{2}\sqrt{\frac{1-q_a}{q_a}t_A t_B} - \frac{1+q_a}{2q_a}t_B\right) \leq 0$$

Proof: Let $\overline{p_B^a}$ and $\overline{\prod_B^a}$ denote firm B's price and profit when $\overline{p_B^a} > p_A^a$. Firm B cannot penetrate into firm A's loyal customer base when it offers a higher price. Therefore, firm B's new profit function can be stated as:

$$\overline{\prod_b^a} = l\,q\,(1-q_a)\left(1-\frac{\overline{p_B^a}-p_A^a}{t_B}\right)\overline{p_B^a}$$

As $\dfrac{\partial^2 \overline{\prod_B^a}}{\partial \overline{p_B^a}^2} < 0$, $\overline{\prod_B^a}$ is a concave function.

$\overline{\prod_b^a}$ corresponds to the second concave down function given below.



There are two ways to prove that the maximum profit from the second concave down function is always less than maximum profit from the first function.

i. If $\overline{\prod_B^a}$ is non-increasing, then its maximum will be at point $\overline{p_B^a} = p_A^a$. Local optimal solution in lemma I.1 is always more profitable than the solution at $\overline{p_B^a} = p_A^a$.

ii. If $\overline{\prod_B^a}$ is not non-increasing, then its maximum will be at point where $\frac{\partial \overline{\prod_B^a}}{\partial \overline{p_B^a}} = 0$ as it is a concave down function. We'll find out the maximum profit and compare it with the local optimal solution found in lemma I.1. We'll determine the conditions which guarantee that local optimal solution always yield higher profits.

i. If $\frac{\partial \overline{\prod_B^a}}{\partial \overline{p_B^a}} \leq 0$ when $\overline{p_B^a} = p_B^a$, then $\overline{\prod_b^a}$ is a non-increasing function. Therefore, $\overline{\prod_B^a}$ has its maximum value at point $\overline{p_B^a} = p_A^a$.

$$\frac{\partial \overline{\Pi_B^a}}{\partial \overline{p_B^a}} = l\,q\,(1-q_a)(1-\frac{\overline{p_B^a}-p_A^a}{t_B}-\frac{\overline{p_B^a}}{t_B}) \le 0 \Rightarrow$$

$$As\ \overline{p_B^a} = p_A^a \Rightarrow (1-\frac{\overline{p_B^a}}{t_B}) \le 0 \Rightarrow \overline{p_B^a} = \frac{1+q_a}{3q_a}t_A \ge t_B$$

ii. $\overline{\Pi_B^a}$ is maximum when $\frac{\partial \overline{\Pi_B^a}}{\partial \overline{p_B^a}} = 0$, Therefore,

$$\frac{\partial \overline{\Pi_B^a}}{\partial \overline{p_B^a}} = l\,q\,(1-q_a)(1-\frac{\overline{p_b^a}-p_a^a}{t_B}-\frac{\overline{p_b^a}}{t_B}) = 0 \Rightarrow \overline{p_B^a} = \frac{t_B+p_A^a}{2} = \frac{t_B}{2}+\frac{1+q_a}{6q_a}t_a$$

Firm B's profit function can be expressed as:

$$\overline{\Pi_B^a} = \ l\,q\,(1-q_a)\frac{(\frac{t_B}{2}+\frac{1+q_a}{6q_a}t_A)^2}{t_B}$$

Firm B cannot deviate from its pricing strategy as long as $\Pi_B \ge \overline{\Pi}_B$. Therefore,

following inequality must be held.

$$\Pi_B = l\,q\,q_a\,\frac{(\frac{2-q_a}{3q_a}t_A)^2}{t_A} \ge l\,q\,(1-q_a)\frac{(\frac{t_B}{2}+\frac{1+q_a}{6q_a}t_A)^2}{t_B} = \overline{\Pi_B^a}$$

Rearranging the terms yields:

$$q_a\,\frac{(p_B^a)^2}{t_A}-(1-q_a)\frac{(\overline{p_B^a})^2}{t_B} \ge 0 \Rightarrow$$

$$\left[\sqrt{\frac{q_a}{t_A}}(p_B^a)+\sqrt{\frac{1-q_a}{t_B}}(\overline{p_B^a})\right]\left[\sqrt{\frac{q_a}{t_A}}(p_B^a)-\sqrt{\frac{1-q_a}{t_B}}(\overline{p_B^a})\right]$$

The first term is always positive as $\overline{p_B^a} \ge p_B^a \ge 0$. The second term can also be expressed as:

$$\sqrt{\frac{q_a}{t_A}}(\frac{2-q_a}{3q_a}t_A)-\sqrt{\frac{1-q_a}{t_B}}(\frac{t_B}{2}+\frac{1+q_a}{6q_a}t_A)\geq 0\Rightarrow$$

$$0\geq\sqrt{\frac{1-q_a}{t_B}}\frac{3t_B}{2}+\sqrt{\frac{1-q_a}{t_B}}\frac{1+q_a}{2q_a}t_A-\sqrt{\frac{q_a}{t_A}}\frac{2-q_a}{q_a}t_A\Rightarrow$$

$$0\geq\frac{3}{2}\sqrt{\frac{1-q_a}{q_a}t_At_B}-\frac{1+q_a}{2q_a}t_A$$

If either i or ii is held, then firm B doesn't deviate from its pricing strategy. Then the following condition needs to be held in equilibrium:

$$0\geq\min(\frac{3}{2}t_B-\frac{1+q_a}{2q_a}t_A,\frac{3}{2}\sqrt{\frac{1-q_a}{q_a}t_At_B}-\frac{1+q_a}{2q_a}t_A)$$

Lemma I.3: Firm A cannot deviate from its pricing strategy if

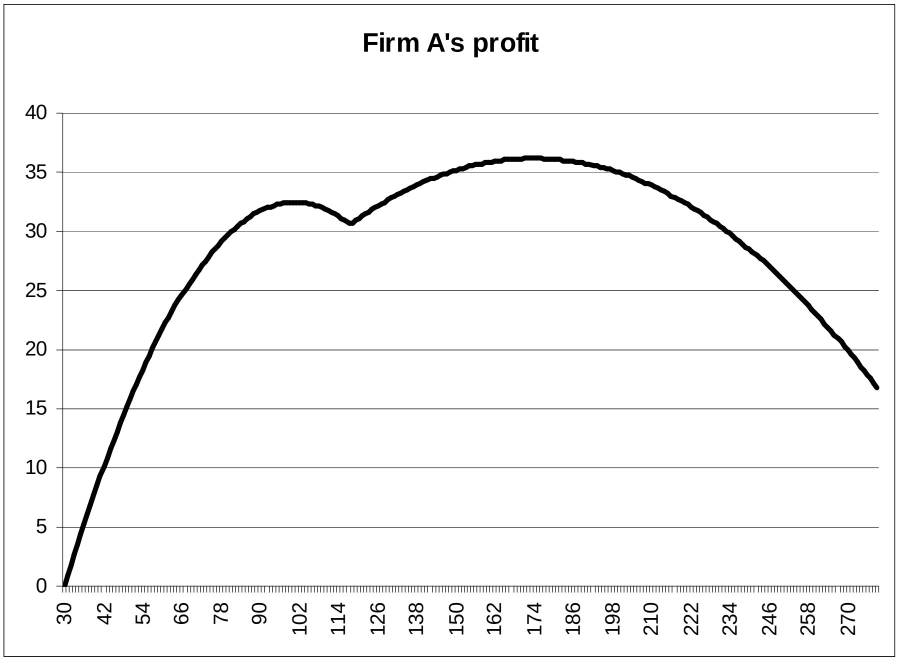
$$0\geq\min(\frac{2-q_a}{2q_a}t_A-\frac{3q_a}{2(1-q_a)}t_B,\frac{2-q_a}{2q_a}t_A-\frac{3}{2}\sqrt{\frac{q_a}{1-q_a}t_At_B})$$

Proof: Let $\underline{p_A^a}$ and $\underline{\prod_A^a}$ denote firm A's price and profit when $\underline{p_A^a}\leq p_B^a$. Firm A can penetrate into firm B's loyal customer base as it offers a lower price. Therefore, firm A's profit function can be stated as:

$$\underline{\prod_A^a}=l\,q(q_a+(1-q_a)\frac{p_B^a-\underline{p_A^a}}{t_B})\underline{p_A^a}$$

As $\frac{\partial^2\underline{\prod_A^a}}{\partial \underline{p_A^a}^2}\leq 0$, $\underline{\prod_a^a}$ is a concave function.

$\underline{\prod_A^a}$ corresponds to the first concave down function given below.



There are two ways to prove that the maximum profit from the first concave down function is always less than maximum profit from the second function.

i. If $\underline{\Pi_A^a}$ is non-decreasing, then its maximum will be at point $\underline{p_A^a} = p_B^a$. Local optimal solution in lemma 1 is always more profitable than the solution at $\underline{p_A^a} = p_B^a$.

ii. If $\underline{\Pi}_A$ is not non-increasing, then its maximum will be at point where $\frac{\partial \underline{\Pi_A^a}}{\partial \underline{p_A^a}} = 0$ as it is a concave down function. We'll find out the maximum profit and compare it with the local optimal solution found in lemma I.1. We'll determine the conditions which guarantee that local optimal solution always yield higher profits.

i. If $\frac{\partial \underline{\underline{\Pi}}_A^a}{\partial p_A^a} \geq 0$ when $\underline{p_A^a} = p_B^a$, then $\underline{\Pi_A^a}$ is a non-decreasing function. Therefore, $\underline{\Pi_A^a}$ has its maximum value at point $\underline{p_A^a} = p_B^a$.

$$\frac{\partial \underline{\underline{\Pi}}_A^a}{\partial p_A^a} = l\,q\,(q_a + (1-q_a)\frac{p_B^a - \underline{\underline{p}}_A^a}{t_B} - (1-q_a)\frac{\underline{\underline{p}}_A^a}{t_B}) \geq 0 \Rightarrow q_a + (1-q_a)\frac{p_B^a - 2\underline{\underline{p}}_A^a}{t_B} \geq 0$$

$$0 \geq \frac{2-q_a}{2q_a} t_A - \frac{3q_a}{2(1-q_a)} t_B$$

ii. If the inequality stated above doesn't hold, $\underline{\Pi_A^a}$ is maximum when $\frac{\partial \underline{\underline{\Pi}}_A^a}{\partial p_A^a} = 0$,

Therefore,

$$\frac{\partial \underline{\underline{\Pi}}_A^a}{\partial \underline{p_A^a}} = l\,q\,(q_a + (1-q_a)\frac{p_B^a - \underline{\underline{p}}_A^a}{t_B} - (1-q_a)\frac{\underline{\underline{p}}_A^a - c_a}{t_B}) \Rightarrow q_a + (1-q_a)\frac{p_B^a - 2\underline{\underline{p}}_A^a}{t_B} = 0 \Rightarrow$$

$$\underline{p_a^a} = \frac{2-q_a}{6q_a} t_A + \frac{q_a}{2(1-q_a)} t_B$$

Firm A's profit function can be expressed as:

$$\underline{\Pi_A^a} = l\,q_a(1-q_a)\frac{(\frac{2-q_a}{6q_a} t_A + \frac{q_a}{2(1-q_a)} t_B)^2}{t_B}$$

Firm A cannot deviate from its pricing strategy as long as $\Pi_A^a \geq \underline{\Pi_A^a}$. Therefore,

following inequality must be held.

$$\Pi_A^a = l\,q\,q_a \frac{(\frac{1+q_a}{3q_a} t_A)^2}{t_A} \geq l\,q(1-q_a)\frac{(\frac{2-q_a}{6q_a} t_A + \frac{q_a}{2(1-q_a)} t_B)^2}{t_B} = \underline{\Pi_A^a}$$

Rearranging the terms yields:

$$q_a \frac{(p_A^a)^2}{t_A} - (1-q_a)\frac{(\underline{\underline{p}}_A^a)^2}{t_B} \geq 0 \Rightarrow$$

$$\left[\sqrt{\frac{q_a}{t_A}}(p_A^a) + \sqrt{\frac{1-q_a}{t_B}}(\underline{p_A^a})\right]\left[\sqrt{\frac{q_a}{t_A}}(p_a^a) - \sqrt{\frac{1-q_a}{t_B}}(\underline{p_A^a})\right]$$

The first term is always positive as $p_A^a \geq 0$ and $\underline{p_A^a} \geq 0$. The second term can also be expressed as:

$$\sqrt{\frac{q_a}{t_A}}(p_A^a) - \sqrt{\frac{1-q_a}{t_B}}(\underline{p_A^a}) \geq 0 \Rightarrow$$
$$\sqrt{\frac{1-q_a}{t_b}}\frac{2-q_a}{2q_a}t_a + \sqrt{\frac{1-q_a}{t_b}}\frac{3q_a}{2(1-q_a)}t_b - \sqrt{\frac{q_a}{t_a}}\frac{1+q_a}{q_a}t_a \leq 0 \Rightarrow$$
$$0 \geq \frac{2-q_a}{2q_a}t_A - \frac{3}{2}\sqrt{\frac{q_a}{1-q_a}t_A t_B}$$

If either i or ii is held, then firm A doesn't lower its price. Then the following condition

needs to be held in equilibrium:

$$0 \geq \min\left(\frac{2-q_a}{2q_a}t_A - \frac{3q_a}{2(1-q_a)}t_B, \frac{2-q_a}{2q_a}t_A - \frac{3}{2}\sqrt{\frac{q_a}{1-q_a}t_A t_B}\right)$$

Lemma I.4: The only condition that needs to be met so that the equilibrium is maintained is the following:

$$\frac{3}{2}\sqrt{\frac{1-q_a}{q_a}t_A t_B} - \frac{1+q_a}{2q_a}t_A \leq 0 \leq \frac{1+q_a}{q_a}t_A, \qquad \text{if } \frac{q_a}{t_A} \geq \frac{1-q_a}{t_B}$$
.
$$\frac{2-q_a}{2q_a}t_A - \frac{3}{2}\sqrt{\frac{q_a}{1-q_a}t_A t_B} \leq 0 \leq \frac{1+q_a}{q_a}t_A \quad \text{,otherwise}$$

Proof: The following three conditions are met in equilibrium:

i. $0 \geq \min\left(L_1 = \frac{3}{2}\sqrt{\frac{1-q_a}{q_a}t_A t_B} - \frac{1+q_a}{2q_a}t_A, L_2 = \frac{3}{2}t_B - \frac{1+q_a}{2q_a}t_A\right)$

ii. $0 \geq \min\left(L_3 = \frac{2-q_a}{2q_a}t_A - \frac{3q_a}{2(1-q_a)}t_B, L_4 = \frac{2-q_a}{2q_a}t_A - \frac{3}{2}\sqrt{\frac{q_a}{1-q_a}t_A t_B}\right)$

iii. $\frac{1-2q_a}{q_a}t_A = L_5 \leq 0 \leq \frac{1+q_a}{q_a}t_A$

The proof hast 2 steps. We prove that

1. If $\frac{q_a}{t_A} \geq \frac{1-q_a}{t_B}$, then $L_1 \leq L_2$ and $L_3 \leq L_4$. Therefore, the following is the necessary condition:

$\max(L_1, L_3, L_5) \leq 0 \leq \frac{1+q_a}{q_a} t_A$.

Then, we prove that $\max(L_1, L_3, L_5) = L_1$

2. If $\frac{q_a}{t_A} \geq \frac{1-q_a}{t_B}$, then $L_1 > L_2$ and $L_3 > L_4$. Therefore, the following is the necessary condition: $\max(L_2, L_4, L_5) \leq 0 \leq \frac{1+q_a}{q_a} t_A$.

Then, we prove that $\max(L_2, L_4, L_5) = L_4$

Step 1: If $\frac{q_a}{t_A} \geq \frac{1-q_a}{t_B}$, then $L_1 \leq L_2$ and $L_3 \leq L_4$.

We first show $L_1 \leq L_2$. Then, we show $L_3 \leq L_4$. Finally, we prove that $\max(L_1, L_3, L_5) = L_1$

$L_2 - L_1 = \frac{3}{2} t_B - \frac{1+q_a}{2q_a} t_A - (\frac{3}{2}\sqrt{\frac{1-q_a}{q_a} t_A t_B} - \frac{1+q_a}{2q_a} t_A)$. Then,

$$L_2 - L_1 = \frac{3}{2}(t_B - \sqrt{\frac{1-q_a}{q_a} t_A t_B}) \geq 0 \Rightarrow L_2 \geq L_1$$

$L_4 - L_3 = \frac{2-q_a}{2q_a} t_A - \frac{3}{2}\sqrt{\frac{q_a}{1-q_a} t_A t_B} - (\frac{2-q_a}{2q_a} t_A - \frac{3q_a}{2(1-q_a)} t_B)$. Then,

$$L_4 - L_3 = \frac{3}{2}(\frac{q_a}{(1-q_a)} t_B - \sqrt{\frac{q_a}{1-q_a} t_A t_B}) \geq 0 \Rightarrow L_4 \geq L_3$$

So far, we proved that If $\frac{q_a}{t_A} \geq \frac{1-q_a}{t_B}$, then $L_1 \leq L_2$ and $L_3 \leq L_4$. The next step is to prove that $\max(L_1, L_3, L_5) = L_1$

We first prove that $L_3 \leq L_5$. Then, we will show that $L_5 \leq L_1$.

$$L_3 - L_5 = \frac{2-q_a}{2q_a}t_A - \frac{3q_a}{2(1-q_a)}t_B - \frac{1-2q_a}{q_a}t_A = \frac{3}{2}q_a(\frac{t_A}{q_a} - \frac{t_B}{(1-q_a)}) \le 0$$

$$L_5 - L_1 = \frac{1-2q_a}{q_a}t_A - (\frac{3}{2}\sqrt{\frac{1-q_a}{q_a}t_At_B} - \frac{1+q_a}{2q_a}t_A) = \frac{3}{2}(\frac{1-q_a}{q_a}t_A - \sqrt{\frac{1-q_a}{q_a}t_At_B}) \le 0 \Rightarrow L_5 \le L_1$$

Therefore, $\max(L_1, L_3, L_5) = L_1$ if $\frac{q_a}{t_A} \ge \frac{1-q_a}{t_B}$

Step 2: If $\frac{q_a}{t_A} < \frac{1-q_a}{t_B}$, then $L_1 > L_2$ and $L_3 > L_4$.

We first show $L_1 > L_2$. Then, we show $L_3 > L_4$.

$$L_2 - L_1 = \frac{3}{2}(t_B - \sqrt{\frac{1-q_a}{q_a}t_At_B}) < 0 \Rightarrow L_2 < L_1 \ .$$

$$L_4 - L_3 = \frac{3}{2}(\frac{q_a}{(1-q_a)}t_B - \sqrt{\frac{q_a}{1-q_a}t_At_B}) < 0 \Rightarrow L_4 < L_3$$

So far, we proved that If $\frac{q_a}{t_A} \ge \frac{1-q_a}{t_B}$, then $L_1 > L_2$ and $L_3 > L_4$. The next step is to prove

that $\max(L_2, L_4, L_5) = L_4$

We first prove that $L_2 < L_5$. Then, we will show that $L_5 < L_4$.

$$L_2 - L_5 = \frac{3}{2}t_B - \frac{1+q_a}{2q_a}t_A - (\frac{1-2q_a}{q_a}t_A) = \frac{3}{2}(1-q_a)(\frac{t_B}{1-q_a} - \frac{t_A}{q_a}) < 0 \Rightarrow L_2 < L_5$$

$$L_5 - L_4 = \frac{1-2q_a}{q_a}t_A - \frac{2-q_a}{2q_a}t_A - \frac{3}{2}\sqrt{\frac{q_a}{1-q_a}t_At_B} = -\frac{3}{2}t_A - \frac{3}{2}\sqrt{\frac{q_a}{1-q_a}t_At_B} < 0 \Rightarrow L_5 < L_4$$

Therefore, $\max(L_2, L_4, L_5) = L_4$ if $\frac{q_a}{t_A} < \frac{1-q_a}{t_B}$

APPENDIX I – Authorization to include previously published paper

Part of this dissertation is already published by Electronic Commerce Research and Applications. This is an Elsevier publication. The screenshot below proves that I have the right to include it in my dissertation. The authorization can be accessed online at http://www.elsevier.com/journal-authors/author-rights-and-responsibilities

Author Use	Author Posting	Definitions

How authors can use their own journal articles

Authors can use their articles for a wide range of scholarly, non-commercial purposes as outlined below. These rights apply for all Elsevier authors who publish their article as either a subscription article or an open access article.

We require that all Elsevier authors always include a full acknowledgement and, if appropriate, a link to the final published version hosted on Science Direct.

For open access articles these rights are separate from how readers can reuse your article as defined by the author's choice of Creative Commons user license options.

Authors can use either their accepted author manuscript or final published article for:	
✓	Use at a conference, meeting or for teaching purposes
✓	Internal training by their company
✓	Sharing individual articles with colleagues for their research use* (also known as 'scholarly sharing')
✓	Use in a subsequent compilation of the author's works
✓	Inclusion in a thesis or dissertation
✓	Reuse of portions or extracts from the article in other works
✓	Preparation of derivative works (other than for commercial purposes)

Figure 11. Rights to publish paper in the dissertation

CITED LITERATURE

Acquisti A., Varian H. R.: Conditioning prices on purchase history. Marketing Science 24: 367-381, 2005.

Armstrong, M. and Vickers, J.:Competitive Price Discrimination. RAND Journal of Economics. 32:579-605, 2001

Awad, N. F. and Krishnan M. S. : The Personalization Privacy Paradox: An Empirical Evaluation of Information Transparency and the Willingness to be Profiled Online for Personalization. MIS Quarterly 30:13-28, 2006

Ba S., Stallaert J., Zhang Z.:Price competition in e-tailing under service and recognition differentiation. Electronic Commerce Research and Applications 6: 322-331, 2007.

Barnett M., The Profilers: Invisible Friends. The Industry Standard: Mar. 13, 2000

Battaglini, M.: Long Run Contracting With Markovian Consumers. American Economic Review 95:637-658, 2005.

Blumenstein, R. Package Plan: AT&T See Wireless as the Key to Its Broader Strategy of Bundling its Services. The Wall Street Journal, Sept. 20 1999.

Borenstein S. Price discrimination in free-entry markets. RAND Journal of Economics 16 :380-397, 1985.

Aberdeen Group, 2006, Business Intelligence in Retail: Bringing Cohesion to a Fragmented Enterprise A Benchmark Report. Available online at: https://www2.microstrategy.com/download/files/whitepapers/open/Aberdeen-BI-in-Retail-Benchmark.pdf

Carlton D.W., Perloff J. M. Modern Industrial Organization. Harper Collins College Publishers, 1994.

Chellappa R.K., Sin, R.G.: Personalization versus Privacy: An Empirical Examination of the Online Consumer's Dilemma. Information Technology and Management 6:181-202, 2005

Chen Y., Narasimhan C, Zhang, Z. J.:Individual Marketing with Imperfect Targetability. Marketing Science 20:23-41, 2001.

Chen Y.:Paying Customers to Switch. Journal of Economics and Management Strategy 6: 877-897, 1997.

Chen, Y., and Iyer, G.: Consumer addressability and customized pricing. Marketing Science 21:197-208, 2002

Chen, Y., and Zhang, Z.:Dynamic Targeted Pricing with Strategic Consumers. International Journal of Industrial Organization 27:43-50, 2009
Choudhary V., Ghose A., Mukhopadhyay T., Rajan U.: Personalized Pricing and Quality Differentiation. Management Science 51:1120-1130, 2005

Clarke, R.:Profiling: A hidden challenge to the regulation of data surveillance. Journal of Law and Information Science. 4;1993

Crawford P.: Transactions Go into a Database; Businesses Compile Dossiers on Customers, Richmond Times Dispatch, July 28, 1996.

Culnan, M. J., & Armstrong, P. K. : Information privacy concerns, procedural fairness and impersonal trust: An empirical investigation. Organization Science, 10: 104-115, 1999

Demsetz H., Joint Supply and Price Discrimination, Journal of Law and Economics 16: 389-405, 1973.

Dewan R.,Jing B., Seidmann A.: Product Customization and Price Competition on the Internet. Management Science 49:1055-1070, 2003

Dickson, P.R., Ginter J.L.: Market Segmentation, Product Differentiation, and Marketing Strategy. Journal of Marketing 51:1-10, 1987.

Dinev T. , Hart P. :An Extended Privacy Calculus Model for E-Commerce Transactions. Information Systems Research, 17: 61-80, 2006.

Esteves R.: Price Discrimination with Partial Information: Does it pay-off? Economics Letters 105:28-31, 2009.

Esteves R. Pricing with Customer Recognition. International Journal of Industrial Organization 28:669-681, 2010.

Fox, S., Rainie L., Horrigan J., Lenhart A., Spooner T. and Carter C., Trust and Privacy Online: Why Americans want to rewrite the rules. Pew Internet & American Life Project, Washington DC. Available online at http://www.pewinternet.org/2000/08/20/trust-and-privacy-online/ [Accessed on June 18th, 2008]

Frank R.E., Massy W. F., Wind Y. :Market Segmentation, Englewood Cliffs, NJ: Prentice Hall, 1972

Fudenberg D, Tirole J. : Poaching and Brand Switching. The RAND Journal of Economics 31: 634-657, 2000

Gartner Group. IT Definitions and Glossary, Available online at http://www.gartner.com/technology/research/it-glossary/ 2004

Ghose, A. and Huang K.W. :Personalized pricing and quality customization. Journal of Economics and Management Strategy 18:1095-1135, 2009

Grochowski E. and Halern R.D. :Technological Impact of Magnetic Hard Disk Drives on Storage Systems. IBM Systems Journal 42:338-346, 2003.

Hansen E.: DoubleClick postpones data-merging plan. March 2, 2000, Available online at http://news.cnet.com/2100-1023-237532.html

Hill K., 2012, How Target Figured Out A Teen Girl Was Pregnant before Her Father Did, Available online at http://www.forbes.com/sites/kashmirhill/2012/02/16/how-target-figured-out-a-teen-girl-was-pregnant-before-her-father-did/

Hoffman, T.:Banks turn to it to reclaim most profitable customers. ComputerWorld, Dec 7 1998.

Holmes T.: The effects of third degree price discrimination in oligopy. American Economic Review 79: 244 - 250, 1989.

Hotelling, H.:Stability in Competition. Economic Journal 39:41-57, 1929

Hui, K.L., Teo, H., Lee, S.T.: The Value of Privacy Assurance: An Exploratory Field Experiment. MIS Quarterly 31:19-33, 2007.

Johnson J.P.; Myatt D.P. : Multiproduct Quality Competition: Fighting Brands and Product Line Pruning. The American Economic Review 93: 748-774, 2003

Kasanoff B.: Making It Personal: How To Profit From Personalization Without Invading Privacy, Basic Books, 2001.

Kennan, J.: Repeated Bargaining with Persistent Private Information. Review of Economic Studies 68:719-755, 2001.

Kotler, P.: Marketing Management: Analysis, Planning, Implementation and Control, 9th ed. Upper Saddle River, NJ: Prentice-Hall. 1997.

Krugman P.:What price fairness?. New York Times, Oct 4, 2000.

Krugman, P R., Obstfeld, M.:International Economics: Theory and Policy (6th ed.). Boston: Addison-Wesley, 2003.

Kumar V. , Reinartz W.: Customer Relationship Management: A Databased Approach, Wiley; 1 edition, 2005.

Lancaster, K.:Variety, equity, and efficiency. Columbia University Press, New York 1979.
Lapowsky I.:The Power of Loyalty-Card Apps. Available online at http://www.inc.com/magazine/20101101/the-power-of-loyalty-cardapps.html, 2010

Lee, D.J., Ahn, J.H., Bang, Y.: Managing consumer privacy concerns in personalization: a strategic analysis of privacy protection. MIS Quarterly 35:423–444, 2011.

Litman J, : Symposium: Cyberspace and Privacy: A New Legal Paradigm?. Stanford Law Review 52: 1283-1313, 2000.

Liu Q, Serfes, K.: Imperfect price discrimination, market structure, and efficiency. Canadian Journal of Economics 38: 1191–1203, 2005

Liu Q, Serfes, K.: Imperfect price discrimination in a vertical differentiation model. International Journal of Industrial Organization 23:341-354, 2005.

Liu Q, Serfes, K.: Quality of Information and Oligopolistic Price Discrimination. Journal of Economics & Management Strategy 13: 671-702, 2004.

Lohr S. : A $1 Million Research Bargain for Netflix, and Maybe a Model for Others, Available online at http://www.nytimes.com/2009/09/22/technology/internet/22netflix.html, 2009

Malhotra N. K., Kim S. S., Agarwal J. : Internet users' information privacy concerns (IUIPC): The construct, the scale, and a causal model. Information Systems Research 15:336–355, 2004.

Mohammed R.: The Art of Pricing: How to Find the Hidden Profits to Grow Your Business, Crown Business 2005.

Narasimhan C.: Competitive Promotional Strategies. Journal of Business 61: 427-449, 1988.

Pettey C., Goasduff L.: Gartner EXP Worldwide Survey of More than 1,500 CIOs Shows IT Spending to Be Flat in 2009, Available online at http://www.gartner.com/newsroom/id/855612 , 2009

Phlips L.:Economics of Price Discrimination.Cambridge University Press, 1983.

Pigou, A. C.:The Economics of Welfare. 4th edition. London: Macmillan 1920.

Post R.C.:Three Concepts of Privacy. Georgetown Law Journal 89,2001.

Prior K. O.: Merging Shopping with Social Sharing, available online at http://www.entrepreneur.com/article/217761, 2010

Ricadela A. Principal Financial Group Targets 15 Million Customers. InformationWeek 1105, 2006.

Shaffer G., Zhang Z J.: Competitive Coupon Targeting. Marketing Science 14:395-416,1995.

Shaffer G., Zhang Z J.: Competitive One-to-One Promotions. Management Science 48:1143-1160, 2002.

Shaffer G., Zhang Z J.: Pay to switch or pay to stay: Preference-based price discrimination in markets with switching costs. Journal of Economics & Management Strategy 9:397-424, 2000.

Shapiro A.L., Privacy for Sale: Peddling Data on the Internet. Available online at http://connection.ebscohost.com/c/articles/1552331/privacy-sale-peddling-data-internet., 1999

Shapiro C., Varian H. R., Information rules: a strategic guide to the network economy. Harvard Business Review, 1998

Smith, W.R.:Product differentiation and market segmentation as alternative marketing strategies. The Journal of Marketing 21:3-8, 1956

Steiner, P. O.: Peak loads and efficient pricing, Quarterly Journal of Economics 71: 585-610, 1957

Stewart, K.A., and Segars, A.H..: An Empirical Examination of the Concern for Information Privacy Instrument. Information Systems Research 13:36-49, 2002

Stigler, G.:Theory of Price. Macmillan, New York,1987.

Stigler, G.J.:An introduction to Privacy in Economics and Politics. Journal of Legal Studies 9: 623-644, 1980.

Stole, L.A.: Price Discrimination and Competition. M. Armstrong and R. Porter (ed.). Handbook of Industrial Organization 3;2221-2299, 2007

Streitfeld D.: On the Web, Price Tags Blur. Washington Post. Sep 27, 2000.

Syam, N., & Kumar, N.: On customized goods, standard goods and competition. Marketing Science 25: 525–537, 2006

Taylor, C.:Price Discrimination in Markets with Repeat Purchases. RAND Journal of Economics 34:223-246, 2003

Thisse J. F., Vives X.: On The Strategic Choice of Spatial Price Policy. The American Economic Review 78: 122-137, 1988.

Villas-Boas, J.M.: Price Cycles in Markets with Customer Recognition. RAND Journal of Economics 35: 486-501, 2004

Wedel, M., Kamakura, W.A.: Market Segmentation: Conceptual and Methodological Foundations. 2nd edn. Kluwer Academic Publishers, Boston, MA, 2000

Westin, A. F.:Privacy and freedom.New York, NY: Atheneum,1967.

VITA

Ferdi Eruysal

Education

Degree	*Discipline*	*Institution*	*Year*
Ph.D.	Management Information Systems	University of Illinois at Chicago, IL,USA	2014
M.S.	Information Technology	Bahcesehir University, Istanbul, Turkey	2003
B.S.	Industrial Engineering	Middle East Technical University,Ankara, Turkey	2000

Work Experience

Rank/Position	*Department*	*Organization*	*Period*
Research Assistant	Cooperative Information Systems and Organizational Research and Services Laboratory	University of Illinois at Chicago	Sep'11 – May' 14
IT Assistant	Center for Pharmaceutical Biotechnology	University of Illinois at Chicago	Aug '08 – Aug '11
Teaching Assistant	Information and Decision Science	University of Illinois at Chicago	Aug '03 – Aug '07
Instructor	Industrial Engineering	Bahcesehir University	Jul '01 – June '03
Business and System Analyst	IT Consulting	Meta Group	Jun '00 – June '01

Teaching Experience ("T" = TEACHING ASSISTANT , "I" = INSTRUCTOR)

University of Illinois at Chicago

Business Model Simulation (T)
Introduction to Management Information Systems (T)
Operations Management (T)

Bahcesehir University

Database Management (I)
Introduction to Management Information Systems (I)

Information Systems Analysis and Design (I)
Introduction to Programming (T)

Publications

Journal Publications

Ouksel A.M., Eruysal F., Loyalty intelligence and price discrimination in a duopoly, Electronic Commerce Research and Applications (2011).

Conference Proceedings

Eruysal, F., Ranganathan, C., and Ye, C. “Association between IT-based Supply Chain Management Initiatives and Operating Performance,” Proceedings of 15th International Purchasing and Supply Education and Research Association Annual Meeting (IPSERA), April 2006, San Diego, California.

Workshops

Eruysal, F., Ouksel A.M, “Marketing intelligence, product cost and price discrimination in a duopoly,” Workshop on e-Business, December 2012, Orlando, Florida.

Service Activities

Reviewer

3rd International Conference on Service Oriented Computing, 2005

International Conference on Next Generation Web Services Practices, 2005

Hawaii International Conference on Systems Sciences (HICSS), 2006

International Conference on Information Systems (ICIS), 2006

Hawaii International Conference on Systems Sciences (HICSS), 2007

IEEE 23rd International Conference on Data Engineering, 2007

The IEEE/IPSJ Symposium on Applications and the Internet, 2007