Understanding Jumps in the High-Frequency VIX

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THESIS

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LIST OF ABBREVIATIONS

BPV	The bipower variation
CBOE	The Chicago Board Options Exchange
CME	The Chicago Mercantile Exchange
CT	Central Time (Chicago)
FOMC	The Federal Open Market Committee
RV	Realized variance
S&P 500	Standard and Poor's 500 composite stock index
SVCJ	The stochastic volatility model with contemporaneous jumps in price and volatility
SVJ	The stochastic volatility model with jumps in price
SVICJ	The stochastic volatility model with independent and contemporaneous jumps in price and volatility
VIX	The volatility index disseminated by the CBOE

SUMMARY

This thesis provides a comprehensive nonparametric study of volatility jumps and the leverage effect by examining high-frequency data on the VIX and S&P 500 from 1992 to 2010. It is found that the VIX data prior to 1998 are too noisy to provide a reliable inference. After 1999, the dataset is cleaner but still controversial. More specifically, the high-frequency dynamics of the VIX jumps challenges the assumptions of commonly used stochastic volatility jump-diffusion models. I explain this phenomenon by hypothesizing that most jump-like movements in the VIX are "*pseudo-jumps*," i.e., these jumps are large but temporary deviations from fundamental values.

CHAPTER 1

INTRODUCTION

The volatility index (VIX) has attracted a lot of attention from both industry and academia due to a number of outstanding features. The VIX incorporates the market's expectation of stock market volatility over the next 30 calendar days. The Chicago Board Options Exchange (CBOE) disseminates the VIX every 15 seconds in real time. Since its introduction in 1993, the VIX has been one of the most commonly used estimates of the latent volatility process. The most remarkable and widely recognized fact is that the changes in the VIX are strongly negatively correlated with the corresponding changes in the underlying S&P 500 index. This strong negative correlation is even more pronounced in down markets, which makes derivatives on the VIX highly attractive instruments for portfolio diversification and risk management (see (1), (2) among others). The VIX has been so successful that its methodology has become an industry standard for many financial and commodity markets worldwide. In academia, the VIX has contributed to volatility forecasting, variance pricing, and other volatility related studies. Because of this popularity, it is important that researchers and practitioners gain a precise understanding of the VIX dynamics.

This study is also motivated by the necessity of nonparametric information about volatility jumps. The correct interpretation of jumps has important implications for volatility modeling, developing hedging strategies, and specification of risk premia. References (3), (4), and (5) are among the first who have substantiated the inclusion of volatility jumps in option pricing models. Further empirical justification has been obtained through the estimation of structural models in (6), (7), (8), (9), and (10) among others. However, such evidence is inconclusive since the results largely depend on assumptions about other parameters of the model.

I analyze the data within the theoretical framework of a classical jump-diffusion stochastic volatility model while remaining completely nonparametric about jumps and leverage effect dynamics. I consider the model in a very general form: independent and dependent jumps in price and volatility along with the leverage effect[1] specified through diffusive and jump components. I detect jumps using a nonparametric test similar to the test of Lee and Mykland (12). Thus, I define jumps as unusually large (relative to the current volatility level) changes in the level of a financial instrument over a short time interval. The data used comprise high-frequency records of S&P 500 futures and spot VIX from January 1992 to June 2010. Also I employ daily data on the VIX: open/close/min/max.

I find that the first part of the data (1992 - 1998) is too noisy to be used for inference because of the monotonic time trend in several features, such as the leverage effect, annual occurrence of jumps, and microstructure noise. This time trend is independent of market conditions but is consistent with the overall improvement in the quality of the option data.

The most important part of my contribution comes from the second, stationary part of the dataset (1999 - 2010), where I observe that the high-frequency dynamics of VIX jumps

[1]Negative correlation between return and volatility innovations has been well documented in the literature and is often referred to as the leverage effect (e.g., (11)). In this study, I use both terms interchangeably.

challenges standard stochastic volatility models. First, such jumps rarely correspond to any economic event. Second, the time series of these jumps has a strong negative autocorrelation, which seems to be a characteristic of noise. Moreover, the smaller the time period between such jumps the stronger is the negative autocorrelation. Third, these jumps are not followed by the change in the spot volatility. Interestingly, the more counterintuitive are the properties of these jumps the less strong is the leverage effect transferred by these types of movements. Noteworthy, such controversial features belong only to the independent volatility jumps, while the dependent volatility jumps seem to agree with the model.

These controversial features of extreme movements in the VIX coupled with the overall lower quality of the dataset lead me to the hypothesis of "*pseudo-jumps*" in the VIX. These are jumps, which take place in the observable VIX but do not represent real movements in its fundamental value. In other words, pseudo-jumps are not a feature of the true volatility or the true VIX but rather a peculiarity of the VIX estimate, which is a time series of numbers computed by the CBOE.

One example of many days with such suspicious movements is shown on Figure 1. From the beginning of the day until roughly 10:30 a.m., the VIX undergoes five abrupt changes in the level as large as 8.3%. Two of the changes reverse almost immediately while the remaining three movements will be identified as jumps by almost any jump estimation procedure. However, all these movements are more likely to be pseudo-jumps because they bring the level of the VIX outside of the daily minimum and maximum also reported by the CBOE.

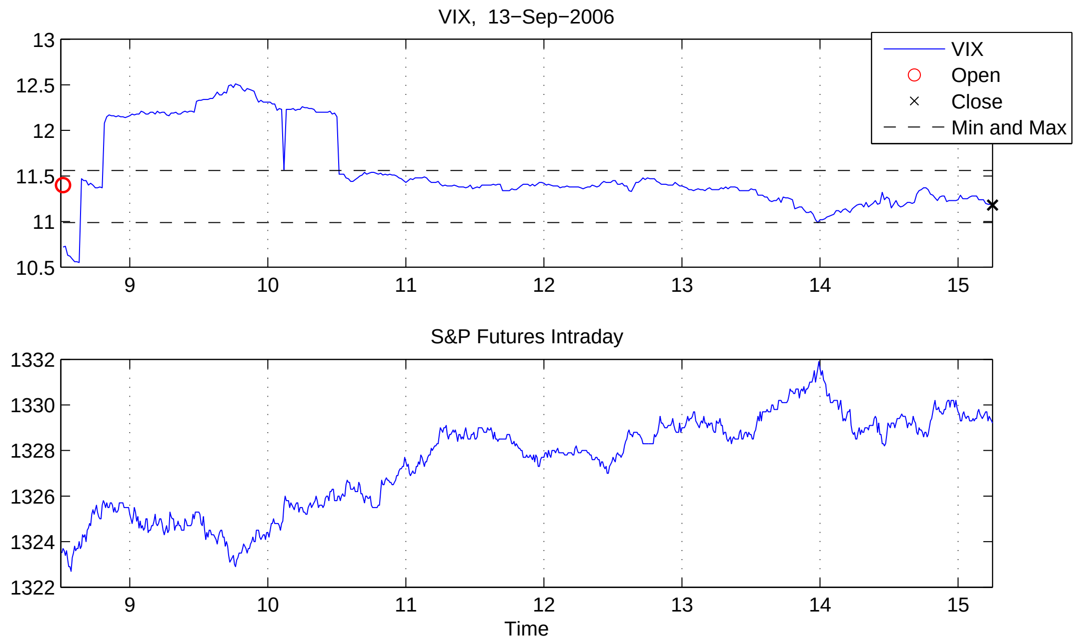

Figure 1. An example of potential pseudo-jumps in the VIX.

In the upper panel, the solid line shows the intraday level of the VIX from the high-frequency dataset, while the dashed lines mark daily max/min levels from the daily dataset. The "o" and "x" correspond to the opening and closing values. The opening price and intraday variations from the high-frequency dataset are inconsistent with the daily open/close/max/min price series also provided by the CBOE.

My results confirm the existing critique of the VIX. Reference (13) show that the CBOE computation procedure might sometimes provide an incorrect estimate of the VIX due to approximation errors. Reference (14) recompute the VIX from options quotes and discover "spurious breaks or artificial jumps" in its level due to regimes switching in the computation method of the CBOE. My contribution is to infer the presence of errors in the VIX from historical data only without revising the computation method, which is extremely time-consuming and data-extensive. Thus, my approach allows examination of all available data on the VIX, while reference (14) considers only two years.

This study also contributes to the literature on volatility behavior by providing evidence on volatility jumps and their relationship with jumps in the underlying price process. It is found that the distribution of jumps in volatility appears to be symmetric, i.e. negative jumps in volatility are as common as positive jumps. Furthermore, the volatility of volatility changes significantly after the arrival of a jump. In addition, the leverage effect is channeled by both extreme and diffusive types of movements. Finally, I find the evidence that only simultaneous jumps in price and volatility are possible. In the literature, these jumps are also referred as co-jumps.

The rest of the dissertation is organized as follows. Chapter 2 provides general information about the VIX. Chapter 3 describes data. Chapter 4 undertakes comparative study of the S&P and VIX. Chapter 5 introduces methodology and hypotheses to test. Chapter 6 reports the estimation results. Chapter 7 discusses these results with respect to the hypothesis of pseudo-jumps and provides a robustness check. Chapter 8 concludes.

CHAPTER 2

LITERATURE REVIEW

2.1 The VIX: Background and critique

The computation of the VIX is based on the concept of model-free implied volatility.[1] The theoretical formula contains integrals over continuous strike values from zero to infinity (18):

$$\sigma_T^2 = \frac{2e^{r_f T}}{T}\left[\int_0^{F_0} \frac{P(T,K)}{K^2}\,\mathrm{d}K + \int_{F_0}^{\infty} \frac{C(T,K)}{K^2}\,\mathrm{d}K\right] = \frac{2e^{r_f T}}{T}\left[\int_0^{\infty} \frac{Q(T,K)}{K^2}\,\mathrm{d}K\right] \tag{2.1}$$

$$Q(T,K) = \min\{C(T,K), P(T,K)\} \tag{2.2}$$

where r_f is the risk-free interest rate, T is time-to-maturity, F_0 denotes the forward price at maturity T, $P(T,K)$ and $C(T,K)$ are the mid-quotes for European put and call options with strike K and time to maturity T, and $Q(T,K)$ denotes the out-of-the money option (call or put) at strike K. In practice, the CBOE replaces the continuous integration with the following finite sum:

$$\hat{\sigma}_T^2 = VIX^2 = \underbrace{\frac{2e^{r_f T}}{T}\sum_{i=I_l}^{I_u} \frac{\Delta K_i}{K_i^2} Q(K_i)}_{\text{DiscreteApproximation}} - \underbrace{\frac{1}{T}\left[\frac{F_0}{K_0} - 1\right]^2}_{\text{CorrectionTerm}} \tag{2.3}$$

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[1]The fact that “the fair value of total variance” is given by the value of infinite strip of European options in a model-free way has been originally noted by (15) and then by (16). For a more complete literature list please see (17) and (13).

where K_{I_l} and K_{I_u} denote the minimum and maximum strikes included in the computation, K_0 is the first strike price available below the forward price F_0, and the second term in (Equation 2.3) reflects a correction for the discrepancy between K_0 and this forward price.

The CBOE introduced the above method on September 22, 2003. This has been a break-through improvement over the old VIX (now cited as VXO), which was based on the Black- Scholes-Merton model[1] and mimicked the implied volatility of at-the-money one-month option on the OEX index. Instead, the new VIX incorporates information from the entire volatility skew with no reliance on any model as long as the underlying process is a diffusion. In addition, the new VIX is based on the S&P 500 index (SPX) which has become much more popular than the OEX index. The purpose of the CBOE was to "provide a more precise and robust measure of expected market volatility and to create a viable underlying index for tradable volatility products" (18).

In spite of apparent advantages of the new VIX, the CBOE approximation formula has been criticized in the literature. Reference (13) is the first to point out a number of implementation flaws in the CBOE computation procedure arising from

> (i) truncation errors, due to minimum and maximum strike prices being far removed from zero and infinity in (Equation 2.3); (ii) discretization errors, due to the lack of numerical integration in (Equation 2.3); (iii) expansion errors, due to the Taylor series approximation of the log function used by the CBOE in defining

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[1]See (19) and (20).

the correction term in (Equation 2.3); and (iv) interpolation errors, caused by the interpolation of maturities.

According to (13), these errors are economically significant and can be as large as 198 index basis points.

Further, reference (14) finds that the truncation errors are highly important because they often cause a substantial bias in the VIX, and, worse, abrupt shifts in it. Specifically, the CBOE uses a certain rule to truncate these "tails" to obtain the effective range of options $(K_{I_l} - K_{I_u})$ to be used in (Equation 2.3). These authors reveal occasional and abrupt changes in the effective range provided the CBOE's rule is followed and claim that the "regime switching in a computation procedure may induce spurious breaks or artificial jumps" in the level of the VIX index. They offer an improvement to the existing routine, a novel Corridor Implied Volatility index (CX), based on an economically invariant strike range (21). They recommend using the CX instead of the VIX to analyze frequency of jumps in the implied volatility and the high-frequency leverage effect.

I complement the above critique by inferring the existence of errors in the VIX from statistical properties of the data, without revising the computation method. Recomputing the VIX is a complicated time-consuming and data-extensive procedure, which requires high-frequency data on options of all quoted strikes. My advantage is that I can analyze a much longer time period: all available data since the introduction of the VIX as opposed to two years in (14).

Similar to (14), I also find large unexplainable moves in the data, some of which are shown on Figure 1 and Figure 19. However, the proof of the exact correspondence is beyond the scope of my study.

CHAPTER 3

DATA

3.1 Data description and preliminary modification

The VIX index is obtained from the CBOE (January 1992 - June 2010) at a one minute frequency. The transaction data on the Standard and Poor's 500 (S&P 500) composite stock index futures contracts (January 1992 - June 2010) are obtained from the Chicago Mercantile Exchange (CME).

I consider only the days when both indices have been reported. Thus, the total period consists of 4629 trading days. The time series of daily closing values for both indices are presented on Figure 2. I ignore the price changes between trading days (overnight and over-weekend returns) and focus on the transactions that occur during the normal trading hours. Descriptive statistics are presented in Table XII and Table XIII, Appendix B.

There are two issues with the VIX data that should not be ignored. These problems differ for two parts of the dataset, which also are not similar by construction. Prior to September 22, 2003, the VIX has been back-filled from option historical records. Since then, the VIX is computed and recorded in real time. The first problem is that the recomputed VIX is always constant before 9 a.m. and after 3 p.m. These pieces of constant values are incompatible with other data and often cause jump-like returns around 9 a.m. and 3 p.m. The second problem is that the intraday data obtained in real time are often inconsistent with the daily

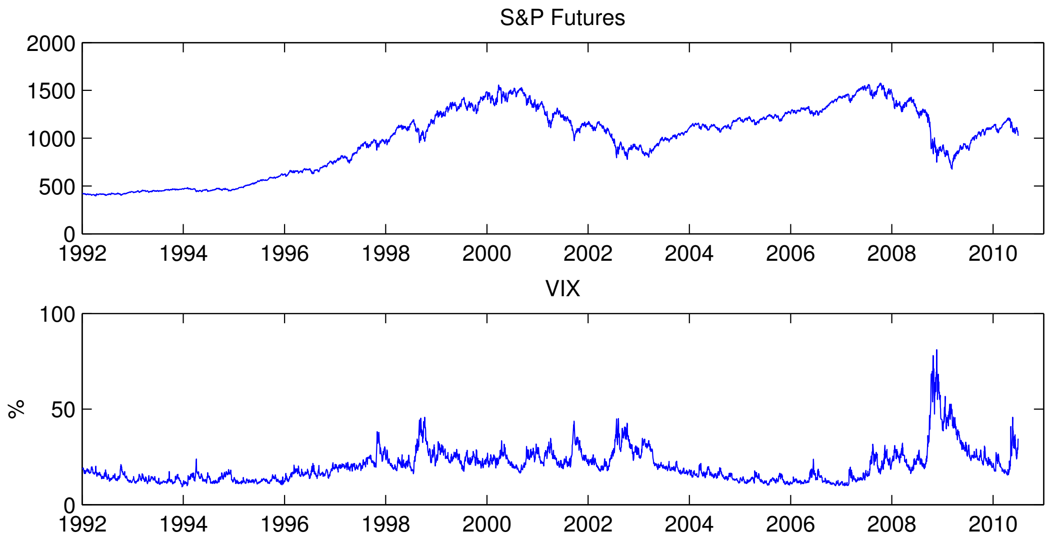

Figure 2. Daily closing values for the S&P 500 futures and the VIX. The VIX is reported in the annualized percentage form.

min/max/open/close, which can be downloaded from the CBOE web-site, as illustrated, for instance, in Figure 1. More details are provided in Appendix B.

To address the aforementioned shortcomings I undertake the following steps. I consider only the time window between 9:01 a.m. and 3:00 p.m. Central Time, which also helps avoid issues with the beginning and closing of trading. Further, I pre-filter data by omitting prices outside of daily min/max. Though the number of thus removed one-minute price records is relatively small 124 (0.0074% of all data analyzed), this correction seems to eliminate a significant number of large movements in the VIX, because the total amount of detected jumps in the VIX decreases by about 23%. This effect can be also illustrated by the substantial fall of the kurtosis. For example, in year 2004, the VIX kurtosis falls from 3760 to 200. One can see this effect by comparing descriptive statistics before and after pre-filtering in Table XIII and Table XV, Appendix B.

CHAPTER 4

COMPARATIVE QUALITY STUDY OF THE VIX AND S&P 500 DATASETS

The previous chapter has revealed certain problems with the VIX data. The present chapter investigates how the data quality affects the inference from the data. This chapter also compares the data quality of two datasets and shows that certain properties of the VIX cast doubt on the reliability of this dataset. It is justified that the S&P 500 futures data are trustworthier and can be used to filter the information from the VIX, for example, using the stochastic volatility jump diffusion model in Section 5.1. Besides, several important practical choices are justified here as well, such as the use of a 5-minute sampling frequency.

4.1 Two ways of measuring of the return-volatility relationship

The primary concern of this section is whether the data allow to measure the leverage effect using high-frequency observations on the VIX and S&P. This section considers the return-volatility relationship (leverage effect) for all returns while Section 6.4 is devoted to the correlation of jumps. The main messages of the current section are: 1) the leverage effect monotonically increases in years; 2) the VIX seems to have a lot of large erroneous movements that distort the measurement of the leverage effect. I focus only on the contemporaneous relationship between the VIX and S&P, because it is much stronger than any lagged dependence as shown by the following cross-correlation analysis.

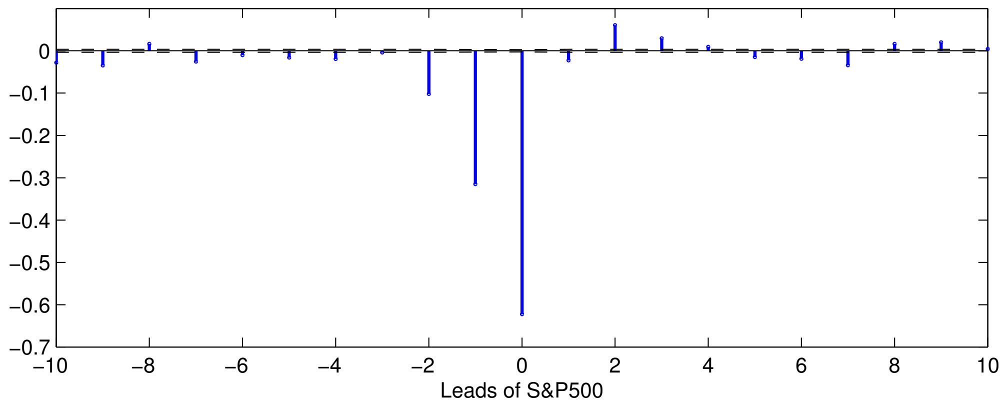

Figure 3. Sample crosscorrelations of the VIX and the S&P.
Sampling frequency is five minutes. The standard error is 0.0018.

Figure 3 provides sample cross-correlations between the VIX and the S&P at a 5-min sampling frequency. This plot shows that the series are concurrently correlated and the contemporaneous dependence is the strongest one. Also the changes in the VIX (ΔVIX) are correlated with several past values of changes in the S&P (ΔS&P). The first lead of the ΔS&P also has statistically significant correlation with contemporaneous value of ΔVIX, but the value itself is so small that hardly could be considered practically important. Thus, at this sampling frequency, the leverage effect appears to be almost instantaneous. My results are in line with the nonparametric study of (22) who documents that at a 5-minute frequency the "contemporaneous return is the most important factor that determines changes in current implied volatility" and "several lagged values of the return are also significant."

It is widely acknowledged that the strength of relationship between the VIX and S&P depends on the magnitude of returns. For example, reference (22) distribute S&P returns into sub-samples based on their size, compute correlation with the contemporaneous changes in the VIX and report that "the return-implied volatility relation is dominated by extreme return situations."[1] I repeat their result and make one important extension: I also compute correlation when returns are grouped based on the size of the VIX returns. This exercise is very illustrative of problems with the VIX, because it gives quite different estimates of the leverage effect. Details are as follows.

The procedure of computing correlations for sub-samples of the S&P is as follows. First, I normalize intraday changes in both indices using intraday periodicity volatility "factors"[2] following the approach of (14). I further normalize intraday changes in both indices by their respective daily standard deviations determined as in Section 5.4. Thus, I effectively make all changes comparable by measuring them in multiples of daily standard deviations or "sigmas." I discard all VIX changes greater than 10 sigmas in absolute value to avoid the influence of extreme observations. For each year, I divide a sample of the normalized S&P returns into negative and positive parts and then segregate each part into thirds of equal counts. Thus, I obtain six groups of returns for each year: large, medium, and small positive, and negative

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[1]Similar effect is observed with uni-variate high-frequency volatility estimation. (24) states that "large (absolute) returns are inherently more informative of the underlying volatility than small returns."

[2]This accounts for a varying trading activity during a day.

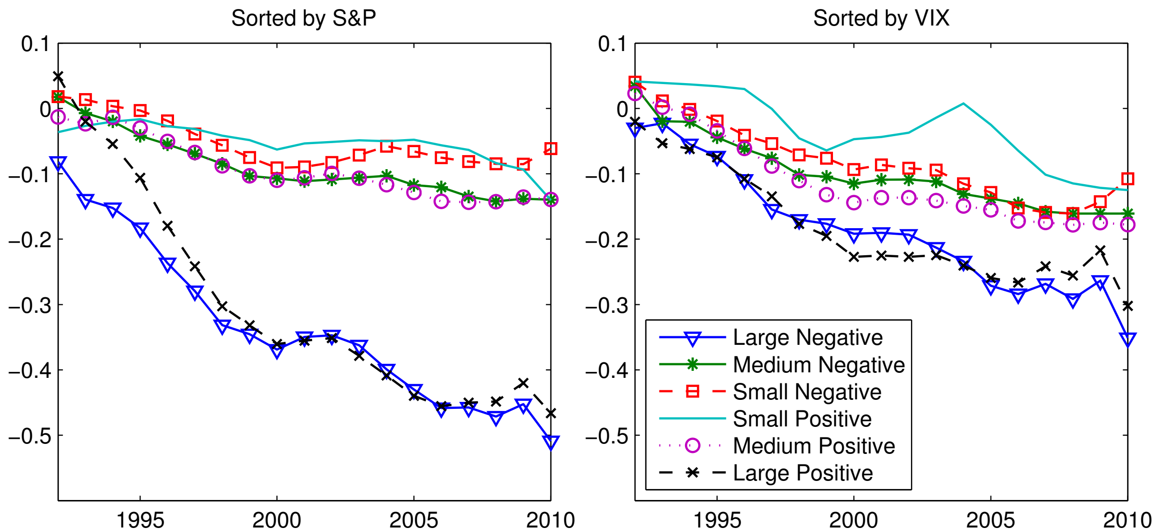

Figure 4. Annual sample correlations by size of returns.

The left panel presents correlations when pairs of returns are sorted by the size of the S&P returns. The right panel refers to the case when pairs of returns are sorted by the size of the VIX returns. Time window is from 9:30 a.m. to 3 p.m.

returns. Finally, I calculate the annual correlation coefficients between these S&P groups and concurrent VIX changes.

Specifically, to compute correlation ρ_s^{LN} for group "Large Negative S&P returns" I choose pairs of VIX and S&P returns: (s_i, v_i) such that $s_i \in$ (Large Negative S&P returns). Thus, I obtain six correlation coefficients for each year: ρ_s^{LN}, ρ_s^{MN}, ρ_s^{SN}, ρ_s^{SP}, ρ_s^{MP}, and ρ_s^{LP}, where subscript s means that correlations is computed based on sorting by the S&P returns.

The correlations when pairs of changes are sorted by the S&P. Annual estimates of these coefficients are presented in the left panel of Figure 4. The correlation seems to increase in absolute value starting from -0.1-0.2 in 1992 and reaching the values below -0.5 or -0.6 in 2010. As expected, large positive and negative returns appear to have a larger correlation.

The correlations when pairs of changes are sorted by the VIX. I repeat the same exercise for the VIX. For example, to compute ρ_v^{LN} for group "Large Negative VIX returns" I choose pairs of VIX and S&P returns: (s_i, v_i) such that $v_i \in$ (Large Negative VIX returns). Again I obtain six correlation coefficients for each year: ρ_v^{LN}, ρ_v^{MN}, ρ_v^{SN}, ρ_v^{SP}, ρ_v^{MP} and ρ_v^{LP}. The right panel of Figure 4 represents yearly dynamics of these correlations. Surprisingly, the correlations for the larger in absolute value changes in the VIX are not the strongest in contrast to those measured based on the S&P grouping:

$$\rho_v^{LN} < \rho_s^{LN} \text{ and } \rho_v^{LP} < \rho_s^{LP}.$$

Moreover, the correlations for medium and small positive and negative returns seem to be slightly larger:

$$\rho_v^{MN} > \rho_s^{MN},\ \rho_v^{MP} > \rho_s^{MP},\ \rho_v^{SN} > \rho_s^{SN} \text{ and } \rho_v^{PN} > \rho_s^{PN}.$$

What could be so different for the VIX? Perhaps, the VIX data may have errors distorting the ordering of returns. For example, a moderate VIX return with a moderate correlation by mistake is recorded as a large return or even a return with a different sign. This explains why the strength of correlation is lower when it is measured based on ordering of the VIX returns. Apparently, the magnitude of these errors should be large because both extreme thirds are affected as well. Similar information can be obtained from Table I, which reports the

TABLE I

LEVERAGE EFFECT FOR LARGE MOVEMENTS, BY SIGN AND ABSOLUTE VALUE

Sorted by	Size of change, sigma	Correlations[a] Negative	Correlations[a] Positive	Count Negative	Count Positive
VIX	(5,6)	0.09	0.13	685	934
	(6,7)	0.08	0.11	442	631
	(7,8)	0.09	0.07	261	399
	$(8,\infty)$[b]	0.12	0.03	184	302
S&P	(5,6)	-0.15	-0.47	113	69
	(6,7)	-0.24	-0.6	45	28
	(7,8)	-0.27	-0.6	22	12
	$(8,\infty)$[b]	-0.46	-0.6	19	12

[a] Subcolumns of column "Correlations" refer to the signs of movements (positive and negative). Correlation means the same as the leverage effect.

[b] The correlation for the returns exceeding 8 sigmas in absolute value may be unreliable because of small sample sizes.

distribution of correlations across size and sign of returns. The larger the returns on the S&P the higher their correlation (in absolute value) with the contemporaneous returns in the VIX. In contrast, large movements in the VIX have weak correlation with simultaneous returns in the S&P.

In conclusion, this section indirectly shows that there might be a lot of VIX jumps that do not coincide in time with jumps in the S&P, which is later demonstrated in Section 6.4.

4.2 Microstructure noise in the VIX is higher than in the S&P 500

It is a commonly accepted fact that high-frequency data are contaminated with microstructure noise, which distort the inference. For this study, it is important that, apart from microstructure

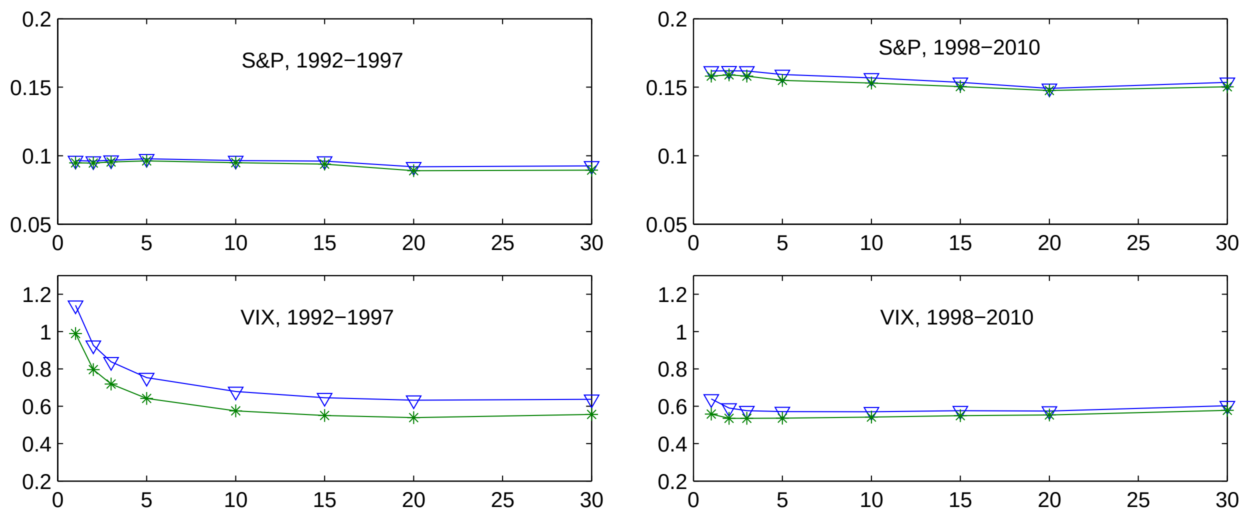

Figure 5. Signature plots of annualized volatility.

The triangles denote $\sqrt{BPV}$ and the asterisks mark $\sqrt{RV}$.

effects (market imperfections) common in most financial data such as bid-ask spread and price discreteness, the VIX has extra complications:

1. The size of the bid-ask spread varies across option strikes and can be quite wide for illiquid strikes. In such cases, the quote midpoint used for the VIX computation is not reliable.
2. Market makers do not always update all option prices at the same frequency. Less liquid options are updated less often.
3. Mislabeling (for example, wrong maturity) and other recording errors are also possible.

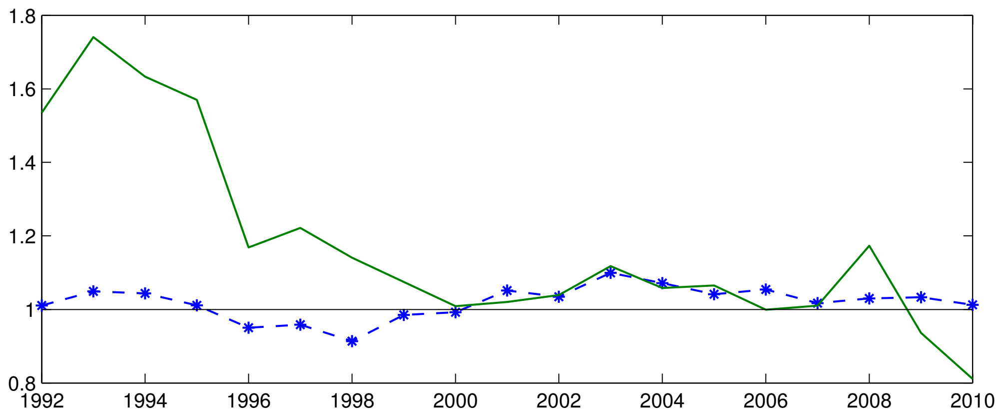

Figure 6. Ratio of BPV computed on data sampled at a one and five minute frequencies. The solid line refers to the VIX while the dotted line with asterisks represents the S&P.

To analyze the level of microstructure noise, I create signature plots for each index. As shown on Figure 5, the S&P and VIX differ much. The S&P signature plot is flat within considered sampling frequencies and reports average values of annualized volatility of roughly 10% in 1992-1997 and around 18% in 1998-2010. The jump component of variance (excess of realized variance (RV) over the bipower variation (BPV)) is negligible. In contrast, the VIX shows an increase in volatility at higher sampling frequencies during both subperiods. In the earlier, noisier, period, this augmentation is higher. The jump component of variance is substantial for the VIX, especially in the earlier period. For these data on the VIX, before 1998, the highest appropriate sampling frequency is ten minutes and five minutes since 1999, while one-minute data on the S&P are clean enough during the whole period under investigation. In this study, I use a five-minute sampling frequency for the entire period for both time series. The same idea comes from the time series of the ratio of BPV calculated at one and five minute sampling frequencies. On Figure 6, this ratio for the VIX is monotonically decreasing from 1992 to roughly 1999. In contrast, this metric for the S&P exhibits no time trend.

4.3 The S&P 500 futures dataset is more reliable than the VIX dataset

Table II compares the quality of the S&P 500 futures and VIX datasets. First of all, the S&P 500 futures data come as transaction records from the world's most liquid stock market, while the VIX data arrive from the less liquid option market through a complicated formula with influential implementation challenges.

Besides, several properties of the VIX, in contrast to the S&P, have a pronounced monotonic time trend. First, the number of extreme returns or jumps in the VIX is monotonically decreasing over time while the number of extreme returns or jumps in the S&P is roughly constant during the same period (see Section 6). Second, Section 4.2 demonstrates that the level of microstructure noise in the VIX is decreasing over years while in the S&P such trend is not present, at least at the same sampling frequency.

Noteworthy, periods with different market conditions such as bull/bear market and financial crises, which involve volatile and quiet periods, appear to not affect this monotonic pattern. Thus, this time trend can be explained by a gradual improvement of the VIX data. Indeed, the option market becomes more liquid (more strikes are available, spreads are smaller, quotes are updated more frequently) and constant technology improvement reduce data errors. Of course,

TABLE II

COMPARISON OF THE S&P AND THE VIX DATASETS

Type of information	S&P 500 Futures	VIX
	Facts	
Data origin	Transactions from the world's most liquid market.	Computed from a large number of quotes using a complex formula. Less liquid market
Data source	CME	CBOE
	Empirical findings of this study *1. Properties of jumps*	
Frequency of jumps	Rare (4.9 per year)	Very frequent (40-90 per year)
Matching of jumps in time	Usually correspond to large changes in the VIX	Most jumps correspond to nothing substantial in the S&P 500
	2. Time trend at a 5-min sampling frequency	
Microstructure noise	Almost constant	Monotonically decreasing
Number of jumps	Almost constant	Monotonically decreasing

the S&P microstructure has been also improving, but at a five-minute sampling frequency this effect is not noticeable.

The increasing quality of the VIX data might also be responsible for the similar time trend in the leverage effect. This is illustrated in Section 4.1, which shows that the negative correlation between the VIX and the S&P 500 is relatively moderate at the beginning of the period but becomes more pronounced in the second half of the period.

CHAPTER 5

METHODOLOGY

This chapter provides the methodological framework: a standard option pricing model with dependent and independent jumps in price and volatility (Section 5.1) and a nonparametric jump test (Section 5.5). However, the classical model describes the spot volatility, which is not the same as the VIX. That is why, Section 5.2 gives the short summary of the literature on the link between the VIX and the spot volatility and discusses to what extent the predictions of different models can be applied to the analysis of the VIX.

5.1 The standard option pricing model. Hypotheses to test

I analyze the data within the theoretical framework of a classical jump-diffusion stochastic volatility model, while being completely nonparametric about jumps and the leverage effect dynamics:

$$\mathrm{d}(\log p_t) = \mu_t \mathrm{d}t + \sqrt{V_t}\left[\rho_t \mathrm{d}W_t^1 + \sqrt{1-\rho_t^2}\mathrm{d}W_t^2\right] + \xi_t \mathrm{d}J_t^p + \xi_t^c \mathrm{d}J_t^c \tag{5.1}$$

$$\mathrm{d}(V_t) = \kappa(\theta - V_t)\mathrm{d}t + \sigma_t^v \sqrt{V_t}\mathrm{d}W_t^1 + \psi_t \mathrm{d}J_t^v + \psi_t^c \mathrm{d}J_t^c, \tag{5.2}$$

where p_t is a stock price at time t; V_t is a spot variance; $\boldsymbol{W}_t = \{W_t^1, W_t^2\}$ is a bivariate standard Brownian motion vector; $\boldsymbol{J}_t = \{J_t^p, J_t^v, J_t^c\}$ is an independent (of $\boldsymbol{W}_t$) trivariate vector of mutually independent Poisson processes with finite[1] positive intensities λ_t^p, λ_t^v, λ_t^c; ξ_t and ξ_t^c are jump sizes in price; and ψ_t and ψ_t^c are jump sizes in volatility. The superscript c refers to the jumps that arrive simultaneously to the price and volatility.

This specification incorporates several popular stochastic volatility models for option pricing (see (26), (7)). It is very flexible in treatments of jumps and correlations between shocks to price and shocks to volatility (the leverage effect).

Models with constant coefficients:

- SV: Heston's (1993) model $\lambda_t^p = \lambda_t^v = \lambda_t^c = 0$.
- SVIJ: model with independent jumps in price and volatility $\lambda_t^c = 0$.
- SVCJ: model with contemporaneous jumps $\lambda_t^p = \lambda_t^v = 0$ and correlated jump sizes $\xi^c \mid \psi^c \sim N(\mu_\xi + \rho^J \psi^c, \sigma_\xi^2)$.
- SVICJ: model with independent jumps and correlated contemporaneous jumps. The same as above but $\lambda_t^p > 0$ and $\lambda_t^v > 0$.

Models with time-varying coefficients:

- (10) use SVICJ but treat intensities and sizes of jumps, correlation, and volatility of volatility σ_t^v as functions of the spot variance level.

In all above models, the leverage effect is described by coefficient ρ_t (further: diffusive correlation) before the diffusion terms in Equation 5.1. Besides, the SVICJ and SVCJ models

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[1]This is not the only known approach to model volatility jumps. For example, (25) have shown that the volatility is a jump process of infinite activity. Such kind of jumps is outside of my scope and might be addressed in future research.

allow for the additional channel for the leverage effect through correlation ρ^J between jumps in price and volatility (further: jump correlation).

To test whether the data agree with the predictions of the classical model, let us consider the following hypotheses:

Hypothesis I: The spot volatility changes significantly after a jump in volatility.

This follows from the model. For example, a positive jumps in Equation 5.2 increases the level of V_t, the square root of which is the factor before the diffusion term in Equation 5.1.

Hypothesis II: Jumps are independent in time. This also follows from the model.

Hypothesis III: Independent jumps in price and volatility do exist, i.e., $\lambda_t^p > 0$ and $\lambda_t^v > 0$.

Hypothesis IV: Jumps correspond to news arrivals. This does not follow from the model but it is a widely accepted stylized fact.

Hypothesis V. Null hypothesis: The high-frequency VIX dataset is contaminated with noise or pseudo-jumps. **Alternative hypothesis:** The standard options pricing model should be reconsidered with respect to the specification of volatility jumps.

Definition. *Pseudo-jumps* are jumps, which take place in the observable VIX but do not represent real movements in its fundamental value.

Hypothesis V cannot be formally tested. I assume that the evidence in its favor can be build upon the rejection of Hypotheses I, II, III, and IV combined with additional considerations such as overall poor data quality and the critique of the VIX in the literature.

5.2 The link between the VIX and spot volatility

To be able to use insights from the model in Section 5.1, I need to verify that inferences obtained from the high-frequency VIX can be extended to the properties of the unobservable spot volatility, or vice versa. As illustrated below, these processes are not equivalent, but the information about distribution of jumps, especially jump times, and the leverage effect is similar for both processes. This section provides only basic ideas along with references to sources where formal proofs can be found.

By definition, the VIX squared is an option implied estimate[1] of a value of a forward contract on the total quadratic variation of the logarithmic price of the underlying asset over the next 30 calendar days:

$$\mathrm{VIX}_t^2 \approx \frac{1}{T}\mathbb{E}_t^{\mathbb{Q}}\left([S,S]_T\right), \tag{5.3}$$

where $\mathbb{Q}$ - the risk neutral probability measure, and $[S,S]$ is the quadratic variation process associated with $S_t = \log p_t$. Since (27), it is a common practice to consider the quadratic variation as consisting of continuous and discontinuous parts:

$$[S,S]_T = \int_t^{t+T} V_u du + \sum_{i=1}^{N_J} (\xi_i)^2 + \sum_{i=1}^{N_J^c} (\xi_i^c)^2, \tag{5.4}$$

where N_J - number of independent price jumps and N_J^c - number of price jumps, contemporaneous to volatility jumps, over period T. The first term represents the integrated variance, which

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[1]The VIX valuation formula based on options is provided in Section 2.1.

includes volatility jumps along with other path-wise features of volatility. The second and third term relate the contribution of the price jumps. Thus, the relationship between the VIX and spot volatility is highly complex and, as shown further, varies for different models.

Affine models. For the affine models, (28) show that the link between the VIX and spot volatility can be expressed in a linear form:

$$\mathrm{VIX}_t^2 = aV_t + b, \tag{5.5}$$

where $a > 0$ and $b > 0$ are functions of model parameters. Thus, within this class of models, jumps in the VIX and volatility arrive at the same time with spot volatility jumps and in the same direction. Sizes of movements might be different, but fortunately, due to the linearity, the movements in the VIX should yield the same strength of the leverage effect as do the movements in unobservable volatility.

Non-affine models. The relationship between the VIX and spot volatility is much more complicated for this class of models. Papers (25) and (29) derive this relationship for non-affine exponential stochastic volatility models. Particularly, by Theorem 1 of (25), the jump times should coincide for both the VIX and spot volatility. This conjecture also includes models that accommodate the long-memory property of volatility through a fractionally integrated Brownian motion.

In conclusion, most available models allow for the univocal relationship between the VIX and the unobservable spot volatility. Specifically, the nondecreasing and monotonic nature of this relationship allows to assume that bigger movements in the spot volatility should always correspond to bigger movements in the VIX. In other words, an extreme movement in the spot volatility should also correspond to an extreme movement in the VIX. In addition, jump times are the same for both the VIX and spot volatility.

5.3 Classification of jumps

In this section, I suggest the classification of realized jumps, according to the standard option pricing model in Equation 5.1 and Equation 5.2. The model has three independent compound Poisson processes, and I classify jump into three groups.

Classification rule. Isolate all movements in the S&P and the VIX based on whether they are contemporaneous to jumps in the other time series. Thus, I obtain three categories of jumps and the fourth category for diffusive movements as shown on Table III:

Type I. Matched jumps ($\xi_t^c \mathrm{d}J^c$ and $\psi_t^c \mathrm{d}J^c$). These jumps arrive simultaneously to both the S&P and VIX.

Type II. Unmatched jumps in the S&P ($\xi_t \mathrm{d}J^p$). The S&P jumps, but the VIX does not jump during the same time interval.

Type III. Unmatched jumps in the VIX ($\psi_t \mathrm{d}J^v$). The VIX jumps, but the S&P does not jump during the same time interval.

Type IV. Non-jump returns. Diffusion terms in Equation 5.1 and Equation 5.2. Neither the S&P nor the VIX jumps.

TABLE III

CLASSIFICATION OF JUMPS IN FINANCIAL VARIABLES

<table>
<tr><td colspan="4">Finite activity jumps</td></tr>
<tr><td colspan="2">Volatility jumps ≈ jumps in the VIX</td><td colspan="2">Price jumps = jumps in the S&P</td></tr>
<tr><td>Unmatched jumps
Type III[a]</td><td>Matched jumps
Type I</td><td>Matched jumps
Type I</td><td>Unmatched jumps
Type II</td></tr>
<tr><td>Pseudo-jumps[b]</td><td colspan="3">Real jumps</td></tr>
</table>

[a] The classification suggested in this study is marked in bold.

[b] The pseudo-jumps are introduced according to Hypothesis V in Section 5.1. As shown further, they are most likely belong to Type III jumps.

In the literature, unmatched jumps are also referred as “independent” jumps or “idiosyncratic” (10) and matched as “contemporaneous” or “co-jumps.”

5.4 Volatility measures

Let assume that the logarithmic price process S_t is observed at $N+1$ discrete equidistant points in time $0 \le t_0 < t_1 < ... < t_N < 1$ over a given period (for example, trading day). Then $\Delta S_i = S_{t_i} - S_{t_{i-1}}$ is a logarithmic rate of return over time interval $\Delta t_i = t_i - t_{i-1}$, $i = 1, ..., N$.

This study uses three well-known high-frequency measures of volatility:

- A realized variance (RV) of (30):

$$\sigma^2(t_0, t_N) = RV_{1,N} = \sum_{i=1}^{N} (\Delta S_i)^2$$

- The bipower variation (BPV) of (31):

$$\sigma^2(t_0, t_N) = BPV_{1,N} = \frac{\pi}{2}\left(\frac{N}{N-1}\right)\sum_{i=1}^{N-1} |\Delta S_i||\Delta S_{i+1}|$$

- The robust volatility of returns based on quantiles of the normal distribution:

$$\hat{\sigma}^{robust}(t_0, t_N) = \frac{\hat{Q}_{1-p/2} - \hat{Q}_{p/2}}{2F^{-1}(1-p/2)}, \tag{5.6}$$

  where $\hat{Q}_p$ is a p^{th} sample percentile, which is determined from a sample of returns. In this study $p = 20\%$. F^{-1} is the inverse gaussian cumulative distribution function. This volatility measure is robust to extreme returns but, for smaller returns, remains dependent on the normality assumption as it assumes that the returns below a specified percentile are normally distributed.

5.5 Nonparametric jump detection

With the recent availability of high-frequency data, a number of statistical tests for jumps has been developed. I have applied five tests described in (32), (33), (34), (12), (35), and (24). Overall, all tests detect more jumps in the VIX than in the S&P and agree on the presence of the monotonic time trend in the number and magnitude of the VIX jumps.

In this study, I report only the results of the jump detection technique similar to the Lee and Mykland jump test described in (12). This test has a valuable advantage because unlike other jump tests it conveys the exact time of a jump up to the smallest available sampling interval (five minutes in this study), jump size and direction. I define jumps as large moves relative to the current volatility level:

$$\mathcal{L}(i) = \frac{|\Delta S_i|}{\hat{\sigma}_d^{robust}/\sqrt{N}} > C_{crit}, \tag{5.7}$$

where $\hat{\sigma}_d^{robust}$ is the volatility estimate on day d, computed according to Equation 5.6 in Section 5.4, and N is the number of intraday returns. I use only returns from the same day to determine the volatility. Thus, I use all data available during the day, before and after a jump. I scale all returns to allow for the intraday periodicity of volatility estimated as in (14). I choose the critical value $C_{crit} = 6$ as a commonly used empirical cut-off for unusual movements in financial variables.

CHAPTER 6

ESTIMATION RESULTS

This section reports estimation results of parameters relating to jumps in the most general model presented in Section 5.1 - SVICJ and compares them to the literature in Table V. I estimate all parameters nonparametrically and independently from each other. The jumps are detected with the test described in Section 5.5, and as such they are essentially all five-minute returns in absolute value greater than the threshold of the test, which is six sigma.[1] This section also evaluates plausibility of hypotheses stated in Section 5.1.

6.1 Distribution and time series of jumps

The realized jumps along with their annual occurrence are reported by Figure 7. Table XI from the Appendix provides exact quantities of jumps detected every year. Descriptive statistics and histograms of jumps are given by Table IV and Figure 8.

Structural break in data. Similar to the analysis of the leverage effect and microstructure noise in Chapter 4, I observe two rather distinct time periods: from 1992 to 1998 (Period 1) and 1999 - 2010 (Period 2). Again during Period 1, the data show a substantial nonstantionarity, but only for the VIX. According to Figure 7, the number of jumps in the VIX is monotonically decreasing until roughly 1998 and then levels off. In 1992, the test reports 268 jumps in the

[1]The sigma is an estimate of a daily volatility (see Section 5.4) divided by the square root of N, which is the number of observations per day.

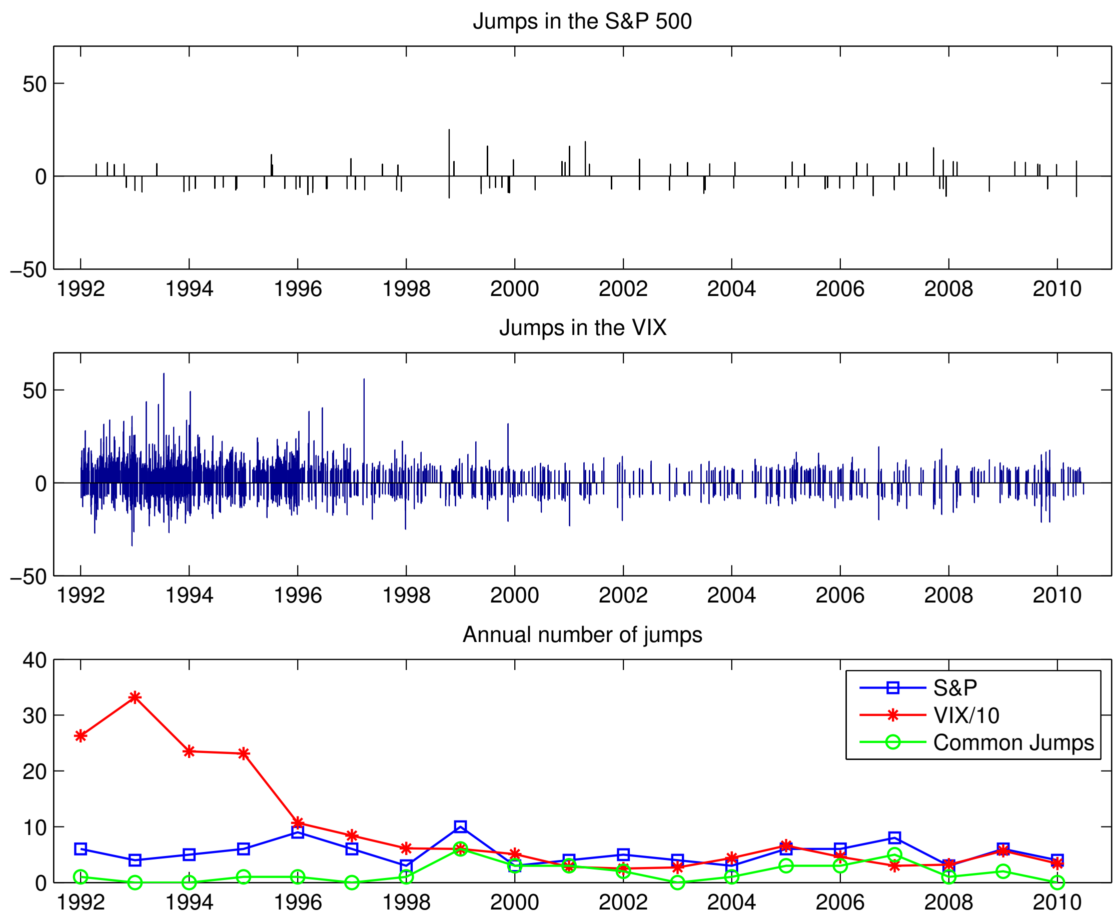

Figure 7. Realized jumps in both indices over time.

The top panel shows jumps in the S&P while the middle panel refers to jumps in the VIX. Jumps are detected with the jump test described in Section 5.5. Jump size is measured in multiples of standard deviations of five-minute logarithmic returns from the same day.
Bottom panel: The asterisks represent the number of jumps in the VIX divided by 10. The "boxes" refer to the number of jumps in the S&P. The "circles" show the number of jumps that happen in both indices at the same time. The number of jumps reported in 2010 is the double of what is measured in the first half of the year because the data are available only until June 30, 2010.

VIX and 58 in 2009 while the number of jumps in the S&P is only 5 and 6 in respective years. Noteworthy, such monotonic time pattern in Period 1 is found in other features of the VIX and is likely to be attributed to the poor quality of the VIX dataset. This conjecture is supported in Section 4 with several empirical observations. Specifically, I observe the substantial decrease in the level of the microstructure noise with the commensurate and pronounced strengthening of the leverage effect. So, all further analysis focuses on Period 2 while Period 1 is sometimes mentioned for completeness.

Descriptive statistics. The descriptive statistics of jumps by their type is presented in Table IV. It seems that the unmatched jumps in the VIX (Type III) are more frequent and larger, especially in the first period. The size of Type III jumps is from -34.0 to 58.9 sigmas while other types of jumps are in range of (-23.2; 25.7) at most. It is interesting that both series have a lot of negative and positive jumps as shown on Figure 8 with very little skewness. This finding is not surprising for the S&P but contradicts popular specifications of stochastic volatility models, which allow only for positive jumps in volatility. This result is consistent with (14) who use much shorter period. The distribution of jumps in the S&P appears to be slightly positively skewed. For the VIX: the distribution of Type I jumps is slightly negatively skewed while the skewness of Type III jumps is not significantly different from zero.

Frequency of jumps. Volatility jumps occur 5-10 times more frequently compared to parametric models, due to the much higher number of Type III jumps. Table IV reports the total number of jumps in Period 2 for the S&P 29+31= 60 and for the VIX 31+454 = 485, which corresponds to 5.2 and 42.2 jumps per year. Thus, about 94% of jumps in the VIX are

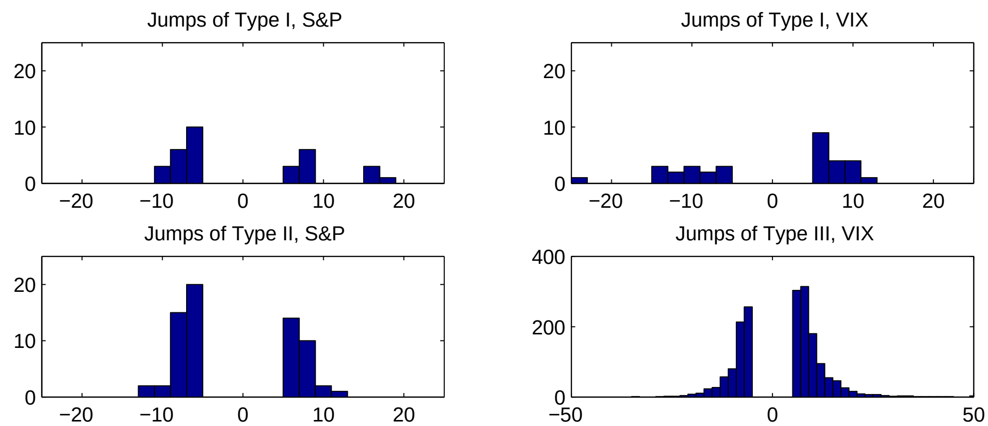

Figure 8. Histogram of jumps, by type.

Jump size is measured in multiples of five-minute daily standard deviations of logarithmic returns.

Type III jumps. As shown in Table V, my estimates of price jumps intensity resemble those in the literature while the intensity of volatility jumps is much higher. Though (10) report a greater annual number of unmatched[1] volatility jumps 13, it is still quite far from observed in my analysis $454/11.5$ years ≈ 39.

In conclusion, the distribution of all types of jumps is roughly symmetric. The frequency of Type I and II jumps agrees with the literature. In contrast, Type III jumps are much more frequent. Such discrepancy in frequency of jumps between the S&P and VIX might not be a problem because it can be explained by the high kurtosis of the VIX at intraday

[1](10) use term "idiosyncratic."

TABLE IV

DESCRIPTIVE STATISTICS OF JUMPS

Type of jumps[a]	Count	Mean	Std.Dev.	Median	Min	Max	Skew
Period 1 (1992-1998)							
Type I. Matched Jumps in the S&P	4	0.1	16.7	-7.3	-10.0	25.1	1.1
Type II.Unmatched Jumps in the S&P	35	-2.3	7.1	-6.5	-11.7	11.6	0.7
Type I. Matched Jumps in the VIX	4	6.5	15.5	6.4	-12.3	25.7	0.0
Type III. Unmatched Jumps in the VIX	1309	3.5	10.9	6.9	-34.0	58.9	0.1
Period 2 (1999 - 2010)							
Type I. Matched Jumps in the S&P	29	0.5	9.4	-6.1	-10.9	18.5	0.5
Type II.Unmatched Jumps in the S&P	31	-0.5	7.7	-6.2	-11.1	9.1	0.0
Type I. Matched Jumps in the VIX	29	-0.4	10.2	6.5	-23.2	12.5	-0.5
Type III. Unmatched Jumps in the VIX	454	0.5	8.7	6.1	-21.2	31.8	-0.0

[a] Jump size is measured in multiples of five-minute daily standard deviations of logarithmic returns.

sampling frequencies. What seems to be suspicious is that (as shown later) all these "heavy tail" movements differ in their properties: the leverage effect, volatility change and serial independence.

6.2 Hypothesis I. Change of volatility after a jump

To test Hypothesis I about the change of volatility after a jump, I use two versions of the homogeneity of variance test: (39) and (40). I consider only days with a single jump and with enough data before and after a jump to perform the test. The tests are applied to returns scaled according to the intraday pattern of volatility estimated as in (14).

The stylized fact that the volatility should change after a jump agrees with the literature ((17) and (41) among others). Besides, a visual inspection of intraday plots also confirms this idea. Figure 1 and Figure 11 show examples of matched and unmatched jumps. On Figure 11,

TABLE V

COMPARISON OF PROPERTIES OF JUMPS OBTAINED IN THIS STUDY WITH THOSE FROM THE LITERATURE

Source	Data	Period	λ^p	λ^v	λ^c	ρ	ρ^J	Models
Structural models								
"The Impact of Jumps..." (7)	S&P daily	1980-1999			1.5	-0.48	- 0.60	SVCJ
"An empirical investigation..." (36)	S&P daily	1980-1996	4.83			-0.39--0.32		SVJ
"Price and Volatility Co-Jumps" (10)	S&P daily, 5-min	04.1982-02.2009	0.86	13	0.89	depends on volatility	-1	SVICJ
"VIX Dynamics..." (37)	VIX daily	1990-2010		2.26				SVJ
Nonparametric								
"Cross-section of jumps..." (38)	US stocks[a] daily	1971-2007	3.19					
"Volatility Jumps" (25)	VIX, S&P 5-min	09.2003-2008					-0.7	
"...Corridor fix for the VIX" (14)	VIX, S&P 1-min	06.2008-06.2010				-0.50	-0.53	
This study, Jumps of Type I	VIX	1999-	–	–	**2.5**	–	**-0.95**	SVICJ
Jumps of Type II	S&P	06.2010	**2.8**	–	–	–	**-0.87**	
Jumps of Type III	5-min		–	**39.4**	–	–	**-0.41**	
Type IV, diffusion			–	–	–	**-0.66**	–	

λ^p and λ^v- annual number of independent jumps in price and volatility, respectively.
λ^c - annual number of contemporaneous jumps in price and volatility.
ρ - correlation between diffusive changes in price and volatility.
ρ_J - correlation between jumps in price and volatility.
SVJ - the stochastic volatility model with jumps only in price.
SVCJ - the stochastic volatility model with contemporaneous jumps in price and volatility.
SVICJ - the stochastic volatility model with independent and contemporaneous jumps in price and volatility.

[a] Average across 25,666 firms listed in US.

the price trajectory seems to become more erratical after the jump; in contrast, the unmatched jumps on Figure 1 are not followed by a visible change in the variability of the price.

To the best of my knowledge, the only nonparametric test for the change of volatility relating to the jump occurrence have been developed by (41). These authors document that a jump in the S&P is followed by the change in the volatility, using one minute S&P 500 index futures (1997-06.2007). My study differs in following aspects. First, the period under my study is longer, from January 1992 to June 2010. Second, I check the change of the volatility for each jump separately while their test detects whether, in a certain time interval (a trading week), there are common arrivals of jumps to both jumps and volatility. In addition, I study the behavior of "volatility of volatility" after a jump arrival, that is, I investigate the change of the VIX volatility following a jump arrival in the VIX. So far, nonparametric evidence on this phenomenon has never been reported. Finally, I do not use the data normality assumption, which seems to be crucial for these data because the Jarque-Bera test rejects normality for 76.4% of day-long samples of the VIX at a 5% significance.

The test results are provided in Table VI. It reports the fraction of jumps, for which the volatility change is significant at $\alpha = 0.05$. This table refers to the volatility of the S&P and the volatility of the VIX, which is essentially the "volatility of volatility." Both tests report that jumps of Type III have a lower fraction of jumps, which are followed by statistically significant changes in volatility. For example, according to Bartlett's test, after 58% of Type I and only after 20% of Type III jumps in the VIX, the volatility of the VIX changes significantly. The tests for the difference in proportions between Type I and III jumps, and between Type II and

TABLE VI

CHANGE OF VOLATILITY AFTER A JUMP

Type of Jumps	Count	Bartlett's test[a]		Levene's test[a]	
		S&P	VIX	S&P	VIX
1992-1998					
Type I. Matched Jumps in the S&P	2	1.00	1.00	1.00	1.00
Type II.Unmatched Jumps in the S&P	32	0.59	0.47[b]	0.44	0.22[b]
Type III. Unmatched Jumps in the VIX	290	0.18[b]	0.34	0.10[b]	0.15
1999-2010					
Type I. Matched Jumps in the S&P	24	0.58	0.75	0.50	0.50
Type II.Unmatched Jumps in the S&P	20	0.55	0.65[b]	0.35	0.25[b]
Type III. Unmatched Jumps in the VIX	215	0.20[b]	0.31	0.15[b]	0.14

[a] Proportion of jumps followed by significant changes (p-value $< 5\%$) in volatility based on the homogeneity of variances test results.

[b] According to the classification, the indices are not supposed to have jumps here; however, test statistics of volatility change are statistically significant.

III jumps are highly significant with p-values less than 10^{-4}. The numbers for Type I and II are almost similar for the S&P.

The Levene's test is more conservative because it is robust to deviations from normality. It reports smaller proportions of significant changes for all categories of movements. Moreover, for the less normal VIX, the drop in the number of significant volatility changes is higher. For example, the first row of Table VI shows that Type I movements in the S&P lose only 0.08 (from 0.58 to 0.50) while the movements in the VIX of the same type have the proportion reduced by 0.25 (from 0.75 to 0.50). Further, I report only results of Levene's test.

The percentage of statistically significant volatility change is relatively low from what is expected because the change of volatility is hard to detect. Indeed, the change of volatility can be detected only if it exceeds the confidence bounds of the volatility estimator. Therefore, the accuracy of such tests is always inferior to the accuracy of the volatility estimator. Apparently, due to the same reason, the test performed by (41) does not show a higher percentage of significant cases. They find that "(i) in approximately 40% of the weeks there is strong evidence for common price and volatility jumps, (ii) in around 20% of the weeks there is evidence for disjoint jumps, and (iii) for the rest of the weeks the tests are inconclusive."

I summarize this as follows. Hypothesis I, which states that the volatility should change after a jump, appears to hold for much higher percentage of Type I and II jumps than for Type III jumps.

6.3 Hypothesis II. Serial correlation of jumps

The standard option model assumes no serial correlation of jumps. More complicated models introduce self-excited jumps corresponding to the hypothesis that jumps tend to appear in clusters. For example, (42) allow the jump intensity to escalate in periods of crises in response to major market shocks.

To investigate the serial dependence of jumps, I plot autocorrelation functions on Figure 9. Type I and II jumps do not show significant serial dependence. In contrast, Type III jumps have autocorrelation of roughly -0.25 at the first lag. Moreover, Table VII shows that the smaller the time period between these jumps, the stronger is the negative autocorrelation. Specifically, for multiple jumps per day the autocorrelation is -0.64 and for jumps following each other within

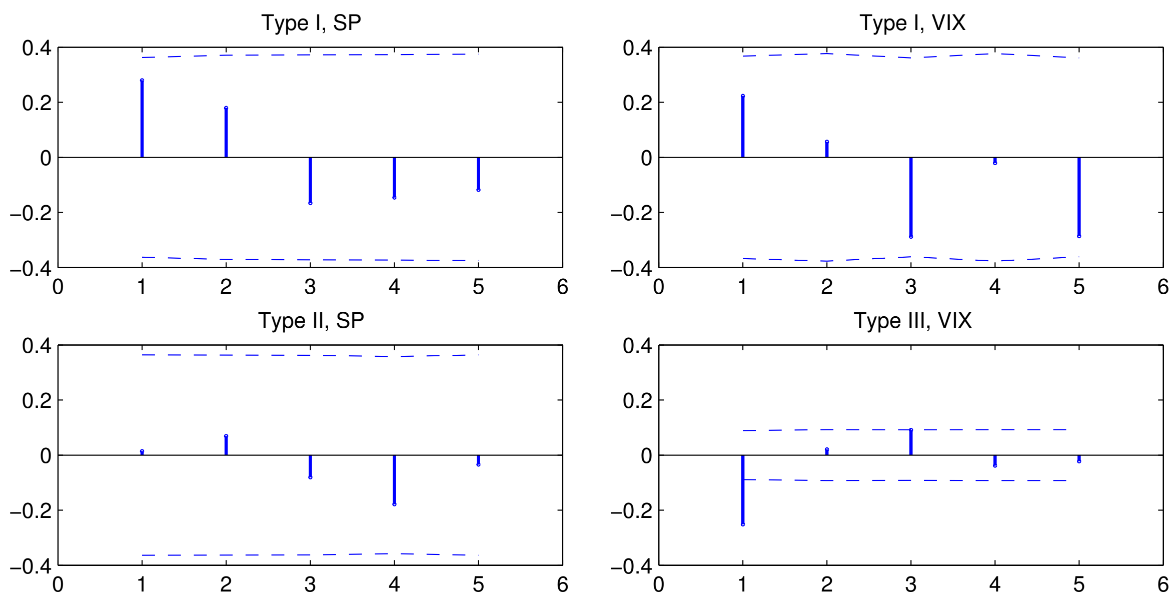

Figure 9. Autocorrelation of jumps of types I, II, and III. Period 2 (1999-2010).

Dashed lines show 95% confidence intervals. Only autocorrelation of Type III jump is significantly different from zero and equal to -0.25 with standard error 0.047.

one week the autocorrelation becomes weaker -0.18. The autocorrelation becomes insignificant when a jump occurs later than one week from the previous one.

In conclusion, Hypothesis II, which states that jumps are serially independent, holds for Type I and II jumps but is rejected for Type III jumps. Specifically, jumps of Type III have statistically significant negative autocorrelation with preceding jumps, and this autocorrelation is much stronger for multiple jumps per day. Such fast mean-reversion of jumps contradicts to

TABLE VII

PROPERTIES OF TYPE III JUMPS DEPENDING ON THE TIME SINCE THE PREVIOUS JUMP

Time span[a]	< day	week	month	> month
Median time	0.01	2.95	11.17	36.03
Count	155	100	147	51
Autocorrelation	-0.64	-0.18	-0.01	-0.06
SE	0.06	0.08	0.08	0.08
Leverage effect[b]	-0.24	-0.45	-0.54	-0.33
SE	0.08	0.07	0.07	0.08

[a] To compute the time between jumps, the pairs of jumps were formed. Every jump, except the first one and the last one, is a member of two pairs. For example, three jumps: # 1: Feb 01 at 9:30 am; # 2: Feb 01 at 1:30 pm; # 3: Feb 10 at 12 pm, form two pairs: (# 1, #2) and (# 2, #3) with time spans: 5.24 days and 9.10 days.

[b] The leverage effect is computed with the simultanuous changes in the S&P.

the common perception of volatility as a persistent process and more likely to be an evidence of noise in the VIX.

6.4 Hypothesis III and the leverage effect

Along with testing Hypothesis III, this section also provides a nonparametric estimate of the diffusive correlation ρ and the jump correlation ρ^J in the model described in Section 5.1. Based on this model, ρ is the correlation of Type IV movements with simultaneous changes in other time series while ρ^J is the correlation for Type I jumps. Noteworthy, the model assumes no correlation for jumps of Type II and III.

Figure 10 presents scatterplots for each type of jumps with least squares regression lines superimposed. Jumps of Type I have the strongest correlation of -0.95 (left upper panel).

Type II jumps have also large in absolute value correlation of -0.87 (left bottom panel). The difference between these two values of correlation is not statistically significant. This might be the evidence that Type II jumps are misclassified Type I jumps and all jumps in price should be modeled as strongly negatively correlated with simultaneous changes in volatility. In our model, it means that the intensity of independent or unmatched price jumps vanishes: $\lambda^p = 0$ and Hypothesis III does not hold for price jumps. In contrast, the weakest correlation of jumps of Type III (right bottom panel) of -0.41 is significantly different from correlations of other types of movements and may be the evidence that the unmatched jumps in the VIX should be accounted in the model $\lambda^v > 0$.

The above decomposition of the leverage effect by categories of stock changes is important because the literature has not agreed yet on the relative contribution of jumps and diffusive components. On the one hand, the recent non-parametric study in (25) argues that "jumps are an important channel for generating leverage effect." This claim is consistent with models generating dynamic leverage effect through jumps, for example, (43) and (44), in which "a negative price jump leads to an increase in the future volatility." On the other hand, (45) argue that "the leverage effect, or asymmetry between returns and volatility, works primarily through the continuous volatility component." Parametric and nonparametric models in Table V, suggest that the jump dependence is slightly stronger than that of diffusive returns, which corresponds to all my results except Type III jumps.

Thus, Type III jumps is the only type of movements in the VIX, which seem to contribute little or diminish the leverage effect. In itself, correlation of -0.41 is not very small in absolute

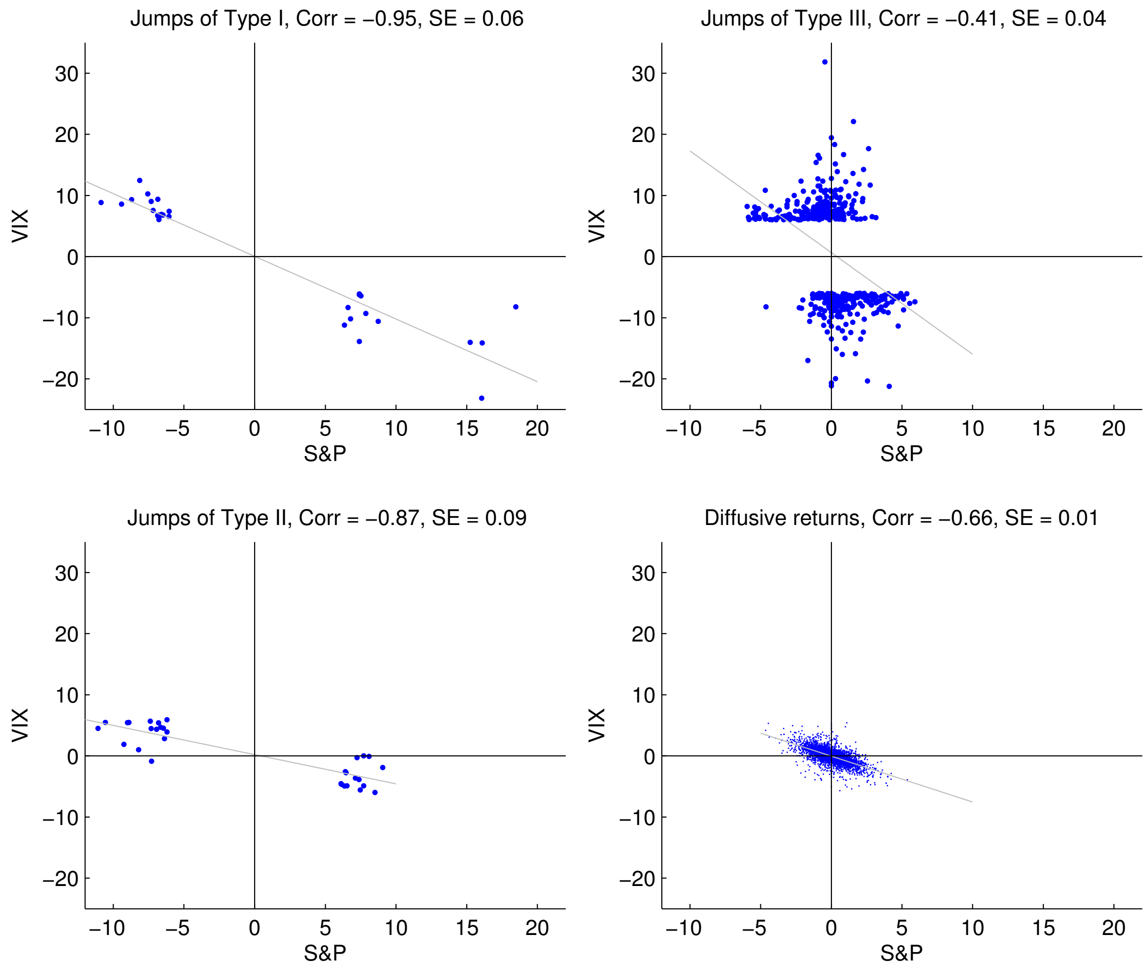

Figure 10. Scatterplots by types of movements in the VIX and S&P 500 (1999-2010). A least-squares line is superimposed on each plot.

value. But let us consider what constitutes this estimate. First, this group contains jumps, which fail to pass the threshold of the jump test only by a little bit (see Figure 16). These jumps are highly correlated with simultaneous returns and should increase the strength of the correlation for Type III jumps. Second, in our model each return always has a diffusive component which has correlation of -0.61 as mentioned before. This fact will also push up the absolute value of the correlation. Therefore, to obtain correlation of -0.41, we should expect a significant amount of Type III jumps which have zero correlation with the simultaneous changes in the S&P, i.e., $\lambda^v > 0$.

Now let us consider whether unmatched jumps in the VIX are possible. On the one hand, the value of the leverage effect for Type III jumps disagrees with the empirical knowledge about the VIX. The strong negative correlation of the changes in the VIX with the returns on the S&P is the most famous and important property of the VIX, the reason why it is called a "fear gauge." This argument has been widely used for portfolio hedging and explains the popularity of VIX derivatives. The CBOE website provides annual estimates of the sample correlation between the two time series in the (-0.85;-0.75) range computed from daily observations during 2004-2009. All literature sources in Table V provide correlation of jumps from -1.0 to -0.53. According to my results, Type I, II and IV are consistent with these estimates.

On the other hand, the literature on parametric models does not completely reject the possibility of independent or unmatched jumps in volatility (hence - not correlated with returns), though some preference is given to simultaneous jumps. Following the model of (26), references (6), (7), (8), (9), and (10) among others, have accounted for the matched jumps in price and volatility along with unmatched jumps. Particularly, using daily settlement prices, reference (8) has shown that the model with correlated jumps in stock prices and stock price volatility is "better in fitting options and returns data simultaneously."

Thus, the literature does not refute the existence of Type III jumps, i.e., it is possible that $\lambda^v > 0$. However, deeper analysis of Type III jumps certainly gives more insight against the plausibility of such extreme and uncorrelated movements. Particularly, it is found that if Hypothesis I and II do not hold for some movements, then the leverage effect is also weaker for those movements. For example, as shown in Table VII, more frequent jumps are characterized by the significantly weaker leverage effect. Multiple jumps per day have the weakest leverage effect of -0.24 while jumps which are one week to one month apart from each other have the leverage effect of -0.54. Therefore, if it is assumed that multiple jumps per day are likely to be noise, then it follows that the strong leverage effect is a property of a real jump. Similar argument also applies to the volatility change after a jump. According to Table VIII, Type III jumps with the significant change of volatility also have a very strong leverage effect (-0.73; -0.67), while the average value of the leverage effect is for Type III jumps is -0.41 (Figure 10). And this correlation is not stronger because these jumps are larger. Conversely, the upper panel of this table shows that their standard deviation of 8.3 is somewhat smaller than 8.7, the metric for all Type III jumps.

In conclusion, Type I and II jumps are characterized by the strong and similar leverage effect. This appears to be the evidence that Hypothesis III does not hold for price jumps and they should be modeled as matched with volatility jumps. The diffusive leverage effect is also

TABLE VIII

DESCRIPTIVE STATISTICS OF TYPE III JUMPS THAT WERE FOLLOWED BY THE SIGNIFICANT CHANGE OF VOLATILITY

Type of jumps[a,b]	Count	Mean	Std.Dev.	Median	Min	Max	Skew	Corr[d]
A. All Type III[c]	454	0.5	8.7	6.1	-21.2	31.8	-0.0	-0.41
B. Including Type III with Signif. Vol. Change[e]								
Type III. SP (volatility change)	33	-0.1	3.1	0	-5.4	5.5	0.15	-0.73
Type III. VIX (volatility change)	33	1.4	7.5	6.2	-13.5	10.8	-0.5	-0.73
Type III. SP (vol of vol change)[f]	31	0.13	3.2	0	-5.9	5.5	-0.01	-0.67
Type III. VIX (vol of vol change)[f]	31	-0.7	8.3	-6.1	-16.0	10.8	-0.1	-0.67

[a] Period 2: 1999-2010.

[b] Jump size is measured in multiples of five-minute daily standard deviations of logarithmic returns.

[c] Panel A: Descriptive statistics for all Type III jumps in Period 2.

[d] "Corr" refers to the leverage effect.

[e] Panel B: Only jumps for which the significant change of volatility after a jump is observed.

[f] "Vol of vol" means "volatility of volatility."

very strong -0.66 and very close to that estimated on daily data (-0.85, 0.75). Only Type III have the weakest leverage effect of -0.41. Moreover, the less counterintuitive are the properties of these jumps the weaker the leverage effect is. This might work against the plausibility of Type III jumps, i.e., it is highly probable that Hypothesis III does not hold for volatility jumps too.

6.5 Hypothesis IV. Do jumps correspond to news announcements?

This section analyzes whether detected jumps correspond to important events and provides the most representative examples of jumps on intraday plots. Table IX shows an attempt to match 10 largest jumps of each type with news announcements, using the Google search engine. Similarly to previous sections, only Type I and II have more reasonable behavior, i.e. it is possible to relate them to news releases. In contrast, jumps of Type III, though very large, seem to happen with no apparent reason. This table also provides time of jumps, which shows that interest rate related announcements usually reach the market around 1:10-1:30 PM Central Time (CT). More detailed description of some days with jumps and related events is given below.

Type I jumps. Figure 11 depicts jumps of opposite sign in both the S&P and VIX at about 12:40 p.m. CT on September 29, 2008. These abrupt movements correspond to a major economic event - the vote of the Congress against Lehman-Brothers' bailout, which has been elaborated on the day before. On Figure 12, the negative jump in the VIX at 12:15 p.m. CT on January 03, 2001 corresponds to the positive jump in the S&P. At about this time the market has been surprised by the unexpected news about the interest rate cut (CNNMoney.com). This day is also indicative because of a negative jump in the VIX, which apparently corresponds to the calming effect of this news. Negative volatility jumps are not specified in commonly used option pricing models; however, this one is a good evidence for models to be less restrictive on the direction of big volatility movements.

TABLE IX

TEN LARGEST JUMPS OF EACH TYPE

Date	S&P, sigma	VIX, sigma	S&P, %	VIX, %	P-v.[a]	Event
Jumps of Type I[b]						
Dec. 11, 2007 1:15 PM	-10.9	8.8	-1.0	5.7		FOMC[c]: decrease target rate
May. 18, 1999 1:10 PM	-9.4	8.6	-0.8	2.7		FOMC: asymmetric directive toward tightening
Dec. 11, 2007 1:20 PM	-8.7	9.3	-0.8	6.0		FOMC: decrease target rate
Sep. 29, 2008 12:40 PM	-8.1	12.5	-3.0	9.2	0.0	House voted against Lehman-Brothers bailout
Nov. 06, 2002 1:30 PM	-7.6	10.2	-1.1	3.5	0.1	Fed Res: interest rate cut
Dec. 21, 1999 1:10 PM	8.7	-10.6	0.9	-3.4	0.0	FOMC: meeting
Sep. 18, 2007 1:15 PM	15.2	-14.0	1.2	-8.8	0.0	Fed Res: interest rate cut
Jan. 03, 2001 12:15 PM	16.1	-23.2	3.7	-11.6	0.3	Fed Res: interest rate cut
Jun. 30, 1999 1:15 PM	16.1	-14.1	1.5	-4.1	0.0	FOMC: symmetrical directive.
Apr. 18, 2001 9:55 AM	18.5	-8.2	2.6	-3.6	0.4	Fed Res: rate cut
Jumps of Type II[b]						
May. 06, 2010 1:40 PM	-11.1	4.5	-2.6	9.1		“Flash Crash”
Aug. 08, 2006 1:20 PM	-10.6	5.5	-0.7	2.0	0.2	Fed skips interest-rate increase
Jun. 25, 2003 1:20 PM	-9.2	1.9	-0.8	0.5	0.0	FOMC: interest rate cut
Nov. 26, 1999 11:50 AM	-9.0	5.5	-0.4	1.4		–
Nov. 16, 1999 1:20 PM	-8.9	5.5	-0.8	2.1	0.0	FOMC: meeting
Mar. 18, 2009 1:20 PM	7.7	-4.9	1.7	-2.0	0.0	FOMC: press release
Jan. 30, 2008 1:15 PM	7.7	0.0	1.0	0.0	0.0	Fed Res: interest rate cut
May. 06, 2010 1:50 PM	8.1	-0.1	1.9	-0.2		“Flash Crash”
Nov. 23, 2007 11:55 AM	8.5	-6.0	0.4	-1.6		Fed Res Statistical Release H.4.1[d]
Apr. 18, 2002 11:20 AM	9.1	-1.9	0.9	-1.0		Fed Res Statistical Release H.4.1[d]
Jumps of Type III[b]						
Sep. 14, 2009 1:25 PM	4.1	-21.2	0.3	-4.3		–
Nov. 09, 2009 12:20 PM	0.0	-21.1	0.0	-4.9		–
Nov. 15, 1999 11:05 AM	0.0	-20.7	0.0	-5.3		–
Dec. 24, 2001 12:10 PM	2.6	-20.3	0.1	-2.5		–
Sep. 15, 2006 2:00 PM	0.3	-20.0	0.0	-6.1		–
Nov. 09, 2009 12:35 PM	2.6	17.7	0.2	4.3		–
Nov. 12, 2007 2:05 PM	0.2	18.3	0.0	16.3		–
Sep. 15, 2006 1:55 PM	0.0	19.5	0.0	6.3		–
Apr. 12, 1999 9:25 AM	1.6	22.1	0.2	9.7	0.4	–
Nov. 15, 1999 11:00 AM	-0.4	31.8	-0.0	8.8		–

[a] P-value for the test of volatility change after a jump (see Section 6.2).

[b] Each type of jumps is represented by 10 largest in absolute value jumps measured in sigma.

[c] FOMC: Federal Open Market Committee.

[d] Factors affecting reserve balances of depository institutions and condition statement of federal reserve banks.

Type II jumps. Figure 13 shows the infamous May 6, 2010 Flash Crash, during which a large drop at around 1:40 p.m. CT is followed by almost immediate recovery 10 minutes later. Figure 14 depicts drop in the S&P at approximately 1:20 p.m. on August 8, 2008 when the news comes that the Federal Reserve would skip the interest rate increase anticipated by the market.

Type III jumps. Figure 1 from Section 1 reports an intraday plot for September 13, 2006. The VIX makes five large moves before 11 a.m. while nothing similar happens in the S&P. The jump test must detect three[1] jumps in the VIX and no jumps in the S&P. The Google search engine provides no information on any event on this day, which can be associated with such movements. Fortunately, on this particular day, the fact that these movements bring the level outside of the daily range as defined by daily min/max reported by the CBOE helps filter these movements out as discussed in Section 3.1 and Appendix B. However, such noise filtering does not help to avoid all mistakes in the data. Another example of unmatched jump in the VIX is depicted by Figure 15. On May 08, 2005 at 11:45 p.m. the VIX suddenly increases from 12.0 to 12.4 but remain in daily bounds (not shown here). In fact, such unmatched jumps which cannot be eliminated based on daily bounds appear quite often especially in the first half of the dataset.

Misclassified jumps. The random nature of the jump test will always produce some misclassification. Figure 16 presents one example of it. The jump in the VIX on February 27,

[1]Only three jumps, because the drop, which is followed by the almmost immediate “recovery” to almost the same level, might not be detected at a 5-minute sampling frequency.

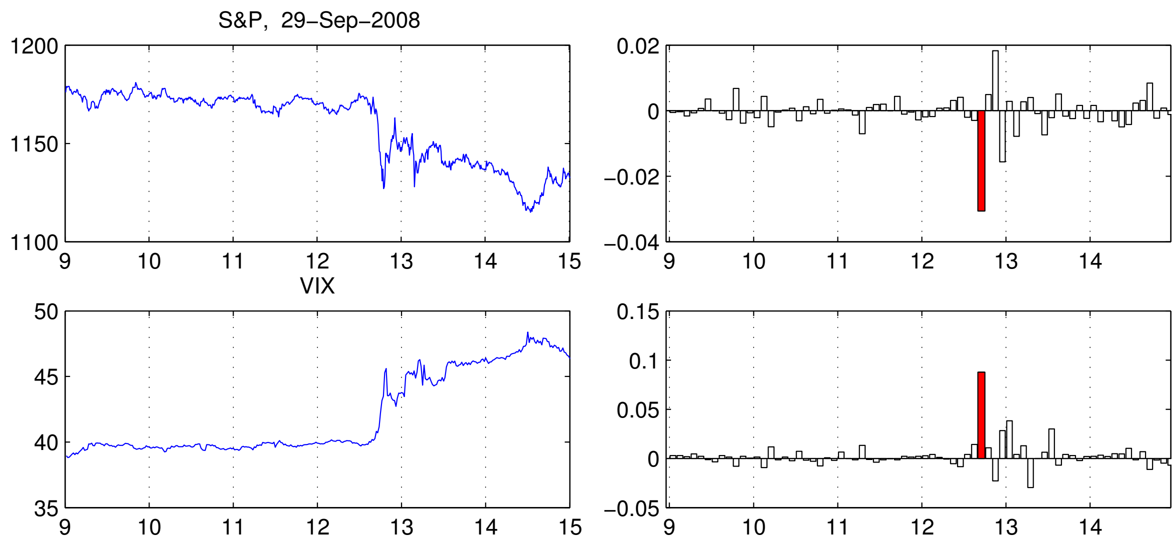

Figure 11. Type I jump on an intraday plot. Matched jumps in the S&P and VIX. September 29, 2008.

The two upper panels refer to the level of the S&P and its 5-min logarithmic returns. The two lower panels represent the same for the VIX. The red bar marks a detected jump. The S&P drops and the VIX raises around 12:40 p.m. These rapid movements appear to correspond to the news arrival that the House has voted against the Lehman-Brothers bailout elaborated by the Congress on the day before.

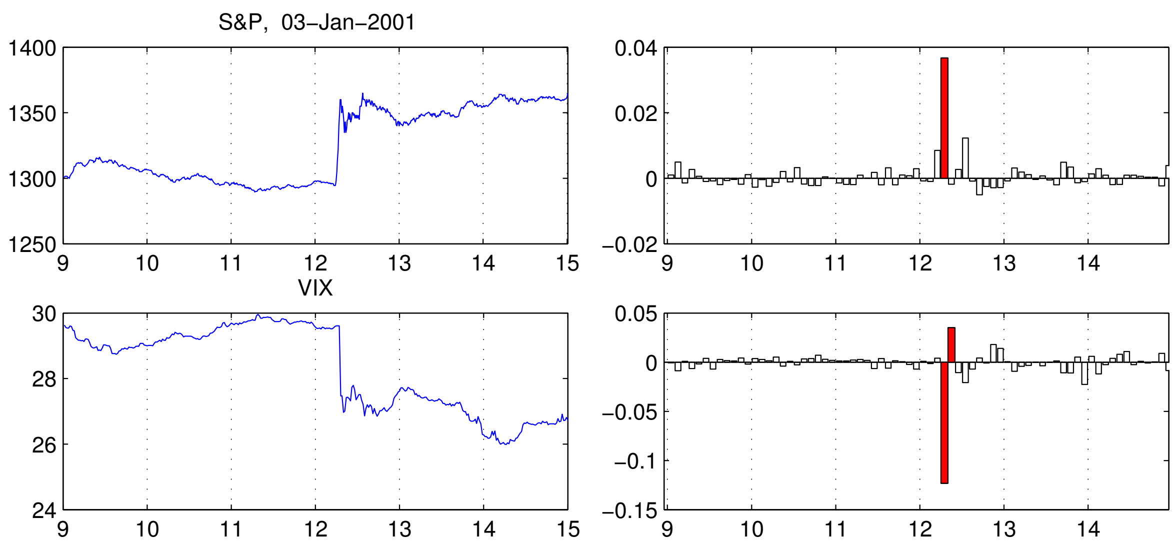

Figure 12. Type I jump on an intraday plot. Matched jumps in the S&P and the VIX. January 03, 2001.

The two upper panels refer to the level of the S&P and its 5-min logarithmic returns. The two lower panels represent the same for the VIX. The red bar marks a detected jump. The negative jump in the VIX at 12:15 p.m. CT corresponds to the positive jump in the S&P. The Federal Reserve made a surprise announcement about the interest rate cut (CNNMoney.com).

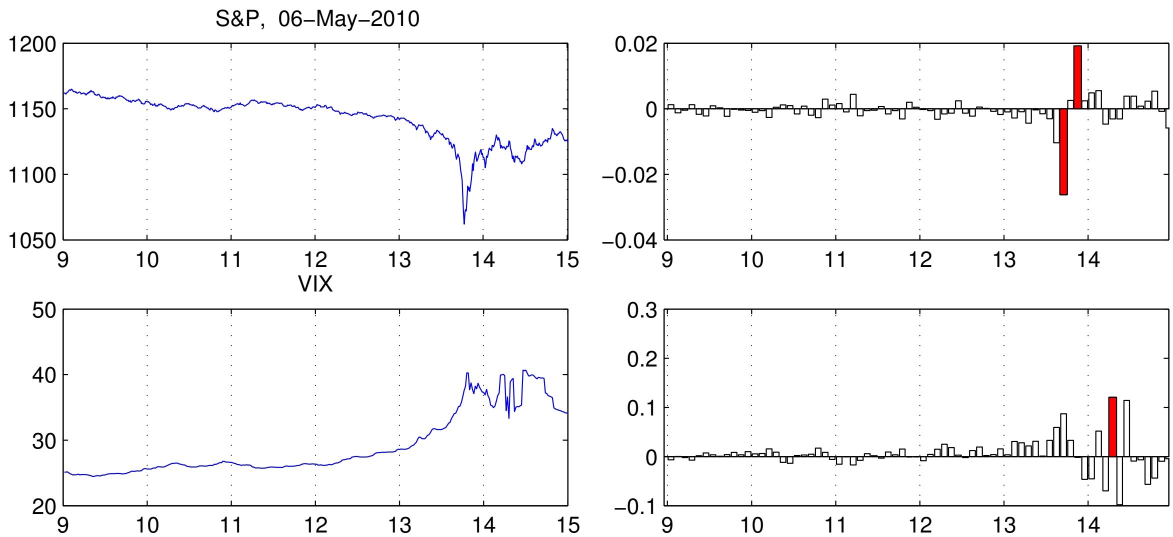

Figure 13. Type II jump on an intraday plot. Unmatched jumps in the S&P. The May 6, 2010 Flash Crash.

The two upper panels refer to the level of the S&P and its 5-min logarithmic returns. The two lower panels represent the same for the VIX. The red bar marks a detected jump.

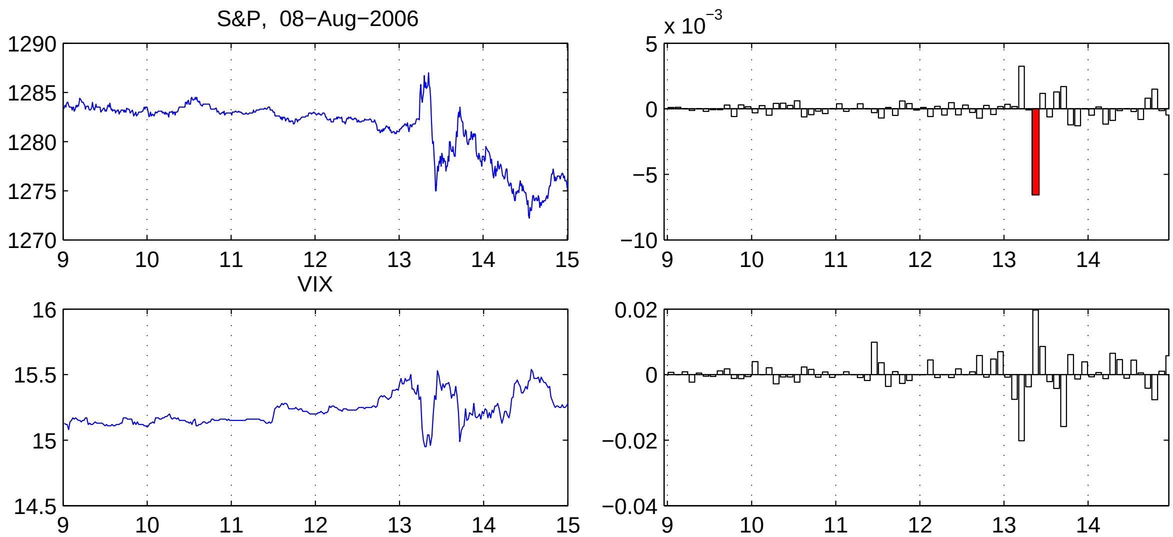

Figure 14. Type II jump on an intraday plot. Unmatched jumps in the S&P on August 8, 2008.

The two upper panels refer to the level of the S&P and its 5-min logarithmic returns. The two lower panels represent the same for the VIX. The red bar marks a detected jump.

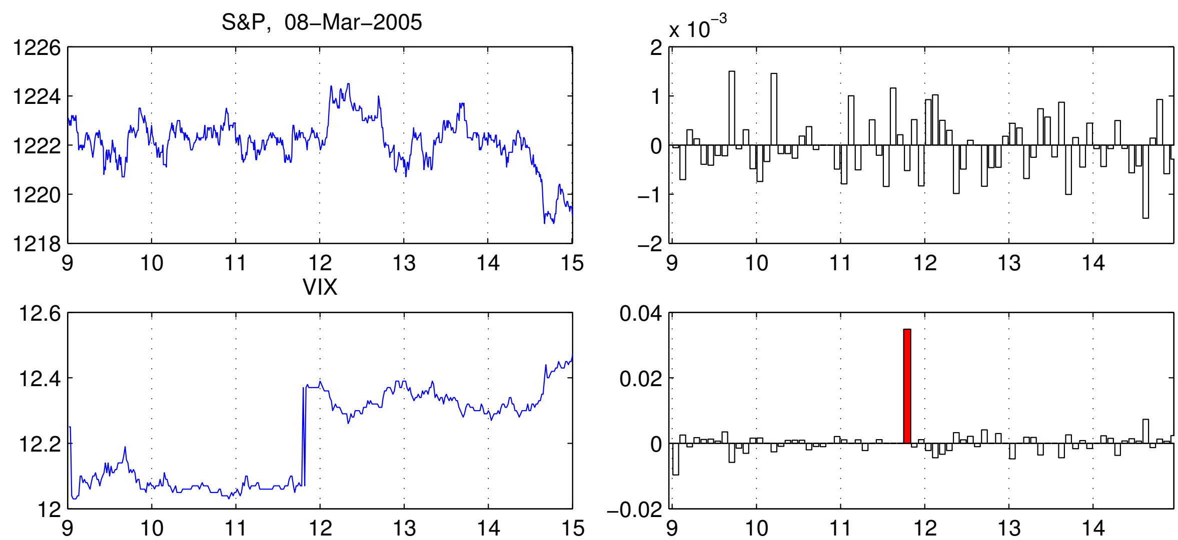

Figure 15. Type III jump on an intraday plot. Unmatched jump in the VIX). March 08, 2005.

The two upper panels refer to the level of the S&P and its 5-min logarithmic returns. The two lower panels represent the same for the VIX. The red bar marks a detected jump.

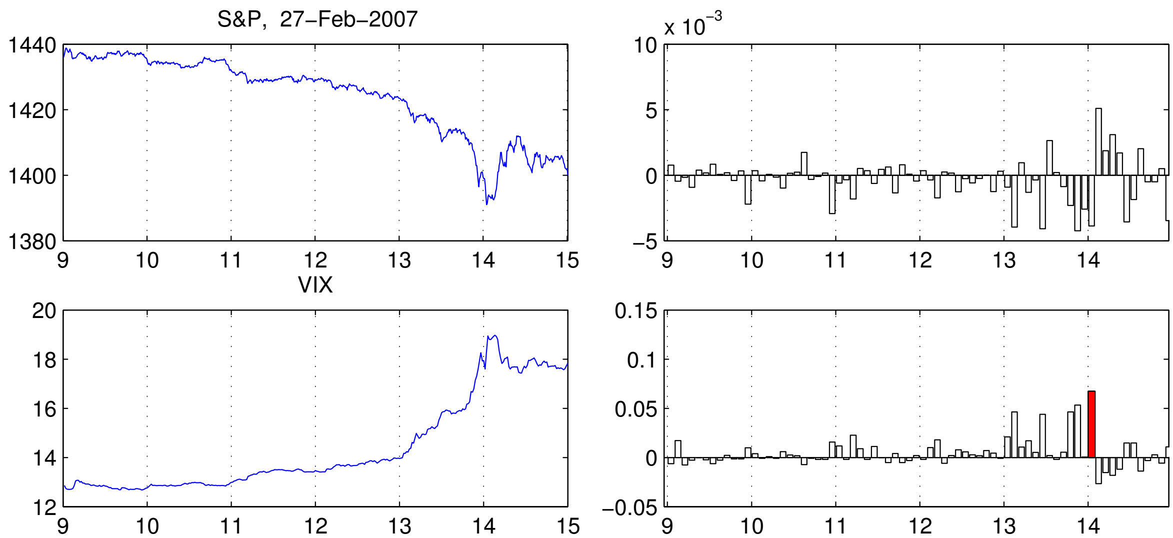

Figure 16. A jump of Type III, which can be classified as Type I. February 27, 2007.

A downward correction in the stock market after a nearly eight-month rally, which pushed the Dow Jones to record highs and and S&P 500 to more than six-year highs. The two upper panels refer to the level of the S&P and its 5-min logarithmic returns. The two lower panels represent the same for the VIX. The red bar marks a detected jump.

2007 has been classified as a Type III jump but could also belong to Type I for the smaller thresholds of the jump test. Indeed, around 2:00 p.m. CT both VIX and S&P move in opposite direction. For the Dow Jones industrial average it was the seventh biggest one-day point drop ever. This massive selling has been triggered by “a big decline in Chinese stocks, weakness in some key readings on the U.S. economy and news that Vice President Dick Cheney was the apparent target in a Taliban suicide bombing attack in Afghanistan” (CNNMoney.com).

In conclusion, jumps of Type I and II can be associated with events, while most Type III jumps cannot. Noteworthy, it seems that if Type III jumps nevertheless can be traced to events, they could be also classified as Type I jumps with the lower threshold of the jump test, as was discussed in the last example about February 27, 2007.

CHAPTER 7

DISCUSSION AND ROBUSTNESS CHECK

7.1 Hypothesis V. Are there *pseudo*-jumps in the VIX?

Table X summarizes the most important findings of the previous section. It is shown that Type I and II jumps, i.e., both matched and unmatched jumps in the S&P, appear to agree with predictions from the standard option pricing model in their frequency, leverage effect, volatility change and serial independence. In contrast, Type III jumps (unmatched jumps in the VIX) challenge the model. Specifically, Hypothesis I (Volatility should change after a volatility jump) and Hypothesis II (Jumps are serially independent) are rejected for Type III jumps. Furthermore, Section 6.5 shows that Type III jumps unlike other types of jumps seem do not correspond to events which are commonly considered to strongly influence the stock market. Besides, the VIX has been criticized in the literature as shown in Section 2.1. Importantly, (14) also report large unexplainable moves in the VIX. In addition, Chapter 4 demonstrates overall inferior quality of the VIX data. Therefore, I suggest that Hypothesis V is true, i.e., Type III jumps contain a large amount of pseudo-jumps.

Alternatively, one can still assume that Hypothesis V is false and all movements in the high-frequency VIX are reliable. Then one should include in the model the unmatched volatility jumps with following properties. First, these jumps are at least eight times more frequent than price jumps. Second, these jumps have strong negative autocorrelation, which is even more negative if the time between jumps is smaller, especially within one day. Finally, the spot volatility does not change after these volatility jumps.

7.2 Robustness check

The characteristics of jumps documented in preceding sections are qualitatively robust under different test designs as shown in Table X.

First, I relax the critical values of the jump test. The need for this adjustment can be justified by examining Figure 16. All jumps in the VIX shown on this figure are classified as Type III jumps because the counterpart in the S&P is not large enough to be selected as a jump, but would be selected for smaller test thresholds. Thus, in Table X "Test threshold = 6/5" means that the matched jumps (Type I) include all following pairs of movements:

- The S&P has a jump > six sigmas in absolute value and the VIX has a simultaneous movement > five sigmas.
- Similarly, the VIX has a jump > six sigmas in absolute value and the S&P has a simultaneous movement > five sigmas.

Panels C, D, and E of Table X reveal that Type III jumps remain different from other types of movements. Overall, relaxing the test threshold brings in more jumps of smaller sizes and decreases the percentage of jumps followed by the volatility change.

Second, I use a different volatility measure, the "trimmed" RV[1]. Panel F of the same table shows the substantial increase of the total number of all jumps in comparison to the standard

[1]Several extreme returns are discarded from the RV estimation. Then RV is augmented as if the tails are cut from a normal distribution.

TABLE X

SUMMARY OF RESULTS FOR DIFFERENT TEST SETTINGS. PROPERTIES OF 5-MIN LOG CHANGES IN THE VIX AND S&P BY TYPES OF MOVEMENTS

Type	S&P	VIX	Count	St. deviation		Leverage Effect	Volatility change[a]		Autocorr	Time
				S&P	VIX	(SE)[b]	S&P	VIX	(SE)	trend
A. Period 1.[c] Test threshold = 6/6										
I	J[d]	J	4	16.69	15.52	-0.80 (0.43)	1.00	1.00	0.00 (0.00)	No
II	J	NJ[e]	35	7.10	3.05	-0.72 (0.12)	0.44	0.22	-0.09 (0.10)	No
III	NJ	J	1309	1.31	10.93	-0.11 (0.03)	0.10	0.15	-0.18 (0.03)	Yes
IV	NJ	NJ	125732	1.05	1.20	-0.24 (0.003)				
B. Period 2. Test threshold = 6/6										
I	J	J	29	9.40	10.16	-0.95 (0.06)	0.50	0.50	0.28 (0.18)	No
II	J	NJ	31	7.73	4.23	-0.87 (0.09)	0.35	0.25	0.01 (0.19)	No
III	NJ	J	454	2.13	8.66	-0.41 (0.04)	0.15	0.14	-0.25 (0.05)	No
IV	NJ	NJ	205694	1.03	1.15	-0.66 (0.002)				
C. Period 2. Test threshold[f] = 6/5										
I	J	J	58	7.94	8.53	-0.95 (0.04)	0.40	0.38	0.21 (0.13)	No
II	J	NJ	23	7.53	3.62	-0.83 (0.12)	0.31	0.25	0.15 (0.13)	No
III	NJ	J	433	1.83	8.74	-0.38 (0.04)	0.14	0.13	-0.32 (0.05)	No
IV	NJ	NJ	205694	1.03	1.15	-0.66 (0.002)				
D. Period 2. Test threshold = 6/4										
I	J	J	89	7.21	8.22	-0.90 (0.05)	0.33	0.26	0.18 (0.11)	No
II	J	NJ	13	7.61	2.44	-0.72 (0.21)	0.27	0.45	0.08 (0.30)	No
III	NJ	J	412	1.57	8.74	-0.35 (0.05)	0.15	0.14	-0.31 (0.05)	No
IV	NJ	NJ	205694	1.03	1.15	-0.66 (0.002)				
E. Period 2. Test threshold = 5/5										
I	J	J	68	7.64	8.17	-0.95 (0.04)	0.38	0.38	0.40 (0.11)	No
II	J	NJ	68	6.06	3.39	-0.86 (0.06)	0.31	0.24	-0.09 (0.12)	No
III	NJ	J	828	2.00	7.35	-0.47 (0.03)	0.10	0.09	-0.22 (0.03)	No
IV	NJ	NJ	205244	1.02	1.12	-0.67 (0.00)				
F. Period 2. Trimmed RV. Test threshold = 6/6										
I	J	J	57	9.57	10.47	-0.90 (0.06)	0.45	0.40	0.25 (0.13)	No
II	J	NJ	48	7.33	3.79	-0.83 (0.08)	0.38	0.32	0.03 (0.15)	No
III	NJ	J	723	2.29	8.96	-0.45 (0.03)	0.13	0.13	-0.27 (0.04)	No
IV	NJ	NJ	205380	1.08	1.24	-0.66 (0.00)				

[a] Fraction of jumps for which the change in volatility is statistically significant at 5%.

[b] SE - standard error.

[c] Period 1: from Jan 01, 1992 to Dec 31, 1998. Period 2: from Jan 01, 1999 to Jun 30, 2010.

[d] J - this return is selected as a jump by a jump test.

[e] NJ - this return is not selected as a jump by a jump test.

[f] Test threshold = 6/5 means that, a jump is of Type I if its size >= 6 sigma while its counterpart >= 5 sigma.

test setting in Panel B. Nevertheless, the statistical properties of Type III jumps are still stand out against other types of jumps.

CHAPTER 8

CONCLUSION

Using a long sample of high-frequency data, I conduct a comprehensive study of jumps in the VIX and document their statistical properties. I find that 94% of jumps in the VIX exhibit a number of puzzling characteristics, which challenge assumptions of canonical option pricing models. Particularly, the time series of these jumps has a strong negative autocorrelation, which seems to be a characteristic of noise. Moreover, the smaller the time period between such jumps the stronger is the negative autocorrelation. In addition, these jumps do not correspond to the change in the spot volatility. Furthermore, these jumps unlike other types of jumps seem to not correspond to events which are commonly considered to strongly influence the stock market.

I run various diagnostic tests and consider alternative explanations for my findings. I conclude that the most plausible explanation is that many unmatched jumps in the VIX are measurement errors, or *pseudo-jumps*, which might stem from flaws in calculation of VIX reported in the literature. The presence of these highly influential outliers might considerably distort the inference about volatility dynamics. For example, the jumpiness of volatility might be overstated while the leverage effect might be understated. As a result, high-frequency studies of VIX might benefit from preliminary data cleaning. I leave developing such procedure for future research. This task is very important because would allow to use the complete dataset.

This study has one important limitation, which could be addressed in the future. The methodological framework assumes that the behavior of the VIX is completely determined by the dynamics of the underlying process under the physical probability measure. Such approach assumes that jumps in the unobserved spot volatility and the VIX should happen at the same time and their sizes are related through a nondecreasing function. Furthermore, I assume that properties of volatility jumps are similar to those of VIX jumps. However, according to some recent studies such as (46), option prices may bear an extra risk factor relating to changes, for example, in the market risk aversion. In theory, this factor may cause more complex dynamics of the VIX resulting particularly in jumps only in the VIX with no relation to the spot volatility. Nevertheless, to reject the hypothesis of pseudo-jumps these extra jumps in the VIX should bear all strange properties of pseudo-jumps - occur several times per day, have the negative autocorrelation, and do not associate with news arrivals. Importantly, these unmatched jumps should be inhomogeneous with respect to these properties. For example, the autocorrelation should be more negative if the jumps are closer to each other.

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APPENDICES

Appendix A

ANNUAL NUMBER OF JUMPS

TABLE XI

ANNUAL NUMBER OF JUMPS

Year	S&P	VIX	Common jumps	S&P[a] 6-9	S&P[a] 9-12	S&P[a] >12	VIX 6-9	VIX 9-12	VIX >12
1992	5	268	1	5	0	0	140	48	80
1993	4	330	0	4	0	0	181	69	80
1994	5	238	0	5	0	0	127	48	63
1995	5	231	1	4	1	0	124	59	48
1996	9	113	1	7	2	0	67	20	26
1997	5	80	0	5	0	0	55	15	10
1998	3	59	1	1	1	1	40	11	8
1999	9	57	5	7	1	1	44	6	7
2000	3	51	3	3	0	0	42	8	1
2001	4	28	3	2	0	2	16	7	5
2002	5	24	2	4	1	0	20	4	0
2003	5	27	0	4	1	0	21	6	0
2004	3	41	1	3	0	0	34	5	2
2005	6	65	3	6	0	0	51	10	4
2006	6	45	3	5	1	0	34	8	3
2007	9	30	5	7	1	1	16	7	7
2008	3	31	1	3	0	0	24	4	3
2009	6	58	2	6	0	0	45	5	8
2010	2	15	0	1	1	0	13	2	0
Total	97[b]	1791	32	82	10	5	1094	342	355

[a] Number of jumps of different sizes measured in multiples of robust standard deviation.

[b]This small amount of jumps in the S&P seems to agree with structural models which reveal rare jumps ($\sim$ 5.1 per year).

Appendix B

DATA ISSUES. AGREEMENT WITH THE CBOE DAILY DATA

I compare the high-frequency VIX (henceforth: HF VIX) to daily min/max/open/close downloaded from the CBOE web-site (henceforth: Daily VIX). The HF VIX has 32 days less than the Daily VIX. Though the CBOE guarantees no accuracy of the Daily VIX data, it seems that most times this information helps detect erroneous observations in the HF VIX data set. I find that both VIX data sets agree until the start of the computation of the VIX in real time on September 22, 2003. Apparently, the early Daily VIX is extracted from the HF VIX, which is in turn computed from historical option prices. However, in a "real-time era" the Daily VIX and HF VIX are quite often inconsistent. Moreover, I discover that omitting the HF VIX data values outside the daily range records of the Daily VIX is quite reasonable because most of such values disagree with adjacent data and market events. As a result of this procedure the VIX kurtosis lowers significantly from 3760 to 200 in year 2004, for example. Tables Table XIV and Table XV report the descriptive statistics for the raw data while tables Table XII and Table XIII report the same for the data pre-filtered as described below:

1. Omit open prices of the VIX, which are incorrect quite often.

2. Omit the first 30 minutes of all trades before September 22, 2003.

3. Omit values outside the daily range.

I observe following issues with the data:

1. From the beginning of my sample period through September 19, 2003 (inclusive) the VIX has strange pieces of constant values before 9 a.m. and after 3 p.m. An example of this behavior is demonstrated on Figure 17. These flats are often inconsistent with other data, i.e. the sample

path is far from being smooth at exactly 9 a.m. and 3 p.m. Such peculiarity of data may induce spurious jumps.

2. Since September 19, 2003 the VIX is available before 9 a.m. but this start of the day period has a lot of erratic movements which make the intraday jumps indistinguishable.

3. The movements in the first 30 minutes are so erratic that they disagree with data in adjacent time periods. For example, see Figure 1 and Figure 19. As shown on Figure 19 we sometimes can avoid errors in the VIX as large as 250% of its daily value.

4. The lack of the VIX data after 3 p.m. is persistent through the whole data set, even after it starts to be computed in real time on September 22, 2003.

Several exemplary days are shown on Figure 17. The lower panel of this figure represents intraday evolution of the S&P futures and the VIX from the HF VIX. On October, 28, 1997 the VIX stays flat at the level of 45% within first 30 minutes after opening and around 9 a.m. makes an abrupt upward movement to almost 50. It appears that the S&P follows relatively smooth path during this time, without flats and nothing noteworthy at 9 a.m. For these four days, open and closing prices of the VIX correspond to those downloaded from the CBOE website as well as daily bounds.

Figure 18 gives an example of a day when open and closing prices are inconsistent. Specifically, the open price of the VIX on October 24, 2008 is equal to the closing price on the day before which disagrees with the HF VIX. In this case, the HF VIX looks more reasonable because the S&P makes an overnight drop and, therefore, the VIX should increase overnight as well. Further, the lower bound on October 24, 2008 is derived from the wrong open price. On October 27, 2011 the intraday VIX values reach neither the lower nor upper bounds downloaded from the CBOE website. I make no changes to my data in this case.

Appendix B (Continued)

Total number of cases when the HF VIX is outside the daily range from Daily VIX is 70. Such instances are observed starting from 21-Jan-2004 to the end of the dataset. Most of these cases resemble the one shown on Figure 19. It is suspicious that the VIX reaches 34% on July 25, 2005 immediately after opening while the S&P slowly fluctuates between 1230 and 1240 within less than 1% of its level. Moreover, the plots on Figure 19 show that the VIX is around 10-12 all other days. Thus such deviations of the VIX up to 34% are suspicious. There are a lot of days when only the opening price is outside the range. Most of such cases happen in the first 15 min of the trading day though some are in the middle of the day. But middle day instances always look like the one on Figure 20 (lower panel).

In conclusion, omitting the intraday VIX values outside the daily range provided on the CBOE website seems to be quite reasonable.

Appendix B (Continued)

TABLE XII

DESCRIPTIVE STATISTICS AFTER PRE-FILTERING. LEVEL

	S&P 500 futures, 30 sec[a]				VIX, 1 min[a]			
Year	Mean	St.Dev.	Min	Max	Mean	St.Dev.	Min	Max
1992	416	8.55	390	444	15.5	2.14	10.3	25.1
1993	452	10.2	427	473	12.7	1.31	8.89	18.3
1994	461	9.59	435	483	13.9	2.1	9.59	28.3
1995	544	46.4	460	630	12.4	1.01	10.1	17
1996	673	38.5	598	767	16.4	1.91	11.1	27.1
1997	878	76.9	733	992	22.4	4.16	16.4	48.6
1998	1091	67.7	918	1257	25.7	6.95	16.1	49.5
1999	1335	58.5	1210	1490	24.4	2.87	17.1	33.7
2000	1440	57.6	1271	1574	23.4	3.43	16.3	34.3
2001	1198	88.5	939	1390	25.9	4.88	18.7	49.4
2002	994	115	768	1178	27.4	6.95	17	48.5
2003	960	77.4	788	1111	22.2	5.27	15.5	35.7
2004	1130	32.2	1060	1220	15.5	1.89	11.1	22.6
2005	1210	30.3	1137	1284	12.8	1.42	9.89	18.6
2006	1316	51.6	1230	1444	12.8	2.18	9.39	23.8
2007	1483	45.3	1372	1586	17.5	5.36	9.71	37.4
2008	1230	188	740	1480	31.9	15.9	15.8	87.8
2009	945	115	666	1126	31.6	9.1	19.3	57.4
2010	1126	44.5	1024	1216	23.2	6.45	15.2	48.2

[a] Sampling interval.

TABLE XIII

DESCRIPTIVE STATISTICS AFTER PRE-FILTERING. LOGARITHMIC CHANGES

Year	S&P 500 futures, 30 sec[a]							VIX, 1 min[a]						
	Mean 10^{-7}	St.Dev. 10^{-4}	Min 10^{-3}	Max 10^{-3}	Skew.	Kurt.	Zeros[b] %	Mean 10^{-6}	St.Dev. 10^{-3}	Min 10^{-2}	Max 10^{-2}	Skew.	Kurt.[c]	Zeros[b] %
1992	3.91	2.29	-4.79	5.12	-0.0975	9.3	26.9	47.5	4.22	-8.57	15.8	5.64	198	53
1993	1.31	1.99	-3.26	3.49	-0.114	7.06	28.2	9.01	5.32	-15.7	19.5	2.1	131	51.9
1994	-1.52	2.23	-11.4	2.97	-0.792	41.9	24.9	26.7	4.6	-9.34	18.6	3.14	141	43.7
1995	11.8	1.94	-4.46	2.73	-0.191	8.35	23.4	29.8	4.55	-16.7	16.2	1.54	131	43.5
1996	3.38	2.63	-5.57	4.06	-0.245	13.3	18	22	2.77	-16.5	18	3.67	478	35.5
1997	5.63	3.67	-7.83	5.65	-0.133	11.8	16.7	-3.38	2.43	-12.8	16.3	1.83	382	25.2
1998	5.69	4.25	-9.57	9.57	-0.0461	16.4	17.9	-6.72	2.24	-4.59	5.58	0.621	66.7	22.8
1999	-0.928	3.75	-11.6	8.1	-0.248	12.6	17.7	-1.98	2.17	-3.84	12.2	4.58	272	23.5
2000	-8.14	4.44	-12.3	11	-0.164	12.9	21	-11.4	1.89	-6.52	4.64	-0.275	52.2	25
2001	1.4	4.35	-15.9	21.6	0.601	60.8	20.8	-27.6	1.78	-7.5	5.98	-0.244	86.1	22
2002	-5.47	4.98	-6.27	5.69	-0.000906	7.01	19.3	-21.2	2.02	-3.49	3.88	0.00546	20	19
2003	9.22	3.64	-6.63	4.85	-0.0564	7.45	23.4	-27.1	1.68	-4.27	3.56	0.105	33.8	22
2004	3.67	2.43	-2.25	3.13	-0.046	6.08	26.4	-20.1	2.22	-11	7.33	-1.91	200	24.8
2005	-4.06	2.19	-1.72	2.13	0.0373	5.8	27.6	-6.65	2.57	-14.2	16	1.51	302	27.2
2006	3.02	2.15	-3.85	2.36	-0.0122	7.57	28.3	-18.8	2.37	-8.99	8.86	0.186	109	28.3
2007	-2.15	2.96	-8.24	7.31	0.042	17	25.3	-9.73	3.05	-21.3	21.2	1.08	738	19.5
2008	-12.7	7.38	-14.1	12.9	0.146	19	22.5	-11.9	3.19	-21.8	21.3	0.978	693	14.6
2009	13	4.96	-5.99	8.96	0.0983	9.69	32.8	-18.7	2.18	-6.88	12.2	2.37	237	19.4
2010	-5.5	3.78	-12.1	9.21	-0.336	41.8	39.8	-8.36	3.04	-21.1	20.7	-0.469	1.34e+03	22.5

[a] Sampling interval.

[b] Percentage of zero returns. The staleness of the S&P is higher in later years because the liquidity is gradually moving to Globex and sometimes transactions are made without change of the price.

[c] Kurtosis of the VIX is lower in comparison to Table XV, Appendix B, where the descriptive statistics before pre-filtering are presented.

Appendix B (Continued)

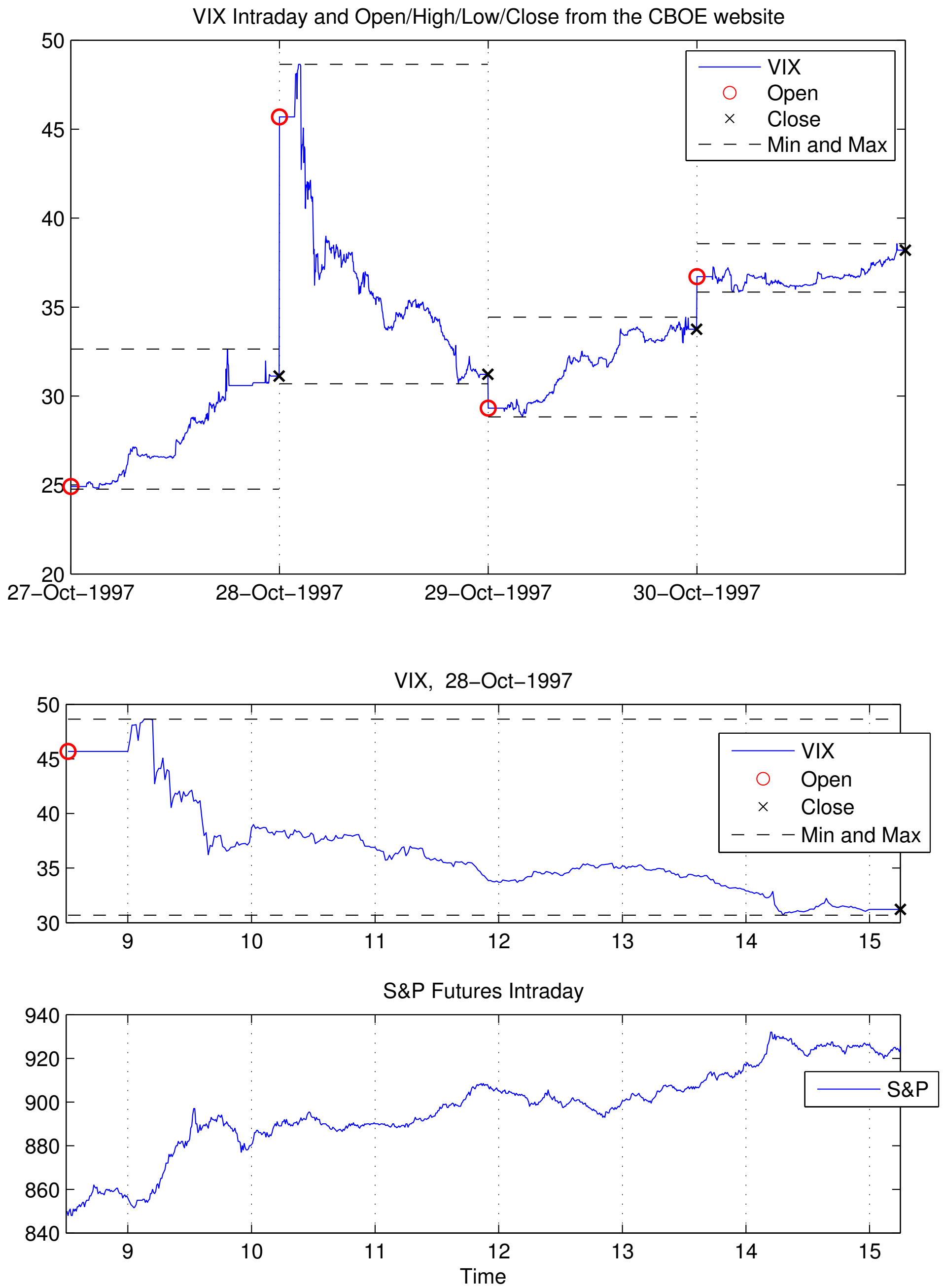


Figure 17. VIX. An example of a "flat" price in the morning.

The first 30 minutes of the VIX are almost always flat (from 8:30 to 9:00 a.m.). The same applies to the last 15 minutes (from 3:00 to 3:15 p.m.) This problem persists until September 19, 2003 (the last day before the change of the VIX valuation method) and often after.

Appendix B (Continued)

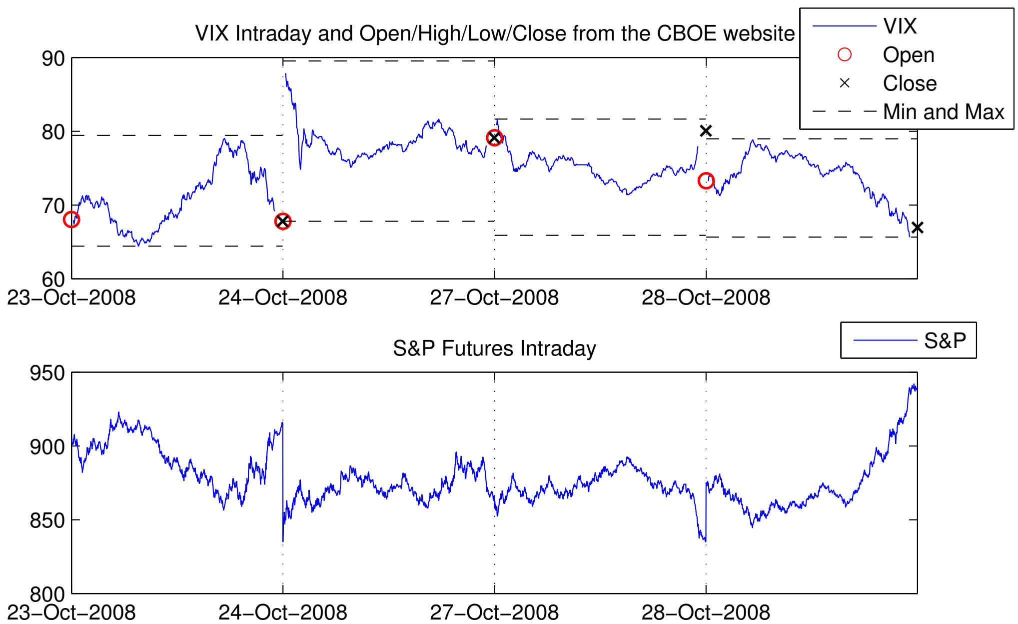


Figure 18. VIX. An example of disagreements of the CBOE data.

Upper panel: On most volatile days the CBOE daily Open/Close/Min/Max disagree with the high-frequency dataset on the VIX.
Lower panel: the S&P 500 futures.

Appendix B (Continued)

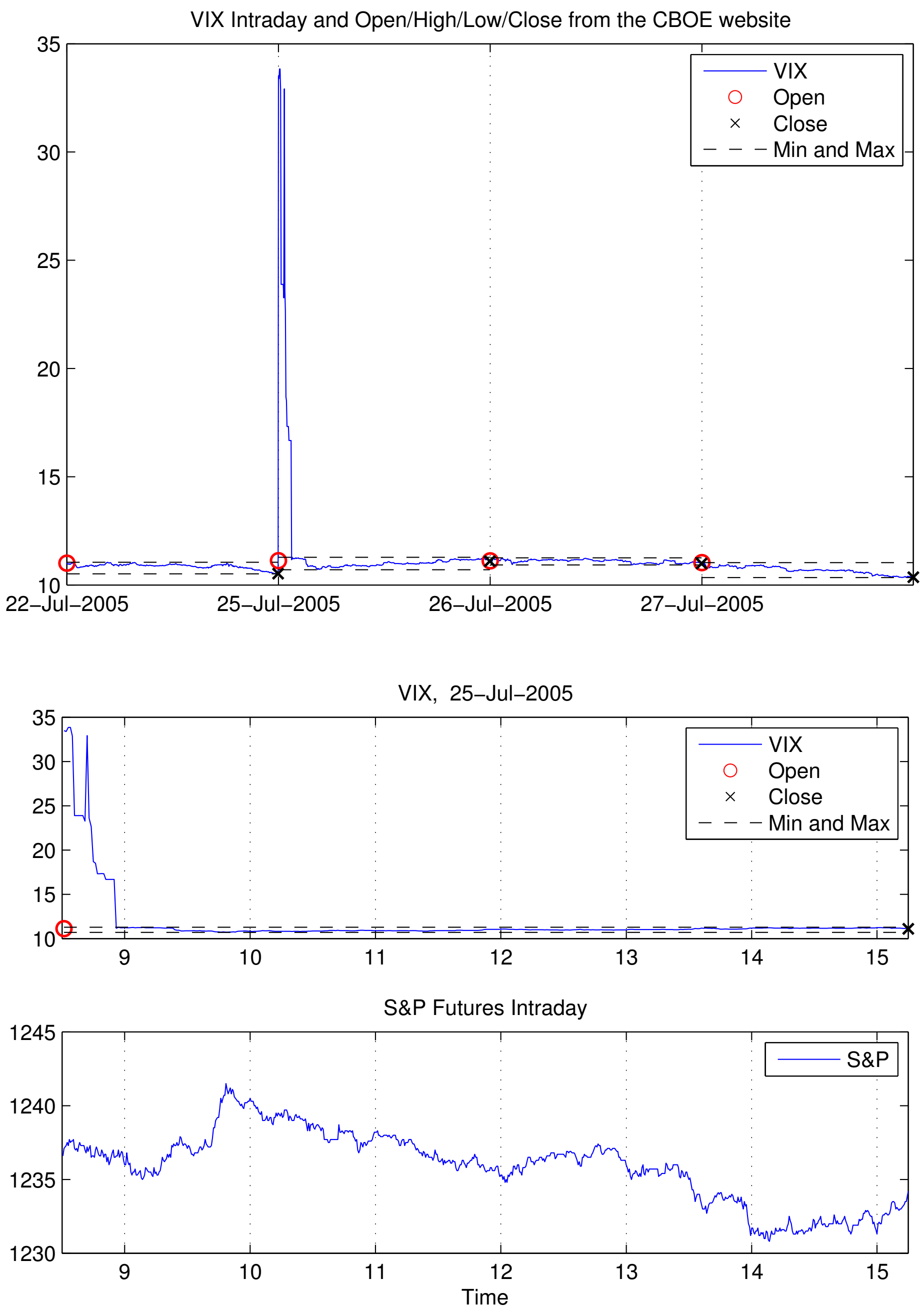
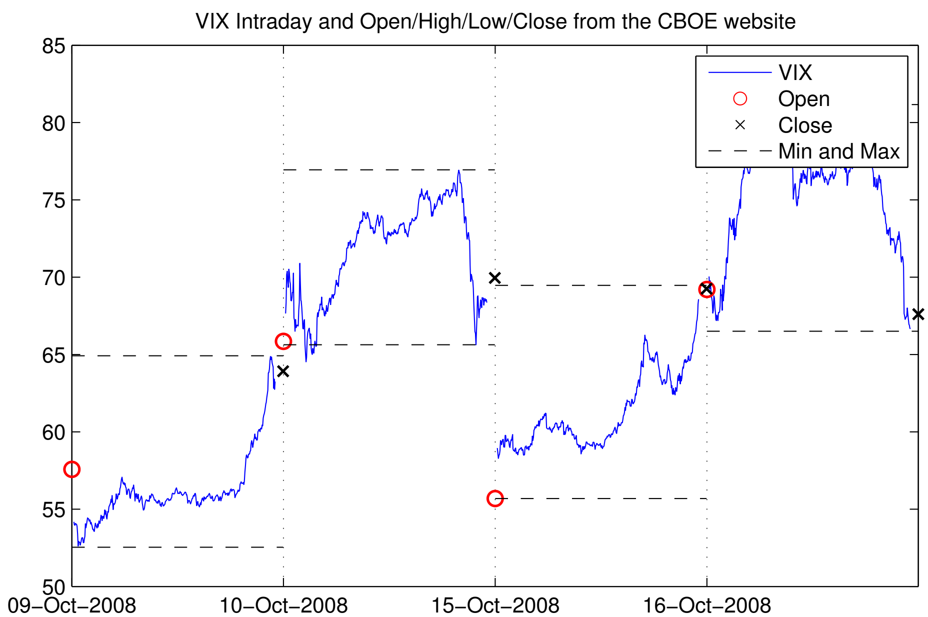


Figure 19. VIX. Example of a large strange movement.

The morning spike on July 25, 2005 appears to be suspicious.
The upper panel shows how the intraday trajectory of the VIX on July 25 compares with the days before and after. The middle panes shows only July 25. The lower panel reports the S&P 500 futures.

Appendix B (Continued)



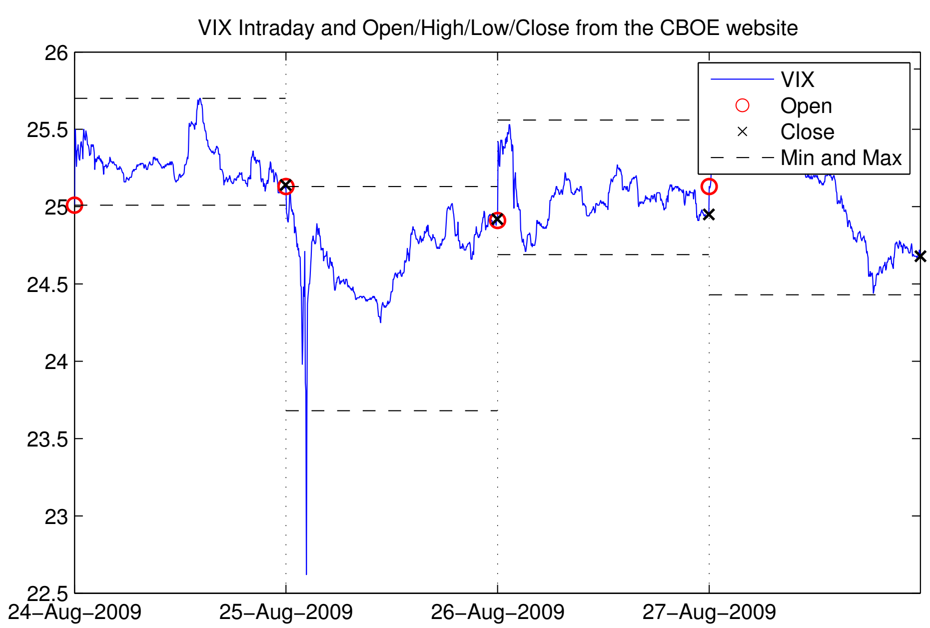


Figure 20. VIX. Minima are below than those reported by the CBOE in the daily dataset.

Upper panel: Minima in the HF VIX data on October 10, 2008 and August 25, 2009 are below those in the Daily VIX data. Lower panel: The downward spike of the HF VIX data on August 25, 2009 is outside of the range of the Daily VIX data.

Appendix B (Continued)

TABLE XIV

DESCRIPTIVE STATISTICS BEFORE PRE-FILTERING. LEVEL

	S&P 500 futures,[a] 30 sec[b]				VIX,[a] 1 min[b]			
Year	Mean	St.Dev.	Min	Max	Mean	St.Dev.	Min	Max
1992	416	8.55	390	444	15.5	2.14	10.3	25.1
1993	452	10.2	427	473	12.7	1.31	8.89	18.3
1994	461	9.59	435	483	13.9	2.1	9.59	28.3
1995	544	46.4	460	630	12.4	1.01	10.1	17
1996	673	38.5	598	767	16.4	1.91	11.1	27.1
1997	878	76.9	733	992	22.4	4.16	16.4	48.6
1998	1091	67.7	918	1257	25.7	6.95	16.1	49.5
1999	1335	58.5	1210	1490	24.4	2.87	17.1	33.7
2000	1440	57.6	1271	1574	23.4	3.43	16.3	34.3
2001	1198	88.5	939	1390	25.9	4.88	18.7	49.4
2002	994	115	768	1178	27.4	6.95	17	48.5
2003	960	77.4	788	1111	22.2	5.27	15.5	35.7
2004	1130	32.2	1060	1220	15.5	1.91	2.85	36.5
2005	1210	30.3	1137	1284	12.8	1.44	9.89	33.8
2006	1316	51.6	1230	1444	12.8	2.18	8.6	41.6
2007	1483	45.3	1372	1586	17.5	5.36	9.71	37.4
2008	1230	188	740	1480	31.9	15.9	15.8	87.8
2009	945	115	666	1126	31.6	9.1	18.5	57.4
2010	1126	44.5	1024	1216	23.2	6.45	15.2	48.2

[a] Only records within first 30 min of every day are removed before Sept 22, 2003.

[b] Sampling interval.

TABLE XV

DESCRIPTIVE STATISTICS. LOGARITHMIC CHANGES. WITHOUT OVERNIGHT RETURNS AND FIRST 30 MIN BEFORE SEPT 22, 2003

Year	S&P 500 futures, 30 sec[a]							VIX, 1 min[a]						
	Mean 10^{-7}	St.Dev. 10^{-4}	Min 10^{-3}	Max 10^{-3}	Skew.	Kurt.[b]	Zeros[c] %	Mean 10^{-6}	St.Dev. 10^{-3}	Min 10^{-2}	Max 10^{-2}	Skew.	Kurt.[b]	Zeros[c] %
1992	3.91	2.29	-4.79	5.12	-0.0975	9.3	26.9	47.5	4.22	-8.57	15.8	5.64	198	53
1993	1.31	1.99	-3.26	3.49	-0.114	7.06	28.2	9.01	5.32	-15.7	19.5	2.1	131	51.9
1994	-1.52	2.23	-11.4	2.97	-0.792	41.9	24.9	26.7	4.6	-9.34	18.6	3.14	141	43.7
1995	11.8	1.94	-4.46	2.73	-0.191	8.35	23.4	29.8	4.55	-16.7	16.2	1.54	131	43.5
1996	3.38	2.63	-5.57	4.06	-0.245	13.3	18	22	2.77	-16.5	18	3.67	478	35.5
1997	5.63	3.67	-7.83	5.65	-0.133	11.8	16.7	-3.38	2.43	-12.8	16.3	1.83	382	25.2
1998	5.69	4.25	-9.57	9.57	-0.0461	16.4	17.9	-6.72	2.24	-4.59	5.58	0.621	66.7	22.8
1999	-0.928	3.75	-11.6	8.1	-0.248	12.6	17.7	-1.98	2.17	-3.84	12.2	4.58	272	23.5
2000	-8.14	4.44	-12.3	11	-0.164	12.9	21	-11.4	1.89	-6.52	4.64	-0.275	52.2	25
2001	1.4	4.35	-15.9	21.6	0.601	60.8	20.8	-27.6	1.78	-7.5	5.98	-0.244	86.1	22
2002	-5.47	4.98	-6.27	5.69	-0.000906	7.01	19.3	-21.2	2.02	-3.49	3.88	0.00546	20	19
2003	9.22	3.64	-6.63	4.85	-0.0564	7.45	23.4	-27.1	1.68	-4.27	3.56	0.105	33.8	22
2004	3.67	2.43	-2.25	3.13	-0.046	6.08	26.4	-18.4	7.03	-85	170	111	3.76e+04	24.9
2005	-4.06	2.19	-1.72	2.13	0.0373	5.8	27.6	-68.2	6.13	-81.6	34.7	-67.2	6.9e+03	27.2
2006	3.02	2.15	-3.85	2.36	-0.0122	7.57	28.3	-43.2	5.83	-113	23.2	-148	2.83e+04	28.3
2007	-2.15	2.96	-8.24	7.31	0.042	17	25.3	-19.1	5.06	-87.9	60.8	-43.1	1.25e+04	19.5
2008	-12.7	7.38	-14.1	12.9	0.146	19	22.5	-15.3	3.79	-32	22.7	-6.88	1.6e+03	14.6
2009	13	4.96	-5.99	8.96	0.0983	9.69	32.8	-12	2.64	-15.5	15.5	3.85	739	19.4
2010	-5.5	3.78	-12.1	9.21	-0.336	41.8	39.8	-8.36	3.04	-21.1	20.7	-0.469	1.34e+03	22.5

[a] Sampling interval.

[b] Kurtosis. It is strange:

1. the S&P has very high percentage of zero returns in 2009-2010 and relatively large kurtosis.
2. the VIX in 2004-2010 has also very high kurtosis. After removing values outside of the range reported CBOE website the kurtosis is much lower (Table XIII).

[c] Percentage of zero returns. The staleness of the S&P is higher in later years because the liquidity is gradually moving to Globex and sometimes transactions are made without change of the price.

VITA

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EDUCATION

Baltic State Technical University (former Mechanical Institute of Leningrad), St. Petersburg, Russia

M.S., Aerospace Engineering , 1990, *Summa cum Laude*, GPA 4.0/4.0

St. Petersburg State University, St. Petersburg, Russia

M.S., Economics: World Economy, 1998

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Ph.D., Business Administration: Business Statistics, 2014

PUBLICATIONS

Arslan-Ayaydin Ö. and Khagleeva I.: Dynamics of Crude Oil Spot and Futures Markets. In Energy Economics and Financial Markets, eds. A. Dorsman and W. Westerman, pp. 159-174. Berlin Heidelberg, Springer-Verlag, 2013.

Arslan-Ayaydin Ö. and Khagleeva I.: Geopolitical Market Concentration Risk of Turkish Crude Oil and Natural Gas Supply. In Perspectives on Energy Risk, Springer-Verlag, 2014.

PRESENTATIONS

An Analysis of Exchange Rates Data Using Multivariate Time Series Models.

International Conference "Computational Mathematics, Differential Equations, Information Technologies", Ulan-Ude, Russia, August 2009.

The Information Content of the Volatility Index (VIX) in the High Frequency Domain.

Midwest Economic Association, 75th Annual Meeting St. Louis, USA, March 2011.

Understanding Jumps in the High-Frequency VIX

- *Midwest Finance Association Annual Meeting*, New Orleans, USA, February 2012.
- *SIAM Conference on Financial Mathematics and Engineering*, Minneapolis, USA, July 2012.
- *Research Seminar*, International College of Economics and Finance (ICEF)/ Higher School of Economics (HSE), Moscow, Russia, January 2013.
- *Research Seminar*, Spot Trading LLC, Chicago, USA, April 2013
- *Conference on Mathematical Finance and Partial Differential Equations at Rutgers University*, New Brunswick, New Jersey, USA, November 2013

HONORS AND AWARDS

Second prize, Soviet Union Students' Olympiad in the Strengths of Materials, Russia, 1986

Liautaud Scholarship, University of Illinois at Chicago, 2007-2011

Travel grant, Midwest Finance Association Annual Meeting, 2012

Travel grant, Conference on Mathematical Finance and Partial Differential Equations at Rutgers University, 2013

TEACHING EXPERIENCE

Lecturer, University of Illinois at Chicago

Business Statistics I, Fall 2008

Business Statistics II, Spring 2012, large class of 110 students

Teaching Assistant, University of Illinois at Chicago

Investments, Business Statistics, Portfolio Management, Corporate Finance 2007-2011

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Research Assistant, University of Illinois at Chicago

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Nonparametric analysis of jump processes in price and volatility

OTHER PROFESSIONAL EXPERIENCE

Chief Accountant, Balt-Invest Plus Ltd, - St. Petersburg, 1993 - 1999

Chief Financial Officer, Petroagroprom Ltd., - St. Petersburg, 1999 - 2006

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Construction Company "20th Trust" Inc., - St. Petersburg, 2000 - 2001

Tax and accounting consultant, Joystik Ltd., - St. Petersburg, Russia, February 2007 - July 2007

PROFESSIONAL MEMBERSHIP

AFA, AWM, FMA, MFA, SIAM