Student Ratio Use and Understanding of Molarity Concepts Within Solutions Chemistry

BY

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THESIS

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This dissertation is dedicated to my late grandmother, Margaret (Taylor) Cunningham, who always believed that I was a firework. Even in death she reminds me constantly of my capabilities. May she rest in peace.

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LIST OF ABBREVIATIONS OR NOMENCLATURE

$CaCl_2$	calcium chloride
DVSM	Different Volume Same Molarity
SVDM	Same Volume Different Molarity
PR	Proportional Reasoning
CD	Chemistry Diagnostic
SHDC	Same Height Different Color
DHSC	Different Height Same Color
M	Molarity (moles/liter)
g	grams

SUMMARY

This dissertation presents results and conclusions about student ratio use and understanding of molarity concepts within solutions chemistry. Data were collected using a structured interview approach. Grounded theory was used to analyze student responses with particular emphasis on ratio in molarity to develop theoretical statements. The purpose of this inquiry was to determine ways in which conceptions of ratio affected students' understanding and use of ratio within solutions chemistry. A ratio in this context is the idea of two measured quantities in relation to each other (e.g. density is measured in grams per milliliter). An example of a ratio within solutions chemistry is molarity, which is measured in moles per liter and represented by the capital letter M. Results from this study indicate that most students do not have an intensive view of molarity and interpret M to mean moles. This caused students to have difficulty reasoning through the Different Volume/Same Molarity (DVSM) task. Students were able to represent concentration as an intensive quantity qualitatively through structurally similar tasks. Students in this study were successful proportional reasoning problem solvers in the direct proportion problems. Students attempted to use direct proportions for inverse relationships, which led to incorrect answers. Recommendations include explicit connections between molarity and the structurally similar tasks so that the intensive nature of molarity is emphasized.

I. INTRODUCTION

In this dissertation, research is presented about student ratio use and understanding of molarity concepts within solutions chemistry. Data were collected using a structured interview approach. Grounded theory was used to analyze student responses with particular emphasis on ratio in molarity to develop theoretical statements. The purpose of this inquiry was to determine ways in which conceptions of ratio affected students' understanding and use of ratio within solutions chemistry. This study used Clark, Bereneson, & Cavey (2003)'s pedagogical roadmap as a theoretical framework to guide the view of molarity as a ratio. A ratio in this context is the idea of two measured quantities in relation to each other (e.g. density is measured in grams per milliliter). An example of a ratio within solutions chemistry is molarity, which is measured in moles per liter and represented by the capital letter M. Results from this study include students' interpretations of molarity in several situations. The results give insight into how students view ratio in solutions chemistry and how students use ratios in molarity calculations.

A. **Research Questions**

This dissertation has an overarching question of

•What are the roles of ratio and chemical concepts in student understandings of molarity?

Specifically this study will address the following research questions:

- •R1: Do students' understandings of ratio vary from domain specific tasks to structurally similar tasks?
- • R2: What are students' interpretations of molarity in solutions chemistry?

•R3: Are there patterns in how students use ratio in their solving strategies for molarity problems?

This dissertation will systematically look at the three research questions beginning with the domain general and ending with more complex domain specific research questions. Each chapter presents data to build an overall understanding of individual research questions, ultimately ending in conclusions about the overarching question with implications for pedagogy and standards.

B. **Dissertation Layout**

In this chapter, ratio and chemistry are discussed in terms of their prevalence in standards and chemical education. The theoretical frameworks that guide the research are then provided. A literature review is given in Chapter II with a discussion of ratio in mathematics literature as well as descriptions of solutions studies to situate this project within the literature base. Methods, including sampling and specific interview session protocols are described in Chapter III.

Chapters IV, V, and VI present detailed reports on the data obtained for Questions 1, 2, and 3 respectively. The three questions are separated into different chapters because they build from the domain-general to the domain-specific ultimately ending in conclusions for the overarching research question in Chapter VII.

Representative data are presented in Chapters IV-VI. Additional examples of student data for each of the Theoretical Statements derived from a grounded theory analysis can be found in Appendices A-C respectively. In Chapter IV, the data from the interviews are analyzed with respect to R1 in order to characterize student ability with qualitative reasoning about concentration. Chapter V presents an analysis of student understandings of the concept of molarity in the domain-specific chemistry context. Student solving strategies both in the Proportional Reasoning Diagnostic and in the Molarity problems are presented in Chapter VI. Finally, in Chapter VII, each of the Theoretical Statements is discussed with respect to the overarching research question along with potential implications for instruction and policy.

The layout of the Introduction chapter is to first present the research questions for this study and then give the reader a background into why numbers are an important component of chemistry, especially the ratio of molarity in solutions chemistry and its relation to the research questions of this study. Molarity as a ratio is discussed in two major contexts pertaining to student learning. First, the field of chemistry and specifically chemistry education instruction are discussed in terms of ratios and molarity. Second, standards are discussed at a general level and then specifically at the K-12 science education level. This discussion serves to validate the importance of ratio and molarity to a student's understanding of chemistry and other disciplines that rely on a robust understanding of chemistry. The introduction chapter ends with a discussion about the theoretical frameworks that guide the research questions.

C. **Terminology**

This dissertation utilizes discipline-specific vocabulary and this section intends to define key terms for reader understanding. A key concept within the field of chemistry is that of the *mole.* Because there are massive amounts of atoms in compounds, scientists use the mole as a unit for counting (Zumdahl 2008). The standard definition (SI) for a mole (as well as

Zumdahl's) is the the number equal to the number of carbon atoms in exactly 12 grams of pure ^{12}C. Because the mole is a counting unit, it allows chemists to discuss relative amounts of atoms and molecules at a measurable level. For example, if a solution has 6 x 10^{23} molecules of water (H_2O), it has one mole of water, two moles of hydrogen atoms, and one mole of oxygen atoms. A mole is an example of something that varies with the size of the sample. That is, the larger the sample, the more moles are present. A synonym for size is extent and for this reason moles are an example of an extensive quantity. Another example of an extensive quantity is mass. The mass of table salt (sodium chloride) increases as the amount of table salt increases.

Concentration as a concept refers to the amount of something in a mixture. It can be expressed using many different units. In chemistry, it is common to measure concentration using the number of moles of a solute dissolved in the total volume of solution. In chemistry, we call this concentration by a different term: *molarity.* An example of a definition of molarity in a chemistry textbook can be found in Masterton & Hurley's text (2008) where they state that the "concentration of a solute in solution can be expressed in terms of its molarity." Their chapter on molarity discusses how one can use molarity for calculations of (1) volume given moles and molarity and (2) moles given volume and molarity. Molarity is later referred to as a concentration unit with the following equation:

$$\text{molarity (M)} = \frac{\text{moles of solute (mol)}}{\text{liters of solution (L)}}$$

Scientists often refer to a solution with a certain molarity as a *molar* solution. For example, a scientist may refer to a 0.05 M KCl solution as "a 0.05 molar solution of potassium chloride".

Molarity is considered an *intensive quantity* because it is composed of two *extensive quantities*, moles and volume. An example of this can be found in Wink et al. (2003), where they define an intensive quantity as one that "does not depend on the size of the sample and is the same whether large or small." An example of an intensive quantity is density. The density of water is the same no matter if the sample is five gallons or one milliliter.

The intensive quantity of molarity has a different relationship to moles and to volume. An example of these relationships can be found in the mathematics literature, where Davies (1865) provides an example of a definition for the terminology. First, there is a *directly proportional* relationship between the number of moles of solute and the molarity of the solution. That is, as Davies (1865) describes, "two numbers are directly proportional, when they increase or decrease together." Because they increase or decrease together, "the ratio between them is always the same". The second relationship is between the total volume of the solution and the molarity of the solution which is an *inversely proportional* relationship. That is, as Davies (1865) describes, "two numbers are inversely proportional, when one increases as the other decreases." Because volume is increased, the molarity is therefore decreased if the numerator is held constant.

To summarize, molarity is an intensive quantity representing the concentration of a solution and is composed of two extensive quantities: moles and volume. The relationship between volume and molarity is inversely proportional and the relationship between moles and molarity is directly proportional. An example of this in everyday life would be in the creation of orange juice from concentrate. To make orange juice at an acceptable taste (intensive quantity), water and orange juice concentrate (extensive quantities) are necessary. The taste (molarity) increases as more concentrate is added with the same amount of water (directly proportional).

The taste (molarity) decreases as more water is added with the same amount of concentrate (inversely proportional).

These concepts are explored through this work as molarity and concentration as a generic concept are explored within student encounters in the interview.

D. **Numbers as an Important Component of Chemistry**

Numbers are an important component of chemistry and can represent different things in different contexts. A common adage among chemistry teachers is that “A number with no units has no meaning.” For example, the number “2” could represent the charge on an ion or it could represent a stoichiometric coefficient in a balanced equation. Since number can only be interpreted in a context, studying how students understand number in specific chemical contexts is necessary.

One important context for number in chemistry is solutions. Solutions chemistry, the context of this project, is remarkable as a source of insight because solutions add the important additional factor of mixture to studies of chemical substances. In solutions, two or more substances are mixed and since different amounts of substances are present in different solutions, the relative amounts are important. Relative amounts introduce concepts, such as concentration, known as molarity to chemists, in particular the ratio of moles of substance per liter of solution.

There are two compelling reasons for the study of molarity as a ratio in solutions chemistry, both of which will be developed further in this introduction chapter: chemistry instruction and standards. Better insight into student understandings of molarity as a ratio in solutions chemistry could lead to revised standards and more effective chemistry instruction.

Chemistry and chemical education instruction will be discussed in relation to molarity. Then, standards will be discussed generally and then with respect to molarity.

1. **Chemistry and Chemistry Education Instruction**

The importance of ratio and molarity is also reflected in textbooks. An analysis of a representative preparatory college chemistry text, *The Practice of Chemistry* (2003), yielded several types of ratio within it. There are many ways in which ratio plays a central role in chemistry. Examples of ratio within the text are:

•Ratio as it relates to periodicity by combining ratios of elements in compounds to create the periodic table by ratios of elements (Mendeleev) and various intrinsic properties such as density (mass per volume).

•Ratio is used in chemical equations and chemical formulas. For example, a chemical formula tells chemists how many molecules of hydrogen there are to oxygen in water.

•Ratios can also be found in chemical formulas in the form of stoichiometric coefficients to be used in stoichiometric calculations. For example, in the combustion of methane, there is a 2:1 relationship between the number of molecules of oxygen necessary for one molecule of methane to combust.

•Ratio is used to calculate moles from molar mass. For example, a student can be given 15 g of calcium chloride and they can calculate the number of moles by dividing by the molar mass of 110 g to find the number of moles (0.136 moles of calcium chloride)

•Ratio is used to represent the concentration of a given solution through molarity, a ratio of moles of solute per volume of solvent. For example, a student may be asked to calculate the molarity of a solution if they had 1L of water and 0.5 moles of calcium chloride. They would calculate molarity by dividing 0.5 moles by 1L of water yielding a 0.5M solution.

The particular focus of this dissertation is ratio as it is manifest in molarity. Molarity is an intensive quantity that is composed of two extensive quantities: moles of substance and liters of solution. The units for molarity are moles per liter (moles/L) but are often represented simply as capital M.

Since ratios and proportions are an essential component of chemistry, students are often tested on their algorithmic skills to show proficiency. However, as studies show, students can perform these tasks algorithmically and have no understanding of the concept being tested (Beal & Prescott 1994; Gabel 1998; Lythcott 1990; Wandersee et al. 1994; Dori & Hameiri 3003; BouJaoude et al. 2004; Nakhleh 1993; Nakhleh & Mitchell 1993; Schmidt & Jigneus 2003; Pinarbasi & Canpolat 2003; Zoller et al. 1995; Schmidt 1994; O'Grady-Morris 2008; Groves 1995; Songer & Linn 1991). These will all be discussed more in depth in Chapter II. This is problematic because students who hold an algorithmic understanding of molarity and concentration could potentially hold a simplistic (extensive) understanding of molarity that doesn't account for changes in volume. A person with this type of understanding of molarity could give incorrect dilutions of intravenous fluid to patients.

Given that ratios are present in a variety of algorithms in chemistry, represent a variety of things in chemistry, and are possibly used in different ways depending on context, it is possible that the algorithmic preference of students reported by O'Grady-Morris (2008) contributes to the memorization of numbers rather than conceptualization of them.

Student performance without understanding suggests that a study focused on students' understandings of molarity would be worthwhile. A study is needed to explore more carefully the meanings that students apply to ratios in thinking about chemistry. The ratio of molarity was chosen for this study because of the vast applications in other fields where an implicit understanding is assumed from students. As the mathematics literature suggests (Chapter II), students have different levels of sophistication when it comes to the use of ratios (e.g. Clark, Berenson, & Cavey 2003). A study of how students are solving various molarity problems that explicitly examines the use of ratio could also be revealing in the diagnostic sense as an indicator of student understandings of molarity.

2. **Standards in K-12 Science Education**

To assess student understandings of solutions, the standards provide benchmarks for teachers to use to gauge student learning. Presumably, students graduating high school are deemed proficient in the state standards. However, college students are unable to describe molarity and other scientific concepts at a conceptual level (as shown in Figures 4 and 5 later in this chapter). Students are assessed through knowledge of algorithms and facts but not of conceptual understandings. Therefore, a student could be deemed proficient in a standard by rote memorization of an equation with little to no understanding of the concept represented by the equation. This can lead to assumed understandings of concepts, such as molarity as an intensive quantity, when such understandings are not present. To do more than this algorithmic and fact-based assessment and to better support conceptual learning, better information on how students understand phenomena is needed.

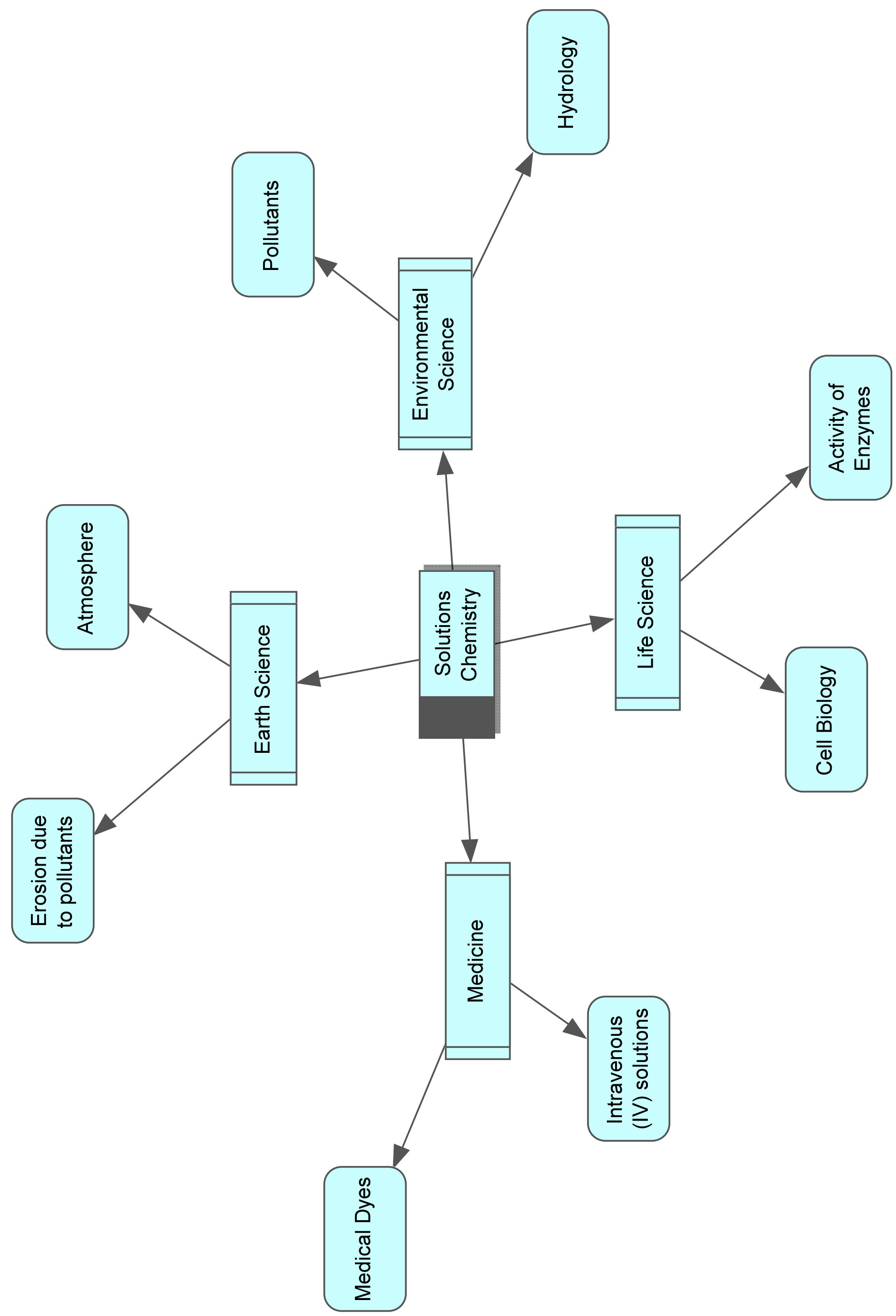
The current standards for instruction in science in individual states are guided by the national standards. At the national level, the National Science and Education Standards (NSES) suggest that students in grades K-12 should understand the unifying concepts of 1) systems, order, and organization; 2) evidence, models explanation; 3) change, constancy, and measurement; 4) evolution and equilibrium; and 5) form and function. For grades 9-12 in the physical sciences specifically, the NSES indicates that students should understand 1) structure of atoms; 2) structure and properties of matter; 3) chemical reactions; 4) motions and forces; 5) conservation of energy and increase in disorder; and 6) interactions of energy and matter. Ratios are implicitly present in many of these standards.

For example, the State of Illinois has two sets of standards for K-12 instruction: the Illinois Learning Standards (ILS), which are the basis of the Illinois Science Assessment Framework (ISAF) (Illinois State Board of Education 2005). Since these have been in place since before 2003, presumably, students entering college have had standards based instruction for at least middle and high school. Several standards involve ratio within chemistry, such as ISAF Standard 12.11.58. It states that students should be able to "understand that the chemical quantity called 'one mole' is set by calling the number of atoms in exactly 12 grams of carbon-12 atoms one mole. This number turns out to be 6.02×10^{23}, also known as 'Avogadro's Number'. However, only one standard specifically talks about ratio in solutions chemistry, including the ratio of molarity. ISAF Standard 12.11.63 states that students should be able to "Distinguish between chemical compounds and solutions and mixtures. Differentiate between solute and solvent. Understand the concentration of a solute in terms of molarity, parts per million, and percent composition." It is important to note that the standards do not explicitly address student understanding of molarity as an intensive quantity.

3. <u>**Importance of Solutions to Other Science Learning**</u>

Solutions are important to reactions, concentration, and identity of chemical systems. Solutions chemistry and specifically molarity are important because they pertain to life, environmental and earth sciences as well as the field of chemistry and its subfields (e.g. biochemistry). Figure 1 shows the conceptual connections between solutions chemistry and various other fields with examples. For instance, in the life sciences, cell biology studies the interactions of solutions inside and outside the cell across a semi-permeable membrane, so an understanding of solutions chemistry is necessary to understand how the semi-permeable membrane functions. Another example comes from the field of medicine. Intravenous fluids are solutions containing various medicines or vitamins to be delivered into the bloodstream of a patient, and nurses and doctors are often required to calculate dilutions of solutions for proper dosages on the spot (Noss et al. 2002).

Figure 1: Solutions chemistry and how it relates to other fields of science



Another example hails from biochemistry in the measurement of cholesterol from aliquots of blood or in foods. To determine the concentration of cholesterol, technicians prepare a serial dilution of a patient's sample and then compare the sample to standard solutions to determine the concentration of cholesterol in the patient sample.

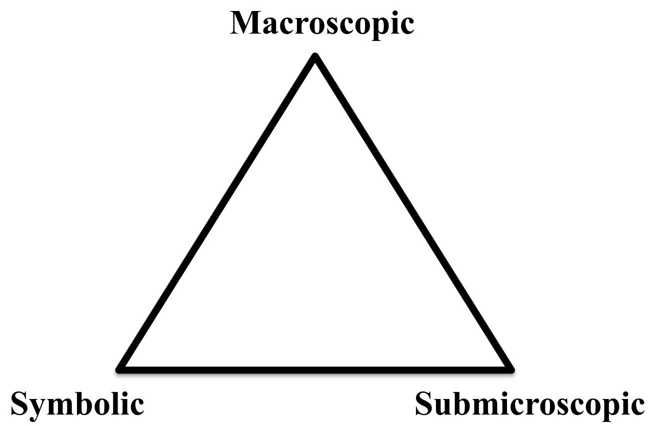
Cliff (2009) points to misunderstandings of chemistry affecting student understandings of physiology. In that study, a diagnostic question was included regarding concentration as it relates to equilibrium. 42% of the students incorrectly predicted that the concentration of a second reactant would not change as the concentration of the first reactant increased because there wasn't a change to the second reactant explicitly in the problem. As shown in Figure 1, understandings of molarity are applicable not only to high school and college students interested in chemistry major but other majors as well.

Because molarity is a ratio between two extensive quantities (moles and volume), an understanding of molarity that meets standards in chemistry and the use of chemistry in other sciences in turn relies on an understanding of ratio. For example, semi-permeable membranes involve concentration, which is typically represented using molarity. The intravenous fluid example mentioned earlier clearly involves ratio as dilutions of solutions require an understanding of volume and concentration. In the case of intravenous fluids, the amount of substance (the drug) remains constant while the amount of solvent (volume of solution) is varied. Misuse of a direct proportion to this inverse relationship could yield lethal results.

4. **Current Ongoing Research on Student Understandings of Solutions**

This research stems from related work on a project titled "Student Understandings of Solutions", which seeks to answer the question "Does quantification help or hinder student conceptions of solutions chemistry?" That project, which will be referred to as the "Solutions Project", builds on Johnstone's triangle of representations (1982, 2000, 2010) in chemistry. Briefly, Johnstone suggested that there are three ways that scientists represent phenomena in chemistry: macroscopic, symbolic, and submicroscopic/particulate (Figure 2). An example of a macroscopic description of concentration would be "This one is more concentrated because it is bluer." A symbolic representation of a reaction would be a chemical equation. A particulate (or submicroscopic, or atomic / molecular) representation would be a picture that shows atoms, ions, and molecules depicted as balls or a similar drawing. Scientists (experts) can represent a given chemical phenomenon using all three of these representations and can do so seamlessly (Kozma & Russell 1997). Expert use of multiple representations can be problematic because students do not always understand the role of the representation assumed by the teacher (Treagust et al. 2003). Bodner & Domin (2000) also found that the use of more representations is correlated with more success (with success meaning getting the problem right).

Figure 2: An adaptation for the "Solutions" project of Johnstone's triangle

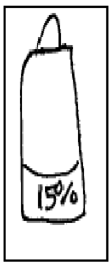


Throughout the interviews and coding in the "Solutions" project, it was found that students' descriptions of what the numbers *meant* were varied. Originally, student utterances were coded for correctness but upon closer examination, it was clear that within those correct and incorrect responses were different conceptions of molarity in chemistry. For example, students were interpreting the 0.15 in front of molar to mean that it was 15% instead of as 0.15 moles per liter (Figure 3). Percentage is also a ratio but is not the same as the ratio of molarity. This was intriguing because it wasn't something that the researchers had considered. Furthermore, Figure 5 (can be found later in this chapter) shows additional preliminary research data involving alternative conceptions of molarity with respect to proportions of ions in solutions chemistry.

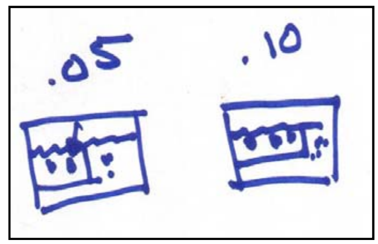
Figure 3: Examples of student responses in the "Student Understandings of Solutions" project in response to a question probing understanding of molarity

(A) Student response to a question probing student understandings of molarity in a 0.15M solution of $CaCl_2$. The student thought that 0.15M meant that 15% of the solution was $CaCl_2$.
(B) Student response to a question asking the difference between 0.05M and 0.10M $CaCl_2$. Student thought that because 0.10 had two significant figures and 0.05 only had one, that the 0.10M was more important than 0.05M $CaCl_2$.
(C) Transcript of a student response to question asking the difference between 0.05M and 0.10M $CaCl_2$. Student believed that 0.05M had 5% less $CaCl_2$ than 0.10M $CaCl_2$.

(A)



(B)

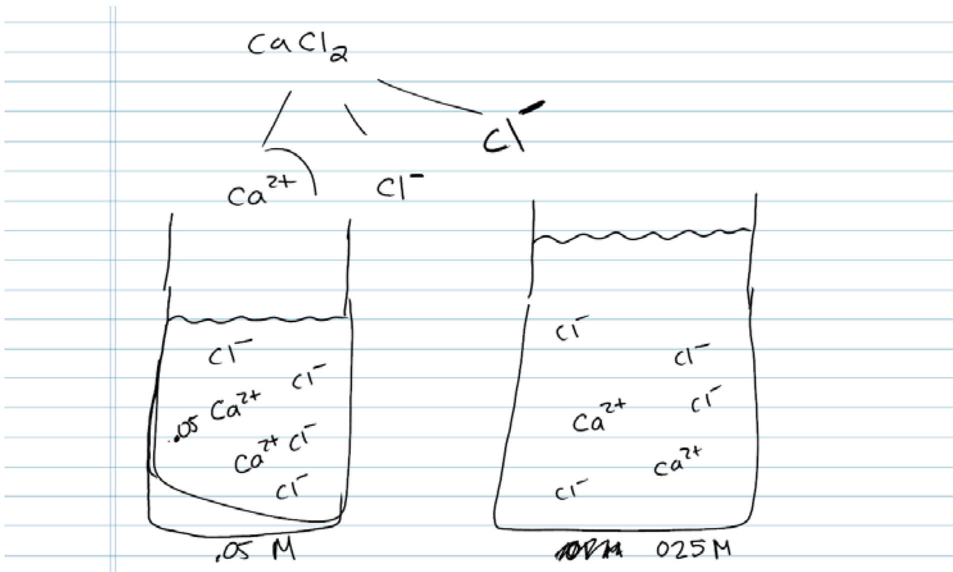


(C) [00:00:55.05] Student: Um it just means that um like this one has five percent less moles per milliliter than this one does.

Figure 4 shows that some students have various interpretations of molarity in solutions chemistry. The two students whose work is shown in Figure 4 represent molarity as a percentage. This is an intensive quantity, but it does not relate moles and solution volume. These different interpretations could be explained by how students understand and use ratios and the proposed research intends to characterize those interpretations with respect to ratio use in different contexts. Understanding the different interpretations requires that we understand better how they understand ratio in different contexts. Therefore we need to characterize student use of ratio in a domain-general context as well as the domain-specific context of molarity.

Figure 4: Additional example of student descriptions of solutions chemistry phenomena involving ratio.

Student uses number to indicate a charge on the calcium ion; student also uses number to indicate molarity of a solution by keeping the number of calcium and chloride ions constant while changing the volume.



E. **<u>Theoretical Frameworks</u>**

Two theoretical frameworks guide this research project both in the construction of individual tasks and in the analysis of the interview data. The first is a means of characterizing student knowledge: constructivism. The second framework describes student uses of ratio within mathematics. Both theoretical frameworks will be described as to why it was chosen, how they interrelate, and how each framework impacts the construction and analysis of the tasks for this project. Frameworks for methodology will be described in Chapter III.

1. **Constructivism and Transfer**

This dissertation has constructivist roots in that it seeks to characterize student initial constructions of the concept of molarity to act as a starting point in the ultimate move toward conventional or accepted scientific views. Constructivism is a learning theory that frames knowledge as something students construct when faced with a phenomenon; therefore it is possible for every student in a classroom to have different interpretations of the same phenomenon. As Bodner et al. (2001) suggest: "Constructivist theories of knowledge are based on a fundamentally different assumption [than other learning theories] Knowledge is constructed in the mind of the learner". They go on further to state, "*Knowledge is seldom transferred intact from the mind of the teacher to the mind of the student*. A second, more radical form of the constructivist theory has been summarized as follows: *Useful knowledge is never transferred intact*. (italics in original)" Constructivism rejects the notion that scientific truths are available to be acquired by students directly and instead posits individual construction of knowledge can be affected by the nature of the task and the prior knowledge of the learners (Schunk 2007).The tasks in this study were created to show both a general situation involving number and a domain specific task to explore student interpretations of number with respect to the situation.

Bodner et al. (2001) and the other papers in this constructivism section of this dissertation are about pedagogy with pedagogical constructivism. Wink (2006) offers two definitions of constructivism: pedagogical and epistemological. He suggests that pedagogical constructivism views the individual learner as the only location where knowledge is constructed. The knowledge is tied to the learner. Epistemological constructivism views knowledge as something that individuals and groups construct from their own choices even in interactions with inanimate objects. In this case, knowledge can be known in different ways depending on the context or the need.

Constructivism as a pedagogy does not accept any answer that a student constructs as correct but rather it uses their knowledge constructions as starting points to move toward accepted scientific views, which are governed by community norms and epistemological constraints on how the discipline establishes knowledge claims. Constructivism was chosen as a theoretical framework because it allows for the characterization of how each student constructs knowledge in his or her own way and allows an understanding of if they have knowledge in different ways depending on the context. As Noss & Hoyles (1998) suggest, it is impossible to design an effective learning environment without having a very good understanding of what students know when they enter the classroom. Results emerging from this research can show what the students know about molarity and ratio when they encounter it within the chemistry classroom, which can inform researchers and instructors how to help learners make connections between old and new knowledge for stronger connections (Bransford et al 2000). Teachers can provide scaffolding so that students can "engage in the structure of a mature performance" (Schwartz et al. 2008) in the new domain.

One type of connection between old and new knowledge can also be described as transfer. Bransford, Brown & Cocking (2000) define transfer as "the ability to extend what has been learned in one context to new contexts." Mathematics, which Schwartz et al. (2008) indicate, is "general and widely applicable across many domains". That is, a student can learn a skill such as recognizing a ratio in mathematics and then transfer that skill to a new context such as chemistry by recognizing a ratio within molarity. This type of transfer is also called similarity transfer the tasks "share some type of similarity in the experimenter's view" (Brown 1989) in hopes that students recognize the similarity. Schwartz et al. (2008) define similarity transfer as when people have "knowledge gained in one context that is sufficient for application to another" and points to the underlying questions of whether students will use the similar knowledge in the new context and whether they will recognize the similarity between the tasks. They suggest that there are two types of a similarity transfer: surface level and deep level. Surface level transfers can lead to incorrect features being transferred whereas deep features address the underlying features and can be transferred even when there are surface level differences between the tasks. Contrasting cases can be used to alert students to the similarities or differences between cases that may not have been obvious at the start (Bransford et al. 2000). For example, Schwartz et al. (in press) used contrasting cases to "highlight empirical regularities" by teaching density using clown crowdedness on buses. Schwartz et al. (2008) state that they utilize contrasting cases because "people detect an internal contradiction or an external impasse and this causes them to search for a new way to solve the problem". However, they point to students performing poorly on transfer tasks if they do not recognize the ratio structure during instruction (Schwartz et al. in press) which is consistent with Bransford et al. (2000) in their assertion that the degree of mastery is the most important factor in successful transfer along with the degree of memorization of a concept.

The concept of transfer raises the question of what knowledge is transferred. A suggestion for this is given by Hammer et al. (2005), who discussed transfer in terms of diSessa's knowledge-in-pieces work (diSessa 1988) in that transfer involves coordinating pieces of knowledge and the resulting transfer is the activation and application of those “resources” (Hammer et al. 2005). That is, students who are deemed as having a positive transfer are activating the resources necessary to solve the problem.

A further distinction of types of transfer is offered by Ferrante (2007) in his dissertation work. He differentiates between goal transfer and domain transfer by suggesting that in goal transfer the domain remains constant but with the different goals. For example, the domain of chemistry remains the same when changing the type of question (goal) from stoichiometry to reactions because the laws of chemistry still apply to the situations. Domain transfer, on the other hand, has the same goal as applied in different domains. An example of this would be learning about ratios in mathematics and using it in another domain such as chemistry in the form of molarity. The present dissertation work can be categorized as a domain transfer task with the goal of recognizing a ratio in the domains of solution chemistry and of a familiar task (painted blocks).

Perkins & Salomon (1987) frame transfer of learning as a pleasant by-product of the learning process by stating, “You learn A and find that performance in B improves as well.” The problem then occurs when “Learning A impairs performance on B”, which is termed negative transfer. Bransford et al. (2000) suggest that fast-paced curriculum that do not allow time for in depth discovery can leave students with a series of isolated facts that may block future learning because they then lack basic foundations for understanding. Similarly, overly contextualized information also poses a problem in transfer because the information is “too tied to its original context” (Bransford & Schwartz 1999). Bransford et al. (2000) suggest that existing prior knowledge can also block new information. As shown earlier, sometimes a contrasting case can prompt a student to amend his or her mental model but the real danger is when students “construct a coherent (for them) representation of information while deeply misunderstanding the new information (Bransford et al. 2000).”

Another definition for negative transfer can be described as merely “repeating an old behavior in a new setting” (Bransford & Schwartz 1999) or use of an inappropriate solution principle for solving the problem (Chen 1989). For example, students can exhibit einstellung or rigidity of behavior (Luchins & Luchins 1959) between tasks even if a better option presents itself (Schwartz et al. 2008; Schwartz et al. in press). Chinn and Brewer (1993) used anomalous data (evidence that contradicts student pre-instructional theories) with the intent of causing change but found seven categories of response with only one that included accepted the anomalous data and changing mental models. Some of the other responses included ignoring the data, rejecting the data, excluding the data or reinterpreting the data.

The transfer body of literature is pertinent to this work and a useful framework for design and analysis because molarity is an example of domain transfer or similarity transfer. Understanding how students may (or may not) be activating the pieces of knowledge or recognizing (or not) the similarities between the tasks is an important feature of this study. In the case of this work, problems in the general mathematics domain for direct proportional reasoning have been termed domain-general. As Schwartz et al. (2008) notes: “mathematics is general and widely applicable across many domains, but people do not always transfer mathematics when relevant.” Therefore, this study considers mathematics tasks as domain-general. As will be shown later, situations that are considered familiar to everyday life are also termed as domain-general. Problems and contexts that are specific to chemistry are termed domain-specific tasks. This research study seeks to see if students transfer their recognition of ratios from a domain-general context to a domain-specific context. An analysis of solving strategies and mental models on the domain-specific tasks also seek to identify sources of negative transfer.

2. **Student Use of Ratio within Mathematics**

Chemistry assumes a basic understanding of ratio as reflected in mathematics curricula. Indeed, students are exposed to number early in their education and gradually build their understandings over time. According the NCTM in its publication, *Principles and Standards for School Mathematics*, students should be able to use fractions, percents, and decimals interchangeably and appropriately in the middle grades (6-8). Specifically:

> As they solve problems in context, students also can consider the advantages and disadvantages of various representations of quantities. For example, students should understand not only that 15/100, 3/20, 0.15 and 15 percent are all representations of the same number but also that these representations may not be equally suitable to use in a particular context. For example, it is typical to represent a sales discount as 15%, the probability of winning a game as 3/20, a fraction of a dollar in writing a check as 15/100, and the amount of the 5 percent tax added to a purchase of $2.95 as $0.15. (p 216)

Ability to represent a single rational number in multiple ways causes confusion. For example, 1/4 can be also be represented as 0.25 or 25% in the right context. A molarity of 0.25M cannot be represented as 1/4 or 25% because it actually represents 0.25 moles divided by 1 liter.

Students in the middle grades should also be attending to addition, multiplication, division and subtraction of fractions, decimals and percents. In these grades, the NCTM also points to student difficulty in comparing fractions and in dealing with percents at both ends of the spectrum in terms of magnitude (less than 1% and more than 100%). The middle grades are also when teachers are encouraged to teach proportionality at a deeper level than “setting two ratios equal and solving for the missing term (p217)”. The report singles out proportionality as “an important integrative thread that connects many of the mathematics topics studied in grades 6-8.” This is not only true for mathematics but many science concepts as well. Lesh, Post, & Behr (1988) argue that proportional reasoning is the cornerstone of all that is to follow. They do not indicate if this statement is intended to span only mathematics, but it is reasonable to consider that this extends to science and chemistry as well.

The skills described in the NCTM mathematics examples are all skills essential to understanding the concept of molarity and for calculations involving molarity. To parallel the provided NCTM example in a chemistry context, it is typical to represent the percent composition of titanium in an ore as 15%, the concentration of sucrose as 3 g sucrose/20g water, and the concentration of $CaCl_2$ as 0.15M. In chemistry it is also important for a student to consider the advantages and disadvantages of the representation’s units because in the case of the concentration of $CaCl_2$, 0.15M cannot be written as 15% $CaCl_2$. Molarity is a representation of two measurement systems: moles and volume.

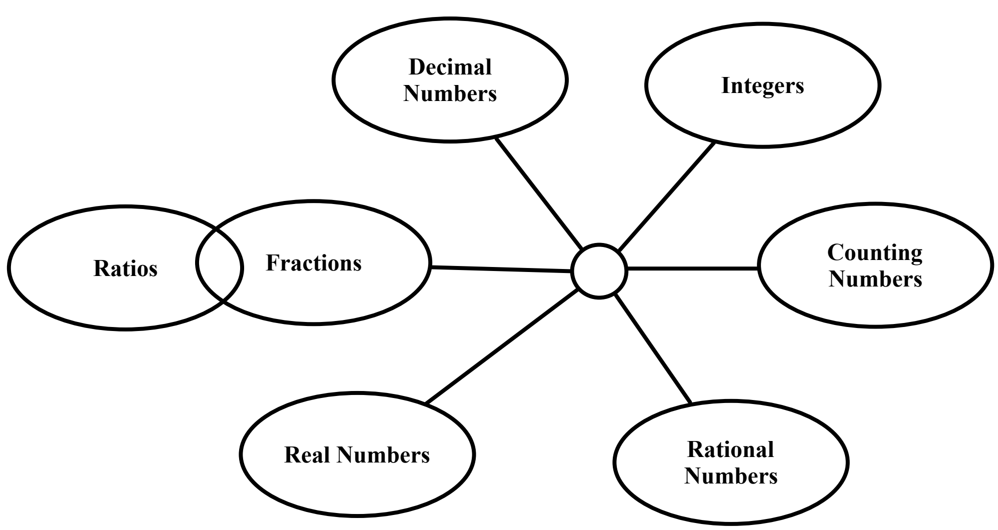
By high school, the NCTM standards don’t mention ratio, which indicates that students should master this skill in the middle grades. The only NCTM mathematics standard applicable to ratio use is found in the “Number and Operations” section where students should understand that “properties that hold in some systems may not hold in others.” For example, in the case of molarity, the relationship between moles and molarity is a direct relationship and as will be shown in this dissertation. Students may mistakenly assume that a direct proportional relationship holds with volume and molarity just as it does with moles and molarity.

Vergnaud (1982) suggests that the fragmented situations in which students learn the concepts of fractions can cause difficulty because their concepts are so narrow. He points to the age at which students learn fractions and ratios and "yet the concept of rational number is a big and long-lasting source of difficulty for 15 or 16 year olds and many adults (33)." Fisher (1988) found that teachers were almost perfect on the performance in calculating direct proportions, but that they still encountered problems with inverse proportions.

Another perspective is exemplified by the work of Clark, Bereneson, & Cavey (2003), where they set out to define how teachers viewed ratios and fractions. They developed a model or a "pedagogical roadmap (p309)" to discuss ratios and fractions and other number types (Figure 5). Each of these types is referred to as a realm. Their roadmap is designed to mirror the optometrist's eye-examination machine where each of the realms is like a lens that can be rotated to overlap with the ratio realm remaining constant. They presented five models of ratio-use that were then used to develop their pedagogical roadmap: ratios as a subset of fractions, fractions as a subset of ratios, ratios and fractions as distinct sets, ratios and fractions as overlapping sets, and ratios and fractions as identical sets. In their workshops with teachers, they found that teachers tended to argue for fractions as a subset of ratios or ratios and fractions as overlapping sets. This is consistent with Streefland (1985) who points to the connections between proportions and ratio with equivalence fractions. Clark et al. (2003) themselves reject the notion that fractions are a subset of ratios because "if the term ratio is all-inclusive it loses its power of discrimination (306)." Clark, Berenson, & Cavey (2003) conclude that ratios and fractions are overlapping sets, meaning that some ratios are fractions and some fractions are ratios, but not all of them are. An example given by the authors to exemplify these overlapping sets is: "1 cup sugar:2 cups flour in the ratio only realm, 1 cup sugar/3 cups ingredients in the intersection, and 1⁄2 cup sugar in the fraction only realm." The authors conclude with a summary that outlines the problem of these multiple realms in student understanding:

> In performing the cross-multiplication procedure to solve a missing-value proportion problem, students' path of decontextualizing ratios has no impact on producing the correct answer, provided that they are consistent with the numbers in the numerators and denominators of their fractions. Given other types of problems, however, students who rely on this and other procedures often fail to reason proportionally. Students who understand ratios and fractions, each in isolation, may recognize ratios, write them as fractions, and operate on those fractions according to memorized numerical rules, producing an incorrect answer...
>
> Successful students move fluidly across the boundaries, able to work in one realm without disconnecting from the others. While solving a problem, a student may move back-and-forth between the ratio-only realm and the intersection of ratios and fractions (or other number-type domain), as the student writes relevant information and formulates a strategy, then move back-and-forth between the intersection and the fraction-only realm, as the student computes an answer, and then move across all three realms to confirm the answer or make adjustments. We see this frequent back-and-forth movement as maintaining context, a critical component of solving problems. For a specific problem, the movement is made easier when the student is able to travel a familiar path, a path previously forged by making connections between concepts. (p 315)

Figure 5: Pedagogical roadmap of ratios and fractions with respect to other number types. Adapted from Clark, Berenson, & Cavey (2003), 308



The "pedagogical roadmap" provided by Clark, Bereneson, & Cavey (2003) links directly to student understandings and uses of ratio in the case of molarity in solutions chemistry. Molarity would fall into the "ratio only" realm. For example, ratio would be used in figuring out the ratio of $CaCl_2$ to water in various volumes with a fixed concentration (0.15M $CaCl_2$ in 100mL, 0.15M $CaCl_2$ in 200mL, and 0.15M $CaCl_2$ in 300mL). Student errors can occur when students view molarity in the "fraction only" realm or as one of any of the other values. If a student views molarity as any of the other realms other than ratio, it will cause issues in his or her understanding. If a student views 0.15M as a decimal or a real number, M as moles per liters is lost on them because they do not recognize that it is a ratio. That is, in terms of Clark et al.'s (2003) pedagogical roadmap, they view it as a decimal-only realm or a real-number only realm, not recognizing it as overlapping with a ratio.

Stavy & Tirosh (1996) presented what they call an intuitive rule that will play a role in understanding the student responses. They present the "the More of A the More of B" rule. They conclude that "conceptions apparently related to specific domains are actually specific instances of this rule". They found that issues occurred when students applied this rule to an inverse relationship. This is pertinent to this dissertation work because there is a direct relationship between molarity and the number of moles (more of moles, more of M) and an inverse relationship between molarity and volume (more volume, less M).

In molarity calculation problems, molarity has often been decontextualized by use of algorithmic strategies without having an impact on the correct answer. Given other types of conceptual problems, students do show difficulty in proportional reasoning with inverse relationships (Figures 3 and 4). It is worthwhile to study student uses of ratio and fractions in the case of molarity to ascertain patterns in solving strategies.

II. LITERATURE REVIEW

First, this study has a major mathematics component underlying the concept of molarity. In this research, different tasks explore both the mathematics component and the related concept of molarity. These tasks are analyzed with respect to ratio and mathematics and, where appropriate, chemistry concepts. It is necessary to present the ratio and mathematics literature because it helps situate this research within the mathematics education body of literature as well as informs the analysis of the data in this research study. As with later chapters, the domain-general will be presented first because it an underlying skill of the domain-specific tasks.

The pertinent domain-specific literature in this project concerns studies of how students understand solutions. In this Chapter, qualitative (symbolic, macroscopic, submicroscopic) solutions chemistry studies will be discussed to emphasize submicroscopic drawings. Then, the few quantitative studies related to solutions chemistry will be described to situate this research study within the existing body of literature.

Studies about conceptual versus algorithmic understandings are then discussed because the concept of molarity is often tested for algorithmic proficiency. The literature relating to this topic guided the construction of the tasks in this research. This study sought to examine the conceptions that students held about molarity, therefore it is necessary to present literature regarding alternative conceptions research.

A. Ratio and Proportional Reasoning

Much of the ratio and proportional reasoning research is found in K-8 research, as these are mathematical concepts taught at a young age. The NCTM standards suggest that by high school, students should have mastered proportional reasoning skills, but their methods of solving problems are varied (Mitchelmoore et al. 2007). Mistakes at that level are largely due to students not maintaining units within their responses. The literature also points familiarity with extensive quantities and to particular issues that arise when students encounter intensive quantities, which will be shown in this section.

Research on student understandings of ratios often occurs in contexts with decimals or fractions. Lachance & Confrey (2002) constructed a curriculum that focused on the grounding of decimal instruction within ratios. They found that elementary school children used their prior experiences with the broader concept of ratio to help them make sense of the narrower concept of decimals. They also found that students were able to move between similar constructs like fractions and decimals after instruction.

Mitchelmore, McMaster & White (2007) conducted a study using Teaching for Abstraction to see if it could be used to teach ratios to year 8 students. They started with relative and absolute comparisons, then the concept of ratio, then calculating with ratios, then fractions and ratios, then the concept of rate and finally calculating with rates. In the higher ability class, approximately 50% of the errors were related to units. The average-ability students' most common error was incorrect division or multiplication (30%). The low-ability students' most common error was failing to reduce a ratio. Student understandings were scaffolded in this study, which is an interesting method that is not used in chemistry. In chemistry, the ability to reason with ratio and rates is assumed to have been mastered at a young age. This study is particularly relevant to this dissertation work in that molarity is a ratio represented by the letter M. M stands for moles/L and if units are an issue for students in domain-general situations, then it could be an issue in domain-specific chemistry tasks.

Proportional reasoning skills are assumed in later years, such as in chemistry courses. As Ochiai (1993) suggests in a commentary, “However, the main problem, at least in entry level chemistry courses, is not the complicated mathematical operations that require computers. Most problems in entry level chemistry require only arithmetic, (+, -, x and /) and a little algebra.” He points to the C1V1=C2V2 algorithmic formula and says that “students try to apply this equation to almost any problem involving calculations of concentrations.” He then goes on to say that there is essentially only one relationship and that is the ratio of the quantities in question. He suggests that ratios are hidden in many chemical calculations and that they are “disguised”.

Cai (2002) reports that gaining information about how students approach the solution to a mathematical problem is more important than a correct solution. He distinguishes ratio problems as process-constrained and process-open. An example of a process-constrained task is when students “need to set up a ratio for the number of pizzas and the number of people and then compare the fraction representations...a process-open task cannot be solved following a standard algorithm (279).” Direct proportional reasoning questions follow this structure. An example of a process-open task would be asking students to finish a pattern of block steps that cannot be solved using an algorithm. The structurally similar tasks within this dissertation follow this structure. This study is in line with this dissertation work because student solving strategies give insight into their understandings.

Heller, Post, Behr, & Lesh (1990) studied the relationship between middle grade students’ reasoning about rates and numerical reasoning on proportion-related word problems. They define a rate to “denote a comparison between elements in two different measure spaces (389).” They found that different contexts may cause different reasoning processes. They also found that it was possible for students to show proportional reasoning without an understanding of direction of rate change, indicating that students can memorize without meaning. This aligns with solving strategies used by students in this dissertation. Depending on the salient features of the question, different strategies were used.

Lamon (1993) identified four semantic problem types for proportions: Well-chunked measures, Part- Part-Whole, Associated Sets, and Stretchers and Shrinkers. She found that in the Well-chunked measures semantic problem type, ten students “used miles per hour with no recognition that two quantities were compared in that ratio (47).” She found that students most often used ratios and proportions to solve associated sets problems; part-part- whole problems did not elicit any proportional reasoning, and almost all students failed to see the multiplicative structure of the Stretcher/Shrinker problem.

Singh (2000) studied two sixth-grade students and the knowledge that was critical to understanding ratio and proportion. She found that one of the students used the “x quarters for y candies relationship as a countable unit to find the answer (282).” This method has also been called the additive method (Fisher 1988; Lamon 1993; Misailidou & Williams 2003) or build-up strategy (Adjiage & Pluvinage 2007).

This study utilizes a proportional reasoning test bank created by Misailidou & Williams (2003). They created, validated, and calibrated a proportional reasoning test bank from the relevant literature that after testing has been narrowed to a 13 item “without models” test. They suggest it be used to measure proportional thinking and to diagnose additive tendencies. It can also be used to diagnose other problems such as “incorrect build up, magical halving/doubling, constant sum and incomplete reasoning errors.” This research study employs this proportional reasoning test as a diagnostic resource for students’ proportional reasoning skills outside the domain of chemistry. In their 2002 paper at the *2nd International Conference on the teaching of Mathematics* at the undergraduate level on the Mr. Short and Mr. Tall problem, they identified three correct strategies used by students: for every and multiplicative strategy, build-up method, and unit value method. Incorrect strategies included: additive strategy, magical doubling, and incomplete strategy. One teacher was found to use a cross multiplication method and several teachers didn’t report their strategies.

Person, Berenson, & Greenspon (2004) examined instructional representations of rate of change of a prospective high school teacher. The teacher verbalized how he wanted to give real life examples for his instruction of the concepts. They found that the map of his concepts moves from ratio, to comparisons, to fractions with a difficulty in reconnecting back to ratios. They find their work is consistent with the framework posed by Clark, Berenson, & Cavey (2003) that was discussed in Chapter I.

Also important are concepts that are lacking in the chemical education literature, but prevalent in the mathematics literature. This includes intensive quantities. Because molarity is an intensive quantity comprised of two extensive quantities, it is difficult to grasp as a concept. Canagaratna (1992) points to student difficulty in calculating and reasoning with intensive properties in chemistry. Gennaro (1981) explored proportional reasoning as it related to density, another intensive quantity in chemistry. He concluded that more research needed to be done to seek contributing factors to learning problems associated with density and solubility. These two studies point to the need for further research on intensive quantities in chemistry and in the case of this dissertation, specifically molarity.

Howe, Nunes & Bryan (2010) discuss the additive combination of extensive quantities and the proportional relations between variables in intensive quantities. They also discuss variable salience and relational focus, which are a focus of their study. They found that children aged 7-12 years old improved their reasoning with age on comparison and missing value format questions but that they were strongly influenced by variable salience and relational focus. They conclude that the difference between intensive and extensive quantities is theoretically significant and that instructors should pay more attention to intensive quantities in instruction.

Nunes, Desli, and Bell (2004) presented obstacles to understanding intensive quantities. First, students are asked to consider two variables at the same time and then they are asked to reason through inverse relations between those variables. They also point to the non-additive nature of intensive quantities. Their study was with younger children and they found that only a few children under the age of 7 years old show an understanding of intensive quantities regardless of familiarity (taste). They found no significant difference between intensive quantities comprised of two extensive quantities to form a whole and intensive quantities where the extensive quantities remain separate. They also found that students were successful on items that required inverse proportional reasoning when the quantities in the problem were extensive and less so when the quantities were intensive. They conclude that intensive quantities must encompass an added difficulty.

Fassoulopouls, Kariotoglou, and Koumaras (2003) investigated pupils' reasoning about intensive quantities, specifically density and pressure. Questionnaires were administered to 300 pupils for four tasks for both density and pressure. They found that a significant percentage of the pupils were inconsistent within their own answers and change their reasoning across the tasks. They recommend intermediate learning steps to help students develop understandings that can be used across tasks.

Van Dooren, De Bock, & Verschaffel (2010) explored the development of students' additive and multiplicative reasoning skills. This study is important to ratio research in that many students attempt additive strategies that are not applicable to intensive quantities. Their study gave a test of missing-values problems to 325 3rd-6th graders with half of the problems containing additive structures and the other half proportional structures. They found an age progression from always applying additive strategies in younger years to always applying proportional strategies in older years. They describe an intermediate state where students use both strategies based upon the salient features and numbers within the problems. Tourniaire (1986) administered proportional reasoning questions to sixty 3rd-6th grade students (with questions similar to Misailidou & Williams 2003). Similar to Van Dooren et al., this study found that students' success rates improved as they progressed to a new grade in school. They attribute this increased success to a better understanding of multiplication, which is necessary for a correct understanding of ratio and proportion due to their multiplicative nature. An insufficient understanding of multiplication can lead to additive strategies of intensive properties, which is incorrect.

Boyer, Levine, & Huttenlocher (2008) conducted a study to find out where young students "go wrong" in their understanding of proportions. They found students have more success when solving proportions involving continuous quantities than with discrete quantities and conclude that “these findings indicate that children go astray on proportional reasoning problems involving discrete units only when a numerical match is possible, suggesting that their difficulty is due to an overextension of numerical equivalence concepts to proportional reasoning problems”.

The literature presented in this section speaks to two issues that are the backbone to this research study. The first is that students do possess direct proportional reasoning skills and they learn them at an early age. This impacted the design of this dissertation work in that student direct proportional reasoning skills were assessed using the Misailidou & Williams (2003) diagnostic to rule out assumptions that students lacked those skills. The second is that students have difficulty reasoning with intensive quantities in mathematics. This extends to their understanding of inverse relationships but only in the context of an intensive quantity. This dissertation work extends the findings in mathematics literature to see if students have the same difficulty in the domain-specific context of molarity in chemistry.

B. **Solutions Chemistry**

The studies within this section focus on solutions chemistry to situate this study within the current body of literature and describes studies that address student use of particulate (submicroscopic) level drawings because they were used to as a way to interpret student understandings of molarity.

A vast majority of the studies involving solutions chemistry have examined students’ qualitative understandings with many concluding that students showed difficulty moving among the three representations of matter (symbolic, macroscopic, and sub-microscopic) proposed by Johnstone (1982).

Mattox, Reisner, & Rickey (2006) used the laboratory model Model-Observe-Reflect-Explain (MORE) Thinking Frame to probe student understandings of electrolytes and non-electrolytes. They used a module titled "What happens when chemical compounds are added to water?" Four student types were identified as those who had: good understandings of macroscopic and molecular levels, correct macroscopic predictions but some incorrect molecular-level descriptions, misconceptions about the molecular-level nature of matter, and the ability to differentiate between macroscopic and molecular levels but not always between electrolytes and non-electrolytes. Mattox et al. (2006) study discusses concentration but the molarities are not varied.

Boo & Watson (2001) conducted a study with students who had successfully passed their year 11 exams and chosen chemistry as their field of study. These students were 16-18 years in age. Their understandings of the concept of a chemical reaction were probed two times total in years 12 and 13 to allow for progression using the reactions of magnesium with dilute hydrochloric acid and a solution of (aqueous) lead nitrate with aqueous sodium chloride. The results showed that students made some progress between the two interviews but that some fundamental misconceptions were still strongly held. For the reaction with magnesium and dilute hydrochloric acid, the students were able to identify reactants and products but they did not show understandings of the energy changes, the process of the reaction, or the driving forces of the reaction. Boo & Watson (2001) found several alternative conceptions that they felt to be incompatible with understanding of chemistry, one of, which is particularly meaningful to this study: water is not involved in the reaction. This finding could give insight into how students view molarity as an extensive quantity not involving water or another solvent.

Liu & Lesniak (2006) concluded that students' conceptual progression on matter is multifaceted, contextual and dynamic. Nakhleh et al. (2005) concluded that students did not have coherent, widely applicable frameworks that could be used to explain many different physical phenomena. This is consistent with Ross & Munby (1991), who showed that students did not have a chemical conceptualization of concentration. They found in their study of concept mapping and misconceptions in student understandings of acids and bases that even a student who held correct understandings of concentration and strong and weak acids still did not understand it in terms of ions within the solution. The student also was unable to answer questions related to pH.

The literature referenced in the preceding paragraphs have a gap in that they do not address the use of number specifically within the concepts. The examples in the following paragraphs are the few studies that do specifically address number.

Eliam (2004) had students compare the number of drops needed for pure water and soap solution to fill a 1-mL cup. Most students did not make the connection that a larger number of drops of soap water meant that a drop of soap solution was smaller than a drop of water. Çalỳk (2005) included a problem on types of solutions (dilute and concentrated) and found that most students failed to use specific reasoning about concentration to predict the relative sweetness of two different solutions.

This is similar to results found in a mathematics education research study using a Piagetian task conducted by Schwartz & Moore (1998) where 6th-grade students were presented with mixing problems and then asked about the sweetness of orange juice between a large cup and a small cup of juice from the same carton. Students were assigned to one of four different groups with increased quantitative and physical information and four categories emerged in the coding: Quantified and Partitioned, Quantified and Non-Partitioned, Non-Quantified and Partitioned, and Non-Quantified and Non-Partitioned with partitioned meaning whether the water and the concentrate were divided into separate ingredients in the response. Findings from the study that are particularly relevant to chemistry include the students' focus on one ingredient, students' beliefs that a larger cup has a stronger flavor, and students' statements that if it came from the same carton it would all taste the same. Schwartz & Moore (1998) conclude that students were unable to reason through the task proportionally because their mathematics understandings were not sophisticated enough to attend to the quantitative information in a meaningful way and therefore were unable to construct ratios for their responses.

A similar task can be found in a study conducted by Gabel & Samuel (1986) where lemonade was used as an analogy to concentration. Students had difficulty relating the two because they had never tried to make weak lemonade stronger through evaporation of the water like you would with a solution in chemistry. Harel, Behr, Lesh & Post (1994) found that sixth grade students "based their judgments of taste of two samples from the same mixtures on the relative volumes of the samples to be tasted, whether the mixture is thought of as consisting of a single ingredient or more than one ingredient, and the relative amount of the ingredients stated in the problem (324)." A slight variation of this task assesses taste as an intensive property. Stavy, Strauss, Orpaz & Carmi (1982) presented students with two cups of the sugar water. The two cups were poured into a third cup and the students were asked would happen to the taste. The

6-10 year olds responded that it would be sweeter because there was more volume. Student performance in these "qualitative reasoning about concentration" tasks gives insight into potential students responses within the tasks of this dissertation.

Findings similar to the mixing task studies presented in the previous paragraph can also be found in the chemistry education literature. For example, Sanger and Greenbowe (1997) included a question regarding voltage changes with regard to a concentration change from 0.01 M CuCl2 to 0.001 M $CuCl_2$. They found that students could often work the Nernst equation (an equation explaining electrochemical cell potentials), but at the same time held the misconception that electrochemical cell potentials were independent of ion concentrations, despite ion concentration appearing as a variable within the equation. Results from these studies showed that students handled quantification algorithmically but did not have deep understandings of what the algorithms meant. This points to the need to help students develop a conceptual understanding of concentration as an intensive quantity to improve their reasoning in other chemistry concepts.

Insight about student reasoning is also present in studies probing student understandings at the particulate level. For example, Bruck et al. (2010) asked the questions "Can students' conceptual understandings of solubility be enhanced by their participation in a conceptual, hands-on activity involving manipulatives?" and "Can a multilevel laboratory activity increase students' ability to transition through Johnstone's microscopic, macroscopic, and symbolic forms of chemical representations more proficiently?" They used pre/post tests with 141 first semester general chemistry students with one control group and three treatment groups. The treatment groups were exposed to submicroscopic interactions between water, molecules and ions in solutions and terminologies such as solvent, solute, and solutions. They were shown solvent effects using color and submicroscopic modeling of solubility as well as symbolic representations of solubility. The pre/post tests and data from a free response question show that the treatment groups made advances in their macroscopic and microscopic abilities. There was a significant interaction between group membership and post-test score.

Kelly et al. (2010) conducted a study with 21 general chemistry students to probe their understandings of precipitation reactions from solutions through oral and drawn descriptions of three molecular equations presented to them on worksheets. These reactions include: silver nitrate and sodium chloride, potassium nitrate and sodium chloride (no reaction occurs), and manganese chloride and silver nitrate. Students involved in this study had a largely symbolic lecture and instructors verbalized an assumption that concepts involved in precipitation reactions had been discussed in their courses. Half of the students drew “simplistic smallest ratios” (for example if one molecule of methane reacts with two molecules of oxygen, they would only draw three molecules in total) of the species reacting to form products while the other half drew detailed aggregate representations of the submicroscopic level. Students were unable to imagine submicroscopic molecules from looking at the equation. More than half of the students included symbolic features of the equation in their submicroscopic drawings. Over half of the students drew wavy lines to indicate water and it is not surprising then that 9 students believed that aqueous was the liquid state with 14 not mentioning water at all in their descriptions of an aqueous solution. The authors recommended connecting the simplified views to the complex aggregate views of the submicroscopic along with addressing the connections between the submicroscopic and symbolic as well as between submicroscopic and macroscopic. They also recommend addressing student misconceptions directly and to address the responsibilities of the students. This gives insight into how students may draw at the particulate level. The researcher can then ask probing questions such as, "What is that wavy line?" or "Why did you only draw three molecules in total? Are there only three molecules in the jar?" This insight into potential student responses strengthens the interview techniques used in this thesis.

When students reason through a change in response to a prompt, they are forced to attend what is in solution, sometimes at a "zoomed in" level. Gabel, Samuel & Hunn (1987) constructed a Nature of Matter Inventory and administered it to prospective elementary teachers. The inventory consisted of pictures of matter with atoms and molecules represented by circles and students were asked to draw a new picture after a physical or chemical change. Students did not attend to the features of particle order or conservation of particles in over 50% of the questions. Particularly relevant misconceptions that arose were: enlargement of atoms between the phase change from liquid to gas, addition of lines to indicate the levels of liquids, and intact grouping of particles after decomposition of a molecule. They conclude that while chemistry courses do seem to touch upon the particulate nature of matter (60% had taken a prior chemistry course), it is not sufficient for a higher-level of understanding. Particularly interesting in the Gabel et al. (1987) study was the finding that students increased the size of the particles during a phase change from liquid to gas. It can be argued that the students were attending to the surface features of the problem: they still had the same substance and it suddenly took up more space. To a novice, with nothing being added, the only other option would be the size of the molecules changing to make up for that space.

Kozma & Russell (1997) studied how expert chemists and novice undergraduate students responded to various representations common to chemistry. The first experiment instructed the participants to group the representations provided in any way that was meaningful to them. Both the expert and novice groups formed meaningful groups with novices forming smaller groups of same-media representations and experts forming larger groups using multiple-media representations. The authors explained that experts had conceptual understandings whereas the novices were reliant upon surface features (salient features) of the representations, such as color. The authors stress the importance of the development of representational competence in chemistry students because surface features of the representations play a large role the understanding of chemistry.

The goal of a study conducted by Williamson et al. (2004) was to address whether question formats prompt an everyday or scientific response, if students are cued by content to answer in particulate terms, and if the data show a correlation of reasoning ability and particulate responses. Their findings suggest that instructors can ask less abstract questions and merely include the words atoms or molecules to get particulate level responses. That is, if particulate level language is a salient feature of the question, students will respond at the particulate level.

This detailed literature review shows that certain things are well-established in the literature. In the first half of this section, studies reported student difficulty reasoning through solutions chemistry and concentration. These concepts include quantities of intensive ratios and because of this fact we cannot rule out the possibility that this is why students do not understand concepts such as molarity. The studies presented from the literature do not say what students do know about the concepts. Knowing the alternative conceptions that students hold regarding intensive quantities, such as molarity, would be useful in terms of formative assessment.

The second half of this section shows that student reasoning can be revealed through particulate drawings. The design of this research study capitalizes on this type of reasoning and is used in the analysis of student responses. The literature, especially Kelly et al (2010) also guided the types of questions used for probing student understanding in this study, such as asking students to elaborate on what their wavy lines meant in a drawing or asking them to explain their simplest ratio drawings. The studies presented in the first half of this section spoke to the fact that students do not have deep understandings of solutions chemistry at the conceptual level. This research study attempts to bridge what is known from mathematics literature (students have difficulty with intensive quantities) and see if it applies to the mistakes and misunderstandings that students have in chemistry with respect to the intensive quantity of molarity.

C. <u>**Conceptual versus Algorithmic**</u>

Simple algorithmic proficiency does not provide any information about student understanding of a concept, as evidenced previously by Sanger and Greenbowe (1997). There is good evidence within the literature that students who can successfully do algorithmic problems lack a conceptual understanding (Gabel 1998). This assertion is particularly important to the molarity tasks within this dissertation as students can solve problems algorithmically with no reasoning through the task. The papers within this section speak to this disconnect between algorithmic proficiency and conceptual understanding.

Lythcott (1990) investigated the relationship between chemical knowledge and problem solving approaches in high school chemistry students with problems about mass in chemical reactions. One class was given a set of rules to follow for solving the problems and the second class was given a learning strategy to solve the problems. 34.2% of the problems in the algorithm group were solved correctly (21% perfect scores) and 18.4% had inadequate solutions. 32.6% of the problems in the learning group were solved correctly, 17.4% with perfect scores. Only two students had a clear understanding of the proportionality of coefficients and only 5 students were able to express with complete confidence the existence of particles of water. Only 5 students were able to represent a balanced chemical equation with atoms and only 6 students could define a mole. The scores between the two groups were similar in terms of correctness, but note that this is with a lack of chemistry knowledge, indicating that algorithmic questions do not assess conceptual understanding.

BouJaoude et al. (2004) studied grade 11 students in Lebanon with their performance on conceptual versus algorithmic problem solving. Students performed significantly better on algorithmic problems. Their learning approach questionnaire (LAQ) was a Likert type instrument designed to measure students' orientations to learning ranging from meaningful to rote and they include a question on ratio involving oranges to cups of juice. Meaningful learners outperformed rote learners on a conceptual test where no significant differences existed on the algorithmic questions.

Discrepancy between performance on algorithmic and conceptual problems is well explored by Nakhleh and colleagues. Nakhleh (1993) used five pairs of questions on the chemistry concepts of: gas laws, equations, limiting reagents, empirical formulas, and density.

Each pair had an algorithmic and a conceptual question. She began with the hypothesis that the remedial course would have a higher population of conceptual thinkers and that the honors students would be both conceptual and algorithmic thinkers, and that the science and engineering majors would mostly consist of algorithmic thinkers. Data from the study indicate that students can get the algorithmic question correct but not answer the conceptual question of the same topic. Nakhleh & Mitchell (1993) wanted to ascertain what students think about while solving conceptual and algorithmic problems. They used paired exam questions on gas laws with 60 freshmen introductory chemistry students. Students were identified as either conceptual or algorithmic problem solvers. Success or failure on the items led to four different categories: HA/HC (43.3%); HA/LC (41.7%); LA/HC (5%); LA/LC (10%). To further understand the student responses, they interviewed six students, two from three of the four categories (none were available from LA/HC because of low numbers). More than 50% of the students who took the assessment fell into the low conceptual category.

This conceptual versus algorithmic issue was explained with respect to solutions by Pinarbasi & Canpolat in their study (2003) where they explored concepts related to solutions chemistry such as saturation, physical properties, and gas solubility. Students were given a diagnostic test of 4 MC questions and were asked to write an explanation for each response. 107 students were given the test with 7 participating in an informal interview. Their analysis suggested that the majority of the students correctly stated the definitions of the concepts and that many students tended to leave explanations blank or repeated responses among questions. They identified several major misconceptions. They suggest that a great proportion of the students were unable to apply their chemical knowledge to real-life situations. This relates to both the domain-general and domain-specific tasks of this dissertation work in that the design of the study tests student ability to recognize the similarity between a domain-general tasks and transfer that skill to a domain-specific task.

Schmidt (1994) conducted a descriptive study to create and test questions involving stoichiometry with number ratios for quick mental calculations to identify problem solving strategies. They found five different strategies for calculations on their further probing. They conclude that students' successful solving strategies have to be known in order to decide the effectiveness of items. This study is applicable because this dissertation work seeks solving strategies for both the direct proportional reasoning problems (domain-general) and the molarity problems (domain-specific).

The studies presented in this section speak to the notion that students can be successful at algorithmic problem solving but that it is not an indicator of student conceptual understanding. The literature (Schmidt 1994) also shows that students may have different solving strategies while solving problems. Both of these findings guide the design of this research study in that student solving strategies are specifically sought out as well as whether students have conceptual understandings of molarity.

D. **Alternative Conceptions**

Alternative conceptions are defined as deviations from accepted conceptions held by the larger professional community (Garnett et al. 1995). This is consistent with Fleer's (1999) description of alternative means: alternative to Western science. Alternative conceptions is preferred to misconceptions because it is non- judgmental (Fisher & Lipson 1986).

Molarity is an intensive quantity but students can hold alternative conceptions that involve molarity as an extensive quantity. There is a literature base describing alternative conceptions held for various topics within chemistry, but no one has ever looked at how students interpret ratio or other numbers within solutions chemistry. The literature presented in this section serves to situate this dissertation research in a gap within the current research.

For a variety of reasons, including everyday language confusions (Gilbert et al. 1982) or oversimplifications (Bodner 1991), students can form conceptions of phenomena that can be alternatives to the canonical conceptions of a field. The study of alternative conceptions, though, has been restricted to qualitative representations; no studies considered that the ways in which numbers are used to represent many things in chemistry could also be the source of confusion in some tasks.

O'Grady-Morris (2008) reported that students in her study held alternative conceptions stemming from an overgeneralization of theory related to the particulate level. Others found that the use of multiple definitions can cause student confusions (Carr 1984; Garnett et al. 1990b).

Alternative conceptions research has been conducted with regard to students understandings of the particulate nature of matter (Andersson 1990; Gabel et al. 1987; Gilbert et al. 1982; Novick & Nussbaum 1978, 1981; Renstrom et al. 1990; and Tveita 1993), students' ability to balance equations (Niaz & Lawson 1985; Savoy 1988; and Staver & Jacks 1998) and the information found in chemical equations (Ben-Zvi et al. 1987; Garnett et al. 1992; Nurrenbern & Pickering 1987; and Yarroch 1985) but not ratio on its own.

Nurrenbern & Pickering (1987) focused on limiting reagent problems that, while they are quantitative, do not focus on understandings of number at the most basic level. Yarroch (1985) had students balance equations and then draw them at the particulate level. Students were asked to describe their processes, but the study focused on the correctness of their process and not what the students' conceptions of the numbers were. Garnett et al. (1992) focused on stoichiometry and problems but not on the balancing of the equation itself. Similarly, Nakhleh (1992, 1993) focused on the particulate view of the chemical equation and the limiting reactant extension of it. In the topic area of balancing equations, no one has looked at the actual numbers themselves.

Lastly, students may use language that appears to be conceptual but at the same time hold alternative conceptions. Jaisen (2010) explored what students really meant when they said that something was "neutral". While the topic of acids and bases is loosely related to molarity, the notion that students could be using words without proper meanings behind them is intriguing. Students may use the words of moles, molarity, and concentration and potentially be deemed proficient because of their extensive vocabulary when in reality they may hold an extensive view of molarity. Jaisen (2010) pertains to this study in that the goal of this study is to find out what students mean when they talk about concentration and molarity. There is a gap in the literature that does not look at alternative conceptions about molarity as an intensive quantity. This study intends to fill that gap.

III. METHODS

This chapter serves to describe the experience of the participants, including who the participants were, the materials and questions asked of the students to depict their exact total experience and how their responses were turned into data and codes.

A. **Participants**

A stratified sample of 24 students enrolled in general chemistry at a large midwestern university participated in this study. In order to ensure that students had not received college-level instruction in molarity prior to participation in the study, students were interviewed in the summer prior to the start of their first year of college. The sample was defined through a series of steps that took advantage of the screening placement exam administered at the university where the study was conducted. This screening test is a secure exam that assesses students on Quantitative and Chemical Conceptual ability. This enabled sampling of four distinct groups with a diverse set of academic levels for the students.

Some groups of entering students were not sampled. First, students who placed into Chemistry 116 (Honors Chemistry) already had a high level of the skills necessary and were unlikely to show variance in their understandings. Conversely, relatively few students who placed into Chemistry 101 had high Quantitative score, making them unusual in a way that would make their participation less meaningful. Finally, students who were instructed to postpone chemistry until math is remediated did not have enough background knowledge to complete the tasks and were therefore not included in the study.

There was a sufficient number of students with diversity of placement exam scores in the group of greatest interest for instruction that four different groups were sampled. They were chosen to provide groups that vary by chemistry knowledge (Above C/Below C) and by quantitative ability (Above Q/Below Q), giving four groups: Above Q/Above C (1), Above Q/Below C (2), Below Q/Above C (3), and Below Q/Below C (4). Six students from each category were interviewed to obtain category diversity, yielding 24 students total. (Table 1). Students in each category were labeled with a hyphen and the number of their category. For example, Student 1-3 was the first participant in the study and he belonged to Group 3 (Below Q/Above C).

Table 1: Stratified sampling categories with numbering system

	Concept Above 486 (Range 0-485)	**Concept Below 486 (Range 487-780)**
Quantitative Above 496 (Range 497-631)	Above Q/Above C (1) *6 students*	Above Q/Below C (2) *6 students*
Quantitative Below 496 (Range 331-495)	Below Q/Above C (3) *6 students*	Below Q/Below C (4) *6 students*

Based on the results of the placement test, students were recruited through a multi-step process. The list of students was provided in an anonymous way by the chemistry department such that exact scores on the placement exam sections were unknown to the researcher. Rather, to protect student data, lists of students for each category were generated following the stratified sampling guidelines provided. Fifty students were recruited randomly from each category based on their placement exam scores. First, eligible students were contacted via email, then by postal mail, and finally by a phone call. If less than 6 students responded from a category, 50 more students were contacted until the numbers were met for each category. Students were offered a chance to win a *Nintendo Wii*® for participation. A drawing was held for every set of 12 students. Students were interviewed using a semi-structured interview to look at student interpretations of molarity.

The original goal was to complete the interviews prior to the start of classes at the university. Due to inadequate student response over the summer, recruitment continued into the week prior to the first day of classes. Therefore, some interviews were necessarily scheduled during the first week of classes. This affected three student interviews, students 22-3, 23-4 and 24-3. Those three students were questioned about the chemistry instruction that they did receive during the first week of courses. Students 22-3 and 23-4 had one lecture of chemistry not pertaining to molarity or solutions chemistry. Student 24-3 had two lectures and one lab involving solutions, but the student did not relate the lab to interview tasks.

B. **Design and Materials**

A structured interview approach was chosen for this research because it embodies the type of research necessary to answer the research questions guiding this study. It is intended to help a researcher who is "interested in understanding how participants make meaning of a situation or phenomenon, this meaning is mediated through the researcher as an instrument, the strategy is inductive, and the outcome is descriptive" (Merriam 2002). Because the aim of this research is to characterize students' conceptions of molarity, this type of research suits my research questions well. Data were collected through a combination of videos of structured interviews and student drawings that were then analyzed to identify recurring themes throughout the student interviews.

To accomplish this, interviews were conducted with students using a series of molarity tasks, described in depth later, that were designed to elicit responses that reveal how students think about and reason with the different aspects of the numbers and units involved in molarity. A series of structurally similar tasks were also used to see how students engage in this type of reasoning outside the domain of chemistry. An attempt to obtain information on how students approached these problems and general chemistry content knowledge was made through a Chemistry Diagnostic (CD), which will be explained further in this chapter. Student responses were then analyzed using a constant comparative analysis to develop grounded theory.

When developing theory that is grounded in the data, the researcher allows the theory to emerge from the data rather than speculating how it *should* work based upon his or her own experience (Strauss & Corbin 1998). It involves systematic data collection that could be used to develop new theories that address the various interpretive realities of each participant (Suddaby 2006). This affords both the construction of categories based upon student answers and the ordering of those categories with the explicit goal of not forcing known categories. These categories are then compared using constant comparative analysis to identify variations in the patterns within the data (Strauss & Corbin 1998).

C. **Procedure**

Two studies in particular were relevant to the evolution of the research study: one about student conceptions of matter (Renstrom et al. 1990; Renstrom 1988) and another about student conceptions of solubility (Ebenezer & Erickson 1996). The questioning style to point students in the direction of a particulate-level response found in Renstrom et al. (1990) and Renstrom (1988) was used in this study. Students were asked to draw if they had a special camera that could zoom in "really small", potentially prompting students to draw at the submicroscopic level. The study found that students had different understandings of matter but only because of the types of questions asked by the researcher. The research did not focus on whether they were right or wrong but rather on what their understandings truly were. Similarly, Ebenezer & Erickson (1996) conducted a study on students' conceptions of solubility and pointed to discrepancies in intended meaning versus interpreted meaning. This study guided the research in that careful attention was paid to making the researcher's intended meaning explicit to avoid wrong interpretations of the questions.

Research subjects were interviewed with a protocol that had seven distinct parts that are presented in the order that the participants experienced them:

1. Domain-specific "Think-aloud" diagnostic in chemistry content (Chemistry Diagnostic or CD)
2. Domain-general "Think-aloud" diagnostic in basic direct proportional reasoning by Misailidou & Williams (2003) (Proportional Reasoning Diagnostic or PR)
3. Domain-specific task with four jars all containing the same volume but different molarities (Same Volume Different Molarity or SVDM)
4. Domain-general structurally similar task with five wooden blocks of the same height but with varied amounts of red and white paint (Same Height Different Color or SHDC)

5. Domain-specific task with three jars containing different volumes of solution all with the same molarity (Different Volume Same Molarity or DVSM)
6. Domain-general structurally similar task with five wooden block of varying heights all painted the same color (Different Height Same Color or DHSC)
7. Domain-specific problems involving molarity (Molarity Problems)

The rationale for the specific ordering of tasks stemmed from the desire to keep recency effects and learning during the protocol at a minimum. For example, if the structurally similar tasks occurred prior to the molarity tasks, students may notice the connection between the tasks and learn from them leading to altered responses. The Chemistry Diagnostic and Proportional Reasoning Diagnostic were given first to get an initial understanding of student knowledge of chemistry and ratios without being "contaminated" by experience within the protocol.

For the molarity and paint tasks the students were presented with phenomena (labeled jars of liquid, painted blocks) and prompted verbally with questions. Their responses were audio-recorded and their writing and drawings were recorded with the SmartPen system. An interview guide was employed that was a hybrid between a general interview guide and a standardized interview guide so that students were asked the same questions but also allowed for flexibility when a situation arose where it was need (Patton 1990). For example, in the case of a student contradiction to a previous utterance, Green (2005) suggested saying "It is interesting to me that earlier you noted that X was significant but later you talked about Y. These seem to contradict each other. Can you tell me about this (p37)." Following these suggestions ensured that each student participated in the same interview so that ambiguity of student answers is minimized (Bowden 2005).

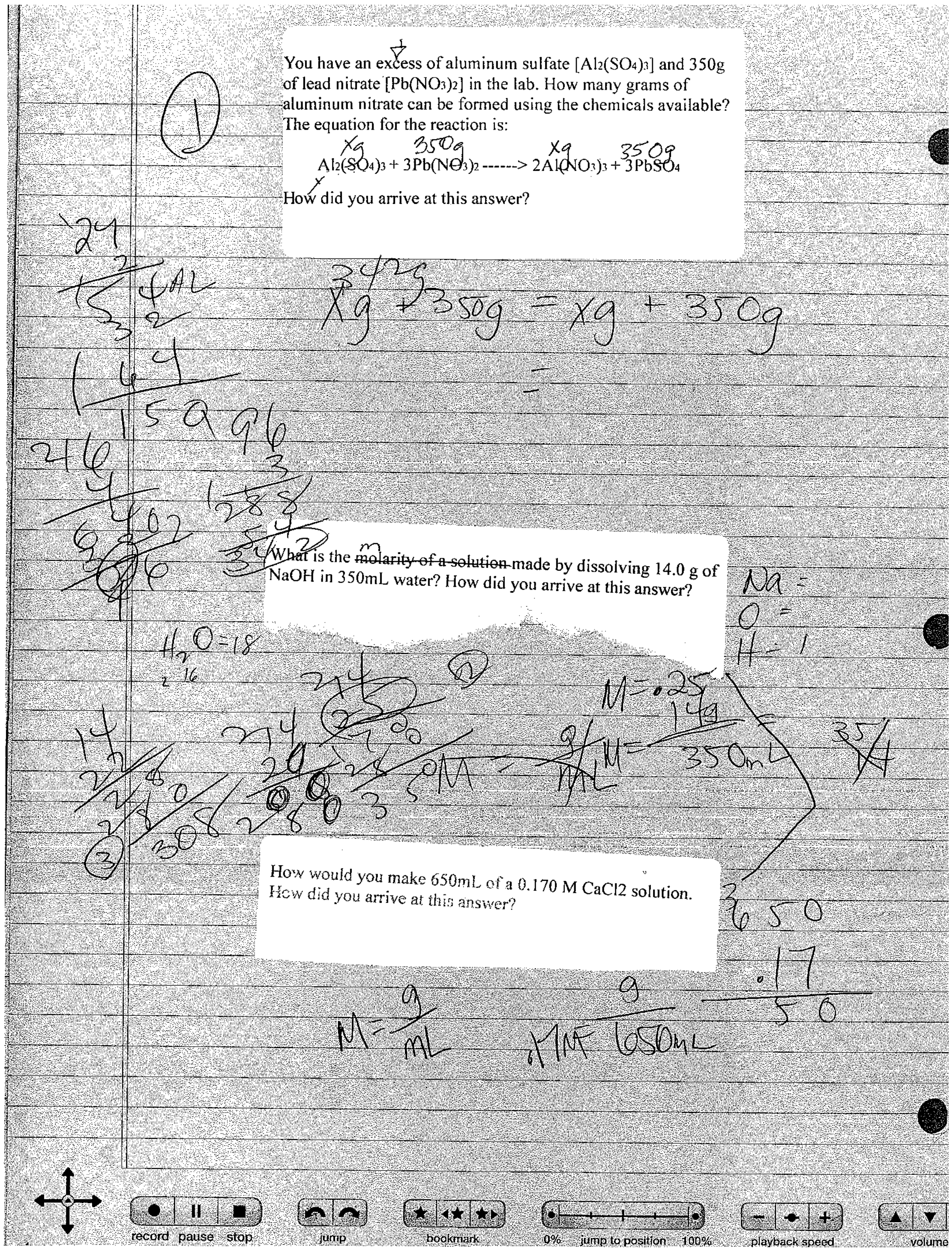
The series of tasks took approximately two hours of interview time depending on the length of student responses. The first four interviews produced little data in the seventh task, Molarity Problems, because students were unable to solve the problems algorithmically. In the remainder of the interviews, the students were asked to attempt the problems and, if they were unable to solve them, they were given hints. The hints consisted of the following: 1) M stands for molarity 2) Molarity is moles/L and 3) an example of a mole calculation using #1 from the Chemistry Diagnostic. These were chosen as hints because without these basic understandings or processes, a student would inherently be unable to solve these molarity problems.

To address the research question of student interpretations and use of ratio as it pertains to molarity, the Same Volume/Different Molarity and Different Volume/Same Molarity tasks were adapted from the "Solutions" project to include parallel structurally similar tasks that were domain-general. They have been used in this research project because they are an efficient way to find student understandings of molarity and specifically ratio as it pertains to molarity. The structurally similar tasks (Wooden Blocks Different Color/Same Height and Wooden Blocks Same Color/Different Height) were created to explore proportional reasoning outside the domain of chemistry.

Students used a SmartPen while solving problems or drawing and were encouraged to think aloud and explain their reasoning for each question or drawing. Figure 6 shows an example of Student 1-3's notebook. As shown in Figure 6, the students were given the problems in the form of labels placed in the notebook. The SmartPen recorded pen strokes at the same time as audio so that order of operations could be determined. This data collection method has been shown to be useful in Linenberger & Bretz (in press) detailing its benefits to qualitative research.

Each of the student interviews were transcribed in their entirety. After the videotapes were transcribed, the SmartPen audio was aligned to the video transcripts. Then, the SmartPen data was annotated into the original transcript. This SmartPen data was used to clarify the steps used by students in solving problems.

Figure 6: A page from Student 1-3's SmartPen notebook from the interview.



1. **Chemistry Diagnostic Paper and Pencil Test**

A diagnostic probing students' prior understandings of chemistry was administered to the participants at the beginning of the two-hour research interview. This will be referred to as the Chemistry Diagnostic or CD. Figure 7 shows the domain-specific areas tested involving molarity and ratios including dilution, molarity calculations, mole to mole conversions, and mole to gram conversions. These questions were chosen because they were typical algorithmic questions in chemistry courses that also involve ratios and molarity. This acted as a starting point for comparison and showed if students had algorithmic understandings in chemistry.

Figure 7: Chemistry Diagnostic probing student understandings of mole/mole, mole/gram, and molarity calculations.

1) Which of the following has the most moles? How did you arrive at this answer?
   - A) 12 g of Helium (He)
   - B) 50 g of Cobalt (Co)
   - C) 200 g of Mercury (Hg)
   - D) 100 g of Titanium (Ti)

2) A reaction produces 175 grams of MgO. How many grams of O_2 were reacted react using the following equation: $2Mg + O_2 \text{ ---> } 2MgO$ How did you arrive at this answer?

3) You have an excess of aluminum sulfate [$Al_2(SO_4)_3$] and 350 g of lead nitrate [$Pb(NO_3)_2$] in the lab. How many grams of aluminum nitrate can be formed using the chemicals available? The equation for the reaction is:

$$Al_2(SO_4)_3 + 3Pb(NO_3)_2 \text{ ------> } 2Al(NO_3)_3 + 3PbSO_4$$

How did you arrive at this answer?

4) What is the molarity of a solution made by dissolving 14.0 g of NaOH in 350mL water? How did you arrive at this answer?

5) How would you make 650mL of a 0.170 M $CaCl_2$ solution. How did you arrive at this answer?

6) How would you make 550.0 mL of 0.220M NaOH from a 6.00 M stock solution? How did you arrive at this answer?

2. **Proportional Reasoning Diagnostic Paper and Pencil Test**

To understand how students used ratios and proportions in their reasoning with molarity, it was important to first document and categorize the types of solving strategies they used in other types of problems. This was important to determine whether the students did indeed have direct proportional reasoning skills and whether they applied the same strategies to problems where there was not a direct proportion, such as a molarity dilution problem.

After the Chemistry Diagnostic (CD), students were given a diagnostic probing their general ratio and direct proportional reasoning abilities. Figure 18 shows the Proportional Reasoning (PR) diagnostic created by Misailidou & Williams (2003). It consists of 13 questions that have been validated and calibrated by the authors who created it. The questions are intended for younger children than for incoming freshmen, for the concepts of ratio and proportion should be mastered in youth. Still, the questions are pertinent here. Having students complete this also permitted the characterization of some of their general proportional reasoning strategies for comparison with those that they used in the domain-specific problems in chemistry. For example, if a student used direct proportions while solving a dilution problem, a negative transfer would be revealed. The titles of the questions presented in Figures 8 and 9 were not provided to the students in the Proportional Reasoning diagnostic to not give hints pertaining to the critical features of the problems.

Figure 8: "Without Models" Diagnostic Test by Misailidou & Williams (2003) Part 1

The title of the questions were not revealed to the students.

1. 'Class'
Mrs. Green put her students into groups of 5, with 3 girls in each group. If Mrs. Green has 25 children in her class, how many boys and how many girls does she have?

2. '1 Eels'
There are 3 eels, A, B and C in the tank at the Zoo.
A: 15 cm long
B: 10 cm long
C: 5 cm long
The eels are fed sprats, the number depending on their length. If C is fed 2 sprats, how many sprats should B be fed to match?

3. '2 Onion Soup'
An onion soup recipe for 8 persons is as follows:
8 onions
2 pints water
4 chicken soup cubes
12 dessertspoons butter
1/2 pint cream
I am cooking onion soup for 2 people.
How many dessertspoons of butter do I need?

4. '6 Onion Soup'
An onion soup recipe for 8 persons is as follows:
8 onions
2 pints water
4 chicken soup cubes
12 dessertspoons butter
1/2 pint cream
I am cooking onion soup for 6 people.
How much cream do I need?

5. 'Fruits' Price'
At a fruit stand, 3 apples cost 90 pence. You want to buy 7 apples. How much will they cost?

6. 'Books' Price'
There is a sale at a bookstore. Every book in this sale costs exactly the same. Mary bought 6 books from the sale and paid 4 pounds. Rosy bought 24 books from the sale. How much did Rosy pay?

Figure 9: "Without Models" Diagnostic Test by Misailidou & Williams (2003) Part 2

The titles of the questions were not revealed to the students.

7. '1 Paint'
Sue and Jenny want to paint together. They want to use each exactly the same color. Sue uses 3 cans of yellow paint and 6 cans of red paint. Jenny uses 7 cans of yellow paint. How much red paint does Jenny need?

8. '2 Paint'
John and George are painting together.
They want to use exactly the same color.
John uses 3 cans of yellow paint and 5 cans of green paint. George uses 20 cans of green paint. How much yellow paint does George need?

9. '1 Campers'
10 campers have camped at the "Blue Mountain" camp the previous week. Each day there are 8 loaves of bread available for them to eat. The loaves are provided by the camp's cook and the campers have to share the bread equally amongst them. This Monday 15 campers camped at the "Blue Mountain" camp. How many loaves are there available for them for the day?

10. '2 Campers'
10 campers have camped at the "Blue Mountain" camp the previous week. Each day there are 8 loaves of bread available for them to eat. The loaves are provided by the camp's cook and the campers have to share the bread equally amongst them. The camp leader told the cook that for next Monday she should prepare 16 loaves of bread. How many campers will be at the camp next Monday?

11. 'Mr. Short'
You can see the height of Mr. Short measured with paper clips. Mr. Short has a friend Mr. Tall. When we measure their heights with matchsticks:
Mr. Short's height is four matchsticks. Mr. Tall's height is six matchsticks. How many paper clips are needed for Mr. Tall's height?

12. 'Printing Press'
A printing press takes exactly 12 min to print 14 dictionaries. How many dictionaries can it print in 30 min?

13. '1 Rectangles'
These two rectangles have exactly the same shape, but one is larger than the other.

3. **Same Volume Different Molarity (SVDM) Task**

This task was designed to address the second research question (R2): What are students' interpretations of molarity in solutions chemistry? The purpose of this task was to explore student understandings of molarity as it relates to the extensive properties of concentration. This task was designed to elicit student responses to a change in one of the factors of molarity: change in substance.

In the Same Volume/Different Molarity task, students were presented with four identical bottles with the same volume but with varying molarities of 0M, 0.05M, 0.10M, and 0.15M $CaCl_2$. The volume was observable by the students. The molarities were given as labels on the bottle. Students were asked questions following the guide in Figure 10. A possible "correct" answer would be one that is accepted by the larger scientific community. In this case, the "correct" answer would be that molarity is defined as the moles of solute per liter of solution and that 0.10M is two times as concentrated as 0.05M meaning that, since the volumes are the same, there are twice as many calcium ions in the 0.10M solution than in the 0.05M solution[1]. A possible student answer could have involved percentages, as shown in Figure 20. Another student could have described molarity using molecules depicting the relationship between the bottles using a ratio. Yet another response could have been a generic algorithmic manipulation. The task was designed to allow for these different types of responses and allow different categories describing various ways in which students understand molarity.

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[1] For every one calcium ion there are two chloride ions, but this additional understanding was not probed.

Figure 10: Same Volume/Different Molarity Task (SVDM)

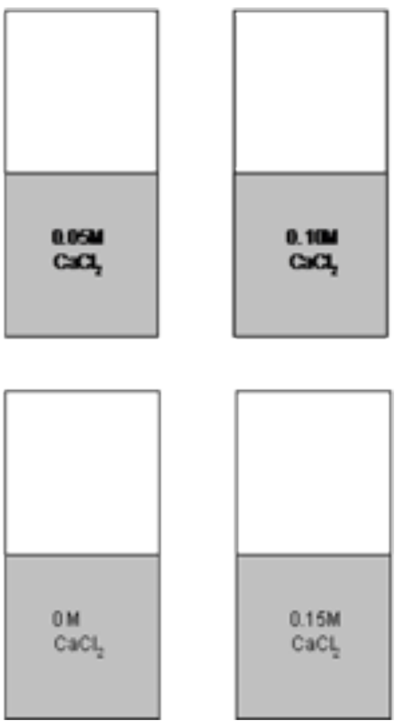
Domain Specific Task

Interview Questions

- Could you read the labels on the bottles for me? What do the labels mean to you?
- [if student says Molarity]: What is Molarity?
- What is different between the two bottles and what is the same ?
- How much more is 0.05M than 0.10M?
- What do the numbers mean?
- If you had a special camera to zoom in, what would these two solutions look like?
- If student hasn't already done so, ask them to order the bottles from lowest to highest amount of calcium chloride.

Possible "Correct" Answer

The broader scientific community would agree that molarity is defined as the moles of solute per liter of solution and that 0.10M is two times as concentrated as 0.05M meaning that there are twice as many calcium ions in solution. For every one calcium ion there are two chloride ions.



Possible Student Answer

Student 14: Um it just means that um like this one has five percent less moles per milliliter then this one does. [00:00:55.05]

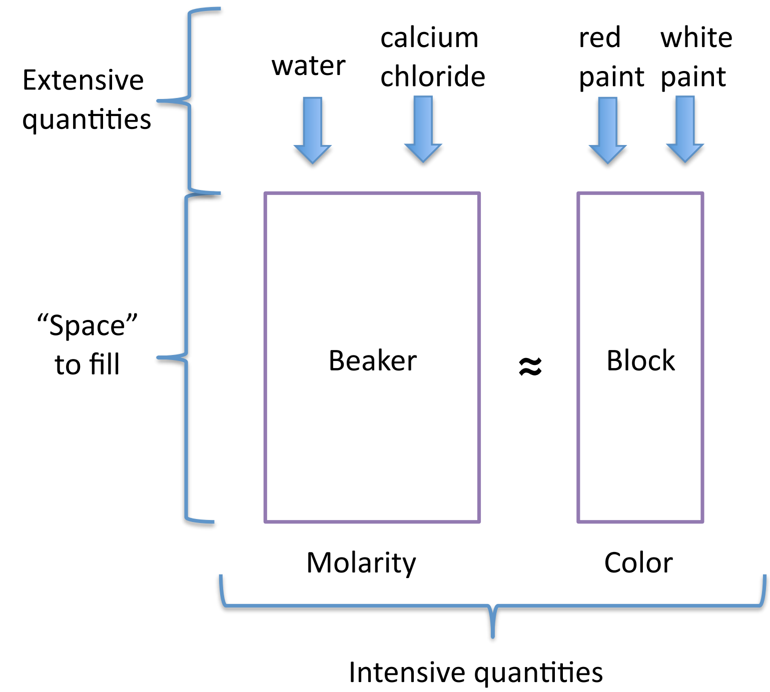
4. **Same Height Different Color (SHDC) Wooden Blocks Task**

Because the research is focused on student understandings of ratio in the context of molarity, it was also important to study student understandings and uses of ratio in structurally similar "real-life" situations. These real life situations are termed the domain-general because they are familiar domain for the students. For a discussion on the structural similarity to the molarity tasks, see Figure 18. This study assumes that students interpret number differently within solutions chemistry and molarity problems than they would while baking or shopping. The "real-life" tasks are referred to as structurally similar tasks within this study and they served as additional pieces of data. These similar tasks were also used to reveal patterns of student use of ratios in solving problems allowing to more clearly see the specific importance of molarity in R2. Students may possess ratio reasoning skills outside the domain of chemistry, but not transfer them when faced with a molarity problem because they might not recognize the similarity.

The domain-general and domain-specific tasks were designed to elicit student understandings on logically similar tasks in chemistry and in a more familiar setting. Molarity is an intensive property that is the ratio of two extensive properties: amount of substance and volume. In the domain-specific tasks, students were presented with changes in both the amount of substance and the volume with regards to molarity. Similarly, the color of the wooden blocks is an intensive property and the amount of paints used for the blocks and the size (including height) of the blocks are extensive properties. In the domain-general tasks, students are presented with changes in both the color and the height of the blocks. This variance in height can also be referred to as a change in the size of the blocks. For the purposes of this dissertation, unless explicitly stated by a participant, the change will be referred to as a change in height.

The only difference between the domain-specific and the domain-general tasks is that the indicator of the property shifts from being a volume of solution or a numeric decimal label (molarity) to the size of a block and its color. The intensive property is molarity in one case and in the other, it is the depth of color. Students are asked to translate the depth of color into a numeric representation using red and white cubes to represent the ratio and amount of the colors used to create the blocks. This is similar to having the students draw at the particulate level the amount of calcium chloride in the jars to represent the molarity. Figure 11 offers a comparison to show the structural similarity of the tasks. The beaker and the wooden block are both spaces to be filled; in the case of the beaker, it is going to be filled with a certain volume of liquid. In the case of the wooden block, it is going to be covered with paint. The intensive quantity for the beaker is molarity and it is made of two extensive quantities: water and calcium chloride. The intensive quantity for the wooden block is the depth of color and it is composed of two extensive quantities: red paint and white paint.

Figure 11: Structural similarity between the molarity tasks and their painted block counterparts



The use of blocks for the structurally similar tasks was inspired by a student response during the "Solutions" project, shown in Figure 12. While she mentioned proportion and varying amounts of calcium chloride, she had an interpretation of molarity that involved space that was revealed when she drew blocks to explain proportions. It affords more explanation of a student's interpretation of ratio within molarity. Blocks such as the ones shown in Figure 12 are common manipulatives in teaching and assessing students' proportional reasoning in elementary school. Therefore, this task is similar to an authentic math-specific structurally similar task--removing chemistry without removing the proportional reasoning.

Figure 12: Solutions Student 25's response that led to the development of the structurally similar molarity task and the proportion section of the number task

The structurally similar task for the Same Volume/Different Molarity task is called Wooden Blocks Same Height/Different Color and was designed to answer the first research question (R1): Do students' understandings of ratio vary from domain specific tasks to structurally similar tasks? The varied amounts of red paint blended with white paint are structurally similar to the varied amounts of substance (calcium chloride) in the first task. Five pieces of wood of the same height were painted in varying proportions of red and white yielding: red, light red, pink, light pink and white. Students were asked questions guided by those in Figure 13. Students were then asked to represent the painted wooden blocks using red and white cubes to show how much red paint and white paint was used to paint each block. An example of a correct answer would show that pink is 50% red and 50% white and involves different ratios of red and white paint to obtain the other shades of red. A possible alternate answer would be one that has no discernible pattern. This task was designed because a student could respond in a similar way as their response to the domain specific task, but they could also respond in a different way. For example, a student could have noticed a ratio with color but not within the domain specific tasks and vice versa.

Figure 13: Wooden Blocks Different Color/Same Height (DCSH) Task (Structurally Similar task for Same Volume/Different Molarity task)

Interview Questions

- What is the same with these blocks of wood?
- What is different?
- If I told you that I made these blocks with only red and white paint, how much red and white paint do you think that I used?
- Could you use these blocks to represent the five pieces of wood?
- Could you describe what you have constructed?
- What does each block represent?
- How do the towers relate to one another?
- If student hasn't already done so, ask them to order from the least amount of white paint to the most.

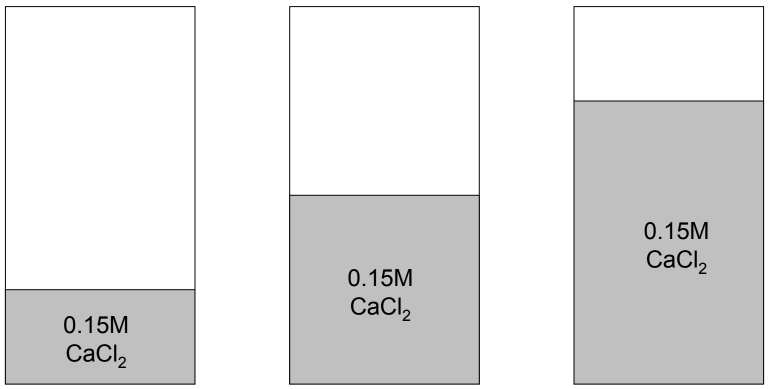
Possible "Correct" Answer **Possible Alternate Answer**

5. **Different Volume Same Molarity (DVSM) Task**

In the "Solutions" project, the researchers found that they needed another task than the SVDM task to better understand student interpretations of molarity. Thus, a contrasting cases task was developed where students were presented with 3 bottles of varying volumes but with the same concentration of $CaCl_2$, thus adding another dimension to the student's interpretation of molarity. This task was designed to address R2: What are students' interpretations of molarity in solutions chemistry? In this task students were presented with a variation in another aspect of molarity: volume. The purpose of this task was to see how students negotiated their understanding of molarity with a change in volume. It also served to explore student understandings of molarity to see if they were static or fluid depending on different variables and if they were intensive or extensive.

Students were presented with three bottles labeled 0.15M $CaCl_2$, but with differing volumes (100mL, 150mL, and 200mL). Students were asked questions guided by those in Figure 14. The broader scientific community would agree that the three bottles all contain calcium chloride and have the same concentration. The bottles had different volumes, which means that there are varying amounts of moles of calcium chloride in each bottle with the largest volume containing the most moles of calcium chloride to maintain the ratio of molarity given. A possible student answer to this task could be that 0.15M means that all three bottles have the same amount of moles of calcium chloride in each bottle with just more space between the molecules as the volume increases. Students could also respond by describing molarity as a percentage, as a ratio using molecules describing the relations between molecules, or as a generic algorithm. The task was designed to allow for these different types of responses and allow different categories describing various ways in which students understand molarity. In this case, students may exhibit different understandings than in the first task because the volume has visibly changed.

Figure 14: Different Volume/Same Molarity Task



Interview Questions

- Could you read the labels on the bottles for me?
- What do the labels mean to you?
- [if student says Molarity]: What is Molarity?
- What is different between the bottles and what is the same?
- What do the numbers mean?
- How can they all three be 0.15M with different volumes?
- Which bottle has the most calcium, or do they all have the same amount?
- If you had a special camera to zoom in, what would these three solutions look like?

Possible "Correct" Answer

The broader scientific community would agree that the three bottles all contain calcium chloride and have the same concentration. The bottles have different volumes, which means there are varying amounts of moles of calcium chloride in each bottle with the bottle with the largest volume containing the most moles of calcium chloride because concentration is a ratio.

Possible Student Answer

A sample student answer could be that students think that 0.15M means that all three bottles have the same amount of moles of calcium chloride in each bottle with just more space between the molecules as the volume increases.

6. **Different Height Same Color (DHSC) Wooden Blocks Task**

The structurally similar equivalent task to the Different Volume/Same Molarity task is called Wooden Blocks Different Height/Same Color and was designed to answer the first research question (R1): Do students' understandings of ratio vary from domain-specific tasks to structurally similar tasks? This task was designed to be structurally similar to the domain-specific task involving varied volumes with the same molarity. The changes in the length of the blocks are analogous to the varied amount of total solvent in the molarity task. The purpose of this task was to explore student understandings of ratio when one aspect is varied in a structurally similar concentration task.

The task involved five pieces of wood that varied in length but were all painted the same shade of pink. The shade of pink was the same from the middle block Same Height/Different Color (SHDC) task where the ratio of red paint to white paint was 50:50. Students were asked questions that were guided by the ones in Figure 15. Students were then asked to represent the painted wooden blocks using red and white cubes to represent the amount of red and white paint used to paint each block. Again, this provides another venue for the students' to share their interpretation of the ratio using the cubes. A possible "correct" answer would show a consistent ratio of 50% red to 50% white with different total numbers of cubes. An alternate answer could mimic the student in Figure 15 who kept one color constant while varying the other. This task was designed because a student could respond in a similar way as their response to the domain specific task, but they could also respond in a different way. For example, a student could have noticed a ratio with color but not within the domain specific tasks and vice versa.

Figure 15: Wooden Blocks Different Height/Same Color (DHSC) Task (structurally similar task for Different Volume/Same Molarity

Interview Questions

- What is the same with these blocks of wood?
- What is different?
- If I told you that I made these blocks with only red and white paint, how much red and white paint do you think that I used?
- Could you use these blocks to represent the four pieces of wood?
- Could you describe what you have constructed?
- What does each block represent?
- How do the towers relate to one another?
- Ask student to order blocks from least amount of red paint to the most amount of red paint.

Possible "Correct" Answer

Possible Alternate Answer

7. **Molarity Problems Task**

The Molarity Problems were designed to answer R3: Are there patterns in how students use ratio in their solving strategies for molarity problems? They were designed to explore how students solve molarity problems and to discern any strategies that might be present, including their use of ratio. Students were given the questions on printed labels and were asked to answer them on paper using a SmartPen in a think-aloud fashion. These questions are shown in Figure 16 and were chosen to represent various types of problems commonly used in chemistry courses to assess understanding. These included:

- calculation of moles from molarity (M1)
- calculation of volume from molarity (M2)
- calculation of molarity from grams (M3)
- calculation of grams from molarity (M6)
- calculation of molarity using a stock solution (M4)
- a dilution of molarity calculation. (M5)

These problems were interview-based think-aloud problems and they differed from the diagnostic questions in that specific solving strategies were sought, not success of knowledge of chemistry.

Figure 16: Molarity Problems Task

M1. You have a 10mL solution of 0.75M $CaCl_2$. How many moles of $CaCl_2$ do you have?

M2. You have 10 moles of $CaCl_2$. How much water would you need to create a 1M solution assuming that you must use all of the solid $CaCl_2$?

M3. You have 15 g of $CaCl_2$ in 25mL of distilled water. What is the molarity of the solution?

M4. You have a 1M stock solution of $CaCl_2$ in the cabinet and need to make 16 mL of a 0.42M $CaCl_2$ solution. How much stock solution do you need?

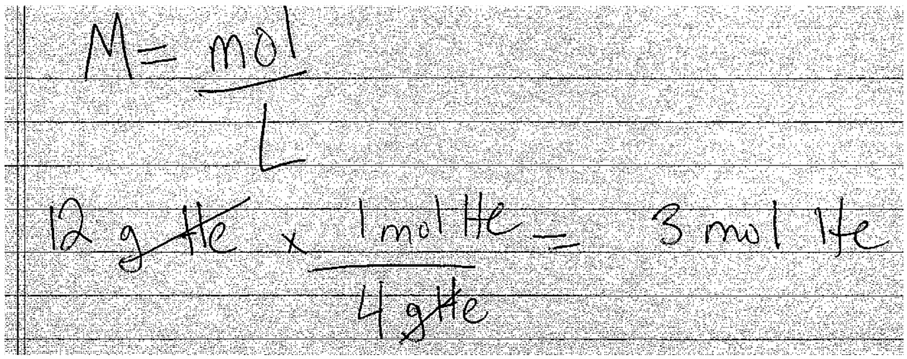
M5. You have a 100 mL of 0.6M $CaCl_2$ solution. Your lab mate adds 100 mL of deionized water to your solution. What is the resulting molarity?

M6. You have 250 mL of a 0.85M $CaCl_2$ solution. How many grams of $CaCl_2$ are in the solution?

This could give insight into their understanding of molarity and their understanding of ratio. For example, a student can write down an algorithm and solve it accordingly, but have difficulty knowing where various factors play into the algorithm. This would be considered a case where a student could correctly answer a problem procedurally but with no conceptual explanation. Different solving strategies may also relate to ratio ability. Very few of the students interviewed had successful solving strategies and there were a variety of different unsuccessful strategies that will be discussed in the analysis chapter (Chapter VI). If certain strategies are indicative of certain ratio abilities or understandings of molarity, teachers could use it as a diagnostic to find the level of students and understand the misconceptions that students hold.

In the first four interviews, three out of the four students did not attempt a vast majority of the molarity problems at the end of the interview. As a whole, the students did not have much success with problem solving in the chemistry tasks and had incomplete and fragmented mental models for molarity. Therefore, if they were unable to solve a problem from a memorized definition or through a conceptual one, they could not move forward in the interview. The researcher and the LS faculty member decided it was necessary to offer hints to students in the molarity problems after they had attempted all six problems so that students could attempt the problems. Three hints were available: 1) how to calculate a mole given molar mass and 2) the definition of molarity with units and 3) M stands for molarity. An example of this hint is shown in Figure 17. The writing in the figure is that of the researcher. The researcher first related the student's definition of molarity with the correct definition and then provided the units and the algorithm for the student. The researcher then used question 1 from the CD as an example of how to calculate the number of moles. Students were not offered a hint if they already had shown success in that area with correct reasoning. For example, if a student guessed the answer correctly or used faulty reasoning to obtain the correct answer, that student was given a hint.

Figure 17: Hint from the researcher in interview with Student 9-2



Because the interviews were two hours long and students had different amounts of time per task based on problem solving time, not all students were able to attempt all problems after the hint without exceeding the time allotment. If this occurred, the researcher asked to students to only reconsider M2 (Volume from Molarity) and M5 (Dilution). An example of this is found in Student 14-1's interview where he was so descriptive of his solving strategies that time was short for the Molarity Problems. He was only able to complete M5.

D. <u>Summary of the Tasks with respect to Transfer</u>

To summarize, the interview was designed to contain similarity transfer tasks between domain-general or familiar contexts involving concentration. The intent was to assess whether students were able to recognize the intensive nature of concentration and transfer it to a new domain (chemistry). The SHDC tasks paralleled the SVDM task and the DHSC tasks paralleled the DVSM task. Students were given the PR diagnostic to ascertain their direct proportional reasoning solving strategies and these were later used to show cases of negative transfer.

E. **Limitations**

This study considered student use of direct proportions in a general setting on the PR diagnostic. However, the PR diagnostic did not address student use of inverse proportions. Inverse relationships between molarity and volume were not explicitly probed, but they were inherent within the domain-specific tasks. The only questions that directly assessed student use of inverse proportions were those that involved dilutions or stock solutions. Because there is no domain-general inverse proportional reasoning counterpart, the author cannot state whether students had this skill outside the domain of chemistry.

Another limitation of this study is that the “domain-general” task still has a familiar domain to students: painting. It is therefore not truly general, but it is less specific than the chemistry task. Lastly, the tasks were only structurally similar and not structurally isomorphic. As shown in Figure 18, the SHDC and SVDM tasks both share a concentration that is varied and given. They differ in that the SHDC task has components that vary (not given) with the same same total (given) and a fixed length whereas the SVDM task has moles that vary (not given) and a volume that is fixed and given. A structurally isomorphic task would have a varied concentration (given) that has varied drops of red paint (not given) and a fixed amount of white paint (given). Likewise, the DHSC is structurally similar to the DVSM task. It differs in structure in that the concentration is fixed (given) and the ratio of the components (red and white paint) is fixed (not given) with the total being varied and the length is also varied (given) whereas the DVSM task has a fixed molarity (given) with varied moles (not given) and a varied volume (given). A structurally isomorphic task would have a fixed concentration (given) with varied drops of red paint (not given) and a varied amount of white paint (given). The similarity in structure allows an assessment of a student's qualitative understandings of concentration. A structurally isomorphic task would have allowed for more comparison.

Figure 18: Equations to show structural similarity and structural isomorphism

$C_{varied,\ given}=\frac{R_{varied}+W_{varied}}{L_{fixed,given}}$	SHDC (Structurally Similar)
$M_{varied,\ given}=\frac{moles_{varied}}{V_{fixed,\ given}}$	SVDM
$C_{varied,\ given}=\frac{drops\ of\ R_{varied}}{W_{fixed,given}}$	Structurally Isomorphic
$C_{fixed,\ given}=\frac{[R+W]_{varied}}{L_{varied,given}}$	DHSC (Structurally Similar)
$M_{fixed,\ given}=\frac{moles_{varied}}{V_{varied,\ given}}$	DVSM
$C_{fixed,\ given}=\frac{drops\ of\ R_{varied}}{W_{varied,given}}$	Structurally Isomorphic

F. **Scoring and Coding**

Student utterances were analyzed through microanalysis using open coding to discover categories of emergent themes. Twelve students were open coded with a Learning Sciences/ Chemistry faculty member and an open code consensus was reached dialogically for each of the three research questions. The other twelve students were coded using that consensus by the researcher alone. If any utterances were vague and the researcher needed assistance, the faculty member was consulted for consensus. This consensus style of coding was conducted similarly to Varelas et al. (2005) in their study of beginning teachers' identities in science. Their description involved two co-researchers and a labor-intensive coding scheme for both of them. It was sufficient to have two coders to establish the basis for the remaining twelve students.

It is important to note that this dissertation is constructing theory grounded within data, denoted by a lowercase g. This is similar to the Grounded Theory described by Strauss & Corbin (1998). The difference between grounded theory and Grounded Theory for the purposes of this dissertation is in the coding process. Instead of traditional axial codes, relevant open codes were used to situate students on a tree diagram as explained later. Open codes were found and organized to find larger patterns, similar to the axial codes in Grounded Theory. Those patterns were then used to construct larger theoretical statements.

To assist in the search for patterns among the data, partially ordered tree diagrams (Miles & Huberman 1994) were constructed as a visual aid and as an efficient organizer of the data. An example of this type of diagram is shown in Figure 19 using Student 1-3's utterances.

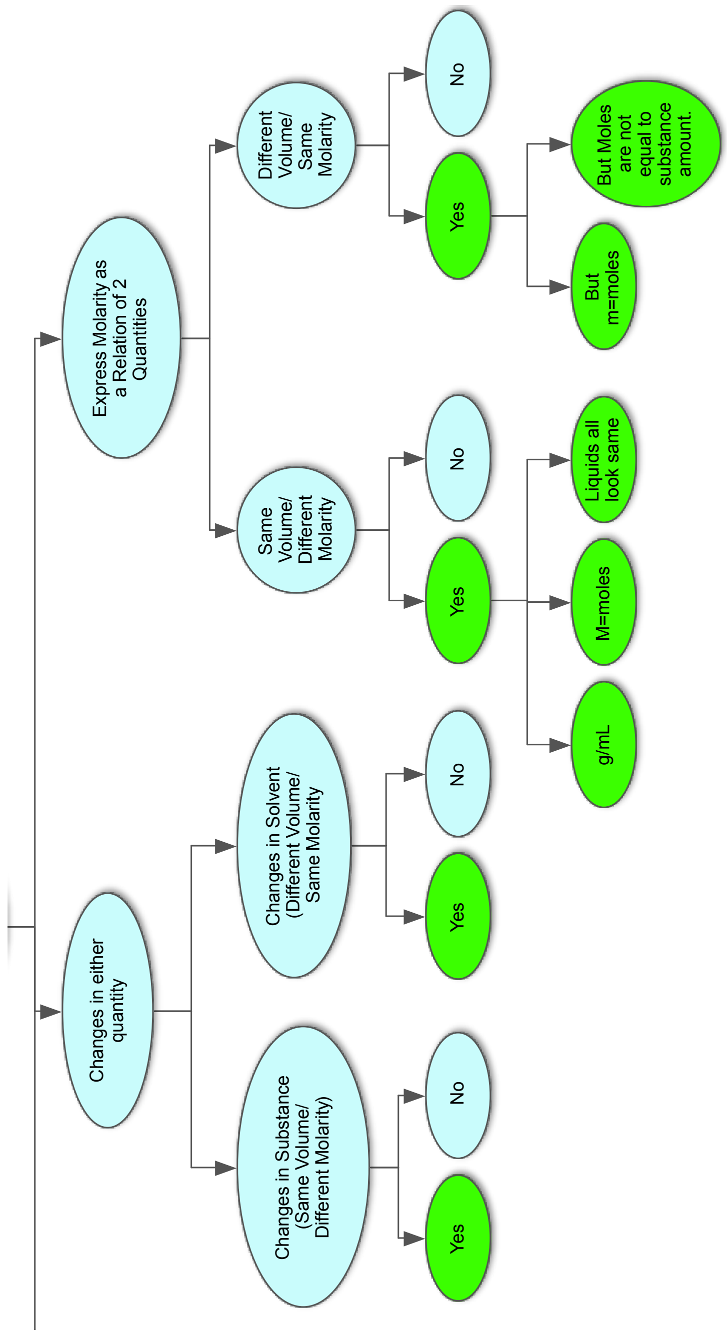
The molarity tasks and their structurally similar counterparts can be divided into three categories: Procedural, Changes, and Ratios.

- Procedural understanding of molarity corresponds to algorithmic solving of a calculation of a mole problem. A student can either get the problem incorrect, correct with a conceptual description or correct without a conceptual response
- In the case of Changes, there can be changes in the amount of substance or the volume. In each of these cases, students can correctly identify that a change has occurred either in the substance of the volume or not identify that a change has occurred. (Figure 19)
- In the case of Ratio, the student can either recognize that the change in substance or change in volume involves a relationship between two extensive properties or they do not recognize such a relationship exists. (Figure 19)

The open codes from the molarity tasks and their structurally similar tasks yielded a variety of conceptions that fall under these categories. The trees were constructed to show the parallels between the molarity task and its structurally similar painted block task. If a student is able to reason through the block task and not in the molarity task, it is easy to see the open codes and conceptions that the student held. The tree for R2 was altered to show patterns in solving strategies before and after hints. Large Xs are used to indicate that the problem was not attempted. All 24 students for all three research questions went through this stage of coding with an additional Learning Sciences graduate student. Because the open codes were agreed upon previously, there was little disagreement in this stage. However, if a disagreement occurred, a consensus was reached between the two coders. These relationships between the data were then used to construct theoretical statements that are presented in the Analysis chapters (Chapters IV-VI). It is important to note that this is not the imposing of codes onto the data, but rather a reorganization of the student utterances for easier comparison.

Figure 19: An example of the analysis trees for the changes in quantity and ratio as shown by Student 1-3's tree with respect to R2

Green and red were used for visual cues for the axial coders.



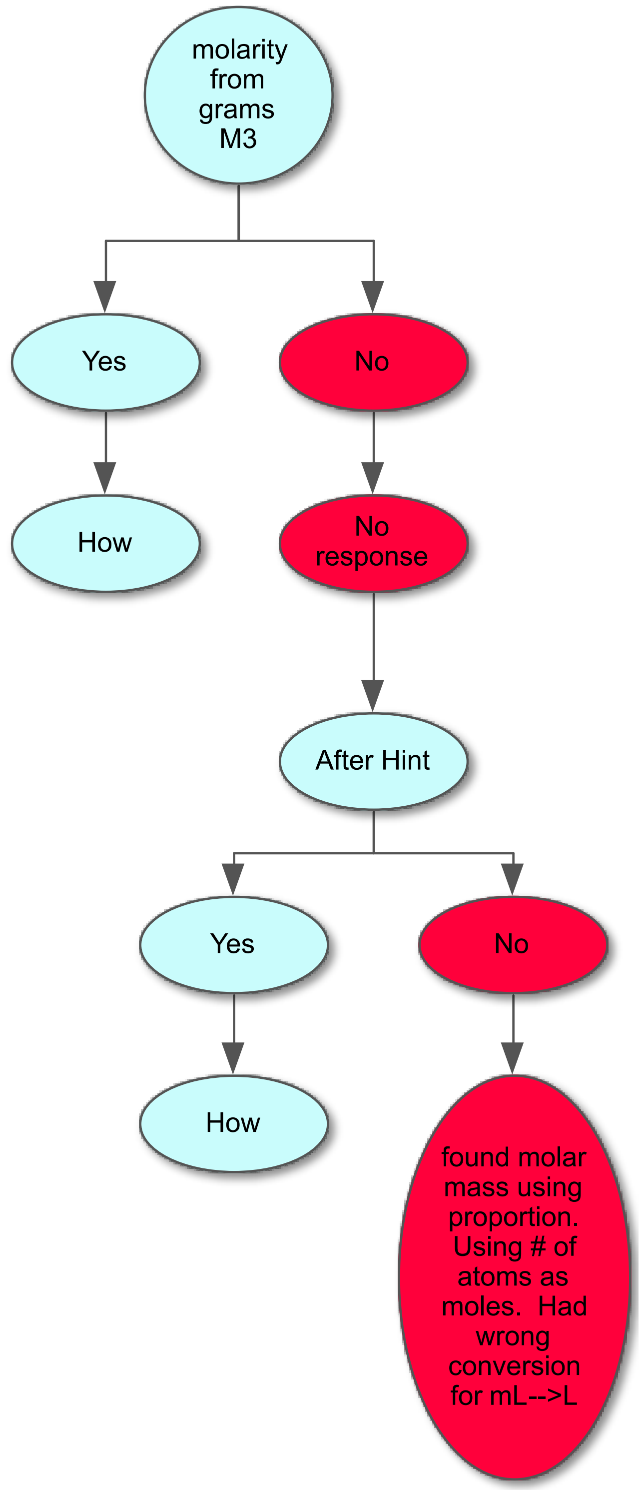
Reliability was enhanced and maintained by designing the study such that the researcher's questions were as clear as possible to minimize unclear expectations or confusions about prompts. If a student response was vague, the researcher asked the student for clarification if she felt that her interpretation might influence the interpretation. Reliability was also maintained because the protocol was designed such that students were not judged upon correctness of their responses and they were given many opportunities to describe their thoughts in their own words, pictures, and towers of cubes. The researcher also did not note student sampling categories prior to the interview so these would not influence her expectations of responses. In the coding process, two outside coders were consulted so that the codes remained as close to the data as possible.

Each research question was open-coded separately and completely before developing open codes for the next research question. Open codes have a prefix indicating to, which research question they refer. For example, Code 02-52 indicates that it is related to R2. Because students were potentially given a hint in the Molarity Problems for R3, the suffix -a was added to codes that applied to a student utterance following a hint.

Axial coding did not begin until all open coding for all research questions was completed. Throughout the open coding process for all three research questions, some codes were found to be similar and were therefore condensed into a single code. An example of collapsed codes occurred with Code 02-52 (Student indicates that the size of the molecules change) and Code 02-34 (Student indicates that the calcium chloride molecules are the same size but they have a different molarity). Both of these codes are similar in nature and both address the same type of discussion: size of molecules. They were collapsed to form the new code 02-34 (Student discusses size of molecules). A finalized list of open codes was created for each research question and can be found at the end of this chapter with an example student utterance for each open code.

The open codes were then analyzed for patterns and placed into tree diagrams for easier analysis and display. R1 and R2 yielded similar analysis trees such as the one shown in Figure 19. The analysis trees for R3 were different in structure than those for R1 and R2 in that they step through solving strategies for the molarity problems before and after a hint. If a problem was not attempted, an X was placed through the bubble. The X served as a way to indicate whether a student wasn't offered a problem or a hint versus the student not attempting the problem. An example of an R3 analysis tree is shown in Figure 20. The axial coders then analyzed these diagrams for categories that could be used to group the students. Then, the axial coders analyzed the transcripts to ground the categories in particular student utterances. The axial codes have been given an alphanumeric code for the reader but were not referred to in this way by the axial coders. Rather, in the analysis trees, these phrases or chunks of these phrases appeared in bubbles within the tree.

Figure 20: Example of branch of organizational tree for R3 for Student 11-1



A set of axial codes was developed for each research question and is presented at the end of the chapter following the open codes from, which they originated. Briefly, the open codes for R1 can be found in Table 2 and the axial codes can be found in Table 3. The open codes for R2 can be found in Table 4 and the related axial codes can be found in Table 5. The open codes related to the PR diagnostic can be found in Table 6. The most common solving strategies were then condensed to form the axial codes found in Table 7. Finally, open codes related to R3 are presented in Table 8 and the related axial codes are presented in Table 9. The axial codes were then used to develop theoretical statements that will be presented with evidence from the data in the analysis chapters.

It is important to note that many open codes were created that are not related to the research questions of this dissertation. These open codes may be analyzed in the future to answer other questions but because they do not address the question of students' solving strategies or molarity, they will not be addressed in this dissertation. An example of this type of code would be 01-56 (Student indicates that the zoomed in picture of the particles in solution would all look similar because they are liquids.) While this is an interesting student utterance, it does not give insight into the student's understanding of molarity. It does not address the research questions of this research study. Codes that only had one student with utterances were also set aside because they were extraneous cases such as 01-91 (Student thinks molarity means math) that was only stated by Student 2-1.

A mapping of specific open codes as they relate to axial codes for each of the three research questions can be found in Appendix E. Scoring Tables can be found in Appendix G.

Table 2: Open Codes for R1		
Code	**Statement**	**Student Example**
1-24	Student indicates that the same amount of paint would be needed to paint each block.	Student 19-2 [01:18:40.28] Well, I needed it, I knew that they all needed to have the same like end number because they were all like the same. They weren't all different so they needed all have the same like number at the end. Like so it all equals four.
1-25	Student indicates that there would be different amounts of paint needed to paint each block.	Student 20-2: [01:12:31.08] Cause this one, I mean obviously requires more paint cause it's more paint than this one.
1-26	Student indicates that that the pink block would be 50/50 red/white.	Student 22-3: [01:31:20.27] It looks like the same amount, equal amount of red and white.
1-27	Student indicates that there are different amounts of white paint in each block.	Student 13-4 [00:58:43.03] Ok this one is just pure red. This is pure white. And then this one is um a tint of red so it'd have less white in this one. And this one is um guess for this one probably would say probably an even amount this one would be like an even amount cause it kind of look like. And then this one a shade of white. This one is really really tinted so this one is real, you used more you used more white than red in this one. And this one you used more more red than white and for this one is equal amount of red and white
1-28	Student indicates that there are different amounts of red paint in each block.	Student 23-4 [01:00:49.23] Ok alright I was just showing that you're still going to use like half and half like the same paint. Like no matter how much you use but I'm also showing that you need more amount of paint to actually paint the blocks.
1-29	Student mentions a life experience	Student 4-2 [01:10:32.01] I feel like I'm in and AT&T commercial
1-31	Student uses the same total amount of cubes to represent each block.	Student 17-3 [01:02:58.18] okay so theres seven cubes of red paint and then I just like minused some red and added white as I got further down.
1-32	Student uses a different total amount of cubes to represent each block.	Student 23-4 [01:00:49.23] Ok alright I was just showing that you're still going to use like half and half like the same paint. Like no matter how much you use but I'm also showing that you need more amount of paint to actually paint the blocks.
1-33	Student discusses "half cubes".	Student 24-3 [00:53:36.27] Yeah one and two and half but not really cause like this one is half. And then this is just being a whole
1-39	Student orders the blocks based upon color or shade.	Student 8-4 [00:44:53.10] Put them in order of shade .
1-40	Student orders the blocks based upon height.	Student 3-4 [01:07:18.06] I just put it from smallest to biggest.

Table 2: Open Codes for R1		
Code	**Statement**	**Student Example**
1-41	Student changes the amount of cubes used to represent the paint from an odd number to an even number.	Student 2-1 [01:05:48.19] um because I felt okay maybe it will be easier an example would be this one. I felt that this one is kind of in the middle between these two like the shading.
1-42	Student mentions a pattern in the length of the blocks.	Student 20-2 [01:12:55.03] Well, this one is a half. This one is three of these. This one is four of these. This one is five of these.
1-43	Student represents the length of blocks using cubes.	Student 6-3 [01:50:21.26] And this is more about like, like how much of this the smallest one is.
1-44	Student represents the color, tone or shade of the blocks using cubes.	Student 1-3 [01:36:10.03] Oh I tried to show the amount of red you would need, the amount of red and white you would need in relation to amounts you would need to paint each block of wood. And since they're different sizes you would need different amounts of both red and white to paint each one.
1-53	Student describes purity	Student 6-3 [01:20:36.11] Ok let's see. Um this would be this I think. This is red. And this would be just straight white.

Table 3: Axial Codes for R1	
Code	**Statement**
1-A	Student had an intensive view of the concentration of color
1-B	Student had an extensive view of the concentration of color
1-C	Student used “half-cubes”
1-D	Student required a prompt
1-E	Student represented the color of the block using cubes
1-F	Student represented the length of the block using cubes

Table 4: Open codes relevant to R2		
Code	**Statement**	**Example Student Utterance**
02-02	Student indicates that a bigger number in front of M should have more volume in the jar (example 0.10 should have more volume than 0.05).	[00:48:28.02] Student 4-2: so if it was volume it would be like, that like half and this one be like full.
02-03	Student indicates that a different number in front of M means a different amount of substance.	[01:18:52.28] Student 1-3: Um well earlier I had used molarity equals grams per milliliter of in this case CaCl. So I'm guessing um there are more grams of CaCl in the point one five mole solution than any of the other ones.
02-04	Student indicates that to make the solution, substances are added to each other.	[00:41:36.04] Student 24-3: A solution is when you I say when you mix two see I lost the word I can't like.
02-05	Student relates concentration or molarity to dilution.	[00:22:25.20] Student 7-4: Um I would go with the bigger number because the more you dilute something one gets diluted but you have more like quantity. Like if I had a gallon of chlorine, the more water I put into it the more I have but the less chlorine there is in the solution.
02-06	Student discusses strength.	[01:30:59.14] Student 16-2: Well, I, it's possible, anything's possible in life. Um, you get a different amount of liquid. I'm thinking just the point fifteen doesn't represent how much. I'm actually, I'm kind of changing it. It doesn't represent the quantity of it. It represents the strength of the chemical.
02-07	Student indicates that concentration means the amount of something in something.	[01:20:53.09] Student 7-4: (SR: What's concentrated mean?) I shouldn't have said that. I knew you were gonna ask me. Um how much of something that you have in something
02-08	Student indicates that M represents volume.	[00:50:10.02] Student 8-4: (SR: OK and how much is in each bottle?) Point fifteen. I want to say I know it's not milliliters cause ml so I know it's milligrams it's one of them. I think milligrams is mg. Um it's one of them I know that much.
02-09	Student views the number in front of M as a decimal.	[01:13:23.25] Student 12-1: Might, um, that's the one, since like they're all decimals then that's only like a small amount of, not a small but only like part of like, ok since um ok so like different compounds and different um elements and stuff have like different mass weights, is that how you call it? Mass weights? Weights. And um and like they use the moles to find out like um how much would each element or compound weigh.
02-10	Student views the number in front of M as a percentage.	[00:37:09.14] Student 8-4: I know like I think it's like point fifteen percent. I know not it's not fifteen percent, I know it's not fifteen percent I think it's point fifteen percent inside
02-11	Student view the number in from of M as parts or a fraction.	[01:05:56.11] Student 22-3: (SR: What's that number in front of it mean?) That's the amount, well it's not a full mole. Zero point something so it's part of a mole. mole of calcium chloride.

Table 4: Open codes relevant to R2		
Code	**Statement**	**Example Student Utterance**
02-15	Student indicates that the labels on the bottle are the concentration.	[01:12:17.09] Student 10-1: (SR: OK what do these labels mean to you) The same concentrations.
02-16	Student indicates that M is an element.	[00:55:44.15] Student 17-4: (SR: the smallest thing. How do you think that (atoms) relates to the number of M or does it) I think it would cuz like tell you like how many atoms of M would be needed in order to make up that the solution in the bottle.
02-19	Student discusses volume.	[00:48:28.02] Student 4-2: so if it was volume it would be like, that like half an this one be like full.
02-21	Student indicates that M means meters.	[00:53:31.12] Student 19-2: Uh huh. Um, Oh M calcium or calcium chlorine two. Um, so, I think that's zero meters. Um, point one meters calcium chlorine two. Point one five calcium chlorine two. Point zero five meters calcium chlorine two.
02-23	Student indicates that M means molarity.	[01:20:07.09] Student 14-1: (SR: Ok um and then the M means?) Molarity, which is moles per liter
02-24	Student indicates that M means moles.	[00:58:53.11] Student 15-1: Oh ok. Zero moles of CaCl2. Zero point zero five moles of CaCl2. Zero point fifteen moles of CaCl2. And zero point zero five moles of CaCl2.
02-26	Student indicates that molarity and concentration are the same thing.	[00:55:20.27] Student 10-1: Mole is like the amount of, not mass. It's like, well molarity is concentration, which is the strength of the substance. Mole is the amount of the substance.
02-31	Student indicates that Molarity is related to moles.	[00:08:03.19] Student 11-1: (SR: Ok what's molarity?) Uh, I guess something to do with moles. Mmm. Something, I'm not a hundred percent. Actually just know it's something to do with moles is all.
02-33	Student indicates that molarity is the amount of moles in 1 liter of water or other solvent.	[01:19:55.04] Student 14-1: (SR: Oh ok. Um so what does the number mean? What does zero point five stand for? What's it mean?) For every liter of water that's in this, there's zero point zero five moles of calcium chloride.
02-34	Student discusses size of the molecules.	[01:28:08.24] Student 22-3: (SR: Ok and are they all the same size? Because last time you had them as different sizes.) Um I think it's , it can be either both actually because the amount of liquid vary in each jar. One hundred fifty, two hundred and one hundred I think the size and the number of atoms in each jar is different.
02-35	Student indicates that Moles or molarity equals g/mL.	[00:22:58.22] Student 1-3: Uh fourteen was the amount of grams given in the question. Fourteen grams of NAOH. And three hundred fifty is the amount, the volume of water that we were given. So I used the moles is equal to grams over milliliters equation. And just put them over each other. Divided the two and came up with point two five for the molarity.
02-37	Student indicates that the number before M is concentration.	[01:02:39.18] Student 3-4: (SR: Ok and they're all point one five. What does that say?) They all like equally the same like concentration-wise except one of them has more than the other.

Table 4: Open codes relevant to R2		
Code	**Statement**	**Example Student Utterance**
02-38	Student indicates that the number before M is how much or quantity of a substance other than water.	[00:50:04.07] Student 20-2: (SR: Ok. What's that number in front of the M mean then?) that's how many moles.
02-39	Student indicates that the number before M is molarity.	[01:29:22.01] Student 6-3: (SR: Ok. Um so what do those numbers mean- the zero point one five in each of the bottles?) Uh the molarity of the solution
02-40	Student indicates that the label with M on it tells nothing about how much calcium chloride is in solution.	[01:42.07.11] Student 7-4: (SR: Ok so how can they all be point one five with different volumes?) Maybe it doesn't mean anything. Cause sometimes not every number that you need to solve a problem. (SR: Ok why would we put it on the label then?) To throw off
02-44	Student views molarity or concentration as a ratio.	[01:28:46.15] Student 1-3: I think that you would use, like how you created each one. Um I think with each, with this one, for example, the middle one, you would use um lesser amounts of each. I think you said you used water and calcium chloride solid to make this. So I would think you would need to use lesser amounts of each to create this one and then increasing amounts to replicate the same moles in each one.
02-45	Student view molarity or concentration as a proportion.	[01:36:01.16] Student 14-1: Ok and then like this one would have proportionately more water and more calcium chloride.
02-46	Student indicates that there is a constant amount of calcium chloride in each jar.	[00:56:17.19] Student 23-4: They all have the same amount it's just this one is more concentrated cause it's there's less liquid to dissolve it in.
02-48	Student indicates that there are varying amounts of calcium chloride in each jar.	[00:56:10.10] Student 19-2: (SR: Ok, um, so what's different about all of these?) Um the different amount of moles in each one that the label said.
02-49	Student indicates that the solutions differ in their packing.	[01:09:36.03] Student 22-3: Um I think the atoms between the calcium and chloride in the point five would be more loosely packed than the ones in the point one five.
02-54	Student states that solution contains water in it.	[01:14:51.04] Student 12-1: Like um what I'm thinking is that um yeah like for example they might make the compound but only take like a small amount of compound and like add to the solution.
02-55	Student indicates that molarities that are decimals have less calcium chloride than a zero molar solution.	[00:51:06.19] Student 3-4: Like I don't know. Like little atoms I guess you could say. Um like this would be zero, there would be like some around the whole liquid like and then the point oh five they would be like I guess a little less than the one before. And then so on and so forth getting like just smaller and smaller.
02-57	Student indicates that they do not know what molarity means.	[00:05:22.01] Student 4-2: okay I don't even know what molarity means.

Table 4: Open codes relevant to R2		
Code	**Statement**	**Example Student Utterance**
02-58	Student indicates that M is molar.	[01:32:33.25] Student 14-1: yeah this is, they're all the same with point one five molar CaCl2. This one has about, let's see, that's sixty seventy eighty two hundred, the little lines here means that approximately
02-73	Student indicates that M is a way to measure liquid.	[01:06:05.19] Student 17-3: if they're all the same label. You would think that they would all have the same amount.
02-75	Student indicates that there is a constant amount of "something else" in each jar.	[01:03:02.09] Student 15-1: A solid. Ok then there's the same amount of water.
02-76	Student indicates that there are varying amounts of "something else" in each jar.	[01:0053:.28] Student 10-1: (SR: So the circles are molecules of calcium chloride?) I guess so yeah basically. And the X is where you added to it to make it more quantity.
02-78	Student indicates that M represents mass.	[01:17:00.02] Student 16:2: Yeah, something with percentage or, not percent, just how much of that quantity, like the mass of it, just how much of it it is.
02-80	Student indicates that they do not know what M means.	[00:12:36.21] Student 18-4: I haven't saw M before.
02-83	Student indicates that M means molecules.	[01:00:10.17] Student 4-2: um point fifteen molecules of calcium chloride, it's the same for all of them.
02-84	Student indicates that the number before M is basicity or acidity.	[01:04:44.12] Student 18-4: Um oh wait no. Um dividing was different. Sorry. Um they have the same acidic but the volumes different.
02-85	Student uses the terms "pure" or "purity" when discussing concentration or molarity.	[00:40:16.15] Student 8-4: Yeah so it would be like cause it's point zero five be closer to becoming pure, I don't want to say pure water because I don't know what that is. So it'd be closer to becoming pure water so this one is a little bit more diluted.

Table 5: Axial codes for R2	
Code	**Description**
2-A	Discusses water but doesn't factor into number.
2-B	M=moles
2-C	M is related to the amount of $CaCl_2$
2-D	Student discusses strength when referring to molarity.
2-E	Student discusses size of molecules and/or packing of molecules.
2-F	M is related to something other than calcium chloride-includes M as an element or related to volume.
2-H	Student recognizes a relationship between water and calcium chloride.
2-I	Student is unsure.
2-J	Student has backwards definition of decimals- thinks that decimals are less than 0.
2-K	Student uses concentration language.

Table 6: Open Codes for the PR Diagnostic		
Code	Statement	Student Example
1-1	Student solves for one item and then multiplies to find answer.	Student 15-1 [00:35:48.23] Um I did similar, something similar to what I did here. I did point five pints of cream divided by eight people, which gives me for one person in the recipe and then I times that by six.
1-2	Student uses "Apples/ apples to Oranges/ oranges" method to find answer.	Student 12-1 [00:56:36.05] So I like on the top is for one person and the bottom was for another.
1-3	Student uses "Apples/ Oranges to Apples/ Oranges" method to find answer.	Student 11-1 [00:36:13.05] So set it up as a ratio again. You get fifteen for the base of the longer, larger rectangle.
1-4	Student uses "Apples/ Oranges to Oranges/ Apples" or its reverse to find answer.	Student 15-1 [00:48:29.29] Um I uh cause I realized I found, I set up like the equation wrong. I, this is telling me how much each person needs

Table 6: Open Codes for the PR Diagnostic		
Code	Statement	Student Example
1-5	Student uses multiples of number given and solves for one to find the rest.	Student 8-4 [00:18:44.11] ok so if three apples cost ninety cent I just divided three into ninety cent so that would give me thirty cent.
1-6	Student uses multiples of number given and estimates to find them rest.	Student 8-4 [00:12:06.29] Ok so ok so um so twenty-eight that's two, two minutes of so, yeah thirty. Nah I didn't need it. Yeah I'll stick with my answer. It's be between thirty-two, probably thirty-four.
1-7	Student uses multiples of number given and halves number given to find the rest.	Student 7-4 [00:50:43.02] So if it's if it takes point one two five to make two people and point two five to make it for four people then when I add them I can find out how much it takes to make it for six people.
1-8	Student adds the difference between two given numbers to find the unknown.	Student 22-3 [00:59:14.18] Well it says for Mr. Short's height is four matchsticks and the amount of paperclips you have for Mr. Short is six and for Mr. Tall it's six matchsticks, which is two more. Which would be two more paperclips.
1-9	Student states that they do not know how to solve the problem.	Student 2-1 [00:16:50.11] all right so now that it's for six people it would be um everything would be cut down to three fourths [writes three underlined over 4] and cream I don't really know what three fourths of a half pint of cream is
1-10	Student recognizes a multiplier in the problem and uses that to solve the problem.	Student 1-3 [00:56:01.18] And then since they were using, or they wanted to use the exact same amount of paint Jenny would need to use fourteen gallons or fourteen cans of red paint per every seven.
1-11	Student uses the term proportion.	Student 2-1 [00:21:03.00] okay so I would I guess I would think of think of everything in a proportion I guess
1-12	Student creates a drawing to decide how many boys and girls are in Mrs. Green's class.	Student 15-1 [00:27:53.10] [Student writes 5 5 5 5 5 then draws two lines coming off of each 5 and writes 2 2 2 2 2 2 2 2 2 2. Then writes 3x5=15girls. Writes 2x5=10 boys] Three times five equals fifteen girls. Two times five equals ten boys.
1-13	Student multiplies 3 girls by 5 groups to get 15 girls and subtracts from 25 to get 10 boys in Mrs. Green's class.	Student 6-3 [00:23:48.15] Even without doing that, you know that leaves two guys to be in each group. Two times five is ten
1-15	Student reports answer as a decimal.	Student 1-3 [00:53:34.08] Cool. Um. Well using that original answer I multiplied point six six by twenty-four because we were told that every book costs the same in the bookstore sale. And since Rosie bought twenty-four books I multiplied those two numbers and came up with fifteen dollars and eighty-four cents but because I was just told that it came out to an even number I have to figure out what I did wrong. Um I'm gonna go onto the second question. [writes . 664]

Table 6: Open Codes for the PR Diagnostic		
Code	Statement	Student Example
1-16	Student reports answer as a fraction.	Student 12-1 [00:47:46.14] But like, so for example if somebody was making this and um like they probably wouldn't know what seventy-five over two hundred would be so just to make it easier like three-eighths would be better.
1-17	Student subtracts number from total and uses that number to divide to find how many.	Student 9-2 [00:28:28.13] Because there's 6 less people for the recipe.
1-18	Student uses correct reasoning but has a decimal error.	Student 10-1 [00:37:24.12] Because I forgot about the zero in front of the three. The decimal is point zero three seven five not point three seven five.
1-19	Student estimates a relationship between the number of paperclips it takes per matchstick using the drawing.	Student 4-2 [00:36:34.28] so if it's for matchsticks for Mister short and it six matchsticks or matchsticks for Mister tall it would be one two three four five six [Student talks while writing 6+ 1 ½=7 ½] paperclips plus one and half so he'd be seven and a half paperclips tall.
1-20	Student states that they know an answer is wrong by looking at it.	Student 1-3 [00:47:00.00] Point six five pints of cream but because that's higher than the original recipe that wouldn't make good soup.
1-22	Student has an unnoticed arithmetic error.	Student 10-1 [00:37:24.12] Because I forgot about the zero in front of the three. The decimal is point zero three seven five not point three seven five.
1-23	Student reduces the numbers to determine the answer.	Student 18-4 [00:32:29.12] So right now I'm still guessing like five times four is twenty so three times four have to be twelve [Student writes 12] and if I reduce that it would be the same.
1-34	Student states that they do not understand what the question is asking.	Student 4-2 [00:25:57.21] um I don't understand that like if Sue uses hmm I don't know it would guess it help if they said um.
1-35	Student misinterprets what the question is asking.	Student 17-3 [00:40:31.28] because it's eight loaves and then there's fifteen campers and have to have the same amount of bread so I'd divide it equally into that.
1-36	Student uses : notation.	Student 4-2 [00:39:11.29] um they both have the same shape but once larger so their the same proportion [Student writes 6:] but(laughs) same problem with all the other proportions.
1-37	Student states that if a relationship exists on one side (numerator larger than denominator) then the same must hold true for the other side of the proportion.	Student 4-2 [00:34:34.06] um, cause like I did the same thing for each one in like I didn't realize my mistake until the next one and I realized when one is less the other one was more in the next side it was like opposite.

Table 6: Open Codes for the PR Diagnostic		
Code	Statement	Student Example
1-45	Student writes 3 five times to solve for Mrs. Green	Student 23-4 [00:21:22.03] Oh I put I made five groups of three, added this together and it's all the girls and subtracted the amount of girls from the total class number.
1-47	Student indicates that a formula can be used.	Student 1-3 [01:05:48.15] For the second question um it's almost the same as the first except finding out how many people can be fed with sixteen loaves of bread. And using point eight from the first equation, which you could find out the same way in the second one um I figured out I guess and checked uh, which number multiplied by point eight would equal sixteen since we're given sixteen loaves of bread and that came out to be twenty.
1-48	Student divides total amount by the number needed to find the answer.	Student 18-4 [00:17:38.29] Um next one says an onion soup recipe for eight people is as follow eight onions four pints water, four chicken soup cubes, four dessert spoon butter, half pint cream. I am cooking for two for soup of two people how many dessertspoons of butter do I need? So since they are since this is a recipe for eight people I would divide by two in each one. But then um yeah I think so. So for two people it would be two onions [Student writes 2 onions] and since it's two pints of water it would be wait I did that wrong. So right now I'm thinking so two pints of water for eight people, how's that four two? So two pints for eight So I'm thinking so two pints for eight how many pints is in so or how many ounces is in a pint. Then I would remember um I forgot the ounces but since they're not asking for that one, I could just go to the dessertspoon butter. And just divide it by two. I mean
1-49	Student provides multiple answers for the same problem.	Student 8-4 [00:14:24.19]Yeah I mean it's either four ok so need it for two. One goes over something I know that much. [Student writes 2 and underlines it. Writes 12 below the line.] So it's either two over twelve, it'd probably be that'd be one sixth. [Writes 1/6] Either twelve over two and it'd be six. [Writes 12/6. Circles the 6. Then circles 1/6] I know that much. But I just can't remember, which one it is. It's either yield over proportion or proportion over yield. [Student writes yield/portion. Then writes portion/ yield] It's one of them. I know that much
1-50	Student indicates that they are using a ratio.	Student 1-3 [01:09:01.14] Ok um [Student writes 1.5] I just rewrote some of the information that was already given for the rectangle on the opposite sides because the opposite sides of a rectangle are both parallel and equal in length. Um and we're trying to figure out the length of the base. So the sides here is um you would figure out the ratio between the two the same way that you would figure out the ratio for Mr Tall and Mr Short. So you get the same answer of one point five and the base that we're given for the smaller rectangle is ten. So you multiply ten by one point five [Student writes 10.00 above 1.5 and underlines.]
1-51	Student indicates that proportions and ratios are the same.	Student 22-3 [01:03:12.19] They're actually kind of the same cause you are comparing like I did here. Like if this were two and four and this was four and eight, the ratio is two to four four to eight.
1-52	Student uses the term variable.	Student 6-3 [00:23:54.27] So I started thinking well, hm, let's add some variables to it and

Table 6: Open Codes for the PR Diagnostic		
Code	Statement	Student Example
1-54	Student writes 2 five times to solve for Mrs. Green's class.	Student 1-3 [00:39:34.23] Alright question number three on page three. Ms Green is putting, [writes a 3 and circles it] she's putting twenty-five kids into groups of five. Uh so twenty-five divided by five means she has five groups of children and in each group she has three girls. So that means in each group there would have to be [Writes five circles] uh assuming she's putting exactly three girls in each group, that would leave room for two boys in each group. [Writes 2s inside each of the five circles] Meaning, two four six eight ten, she would have ten boys and um fifteen girls in her class. [Writes 10 and 15-I can't tell if he circled it or if the circle was already on the page]
1-75	Student finds the multiplier by dividing two similar things and then multiples to find the answer.	Student 2-1 [00:30:31.20] because like if I were to divide this seven yellow by the three that Sue uses [writes 7 inside division box and 3 outside box] but I guess it would be 2.5 [writes 2.5 on top of division box] so Jenny uses 2 1/2 more than Sue and and if I times [writes 2.5 over x 6 and underlines it. Below line writes o. Writes 2 above the 2.5. Below the line writes 15. in front of the 0. Then writes red after 15.0] 2.5 times the six Sue uses so then Jenny would use 15 red cans
1-55	Student uses fractions to solve the problems.	Student 10-1 [00:40:20.23] Cause they want seven apples and three apples cost ninety cents. So I just multiplied ninety by two and then I figured out that a third of ninety cents is thirty cents. So I just added one eighty plus thirty, which is two ten.
1-56	Student divides in half and then divides that in half to find the answer.	Student 6-3 [00:58:21.27] As in just um dividing by half each time, you're gonna get down by two to two, so that's what I did for that.
1-57	Student realizes a mistake and changes it.	Student 6-3 [00:58:33.02] Cause I like, I thought about it because it didn't seem right that I was getting five for this. So I tried to basically look at what I had here, kind of like checking what I got as an answer.
1-61	Student indicates that they used a process of elimination.	Student 18-4 [00:08:38.21]Yeah she got fifteen boys ten girls. She got five groups three girls each group so process of elimination, three three three three three. Seems elementary but that helps. So I'm like ok if she got five girls three girls each group that's ten boys cause that's the same.
1-62	Student uses top or bottom in their explanation.	Student 11-1 [00:34:24.17] Uh the six for the matchsticks is down here, below the four. And then the paperclip one is up here. That'd be Mr. Short's paperclips.
1-63	Student uses denominator or numerator in their explanation.	Student 19-2 [01:05:00.01] Um cause you cause in order to show like what is perspective to what unit common denominator. Like a common value.
1-64	Student uses over or under in their explanation.	Student 17-3[00:50:41.16] I set up another one of the proportions so four centimeters over ten and then six over X for the larger one.

Table 6: Open Codes for the PR Diagnostic		
Code	Statement	Student Example
1-65	Student gets right answer but doesn't recognize it.	Student 21-2 [00:44:32.08] Um four out of six is reduced to two over three. But I need to know two-thirds of ten. Hm. Ok I'm not sure.
1-66	Student uses "flipping" in their explanation.	Student 1-3 [01:12:16.00] Yeah I came up with a um number larger than the one we already were given so I flipped the two numbers. [Student writes .00 next to the 2. On top of the box student writes 0.4] So I'm dividing two by five. So for every centimeter of eel there are fed point four sprats. So eel B is fed four sprats.
1-67	Student divides to find the answer.	Student 1-3 [00:40:53.19] Alright so we're given a recipe that will make enough soup for eight people but whoever's making it only wants it for a couple of people, two people. So you would just divide the entire recipe by four leaving you with two onions, [writes 2] half a pint of water, [writes .5] one cube of bullion, [writes 1] and three dessert spoons [writes 3 and draws an arrow to the label], which is the answer to the question. Three dessert spoons. [writes 1 next to the problem and circles it]
1-68	Student multiplies to find the answer.	Student 1-3 [00:49:16.18] Oh oh the recipe originally makes soup for eight people but she's cooking for six people so I tried to find a way to combine those two numbers, which would be division in this case and then multiply that number by the rest of the recipe
1-69	Student uses the phrase "goes into" in their explanation.	Student 17-3 [00:34:20.04] because for every six books it costs four dollars and Rosie bought twenty-four and six goes into twenty-four evenly so it would have been four times four, which equals sixteen dollars
1-70	Student refers something to something.	Student 11-1 [00:20:40.12] Well basically what they give you is twelve minutes to fourteen dictionaries and then I just did that as fraction and just did the same thing as I did on top. So uh and I kind of just learned it that way.
1-71	Student uses cross multiply in their explanation.	Student 20-2 [00:33:49.02] So for six books, she paid four dollars. For twenty, for twenty-four books, how many dollars would Rosie pay? So I just did the same thing. Cross multiply the whole scale or example and then it lead me to sixteen.
1-72	Student indicates that they knew something inherently or "off the top of their head"	Student 8-4 [00:31:30.01] Yeah uh cause I was like this was like off the top of my head. I knew that much so I didn't really have to set that up like this one. And I know for this one like it helps to know this goes with this, this goes with this.
1-73	Student solves for an answer that is not asked in the question.	Student 14-1 [01:15:49.16] Five equals five over two equals fifteen times fifteen divided by X.
1-74	Student finds a number by dividing by 2 similar things.	Student 17-3 [00:34:12.00] um divided twenty-four by six and just timesed that answer by four.

Table 6: Open Codes for the PR Diagnostic		
Code	Statement	Student Example
1-76	Student finds multiplier using algebra.	Student 6-3 [00:27:38.24] Ok, um let me see. Thirty minutes and X dictionaries. [Student writes 30 min. Student then writes x dictionaries]Twelve minutes. Fourteen. [Student writes 12 min=14. Student writes 1).] Ok let's see. Hm. How many dictionaries can it print in thirty minutes? Let's see if this works. [Student writes 12x=14. Below it student writes 30x=] No that doesn't work. [Student scratches out 12x and 30x] Um. Ok. [Student writes x+12=14. Below it student writes x+2*5x= . Then Student scratches out the problem. Student writes x+ but then changes the + into =. Student writes 14 after the equals sign. Below it student writes 2*5x=. Student goes back and changes the 14 into a 12. Writes x=12. Below it writes 2*5x=30. Student rewrites x=12 and below it writes 2*5x=30. Student scratches it out] Nope that's not right either. Yeah I'm trying to figure out how to put together the variables but that's not working out too well.
1-77	Student uses the term "Convert".	Student 12-1 [00:42:19.21] Um since it's a, since um in the recipe yeah the question shows um a fraction, I'll just convert it into a fraction.
1-78	Student divides to find one	Student 1-3 [00:37:09.02] Well the first, the first um rate that I tried to figure out, which was twelve over fourteen would have ended up being twelve minutes over fourteen dictionaries as [Writes min next to the original 12 and dic next original 14] opposed to fourteen dictionaries over twelve minutes, I want to figure out how many dictionaries can be printed per minute as opposed to my first one, which was how many minutes per dictionary. I don't know what that would have given me.
1-79	Student uses a percent to solve problem.	Student 10-1 [00:48:24.05] Twenty campers. [Student writes 2o campers] You can see the height of Mr. Short measured with paperclips. One, two, three, four, five, six. Six. [Student writes something p] Mr. Short has a friend, Mr. Tall. When we measure the heights with matchsticks, four short equals four [Student writes Short=4 and below it writes Tall=5 and then writes match after each number] tall equals six match match. I thought as much. How many paperclips? Ok, so that's sixty-six percent again. I don't want to use that formula again. Same exact formula as last time. So four to six equals, no exactly four to six. [Student writes 4/6=6/x] I only thought that, yeah, six yeah it is. Four X equals thirty-six. [Student writes 4x=36] X equals nine. [Student writes x=9] Nine paperclips. [Writes 9 paperclips and draws a box around it].
1-80	Student says that they used a "Guess and Check" method.	Student 1-3 [01:06:22.23] uh I I took a random number that I thought would be close, that I could multiply by eight to get close to sixteen. So I started with seventeen, multiply that by point eight but that didn't, it wasn't close to sixteen. So then I started with eighteen, nineteen, and I got to twenty, which ended up being sixteen on the head.

Table 7: Axial codes for PR Diagnostic	
Code	**Statement**
PR-A	Student uses an apples/apples=oranges/oranges strategy
PR-B	Student uses an apples/oranges=apples/oranges strategy
PR-C	Student uses an apples/oranges=oranges/apples strategy
PR-D	Student solved for one and multiplied
PR-E	Student used a multiplier
PR-F	Student used an additive strategy
PR-G	Student used the magic halving strategy
PR-H	Student made an arithmetic error

Table 8: Final list of open codes for R3.		
Code	**Statement**	**Student Example**
03-01	Student divides the number of grams given by the molecular weight to find the number of moles	Student 1-3 [00:29:40.11] Grams is it. Oh, oh no, no worries. [Writes g next to 48 and g next to 4] So if the atomic mass of helium is four grams and we're trying to figure out the amount of moles inside twelve grams of helium um I would divide [Writes 12 and a 4 under it with a line in between. Underlines the 4 and below the four writes 3M] the twelve grams of helium by the four grams of atomic mass and come up with an answer of three moles.
03-02	Student states that the “one with the most grams has the most moles” and vice versa.	Student 3-4 [00:01:43.27] Mass number is how like the weight of the, of the element or whatever. Um I'm gonna have to go with the bigger number. It’s obviously the, I'm gonna go with C. [Student marks a dot on the paper]
03-03	Student rewrites the chemical equation on the paper.	Student 4-2 [00:01:32.08] Ok. You have excess aluminum sulfate. Hm [Student writes Al2. Writes Al2(SO4) 3+2Pb(NO3)2. Student writes Al2 below the equation just written. Then writes S3. Writes O7. Writes +2PbN2O5. Writes=Al2S3012 2PbN2. Writes Al2S3012+2pbN2]

Table 8: Final list of open codes for R3.		
Code	**Statement**	**Student Example**
03-04	Student finds the molecular weight by adding together all of the components weights and/or multiplying.	Student 10-1 [00:06:25.28] So you find out the excess but I don't think you need to use that here. I'm not sure though. [Student writes x, draws a line and writes mol on top of the line] Ok. Alright. One mole, alright, that's lead. Two oh seven. Two oh seven plus (muttering) fourteen oh sixteen three. [Student writes 207, writes 16 and directly below writes 3 and underlines it] Eight one two one four. [Student writes 8 below the line, writes 2 above 16 and then writes 4 in front of the 8] Alright so forty-eight plus, ok, two oh seven plus forty-eight. [Student writes 48, directly below writes 207, and directly below that writes 48 and underlines it.] Five, two two fifty-five [Student writes 255 below the line and then returns to a previously set up problem and writes 255g] gram, alright. I'm not allowed to use a calculator, right?
03-05	Student uses division to find the answer.	Student 7-4 [00:28:41.10]Ok five fifty divided by six. [writes 550 inside division box and 6 outside it. Writes 8 on top of division box. Writes -48 below 55 and underlines it.] Um um (mumbling) seventy [writes 70] so this does not come out even. I prefer even numbers to odd numbers for some reason. Even seems so much easier to work with. Yeah this isn't going to work [scratches out the division problem
03-06	Student uses multiplication to find the answer.	Student 1-3 [00:35:02.19] Oh. Ok um so instead of having point two two as molarity I would put six [puts a line through .22 and writes a 6] and six instead of point two two. And then uh multiply the two to figure out [Writes 550 mL] the amount of grams of NAOH needed. [Writes a 6 below the 550 mL and underlines it.] And then we'll see what happens with that, yeah
03-07	Student uses addition to find the answer.	Student 1-3 [00:12:40.27] Um, let's see. .[writes Xg plus sign plus sign 32g equals sign] I would get rid of um [Xg plus sign 350g. Below that writes another equals sign.] Well if I were to change it into a math formula, any number that I would use for X would make the equation equal to one so the answer that I would come up with, or any answer that you would, any amount of aluminum sulfate that you would use would react with the three hundred fifty grams of the lead nitrate. But that's definitely wrong.
03-08	Student uses subtraction to find the answer.	Student 22-3 [00:23:48.14] The first part, how would you make five hundred fifty milliliters of two hundred, point two two zero moles from six point zero zero moles. And I subtracted the six point zero zero minus point two zero and got five point seven eight zero. That's how many moles would be left in this solution, in this stock solution.
03-09	Student uses a coefficient to solve the problem.	Student 23-4 [00:05:17.06] I feel like it's useful cause you need to know how many moles of oxygen to use to combine it but I don't know like. Yeah I honestly don't know how, I can't even do the next one. I seriously have no idea what's going on.
03-10	Student uses "X" in their problem.	Student 1-3 [00:12:40.27] Um, let's see. .[writes Xg plus sign plus sign 32g equals sign] I would get rid of um[Xg plus sign 350g. Below that writes another equals sign.] Well if I were to change it into a math formula, any number that I would use for X would make the equation equal to one so the answer that I would come up with, or any answer that you would, any amount of aluminum sulfate that you would use would react with the three hundred fifty grams of the lead nitrate. But that's definitely wrong.

Table 8: Final list of open codes for R3.		
Code	**Statement**	**Student Example**
03-12	Student has an unnoticed arithmetic error.	Student 15-1 [01:30:51.12] Um I did zero point eight five moles of CaCl2 divide by one mole of CaCl2 so the moles cancel out and then times by one twenty grams of CaCl2, which is the atomic, or molar mass.
03-13	Student states that M and moles are the same	Student 11-1 [01:08:24.01] I'm just thinking back to uh, like last year I guess and how I'm reading basically the same thing. Point seven five I guess moles of calcium chloride.
03-14	Student writes down a formula or equation and fills in the values to solve the problem.	Student 1-3 [00:18:26.29] Yeah. Um if so um if that is the equation, I'm sure the chemists out there are cringing cause that's wrong.[crosses out M equals sign g underlined with mL written below it] Using the second equation, um , or the second question, you would just um put fourteen as the amount of grams that we were given over the milliliters that were given, which was three hundred fifty. And then divide by two and that would give you the amount of moles. Or that would give you the molarity of the solution.
03-15	Student uses a calculator	Student 9-2 [00:09:46.26] Oh, ok. So just to double check. So we're at fifty-four and then oxygen's sixteen times four. That is sixty-four and then sixty-four plus thirty-two. Ninety-six. Then multiply ninety-six by three because there's three sulfate oxides. And two hundred eighty-with then you add two hundred eighty-eight plus fifty- four and you have three hundred forty two. Okay. [Student writes 342]
03-16	Student "carries" the decimal.	Student 1-3 [01:45:55.09] It takes one point one, [Student writes 1.1 mL] one point one milliliters of water for every one hundred and ten grams of calcium chloride to make a ten mole solution. [Student writes .0 and moves the decimal over two spaces with a line.] So just move the decimal one more place and you'll, point zero, or point one one milliliters to make one mole of solution.
03-17	Student does not convert mL to L.	Student 22-3 [00:18:30.15]Ok six hundred fifty of [Student writes 650 mL, draws a vertical line then an intersecting horizontal line. Writes 0.170M then scribbles it out] um multiply these two. [Student writes 0.170M then x] Zero point one seven zero. [Student writes 0.170 over 650 underlined. Writes 0 then 0 then 5 then 8 then 0. Writes 0 then 0 then 3 then 0 then 2 then 4 then 10. Underlines it and writes 0 then 0 then 5 then 0 then 1 then 1 then 1. Places decimal] That's uh I got a hundred ten point five um I think it's milliliters. [Student writes mL]
03-18	Student converts mL to L.	Student 10-1 [00:12:56.13]Oh, yeah, it's twenty-three. Three plus sixteen, seventeen, which is forty I believe, yeah. [Student writes something but the paper didn't pick it all up] Ok. So molar mass is moles, gotta divide four have to divide fourteen by forty. Ok, so, fourteen divided by forty, enter. Point three five and then you gotta divide that by, molarity's in liters so it's point three five. [Student writes .35/.035] Well it's just one. Molarity is one, one. [Student writes 1M]

Table 8: Final list of open codes for R3.		
Code	**Statement**	**Student Example**
03-19	Student converts grams into moles by dividing by molar mass.	Student 14-1 [00:02:45.04] Ok. What is the molarity made by dissolving fourteen grams of NAOH and three hundred fifty milliliters of water? How did you arrive at this answer? I've learned from high school chemistry classes molarity is the amount of moles of a solution in one liter of water or moles per liter of solution, not necessarily water. [Student writes mol/L] Um in order to figure out moles I'm getting, I have a set amount of grams of NAOH sodium hydroxide. So I could up the amount the grams of sodium, how much per gram, how many grams are in one mole of sodium and to figure that out I believe it's about two point nine eight but just gonna double check. Sodium has twenty-three grams of, [Student writes 23g then Na] there's twenty-three grams in one mole of sodium and then oxygen is sixteen grams per mole. [Student writes O 16g] Sixty grams of hydrogen in one mole, sixteen grams of oxygen in one mole of oxygen. And hydrogen there's one gram [Student writes H 1g and underlines it] of hydrogen per mole of hydrogen. And then adding those three together will give you molar mass of sodium hydroxide. So that's twenty-three plus sixteen and that is thirty-nine. Plus one is forty. [Student writes 40g] So you have, in order to figure out moles, how many moles of sodium hydroxide you have to divide fourteen that you have divided by forty, forty grams per one mole [Student writes 14/40] of sodium hydroxide.
03-20	Student does not convert grams into moles.	Student 13-4 [01:27:59.25] Ok. (mumbling) twenty-five milliliters of distilled water what is the molarity of the solution. Ok I am not sure so for this one I'm basically gonna do the same thing I did. [Student writes 15/25] Fifteen over twenty-five.
03-21	Student shows work.	Student 23-4 [01:08:48.14] So divide fifteen. I did the other one backwards also. I feel, I think I divided wrong.But alright four so it'd be six. So it'd be zero three ok I got zero point six moles of calcium chloride in the solution.
03-22	Student writes down answer with no work.	Student 11-1 [00:02:32.15] So four times three equals twelve. And um the next one I would check was mercury. No that would take two hundred so never-mind. And uh titanium is the one I wanted to see, uh, helium would be, have the highest moles so it'd be A. Do I just write A in here?
03-23	Student does arithmetic by hand.	Student 7-4 [00:28:41.10] Ok five fifty divided by six. [writes 550 inside division box and 6 outside it. Writes 8 on top of division box. Writes -48 below 55 and underlines it.] Um um (mumbling) seventy [writes 70] so this does not come out even. I prefer even numbers to odd numbers for some reason. Even seems so much easier to work with. Yeah this isn't going to work [scratches out the division problem]
03-24	Student does not attempt question.	Student 11-1 [01:16:29.12] Same thing. I just really don't know what that (M) means, the capital M.

Table 8: Final list of open codes for R3.		
Code	**Statement**	**Student Example**
03-25	Student cancels units	Student 13-4 [01:28:33.24] I guess I can just go um cause I have um fifteen grams of CaCl two and um a twenty-five milliliter twenty-five milliliter of distilled water so then I'm a do twenty-five and then I'm gonna multiply that by twelve point zero one. [Student writes x 12.01] Three and five [Student crosses out 15 and writes 3. Crosses out 25 and writes 5. Writes =36.03/5. Writes 7.206] (humming) Ok so for this one I wasn't quite sure so I did fifteen divided by twenty-five times twelve point oh one and that gives me and I did everything I got some point seven two six molar, moles. [Student writes M] I got the fifteen from the num uh the amount of grams that calcium chloride and um that is twenty it's in twenty-five milliliters off distilled water.
03-27	Student discusses converting.	Student 5-3 [01:20:57.10] Um for this second one it says you add water. I don't know what adding water does. I don't know if it doesn't do anything. I don't know if it keeps it like if it's a neutralizer of it just doesn't do anything. I'm not sure. Um the last one how many grams. I don't know how to convert it because milliliters is um like a measurement and grams is a weight.
03-28	Student uses fractions to solve the problem.	Student 9-2 [01:22:26.15] Yea, well, no. three fifths. Spacing out here, ok, three fifths. Um and three fifths is point six moles. [writes .6M and underlines it]
03-29	Student uses decimals to solve the problem.	Student 1-3 [01:45:55.09] It takes one point one, [Student writes 1.1 mL] one point one milliliters of water for every one hundred and ten grams of calcium chloride to make a ten mole solution. [Student writes .0 and moves the decimal over two spaces with a line.] So just move the decimal one more place and you'll, point zero, or point one one milliliters to make one mole of solution.
03-31	Student reports their answer in a decimal.	Student 14-1 [00:43:30.03] It is twelve point one grams. So you'd want to add twelve point one grams [Student writes 12.1 grams of CaCl2 into 650 mL of water] of CaCl2 into six hundred fifty milliliters of water.
03-32	Student uses a proportion or ratio to solve the problem.	Student 14-1 [00:31:33.23] Ok because you, because you have like, because you're trying to find the amount of moles and you have um fourteen grams. Fourteen grams over grams to one mole. [Student writes g then g] Um there's so you have fourteen grams and you can, want to convert grams to moles and then we know that there's forty grams from the previous problem from adding up the molar masses of other, the components of sodium hydroxide. You, one can figure out the, so it's like you have fourteen grams equals so many moles. You're trying to figure that out. And one mole is forty grams and then thirty is at proportion so it'd be four, fourteen grams or X amount of moles [Student writes 14g/ x mols=40g/ 1mol] equals forty grams per one mole and then if you were to solve it as a proportion. You'd do fourteen times one, which is fourteen and that, you divide that by forty grams per X amount, multiply it X times forty, which is yeah equals. So fourteen equals[Student writes 14 and underlines it. Then Writes =40x] forty X because thats a proportion. And then you divide both sides by forty. [Student writes 40 then 40

Table 8: Final list of open codes for R3.		
Code	**Statement**	**Student Example**
03-34	Student equates grams with moles.	Student 16-2 [00:14:38:.03] oh, that's calcium. Ok, that's calcium and Cl is chlorine. And point one seven zero. [Student draws an upward arrow] I'm I'm guessing that's how much, the weight of it. Something to do with something like that. So how would you make six hundred fifty point one seven zero, um. Well my only method is to. how'd you make a solution. How would you make. I'm just confused about what it's trying to ask you. It's like you want to get six hundred fifty ml, so milliliters some type of measurement of that out of a point one seven zero M. So the M represents mass of CaCl2 solution so you'd want to see how much of that is equal to six fifty, so, How would you make. I'm just guessing to multiply, so.
03-35	Student answers in words on paper.	Student 20-2 [01:24:02.20]Yeah. Google the equation.
03-36	Student creates a drawing for their answer.	Student 7-4 [02:04:55.21] I'm thinking of like um if you had this is the bottle thing [Student draws a bottle] again and you put a tablet in here then you poured a hundred milliliters of water in it. [Student draws a bottle. Draws lines throughout the water] And then somebody came and poured another hundred hundred milliliters over this thing is weakened by half.
03-37	Student uses estimation to solve the problem.	Student 12-1 [00:04:16.17] Twelve divided by four. [Student writes 12 inside division box with 4 outside and writes 3 mol on top of the box] Three. So helium has three moles. Fifty-two divided by fifty-nine that would be less than one mole. [Student writes 52 inside division box and writes 59 outside it. Writes less than 1] Two hundred grams that's one mole. [Student writes 1 mol] And titanium. [Student writes 100 inside division box and 48 outside it. Writes 2 on top of the box. Writes 96 below 100 and underlines it. Writes 4. Writes .0 next to 100. Writes 0 next to 4 and writes .0 on top of the box. Writes 2 mol] Titanium is about two so I would say that helium has the most.
03-38	Student writes down units with numeric answer.	Student 12-1 [00:02:37.06]Like yeah I will probably still have um point six moles. [Writes =0.6M]
03-39	Student draws "units canceling grid"	Student 15-1 [00:17:41.13]Yeah. um. um. Five times two, seventy, four. [Student writes 0.170 Mol CaCl2. Draws a horizontal line and then a vertical line and writes 1 mol CaCl2. Then writes g CaCl2 above the line. Writes Ca- then 35x2=70. Writes +40 below it and underlines it. Writes 120. Writes g CaCl2. Writes 20.4 and an = in front of it.] Ok um I'm guessing you would add twenty point four grams of CaCl2 to six fifty ml of water. I'm not sure
03-41	Student writes ML for mL.	Student 16-2 [01:46:07.16] Yeah, just so, ten moles of CaCl2. [Student writes 10 moles CaCl2] So how much water is needed? [Student writes how much water is needed to create 1M solution w/] How much water is needed to create one M solution with the ten moles of CaCl2. Right? How much water is needed to create one M solution and you'll have ten moles of CaCl2. Well, how much water is needed? I'm guessing the one M and ten moles, is that how you say it, moles? You just say moles? Ah, how much is needed? Well, ten and one. It's one, I'm just saying ratio. I don't know why but, so I'm just saying one to ten ratio. One going to ten so how much water is needed to create one M solution of ten moles? You need. I'm, I don't know. I don't know.

Table 8: Final list of open codes for R3.		
Code	**Statement**	**Student Example**
03-42	Student mentions an equation or formula	Student 1-3 [00:11:11.12]Um and then the finite number we were given, the three hundred fifty grams of lead nitrate, just to kind of get rid of that whole sentence and make it easier to comprehend, I just wrote that above the rest of the equation.
03-43	Student uses the term variable	Student 1-3 [00:10:20.13] The first thing I did was begin to kind of get rid of some of the extra words that I don't think were needed. So I translated the excess of aluminum sulfate into a variable. Chose the letter X and we were told that we were given three hundred and fifty grams of lead nitrate. So I just wrote that above the equation, that way I didn't have to keep referencing the question.
03-44	Student states that they are confused or do not know what to do.	Student 10-1 [00:26:27.07] Yeah I just don't remember.
03-45	Student uses “over or “under” in description.	Student 1-3 [00:33:25.12] Ok um I used the same imaginary equation m equals grams over milliliters to figure out the first question on page three. [Writes 121 next g] Um since we were given five hundred fifty milliliters of NAOH and we're trying to, or we're trying to end up with a point two two mole solution um I did point two two is equal to five hundred fifty milliliters and solved for the amount of grams that would be needed to, I solved for the amount of grams that would be needed to equal to point two two [Student circles g, then writes g next to 121 and circles that as well. Then writes NAOH next to it.], which came out to be one hundred twenty-one grams of NAOH. And I don't know where the six um the six mole stock solution come into play.
03-46	Student attends to the subscript	Student 7-4 [00:13:30.29]Cause there was um weight volume volume weight and then weight versus volume and then that uh twenty-five grams per milliliters. [writes 25g/mL] Just a hunch because I know there's such a thing as volume over volume, weight over weight
03-47	Student mistakes molecular coefficient for stoichiometric coefficient or subscript.	Student 12-1 [00:12:55.26] So one mole is twenty-seven and there are two of them. So that would be fifty-four grams of aluminum. [Student writes x2=54g] And oxygen there are three of them. [Student writes O=16x3=48g] Equals fifteen that's three grams. Nitrogen there's one so it’s fourteen. [Student writes N= 14 g'24g] Ok then nitrate is there three times so forty-eight plus fourteen. [Student writes 48+14=62g. Writes x3 and underlines it. Writes 186.] Times three. [Student writes 54 below 186 and underlines it. Writes 0, 4, 2. Scratches it out and then rewrites 240 g. Writes aluminum nitrate and draws ' to 240 g] should be um oh I guess two hundred forty grams of aluminum nitrate can be made here
03-50	student changes to right answer after hint	[01:23:03.14]Ok, one mole is (muttering)... Ok this one I think I might actually have an idea. How much stock solution do you need? I think I might.... actually. I am really confused so ... alright it might be a little proportion problem I think so I am going to do... I would have to use, I would do one mole over sixteen. [Student writes 1/16] Then point forty two moles over x. [Student writes .42/x] Then x is the amount of stock solution I need so I do sixteen times forty two. Six point seven two milliliters. [Student writes 6.72 mL]
03-51	Student uses “cross multiply” in discussion	Student 21-2 [01:25:27.06] You can use cross multiplication.

Table 8: Final list of open codes for R3.		
Code	**Statement**	**Student Example**
03-52	Student realizes the number is wrong based upon its size.	Student 7-4 [00:19:34.24] I'm not sure cause I just got a really huge number and I have a feeling the number's supposed to be lower not higher.
03-53	Student uses “top” or “bottom” in description	Student 15-1 [00:09:00.18] If it’s on the bottom, you divide. If it’s on the top you multiply.
03-54	Student uses some form of Avogadro’s number or references it	Student 13-4 [00:03:20.18] ok I did I think moles mass is twelve point zero one I think times um a number of kinds of gram over four. That's the atomic mass.
03-55	Student recognizes multiplier	Student 10-1 [00:11:28.22] Uh cause of the mental analysis, I just needed to figure out the molar mass of the substance and then, so I added the amount of grams of aluminum and amount of grams of nitrate. Then the multipliers, which is three and two and all that.
03-56	Student misunderstood the problem	Student 7-4 [02:03:29.21] I'm thinking D as in (?) decrease make it lower.
03-57	Student discusses relations or relationships	Student 23-4 [01:07:27.23] Well I looked up here and for ten milliliters of water I guess each milliliter has only zero point seven five moles in it so I want to say it’s like like if you had nine moles or whatever, nine moles of calcium chloride to begin with as like a solid and then like dissolved it into ten so like each milliliter of water would only have three in it. Does that make sense? So like this one if you had like ten milliliters of water then each one would have a complete mole of calcium chloride.
03-58	Student does not think the volume of solution affects molarity.	Student 22-3 [01:53:54.14]I think it’s just extra information for this one.
03-59	Student doesn’t use subscript from the chemical formula	Student 22-3 [01:44:26.07] OK. [Student writes 75 division box 150. Writes .2 then 150 then - then underlines and writes 0.
03-61	Student uses “percent”” in discussion.	Student 10-1 [00:24:16.17] I used a percent, um, formula I learned a while ago, which is percent over a hundred equals part over whole. So I used that but normally doesn't work for me. I'm not sure why. It seems to make sense logically in my head but, uh, it doesn't normally work and I can't exactly figure out what formula to use. I remember there's a formula. I just don't remember what it was.

Table 9: Axial codes for R3	
Code	**Statement**
3-A	Student uses algorithm
3-B	Student uses grid
3-C	Student adds or subtracts the numbers
3-D	Student multiplies or divides the numbers
3-E	Student does not convert
3-F	M is moles
3-G	Student uses volume
3-H	Student uses a direct proportion
3-I	Student uses an inverse proportion

IV. ANALYSIS, RESULTS, AND DISCUSSION OF RESEARCH QUESTION 1

A. Chapter Overview for the analysis of R1: Do students' understandings of ratio vary from domain specific tasks to structurally similar tasks?

In this chapter, students examples are presented that cover the findings for student work related to R1. As will be shown, students possessed the ability to represent the concentration of color as an intensive quantity in the SHDC and DHSC tasks. Generally, the students were able to represent the amounts of red and white paint using cubes as they pertained to length.

Information gleaned from the painted wooden blocks structurally similar tasks is interesting in and of itself because it gives insight into student qualitative reasoning about concentration, but it becomes particularly interesting in combination with the molarity tasks found in Chapter V. The nature of these tasks is such that they were analogous to the molarity tasks that will be discussed in Chapter V. The direct links between the logic of the domain specific tasks and the structurally similar tasks were discussed in Chapter III in Figure 11. Specific mappings of axial codes and how they relate to the Theoretical Statement in this Chapter can be found in Appendix F.

B. Analysis of R1

Students were able to reason through concentrations of paint as an intensive property through height changes and color variations. This was true for all the students at some point or another within the tasks. Each student example will be discussed with an in depth discussion regarding their responses to both the SHDC and DHSC structurally similar tasks. Analysis of student data yielded one theoretical statement that is supported by five subcategories of students.

This is presented as Theoretical Statement 1, which is a consistent interpretation of the behavior of all 24 students in some subset. Students are presented by subcategory in Table 10.

Table 10: Theoretical Statement 1: Student reasoning with concentration as a ratio enables successful description and modeling with the SHDC and DHSC

Category (a)				
Subcategory (i)	Subcategory (ii)	Subcategory (iii)	Subcategory (iv)	Subcategory (v)
22-3	4-2	9-2	11-1	8-4
5-3	16-2		6-3	
17-3	18-4			
2-1	19-2			
13-4				
21-2				
23-4				
24-3				
1-3				
14-1				
10-1				
3-4				
15-1				
12-1				
7-4				

1. **Theoretical Statement 1: Student reasoning with concentration as a ratio enables successful description and modeling with the SHDC and DHSC tasks.**

From Chapter III, axial codes with respect to R1 (Table 3) were analyzed for patterns. It was found that all the students exhibited an intensive ratio view of concentration involving two extensive quantities during the structurally similar tasks. Twenty-one students were able to recognize the intensive nature of color between both of the tasks whereas three students were able to recognize the ratio in one task but not the other. Therefore, this category of an intensive view of concentration can be broken into subcategories:

i. Student initially represented concentration of paint as an intensive property for both the SHDC and DHSC tasks
ii. Student initially represented concentration as an intensive property for SHDC but required a prompt to represent concentration as an intensive property for DHSC task
iii. Student required a prompt to represent concentration as an intensive property for the SHDC task but initially represented concentration as an intensive property for the DHSC task
iv. Student represented an extensive property in the SHDC task but represented concentration as an intensive property in the DHSC task
v. Student represented concentration as an intensive property in the SHDC task but represented an extensive property in the DHSC task

Subcategories i, ii and iii all cover the same branches of the analysis trees but differ where prompts from the researcher were required to help students represent concentration and volume both using cubes on both tasks to arrive at those answers. This is shown in Figure 21. A student in these categories indicated a ratio relationship between the red cubes and the white cubes while representing the blocks for both the SHDC and DHSC tasks. Subcategory iv is a variation on this in that the student did not ever indicate a ratio relationship for the SHDC task but does in the DHSC task. This analysis tree can be found in Figure 22. Lastly, the opposite of subcategory iv is subcategory v. A student in this category indicated that there was a ratio relationship within the SHDC task but did not ever indicate a ratio exists within the DHSC task. This is shown in Figure 23.

Figure 21: An example of an analysis tree for a student in subcategory a.i, a.ii, or a.iii

The black boxes indicate, which branches of the analysis tree the student belongs. Students in this subcategory were able to reason about concentration as a ratio in both the SVDM and DVSM tasks.

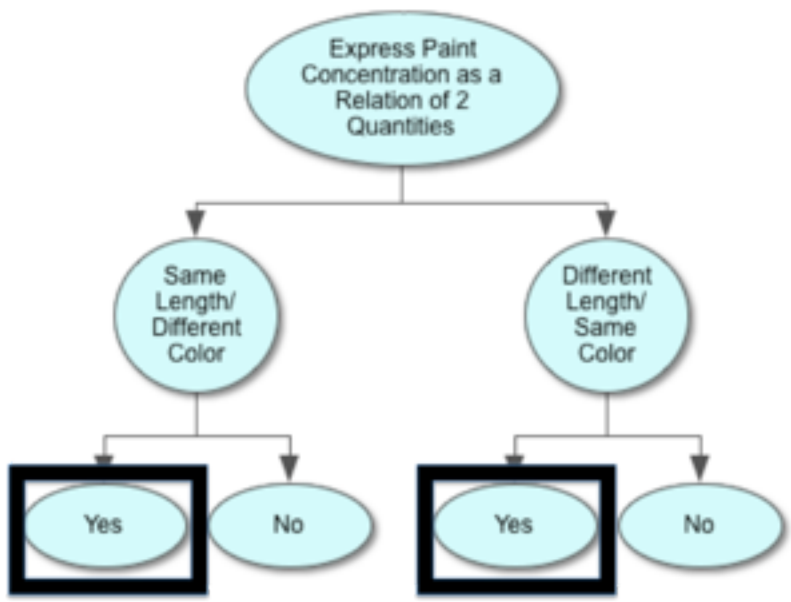


Figure 22: An example of an analysis tree for a student in subcategory a.iv
The black boxes indicate, which branches of the analysis tree the student belongs. A student in this subcategory was able to reason about concentration as a ratio only in the DVSM task.

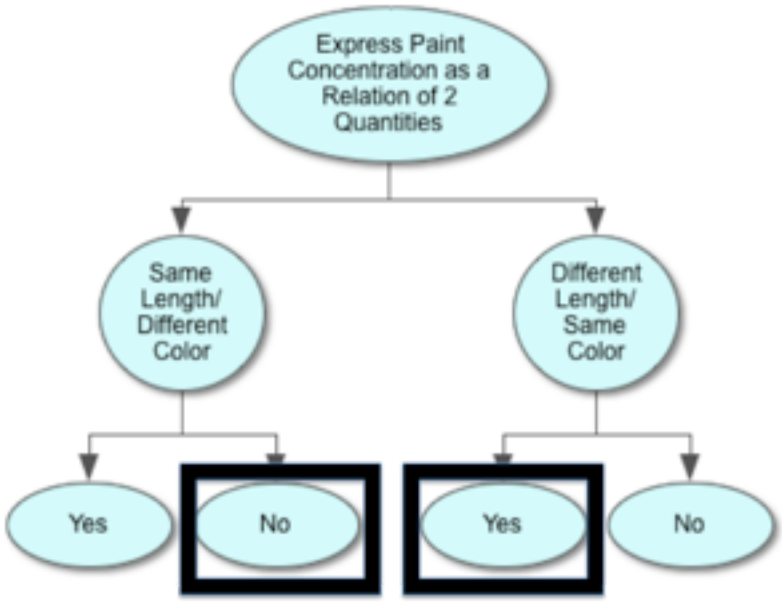
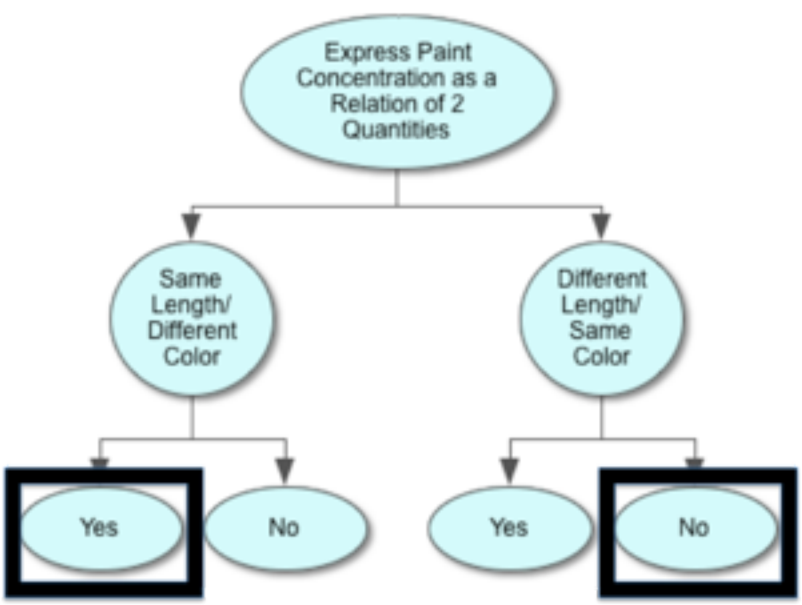


Figure 23: An example of an analysis tree for a student in subcategory a.v
The black boxes indicate, which branches of the analysis tree the student belongs. A student in this subcategory was able to reason about concentration as a ratio only in the SVDM task.



a. **<u>Students hold an intensive view of concentration in the SHDC and DHSC tasks.</u>**

All the students from this study can be categorized as holding an intensive view of the concentration of color as presented in these isolated tasks at one time or another. There were slight variations in student responses, which will be discussed in the following subcategories.

i. **<u>Student initially represents concentration of paint as an intensive quantity for both the SHDC and DHSC tasks</u>**

Students in this subcategory represented the concentration of paint as an intensive property without researcher intervention for both the SHDC and DHSC tasks. This subcategory was applicable to a majority of the students: Student 22-3, Student 5-3, Student 17-3, Student 2-1, Student 13-4, Student 21-2, Student 23-4, Student 24-3, Student 14-1, Student 10-1, Student 1-3, Student 3-4, Student 15-1, Student 2-1, and Student 7-4. Representative examples are provided within this section. Additional student examples for this subcategory can be found in Appendix A.

Students in this group represented both the red paint and the white paint while attending to the volume. In fact, Student 5-3 (Figure 24), Student 17-3 (Figure 25), Student 22-3, Student 2-1, Student 10-1 (Figure 26), and Student 14-1 (Figure 27) all had nearly identical responses. In the SHDC task, the students constructed cube models that had a constant total amount of cubes with variations between the blocks in terms of the number of red and white cubes. In the DHSC task, Student 5-3 describes the need for additional paint for longer length as, “So for the second one it still needs one red and one white but since it's larger it needs an extra one". Similarly, Student 17-3 stated, "Well they all have the same within each block there's the same amount of red and white and it just like I just minused one as each block got smaller". In the DHSC task, she described her mental model as "I started off with this is the smallest block used one red and one white to represent this one and then I gradually just added one more red and white for each one."

Figure 24: Student 5-3's constructions for the SHDC and DHSC tasks

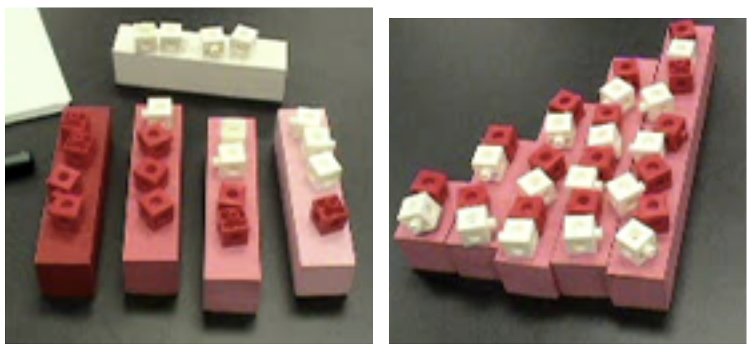
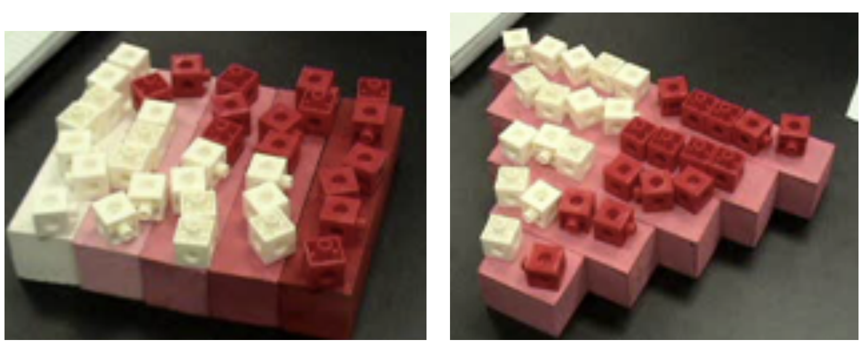


Figure 25: Student 17-3's constructions for the SHDC and DHSC tasks



Some students referred to the proportionality of blocks. Student 10-1 described his cube construction for the DHSC task as follows:

[01:19:06.17] **Interview 10-1:** So it's gotta, so like, this one is gonna have less paint on it than that one but it's still gonna be the same ratio, the same proportions. So it'd be half and half, fifty fifty.

He indicated that as the length of the block increased, there needed to be more paint to cover it but that the increase had to be an increase in both red and white paint with a 1:1 ratio. Student 14-1 had a similar response, but he also noted the proportionality between the length of the blocks.

[01:42:01.29] **Student 14-1:** Well, each of these, well this block I put down as two. And then I just thought, I kind of eyeball measured it and this one appeared that two of these equals one of the blocks, one's the smallest block. I had two cubes and then this one's twice as big as this one. I put four blocks on this one. And then it appeared to me as this smallest block is the two smallest blocks put together equaled the height of the third block in line. So I made this be six. And then it appeared that the third smallest block and the smallest block together were about the same size so I made this one an eight. And then this block and the second to, second largest block and the smallest block appear to be the same size as the largest block so I did eight plus two, which is ten to get this and then with the colors I just thought that I made it look like they were all the same color by using white red.

His cube model combined with his explanation indicated that he chose a 1:1 ratio for the smallest block and because the remaining blocks were multiples of the smallest block he just multiplied to find the total number of cubes for each block.

Figure 26: Student 10-1's constructions for the SHDC and DHSC tasks

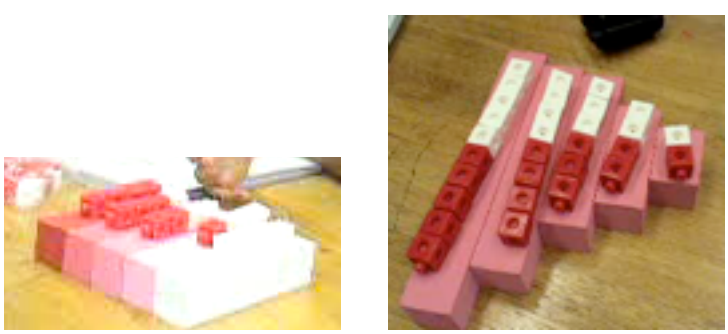


Figure 27: Student 14-1's constructions for the SHDC and DHSC tasks

Students 13-4 and 21-2 also belong in this subcategory as a slight variation. They both had a cube model for the DHSC task consistent with many other students but they both also used "half cubes" in the SHDC task because they used an odd number for their total number. When a student used an odd number of cubes, such as five, to represent color, he or she was faced with a 2.5 to 2.5 ratio of red to white cubes for the middle pink block. The students who did not switch to an even number at this point used "half cubes" indicating that one of the cubes represents only a half of a can of paint rather than a whole can like the other cubes in front of them. These students have the proportional reasoning skills that enable them to reason through the task such that they are able to use half blocks to explain their reasoning.

These students all possessed the reasoning skills to attend to two extensive variables all while keeping a ratio constant in the structurally similar context. This intensive property view of concentration and reasoning would have yielded a correct response in the DVSM task, but many students held a different mental model for the chemistry context. As will be shown in Chapter V, students strongly held onto an "M is moles" belief of molarity as an extensive property and therefore that mental model did not allow for an intensive representation of molarity.

ii. **Student initially represents concentration as an intensive property for SHDC but requires a prompt to represent concentration as an intensive property for the DHSC task**

Another subcategory of students are those who initially represented concentration as an intensive property for the SHDC task but needed researcher prompting to do the same for the DHSC task. This additional probing came in the form of a question asking the student if it would take the same amount of paint to paint each block and then providing an analogy to houses of different sizes in the suburbs getting painted. The following students represented only color first and then added length and in turn a ratio understanding to their responses in the DHSC task after a prompt: Student 4-2, Student 16-2, Student 18-4, and Student 19-2. Student 18-4's responses are shown as a representative example. Further examples of this subcategory can be found in Appendix A.

Student 18-4 had a similar cube model for the SHDC task (Figure 28).

Figure 28: Student 18-4's cube model for the SHDC task

Student 18-4 initially only represented the color as shown in Figure 29. He described his model as:

> [01:07:04.23] **Student 18-4:** Um I would have I would say that you have two blocks of white, those blocks that you just gave me and three blocks of red.

Figure 29: Student 18-4's initial cube model for the DHSC task

The researcher then asked the student to consider the length during the following exchange:

> [01:08:40.16] **Researcher:** OK so it's how much paint. Ok so for each of these blocks I am using three red paints and two white paints. Does it makes sense that I am using the exact same total amount of paint for each of these blocks?
> [01:08:54.21] **Student 18-4:** Yes
> [01:08:54.21] **Researcher:** Why
> [01:08:56.08] **Student 18-4:** Because all this is like same tone of paint.
> [01:08:59.26] **Researcher:** Ok
> [01:09:01.06] **Student 18-4:** So so you wouldn't use anymore because you want it to be the same.
> [01:09:08.07] **Researcher:** Ok so let me rephrase my question. What if we were out in the suburbs and we have five houses. Um they are all different size house like these blocks. Would you need to tell a painter to go buy the same amount of paint for each house? Or would they have different amounts based on the size?
> [01:09:32.01] **Student 18-4:** Uh they would have different amounts of depending on their size.

Figure 30: Student 18-4's second cube model for the DHSC task

It is important to note that he was resistant to ratio use at this point in the task. The student indicated that there was no need to use different total amounts of paint because the tone was represented and having additional cubes would change the color. This utterance implied a lack of a ratio mental model. The student then constructed the model shown in Figure 30. His new cube model now consisted of only white blocks that each represented the tone so that he could show the length. The researcher pushed for the student consider both color and length again:

> [01:11:45.02] **Researcher:** OK so now I'm going to ask you to get a little more specific, ok. So let's say that you, they can't, they don't sell that. They don't sell all this tone. You have to mix two together. How would you mix the red in with that?
> [01:12:00.04] **Student 18-4:** Uh ok so ok um ok more blocks. So each one was, for the first one I would use these ones. And then these two over three, four over six, And this is six over nine. What I'm doing right now is, see double the size of this so multiply two and this is the triple side of this divide by three. Then this is four so I multiply by four, then this is trip, five times the size so I multiply by five.

When faced with a scenario that didn't fit his model (the single color of paint), he immediately switched to a ratio of red to white across the blocks. He considered the proportionality of the wooden blocks for his response and constructed the cube model found in Figure 31. The piles of red cubes behind the blocks were part of his cube model that did not fit on top of the wooden blocks.

Figure 31: Student 18-4's final cube model for the DHSC task

This difficulty in ratio reasoning was shown in his structurally similar tasks to some extent in that he was initially quite resistant to using ratios. With two prompts from the researcher, he was able to construct a cube model that accounted for length, amount of red paint, and amount of white paint.

These students clearly possessed the skills to reason with an intensive property consisting of two extensive properties in the structurally similar task. Because of the nature of the task, it was not immediately obvious to them that the researcher was looking for them to represent a ratio with the cubes. These students were able to immediately switch their responses to account for both variables once they realized this was being requested by the researcher.

iii. **Student requires a prompt to represent concentration as an intensive property for the SHDC task but initially represents concentration as an intensive property for the DHSC task**

In contrast to the students in subcategory (ii), Student 9-2 did not initially represent both length and color for the SHDC task but did initially represent both length and color for the DHSC task. His representations for the SHDC and DHSC tasks can be found in Figure 32. In the SHDC task, he initially indicated that he would use "five red" and "three white" cubes to represent the total amount of paint used for the blocks. When asked to elaborate how many would be designated to each block, he said the following:

> [00:57:27.23] **Student 9-2:** Actually do like Ok, um, for this one I would do three. For that one, for this one I would use two, actually, yeah I would actually do three cause it's like a little but, a slight difference. So I'd use three reds and one white. For this one, I would do two reds and one white. And then for this one, I would do one red and one white.

He only represented the color of the blocks using the cubes and not the length. He used three cubes of red for the all red block and only two whites cubes for the white block. The researcher continued to probe the student's understanding and they had the following exchange:

> [00:59:30.07] **Student 9-2:** Yeah I'm going to change like what like each cube represents.
> [00:59:35.12] **Researcher:** Ok, what did it represent before?
> [00:59:38.18] **Student 9-2:** Just how I didn't really think about that. Um, just like the color. Let's pretend this is like a can of paint.
> [00:59:56.12] **Researcher:** Ok.
> [00:59:58.07] **Student 9-2:** One can would probably cover like a third of this. Two cans and then.
> [01:00:05.26] **Researcher:** Like what you originally had or what you have now?
> [01:00:07.14] **Student 9-2:** What I have now. We'll just start from there. I'm going to remove this one and then do two cans of red paint and one can of white paint for this one. I think I would have to do for this one I think I could mix cans, like mix the colors. So I would do actually have, I would have to use about this is about like halfway between white and red so I think I would have to use about one and half cans of each color. So I would use one and half, like just pretend I'm gonna use half of this. I would half of that and half of white. For this one it's more pinkish so I'd do one can of red, two cans of white. This one just three cans of white.

He changed his representation from solely representing color to include length. He used three blocks as the constant length, which led to the use of "half cubes" to represent the cans of paint. Because he could not create half a cube, he placed the cube half on the block of wood with half hanging off to indicate that only half of it was there.

Figure 32: Student 9-2's cube models for the SHDC and DHSC tasks

In the DHSC task, Student 9-2 initially indicated a proportionality and represented both length and color. He described his model as follows:

> [01:15:20.13] **Student 9-2:** They are proportionate.
> [01:15:22.10] **Researcher:** They are proportionate. What do you mean by that?
> [01:15:26.20] **Student 9-2:** Like, um, if you were to multiply the length of this by a certain number, you'd reach this. You'd get to this length.
> [01:15:36.13] **Researcher:** Ok. And multiply, which one by what?
> [01:15:43.18] **Student 9-2:** If you were to multiply this one by a certain number, you'd equal the length of this.
> [01:15:48.22] **Researcher:** Ok. Ok. How do your piles of paint cans relate to one another?
> [01:15:58.13] **Student 9-2:** Um, since there's more block to paint, there'd be more paint required. So each block represents a can of paint.
> [01:16:14.06] **Researcher:** And this one is three and a half and three and a half. Three, three. Two and a half and two and a half. Two, two. One,one.

He used "half cubes" again and described the proportionality while keeping the ratio of red paint to white paint constant across the varied lengths with more total paint for longer blocks. This student was able to reason through the ratio in the context of a height change but not in the case of a color change.

iv. **Student represents an extensive property in the SHDC task but represents concentration as an intensive property in the DHSC task**

Two students only represented color in the SHDC task even after a prompt: Student 6-3 and 11-1. Otherwise, in their DHSC task they matched the other correct students showing a ratio across the blocks with varying total amounts of cubes. Student 11-1's cube model for the SHDC task is shown in Figure 33. He represented only the color and described his model as:

> [00:53:34.05] **Researcher:** Ok so you said they're all the same length right?
> [00:53:39.10] **Student 11-1:** Yeah
> [00:53:39.10] **Researcher:** Ok so do I need the same amount of paint to paint all of the blocks?
> [00:53:45.00] **Student 11-1:** Oh well no because well you're adding two different types of, like this would have most because you're adding two white and two red to try to get this color, which would be an equal of these two. So you'd be using more just because you want to get more, I mean the color you're trying to achieve I guess.
> [00:54:10.19] **Researcher:** Ok so if these were cans of paint, and I went to the store and I bought one can of paint, is this going to paint my whole house?
> [00:54:18.22] **Student 11-1:** Well no cause it's just one can.
> [00:54:21.11] **Researcher:** Ok so I guess that' kind of what I'm asking if you need four with two and two here.
> [00:54:26.17] **Student 11-1:** Yeah.
> [00:54:28.01] **Researcher:** How many do you need the rest of the way?
> [00:54:30.25] **Student 11-1:** Well I'll just base it off this I guess, so, or add more to this one cause, and then this would probably have one more white and. Um. You're trying to paint the whole house?

The researcher attempted to prompt the student to attend to the length in his model, but he continued to represent solely the color without attending to the volume. In the DHSC task, he constructed a cube model with a ratio of red to white paint with varying total amounts with respect to block height.

Figure 33: Student 11-1's cube models for the SHDC and DHSC tasks

Similarly, Student 6-3 represented only the color in the SHDC task as shown in Figure 34. He described his mental model as:

> [01:19:44.10] **Student 6-3:** Like because for all of these, like this is a zero to one ratio. This is a zero to one ratio. Tells you it's red and white. For this, like you have to use a certain amount of red and a certain amount of white to get this color. And a certain amount of red and a certain amount of white to get that color. And what happens is as you go down the gradient, like the amount of white you're using increases and the amount of red you're using could stay the same as long as you increase the amount of white you're using. Or it could decrease

His initial description involved ratios and two different mental models. The first represented only color and not length whereas the second involved a constant length with varying amounts of colors. As he was assembling his cube model he then said:

> [01:22:40.29] **Student 6-3:** Cause it seems a lot more easier. Ok. Seems easier to have the same, have the same amount of reds for all of them and then increase the amounts of whites so that way there's a constant.

He chose a mental model that did not account for total amount of paint and the length of the blocks.

Figure 34: Student 6-3's cube models for SHDC and DHSC tasks

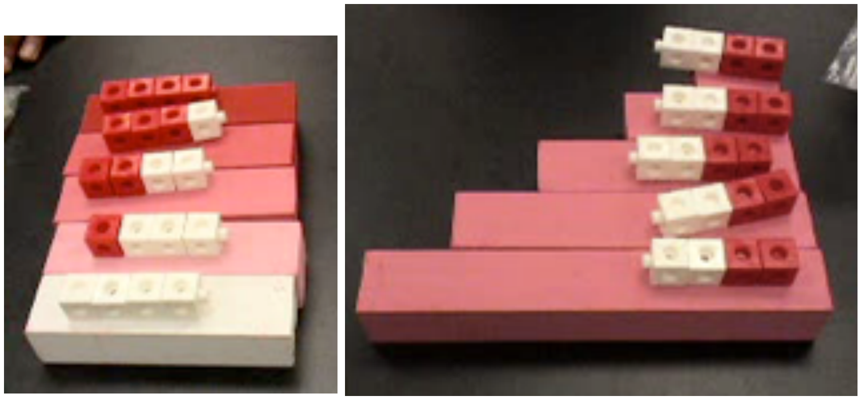
For the DHSC task, he constructed a series of cube towers that increased in total amount with a constant ratio between the red and white cubes.

Students in this subcategory were able to represent concentration of color as an intensive quantity in the DHSC tasks but they did not in the SHDC task. This could be in part due to the salient features of the task combined with personal experience with paint. For example, a student could use one red cube to represent the red block because only red paint is needed and not in a ratio to white paint like the pink blocks. Because the blocks are all the same shade of pink in the DHSC task, this type of response is not elicited.

v. **<u>Student represents concentration as an intensive property in the SHDC task but represents an extensive property in the DHSC task</u>**

One student was able to represent both color and length in the SHDC task and not in the DHSC task even after a prompt. Student 8-4 represented the color of the paint attending to the volume and the relative amounts of red and white paint in the SHDC task as shown in Figure 35.

Figure 35: Student 8-4's cube models for the SHDC and DHSC tasks



For the DHSC task, Student 8-4 represented only the ratio of red to white blocks while ignoring the length of the block, shown in Figure 34 (right). This was similar to the student's response for the DVSM task while attributing the number to the number of calcium chlorides. This student was only attending to the color (structurally similar to the number before M) and not the volume as it related to its extensive quantities. The ratio and total number of cubes that Student 8-4 used in the DHSC task matched the cube model for the pink block of the same color in the SHDC task.

Overall, students referred to proportions and ratios in their language. They exhibited the skills that are necessary for understanding molarity throughout the structurally similar tasks but they did not apply these same mental models to the chemistry tasks, as will be shown in Chapter V.

C. <u>Conclusions regarding R1</u>

The data from this group of twenty-four students suggests that students were able to recognize the ratio nature of paint color concentration. This observation and conclusion involves qualitative conceptual understandings about a quantitative event. This ability to recognize the existence of a ratio relationship between the colors of the paint and the concentration of color enables students to reason through both a change in height and a change in color. A qualitative understanding of concentration is the basis for the understanding of the concept. This finding can be directly applied to the molarity tasks, which will be shown and discussed in length in Chapter V.

Concentration is a critical concept in chemistry, which is one reason why students are introduced to the concept in earlier grades prior to tackling it in high school. If students fail to reason about concentration in high school chemistry courses, teachers might suggest that the students lacked the proportional reasoning skills before joining the chemistry course. If proportional reasoning is critical to the understanding of molarity, then it is helpful to notice that these students generally had that skill in these tasks. As shown in this Chapter, students can reason through concentration of color as an intensive quantity. This enables them to describe and model concentration qualitatively. The salient features of the structurally similar tasks were macroscopic and concentration was not presented as a number to the students.

There were a few cases of students who required prompting to attend to both the length and the color and a few instances where students never represented one of the tasks as an intensive quantity. This tells us that these tasks need to attend to both color and volume.

As discussed previously, the painted blocks tasks were designed to be structurally similar to the molarity tasks that will be discussed in Chapter V. The results of this study indicate that students have at least have a basic conceptual understanding of concentration from their everyday experiences and that they recognize the ratio within concentration in this context.

Students do have facility with intensive reasoning about concentration. Unfortunately, this skill is not transferred to the chemistry context, which will be described in depth in Chapter V.

Chemistry teachers can capitalize on this basic understanding and use it as a starting point for the introduction of molarity concepts. The structurally similar tasks can act as an indicator of sorts for teachers at the beginning of the lesson to see how students reason through concentration at a basic level using colored cubes and painted blocks. It is highly likely that the visual nature of the task cues the students attend to red paint, white paint, and the length of the blocks at the same time. Specific recommendations will be shown in Chapter VII (Implications).

V. ANALYSIS, RESULTS, AND DISCUSSION OF RESEARCH QUESTION 2

A. Chapter Overview for Research Question 2: What are students' interpretations of molarity in solutions chemistry?

This research question examines student interpretations of molarity in domain-specific tasks. This question was sequenced to follow the domain-general tasks so that student qualitative understandings of concentration could be analyzed as a basis of understanding. This chapter serves to analyze the domain-specific tasks. It is important to study this research question because students who do not hold an intensive view of molarity hold a variety of different mental models. To effectively teach students, teachers must first understand their mental model shortcomings. As shown in Chapter IV and will be presented in Chapter VI, these shortcomings were not due to a lack of basic proportional reasoning skills. Specific mappings of axial codes and how they relate to the Theoretical Statements in this Chapter can be found in Appendix F.

B. Analysis of R2

From Chapter III, Axial codes with respect to R2 (Table 5) were analyzed for patterns to develop Theoretical Statements grounded in the data. Students were grouped into two major groups: those who viewed molarity as an intensive quantity and those who viewed molarity as an extensive quantity.

These theoretical statements related to either group can be found in Table 11. Each of the theoretical statements will be described in this chapter along with pieces of evidence for its construction. They are divided into categories of whether students viewed molarity as an intensive or extensive property. Theoretical Statement 2 is unique to students who viewed molarity as an intensive property. This view of molarity is a correct understanding that molarity is a ratio of two extensive properties: amount of substance per volume. Theoretical Statement 3 is unique to students who viewed molarity itself as an extensive quantity. This statement had several subcategories with a majority of the students believing that M stood for moles.

It is important to note that "substance change" in this dissertation does not indicate that the substance has become something new, but rather that the amount of the substance changed. This language was employed by the axial coders so that the molarity tasks and the structurally similar tasks could share an analysis tree structure.

Table 11: Theoretical Statements Related to R2				
Intensive View of Molarity	**Extensive View of Molarity**			
<u>Theoretical Statement 2</u> Students who view molarity as an intensive quantity are therefore able to reason correctly through substance changes.	**<u>Theoretical Statement 3</u>** Students who view molarity as an extensive quantity are therefore able to reason through the SVDM task due the direct proportion relationship between amount of substance and molarity but not in the DVSM task where the relationship between volume and molarity is an inverse proportion.			
a	b			
i	i	ii	iii	iv
1-3	9-2	16-2	17-3	4-2
10-1	12-1	22-3	5-3	
14-1	11-1	24-3	18-4	
	3-4	6-3		
	7-4			
	23-4			
	15-1			
	2-1			
	8-4			
	13-4			
	19-2			
	20-2			
	21-2			

Table 11: Theoretical Statements Related to R2	
Intensive View of Molarity	**Extensive View of Molarity**
Theoretical Statement 4 A conceptual understanding of molar mass as it relates to moles generally enables reasoning with the value in front of M.	
a	b
1-3	11-1
10-1	12-1
14-1	13-4

As mentioned in the introduction to the chapter, students either viewed molarity as an intensive or extensive quantity. Only three students viewed molarity as an intensive quantity and are discussed in Theoretical Statement 2. The remaining students viewed molarity as an extensive quantity and their various subcategories are described in Theoretical Statement 3. Some students showed facility with molar mass and mole calculations, which enabled them to reason through amount of substance changes in the SVDM task. This will be discussed in Theoretical Statement 4.

1. **Theoretical Statement 2: Students who view molarity as an intensive quantity are therefore able to reason correctly through substance changes.**

Students who viewed molarity or M as an intensive quantity were able to reason through substance changes both when the molarity changed (SVDM) and when the volume changed (DVSM). Students with this view of molarity and the understanding that molarity is equal to moles over liters were deemed to have a "robust understanding" of molarity because they recognized the ratio nature and knew the scientific definition of molarity. A student with a robust understanding of molarity would not only know the scientific definition of molarity, but also be able to apply that definition to the tasks in this interview.

a. <u>Student views molarity as an intensive quantity</u>

Three students, Students 1-3, 14-1, and 10-1, were the only students able to reason through the amount of substance changes in both SVDM and DVSM tasks. Students 14-1 and 10-1 were considered to have a robust understanding of molarity. A discussion of Student 1-3's responses will be discussed in a later paragraph.

Student 14-1 describes his view of molarity as follows:

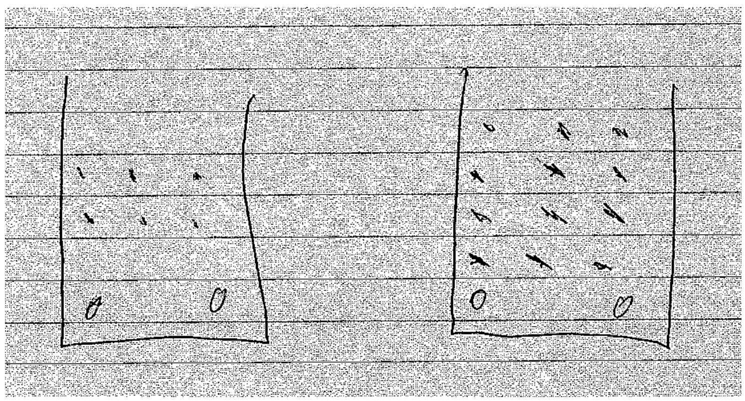
> [00:29:00.00] **Student 14-1:** Ok. What is the molarity made by dissolving fourteen grams of NAOH and three hundred fifty milliliters of water? How did you arrive at this answer? I've learned from high school chemistry classes molarity is the amount of moles of a solution in one liter of water or moles per liter of solution, not necessarily water. ***{truncated quote}***

He indicated his understanding of molarity was that "molarity is the amount of moles of a solution in one liter of solution, not necessarily water". He had a robust understanding of molarity using the correct terminology and knew that not all solutions are created with water. He used this understanding of molarity through the SVDM and DVSM tasks. In the SVDM task shown in Figure 36, described his drawing as follows:

> [01:24:29.23] **Student 14-1:** OK. Ok each of these, each of the circles represent water like they have the same amount of water. But this one point one molar solution because there's twice as many dots of calcium chloride.

He indicated that there were twice as many molecules of calcium chloride in the 0.10M solution than the 0.05M solution and showed that the water (large circles) remained the same throughout both.

Figure 36: Student 14-1's drawing for the SVDM task

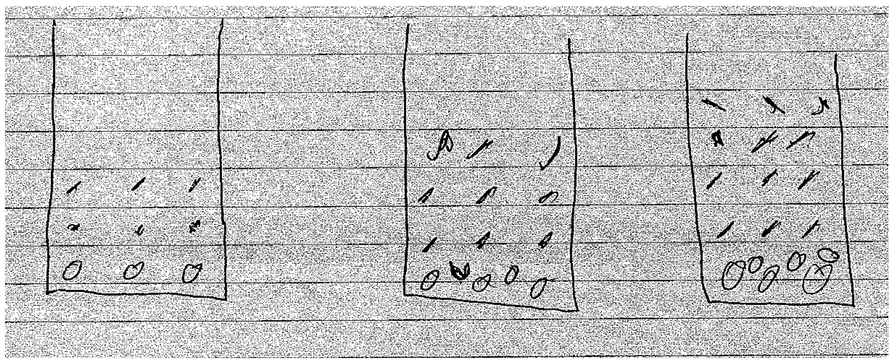


In the DVSM task shown in Figure 37, he described his drawing as follows:

> [01:33:23.29] **Student 14-1:** Um there's, in this one there's, there's the same proportion of, there's the same proportion of like moles to liters in all of these. But in this one it has more moles, the one that has two hundred milliliters would have presumably more moles of calcium chloride than the ones with lesser volume cause there's more volume and since they are the same molarity.

He discussed the proportionality of the jars with the same molarity but different volumes and drew jars indicating more water and calcium chloride as the volume increased.

Figure 37: Student 14-1's drawing for the DVSM task



Similarly, Student 10-1's utterances and drawings through the SVDM and DVSM tasks indicated that he too held an intensive view of molarity. As the following utterance shows, he indicated in the CD that "molarity equals moles over liters".

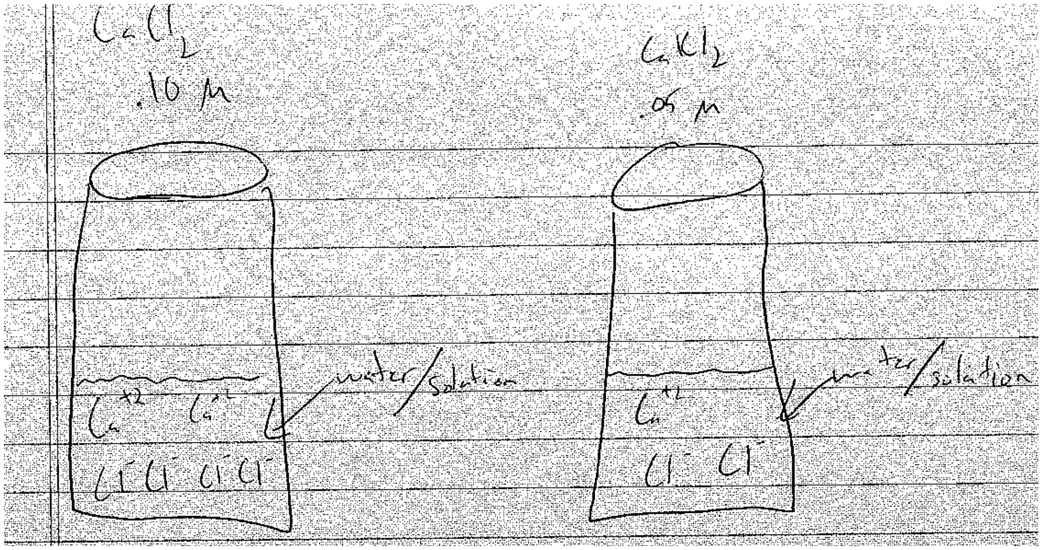
> [00:18:05.16] **Student 10-1:** I used the molarity formula, which is molarity equals moles over liters. And since there's, ah. I did it the wrong way. It shouldn't make a difference. It's gonna be multiplied either way. Six five Yeah it's not gonna make a difference but still.

He used this understanding of molarity throughout the SVDM and DVSM tasks. In the SVDM task shown in Figure 38, he described his drawing as follows:

> [00:59:22.07] **Student 10-1:** (mumbling). Um, I'll just use two calcium chlorides for the point oh one concentration and then I'll use one for the point oh five concentration to represent.
> [00:59:40.18] **Researcher:** ok
> [00:59:40.18] **Student 10-1:** Alright, so, Ca plus two. Cl negative. Ca plus two. Cl negative Um, I guess that would be water. Water.

He indicated that that the 0.05M jar would have half as much [calcium chloride] as the 0.10M jar. He drew a wavy line for water and discussed it as though it was the same.

Figure 38: Student 10-1's drawing for the SVDM task

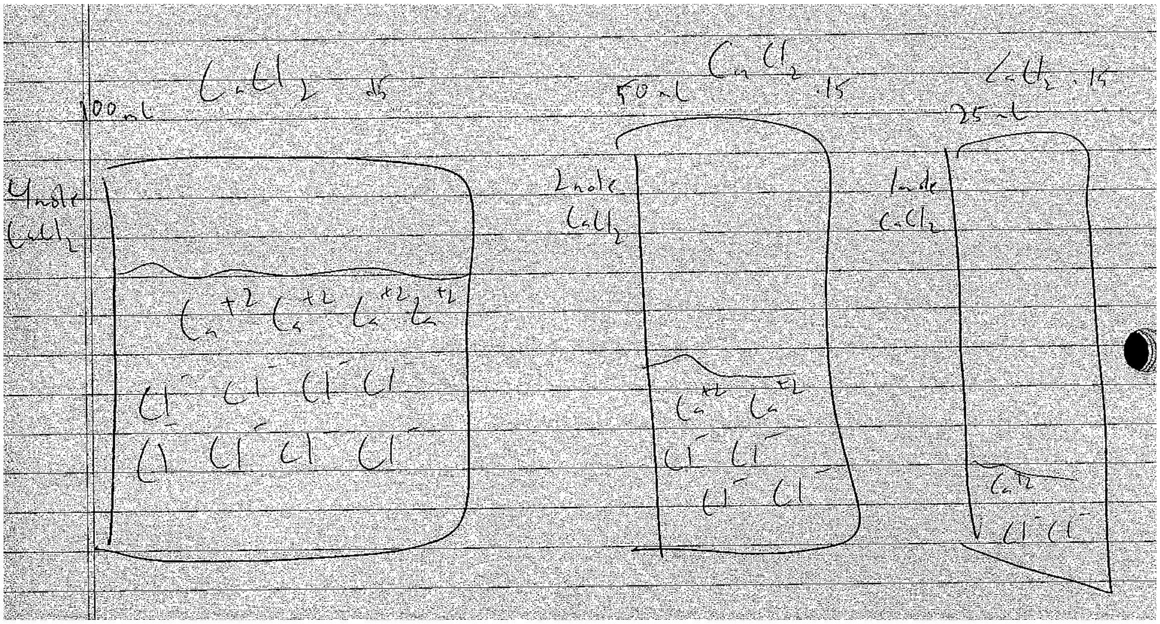


In the DVSM task shown in Figure 39, he described his drawing as follows:

> [01:15:38.10] **Student 10-1:** It's just a random number I picked, just so I could make it make sense a little bit more. So like, I basically made it; the first, the largest container I made a hundred milliliters. And I just put four moles as the biggest number. The second one I just made it fifty milliliters and then, ah, just using two moles. Then the third one just twenty-five milliliters, one mole.

He indicated that he randomly chose the number four for easier division to show the varied amounts of calcium chloride in each of the jars. He correctly showed less and less calcium chloride as the volume decreased and again used wavy lines to indicate water was part of the solution.

Figure 39: Student 10-1's drawing for the DVSM task



As mentioned at the beginning of this section, out of the three students who viewed molarity as an intensive quantity, only two were deemed to have a "robust understanding" of molarity. Student 1-3 was determined to have a partially correct understanding of molarity. His view of molarity was nearly correct in that his belief was that M=g/mL. This is close in that he attended to both the mass and the volume and viewed molarity as an intensive quantity. However, he did not convert the mass into moles or the milliliters to liters. He also faltered in his language interchanging the words moles and molarity, but his description of the relationship between the extensive quantities was correct.

In the SVDM task, he describes the phenomenon as follows:

> [01:18:52.28] **Student 1-3**: Um well earlier I had used molarity equals grams per milliliter of in this case CaCl [sic]. So I'm guessing um there are more grams of CaCl [sic] in the point one five mole solution than any of the other ones.

He correctly indicated that there would be more grams of CaCl [sic] in the 0.15M solution, but calls it mole solution. He failed to attend to the subscript in his symbolic name for calcium chloride.

He described the DVSM task as follows:

> [01:28:46.15] **Student 1-3:** I think that you would use, like how you created each one. Um I think with each, with this one, for example, the middle one, you would use um lesser amounts of each. I think you said you used water and calcium chloride solid to make this. So I would think you would need to use lesser amounts of each to create this one and then increasing amounts to replicate the same moles in each one.

He correctly indicated that as the volume increased there would need to be increasing amounts of calcium chloride to maintain the same "moles" in each one. Again, he interchanged mole and molarity, but the concept behind his language was correct.

Student 1-3's belief was close to the beliefs held by 14-1 and 10-1 in that all three students viewed molarity as an intensive quantity. Because of this, they were able to reason that there was also an amount of substance change as the volume changes in solutions with the same molarity.

2. **Theoretical Statement 3: Students who view molarity as an extensive quantity are therefore able to reason through the SVDM task due the direct proportion relationship between amount of substance and molarity but not in the DVSM task where the relationship between volume and molarity is an inverse proportion.**

Students whose utterances and drawings led to the construction of Statement 3 differ from the students whose utterances and drawings led to the construction of Statement 2 in that they differ in what they thought was in the solution. Many of the students in this category did not attend to water and if they did it was not factored into molarity in a meaningful way. These students did not have a robust understanding of M and viewed molarity as an extensive property.

b. **Student views molarity as an extensive quantity**

Students with an extensive view of molarity could sometimes reason that the amount of substance has changed solely based upon the number in the SVDM task due to the direct proportional relationship between moles of substances and molarity. Furthermore, an extensive view of molarity enables students to recognize a multiplier when asked to draw the difference between a 0.05M and 0.10M solution without an understanding of molarity and without attending to water in a meaningful way. A student could get this type of problem correct on a test and be assumed to understand molarity. Students are also able to get this problem correct with this reasoning due to the direct proportional relationship between moles and molarity. In reality, the same students were unable to reason about molarity when the molarity stayed the same and the volume changed.

However, the shortcomings of their mental models were shown in the DVSM task. Contrary to their successful performance on the SVDM task, 20 out of the 24 students were unsuccessful in the DVSM task. They indicated that the amount of calcium chloride inside the jars was the same in all three jars despite having different volumes of the same molarity. One student was simply unable to reason through either task.

The correct response would be that there is proportionately more calcium chloride in each jar because molarity is a ratio and as one variable increases the other must if they have the same label. The SVDM task has the obvious change in the number before M on the labels all while the volume remains constant.

Three major categories of students surfaced with students who previously were successful in the SVDM task and unsuccessful in the DVSM task:

i. Student had an “M is moles” mental model and believed that the number of moles stayed the same and the amount of solvent changed;

ii. Student switched from an “M is moles” mental model to a belief that the size of the molecules changed to make up for the volume in the DVSM task

iii. Student attributed the number in front of M to something other than the amount of calcium chloride in the solution; and

iv. Student was unable to reason through the task with any method.

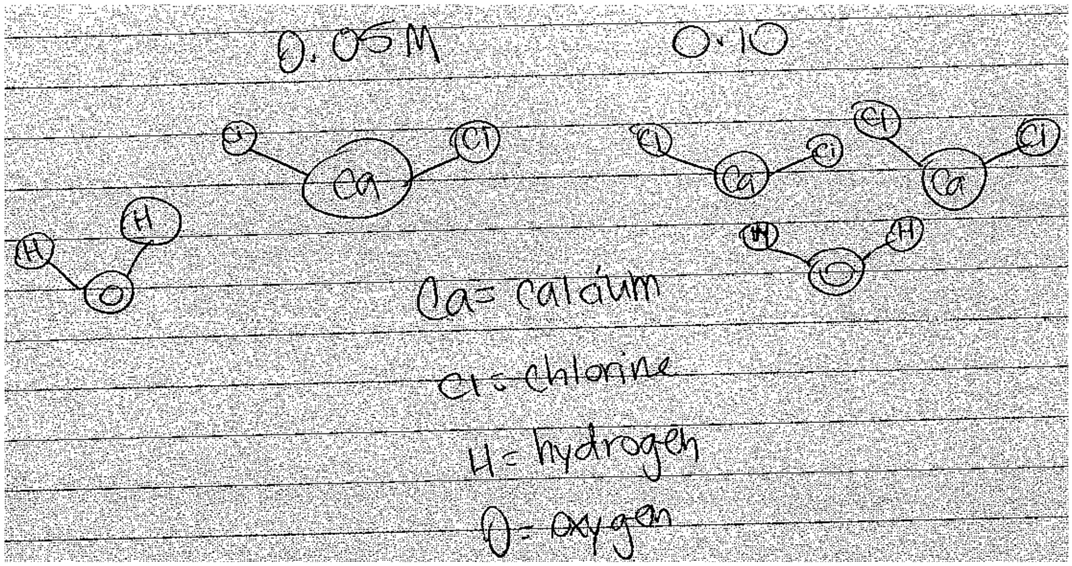
These categories share an extensive view of molarity and the difference in these categories ultimately comes down to one factor: what the student interpreted the M to mean. Students in category (i) believed that M stood for moles and therefore reasoned with that mental model throughout the tasks. In category (ii), students held the “M is moles” belief for the SVDM task and changed their mental model to involve size and spacing of molecules in the DVSM task. Students in category (iii) believed that M represents something other than the amount of calcium chloride in solution, such as acidity or basicity or even possibly another element. One student belongs to category (iv), where she did not know what M meant and was therefore unable to reason through the tasks. These categories will be described at length with student examples below.

i. **Student has an “M is moles” mental model and believes that the number of moles stay the same and the amount of solvent changes**

More than half of the students (13 students) held the belief that M stood for moles and used it as their mental model for reasoning through the tasks consistently throughout the interview unchanged in both SVDM and DVSM tasks. This has been termed an “M is moles” mental model in this dissertation. An interpretation of M as moles indicates that students view M as an extensive quantity. Even though the volume changed, such students didn’t use water in thinking about the concentration of calcium chloride that was present. Students who held this mental model were able to reason through the SVDM task due to the direct proportional relationship between amount of substances and molarity. They then encountered difficulty on the DVSM task when a volume change was introduced. The following students belong in this category: 9-2, 12-1, 11-1, 3-4, 7-4, 23-4, 15-1, 2-1, 8-4, 13-4, 19-2, 20-2, and 21-1. Representative examples are included within this section. Additional student responses can be found in Appendix B.

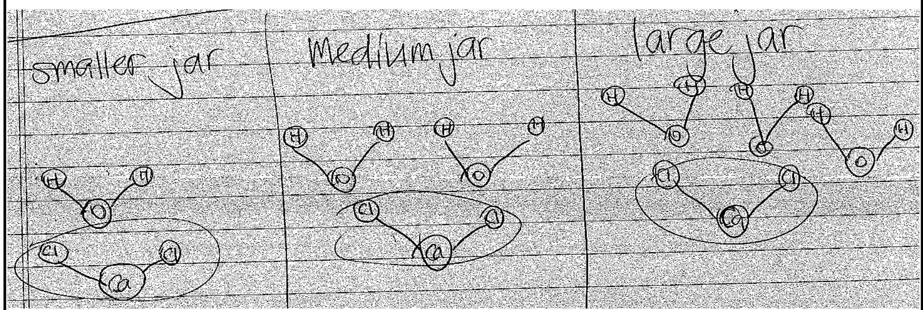
An example of this type of belief is shown with Student 12-1’s transcript and drawings. As shown in Figure 40, she drew a molecule drawing of calcium chloride (non-ionic) and water with the same number of water molecules in the 0.10M and 0.05M jars and different numbers of calcium chloride molecules. The student indicated in the interview that the M stood for “moles”. She drew twice as many calcium chlorides, likely because of the nature of the numbers involved in the task.

Figure 40: Student 12-1's drawing for SVDM task



In the DVSM task, Student 12-1 continued with this mental model and drew three jars with the same number of calcium chloride molecules and varying amounts of water in each jar (Figure 41). The student referred to the jars being "more concentrated then you'll have more of a compound then like smaller amount of solution". She used her experience with concentration to support her incorrect model by saying that the 0.15M with the least volume was more concentrated than the 0.15M with the largest volume. This was a common response that the smallest volume would be the most concentrated and this is consistent with an "M is moles" mental model. If a student believes that M stands for moles, then each jar would have the same amount of calcium chloride with less water in the 100mL than the 200mL jar thus making the smaller volume the most concentrated of the three jars. In this case, her understanding of "concentrated" was reasonable but not true in this situation because the amount of calcium chloride increased as the amount of water increased.

Figure 41: Student 12-1's drawing for DVSM task

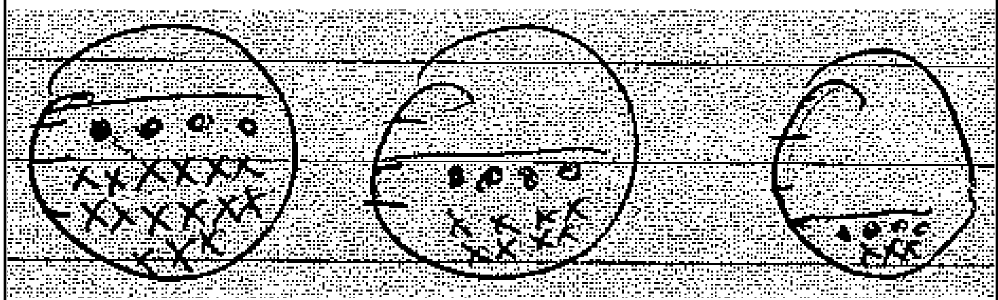


Similarly, Student 11-1's drawing for DVSM was akin to Student 12-1's but instead of water he drew X's to represent the "something else" in the jar. He held the belief that M was moles and his drawing reflected that as he held the amount of calcium chloride (dark circles) constant as the "something else" X amount changed (Figure 42). He earlier stated in the SVDM task that the different number in front of M could be how "potent" it was "how much there is" of calcium chloride added to water. In his discussion of, which would have the most and, which would have the least, an understanding that decimals were less than zero was revealed:

> [00:47:14.08] **Student 11-1:** Just how much of calcium chloride there is. You probably mix more calcium chloride in one these and then just added water for the rest I guess. I'm not sure
> [00:47:25.11] **Researcher:** Ok. So let's work along that one. Um, if that were the case, which one would have the most amount of calcium chloride and, which one would have the least amount of calcium chloride in it?
> [00:47:39.01] **Student 11-1:** Uh this one.
> [00:47:40.09] **Researcher:** This one? Now that has the most or the least?
> [00:47:44.02] **Student 11-1:** the zero Should have the most cause it's the greater number.

He believed that the 0M jar had more calcium chloride because it was a "greater number". Had the questioning stopped at "Do these all have the same amount of calcium chloride or do they have different amounts," his response would've been scored as correct. However, upon further probing, it was found that he held the understanding that decimals were less than zero.

Figure 42: Student 11-1's drawing for DVSM

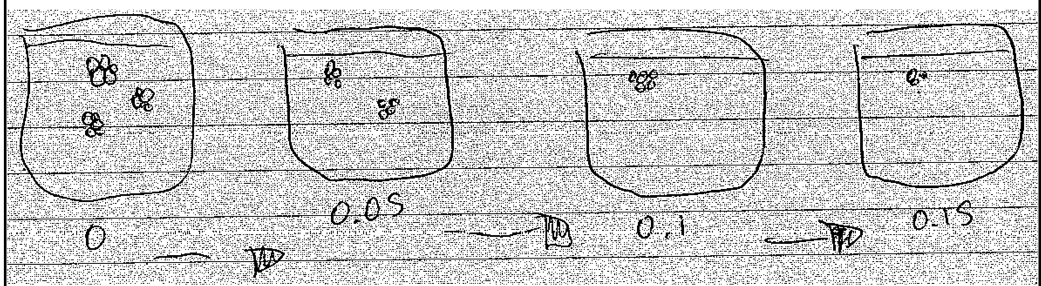


The belief that decimals were less than zero was a belief that was held by Students 3-4, 5-3, and 7-4 held this belief. Student 3-4 indicated that 0.15M would have the least amount of calcium chloride because "it's more more towards like positive side I guess you could say. Cause this is less than zero so they're, there's obviously like less. I don't know I keep right now like going back and forth going back and forth cause I mean these are positive numbers but then again they're like less, less of zero. Cause they're like points like decimal points." Student 3-4's drawing is shown in Figure 43. This student recognized that water was part of molarity after the researcher inadvertently prompted him in the SVDM task by saying:

> [00:48:35.07] **Researcher:** Well if I just used calcium chloride added water to make all three of these, what's the difference between all three?

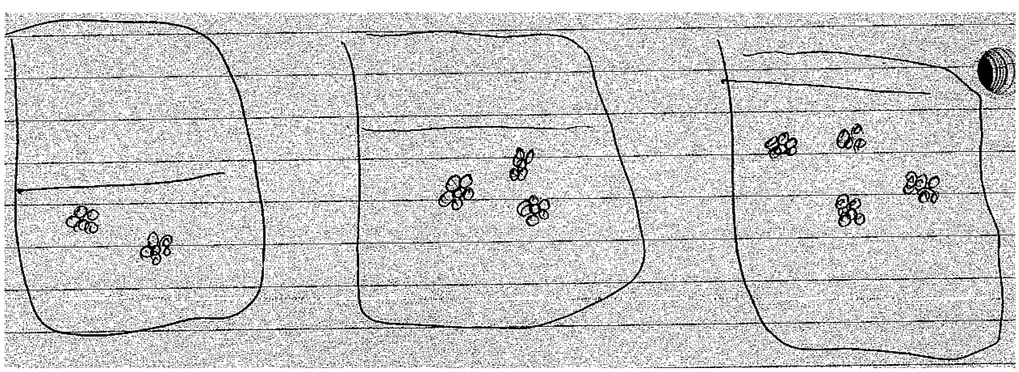
Student 5-3 also suggested that 0.15M had the least amount of calcium chloride stating, "Least is the point fifteen M then point one, point zero five, zero." Student 7-3 revealed this type of understanding while discussing her drawing: "Ok if this was point ten, point oh five then point ten. Maybe the one with point oh five would have more and then the one with point ten if it was being diluted would probably have like less." These understandings that decimals were less than zero were only revealed within the SVDM task because the decimal was a salient feature of the task due to its variation among the jars.

Figure 43: Student 3-4's drawing for SVDM



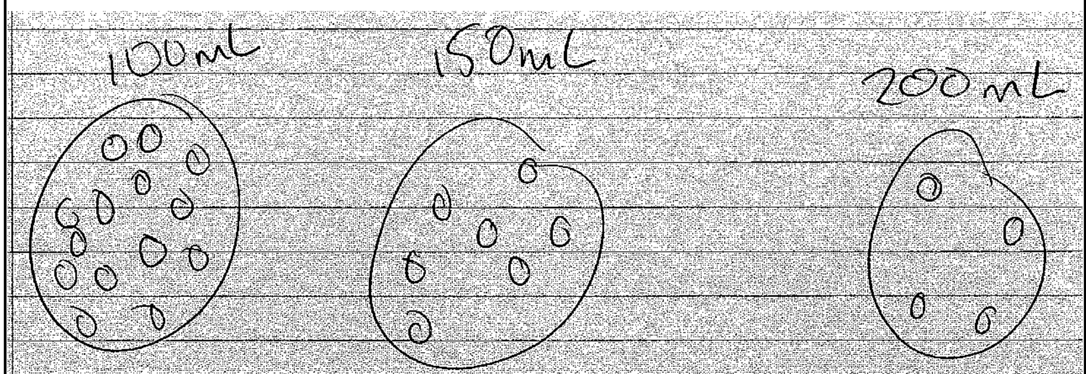
In the DVSM task, while the molarity was a decimal, it was not changing and therefore this understanding did not emerge. In fact, Student 3-4 had a different mental model for the DVSM task that involved personal experience with "coke" and a correct reasoning that with a larger volume there had to be more substance for it to have the same concentration. His drawing can be seen in Figure 44. He utilized the direct proportion between moles and liters.

Figure 44: Student 3-4's drawing for DVSM



Student 15-1 held the view that the smaller volume in the DVSM task would have a higher concentration. Her drawing is shown in Figure 45 for the DVSM task. The larger circle indicated a "zoomed-in" picture. She drew fewer smaller circles in the 100mL view than in the 200mL view indicating "there aren't more, there's just, they're more squished together." Like the other students in this category, she indicated in both of the task that M stood for moles. In the SVDM task, she noted that there was the same amount of water with differing amounts of calcium chloride and in the DVSM task she made no mention of water and just spoke of concentration.

Figure 45: Student 15-1's drawing for DVSM task



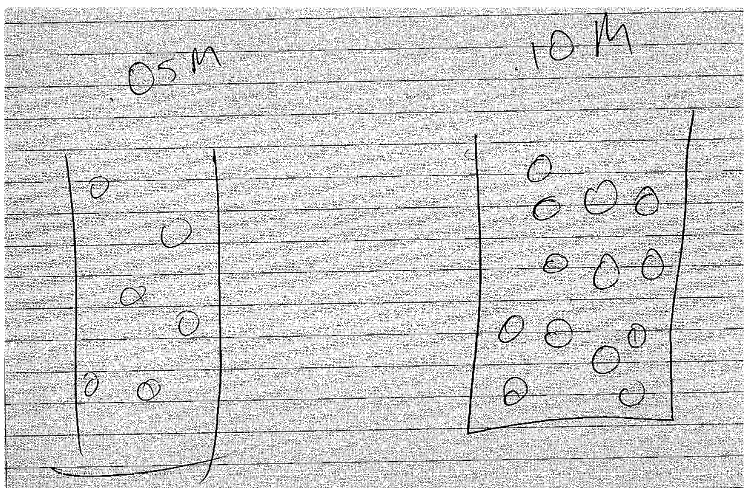
She initially did not produce a drawing when she was presented with the SVDM task until after the DVSM task when she was prompted to draw a similar drawing for the SVDM task. She then drew a proportional increase in circles from 0.05M to 0.10M to 0.15M for the SVDM task and she correctly stated:

> [01:14:50.07] **Student 15-1:** It doesn't mean like how many you have in each. It's just saying that there's more, there are more together like. How do I put that? Since that's, since there's more CaCl2s while it would go up there's just more of them. It keeps adding. It doesn't mean that there's two in zero point five. Or point zero five.

Her view of M as an extensive quantity allowed her to reason with proportions in the SVDM task but not with M itself in the DVSM task. This is interesting because she applied direct proportional reasoning to a chemistry task and because she did not draw a picture for SVDM until after she encountered the DVSM task. This is further evidence that the SVDM task is not an assessment of molarity as an intensive quantity.

As shown in Figure 46, Student 9-2 was able to reason through an amount change in the SVDM task and shows an increase in the amount of circles drawn. Each circle represents calcium chloride and the student referred to the jars being "they are like about the same milliliters" with "this has more moles than this one" and that "there's probably something else that's taking up space in here." The student noted that M stood for "moles".

Figure 46: Student 9-2 drawing for SVDM



He describes his drawing through a discussion with the researcher as follows:

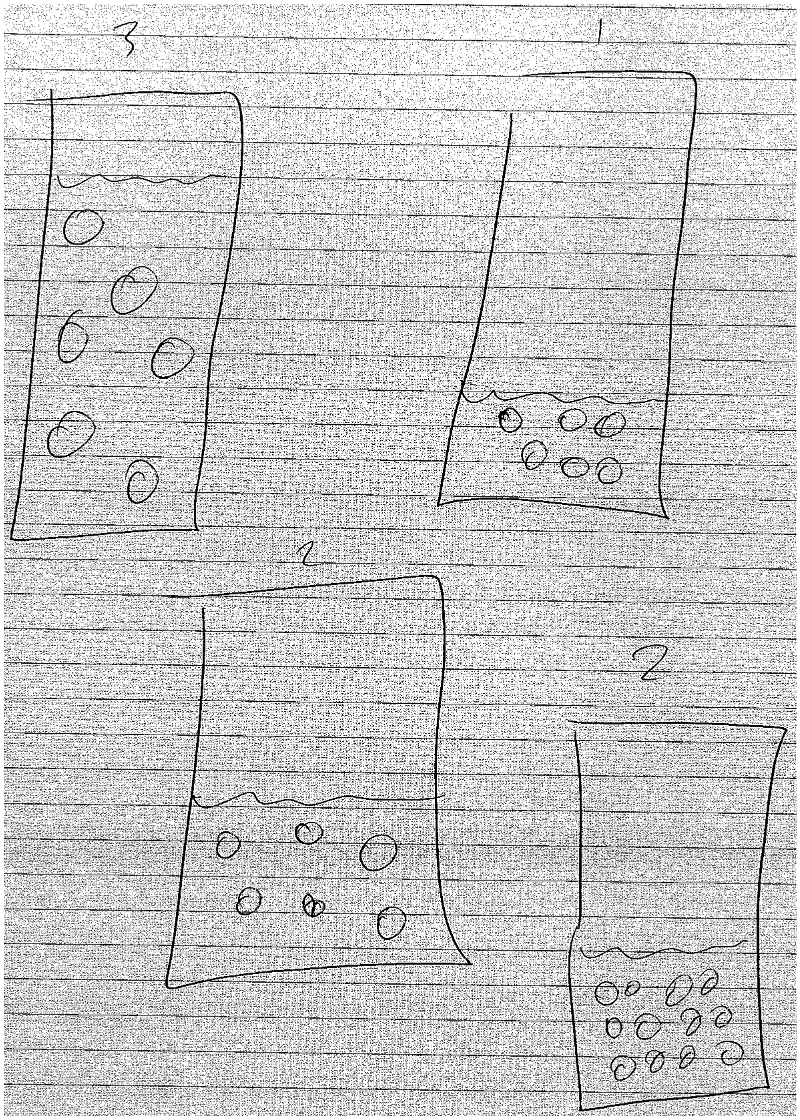
[00:51:39.07] **Researcher:** So this is calcium chloride liquid?
[00:51:41.02] **Student 9-2:** uh huh.
[00:51:43.26] **Researcher:** So there's nothing else in there. Just calcium chloride.
[00:51:46.22] **Student 9-2:** No, there's something else, I'm guessing.
[00:51:47.18] **Researcher:** Like what?
[00:51:49.07] **Student 9-2:** I'm guessing, like water or something. Could be a different solution.
[00:51:55.21] **Researcher:** Ok.You said water. What made you think water?
[00:51:59.06] **Student 9-2:** Just an example. Like it could be mixed with something else, right?
[00:52:01.20] **Researcher:** ok.
[00:52:04.26] **Student 9-2:** Cause they are like about the same milliliters. Yeah, they are about the same and say for example. This has more moles than this one. There's probably something else that's taking up space in here.

Student 9-2 did not hold the misconception that calcium chloride was a liquid and indicated that water or something else was there to "take up the space". He did not indicate that the water played any part in the molarity of the solution and just that it took up space while the amount of moles changed. This "M is moles" confusion is further compounded itself in the DVSM task when the student is presented with a change in volume, but not in molarity.

Student 9-2 continued to use this "M is moles" reasoning and the belief that something else took up space to validate an incorrect reasoning that the moles remain constant within the jars and that only the water changes in the DVSM task. What is unique with Student 9-2 is that he wavered between two competing models in his mind, drawing examples for both. As shown in Figure 47, Student 9-2 drew an example where all three jars contain the same number of moles of calcium chloride, represented by the circles as well as a drawing that the second jar is proportionately more calcium chloride and water than the first jar. The two beakers at the top of the figure labeled 1 and 3 represent the 100 mL and 200 mL beakers respectively. The beaker on the bottom left of the figure represents the 150mL beaker for the "M is moles" mental model. The student's confusion can be seen in the following exchange between the researcher and the student.

> [01:05:47.28] **Student 9-2**: So, that one and that one. Just say they are all fifteen moles. This is the water. This is the water. That's the water. I'm guessing because there's fifteen moles in there, um, let's say there's, hm. Now that I think about it I'm second guessing myself. I'm not really sure how. I'm sure if there's actually just fifteen moles, like fifteen moles equals six calcium chloride. Just an example. There's a six in here, six in here, six in here, or if it's just a concentration.
> [01:06:45.26] **Researcher:** What do you mean by concentration?
> [01:06:48.13] **Student 9-2:** Like say if you are just supposed to just like, scoop up like, maybe a teaspoon of the solution, you would find the same amount of calcium chloride in each teaspoon. So I'm not sure actually now.

Figure 47: Student 9-2 drawing for the DVSM task



Student 9-2 used the "M is moles" model initially until the researcher asked the student to clarify his definition of concentration. The student then initiated a prior experience with concentration that was contradictory to the mental model he currently held[2].

[2] This "teaspoon" answer was used among several students and it involved different types of solutions, including pool water and Coca-Cola®. This type of reasoning was all similar in that a teaspoon of the solution would have the same concentration as a bucket of the solution. Students are likely to have had an experience with concentration in this manner.

In the case of Student 9-2, he had this prior knowledge activated and it clashed with his mental model. He then began to step out both models at the researcher's suggestion. The student stated that "this would be proportions since there's two-thirds. I'd just multiply by three because there's like a third more of solution in this one than that one. I'd multiply six times two." As shown in Figure 47, the student drew a second beaker at the bottom right of the page to represent the proportion model with twelve calcium chloride molecules and indicated that the third [beaker] would have "eighteen [circles]". When asked, which train of thought he felt best described the jars sitting in front of him, he said:

> [01:08:58.27] **Student 9-2:** I'm leaning towards, um, I'm leaning towards the first one.
> [01:09:07.04] **Researcher**: The first one where there's six molecules in each one?
> [01:09:09.07] **Student** 9-2: yea
> [01:09:09.07] **Researcher**: Ok, why?
> [01:09:14.07] **Student 9-2:** Cause, like, to find moles. it's something about molecular, I mean the atomic mass. So um. Okay, actually I don't know anymore. I think I'm confused. I don't know. It's either or for me now.
> [01:09:44.03] **Researcher**: Ok, where are you getting confused?
> [01:09:46.15] **Student 9-2**: Cause if I knew how to find atomic mass, I mean the moles then I would know how to find this problem. like, which one's correct. I just don't know what to do anymore.

His belief that M stood for moles and that water was only present to take up space both combine to an understanding that seems plausible to the student but does not mesh with life experience using the Coca-Cola® example.

Students in this category held an extensive view of molarity due to an "M is moles" mental model. This mental model was incompatible with reasoning about molarity as an intensive quantity, which will be further evidenced by the next category of students.

ii. **<u>Student switches from an "M is moles" mental model to a belief that the size of the molecules change to make up for the volume</u>**

Three students changed their mental model from "M is moles" in the SVDM task to a different extensive mental model after seeing a volume change in the DVSM task: Students 16-2, 22-3, 24-3. Student 6-3 loosely belongs in this category based on his description of the SVDM task. His responses can be found in Appendix B.

The change that occurred was likely because of the salient features of the task: the number stayed the same while the volume changed. This in turn made the students focus on the volume and they then created a reason why the volume could increase while the amount of an extensive quantity (moles) remained the same because it didn't fit their original model. This included students ascribing the M to the extensive property of strength or size of molecules. An "M is moles" mental model involves counting of the extensive quantity and this type of mental model involves counting and size. Size and strength are grouped together as similar understandings because of student descriptions that tend to involve both during the explanation.

To a chemist, the strength of a chemical is an intensive quantity that can be represented by molarity. However, strength is an extensive quantity to these students, as shown in their drawings to represent the tasks. Therefore, a student who believed that the strength is an extensive quantity and who made no mention of water in the solution has to reason through a volume change using only calcium chloride. Because the number before M didn't change, all they are left with to change was the size of the molecule or an indeterminate way of increasing the strength. Students across the categories discussed the "spacing out" or packing of the calcium chloride in the larger volumes, especially in cases involving an "M is moles" type of understanding. This type of response is indicative of an extensive view of molarity in that the student does not account for water in the calculation of molarity.

A student example of this type of understanding was found in Student 16-2's interview. In the SVDM task, shown in Figure 48, the student drew double the amount of circles in the 0.10M jar than the 0.05M jar with the same level water line in the SVDM task. This initially seems like she had an intensive view of molarity because she attended to both water and calcium chloride. In her verbal descriptions, she indicated the following:

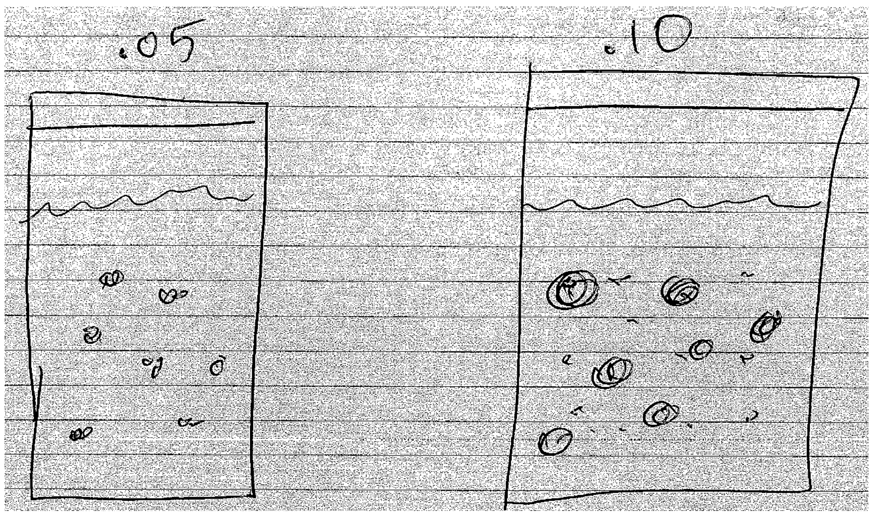
> [01:22:40.06] **Student 16-2:** Ok, I'm just saying that this is just water and this is water. And there's point zero five of that calcium chlorine in it so that's how much of it's in the water. So it's just point zero five and point ten is double the amount of point zero five. So instead of just double the amount.

The student was later faced with a volume change in the DVSM task and changed her mental model as evidenced by the following dialogue between the researcher and the student:

> [01:31:34.23] **Researcher:** Ok, so if you were to draw all three of these jars with the zoomed in picture like you did before, could you do that for me?
> [01:31:43.13] **Student 16-2:** It's gonna look different now.
> [01:31:43.09] **Researcher:** Different now how? You mean you changed your idea of how it looks.
> [01:31:47.26] **Student 16-2:** Yeah. Well, uh, point fifteen. This is point fifteen. Well,
> [01:32:00.14] **Researcher:** I need all three of them.
> [01:32:00.14] **Student 16-2:** Oh. three, ok. Ok, so you have well if you have them going on the quantity of the chemical, they'll all look the same. One, two, one, two, three, four, one, two, three, four. So just say four. But I'm going with the quantity anymore. I'm going with the strength. So with strength, I'm just gonna make them bigger, just say. That represents the strength and going with the previous drawing, the only thing I would change. If you go with this drawing now, I would change that, it means that it doubled but it doesn't mean that doubled in the quantity. It doubled in the strength. So these would stay this size, the seven and there should be only seven here. They'd just be a lot bigger cause of the strength.
> [01:33:02.28] **Researcher:** So there'd still be seven in the second one
> [01:33:06.20] **Student 16-2:** Yeah but just the strength of it's stronger.
> [01:33:10.19] **Researcher:** Ok
> [01:33:11.17] **Student 16-2:** Yeah, so.

> [01:33:14.02] **Researcher:** Where are the water, where's the water level in your point one five drawings?
> [01:33:19.28] **Student 16-2:** Oh, yeah. Well, they are all different. Oops. This one should be like down here. So the water level, this is water. This is water level. This is the water level. Just say this one's filled up. Ok. They are all different.

Figure 48: Student 16-2's drawing for the SVDM task

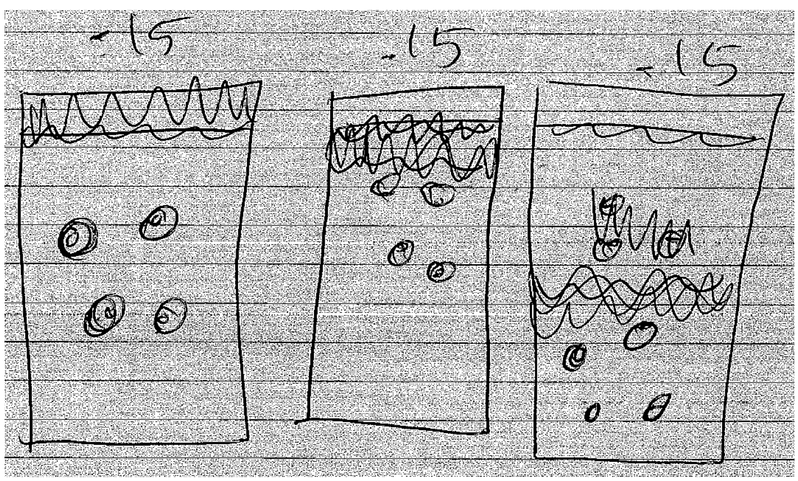


The student went back and drew larger circles in Figure 48 to represent the strength increasing. The researcher questioned the student about this in the following exchange:

> [01:33:46.06] **Researcher:** Ok, so do molecules look different if they are stronger?
> [01:33:52.05] **Student 16-2:** Do molecules look different. No. Do they? No I don't think so.
> [01:33:59.28] **Researcher:** Ok, so when you draw it bigger
> [01:34:00.19] **Student 16-2:** I'm just trying to get it like, a visual of it being. When I think of stronger I think of just like bigger, so, but it doesn't mean it's bigger.

Student 16-2 noted that the molecules' size didn't change but rather the change in the space used for the molecules was a representation of strength. Because she interpreted the molarity as strength and all were all the same in the DVSM task, she drew the same sized circles to represent strength with different water levels (Figure 49). This drawing, combined with her utterance that "they all have the same" amount of calcium chloride in each jar, shows a inconsistent understanding of concentration as an intensive property. While this student was able to say that the size of the molecules that she drew was merely a representational tool, she still used it as evidence to say that the calcium chloride amounts were the same, just stronger. When asked how they got stronger she replied, “I don’t know how they got stronger but they're stronger because it’s point ten instead of point five.” This incomplete understanding was revealed only after the volume was changed. Although she only used size to represent strength and did not believe that the size of the molecules changed, Student 16-2 still belongs within this category because she did use size in general. Had the interview not probed the student’s understanding further, her final drawing would seemingly have involved size as equal to strength.

Figure 49: Student 16-2’s DVSM task drawing



Student 22-3 also uses the size of the molecules to reason through the tasks but in a different way. In her case, size actually represents the relative size of the calcium chloride. Figure 50 shows her drawing for the SVDM task. When asked to describe her drawing, she said the following:

[01:12:20.17] **Student 22-3:** I think they do have the same amount cause each amount, the amount of liquid in each jar is the same. I just think the size of the atoms might be different because they are more closely packed in the point zero.

This student indicated that the M stood for moles and seemed to believe that calcium chloride was a liquid. The student did not mention water at all in her discussion of her drawing and this led her to reason through a number change without a volume change in the SVDM task, but not in the DVSM task. Similar to Student 16-2, Student 22-3 used size to discuss concentration but differed in that she actually believed the size of the molecules really did change.

Figure 50: Student 22-3's SVDM drawing

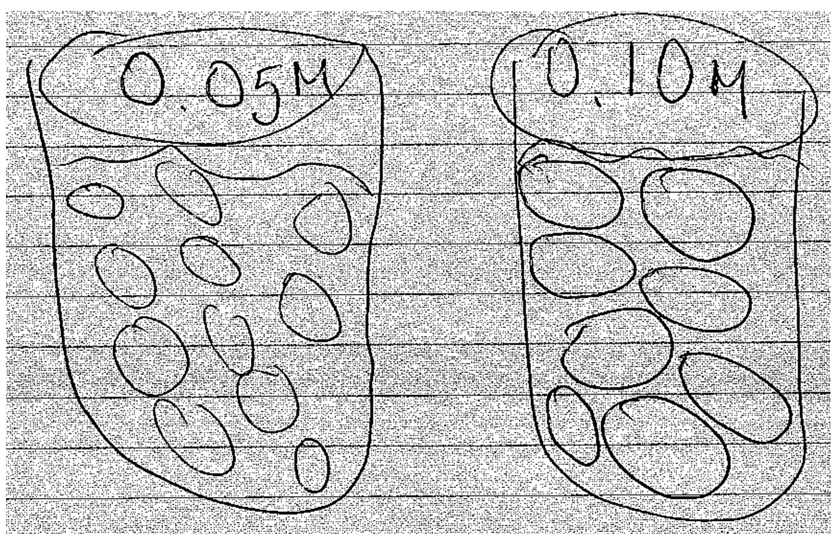


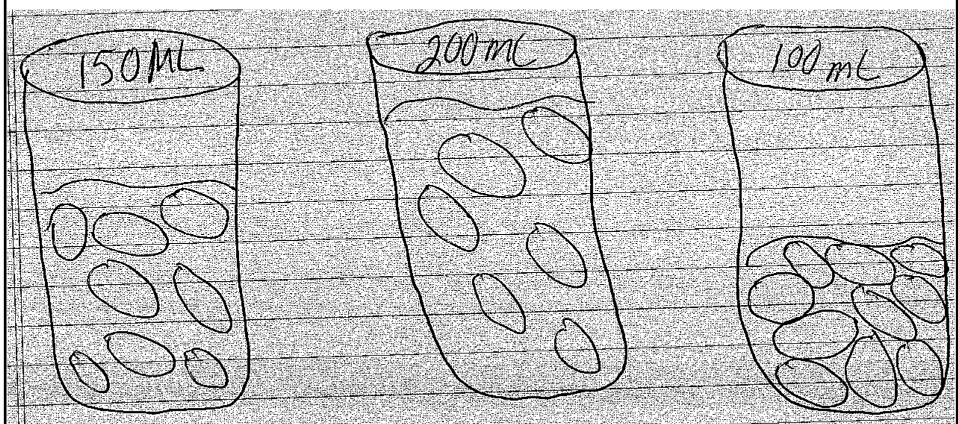
Figure 51 shows Student 22-3's drawing for the DVSM task and she discussed spacing again in her description of her drawing:

[01:27:19.29] **Student 22-3:** Yeah so in the one hundred fifty milliliter one the atoms of the calcium chloride are not tightly packed together. They are somewhat some leeway for them to pass by. There's some, it's loosely packed.
[01:27:38.20] **Researcher:** Ok
[01:27:40.08] **Student 22-3:** And the two hundred milliliters they're even more loosely packed because there's more space for them to move about.

[01:27:49.10] **Researcher:** Ok
[01:27:49.10] **Student 22-3:** And then the one hundred milliliters they are tightly packed together because there's little, smaller amount for them to condense into.
[01:27:57.29] **Researcher:** Ok so are there the same number of atoms in all of them?
[01:28:02.19] **Student 22-3:** Yes
[01:28:02.19] **Researcher:** Ok and are they all the same size? Because last time you had them as different sizes.
[01:28:08.24] **Student 22-3:** Um I think its, it can be either both actually because the amount of liquid vary in each jar. One hundred fifty, two hundred and one hundred I think the size and the number of atoms in each jar is different.
[01:28:32.24] **Researcher:** OK so, which jar has the most atoms?
[01:28:37.03] **Student 22-3:** Um probably the one hundred milliliter because they would have to be smaller to pack into that smaller amount of liquid.
[01:28:49.10] **Researcher:** OK. so does that one have more calcium chloride than the others?
[01:28:58.00] **Student 22-3:** No It's just that these ones have bigger and more space to move around from being broken down so they can fit in a smaller area.
[01:29:08.00] **Researcher:** OK so it has more atoms because it's got a smaller area?
[01:29:12.04] **Student 22-3:** Yeah.

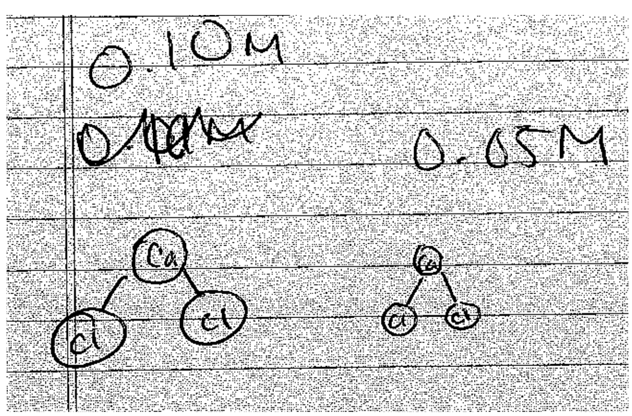
In her description, she described both "packing" and size of the molecules with the notion that a smaller volume would have more molecules because they would be packed closer. She also did not associate number of molecules with amount of moles and indicates that all jars have the same amount of calcium chloride but different amounts of molecules. Because she didn't mention water and viewed molarity as an extensive property she combined her idea of concentration from the SVDM task with spacing to account for the change in volume but not in the number before M.

Figure 51: Student 22-3's DVSM drawing



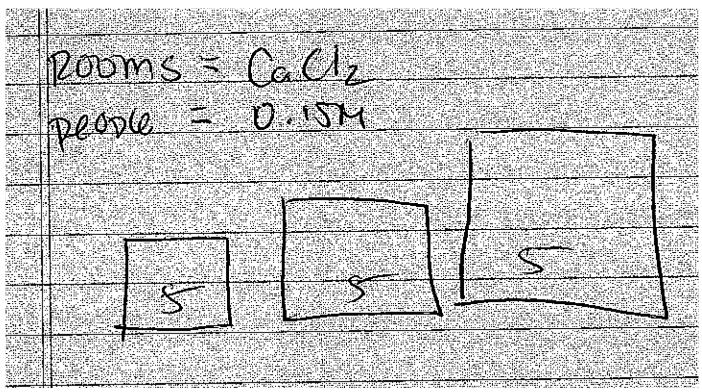
Student 24-3 referred to size in the SVDM task indicating that "I would say it's it would be like same picture but in smaller size." Though she referred to M as molarity, she did not know what it meant. To address the change in number without a change in volume, she focused on the salient feature of the task (the change in the number in front of M) to describe her mental model. The number changed so therefore the change must relate to the substance because her view of molarity involved volume but not as a ratio. Her drawing can be seen in Figure 52. She literally drew a larger molecule for calcium chloride in the 0.10M than her molecule drawn for 0.05M.

Figure 52: Student 24-3's drawing for SVDM task



In the DVSM task, she indicated that the jar with the smallest volume would have the least amount of calcium chloride, which is a correct statement. Her description no longer involved size. However, as her description progressed, she switched to an “M is moles” type of description using an example with people in a classroom but that each person is the molarity. Her drawing for this description is shown in Figure 53.

Figure 53: Student 24-3’s description of the DVSM task



She described this drawing through a discussion with the researcher as follows:

> [00:57:23.25] **Student 24-3:** Because the chloride is only the like I could say the molarity is something different than oh my gosh. I don't know how to explain it. To me it’s like you can have like say you have a big room right and you have a small room. The small room fit’s like five people and you go into a bigger room you're still going to have. Cause the molarity is the five people.
> [00:58:25.08] **Researcher:** OK so the molarity is the five people ok and then
> [00:58:31.13] **Student 24-3:** And the calcium is just the space like the like the room, which is the liquid. five people no matter how big or small the space is.
> [00:58:51.29] **Student 24-3:** yeah ok room equals calcium chloride only. The people it would equal just the molarity zero zero point fifteen molarity. You get me? like I don't understand like so this would be like, I'm just gonna draw a room
> [00:59:17.12] **Researcher:** Ok so this is the small room
> [00:59:17.28] **Student 24-3:** So this is medium room and this is the big room. So no matter how big it is five people only are gonna fit. You get me? Like I don't

know if I explained myself. It’s how much is in there no matter like in there even though it has different volumes in here
[00:59:39.17] **Researcher:** Ok and so ok so the rooms themselves, which is the volume is calcium chloride. And the
[00:59:52.22] **Student 24-3:** The people are the molarity in it like the molarity
[00:59:55.28] **Researcher:** ok so what's molarity? Is molarity something else that could be in the jar besides the calcium chloride?
[01:00:01.24] **Student 24-3:** yes. I would say

This description above suggests that the student viewed molarity as remaining the same across the jars but her example was a mix between constant molarity and constant moles. Student 24-3 seemed to separate the concept of molarity from substance. She discussed molarity as the people in the room and the size of the room as the calcium chloride. This description revealed an extensive view of M. She indicated that the volume of the room was like the calcium chloride, indicating that she believed that calcium chloride was a liquid with no water in the model.

Students in this category held an “M is moles” mental model that changed to involve size in the DVSM task to account for a volume change. This is likely due to the salient features of the task. The number in front of M did not change but the volume did. If a student didn’t factor in water as a component, the difference in volume was attributed in some way to the substance.

iii. **Student attributes the number in front of M to something other than the amount of calcium chloride in the solution**

Two students held beliefs that the letter M represented another element. These elements ranged from magnesium (correct abbreviation is Mg) to mercury (correct abbreviation is Hg) to manganese (correct abbreviation is Mn) to an unknown element with the abbreviation M. One student believed that M stood for the acidity or basicity. Students who believed that M was an element (Code 02-16) attributed the number in front of M to the element

M and not calcium chloride. Interestingly, students with this belief still had ratio understandings of what was occurring in the solution, just between the wrong substances. For example, in the SVDM task, 0.05M and 0.10M were interpreted as differing amounts of element M with calcium chloride being held constant. In reality, calcium chloride is varied with constant amounts of water. In the DVSM task, students who held this belief then attributed the number in front of M to the element M, indicating that the amount of element M was constant with the amount of calcium chloride changing to account for the volume change. In reality, the amount of calcium chloride increases proportionately with the increase in the volume and amount of water. This ratio understanding begins to fall apart as the volume changes making them more similar to students in Statement 2. Students in this category ultimately create a more complicated mixture by introducing M as another compound in solution.

Three students referenced M as being an element. Student 7-4 momentarily discussed M as being mercury in the SVDM task but then stated "I don't know but I have a feeling it's more of moles than it is mercury." She changed her response before moving forward in the task. The other two students, Student 5-3 and Student 17-3, both continued with this mental model.

When asked what he thought M meant, Student 5-3 responded:

> [00:51:14.28] **Student 5-3:** I think the M is just for, oh, now I don't know. Now I know it's not for the amount. Cause all the decimals are different and then but the amount is the same. So M isn't used for the amount. I'm assuming the M is another element, which is not on this paper. But that along with the CaCl two.
> [00:51:44.15] **Researcher:** Ok um so what's different between all of these?
> [00:51:48.12] **Student 5-3:** Um the amount of M is different in each one.
> [00:52:49.11] **Student 5-3:** Um well there's gonna be more of the CaCl two in the with the point oh five M and less in the point one.
> [00:53:11.09] **Researcher:** Ok so you said you think M is another element? How does that play into those two?

> [00:53:18.18] **Student 5-3:** Um that's not, hm, it just hm let me think how to say it. It's like I don't know if the M is necessarily an element but it's like another, it's what's added to the CaCl two so.
> [00:53:34.20] **Researcher:** What do you mean by that? Like if I, if you were making this what do you mean by adding M.
> [00:53:43.24] **Student 5-3:** Like if you add, if you have water and vinegar in the water is the M and the vinegar is CaCl two like adding however much.

While this student's view changes into a discussion involving water, it is important to note that she still thought that the number before M was an extensive quantity not the intensive quantity involving calcium chloride. This is further evidenced by her discussion about the DVSM task:

> [01:03:26.18] **Student 5-3:** Ok so we'll go, what are the differences? We'll go with like a third. So like there's like a third no I don't like that. No a third, two thirds and three thirds of each how it um no I don't like that. Um hm like however much is like if the CaCl is two and the M is one and then for this one it took a little more.
> [01:04:25.21] **Researcher:** Of both?
> [01:04:28.10] **Student 5-3:** Yeah because it'd be the same amount like this that's about doubled so if this used hm. If this had like it used one M and two CaCl twos in this one you used two M and four CaCl twos.

She held onto this belief as the volume changed and in this case she attributed the number before M to the amount of M in solution. Therefore, she believed that the calcium chloride varied while the amount of M remained the same across all three jars. This mental model mirrored responses found in students who believed that M was moles, but with the constant amount being M in this case.

Student 17-3 indicates that she believes M is an element when presented with the four jars in the SVDM task:

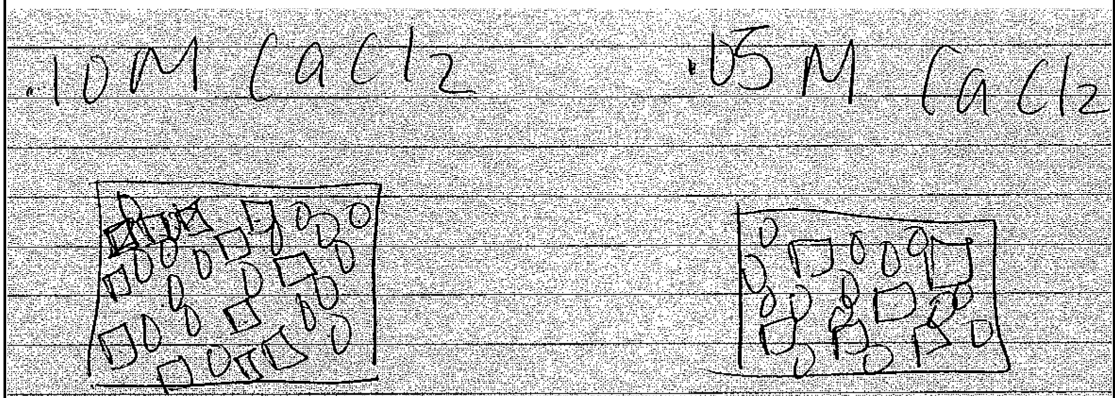
> [00:52:58.05] **Student 17-3:** Like two elements are like put together.
> [00:53:02.01] **Researcher:** Okay what are the two elements in here.
> [00:53:04.27] **Student 17-3:** Wait I think there's three
> [00:53:06.09] **Researcher:** Okay what are the three
> [00:53:07.02] **Student 17-3:** M calcium and isn't CL chlorine. Or chloride either/or those

When asked to draw zoomed in pictures of the two jars, she described her drawing shown in Figure 54 as "I basically used what was it twelve squares to represent the M so then I just put six just to represent half of it." She is attributing the doubling to the amount of M and keeping the amount of calcium chloride constant. When faced with a difference in volume, she begins to get confused because it becomes clear that she was associating M also with volume.

> [01:06:05.19] **Student 17-3:** Because if they have the same amount of the solution wouldn't they all be the same like they would they would all have the same amount in the jar.
> [01:06:15.21] **Researcher:** I'm not. I can't answer that so let's work through what you just said if they all had the same amount. Wouldn't they all have the same amount of solution in the jar
> [01:06:24.28] **Student 17-3:** Yeah
> [01:06:26.24] **Researcher:** So what you think.
> [01:06:30.14] **Student 17-3:** They all they have a different amount of the calcium chlorine.
> [01:06:34.09] **Researcher:** Okay so you think that they have zero point one five M and then in there there are different amount of calcium chlorine.
> [01:06:43.00] **Student 17-3:** But wouldn't that number be represented then as well then.
> [01:06:46.03] **Researcher:** I'm not sure I can't answer that so.
> [01:06:49.01] **Student 17-3:** I'm confused.

She was faced with a discrepant event in the form of a volume change and an alternate understanding began to surface, one that involved volume. When she couldn't think of how it could have different volumes with the same label, she continued with her prior mental model that attributed the number in front of M to the amount of M. The only other thing that could make up for the volume difference would involve the calcium chloride and she says that its amount changed while the amount of M remained constant. Again, this is similar to the discussion of the DVSM task by an "M is moles" student.

Figure 54: Student 17-3's drawing for SVDM



Student 18-4 held the belief that M stood for the acidity or basicity of the solution. This will be described more in depth in Statement 3. When faced with M, students in category (c) wind up introducing something into solution that isn't there inside the jar.

iv. **Student is unable to reason through the task with any method**

One student was unable to reason through the tasks due to her confusion with the letter M on the label on the jars. She had an extensive view of molarity but in a way that was different from the other students in Theoretical Statement 2. Student 4-2 associated the M with the volume and when asked to draw a picture for SVDM, she drew a larger volume with 0.10M than 0.05M even though the jars in front of her were the same volume. Her drawing is shown in Figure 55. She described her drawing and confusion as follows:

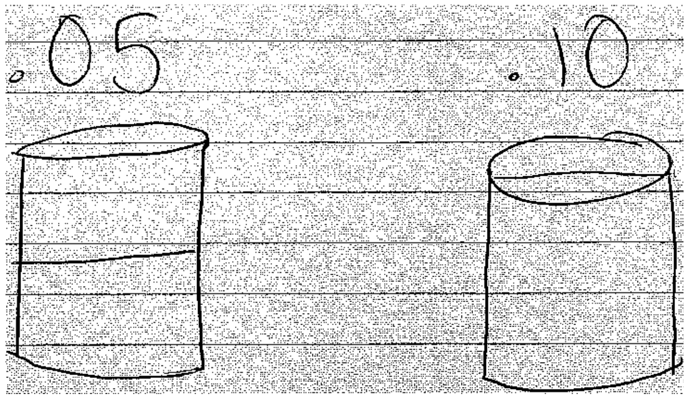
> [00:48:10.28] **Student 4-2:** Oh I am so like point zero five point ten look like this is half of point ten.
> [00:48:25.03] **Researcher:** Okay.
> [00:48:28.02] **Student 4-2:** So if it was volume it would be like, that like half an this one be like full.
> [00:48:42.22] **Researcher:** Okay so but you just told me though that they all four have the same volume.
> [00:48:47.12] **Student 4-2:** Yeah that's why I don't know what that number represents.

The student faced a situation that challenged her understanding of M and the number before it and was unable to reason through it with a new mental model. This confusion continued into the DVSM task where the student was unable to create a drawing at all. She described her confusion as:

> [01:01:02.23] **Student 4-2:** Um I don't know. I guess the number or the M means something different. I have no idea.
> [01:01:14.22] **Researcher:** Okay um I'm just making sure so if any, which bottle has the most calcium and it order they all have the same amount.
> [01:01:35.06] **Student 4-2:** I'm going to say they all have the same amount.
> [01:01:39.17] **Researcher:** Why?
> [01:01:41.09] **Student 4-2:** Because there's not like like there's two chlorine elements there's no like little sub numbers for the calcium.
> [01:01:53.27] **Researcher:** Okay so the label has the same calcium for each one.
> [01:01:56.18] **Student 4-2:** Yeah.
> [01:01:57.12] **Researcher:** So that they should all be the same.
> [01:01:57.12] **Student 4-2:** Yeah I have no idea why they're different volumes.
> [01:02:02.06] **Researcher:** Okay so um if you had a special camera again where you could zoom in and draw um what would these three solutions look like. Would they look the same or with a be different?
> [01:02:16.26] **Student 4-2:** Um I have no idea like well if I knew what the number and like the M represented then I guess I could figure out why they have less and one has more but I don't know how to like write that down to show that they have like different amounts.

She admitted that she didn't know what M meant and she wasn't sure how the three jars all had the same M but different volumes. This was a continuation from her earlier descriptions of M involving volume. Being faced with contradictory evidence did not activate a different mental model for this student. She did however begin to consider that her mental model was potentially wrong.

Figure 55: Student 4-2's drawing for SVDM



An extensive view of molarity as being only related to the volume caused major confusion for this student when she was presented with contradictory evidence.

3. **<u>Theoretical Statement 4: A conceptual understanding of molar mass as it relates to moles enables reasoning with the value in front of M</u>**

Stepping outside of the SVDM and DVSM tasks as well as the SHDC and DHSC tasks, Theoretical Statement 3 examines a skill assessed on the CD as it related to performance on the SVDM task. Their responses to the first question of the CD were analyzed by the axial coders to determine if they had a conceptual understanding of molar mass as it related to moles of an element. This involved students indicating that they knew each element had a different molar mass and therefore required different amounts of grams to create a mole of each substance. Students either got the problem incorrect with no conceptual understanding (responses such as more moles because the number is larger) or correct with conceptual or without conceptual understandings of moles. A student without a conceptual understanding that got the problem correct was considered able to solve the problem algorithmically, but upon further discussion could not articulate why. A student was considered to have a conceptual understanding if he or she was able to articulate that a mole was a different mass for each element and that the molar mass could be used to calculate the moles of a substance. Students 24-3, 11-1, 12-1, 1-3, 10-1, 14-1, 13-4, and 18-4 were coded as having a conceptual understanding of molar mass as it relates to mole.

It was found that if students had a conceptual understanding of the mole, they were able then to reason that a change in the amount of substance had occurred in the SVDM task. This held true unless the student attributed the number in front of M to something other than the amount of calcium chloride. Student 18-4 was able to describe the relationship of mass and the mole at a conceptual level:

> [00:01:38.23] **Student 18-4:** Since like gram is different and one mole, the mole in each gram like like I know that like in chemistry is like every basic element has it own specific type of gram.

But when faced with the substance change in the SVDM task, she indicated that the number before the M was the amount of acidity or basicity in the jar, not the amount of calcium chloride.

> [00:51:55.03] **Student 18-4:** Ok. Like the same amounts is meaning like how much is put into the jar. But like uh um what I'm trying to say is like water is like what I'm comparing to water is like these all have one liter but each one of them has more acid than another.

Therefore, she was unable to recognize that a change in the amount of calcium chloride was present. The same is true for the case of Student 24-3, but for a different reason. As previously discussed, Student 24-3 indicated in her drawing that the number in front of M related to the size of the molecule. Therefore, she was unable to recognize that a change in the amount of substance calcium chloride had occurred.

The fact that students who understood the calculation of moles from molar mass and grams at a conceptual level were able to recognize the substance change for calcium chloride was not surprising as a student with a conceptual understanding of the mole understood one component of molarity (the mole) and how it varied by element. Because they had this deeper understanding of numbers, they were more able to reason through number changes. However, as previously mentioned, the SVDM task can be answered by a student who does not know what molarity means based solely on the multiplier between the jars. Out of these students, only Student 1-3, Student 10-1, and Student 14-1 were able to reason through the DVSM task correctly. Students 11-1, 12-1, 13-4, and 24-3 all held the "M is moles" mental model that didn't account for both variables. Student 18-4 as previously mentioned that M was related to the acidity and therefore held a different unique mental model. If a student only attributed M to one quantity such as moles, they were unable to reason through the volume change correctly because they fail to factor in water to their reasoning.

C. Discussion and Conclusions

1. Conclusions related to Theoretical Statements 2-4

The data presented in this chapter indicated that there were two overall categories of students: those who viewed molarity as an intensive quantity and those who viewed molarity as an extensive quantity. The three students who recognized molarity as an intensive quantity were able to reason through substance changes in both the SVDM and DVSM tasks. This is not surprising as these students attended to both the amount of substance and the volume in their definitions of molarity. A student who viewed molarity as moles per liter would have no difficulty approaching the DVSM task because they would recognize that the amounts of calcium chloride and volume increased proportionately as the volume increased. This is structurally similar to the DHSC task discussed in Chapter IV. Nearly all of the students were able to reason about the concentration of color as an intensive quantity, but only these three students transferred this reasoning to the molarity tasks.

What is more interesting is the examination of the students who did not view molarity as an intensive quantity. Many of these students believed that molarity was an extensive quantity, such as moles. Upon first glance, most of these students noted an amount of substance change in the SVDM task and could be assumed to understand molarity. Their understanding was inadequate to reason about the DVSM task. These students were able to reason qualitatively about the concentration of color in the structurally similar tasks but did not transfer this reasoning to molarity tasks because they did not see M as a concentration. As shown in this chapter, a vast majority of the students interpreted the M to mean moles and therefore adopted an "M is moles" mental model. This mental model allowed them to provide correct responses regarding substance changes within the SVDM task because moles and molarity were in a direct proportion.

The SVDM task proved not to be a good assessment of whether students believed molarity was an intensive quantity. In fact, most students were able to reason correctly about the amount of calcium chloride in the 0.05M and 0.10M jars. The numbers 0.05 and 0.10 are also easily recognized as having a multiplier and thus elicit responses involving doubling. Students were not asked to consider the 0.15M jar, but it is likely that they would've tripled the number of circles from the 0.05M jar. Because of the numbers within the task, students could recognize a multiplier and simply answer that there would be double the amount. Similarly, the student could get this problem correct because of the direct proportion between moles and molarity not because of their understanding of molarity.

It wasn't until the DVSM task that student understandings of molarity became clear. If a student held the "M is moles" mental model, they had two mental pathways they could take when faced with a volume change with the same number on the jar: the size of the molecules made up for the volume change or more water was added to achieve the new volumes. In neither case did they believe that the amount of the substance (calcium chloride) changed as the volume changed. This task allowed for the detection of flaws in reasoning. Had the students only been given the SVDM task, their understandings of molarity would have been deemed sufficient because they were able to determine the amount of calcium chloride in the 0.10M jar as it related to the 0.05M jar.

An interesting finding of the this chapter was addressed in Theoretical Statement 4 that encompassed student performance on Question 1 on the CD that assessed student ability to calculate the number of moles and molar mass given grams. The interesting piece of this finding is the group of students who did not view molarity as an intensive property but were able to calculate the number of moles. One would think that this skill would support an intensive view of molarity but it did not because the students did not view M as a concentration. However, they were able to reason through changes in the number in front of M like the other students in the study. The difference between this group and other groups is the fact that these students have facility in molar mass and mole calculations.

Another interesting outcome from the student data was the finding that several students held the belief that decimal numbers were less than zero. This only presented itself in the SVDM task because it was the salient feature that was visibly changed whereas in the DVSM task the number before remained the same. The students who held this view also held extensive views of molarity so they did not attend to a change in substance for the DVSM task.

2. **Conclusions about proportional reasoning skills in molarity**

In addition to particular conclusions about the Theoretical Statements within this chapter, it is also possible to draw some conclusions pertinent to Theoretical Statement 1 from Chapter IV. Many chemistry teachers suggest that students who cannot reason through molarity problems lack the proportional reasoning skills necessary to complete the problems. However, as shown in Chapter IV and as will also be discussed in Chapter VI, most students did possess proportional reasoning skills. In fact, all of the students were able to recognize the color concentration as an intensive quantity in one or both of the SHDC and DHSC tasks. Only three students represented an extensive quantity and then it was only in one task and not both. Student direct proportion ability will be discussed further in Chapter VI.

It is interesting to compare and contrast student work on the SHDC and DHSC tasks with their performance on the structurally similar SVDM and DVSM tasks. The students were categorized with both types of tasks. There were several subgroups that emerged:

- Student viewed both the SHDC and DHSC tasks as intensive quantities but viewed both the SVDM and DVSM tasks as extensive quantities.
- Student viewed the SHDC task as an intensive quantity but did not view the SVDM task as an intensive quantity

- Student viewed the DHSC task as an intensive quantity but did not view the DVSM as an intensive quantity

The vast majority of the students are described in the first bullet point. For example, Student 23-4 was able to construct cube models representing the color concentration as an intensive quantity but because of her “M is moles” mental model, she fails to reason through the SVDM and DVSM tasks with molarity as an intensive quantity. She did mention water in her descriptions, but did not indicate that water was at all related to the number before M. Therefore, molarity was an extensive property to her. Whereas in the case of the SHDC and DHSC tasks, the salient feature of the task was the color of the blocks ranging from red to white and the student was given red and white blocks to represent the color. Concentration of the color of the paint was more obviously a ratio between two colors. Whereas in the molarity task, the students had to construct this model without the use of physical models and were left on their own to create a model with no constraints.

The salient features of the structurally similar tasks were changes in the length and in color. For the molarity tasks, it was not obvious that the amount of substance is changing inside a clear, colorless solution. The attention remained on the number that changed. In the SVDM task, that number was the molarity, which many students interpreted to mean the moles. In the DVSM task, that number was the volume.

Many students did not attend to water at all in the molarity task. When water was missing from the mental model, the only substance left to fill the volume change was the size of the molecules. When a student did mention water but did not factor it into molarity, the volume change became the salient feature where because “M is moles” and the number is constant, the amount of water must change.

It is interesting to note that the students who held the view that a decimal was less than zero in the SVDM task were able to reason through the SHDC task in the case of a “pure white” or a “pure red” block. These blocks consisting of only one color are similar to the 0M jar in the SVDM task because it had no concentration of calcium chloride and was simply water in a jar. They were unable to reason through the same scenario in the SVDM task likely due to the number because the SHDC task did not have a number associated to the color.

Perhaps the most interesting student interview regarding this comparison is the interview with Student 4-2. Her case was unique because she was the only student unable to reason through either of the molarity tasks (SVDM and DVSM) due to her confusion of molarity as an extensive quantity only related to the volume. In the SVDM task she was unable to reason about the amount of substance changing because she attributed the number before M to a change in the volume. This was so ingrained that she kept this mental model even though the jars that were sitting in front of her had the same volume but different numbers before M. Initially, one could say that she wasn’t able to reason with proportions in these tasks but the researcher would argue that this is not the case. Rather, as shown in Chapter IV, she was able to reason with direct proportional relationships in the SHDC and DHSC tasks. The case is not that she lacks proportional reasoning skills, but that she has absolutely no idea what M stood for or what molarity meant.

If students had applied the same reasoning from the SHDC and DHSC tasks directly to the SVDM and DVSM molarity task, they would have been successful. However, student mental models for the molarity tasks were so pervasive and/or lacking that they were unable to recognize the ratio nature of the tasks.

VI: ANALYSIS, RESULTS, AND DISCUSSION OF RESEARCH QUESTION 3

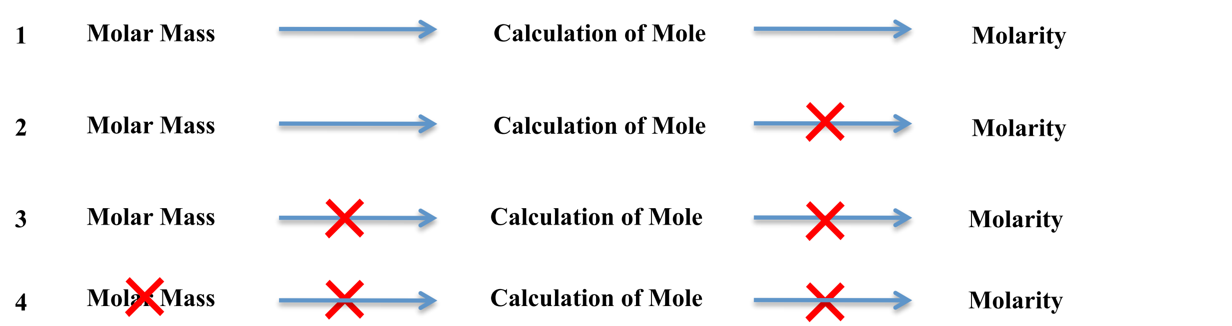
A. <u>**Chapter Overview for the analysis of R3: Are there patterns in how students use ratio in their solving strategies for molarity problems?**</u>

This research question examines student solving strategies on basic direct proportional reasoning problems and on domain specific molarity problems. It is important to study student solving strategies on these types of problems because a successful strategy in one domain may or may not yield incorrect answers in another domain. As shown in Chapter IV and will be further shown in this chapter, student data indicated that they did have direct proportional reasoning skills and the ability to recognize concentration qualitatively as an intensive quantity. The analysis of student solving strategies also speaks to their ability to solve direct proportional reasoning problems. As shown in Chapter V, this skill was not transferred to the molarity tasks for a majority of the students and analysis of their solving strategies in the Molarity Problems indicate that students do not reason about molarity as an intensive quantity. Specific mappings of axial codes and how they relate to the Theoretical Statement in this Chapter can be found in Appendix F.

Students were given a series of proportional reasoning problems on the PR diagnostic from Misaildou and Williams (2003) to gauge basic direct proportional reasoning skills. It was found that students showed success on these types of problems using a variety of different correct strategies. Students then applied these direct proportional reasoning strategies to problems involving inverse proportions in the Molarity Problems to yield incorrect responses. This chapter discusses the different types of successful strategies used by students in both the PR diagnostic and the Molarity problems as well as their unsuccessful strategies.

It will be concluded that there were four different groups of students in the intersection between calculation ability and understanding molarity as an intensive quantity. All students within the groups shown in Figure 56 have a commonality of direct proportional reasoning ability on the PR diagnostic. The first group shown in the figure includes students who have an intensive view of molarity as a ratio are able to calculate the molar mass and then use that molar mass to calculate moles from grams. This is intuitive because an intensive understanding of molarity involves moles. The second group is students who were able to calculate molar mass and use it to calculate moles but do not view molarity as an intensive quantity. A vast majority of these students held the “M is moles” mental model and therefore did not view M as an intensive quantity made up of two extensive quantities. The third group of students were able to calculate the molar mass of an element or a compound but were unable to use it in any way. They possessed the basic skill of addition from values on the periodic table but did not apply the value later in other problems. These students were unable to calculate a mole and did not view molarity as an intensive quantity. Lastly, there were students who did not know how to calculate molar mass. These students were therefore unable to calculate moles from grams and molar mass and did not view molarity as an intensive quantity. These students will be discussed in depth throughout the chapter.

Figure 56: Diagram of the different calculation skills and how they relate to an understanding of molarity as an intensive quantity



To analyze student responses to the Molarity Problems, it was important to analyze their basic direct proportional reasoning skills to draw parallels. Therefore, Section B will first categorize and catalog student solving strategies for basic direct proportional reasoning questions from the PR and then Section C will do the same for the Molarity Problems. Finally, a discussion of the conclusions will occur at the end of the chapter.

B. <u>**Analysis of student performance on the PR diagnostic**</u>

Most students scored well on the PR diagnostic with scores ranging from 9/13 to 13/13 and a median score of 11. A summary of student scoring on the PR can be found in Appendix G. Misailidou & Williams (2003) created their proportional reasoning diagnostic to diagnose various misconceptions and alternative solving strategies. This implied that a proportion was a proportion, no matter the type. That is, the authors did not distinguish between different types of correct direct proportional reasoning strategies. The focus of the research in this thesis was on proportional reasoning skills and ratio use, therefore student proportion use was analyzed at a deeper level than found in Misailidou & Williams (2003) to assess what types of proportions students used. Axial codes with respect to the PR diagnostic (Table 7) were analyzed for patterns by the researcher to create theoretical statements. Students had a variety of successful strategies as well as unsuccessful strategies. These are cataloged in Table 12. Students are not listed for each category for these theoretical statements because categories were not mutually exclusive. Students used a variety of these strategies for the problems and could not be categorized as just one strategy. Each of these categories will be described in this section.

Table 12: Theoretical Statements and categories related to student performance on the PR diagnostic	
Theoretical Statement 5: Students have the ability to solve direct proportional reasoning problems with success using a variety of strategies	
Category (a)	The "$apples_1$:$apples_2$::$oranges_1$:$oranges_2$" strategy
Category (b)	The "$apples_1$:$oranges_1$::$apples_2$:$oranges_2$" strategy
Category (c)	The "solve for one and multiply" strategy
Category (d)	The "multiplier" strategy
Category (e)	Combination strategies
Theoretical Statement 6: A variety of errors are the cause of mistakes on the PR Diagnostic	
Category (a)	The "additive" strategy
Category (b)	The "$apples_1$:$oranges_1$:::$oranges_2$ $apples_2$" strategy
Category (c)	The "random" strategy
Category (d)	The "magic halving" strategy

1. **<u>Theoretical Statement 5: Students have the ability to solve direct proportional reasoning problems with success using a variety of strategies</u>**

Generally, students were successful on the PR diagnostic questions. In the analysis, it was found that there were four types of successful strategies and students occasionally combined different strategies. Students could use the "$apples_1$:$apples_2$::$oranges_1$:$oranges_2$" strategy or the "$apples_1$:$oranges_1$::$apples_2$:$oranges_2$" to successfully solve any of the 13 direct proportion problems on the PR diagnostic. The : notation indicates the word "to" and the :: notation indicates "are to" . Figure 57 shows that with manipulation, both of the strategies are equivalent. Both strategies maintain the units within the problem to maintain the direct proportion.

Figure 57: Proof showing that the "$apples_1$:$oranges_1$::$apples_2$:$oranges_2$" is mathematically the same as the "$apples_1$:$apples_2$::$oranges_1$:$oranges_2$" strategy

$$\frac{\text{apples}_1}{\text{oranges}_1} = \frac{\text{apples}_2}{\text{oranges}_2}$$

Multiply both sides by $\frac{\text{oranges}_1}{\text{apples}_2}$

$$\frac{\text{oranges}_1}{\text{apples}_2} \times \frac{\text{apples}_1}{\text{oranges}_1} = \frac{\text{apples}_2}{\text{oranges}_2} \times \frac{\text{oranges}_1}{\text{apples}_2}$$

Cancel by division

$$\frac{\text{apples}_1}{\text{apples}_2} = \frac{\text{oranges}_1}{\text{oranges}_2}$$

As shown in Figure 56, the "$apples_1$:$oranges_1$::$apples_2$:$oranges_2$" strategy is easily manipulated to become the "$apples_1$:$apples_2$::$oranges_1$:$oranges_2$" strategy and they therefore yield the same answer when solving for $apples_1$. Direct proportion problems generally give three of the variables and require solving for the unknown variable. Figure 58 shows the arithmetic that is necessary to solve for one variable, in this case for $apples_1$.

Figure 58: Arithmetic to solve for one variable in a direct proportion problem

$$\frac{apples_1}{apples_2} = \frac{oranges_1}{oranges_2} \qquad \text{Multiply both sides by } apples_2$$

$$apples_2 \times \frac{apples_1}{apples_2} = \frac{oranges_1}{oranges_2} \times apples_2 \qquad \text{Cancel by division}$$

$$apples_1 = \frac{oranges_1}{oranges_2} \times apples_2$$

This research also uncovered two additional student solving strategies. In one case a student could solve for one (e.g. 1 apple is 2 oranges) and then multiply by this factor rather than setting up a proportion to establish the relationship. In another case, students used multipliers to solve the problems without setting up proportions or any calculations at all other than mental calculations. Some students utilized combinations of these methods for successful solving. Each of these will be discussed with student examples. Representative examples are presented for each category. Additional student examples for each category can be found in Appendix C.

a. The "$apples_1$:$apples_2$::$oranges_1$:$oranges_2$" strategy

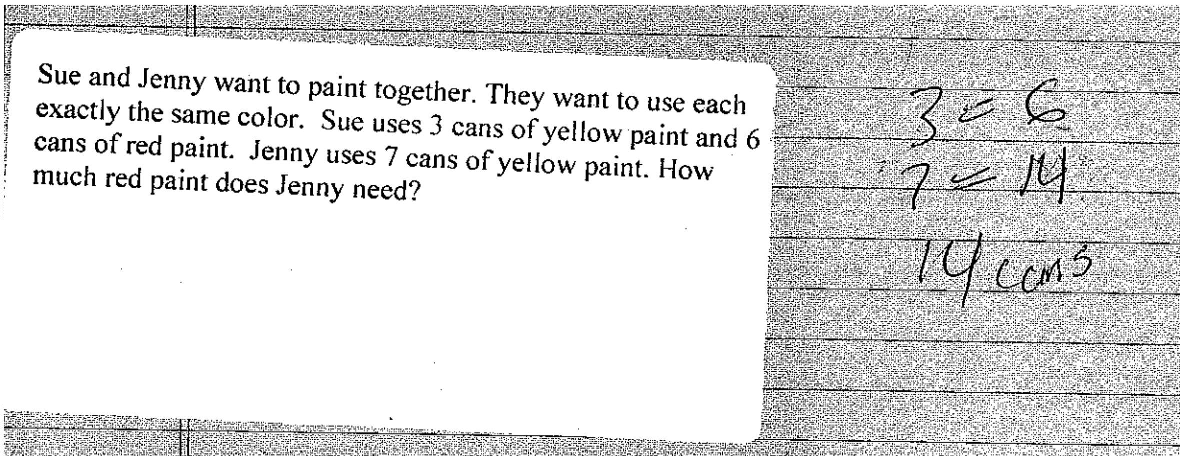
This strategy is termed the "$apples_1$:$apples_2$::$oranges_1$:$oranges_2$" strategy because students kept one thing on one side of the proportion and another thing on the other side of the proportion. This is the resulting equation found in Figure 57. This strategy helps students keep their units straight and therefore yields a correct response in cases where variables are in direct proportion. The open coders determined that a problem that was set up in a parallel nature using equal signs or arrows was the same type of solving strategy as using the divisor lines.

An example of this type of strategy is shown in Figure 59. Student 6-3 used the $apples_1$:$apples_2$::$oranges_1$:$oranges_2$ strategy on P7 of the PR diagnostic. He put the cans of yellow paint on the left side and the cans of red paint on the right side, with Sue's numbers on the top and Jenny's numbers on the bottom. He did not attend to the units on the page while solving the problem until the answer, but he did attend to units in his speech while solving the problem:

> [00:35:39.08] **Student 6-3:** Let's see. Sue and Jenny want to paint together. They want to paint each using exactly the same color. Sue uses three cans of yellow paint and six cans of red. Jenny has seven cans of yellow paint. How much red paint does Jenny need to use each exactly the same color? OK let's read this again. They want to paint together. They want to use exactly the same color. Sue uses three cans of yellow paint and six cans of red paint. Seven cans of yellow paint. Ok, let's see. Yellow seven, ok three and six. That get's doubled. Fourteen would even it out. I think she would need fourteen cans of red paint.

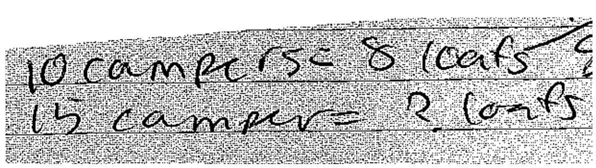
This yielded the correct response of fourteen cans of red paint. This strategy is used by students to keep one unit on one side and another unit on the other side. The student confirmed his answer by using a multiplier.

Figure 59: An example of the "apples$_1$:apples$_2$::oranges$_1$:oranges$_2$" strategy used by Student 6-3 on P7



Another example of this type of strategy is shown in Figure 60. Student 16-2 used the apples$_1$:apples$_2$::oranges$_1$:oranges$_2$ strategy to solve P9 on the PR diagnostic. He placed the number of campers on the left side and the number of loafs [sic] of bread on the right side and kept the numbers pertaining to the first week on the top of the problem (numerator) and the number pertaining to the second week on the bottom (denominator) of the problem. Students who used this strategy on problems with a direct proportion relationship were successful barring any arithmetic errors.

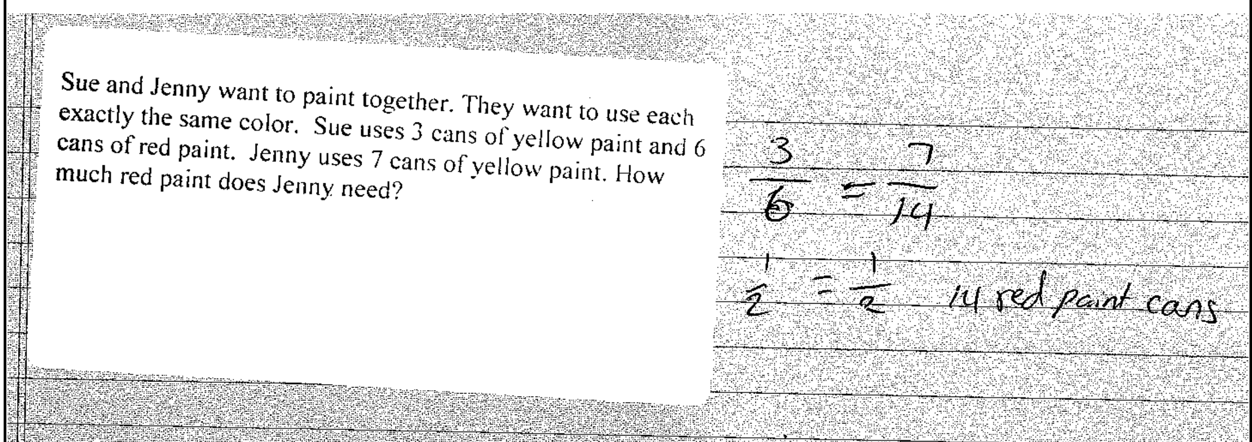
Figure 60: An example of the "apples$_1$:apples$_2$::oranges$_1$:oranges$_2$" strategy used by Student 16-2 on P9



b. **<u>The "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy</u>**

As shown in Figure 57, a mathematical manipulation of the "apples$_1$:apples$_2$::oranges$_1$:oranges$_2$" strategy yields the "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy. Rather than placing the numbers related to one situation on top and the numbers from another situation on the bottom, they are placed on the right and the left of the problem shown in the initial equation in Figure 57. An example of this strategy is shown in Figure 61.

Figure 61: An example of the "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy used by Student 18-4 on P7



Student 18-4 used the "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy on P7 on the PR diagnostic. She placed the number of cans of red paint that Sue uses over the number of cans of red paint that Sue used and set that equal to the number of cans of red paint that Jenny used over the unknown amount of red paint that Jenny should use to keep the same shade of color. Student 18-4 did not explicitly attend to the units but in her description of what she was doing she said the following that indicated she was attending to the units:

> [00:28:53.12] **Student 18-4:** Next one say Sue and Jenny paint want to paint together. They want to use each exact wait they want to use exactly the same color. Sue uses three paint three cans of yellow paint and six cans of red paint. Jenny used seven cans of

> yellow. How many paint red paint does Jenny need? So now I'm like now I'm guessing if Sue used three cans of yellow and six cans of red I would use that as a fraction. So three over six. So if Jenny' paint was to do it together and they want to use exactly the same color I would say that they'd have to be equal to Jenny. So Jenny used seven cans and I'm trying to find a denominator for red paint. So I would so it's fourteen because three over six is one half and seven over fourteen is one half

This student indicated that the denominator needed to be red paint for her proportion. This indicates that she was attending the units in her mind but was not sharing them on paper. [3]

Student 17-3 also used this strategy for this question as shown in Figure 62. She did not use units until the end of the problem. She used an X where the unknown variable was located in this proportion. The student started to use an "$apples_1$:$apples_2$::$oranges_1$:$oranges_2$" strategy and crossed it out as shown in the Figure. She didn't write down units within the problem and didn't discuss them out loud:

> [00:35:57.10] **Interview 17-3:** Cuz I know I have to set up a proportion. I'm just trying figure out how I would set it up so I think I would use Sue's on one side and then Jenny's on the other and cross multiply.

She did tack on a unit at the end of the problem. In her speech she indicated that she was keeping Sue on one side and Jenny on the other. This is language consistent with the "$apples_1$:$oranges_1$::$apples_2$:$oranges_2$" strategy.

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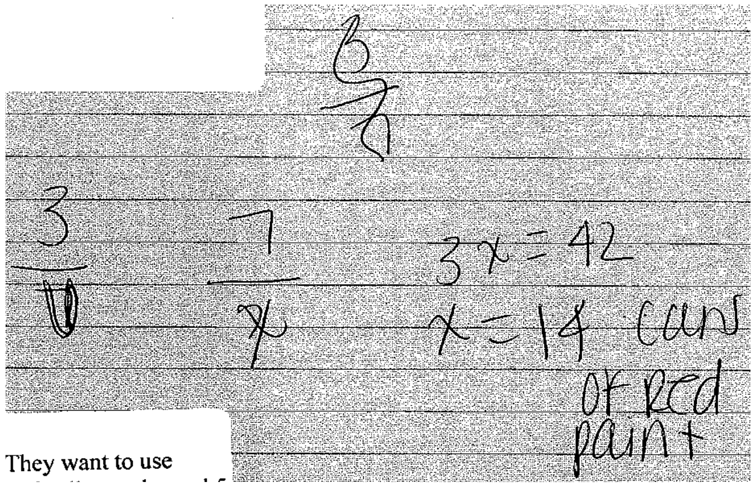
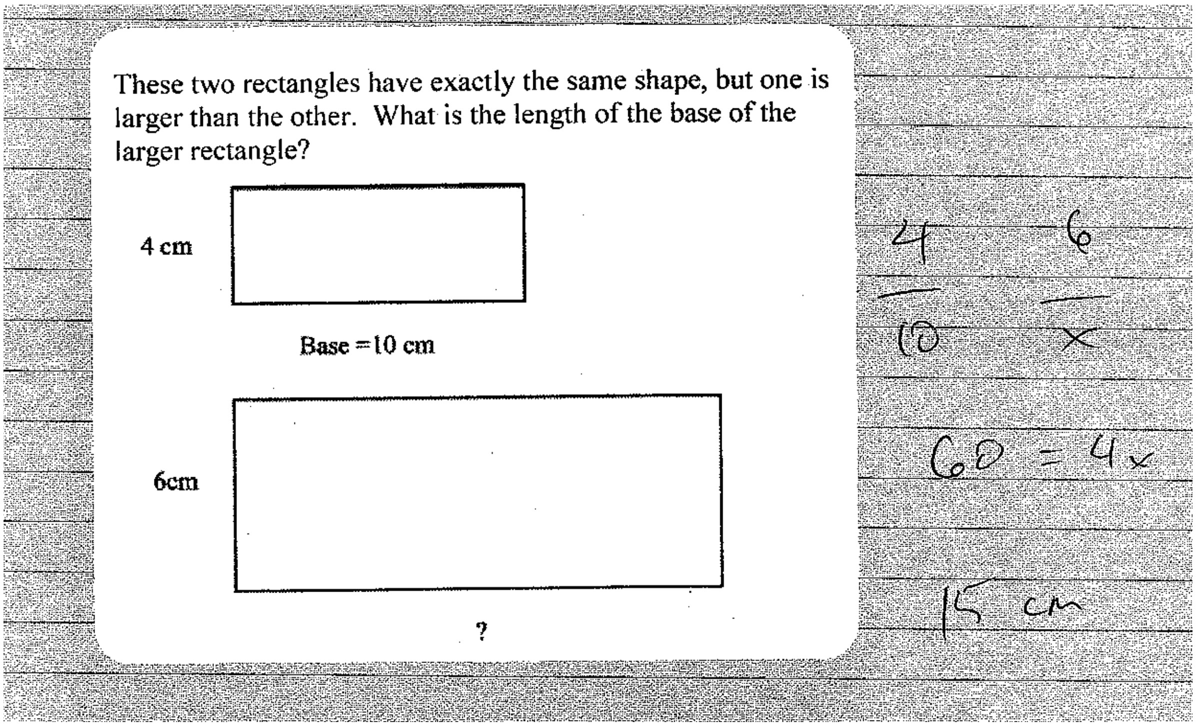
[3] The 1/2=1/2 is not part of Student 18-4's reasoning but rather an extra step to confirm equality.

Figure 62: An example of the "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy used by Student 17-3 on P7



A final example of this common strategy is shown in Figure 63 where Student 10-1 solved P13 on the PR diagnostic. The student placed the height of the smaller rectangle over the length of the smaller rectangle and set it equal to the height of the larger rectangle over the unknown length of the larger rectangle. He did not use units in his proportion but applied the correct unit to the final number from his calculation. Students who used this strategy were successful in solving problems with a direct proportional relationship barring any arithmetic errors.

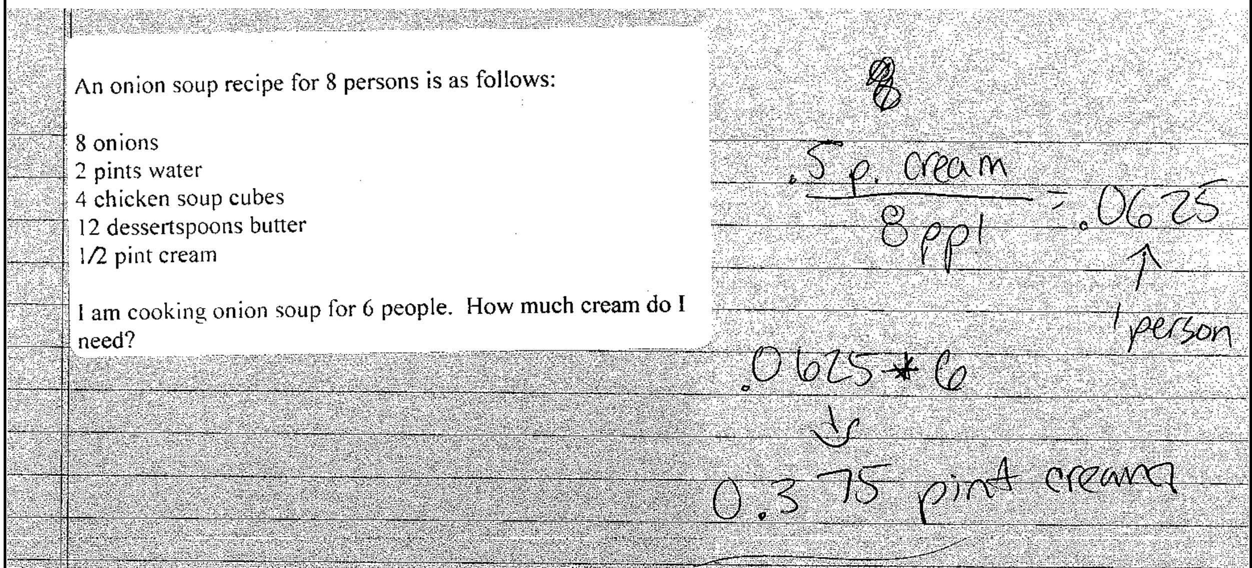
Figure 63: An example of the "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy used by Student 10-1 on P3



c. **<u>The "solve for one and multiply" strategy</u>**

The PR diagnostic questions are set up in such a way that students are given three of four variables and a direct relationship between two of the variables. The intent was to have students set up a proportion of some sort. However, another successful strategy emerged in which students solved for the value of one object and then multiplied to find the answer. This was termed the "solve for one and multiply" strategy in open coding and the named remained as an axial code. An example of this strategy is shown in Figure 64.

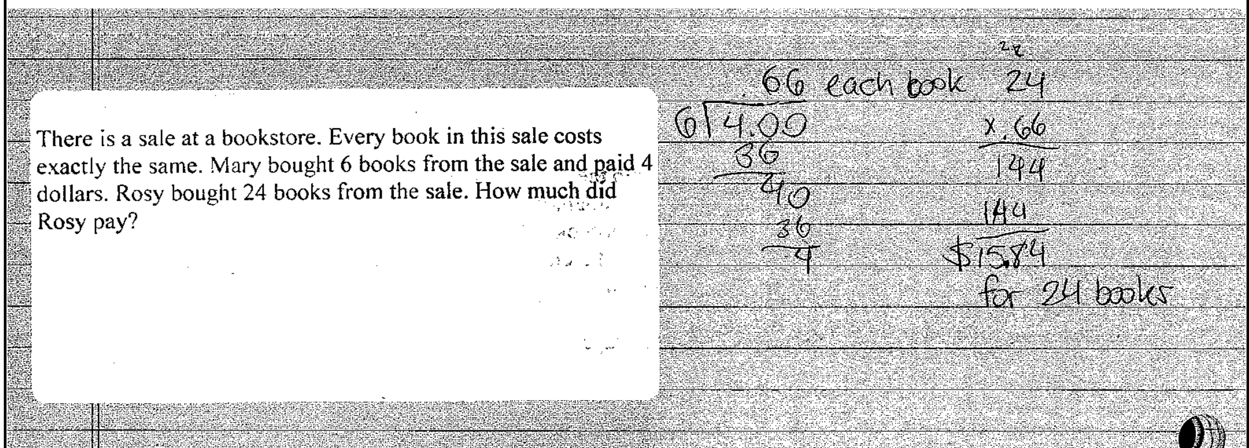
Figure 64: An example of the "solve for one and multiply" strategy used by Student 15-1 on P4



Student 15-1 calculated how much cream would be needed for one person and then multiplied that number by the six people necessary to complete the recipe. This strategy yielded the correct answer. This student showed units within the problem and with the final number.

A final example of this type of strategy is shown in Figure 65 that shows when this strategy is nearly successful. Student 12-1 used this strategy and solved for the cost of one book and multiplied by twenty-four books to get the answer. In this case, the student got $15.84 instead of the correct answer of $16. This was due to a rounding error in determining that one book cost $0.66 when the correct value was "$0.66666.....", which is a drawback of this strategy.

Figure 65: An example of the "solve for one and multiply" strategy used by Student 12-1 on P12



Students who utilized the "solve for one and multiply" strategy could still be successful in their problem solving when a problem had a direct proportion. Minor mistakes such as rounding errors led to nearly correct answers in problems that did not have an exact number for one of the variable of interest. Students who used the "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy or "apples$_1$:apples$_2$::oranges$_1$:oranges$_2$" strategy did not make these types of rounding errors.

d. **<u>The "multiplier" strategy</u>**

As described in Chapter V, students were able to solve the SVDM task with ease due to an obvious multiplier among the jars. The "multiplier strategy" included both multiplying by a factor and dividing by a factor.

Particularly important in the discussion of student responses in the SVDM task and the ability to reason through amount of substance was the performance of students on questions on the PR diagnostic that involved obvious multipliers. Questions P2, P3, P5, P6, P7 and P10 all contained numbers that had an obvious multiplier. P2 asked how many sprats an eel would eat given the relationship between length and sprats for an eel half the size. P3 involved a recipe in which the student was finding ingredients necessary for two people instead of eight, where a student could just divide all ingredients by the multiplier of four. P5 involved calculating the cost of pieces of fruit given that three cost ninety cents. While this could involve "dividing for 1 and then multiplying" to find the rest, that was still considered to be recognition of a multiplier in the intermediate step. P6 involved a book sale and asked how much set of books costs given a relationship between number of books and cost. The amount of the books for one person was four times the amount for the other. P7 involved two different colors of paint to obtain a color for a room of one size and it asks how much paint for one variable was needed if the other is changed. The relationship between the two colors was that twice as much yellow was needed than red. Lastly, P10 involved loaves of bread at a camp given a relationship between bread and the number campers. The amount of loaves in the second case was double the amount in the first case, which yielded double the amount of campers.

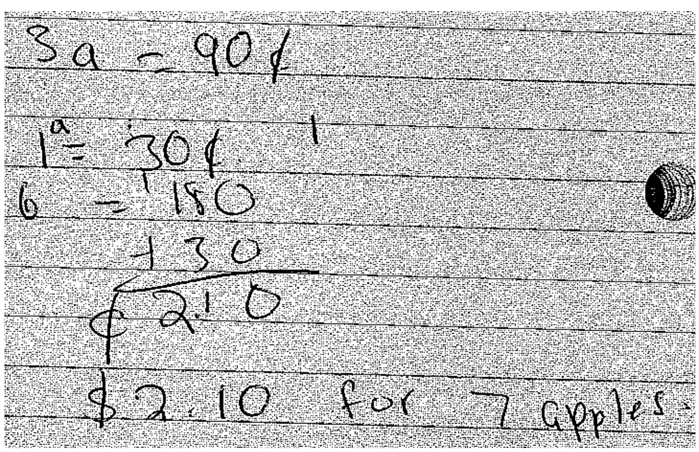
All of these problems could be set up without a proportion and still be answered correctly. This also makes them easier because the multipliers are easily recognizable. Due to the obvious multiplier in the eels task, no students missed that question. Some students immediately recognized the multiplier and responded and some used a proportion to solve it. This problem is analogous to the SVDM task where the students were asked to reason how much calcium chloride was in the 0.05M and 0.10M jars. If a student used six circles for the 0.05M jar they drew double the number of circles (twelves) in the 0.10M jar.

This strategy is a successful strategy due to the multiplier but it can mask alternative conceptions depending on the task. For example, due to the direct relationship between molarity and moles, students were able to correctly respond to the SVDM task because of the multiplier with no understanding behind their responses.

e. Combination strategies

Some PR diagnostic problems allowed for students to use a combination of the strategies shown above. An example was given earlier in the paint task where a student used the “multiplier” strategy in combination with the “apples$_1$:apples$_2$::oranges$_1$:oranges$_2$” strategy. Another example of a combination strategy is shown in Figure 66. Student 7-4 combined the “solve for one and multiply” strategy and the “multiplier” strategy to solve P5 on the PR diagnostic. The student solved for the cost of one apple and doubled the cost given in the problem to find the cost of six and added the cost of one to obtain the correct answer.

Figure 66: An example of a combination strategy used by Student 7-4 on P5



Combination strategies carried the same affordances and limitations as the strategies of which they were composed. Students seemed to use combination strategies when the problems allowed for it by having an obvious multiplier in the problem. In Figure 66, it seemed that the student went as far as she could with the multiplier strategy and then applied a different strategy to finish. This use of a multiplier until it doesn't work anymore was also seen in other tasks, such as the dictionary task where students would use multiples until they reached 28 dictionaries and needed to solve for the remainder. An example of this will be discussed in the Combination Strategies section of Theoretical Statement 6 in Figure 75.

2. **<u>Theoretical Statement 6: A variety of errors are the cause of mistakes on the PR diagnostic.</u>**

Although the students were largely successful on the PR diagnostic, they did make a series of errors. These errors related to direct proportion problems are relevant to how the students approached the molarity tasks as well as how they solved the molarity problems. These will be discussed later in the chapter.

Most students scored well on the PR diagnostic with scores ranging from 9/13 to 13/13 and a median score of 11. When a student did make a mistake other than an arithmetic error it was because of the following unsuccessful solving strategies:

a. an "additive" strategy*

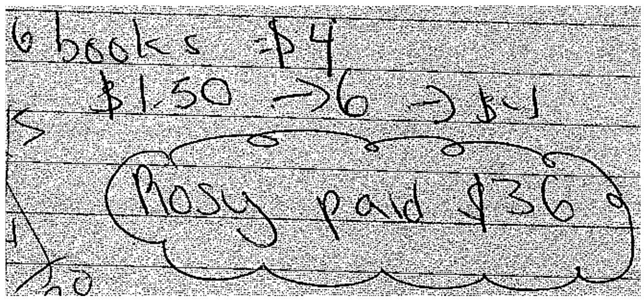
b. an "$apples_1$:$oranges_1$::$oranges_2$:$apples_2$" strategy

c. a "random" strategy

d. a "magic halving" strategy*

The data showed strategies that were unique to this study and two strategies that were described by Misailidou and Williams (2003), which are denoted by an asterisk. An additional error that was not cataloged was the basic arithmetic error, such as the one shown in Student 7-4's solving of the books problem shown Figure 67. As shown, the student divided 6 by 4 instead of 4 by 6.

Figure 67: An example of an arithmetic error made by Student 7-4



An example of each of these mistakes is included with descriptions of student responses that can be categorized as such.

a. **"Additive" strategies**

Misaildou & Williams (2003) reported a solving strategy where students found the difference between two known variables and then added that difference to the remaining number to find the unknown amount. This was termed an "additive strategy". This type of error was found with two students during the PR diagnostic: Student 22-3 and Student 13-4 (several instances).

Student 22-3 used the additive strategy to solve P11, the question that involved two fictional characters being measured both with matchsticks and paperclips. The student described her answer of "8" as follows:

> [00:59:14.18] **Student 22-3:** Well it says for Mr. Short's height is four matchsticks and the amount of paperclips you have for Mr. Short is six and for Mr. Tall it's six matchsticks which is two more. Which would be two more paperclips.

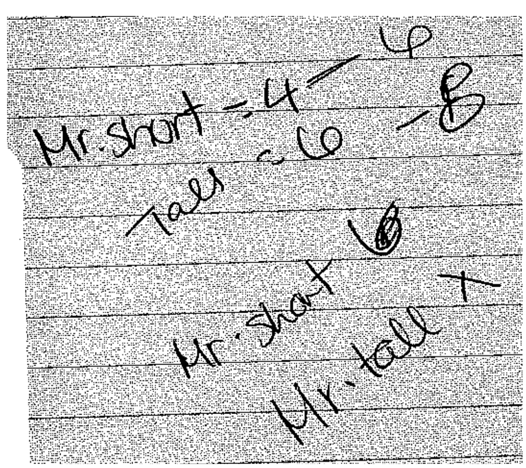
She found the difference between the given number of matchsticks for Mr. Short and Mr. Tall and determined that the difference between the number of paperclips would be the same difference in amount, yielding 8.

This strategy was also used for the P11 paperclip question by Student 13-4 (shown in Figure 68). His response offered the same description as Student 22-3:

> [00:41:57.14] **Student 13-4:** See the height of Mr Short measured. Hm Mr Short equals four Mr Tall six. Mr Short with paperclips three four five six is six paperclips. Mr Tall is X. Four to six . Ok for this one I did um when we when they measured Mr. Short with the um the matchstick they um use four matchsticks but when measured with when measured with the paperclips they use six paperclips. When Mr. Tall when they measured him with the matchstick he uses six so but they didn't tell us how much it would be measured with the um with the paperclips so what I did I did since for Mr. Short it went up by two I plus two when they measured him with um with the paperclips. So for Mr. tall I did plus two for both.

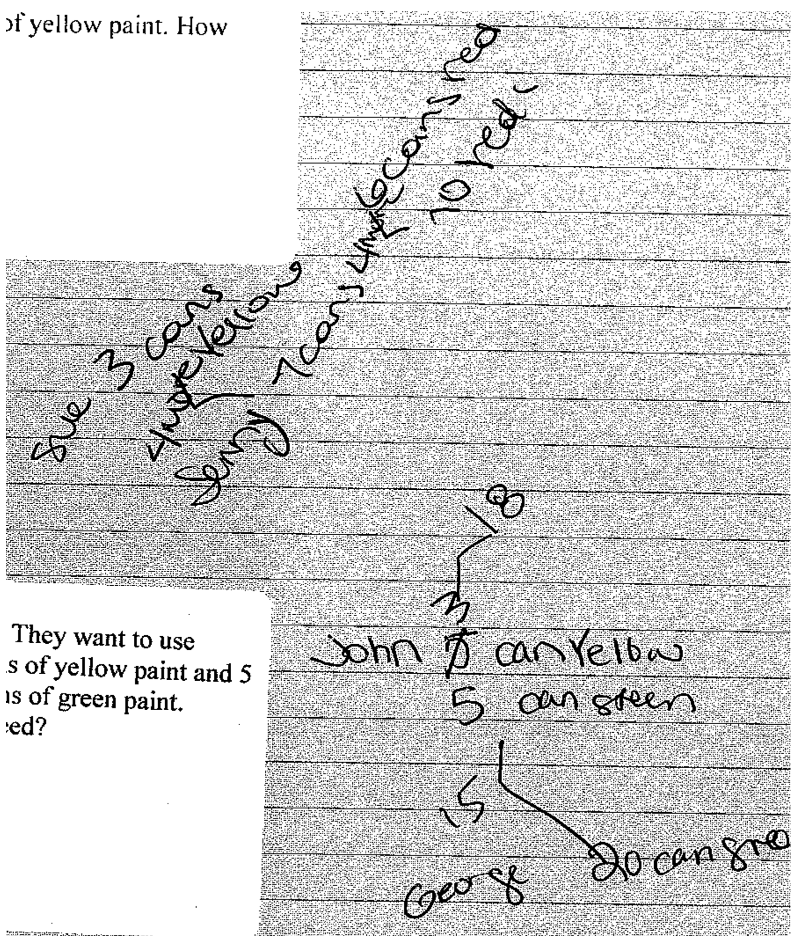
He described the difference between the number of matchsticks used for Mr. Short and Mr. Tall and added the difference to the number of paperclips to find the total number of paperclips for Mr. Tall. The difference between the two students is that Student 13-4 used this strategy two other times in the PR diagnostic.

Figure 68: An example of the "additive" error made by Student 13-4 on P11



Student 13-4 also used this strategy for both of the painting questions (P8 and P9) shown in Figure 69. He shows in the top of the figure for P7 that he found the difference between the number of yellow paint cans that Jenny and Sue used to achieve a certain shade of paint. He then used that difference of four cans and added it to the six red cans given in the problem to find the unknown amount of red cans yielding a response of ten red cans. He used this strategy again in P8 when finding the number of green cans. The difference between the number of green cans given in the problem was 15 and the student took the 15 and adds it to the three cans of yellow given to find 18 total cans of yellow.

Figure 69: An example of an "additive" error made by Student 13-4 on P7 and P8

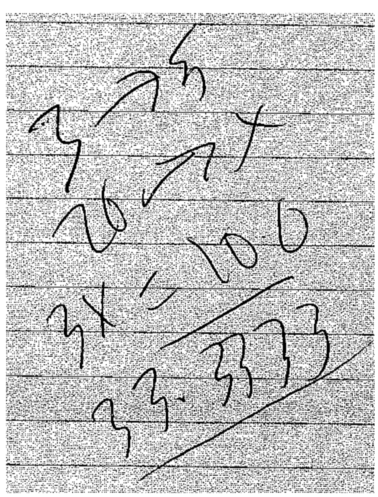


This strategy did not occur for the other questions, which leads the researcher to believe that when faced with an unknown situation where a direct proportion was not obvious or part of an everyday experience, this strategy emerged for these students.

b. **“apples$_1$:oranges$_1$::oranges$_2$:apples$_1$” strategies**

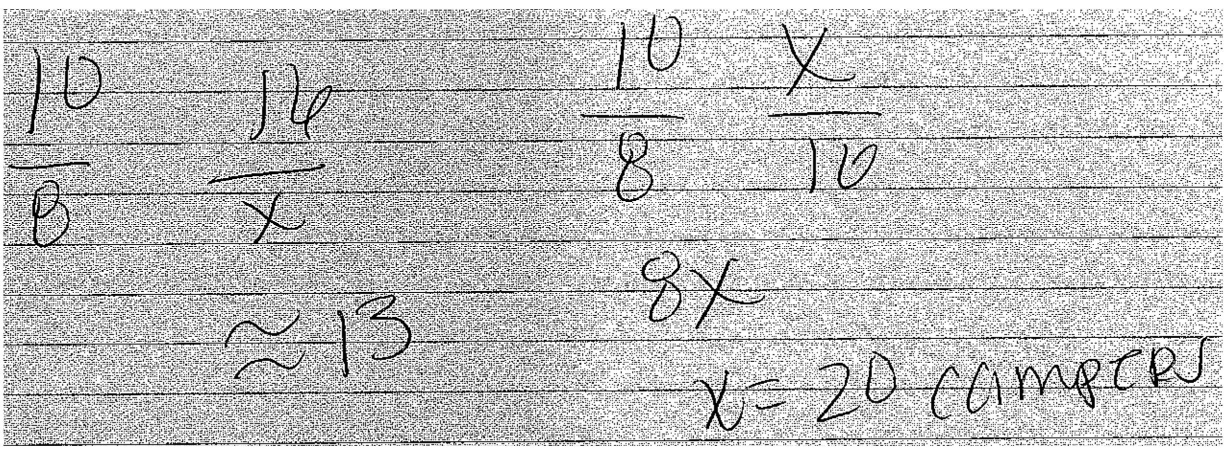
An incorrect variation of the “apples$_1$:oranges$_1$::apples$_2$:oranges$_2$” strategy is where a student does not keep units constant across the equality. The researcher has termed this strategy the “apples$_1$:oranges$_1$::oranges$_2$:apples$_2$” strategy, which as shown earlier in the chapter can be manipulated to be the “apples$_1$:apples$_2$::oranges$_2$:apples$_2$ strategy. A student with this type of error recognized a direct proportion but set it up without one side’s units matching the other side’s units. An example of this type of error is shown in Figure 70 made by Student 20-2 in P8 on the PR diagnostic. The student set up the problem such that the number of yellow cans to green cans was set equal to the number of green cans to yellow cans. This yielded an incorrect answer.

Figure 70: An example of an “apples$_1$:oranges$_1$::oranges$_2$:apples$_2$” error made by Student 20-2 on P8



Similar to the other examples but different in that the student changed her mind is Student 17-3 shown in Figure 71. Student 17-3 solved one of the camper and loaves of bread problems in this manner. She initially set up the problem with the number of people over the loaves of bread equal to the number of loaves of bread over the people. She recognized this error and correctly set up a direct proportion using the “apples$_1$:oranges$_1$::apples$_2$:oranges$_2$” strategy.

Figure 71: An example of the “apples$_1$:oranges$_1$::oranges$_2$:apples$_2$” error made by Student 17-3



This type of error indicates that the students are able to recognize the necessity for a proportion but do not necessarily understand how to use a proportion. The units are not kept constant and this leads to an inverse proportion rather the correct direct proportion. With the number of students not using units while solving problems, it is surprising that this type of error was not more prevalent.

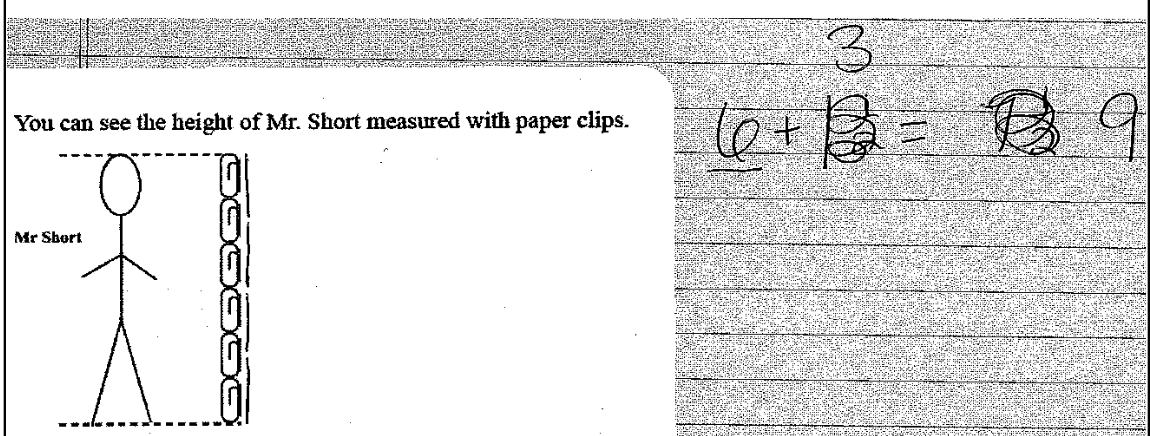
c. **<u>“random” strategies</u>**

Some students made errors that did not belong in the other categories and had nothing in common other than the fact that they are random in nature. An example of this type of error is shown in Figure 72. Student 4-2 created her own relationship between matchsticks and paperclips in P11 on the PR diagnostic by drawing on the label and estimating a relationship. Her description of her work indicated that one matchstick was equal to one and a half paperclips:

> [00:36:34.28] **Student 4-2:** So if it's for matchsticks for Mister short and it six matchsticks or matchsticks for Mister tall it would be one two three four five six paperclips plus one and half so he'd be seven and a half paperclips tall.
> [00:37:10.17] **Researcher:** Okay and how did you get that one more time.
> [00:37:13.24] **Student 4-2:** Um.
> [00:37:14.28] **Researcher:** Because um I saw you kind of tap at what you drew and then write one and one half how'd you get one and one half.
> [00:37:27.19] **Student 4-2:** Um, because of Mr. tall is like six matchsticks four times six yeah, hold on if it's for never-mind three I lied.

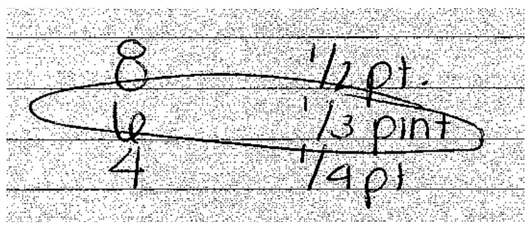
She initially indicated that there would be seven and half paperclips until deciding on the correct answer of nine. She did not use a proportion explicitly to solve the problem, but her drawing was creating a proportion that she used to solve to solve the problem.

Figure 72: An example of a "random" error made by Student 4-2 on P11



Another example of a “random” error is using estimation in to solve the problem. This is shown by Figure 73 showing Student 23-4’s response to P4 on the PR diagnostic. She knew that for eight students a recipe needed half a pint and for four students that would be a quarter of a pint. She knew that between four and eight was the number of people she was looking for and estimated to find the answer. It is curious that she did not choose a number halfway between 1/2 and 1/4, which would have yielded the correct response of 3/8.

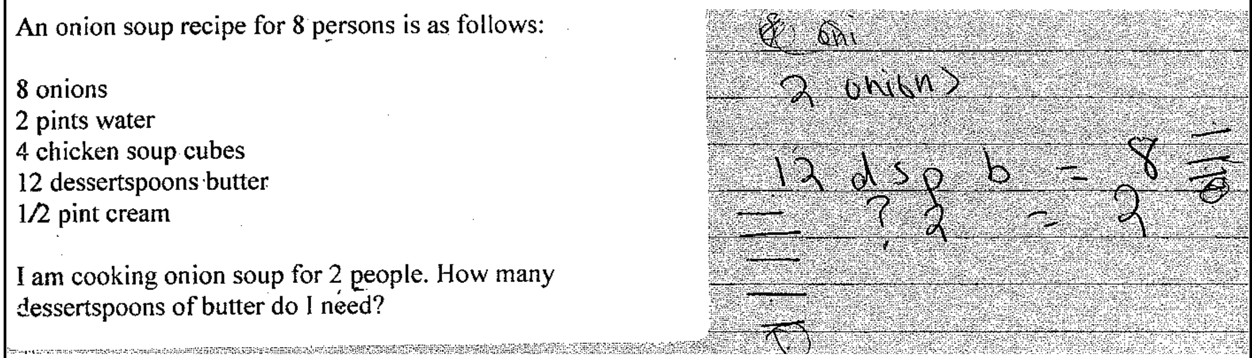
Figure 73: An example of a “random” error made by Student 23-4 on P4



Finally, Student 7-4 made the “random” error of stepping down by two four times to get to two on P3. She indicated that she thought the number four was too easy and came up with this strategy instead. She described her response shown in Figure 74 as follows:

> [00:43:34.27] **Student 7-4:** Because four for once four just seems convenient. I'm thinking as far as I thought a third of eight if I was to take two like how many times can two go into eight, which would be four and then the two, two divided by or twelve divided by two is six. I don't know for some reason four just seems too (?) for two if twelve is for eight, how many for two? Two four six eight twelve ten eight six four two so three four one two three four. I'm going with two.
> [00:44:47.14] **Researcher:** Two dessert spoons?
> [00:44:48.25] **Student 7-4:** Uh huh
> [00:44:48.25] **Researcher:** Why?
> [00:44:52.17] **Student 7-4:** I just did like in my head how many steps it takes to go from eight to two evenly.
> [00:44:59.11] **Researcher:** Ok
> [00:44:59.11] **Student 7-4:** So if from eight then six down four then two and then from twelve I just did the same but counting two down and then after that I just one from here one from here. I don't know how that makes sense but that's what I came up with.

Figure 74: An example of a "random" error made by Student 7-4 on P3


An onion soup recipe for 8 persons is as follows:

8 onions
2 pints water
4 chicken soup cubes
12 dessertspoons butter
1/2 pint cream

I am cooking onion soup for 2 people. How many dessertspoons of butter do I need?

Students who made "random" errors still showed partial understandings of proportions creating relationships between the numbers to work with them. These relationships still demonstrated some direct proportional reasoning and were likely used when faced with an unfamiliar problem or uncertainty with fractions.

d. **"magic halving" strategies**

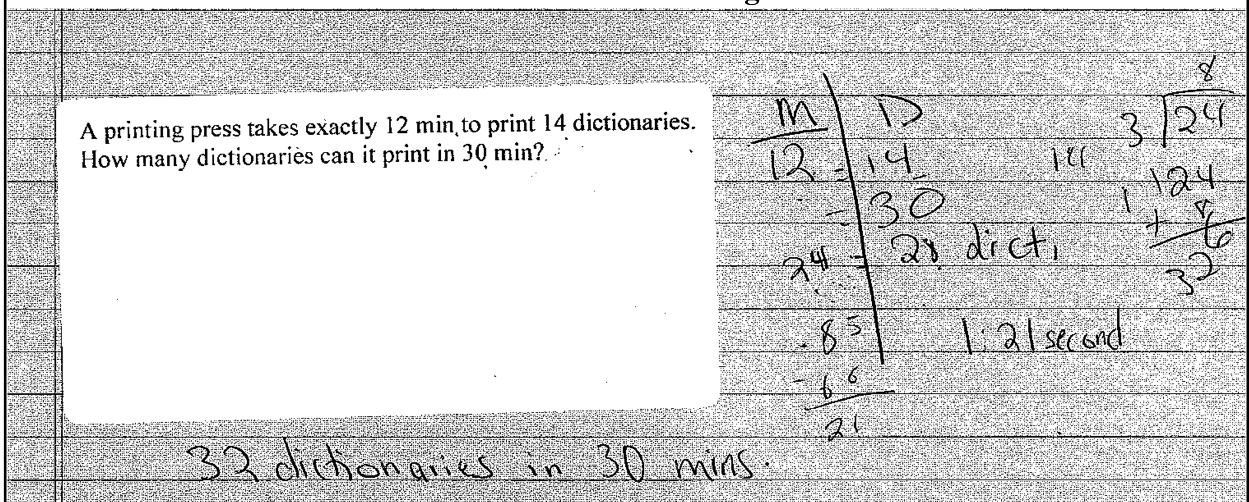
Misailidou and Williams (2003) reported that their students used an erroneous strategy called the "magic halving" error. One student in this study exhibited this type of error on P4 on the PR diagnostic. Student 9-2 halved the amount of pints of cream to decide upon the answer of 1/4. He reduced the number of people from six over eight to three over four but still divided the amount by two instead. This is an example of the "magic halving" strategy. This type of strategy indicated that the student did not view the problem as a direct proportion.

e. **Combination strategies**

Students within this category used a variety of unsuccessful strategies. An example of a series of unsuccessful strategies on P12 made by Student 7-4 is shown in Figure 75.

He placed the minutes on the left side of the line and dictionaries on the right side of the line. He placed 12 minutes under the minutes side and 30 minutes under the dictionary side. This setup yielded a “minutes$_1$:dictionaries$_2$::dictionaries$_1$:minutes$_2$” proportion.

Figure 75: An example of the “apples$_1$:oranges$_1$::oranges$_2$:apples$_2$” error made by Student 7-4 on P12 of the PR diagnostic



Student 7-4 abandoned that proportion set up and then resorted to using a multiplier of two to solve for double the dictionaries. This only yielded the amount of dictionaries made in 24 minutes, when the question asked for 30 minutes. He described his process in the following exchange with the researcher:

> [00:37:18.00] **Student 7-4:** So I doubled the amount that but I just can't get, I have twenty-four minutes I need six, wait I've been looking at the wrong. Labeling could really help. This is minutes. This is dictionaries. So if it takes me twenty-four minutes to make twenty-eight dictionaries. I need to get to thirty minutes and that's six, which is a third of it. One four divided by three is eight so I need twenty-four plus eight. I will have thirty- two dictionaries in thirty minutes.
> [00:38:15.21] **Researcher:** How did you get that?
> [00:38:17.01] **Student 7-4:** It’s just kind of funny cause the difference between twelve and fourteen was two and it’s making me think I'm right cause the difference between the number of dictionaries and the number of minutes I have is exactly two.
> [00:38:31.17] **Researcher:** Ok how did you get the number?

[00:38:35.07] **Student 7-4:** Um after I doubled the minutes because it said thirty minutes I doubled it to twenty-four minutes for twenty-eight dictionaries. And then I took a third of twenty-four.
[00:38:48.03] **Researcher:** Why
[00:38:49.08] **Student 7-4:** Because I'm trying to get to um thirty and it's a six, for a six.

The discussion reveals that the student made an arithmetic error while attempting to find a factor to use the "multiplier" strategy. He intended to find a factor between 6 and 24 and ultimately ended up with 1/3 when 6 is actually 1/4 of 24. He then determined an answer and used an "additive" error to confirm his final incorrect answer.

Errors were rare on the PR diagnostic. The students who made all of these types of errors only did so on a few of the PR diagnostic problems. The lowest score on the PR diagnostic was a 9 and the median score was an 11. Several students had perfect scores. The errors likely result from unfamiliarity with the problems and arithmetic errors for the most part. As shown in Figure Lime, when a student didn't know how to solve a direct proportion problem they may have attempted different strategies. As discussed earlier in this Theoretical Statement 1 section, several students utilized multiple solving strategies to find the correct answers and it makes sense that they would do the same with incorrect strategies.

C. <u>Analysis of student performance on the Molarity Problems</u>

The previous section showed that students did indeed have direct proportional reasoning skills. Not only did they have the skills, but they were fluent in them. Axial codes related to R3 (Table 9) were analyzed for patterns in solving strategies to construct the theoretical statements presented in this section. The Molarity Problems were analyzed for strategies to see if students used similar correct and incorrect strategies to the ones that they had used on the PR diagnostic or if they had chemistry-content specific strategies. Table 13 shows all of the Theoretical Statements that are related to student performance on the Molarity Problems. Theoretical Statements 7-10 correspond to mutually exclusive categories that align the groups of students shown in Figure 55 in the introduction to this chapter. Therefore, they are shown with the students who belong to each category. There were only three students who were found to have an intensive view of molarity and they were therefore able to calculate moles given molar mass and grams. As the Theoretical Statements get larger in number until Theoretical Statement 10, the students have fewer and fewer abilities. The remaining four Theoretical Statements (11-14) do not correspond to mutually exclusive categories and therefore are not broken down by students who belong in that category. Rather, representative student examples will be shown because students could use a variety of different strategies.

The errors that students made in the molarity and dilution problems were mostly the application of a direct proportion to an inversely proportional relationship. In molarity problems, the relationship between moles and molarity is direct whereas the relationship between volume and molarity is inverse. Another complication to the solving of Molarity Problems was the “M is moles” mental model persevering through hints to the contrary.

Lastly, due to the numbers within the problems, students were able to either randomly select numbers yielding correct responses or were able to get a correct answer due to a unit value within the problem. These will all be discussed in depth within this section.

<table>
<tr><th colspan="8">Table 13: Theoretical Statements related to student performance on the Molarity Problems</th><th>TOTAL</th></tr>
<tr><td colspan="9">Theoretical Statement 7: Students with an intensive understanding of molarity can also calculate molar mass and calculate moles given molar mass.</td></tr>
<tr><td colspan="2">14-1</td><td colspan="2">10-1</td><td colspan="2">1-3</td><td colspan="2"></td><td>3</td></tr>
<tr><td colspan="9">Theoretical Statement 8: Student ability to calculate moles from molar mass and grams is not indicative of an understanding of molarity as an intensive property.</td></tr>
<tr><td>11-1</td><td>18-4</td><td>12-1</td><td>9-2</td><td>24-3</td><td>17-3</td><td>13-4</td><td></td><td>7</td></tr>
<tr><td colspan="9">Theoretical Statement 9: Ability to calculate molar mass does not indicate ability to calculate the number of moles.</td></tr>
<tr><td>4-2</td><td>19-2</td><td>6-3</td><td>20-2</td><td>7-4</td><td>21-2</td><td>15-1</td><td>22-3</td><td>8</td></tr>
<tr><td colspan="9">Theoretical Statement 10: Students who are unable to calculate the molar mass are therefore unable to calculate moles from grams.</td></tr>
<tr><td>2-1</td><td>8-4</td><td>3-4</td><td>16-2</td><td>5-3</td><td>23-4</td><td></td><td></td><td>6</td></tr>
<tr><td colspan="9">Theoretical Statement 11: Student proficiency with direct proportion problems in mathematics does no support use of inverse proportions in molarity problems.</td></tr>
<tr><td>Category (a)</td><td colspan="8">The “apples1:oranges2::apples2:oranges1” strategy</td></tr>
<tr><td>Category (b)</td><td colspan="8">The “apples1:oranges1::apples2:oranges2” strategy</td></tr>
<tr><td>Category (c)</td><td colspan="8">The “apples1:apples2::oranges1:oranges2” strategy</td></tr>
<tr><td>Category (d)</td><td colspan="8">The “unit-less one” strategy</td></tr>
<tr><td>Category (e)</td><td colspan="8">The “random proportion” strategy</td></tr>
<tr><td colspan="9">Theoretical Statement 12: An “M is moles” misconception is not dispelled by providing a definition of molarity.</td></tr>
<tr><td colspan="9">Theoretical Statement 13: Students can solve some molarity problems through multiplication of volume and molarity.</td></tr>
<tr><td colspan="9">Theoretical Statement 14: When students do not know how to solve a problem they react differently in chemistry and direct proportional reasoning problems.</td></tr>
</table>

3. **Theoretical Statement 7: Students with an intensive understanding of molarity can also calculate molar mass and calculate moles given molar mass.**

As discussed in Chapter V, only two students were considered to have a robust understanding of molarity. They were the only students able to define molarity with its proper units and had a ratio mental model of molarity that was consistent throughout both the SVDM and DVSM tasks. [4] Not surprisingly, these students were also the only two students to score higher than a 4 out of 6 on the CD. Students 10-1 and 14-1 were able to get all three of the molarity questions on the CD correct except for the stock solutions questions and Student 14-1 was able to complete one stoichiometry problem correctly. They were also able to calculate the number of moles using the molar mass on the CD and in the Molarity Problems. It is possible that stock solutions were not covered in their high school chemistry courses. It is also possible that by the time molarity was taught in chemistry courses that these students developed proficiency in other concepts along the way such as molar mass and the calculation of a mole and stoichiometry in Student 14-1's case.

Student 14-1's work for questions 1 and 2 on the CD Problems (C1 and C2) can be seen in Figure 76. The first question involved the calculation of moles from molar mass that the student obtained from the periodic table. He used estimation to determine, which substance contained the most moles. It indicated that the student had the understanding that each substance contained a different amount of moles based upon its molar mass. He correctly chose response A with helium. Many of the other students ignored A because it was the lowest number, but he

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[4] It is important to note that while Student 1-3 was able to reason through both the SVDM and DVSM tasks with a ratio understanding, he was not deemed as a student with a "robust understanding" of molarity because he could not define molarity as moles per liter. He did, however, hold an intensive view of molarity.

recognized that while it was the lowest number of grams, it also had the smallest molar mass therefore it had more moles than its heavier counterparts. Therefore, he showed facility with molar mass not only with calculating it, but also with application in the calculation of moles using molar mass.

In C2, he used a proportion to solve for the amount of oxygen needed to react with magnesium to produce 175 grams of magnesium oxide. He was able to calculate the number of moles given molar mass and had proportional reasoning skills that he applied to the chemistry problems. Student 14-1 was the one of two students able to solve this problem.[5]

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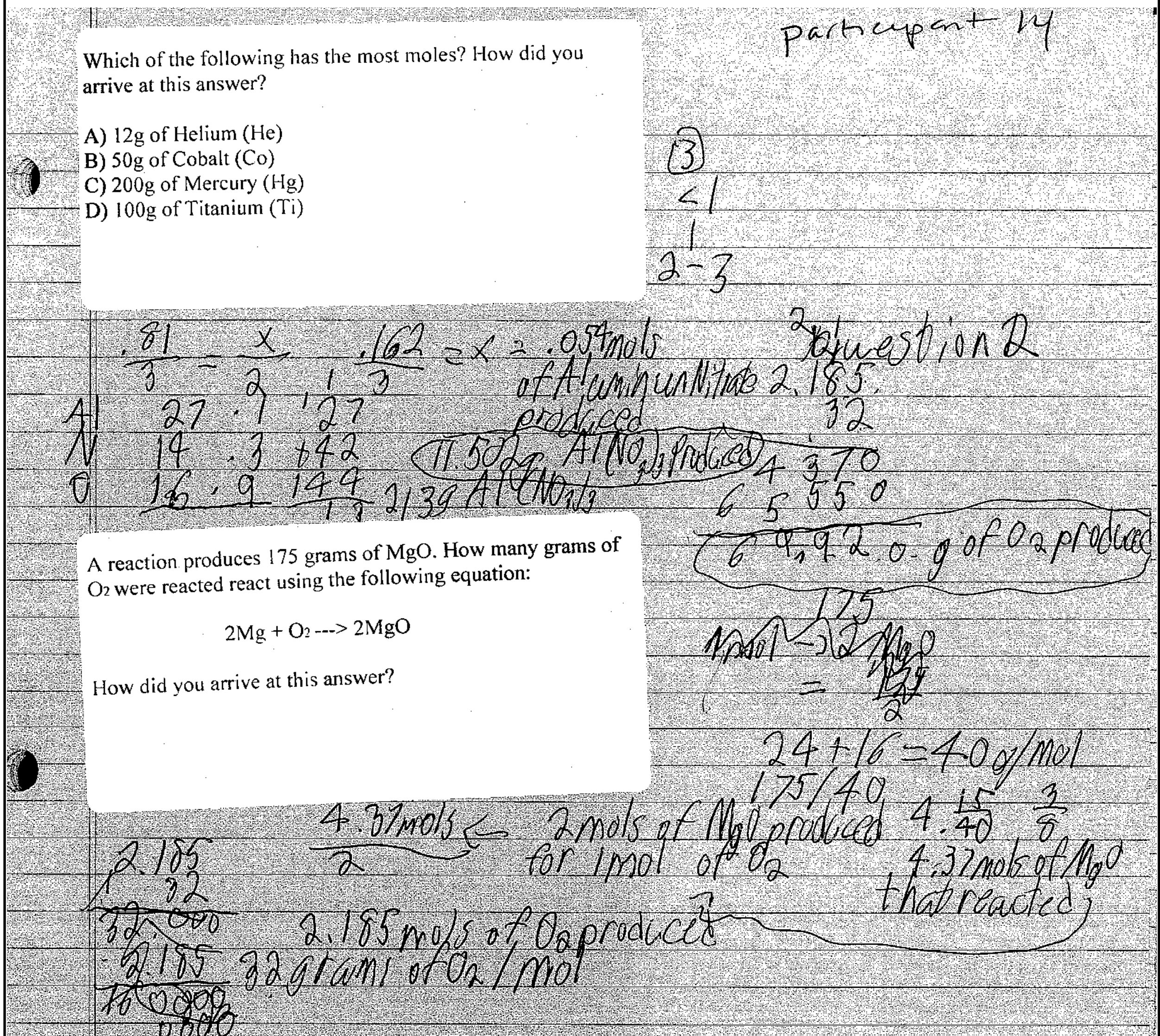
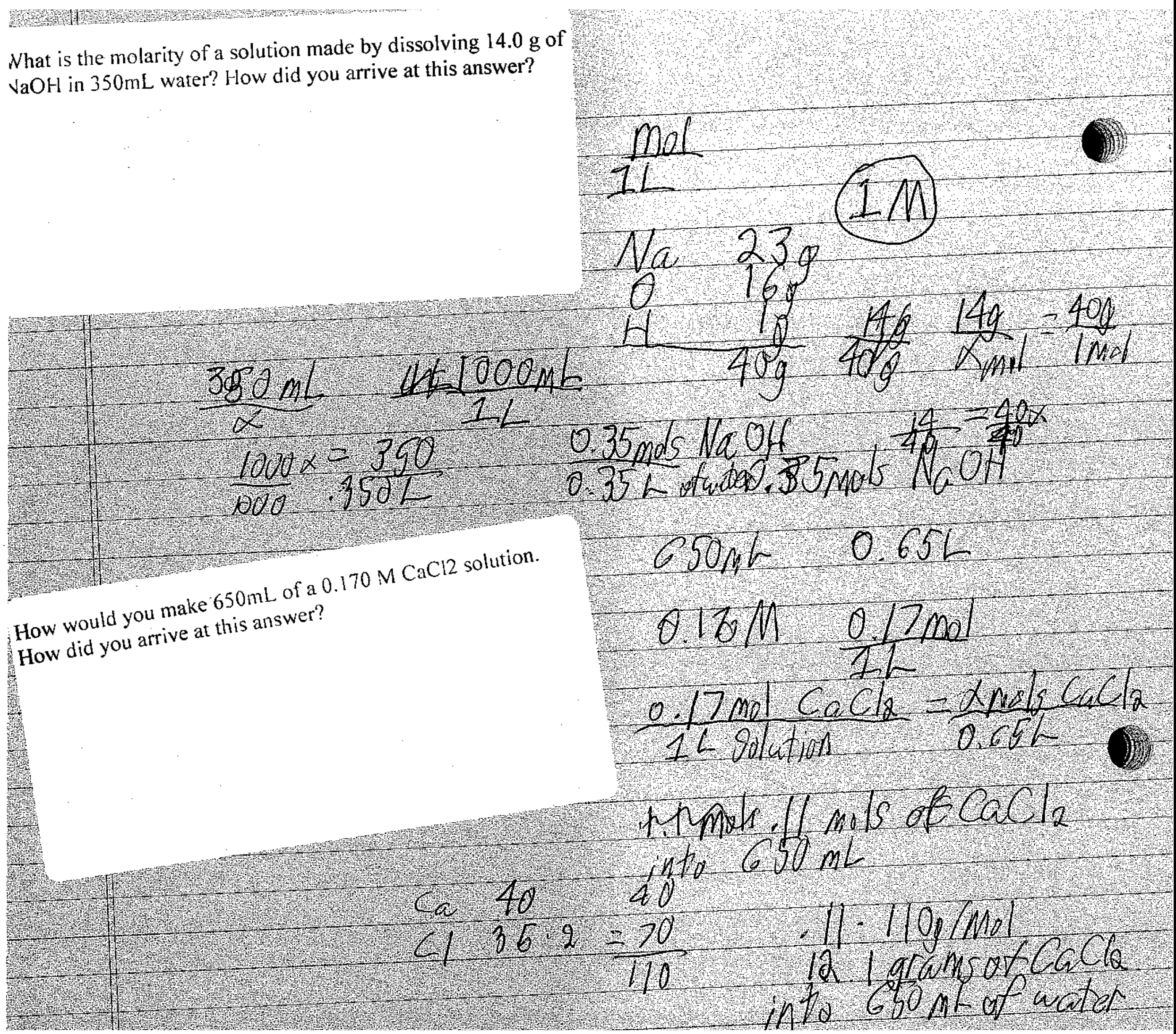
[5] The other student who could solve this problem was Student 15-1. She was unable to solve any of the other problems correctly. She was able to solve the problem by correctly constructing a grid to cancel the units but did not finish the calculation.

Figure 76: Student 14-1's responses to C1 and C2



Student 14-1 was one of two students to correctly solve questions 4 and 5 on the CD. These questions involved molarity. Figure 77 shows Student 14-1's work for these two problems. Question 4 (top question) required the student to convert grams into moles, milliliters into liters and finally to calculate the molarity of the resulting solution by dividing the moles by the liters. Student 14-1 wrote down the units for molarity, which points to his understanding of molarity. He calculated the molar mass for one mole and then created a proportion to find the amount of moles for 14 grams. He did the same in his conversion of milliliters to liters by setting up a proportion using an "$\text{apples}_1\text{:oranges}_1\text{::apples}_2\text{:oranges}_2$" strategy. He then took those two numbers and calculated the molarity correctly.

Figure 77: Student 14-1's responses to the CD questions 4 and 5



Question 5 (the second question in Figure 77) required the student to calculate the number of grams of calcium chloride needed from a given volume and molarity. This question also required an understanding that was needed to create this solution. Student 14-1 converted milliliters into liters and changed the 0.170M into 0.17moles/1L indicating that he recognized that M is an intensive property. He then set up a proportion using an "$apples_1$:$oranges_1$::$apples_2$:$oranges_2$" setup to calculate the number of moles necessary for 0.65L solution. He then used that number and calculated the number of grams necessary using the molar mass of calcium chloride. He correctly attended to the subscript in his calculation of the molar mass, yielding a response of 110 grams. This student had great attention to units, which led to successful problem solving and correctly set up proportions. Student 10-1's responses can be found in Appendix C.

Student 1-3 was able to calculate molar mass and a mole given molar mass, indicating that an intensive view of molarity without proper units enables students to solve these other types of problems. However, he was not successful on the CD as the other two students had been.

For comparison purposes, the two high scoring students also did well on the PR diagnostic, Student 10-1 only missed one question where he was incorrect by a factor of ten in his calculation. Student 14-1 did not miss any of the PR questions. Student 14-1 showed his units on all of the questions and tended to use the "$apples_1$:$oranges_1$::$apples_2$:$oranges_2$" strategy with the exception of the question involving money, where he used the "solve for one and multiply" strategy. Student 10-1 used a variety of strategies across the problems without showing much work or units. Student 1-3 had a lower PR score (missing more than two problems).

It makes sense that a student with an intensive understanding of molarity would be able to carry out tasks related to molarity given various pieces of information of either volume or mass because the student understood that molarity is an intensive property comprised of two extensive variables. These students were also able to reason through substance amounts during a volume change in DVSM where their peers were not. Lastly, these students also, not surprisingly, had conceptual understandings of molar mass and were therefore able to calculate moles using molar mass. This is important because in the next three categories of students it will be shown that ability to calculate molar mass does not indicate ability to calculate moles and the ability to calculate moles does not indicate that students have an intensive view of molarity.

4. **Theoretical Statement 8: Student ability to calculate moles from molar mass and grams is not indicative of an understanding of molarity as an intensive quantity**

The previous Theoretical Statement 7 discussed students who had a conceptual understanding of molar mass and the ability to use the molar mass to calculate moles. This aligns with Group 2 in Figure 56. In Chapter V, Theoretical Statement 4 discussed a group of students who had a conceptual understanding of molar mass as it applies to the calculation of a mole and were therefore better able to reason though a change in the number in front of M. One would think that this would enable an intensive view of molarity at least with a hint. In fact, they cannot as will be shown in this section. This section serves to describe the students who were able to calculate the number of moles using molar mass and grams but were unable to recognize that molarity was a ratio.

Question 1 from the CD required students to use molar mass and grams to calculate the number of moles for various substances. Their responses were then categorized as correct-conceptual, correct, and incorrect. To be categorized as correct-conceptual, the students had to demonstrate an understanding beyond an equation or formula. To solve this problem, a student had to be able to set up ratios using molar mass and grams to calculate unique ratios to find moles. Students 1-3, 10-1, 14-1, 11-1, 12-1, 24-3, 13-4, and 18-4 had conceptual understandings of molar mass and used it to calculate moles. They were therefore were familiar with calculating the number of moles using molar mass and grams. Two students, 9-2 and 17-3, were correct in their solving but were only able to describe their answers at an algorithmic level. Thus 10 students out of the 24 got this problem correct. Of these, Students 1-3, 10-1, and 14-1 have been described in other sections and they demonstrated a ratio understanding in both the DVSM and SVDM tasks and they therefore do not belong in this category.

However, the ability to calculate moles from molar mass and grams does not mean a student can work with molarity as a ratio. Indeed, seven of the ten students who could do calculations with molar mass did not recognize molarity as a ratio and could not reason through the DVSM task.

Two students were able to solve the problem (C1) algorithmically and not conceptually, Student 17-3 was described in Chapter V as attributing the number before M to something other than the amount of calcium chloride in solution. Student 17-3 held the belief that M was an additional element and therefore failed to recognize that molarity was a ratio. Her response to question 1 was correct but with incorrect reason as evidenced by her description:

> [00:04:48.10] **Student 17-3:** Because it had four and I was maybe like guessing that the lowest number out of these would be a mole.

The transcript shows that the student simply chose the lowest number and that number happened to the be the correct response. This is similar to a very common incorrect answer where students choose the largest number because it should have the most grams. However, it differs because picking the element with the lowest number of grams in the problem is not intuitive. Another type of mental model must be at play.

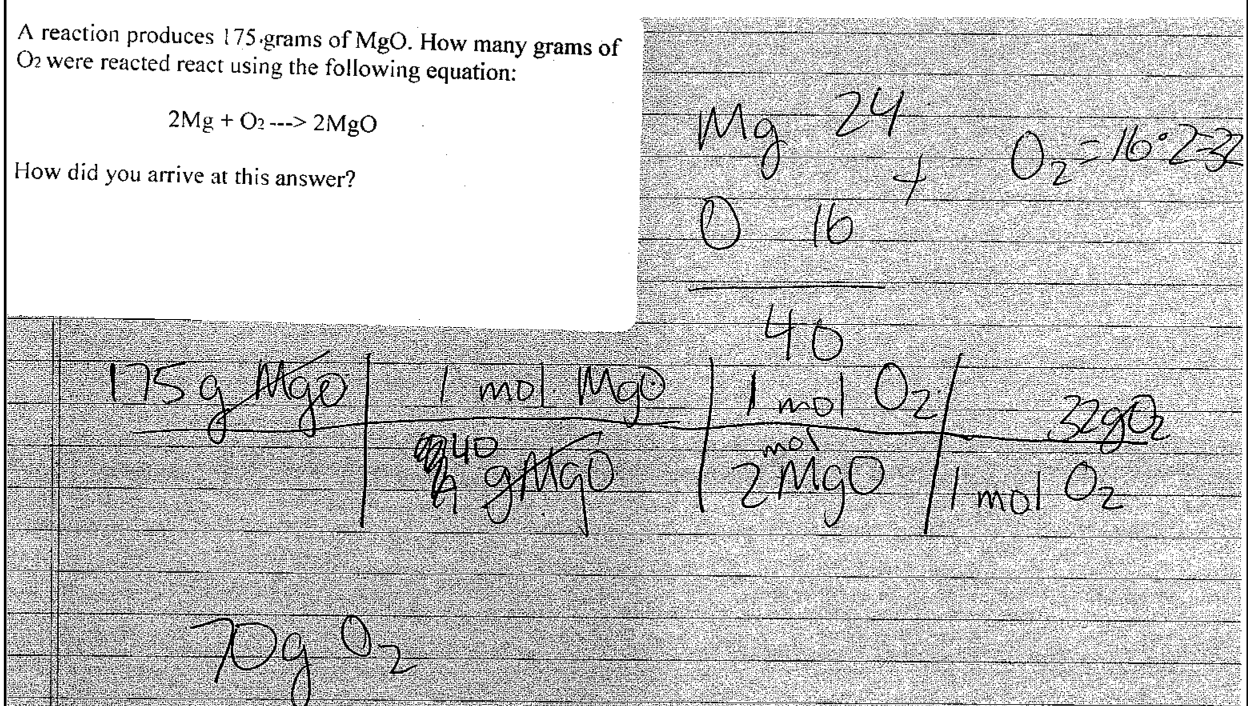
Student 9-2 was unique in that she had two competing descriptions for molarity, one that involved a ratio understanding and one that did not. She ultimately chose an “M is moles” mental model to describe the jars. The “M is moles” mental model also describes the understandings held by Students 11-1, 12-1, 13-4 and 24-3 (as described in detail in Chapter VI). These students all stated that M stood for moles and therefore in the DVSM task would fail to recognize that the number before M was an extensive quantity representing both the amount of substance and volume. This yields a mental model that describes a volume change as just that: a change in volume but not in substance. Student 18-4 was unique in that he had a conceptual understanding of the mole but attributed the number in front of M to the acidity or basicity of the solution and not the amount of substance. Therefore, he was unable to recognize molarity as a ratio of volume and amount of substance.

These students were unable to recognize M as molarity and therefore, were unable to interpret molarity as a ratio. Again, we see some ratio skills in chemistry that are not working in molarity because the students do not recognize molarity as an intensive quantity. This finding is consistent with what was found on other tasks. It can be ascertained then that the ability to calculate moles from grams and molar mass does not indicate the ability to reason about molarity as an intensive quantity.

5. **Theoretical Statement 9: Ability to calculate molar mass does not indicate ability to calculate the number of moles.**

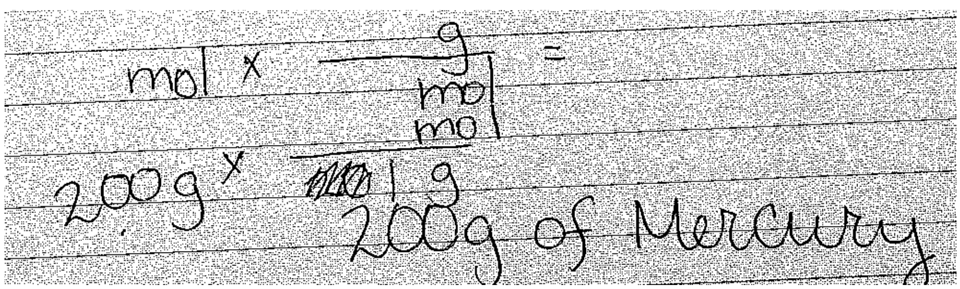
The students in this category align with Group 3 in Figure 55 where a student is able to calculate the molar mass but then is subsequently unable to use molar mass to calculate the number of moles. Since ten students correctly answered Question 1 on the CD and were therefore able to calculate a mole using molar mass and grams, that means that 14 students incorrectly answered it for a variety of reasons ranging from a student not understanding how the solve problem to a student choosing the largest number because it would be the most moles. This Theoretical Statement also includes students who use "grids" to cancel units. This algorithmic skill can be used successfully when applied properly. For example, Student 15-1 was the anomaly student when she was able to solve a stoichiometry problem, but she was able to so because of the grid she set up (Figure 78) to cancel the units. It can be applied incorrectly, as will be shown in this section. She was able to lay out the steps of the grid correctly without solving the problem. Had she completed the multiplication and division steps, she would have found the correct answer.

Figure 78: Student 15-1's successful use of a "grid" on Q2 of the CD



Student 2-1 wrote out a correct formula for calculating the number of moles from molar mass (Figure 79) but then stated, “so since Mercury has the most grams I would think that would have the most moles so I would have to go with Mercury”. This was a common incorrect response among the students. Students 3-4, 4-2, 6-3, 7-4, 15-1, 16-2, 19-2, 20-2, 21-2, 22-3, and 23-4 all responded that Mercury was the answer because it had the largest number and therefore the most amount of moles. Students 5-3 and 8-4 were unable to answer the question.

Figure 79: Student 2-1’s response to Question 1 on the CD

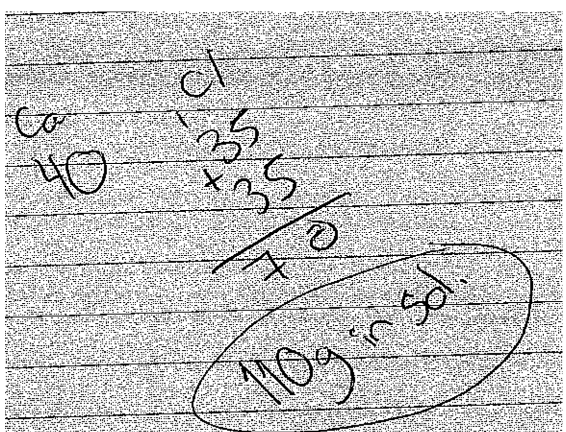


Throughout the axial coding process of the molarity problems at the end of the interview, the second axial coder noted the number “110” appearing as an answer to the Molarity Problems and CD problems. Upon further investigation, it was found that students were calculating the molar mass of calcium chloride (110 grams) and stopping there either because they didn’t know how to continue or they thought the had found the answer. Variations of this response are the calculation for the molar mass of calcium chloride that ignores the subscript yielding a molar mass of “75” and the arithmetic error that yields “120”. Specific examples can be found in Appendix C.

Of the students who had incorrect responses for Question 1 on the CD, four students were able to correctly calculate molar mass of compounds later in the molarity problems: Student 7-4, Student 15-1, Student 21-2 and Student 22-3. Additionally, four students were able to calculate molar mass after getting a hint in the molarity problems: Student 5-3, Student 6-2, Student 19-2, and Student 20-2.

Student 21-2 calculated the molar mass correctly for calcium chloride in M6 in the molarity tasks. She indicated that calcium was 40 g and that there were two Cl atoms at 35 g each and she correctly added the numbers together. However, she indicated that this was her final answer as shown in Figure 80. This is potentially due to the nature of the question in that it asks the student to calculate the number of grams in the solution given the volume and the molarity. Because the student found an answer with grams, she may have assumed that she was finished. However, the solution did not contain one mole of calcium chloride, so the answer is incorrect. This indicates that the student is able to correctly calculate molar mass, but does not know how to apply that further as evidenced by both the problem in M6 and in her incorrect response to Question 1 on the CD.

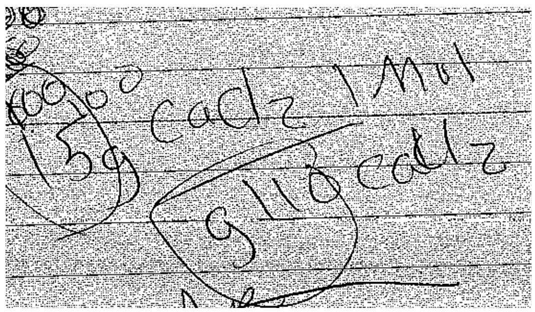
Figure 80: Student 21-2's response to M6 in the molarity problems



As discussed earlier, a hint was provided to the students after a first attempt at the Molarity Problems if they struggled with this task. This hint included a discussion of how to calculate a mole given the molar mass of a substance, in that case of a single element. This led to four students calculating molar mass after a hint without application.

Student 20-2 used the molar mass hint to calculate the molar mass of calcium chloride. Her response is shown in Figure 81.

Figure 81: Student 20-2's response to M3 in the molarity tasks



She found it to be 110 g and set up an algorithm with the number of grams given in the problem divided by 110 g. This was a correct setup to calculate the amount of moles. The student divided the numbers backwards, realized this, but did not continue past that step as shown in the following exchange between the student and researcher:

> [01:38:51.27] **Interview 20-2:** Just dividing um, wait wait. We have fifteen six and then you've got, just have to, Six point oh six moles. And then you divide it by the molarity of the solution. Twenty-five milliliters. This is getting tricky.
> [01:40:17.17] **Stephanie:** Ok go ahead and hit stop. Could you tell me how you got six point oh six or whatever for the moles?
> [01:40:28.25] **Interview 20-2:** Well
> [01:40:30.09] **Stephanie:** Tell me the math you did.
> [01:40:32.07] **Interview 20-2:** It's bad math because now I realize that it should be the other way around.

Student 6-3 was able to correctly calculate the molar mass for calcium chloride for M1 in the molarity problems task. However, he was then unable to apply to it solve the rest of the problem. His final answer was 110 moles, which he found by simply by calculating the value of the molar mass but then ascribed this number to moles. Similarly, Student 5-3 reported his answer to M1 as “110 moles” with no work on the problem. These students are confusing molar mass with mole and although they are able to calculate the numeric value, they were incorrectly using it.

Some students attempted to calculate molar mass and were slightly off due to an addition error (120 grams) or because they did not attend to the subscript (75 grams). They did however set up problems correctly. Several students were able to calculate the molar mass in the molarity task problems without having an understanding of mole. These students were able to add the different atomic masses, but then did not know how to use them to calculate the number of moles. As shown in Figure 56 at the beginning of this section, the ability to calculate molar mass does not have any bearing on student ability to view molarity as an intensive quantity

6. **Theoretical Statement 10: Students who are unable to calculate molar mass are therefore unable to calculate the number of moles from grams.**

The previous three Theoretical Statements all described students who were able to do calculations at different levels from the ability to calculate molar mass but not use it, the ability to calculate moles from molar mass and finally the ability to see molarity as an intensive quantity. These all assume a basic skill of the calculation of molar mass. However as shown in Table 13, one-fourth of the students were unable to calculate molar mass and were therefore unable to calculate the number of moles. They also did not recognize molarity as a ratio.

7. **Theoretical Statement 11: Student proficiency with direct proportion problems in mathematics does not support use of inverse proportions in molarity problems.**

Dilution calculations are important calculations in chemistry. The mathematics of dilution, however, does not involve a direct proportion. Instead, there is an inverse proportion between molarity and volume. Most students in this study who attempted a dilution problem did not recognize the relationship as inverse and treated it as though it were a direct proportion. This indicated that they did not know the units of molarity and that it was an intensive quantity. It is important to note that students may have inverse proportional reasoning skills outside of the domain of chemistry, but the PR diagnostic did not assess that skill. It is likely that their errors stem from the lack of an understanding of molarity as an intensive quantity where molarity has a direct relation to moles and an inverse relation to volume.

To properly perform a calculation in a dilution problem, one first begins with the M1V1=M2V2 relationship. As shown in Figure 82, this equation can be manipulated to create a proportion representing the inverse relationship between volume and molarity.

Figure 82: Dilution Equation Manipulation

$$Molarity_1 \times Volume_1 = Molarity_2 \times Volume_2$$ Divide both sides by $Volume_1$

$$\frac{Molarity_1 \times Volume_1}{Volume_1} = \frac{Molarity_2 \times Volume_2}{Volume_1}$$ Cancel by division

$$Molarity_1 = \frac{Molarity_2 \times Volume_2}{Volume_1}$$ Divide both sides by $Volume_2$

$$\frac{Molarity_1}{Volume_2} = \frac{Molarity_2 \times Volume_2}{Volume_1 \times Volume_2}$$ Cancel by division

$$\frac{Molarity_1}{Volume_2} = \frac{Molarity_2}{Volume_1}$$

To solve for a certain unknown variable, in this case $Molarity_1$, students can multiply both sides of the equation by the volume of the second solution as shown in Figure 83.

Figure 82: Solving for a variable in a dilution problem

$$\frac{Molarity_1}{Volume_2} = \frac{Molarity_2}{Volume_1}$$ Multiply both sides by $Volume_2$

$$Volume_2 \times \frac{Molarity_1}{Volume_2} = \frac{Molarity_2}{Volume_1} \times Volume_2$$ Cancel by division

$$Molarity_1 = \frac{Molarity_2}{Volume_1} \times Volume_2$$

Students exhibited several solving strategies that had proportions in them involving what this dissertation will term: (a) "$apples_1$:$oranges_2$::$apples_2$:$oranges_1$" strategy (b) "$apples_1$:$oranges_1$::$apples_2$:$oranges_2$" strategy, (c) "$apples_1$:$apples_2$::$oranges_1$:$oranges_2$" strategy,

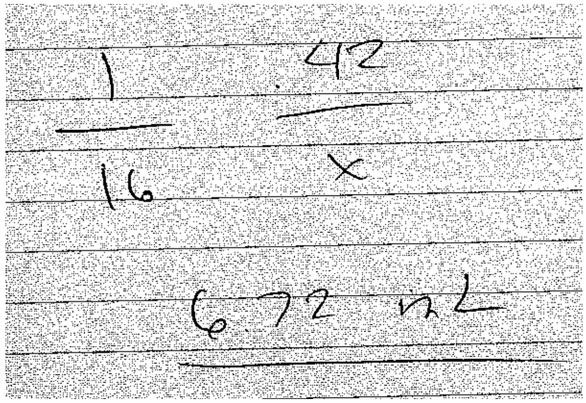
(d) "unit-less one" strategy, and (e) "random proportion" strategy. Different strategies are necessary for successful problem solving and some are entirely incorrect. These will be discussed in each subcategory.

a. **The "apples$_1$:oranges$_2$::apples$_2$:oranges$_1$" strategy**

Upon first glance, the title of this strategy seems counterintuitive to the way proportions are used in general mathematics problems because most mathematics problems involve direct proportions. To correctly solve a dilution problem in chemistry, an inverse proportion must be used to solve for the unknown variable. The "apples$_1$:oranges$_2$::apples$_2$:oranges$_1$" strategy is an inverse proportion strategy that can be used to correctly solve a dilution problem. To do this, a student must take the molarity from the first solution and multiply it by the volume of the second solution and set that equal to the molarity of the second solution divided by the volume of the first solution. This yields a commonly used formula: M1V1=M2V2.

An example of this type of strategy is shown in Figure 84. Student 9-2 used this strategy while solving M4 in the molarity problems task. This question asked students to create 16mL of a 0.42M solution from a 1M solution. The student set up an inverse proportion in which the 1M of the stock solution was divided by the 16mL. This was then set equal to the 0.42M divided by the unknown volume of the stock solution. In this case, the student provided the correct units at the end of the problem. The student did not use units within the problem.

Figure 84: Student 9-2's use of the "apples$_1$:oranges$_2$::apples$_2$:oranges$_1$" strategy in M4



This strategy also would work for the M5 problem where students were asked to determine the molarity of the resulting solution if the amount of the water was doubled. However, none of the students used this strategy on M5. Instead, they either verbally responded that it would be half the molarity because you are only changing the volume number (example Student 10-1) or they wrote out separate problems for each of the scenarios.

Students who used their direction proportional reasoning skills on these two problems had incorrect responses. These will be shown in the "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy and the "apples$_1$:apples$_2$::oranges$_1$:oranges$_2$" strategy later in this section. These are applications of the strategies used to successfully solve the PR diagnostic.

b. **<u>The "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy</u>**

A common solving strategy used in proportional reasoning problems is the "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy. This strategy is termed the "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy because the numerator and denominator are held constant over the equal sign. This type of strategy can be correctly used to solve problems where two variables are in a direct proportional relationship. This includes portions of a molarity related problem. When used in most dilution problems, though these methods are incorrect. These two uses will be described. In a dilution problem this would read: "molarity_1:volume_1::molarity_2:volume_2" and it could be mathematically manipulated to be equal to "molarity_1:molarity_2::volume_2:volume_2". This is shown in Figure 85.

Figure 85: Mathematical manipulation showing the application of "apples_1:oranges_1::apples_2:oranges_2" strategy to a molarity context

$$\frac{\text{Molarity}_1}{\text{Volume}_1} = \frac{\text{Molarity}_2}{\text{Volume}_2}$$

Multiply both sides by $\frac{\text{Volume}_1}{\text{Molarity}_2}$

$$\frac{\text{Volume}_1}{\text{Molarity}_2} \times \frac{\text{Molarity}_1}{\text{Volume}_1} = \frac{\text{Molarity}_2}{\text{Volume}_2} \times \frac{\text{Volume}_1}{\text{Molarity}_2}$$

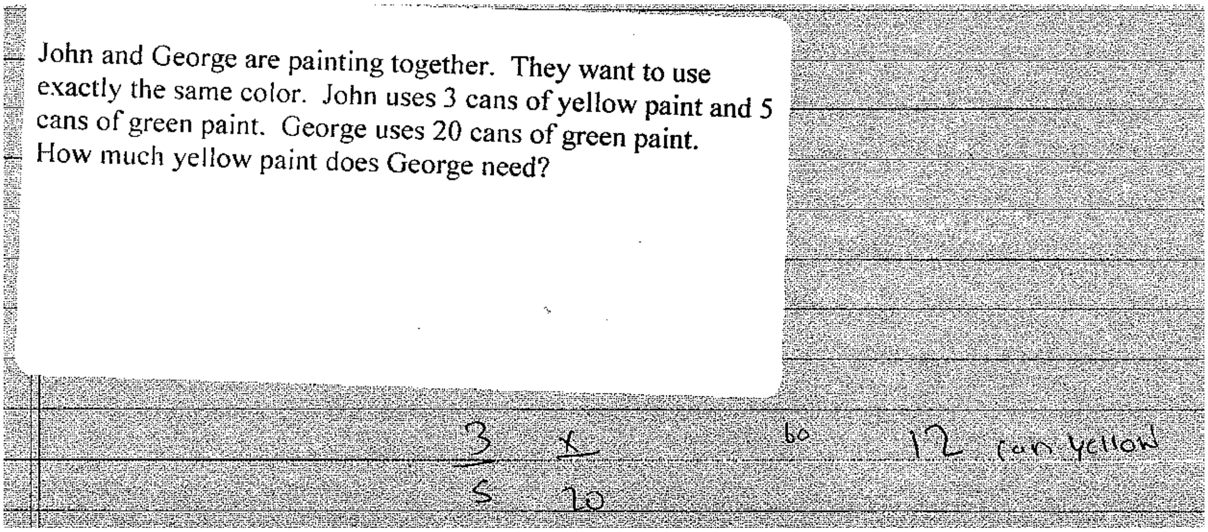
Cancel by division

$$\frac{\text{Molarity}_1}{\text{Molarity}_2} = \frac{\text{Volume}_1}{\text{Volume}_2}$$

To solve for a certain unknown variable, Molarity_1 in this case, the student can multiply both sides by Molarity_2. This yields an equation with an incorrect direct relationship between volume and molarity.

As discussed, this strategy does work in direct proportion problems. For example, this strategy was successful for Student 11-1 on the Paint 2 question in the proportional reasoning diagnostic as show in Figure 86. He was able to set the amount of cans of yellow paint used for John divided by the cans of green used for John equal to the unknown amount of yellow cans used for George divided by the amount of yellow cans used for George.

Figure 86: Student 11-1's successful use of the "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy in the proportional reasoning diagnostic

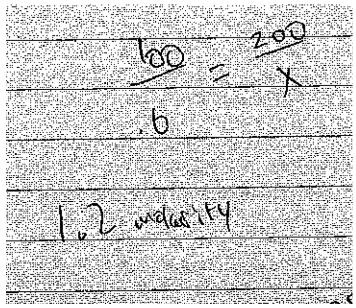


John and George are painting together. They want to use exactly the same color. John uses 3 cans of yellow paint and 5 cans of green paint. George uses 20 cans of green paint. How much yellow paint does George need?

$\frac{3}{5}$ $\frac{x}{20}$ 60 12 can yellow

The strategy does not work in inverse proportion problems. Student 11-1 also used this strategy in M3 following the hint in an attempt to find the molar mass but failed with this strategy. In M3 of the molarity problems task, Student 11-1 used this strategy after the hint to incorrectly solve for the molarity of a solution given the grams and volume. As shown in Figure 87, the student placed the volume of the original solution divided by the molarity of the original solution and set that equal to the volume of the resulting solution divided by the unknown molarity of the resulting solution. This yielded 1.2, double the amount of the original solution, because as the volume doubled, the molarity must double in the proportion as written.

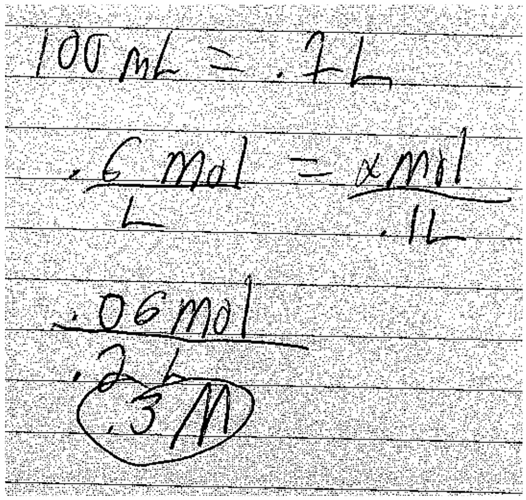
Figure 87: Student 11-1's unsuccessful use of the "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy in the molarity problems task



An example of a correct use of this strategy in a chemistry context is provided by Student 14-1. He turned the problem into a direct proportion problem using his robust understanding of molarity. This enabled him to determine the amount of moles of calcium chloride found in a 0.1L solution as compared to a 1 L solution. He also used a proportion to do his conversion factoring. He then used that number to calculate the correct response. This is shown in Figure 88. This is an example of a correct use of the "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy (from the PR diagnostic) within the molarity problems. He avoided having to construct an inverse proportion because he had a robust understanding of molarity and was able to separate M into moles/L. Because moles and molarity are in a direct proportion, this strategy was successful.

This ability to convert the inverse relationship into a direct proportion correctly because of a robust understanding was unique to Student 14-1. Student 10-1 did not use this strategy. Rather, he presumably used a "multiplier" strategy in his head, simply stating the answer.

Figure 88: Student 14-1's use of proportion in M5 in the molarity problems



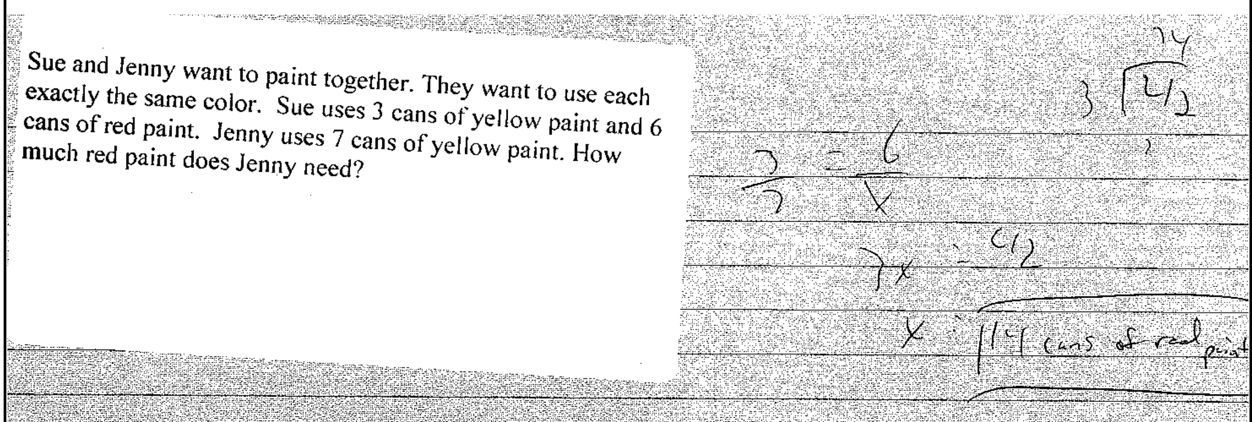
This strategy used without a robust understanding of molarity was unsuccessful for some students. For this strategy to work in the molarity context, the student must recognize molarity as an intensive quantity comprised of moles and liters. Student 14-1 completely sidestepped an inverse proportion by converting M into its extensive components. By doing this, he was able to create a direct proportion between two molarities using moles and liters.

c. The "apples$_1$:apples$_2$::oranges$_1$:oranges$_2$" strategy

Some students did dilution problems with a strategy that placed mL/ mL=M/M, which would have been correct if the given ratio was a direct relationship whereas in a dilution problem the relationship between molarity and volume is inverse. This type of error is the extension of basic direct proportional reasoning in mathematics type problems to a non-analogous inverse situation. If an apples$_1$:apples$_2$::oranges$_1$:oranges$_2$ strategy is used, the student would end up doubling the molarity of the solution.

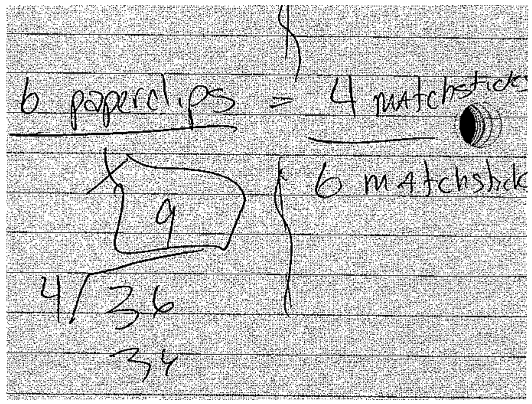
As discussed earlier, this is a good strategy for the direct proportion problems of the PR diagnostic. An example of the successful use of this same strategy can be seen in Figure 89 showing the work of Student 10-1. He set the number of cans of yellow paint used by Sue divided by the number of cans of yellow paint used by Jenny equal to the number of cans of red paint used by Sue divided by the unknown number of cans of red paint used by Jenny. This strategy was successful in general direct proportional reasoning problem and not in the chemical context when one the variables, molarity and volume, are in an inverse proportion.

Figure 89: Student 10-1's response to the Paint 1 question correctly applying the "apples$_1$:apples$_2$::oranges$_1$:oranges$_2$" strategy in the proportional reasoning diagnostic



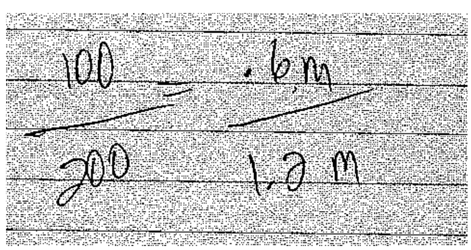
This strategy can be misapplied in the molarity context as in the case of Student 3-4. He was successful using this strategy in several questions on the proportional reasoning diagnostic, including the paperclip question as shown in Figure 90. The student set the number of paperclips on one side equal to the number of matchsticks on the other side. This strategy was successful for him in this direct proportion context.

Figure 90: Student 3-4's response to the Paperclip question correctly applying the "apples$_1$:apples$_2$::oranges$_1$:oranges$_2$" strategy in the proportional reasoning diagnostic



However, in the context of chemistry on M5 (dilution) after the hint, he applied this strategy and was unsuccessful (Figure 91). He set the volumes of the solution on the left side equal to the molarities of the solutions on the right side. In doing so, this yielded double the original rather than half as it should be. Throughout, it is the fact that volume is inversely proportional to molarity that causes these strategies to fail.

Figure 91: Student 3-4's response to M5 after the hint incorrectly applying the "apples$_1$:apples$_2$::oranges$_1$:oranges$_2$" strategy in the molarity problems task

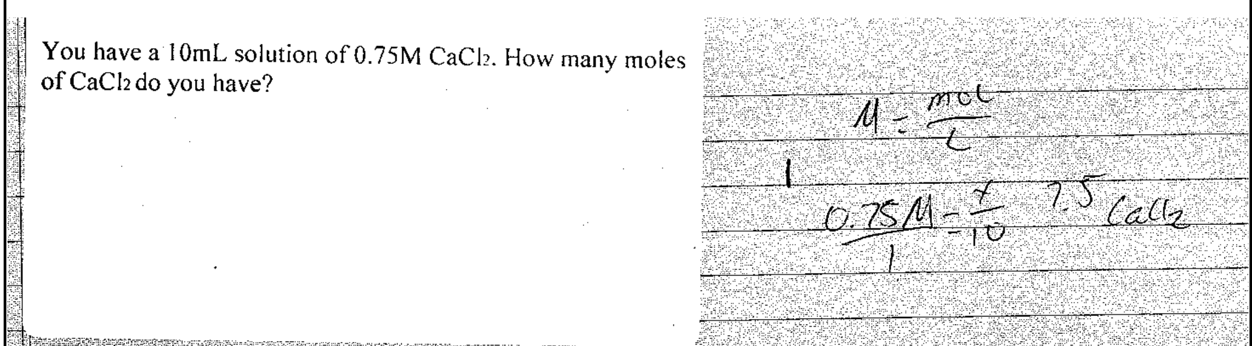


d. **<u>"unit-less one" strategy</u>**

Several students set up problems using the algorithm M=moles/L and filled in the values to solve for the unknown. In doing this, they added a one under the value for

M to set up a cross-multiplication problem. A student may place a one under the variable because they do not see the variable as a ratio itself. The one had no units and therefore is termed “unit-less one” errors in this dissertation. Students who set this up are not likely setting up a proportion consciously but are simply performing a solving strategy for cross-multiplication. This also suggests that even though they can write M=mol/L, they do not understand molarity as an intensive quantity. For example in Figure 92, Student 18-4 used the “unit-less one” strategy to solve for the number of moles of calcium chloride in solution. She was numerically correct because of the numbers in the problem. The 0.75M could be broken down into 0.75 moles per 1 L. This student did not convert mL into L and therefore resulted in an answer that was off by three orders of magnitude. The “M is moles” strategy was still confusing her. She is able to take the right side of the equation and break out the units to be moles/L but did not do the same for the left side for M.

Figure 92: Student 18-4’s use of the unit-less one for M1 in the molarity problems task after hint



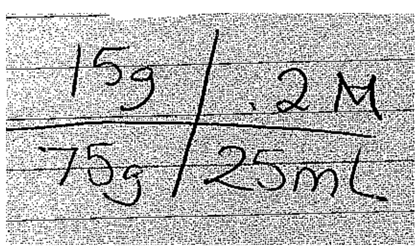
Students who used the “unit-less one” strategy did not recognize molarity or M as a ratio. They were not breaking molarity into its two components and leaving it as M. They then were forcing a "unit-less one" into the problem to allow for cross multiplication. Had the students recognized molarity as a ratio, they could have used a direct proportion to solve the problem.

e. <u>The "random proportion" strategy</u>

Several students employed the use of a canceling grid as shown with Student 22-3 in Figure 93. In the case of Student 22-3, the proportion represented in the grid is not logical. No units can cancel within this grid and the student ends up with g/g and M/mL. This is termed a "random proportion" strategy because the student is attempting to use all of the given numbers in the problem using this type of grid without logic. It may be that the student was randomly placing numbers in the grid to try to work out the problem.

Student 22-3 also used this "random proportion" strategy in M6 as shown in Figure 94. She created a random ratio to cancel M in the grid using 75g/1M. This was likely a misconception involving an "M is mole" mental model and the incorrect molar mass was equal to 1 capital M mole.

Figure 93: Student 22-3's "random proportion" strategy using a grid in M3 of the molarity problems task



Student 20-2 also employed a "random proportion" strategy in her solving of M1 of the molarity problems task as shown in Figure 95. She described the type of equation that she would need to solve this problem and she set up a proportion using molarity on the left and g/mL on the right.

Figure 94: Student 22-3's "random proportion" strategy using a grid in M6 of the molarity problems task

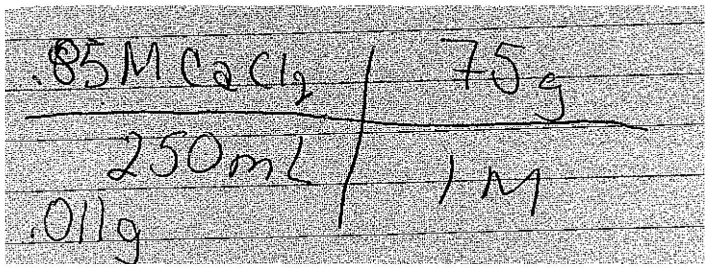
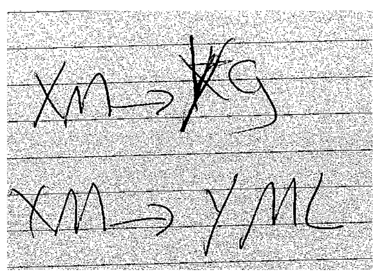


Figure 95: Student 20-2's "random proportion" strategy in M1 of the molarity problems task



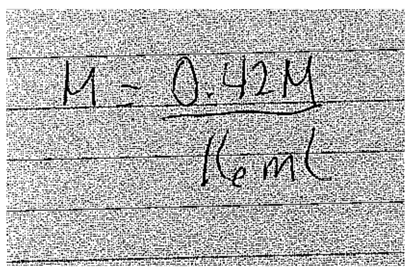
These two students used random ratios in an attempt to cancel out units so that their final answer could end with the desired unit.

8. **<u>Theoretical Statement 12: An "M is moles" misconception is not dispelled by providing a definition of molarity.</u>**

As discussed in Chapter VI, a vast majority of the students held the belief that "M is moles". This misconception persisted even after a hint to the contrary was provided. Several students used the following equation for the dilution and stock solution problems: Molarity=Molarity/Volume. This equation surfaced in student responses only after the hint was given. Students attempted to fit all of the numbers from the problem into the equation.

This strategy was used by Students 22-3, 11-1 and 8-4 on M4 after a hint and by Students 13-4, 16-2 and 18-4 after a hint. An example of this can be seen in Figure 96 for M4. The student set M equal to M/V after learning that M stood for molarity and it was equal to moles per liter. The student ignored some of this new information and attempted to fit all number in the problem into the formula.

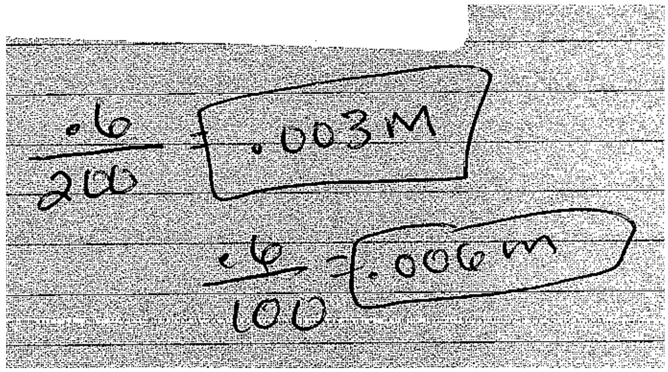
Figure 96: M=M/V strategy as used by Student 22-3 on M4 after a hint in the molarity problems task



Even after being given a hint, the student still placed the molarity in the spot of the mole in the molarity equation. Her "M is moles" mental model persists through a hint.

Student 16-2 continued to hold onto her "M is Moles" mental model after a hint in the Molarity Problems, shown in Figure 97. While the student got the wrong answer because she used the molarity as the moles in the equation, she set up two individual problems to figure out the answer, recognizing that the one number (moles) did not change while the volume number did. If she had viewed M as an intensive quantity, she would have had 0.6 moles per 1 L and been able to reason how many moles would be in .2 L and .1 L respectively using a direct proportion.

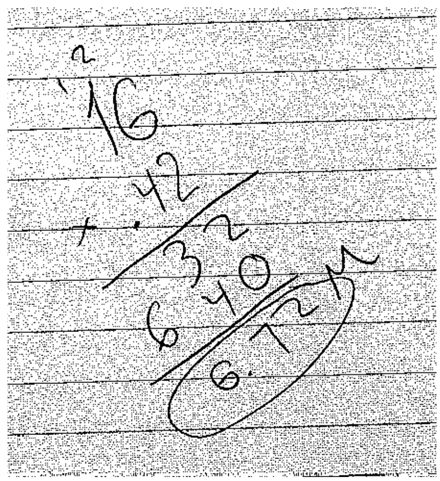
Figure 97: Student 16-2's response to M5 in the molarity tasks



9. **<u>Theoretical Statement 13: Students can solve some molarity problems through multiplication of volume and molarity.</u>**

Due to nature of the numbers in the problem for M4, students were able to simply multiply the 0.42M of the second solution by the 16mL of the second solution to find volume of stock solution necessary to make the desired solution. Student 21-2 was able to use this strategy without a hint as provided by Figure 98. Students 13-4, 15-1, and 20-2 were all able to use this strategy following the hint. No students used this strategy for the dilution problem, likely because the numbers in the problem did not afford this strategy.

Figure 98: Example of the MxV strategy used by students on M4 in the molarity problems task



It is important to note that this student and others were able to use this strategy and get the problem correct only because of the numbers within the problem. Because the stock solution was 1M, which translates to 1 mole/1L, the students encountered a problem in which the number they need was 1. It is possible that the student recognized that because the number was 1, they only needed to multiply by the volume to obtain the answer. This is a problem with a lot of chemistry problems in that many problems have unit values (1) within them.

10. **<u>Theoretical Statement 14: When students do not know how to solve a problem they react differently in chemistry and proportional reasoning questions.</u>**

When stymied in the PR problems, students perform random operations to the number to find an answer. They don't do that in the chemistry problems. Unlike the direct proportional reasoning context, students simply skipped problems that they didn't know how to solve in the chemistry context.

In the proportional reasoning diagnostic context, students did not leave answers blank if they did not know them. They pushed through problems and created their own ways to solve them. This included the “find the difference and add” method when faced with a matchstick and paperclip situation with, which they were unfamiliar. This is similar to the addition and subtraction found when faced with a stock solution problem, which with they are unfamiliar.

Similar to the Bread Question at Camp in the proportional reasoning diagnostic, when students misunderstood the question, they often chose the answer from the numbers in the question. For example:

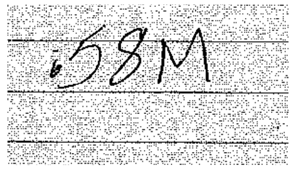
> [00:31:02.18] **Student 4-2**: Okay it says each day there are eight loaves of bread available for them to eat when there are ten campers so I don't know if there would be more because there are more campers or it doesn't sound like they would have more because there's more people it just sounds like. Loaves each day so I don't know. I don't understand what they're asking. It sounds like it just be eight loaves again

Student 4-2 misunderstood the problem and chose the answer from the question. This is similar to a student thinking that the molarity in a dilution problem simply doesn’t change because they don’t recognize that it is a ratio (Student 6-3, 16-2 and 17-3).

When faced with a problem that the student does not know the answer in PR diagnostic problems, the student can do one of four basic operations to the numbers: add, subtract, multiply or divide.

Student 6-3 attempted to solve the M4 stock solution problem by subtracting one molarity from the other (0.42M from 1M), shown in Figure 99. In the M5 dilution question, Student 2-1 and Student 4-2 just added the two volumes together. As described earlier, the student could have forced numbers into equations to use all available pieces of information from the problems. Another example of this can be found with Student 12-1, who creates her own equation for calculating molarity using molar mass multiplied by molarity equals molarity.

Figure 99: Student 6-3's use of subtraction in M4.



Students tended to skip problems that they did not know how to solve in the chemistry specific context. Whereas in the PR diagnostic problems they performed random operations with the numbers that were given within the problem to find an answer. This was a rare occurrence in the chemistry problems.

D. Conclusions regarding R3

This chapter described the theoretical statements along with their grounding in the data for the research question involving student solving strategies for molarity problems. To examine the notion that students may not possess the basic direct proportional reasoning skills necessary to complete the molarity tasks, the strategies used for the PR diagnostic were also analyzed.

This research found that students overwhelmingly had the ability to solve the direct proportion problems on the PR diagnostic. They were fairly uniform in their strategy choices for a correct response. A common strategy was to set up a proportion using the numbers given in the problem and solving for the unknown using either the "$apples_1$:$apples_2$::$oranges_1$:$oranges_2$" strategy or the "$apples_1$:$oranges_1$::$apples_2$:$oranges_2$" strategy. Barring any arithmetic errors, these two strategies yielded correct responses because of the direct relationship between the variables in the problems.

This is not to say that all students used proportions all of the time on the PR diagnostic. In fact, a common solving strategy that was found was the "solve for one and multiply" strategy.

Students who utilized this strategy did not use proportional reasoning to solve the problems. For some of the questions, students recognized an obvious multiplier within the numbers given and simply responded using that strategy. Several students used this strategy in combination with other strategies as a confirmation of their answer. Because of this, all that can be said regarding the “multiplier” strategy is that this strategy involved proportional reasoning for some students and not for others. Some students showed their strategy on paper and some simply stated their responses. It is possible that students who utilized the “multiplier” strategy were constructing proportions in their mind.

Although the students had great success on the PR diagnostic, especially compared to their performance on the CD and Molarity Problems task, students still made a variety of mistakes. These included the “additive” error, “magic halving” error, “random” error and the $apples_1$:$oranges_1$::$oranges_2$:$apples_2$ error. The first three strategies mentioned were more common with problems that may have been unfamiliar to students with fictional situations such as Mr. Short and Mr. Tall being measured with matchsticks and paperclips. The last strategy, apples:oranges::oranges:apples, was an interesting incorrect use of proportions that the student did not use the proportion correctly and because they did not maintain units across the equals sign. As shown in this chapter, students were averse to providing units while solving problems with some students tacking on units to their answers at the end or some showing no units. This lack of attendance to units is likely the major cause of this type of error.

As discussed in Chapter VI, most students did not recognize molarity as a ratio. Without the PR diagnostic, it could have been argued that these students lacked the basic proportional reasoning skills necessary to solve the problems. But through the analysis of the PR diagnostic, the data shows that student did indeed possess proportional reasoning skills. The type of problems that they were asked to solve were direct proportions whereas the molarity dilution problems involve an inverse proportion between molarity and volume. The majority of the mistakes made on the Molarity Problems task involved an application of direct proportion reasoning strategies to inverse relationships.

The data showed that proficiency with direct proportional reasoning problems on the PR diagnostic did not support inverse proportional reasoning in molarity problems. Students either applied the basic direct proportional reasoning strategies used in the PR to the an inverse situation or they used strategies such as a random proportion using a grid, or a unit-less one.

Of course, the two students who had a robust understanding of molarity used the correct strategy representing an inverse relationship with $apples_1$:$oranges_2$::$apples_2$:$oranges_1$. It is not surprising that the two students who had a robust understanding of molarity as an intensive quantity with proper units were able to do well on other chemistry problems. It is possible that by the time they were introduced to molarity in high school chemistry that they had already mastered the other concepts tested on the CD.

VII. CONCLUSIONS AND IMPLICATIONS OF RESEARCH

The previous chapters presented findings developed from data and analyses that speak to each of the three research questions addressed by this research study. This chapter serves to summarize the findings and to situate the findings within the theoretical frameworks discussed in Chapter I and the literature presented in Chapter II. The chapter concludes with implications and plans for future research.

A. **Conclusions**

Specific conclusions are presented in this section as they relate to the research questions of this dissertation work. These include the following:

1. Students who have an intensive understanding of molarity are therefore able to reason that the amount of substance changes as the volume changes with jars of a constant molarity.
2. Students are able to reason qualitatively about concentration as an intensive quantity.
3. Students do not recognize the intensive nature of molarity or the letter M.
4. Students apply direct proportional relationship reasoning to inverse relationships.

These conclusions will be discussed with respect to implications in a later section within this chapter.

The results from this study suggest that students generally were successful problem solvers on direct proportion problems and that they qualitatively viewed concentration as an intensive quantity. These conclusions all speak in one way or another to the theoretical framework and literature review provided in Chapters I and II. Specific articulations will be presented for these.

The structurally similar tasks in this study were designed to elicit a similarity transfer (Schwartz et al. 2008; Brown 1989) between the domain-general context of the concentration of paint to the domain-specific context of molarity. When presented with the structurally similar tasks, students were able to account for the color and the length of the blocks using colored cubes. They could manage the proportions and total number of cubes to show their understanding of concentration and amount.

Student performance on the molarity tasks revealed that a variety of transfers occurred among the students. First, three students recognized the ratio structure (Schwartz et al. in press) within molarity. That is, they recognized that molarity was an intensive quantity that involved a direct proportion to the amount of calcium chloride and an inverse proportion to the volume. This suggests that they had successfully engaged in similarity transfer of the concept of concentration.

With only three students recognizing the intensive structure of molarity, that means that there were 21 out of the 24 students who viewed "M", or molarity as an extensive quantity, with most students thinking that "M is moles." These students failed to recognize the similarity between the tasks and their performances can all be categorized as various forms of negative transfer.

The students who held an "M is moles" mental model fell into two categories. First, a large group of students maintained their "M is moles" mental model through the contrasting cases of the SVDM and DVSM task. These students constructed a mental model that was logical to them and continued to used the same model for the contrasting case despite its incorrectness, an error of the sort foreseen by Bransford et al (2000). These students didn't recognize an internal contradiction because the mental models that they constructed viewed the number before M as representing moles and the volume therefore did not present a contrasting case to them, as seen in a similar proportional reasoning case by Schwartz et al. (in press). These students exhibited negative transfer by first not recognizing the similarity between the tasks and second by transferring an incorrect mental model between tasks. Students who associated M with something other than calcium chloride also exhibited negative transfer.

The second group of "M is moles" students did recognize the contradiction within the contrasting cases and did change their mental models, but to another incorrect model fueled by their "M is moles" mental model. Because they viewed M as representing moles, they were faced with a dilemma in the DVSM task as the volume changed and the number before M remained the same. Similar to the findings of Kelly et al. (2010), these students did not attend to water in their mental models. This further complicated the contrasting case for the students, because if the number remained the same, what could possibly explain the change in volume? These students used the size of the molecules to explain this change. This is a variation of the negative transfer exhibited by the "M is moles" students, in which students interpreted the change in volume to involve the size of them molecules.

Interestingly, all of the students (except for the one student who associated M with volume only) were able to successfully reason that there would be more calcium chloride in the 0.10 M jar than the 0.05 M jar despite their incorrect mental models. In this case, students were able to reason through the SVDM task using these mental models due to the direct proportionality of the relationship between molarity and moles. It isn't until the DVSM contrasting case that the flaws in the students' mental models are fully revealed. The DVSM task forced students to reason with volume and the data show that students reacted similarly to Chinn and Brewer (1993) in that students rejected the contrasting information or changed the data to fit their mental model. In no case did a student face the contrasting case and change his or her model to a correct scientific model. In the cases of the students who use size, they did change their models, but to another incorrect mental model. Similar to Fassoulopouls et al. (2003), this study found that students were inconsistent within their own responses (especially in their use of the word molarity) and they changed their mental models across contrasting cases to other incorrect mental models. Similar to Nakhleh et al. (2005), some students held frameworks that were not widely applicable and therefore needed to change them when faced with anomalous data. This likely stems from students not recognizing M as a ratio because it is "disguised" (Ochiai 1993). That is, the simple unit "M" suggests a simple and therefore, extensive quantity when the units are not readily available to suggest the intensive nature of molarity. This is further evidence for the use of contrasting cases in research studies to focus student attention to other aspects of a concept.

This is further evidence that students do in fact have difficulty reasoning with intensive quantities (Nunes et al. 2004), molarity specifically in this context. Students did not recognize the ratio structure within the molarity task or in the letter M itself. Instead, students recognized molarity or "M" as an extensive quantity, usually moles. They therefore failed to exhibit a similarity transfer of the concept of concentration. This is an example of negative transfer as described by Chen (1989), where the students use an inappropriate solution principle for solving the problem. This is similar to the findings of mixing task studies involving the taste of orange juice (e.g. Schwartz & Moore 1998; Calyk 2005; Gabel & Samuel 1986; Harel et al. 1994; Stavy et al. 1982). Students were able to reason about the concentration of color qualitatively as an intensive quantity but still negatively transferred their direct proportional reasoning skills to an inverse relationship in dilution problems.   Combined with the findings from the third research question, we found that students had an isolated view of concentration in real life but did not transfer the old knowledge to the new situation.

The students had a variety of different levels of success in solving of the Molarity Problems and the CD problems. This ranged from correct responses, to skipping the problems altogether to the ability to calculate molar mass to the ability calculate a mole. A majority of the students had the ability to calculate molar mass but not all of them had an understanding of how to use it. Findings from this study are similar to Ochiai (1993) where he suggests that students use the C1V1=C2V2 when in doubt. In this study, students tended to calculate molar mass when they were unsure what to do. Students with these types of misconceptions can still score well on algorithmic problems (such as the SVDM task).

The data showed that student ability to calculate moles from molar mass and grams did not indicate an understanding of molarity as an intensive quantity. Appendix D shows a mapping of Theoretical Statements by student. Briefly, if a student had an intensive view of molarity (Theoretical Statement 2), they were likely to be successful solvers of molar mass and calculation of a mole problems as assessed by C1 (Theoretical Statement 7). Furthermore, they were likely to be able to reason about an amount of substance change in the SVDM task (Theoretical Statement 4). The students who had an extensive view of molarity (Theoretical Statement 3) had a wider variety of responses (i, ii, iii, and iv) with a variety of problem solving abilities (Theoretical Statements 8-10). However, one pattern emerged with respect to overlap of categories for Theoretical Statement 4 and Theoretical Statement 8. Half of the students in Theoretical Statement 4 were also in Theoretical Statement 7. The other half of the students were in Theoretical Statement 8. It is worth reminding that these students were deemed to have a conceptual understanding of the calculation of a mole, meaning that they were able to explain why they were using the calculations that they chose. There were four other students who were correct in their calculation that did not belong in this Theoretical Statement: 9-2 and 17-3 who were only algorithmic problem solvers and 24-3 and 13-4 who were unable to reason about the amount of substance changing in the SVDM task. The relationships shown by Appendix D provide further evidence that being able to calculate a mole as assessed by C1 is on the path for having ability in molarity for some but no all students. This provides grounding for Table 14 as presented later in this chapter.

A likely reason for these findings is that, while they understand some things about moles and molar mass, they present an "M is moles" mental model that is incompatible with an intensive view of molarity. If a student didn't know what molarity was or did not view it as an intensive quantity, he or she would therefore be unable to reason through molarity tasks in an intensive manner. Students who viewed M as moles would not attend to the volume because they didn't recognize it as a factor. The "M is moles" mental model was so pervasive that it was not dispelled after being provided with the definition of molarity in the hint. This problem meant many students acted as if molarity and volume might be directly proportional, which is the case for moles and volume and for moles and molarity. This helps to explain the SVDM task in Chapter V where several students were able to reason that the amount of calcium chloride had changed as the number before M changed while holding an extensive view of molarity. Because moles and molarity are directly proportional, students were able to answer this question correctly without attending to water. However, in the DVSM task, they maintained an "M is moles" mental model and because moles are directly proportional to molarity and the number in front of M stayed the same, they believed that the change in volume had to be solely the water or potentially the size of the molecules changed.

Another group of students were able to calculate molar mass but were unable to calculate the number of moles using the molar mass and grams. These students had a very basic ability (calculating molar mass) that they were unable to apply in the situations provided in the interview tasks and diagnostic problems. The relationship of the abilities of students with one another can be represented as a hierarchy, as shown by Table 14.

Table 14: Fluency in different features of the tasks and what that means

		Implies Ability in:		
		Molar Mass	**Calculation of Mole**	**Molarity**
Ability in:	**Molar Mass**	√	X	X
	Calculation of Mole	√	√	X
	Molarity	√	√	√

This table is based on the assumption that students did possess proportional reasoning skills as shown in earlier chapters. This implies that successful math students are not inherently successful at solving chemistry math problems. The relationship of abilities presented in the table indicate that if a student had facility in calculating the molar mass of a substance, this was not an indicator that the student had an intensive understanding of molarity as a ratio nor did it indicate that he or she could use the molar mass to calculate a mole. If a student was able to calculate the amount of moles of a substance, this was an indicator that the student did have facility in the calculation of molar mass. This ability would probably support reasoning about molarity as an intensive quantity of moles per liter if a student had a robust understanding of molarity. Short of that students had breakdowns in conceptual knowledge in chemistry that compromise what they can do as shown in the SVDM and DVSM tasks along with the Molarity Problems. If a student had an intensive view of molarity as a ratio, he or she was therefore able to both calculate the molar mass of a substance and apply it to the calculation of a mole of that substance.

As mathematics education researchers are aware (Nunes, Desli, and Bell 2004), intensive quantities such as molarity not only involve a direct proportion relationship but also an inverse proportional relationship. The inverse proportional relationships should be treated carefully, so students do not develop a misconception that all proportional relationships are indirect.

B. **Articulation of findings with literature**

In Chapter II, literature was presented showing a gap in the alternative conceptions research regarding molarity as an intensive quantity. This study has presented theoretical statements grounded in the data regarding student alternative conceptions of molarity and respective solving strategies for molarity problems. This study shows that students do indeed possess proportional reasoning skills, but this does not imply the ability to transfer these skills to a domain-specific context, such as molarity. Student interpretations molarity as an intensive quantity are crucial for this skill to transfer, otherwise students apply direct proportions to inverse relationships.

Many of the mistakes that students made with the molarity problems would be remedied if students held an intensive view of molarity and if they kept track of their units within the problems. This is similar to the findings of Mitchelmore, McMaster & White (2007) that were discussed in Chapter II. Indeed, the results of this study agree with Cai (2002)'s report that gaining information about how a student solves a problem is more important than the correct solution. This dissertation work revealed a variety of solving strategies that may have simply been scored as incorrect. Upon further investigation, the mistakes on the dilution problems were simply a misuse of a direct proportion for an inverse relationship. This finding is consistent with Stavy & Tirosh (1996)'s discussion of improper use of the "More of A then More of B" intuitive rule applied to an inverse relationship.

This dissertation work shows the importance of the intersection between underlying basic domain general mathematical concepts, such as ratio, and domain-specific contexts such as molarity. As suggested by Clark et al. (2003), students have difficulty when they confuse molarity (ratio realm) with the decimal numbers and real numbers realms. This is an extensive view of molarity that is incompatible with reasoning through tasks and problems in solutions chemistry. Therefore, students need to recognize that molarity belongs in the "ratio only" realm and cannot be manipulated like a real number or decimal number, despite it looking like one. This requires careful attention in addressing ratio as a separate entity and then introducing molarity as an example of that type of entity.

Finally, this dissertation work provides further evidence for the inclusion of particulate level drawings in student interviews to reveal student understandings of concepts. As the findings from Kelly et. al (2010) suggested, students held a variety of understandings that were only made evident through particulate level drawings. Student reasoning involving the size of molecules is an example of this type of understanding that was only fully revealed through particulate level drawings in this dissertation work. At several points in the analysis, it was the student drawings that allowed for characterization of student reasoning. An example of this was found in the SVDM task, where student drawings revealed that students were successfully reasoning though the task, despite having an "M is moles" mental model. The task of having the students draw at the particulate level also revealed how students were reasoning incorrectly about the DVSM task.

This dissertation work also gives insight into the literature presented in Chapter II about students understandings of solutions. Many of the studies conclude that students did not have a conceptual understanding of concentration (Sanger & Greenbowe 1997; Ross & Mumby 1991; Calyk 2005; Schwartz & Moore 1998; Gabel & Samuel 1986; Harel et al. 1994 and Stavy et al. 1982) in chemistry contexts or in a general context involving the taste of juice. This research suggests that students did not recognize the concentration (or the taste) as an intensive quantity. A likely reason that students had difficulty with this concept could also stem from the fact that students did not recognize that water was part of the concentration (Boo & Watson 2001; Kelly et al. 2010) because they viewed concentration as an extensive quantity.

C. <u>**Conclusions and Implications**</u>

Based on the work in this thesis, a set of well-grounded conclusions can be presented about the ways in which these students understand molarity in the context of solutions chemistry. These are summarized in the first column of Table 15. Students can hold either an intensive view or an extensive view of molarity. Students who hold an intensive view of molarity and know the units of molarity can reason through a variety of tasks and are considered to have a robust understanding of molarity. Students who hold an extensive view of molarity fail to understand molarity as involving both a solvent and a substance, leading to a variety of misconceptions but mostly an “M is moles” mental model that sometimes involves size of molecules. These same students did have an intensive view of concentration of color in the structurally similar tasks and they had facility with direct proportion calculations. Many of their mistakes with molarity problems involved the application of a direct proportion to an inverse relationship between molarity and volume.

The findings from this study have implications that reach to the two areas mentioned in Chapter I (Introduction): Chemistry Education and Standards. Table 15 catalogues the conclusions that can be made from the data within this study and the implications as they apply to the different areas of interest. Each of these conclusions and implications will be discussed in depth within this chapter along with specific recommendations.

<table>
<tr><th colspan="3">Table 15: Conclusions and Implications</th></tr>
<tr><th>Conclusion</th><th>Area</th><th>Implication</th></tr>
<tr><td rowspan="2">1. Students who have an intensive understanding of molarity are therefore able to reason that the amount of substance changes as the volume changes with jars of a constant molarity [See Appendix D]</td><td>Chemistry Education</td><td>Teachers should assess a student’s understanding of molarity as an intensive quantity and pay careful attention that the numbers in the problems do not include a unit value or a multiplier.</td></tr>
<tr><td>Standards</td><td>Molarity should be explicitly addressed in the science standards as an intensive quantity.</td></tr>
</table>

Table 15: Conclusions and Implications		
Conclusion	Area	Implication
2. Students are able to reason qualitatively about concentration as an intensive quantity.	Chemistry Education	Teachers can teach concentration qualitatively using structurally similar tasks as a precursor to teaching molarity.
	Standards	Concentration should be assessed at a qualitative level so that student understandings of concentration can be gauged.
3. Students do not recognize the intensive nature of molarity or the letter M. [lack of similarity transfer]	Chemistry Education	Explicit connections between structurally similar tasks and molarity need to be made for the students to highlight the intensive nature of molarity. Teachers could also initially avoid using M and instead use mass per volume.
	Standards	Standards should specifically address student interpretations of the letter M and their definitions for molarity.
4. Students apply direct proportional relationship reasoning to inverse relationships. [negative transfer]	Chemistry Education	Teachers should address student direct proportional reasoning skills and describe how they can and cannot be applied to molarity situations.
	Standards	Assessment items should address this alternative conception with distractors for formative use for teachers.

1. **<u>Conclusions 2 and 4</u>**

Students have a lot of personal experience with concentration in their day-to-day lives that can be used as starting points in teaching molarity to students in chemistry. A conceptual understanding of concentration is crucial to understanding molarity and explicit connections from structurally similar tasks can aid in developing an understanding of molarity as an intensive quantity.

Because students also have experience with direct proportions in other subject areas, it is important to focus student attention on appropriate use of direct proportions in cases where two quantities do vary directly. This would then highlight that there are cases where direct variation does not occur. A good place to start this in chemistry is with molarity, since there are many problems, such as those used in the Molarity Problems portion of this research, where the inverse relationship of molarity and volume is critical. These conclusions have implications for Chemistry Education and Standards, which will be discussed below.

a. **Chemistry Education**

Because similarity transfer of prior knowledge about concentration is needed to reason about molarity, it is important that chemistry teachers probe student prior knowledge regarding concentration and use it as a starting point to teach molarity. By focusing on the responses afforded by a visual cue (such as painted blocks in a structurally isomorphic task as described in Figure 18 in Chapter III), teachers can link a qualitative understanding of the intensive view of concentration to an intensive view of molarity. For instance, a teacher can draw a parallel between what the students just constructed using the colored cubes (SHDC task) with molarity using a solution with varied concentrations of a substance that has color, such as copper nitrate. This would be a version of the SVDM but with an added visual cue. The students can then draw parallels between the amount of substance being added to the constant amount of water and see that the color gets more intense with a higher concentration, much like the blocks got redder with more red paint. Finally, this task can then be paralleled with a set of clear, colorless solutions to challenge students' understandings of molarity and concentration (SVDM task).

After these parallels are drawn, a teacher could then do the same for a constant concentration among different volumes by using the structurally isomorphic DHSC tasks with colored blocks and then the color version of the DVSM task, where solutions of different volumes of the same concentration of copper (II) nitrate are used to show that the concentration is the same in the different solutions, and that concentration is an intensive quantity. Then, this can be followed by a set of clear, colorless solutions (such as the DVSM task used in this dissertation research). The students are then faced with a situation that pushes their understanding to include the volume. Students can then connect the ratio relationship of concentration in the structurally similar tasks to the chemistry concept of molarity. Throughout these tasks, students can also be prompted to represent the solutions at the particulate level, to strengthen their understanding that molarity refers to a concentration of particles, not just symbols. This will in turn enable students to understand the more complex topics in chemistry such as equilibrium and other courses in the future.

The macroscopic visual connection made with a numeric concentration of a solution with color, such as copper nitrate, could act as a bridge between the macroscopic level and an intensive understanding of molarity. A future study could include the addition of this task to study student progressions of understanding molarity in this way.

When students are introduced to molarity as a quantitative way to represent concentration, teachers should address direct proportional reasoning skills and describe how they can and cannot be applied to molarity situations. For example, if a student maintains the units of molarity as mol/L in a dilution problem, a direct proportion can be used to find the number of number of moles in a solution with a volume less than 1 L as an intermediate step. That number can then be used to calculate molarity. Otherwise, students need to be aware of the inverse relationship between molarity and volume and that a direct proportion will not work in that context.

b. **Standards**

Specific standards addressing the various relationships between extensive quantities and the intensive quantity of molarity need to be considered. The following are recommended standards that stem from student alternative conceptions of molarity:

- *Students should be able to reason about molarity as a ratio*
- *Students should be able to provide the units for molarity (moles/L)*

Specific attention to molarity as a ratio in the standards would enhance student understandings and they would therefore be able to reason through a variety of molarity situations in other subject areas.

Qualitative understandings of concentration need to used for formative assessment. The findings from this research suggest that the following standard could improve formative assessments of student understanding of concentration.

- *Students should be able to reason about concentration qualitatively as an intensive quantity.*

Students likely possess qualitative understandings of concentration and this type of question can provide a continuum of understanding for a teacher to work with for each student.

2. **Conclusions 1 and 3**

As shown in the data analysis, students who had a robust understanding of molarity were therefore able to reason that the amount of substance changes as the volume changes in the DVSM task. A student was deemed to have a robust understanding if they had an intensive view of molarity and were able to recognize the units. Many students held extensive views of molarity and these types of misconceptions can be addressed while assessing for a robust understanding of molarity. This conclusion has implications for Chemistry Education and Standards, which will be discussed below.

a. **Chemistry Education**

Chemistry teachers can use this conclusion to better assess students' understandings of molarity by probing their understanding of molarity as an intensive quantity. The teacher should pay careful attention to not use numbers within the problem with obvious multipliers or unit values. As shown in this analysis, some students may be able to solve the problems with no understanding of molarity. A student can also correctly answer a question involving a change in the amount of substance when the volume is held constant due to the direct proportion between moles and molarity. To assess a student's understanding of molarity as robust, the teacher should present molarity in three stages: Constant molarity with different volumes (DVSM), varied molarity with constant volumes (SVDM), and different molarities with varied volumes but containing the same amount of substance. Introducing different molarities with different volumes but with same amount of substance forces students to further reason with molarity as a ratio instead of solely the amount of moles (extensive). Teachers can also use particulate representations as an instructional tool as well as an assessment tool. Each of these scenarios assess a different aspect of a student's understanding of molarity and specific standards to assess their understandings are shown in the next section.

b. **Standards**

Currently, standards do not address student understandings of molarity as an intensive quantity. As shown in this study, many students do not hold this view of molarity even though current standards implicitly assume this understanding. The findings from this study suggest that the following standards could assist teachers in pinpointing misconceptions and place a focus on molarity as a ratio.

- *Students should recognize that M stands for molarity*
- *Students should know that the relationship between moles and molarity is direct*
- *Students should know that the relationship between molarity and volume is inverse*

Simply implying that students should be able to reason about intensive quantities that are potentially masked by an abbreviation (such as M standing for moles/L) is not enough. Molarity is a complex topic for students to grasp due to its intensive nature and special attention should be paid that students actually do understand molarity as a ratio.

C. **Future Studies**

Future work could explore this scaffolded approach to the teaching of molarity as an intensive quantity through a curricular unit. A pre/post Molarity Assessment could be administered to high school students before and after use of the scaffolded unit. The unit would consist of:

- structurally isomorphic SHDC task
- SVDM task with visual cues
- SVDM task
- structurally isomorphic DHSC
- DVSM task with visual cues
- DVSM task

Making explicit connections between everyday familiar experiences and molarity could help students understand molarity as an intensive quantity.

Figure 100 shows potential items for a Pre/Post Molarity Assessment that could address the alternative conceptions found in this study. Questions 6 and 7 in Figure 100 probe student understandings of the relationships between molarity and moles and molarity and volume. It is important for a chemistry teacher to know how a student views these relationships so that he or she can help address these misconceptions through teaching. An “M is moles” understanding could be revealed in question 6 if a student responded with B and C in combination with other item responses.

Questions 13 and 14 are related to the SHDC and DHSC tasks with Question 14 acting as a direct link between structurally similar tasks and molarity. Figure 100 contains recommended items associated with a proposed standard involving an molarity as an intensive quantity. Questions 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 15, and 16 all address the various features of molarity as an intensive quantity with distractors that offer insight into student understandings of molarity as an extensive quantity.

Question 3 was designed to assess student understanding of the relationship between moles and molarity as well as to highlight the alternative conception that decimals are less than zero. This question is similar to the SVDM task just with different numbers. If a student chooses B, the teacher can infer that the student believes decimals are less than zero. If a student chooses A, the teacher can infer that the student is able to reason that an increasing number before M indicates more substance.

Questions 8 and 9 were designed similar to the DVSM task to assess student understandings of molarity as an intensive quantity. An “M is moles” understanding is revealed if a student chooses D for Question 8 or any option but D for Question 9. Student beliefs that the size of the molecules change can be revealed if a student chooses A for Question 9.

Questions 1, 2, and 8 were designed to address the scenario where the total amount of substance is the same among the jars but the molarities differ. If a student chose B for Question 1, the teacher would be alerted to the fact that the student is viewing M as an extensive quantity.

To ascertain student conceptions of the letter M, Question 4 was created so that teachers could pinpoint any misconceptions associated with the abbreviation. If a teacher knows through a formative assessment that a student believes the M stands for moles, his or her lesson could change to address this misconception. Equally important to a robust understanding of molarity is to understand the intensive nature of molarity. To do this, a student must know the units of molarity and Question 5 addresses this declarative knowledge piece. To assess student understandings of intensive and extensive quantities, questions 11 and 12 were created. If a student believes that molarity is an extensive quantity that was directly measure (D), then a teacher can address this misconception through activities or discussion.

Finally, molarity calculation problems that did not contain easily identifiable multipliers or unit values were created (Questions 15 and 16). These are similar to the questions offered in the Molarity Problems task but with specific attention to the numbers used in the problems. Random strategies will become more clear with a problem that does not contain a unit value. Direct proportion use on inverse relationships will also become more clear.

The final four questions of the proposed assessment were directly taken from the Molarity Problems task and converted to multiple choice using various student responses as the distractors. They are designed to assess student understanding of molarity as a ratio and also to elicit the various alternate conceptions in the distractors. For example, if a student responds with A on Number 17, the teacher knows that the student likely has an “M is moles” misconception. Student application of a direct proportion to an inverse relationship or an “M is moles” misconception can be detected in Question 19.

Based on evidence from this study, this is a natural connection between student prior knowledge and skills to domain-specific content. The assessment could help teachers situate students based on their understandings of molarity and help address misconceptions that currently go unnoticed in classrooms and in standardized exams.

Figure 100: Potential items to assess robustness of understanding of molarity

1. Which of the following jars contains the most calcium chloride?
   Jar A: 100 mL of 0.05M $CaCl_2$
   Jar B: 200 mL of 0.10M $CaCl_2$

a) Jar B has twice as much calcium chloride as Jar A
b) Jar A has twice as much calcium chloride as Jar B
c) Jar A and Jar B contain the same amount of calcium chloride
d) The amount of calcium chloride cannot be determined from the information given

2. Two students are in lab. Student A dissolves 11 grams of calcium chloride into 125 mL of water. Student B dissolves 22 grams of calcium chloride into 250 mL of water. Which student has a solution with the greatest concentration?
a) Student A
b) Student B
c) Both Student A and Student B have solutions with the same concentration
d) The concentration of the solutions cannot be determined from the information given

3. Place the following jars in order of increasing amounts of calcium chloride: 0.50M, 0.10M, 0.25M, and 0M.
a) 0M, 0.10M, 0.25M, 0.50M
b) 0.50M, 0.25M, 0.10M, 0M
c) 0.10M, 0.25M, 0.50M, 0M
d) 0M, 0.50M, 0.25M, 0.10M

4. What does M stand for in 0.05M $CaCl_2$?
a) moles
b) mercury
c) strength
d) element M
e) molarity

5. What are the units for molarity?
a) mol
b) L
c) mol/L
d) L/mol
e) M/L

6. The relationship between molarity and moles is:
a) an inverse relationship
b) a direct relationship
c) they are the same thing
d) there is no relationship between molarity and moles

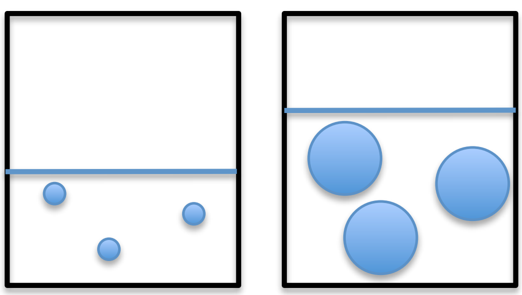
7. The relationship between molarity and volume is
a) an inverse relationship
b) a direct relationship
c) they are the same thing
d) there is no relationship between molarity and volume

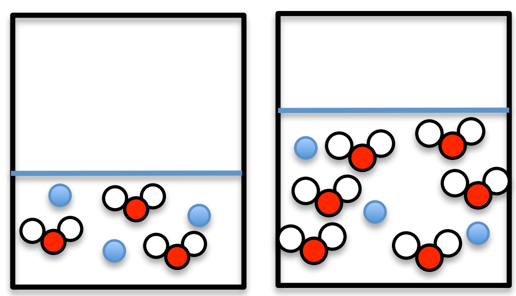
8. , which of the following jars has the most calcium chloride?

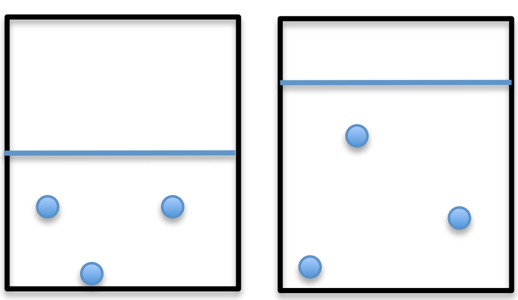
Jar A: 100mL of 0.15M KCl
Jar B: 150mL of 0.15M KCl
Jar C: 200mL of 0.15M KCl

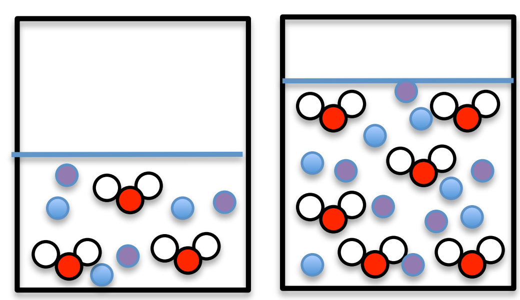
a) Jar A has the most calcium chloride
b) Jar B has the most calcium chloride
c) Jar C has the most calcium chloride
d) All of the jars contain the same amount of calcium chloride
e) There is not enough information to determine the amount of calcium chloride in each jar.

9. Which of the following submicroscopic representations best depicts Jar A and Jar C from Question 8?

A


B


C


D


10. Which of the following is most concentrated?
    Jar A: 100mL with 15g of KCl
    Jar B: 50mL with 10g of KCl
    Jar C: 250mL with 20g of KCl
a) Jar A
b) Jar B
c) Jar C
d) All of the jars have the same concentration of KCl
e) e) There is not enough information to determine the concentration of KCl in each jar

11. What does intensive quantity?
a) one number that is a ratio of two numbers
b) one number
c) a number that can be measure directly

12. Molarity is:
a) an intensive quantity composed of two extensive quantities
b) an extensive quantity composed of two intensive quantities
c) an intensive quantity that was directly measured
d) an extensive quantity that was directly measured

13. One person with a very small house decides to paint his house a shade of pink that requires 3 red cans of paint and 3 white cans of paint to obtain that shade of pink. His neighbor decided to paint her house the same shade of pink but her house is three stories taller than his house. What should her paint order be when she goes to the store?
a) She needs 3 cans of red paint and 3 cans of white paint because it is the same color.
b) She needs 21 cans of red paint and 3 cans of white paint because it is the same color and her house is taller.
c) She needs 12 cans of red paint and 12 cans of white paint because the color is a ratio and as the height increases so does the amount of paint needed.
d) She needs 6 cans of red paint and 6 cans of white paint because her house is three times the size of his and you add three to both.

14. In the painting scenario in Question 13, which of the paint concentration features can be related to molarity of a solution of KCl and how?
a) The red paint is like the KCl and the white paint is like the water. The height of the house is the total volume of solution to be filled.
b) The red paint is the K and the white paint is the Cl. The height of the house is the total volume of solution to be filled.
c) Molarity and concentration of paint cannot be related.

15. You have 0.08 moles of $CaCl_2$ in lab. How much water would you need to create a 0.61M solution?
a) 131mL
b) 49mL
c) 8mL
d) 8000mL

16. You have a 2.4M stock solution of KCL and need to make 17mL of a 0.42M solution. How much stock solution do you need to use to create this solution?
a) 2.975mL
b) 97.14mL
c) 0.059mL
d) 1.98mL

17. You have a 10mL solution of 0.75M $CaCl_2$. How many moles of $CaCl_2$ do you have?
a) 0.75 moles
b) 0.0075 moles
c) 0.075 moles
d) 7.5 moles

18. You have 15g of $CaCl_2$ in 25mL of distilled water. What is the molarity of the solution?
a) 5.45M
b) 0.6M
c) 600M
d) 0.00545M

19. You have a 100mL of 0.6M $CaCl_2$ solution. Your lab mate adds 100mL of deionized water to your solution. What is the resulting molarity?
a) 0.6M
b) 0.3M
c) 1.2M

20. You have 250mL of a 0.85M $CaCl_2$ solution. How many grams of $CaCl_2$ are in the solution?
a) 0.374 grams
b) 374 grams
c) 23.375 grams
d) 32.34 grams

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Wink, D.J., Daubenmire, P.L., Brennan, S.K., Cunningham*, S.A: "Bringing Social and Personal Perspectives into Stardards-based Chemistry Instruction in an Urban School District" in Chemistry and the National Science Education Standards, American Chemical Society, (ed) Stacey Lowry Bretz , 2008.

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Wink, D.J., Brennan, S.K., Cunningham*, S.A., Shippy, S.A., Kassner, R.J. Course Planning Guide and Model Lessons for CPS 10th Grade Chemistry , As part of the High School Transformation Project funded by the Bill & Melinda Gates Foundation, 2007

PEER REVIEWED PUBLISHED PROCEEDINGS (presenters bolded):

Ryan, S.A.C; Wink, D.J.; Goldman, S.R.; Pellegrino, J. (2010). Student ratio use and understanding of molarity concepts within solutions chemistry. In I. Maciejowska & P. Ciesla (Eds.), *10th European Conference on Research In Chemistry Education* (pp 239-240). Krakow, Poland: Pedagogical University of Krakow.

Ryan, S.A.C; Wink, D.J.; Goldman, S.R.; Pellegrino, J. (2010) Student interpretations of number in solutions chemistry. In M. Nodzynska (Ed.), *4th International Conference Research in Didactics of the Sciences* (pp 99-100). Krakow, Poland: Pedagogical University of Krakow.

Ryan, S.A.C; Wink, D.J.; Goldman, S.R.; Pellegrino, J. (2010) Student understandings of solutions. In Gomez, K., Lyons, L., & Radinsky, J. (Eds.) *Learning in the Disciplines: Proceedings of the 9th International Conference of the Learning Sciences (ICLS 2010) - Volume 2, Short Papers, Symposia, and Selected Abstracts* (pp. 460-461) International Society of the Learning Sciences: Chicago IL

Ryan, S.A.C.; Wink, D.J: Student conceptions of number in solutions chemistry. In Gomez, K., Lyons, L., & Radinsky, J. (Eds.) *Learning in the Disciplines: Proceedings of the 9th International Conference of the Learning Sciences (ICLS 2010) - Volume 2, Short Papers, Symposia, and Selected Abstracts* (pp. 423-424) International Society of the Learning Sciences: Chicago IL

ORAL PRESENTATIONS (presenters bolded):

Ryan, S.C.; Wink, D.J: Student interpretations of number in solutions chemistry. 21st Biennial Conference on Chemical Education, 2010, Denton, Texas.

Ryan, S.C.; Wink, D.J; Marchlewicz, S.: After School Matters science37 program in Chicago Public Schools. 21st Biennial Conference on Chemical Education, 2010, Denton, Texas.

Ryan, S.C.; **Wink, D.J.;** Goldman, S.R.: Student Understandings of Solutions. 21st Biennial Conference on Chemical Education, 2010, Denton, Texas.

Cunningham*, S.A., Wink, D.J., Brennan, S.K., Goldman, S.R.: Student understandings of solutions, 20th Biennial Conference on Chemical Education, 2008, Bloomington, IN.

Wink, D.J., Daubenmire, P.L., Brennan, S.K., **Cunningham*, S.A**: Bringing Social and Personal Perspectives Into Standards-Based Chemistry Instruction in an Urban School District, 20th Biennial Conference on Chemical Education, 2008, Bloomington, IN.

Cunningham*, S.A., Wink, D.J, Brennan, S.K., Goldman, S.R.: Student understandings of solutions, 235th American Chemical Society National Meeting, 2008, New Orleans, LA.

Cunningham*, S.A., Wink, D.J., Brennan, S.K., Bertenthal, M., Goldman, S.R.: Development of a tutorial instruction method to study student understanding of solutions. 234th American Chemical Society National Meeting, 2007, Boston, MA.

Dianovsky, M., Cunningham*, S.A., Wink, D.J.: Using CASPiE as an introduction to research for pre-college students. 233rd American Chemical Society National Meeting ,2007, Chicago, IL.

Wink, D.J., **Brennan, S.K., Cunningham*, S.A.**, Daubenmire, .P.L., Shippy, S.A., Kassner, R.J.: Incorporating chemistry in the community in a comprehensive curriculum and teacher professional development program. 233rd American Chemical Society National Meeting, 2007, Chicago, IL.

POSTER PRESENTATIONS (presenters bolded):

Ryan, S.A.C; Stieff, M.; Nighelli, T.; Yip, J: Using Participatory Design to Develop Visualizations for Learning. Gordon Research Conference on Visualization in Science and Education. 2011, Smithfield, RI

Ryan, S.A.C.; Wink, D.J.: Student Ratio Use and Understanding of Molarity Concepts Within Solutions Chemistry. Gordon Research Conference on Chmistry Education Research and Practice, 2011, Davidson, NC.

Ryan, S.A.C.; Wink, D.J.: Student Ratio Use and Understanding of Molarity Concepts Within Solutions Chemistry. 2nd Chemical Education Research Graduate Student Conference, 2011, Oxford, OH

Ryan, S.A.C.; Wink, D.J.: Student Ratio Use and Understanding of Molarity Concepts Within Solutions Chemistry. Transforming Research in Undergraduate STEM Education, 2010, Orono, Maine.

Cunningham*, S.A.; Wink, D.J.; Goldman, S.R.; Vassileva, M.; Pellegrino, J: Student Understandings of solutions. Gordon Research Conference on Visualization in Science and Education, 2009, Oxford, United Kingdom

Cunningham*, S.A., Wink, D.J., Brennan, S.K., Goldman, S.R., Marchlewicz, S., Vassileva, M.: Student understandings of solutions. Gordon Research Conference on Chemistry Education Research and Practice, 2009, Waterville, Maine

Cunningham*, S.A., Wink, D.J., Brennan, S.K., Goldman, S.R., Vassileva, M: Student Understandings of Solutions. Chemical Education Research Graduate Student Conference 2009, Oxford, Ohio

Cunningham*, S.A., Dianovsky, M., Wink, D.J., Gomez, K.: Motivation and self- efficacy of high school chemistry students, 235th American Chemical Society National Meeting, 2008, New Orleans, LA

Cunningham*, S., Wink, D.J., Brennan, S.K., Bertenthal, M., Goldman, S.R.: Studies of student understandings of solution. Gordon Research Conference on Chemistry Education Research and Practice, 2007, Lewiston, ME.

Wink, D.J., Brennan, S.K., Cunningham*, S., **Daubenmire, P.L.**: Bringing Chemistry in the Community to a High School Transformation Project. Gordon Research Conference on Chemistry Education Research and Practice, 2007, Lewiston, ME.

Cunningham*, S.A., Brennan, S.K., Bertenthal, M., Goldman, S.R.: Studies of student understandings of solution. 233rd American Chemical Society National Meeting, 2007, Chicago, IL.

Cunningham*, S.A., Shippy, S.A.: Fast MEKC Analysis of Amino Acids with On-Capillary LIF Detection. The Pittsburgh Conference on Analytical Chemistry and Applieced Spectroscopy, 2007, Chicago, IL.

Cunningham*, S., Fosco, T., Schmeling, M.: Characterization of the Chicago land-lake breeze 2003. 227th American Chemical Society National Meeting, 2004, Anaheim, CA.

* indicates maiden name

PROFESSIONAL EXPERIENCE

Item Writer	American Chemical Society (Fall 2010-Present) Item writer for the 6th Edition of *Chemistry in the Community* Test Bank; aligned previous editions' questions to the new edition; wrote new questions for new sections.
Instructor	Science 101 (Fall 2010-Spring 2011) After School Matters Instructor for Science 101 at Crane High School; focus on forensics in the fall and health/medicine in the spring; started the only science37 program at the school; developed and implemented two separate curricula as the sole instructor. Worked with Chicago Cares as part of a focus on service work in the community.
Instructor	Lab 101 and Lab 102 (Spring 2009 to Spring 2010) After School Matters Co-Instructor for Lab 101 and Lab 102 at Foreman High School; focus on post-secondary options for the students as well as work on their lab skills; had a large role in the planning and implementation of the program for a total of three semesters

TEACHING EXPERIENCE

Fellow	HST Instructional Development System Fellow (Fall 2007-Spring 2009) Loyola University Chicago and University of Illinois at Chicago Was a fellow for the High School Transformation project (funded by the Bill & Melinda Gates Foundation) in Chicago Public Schools; assisted in the development, implementation and assessment of the curriculum; involved in the alignment of the curriculum to the state and national standards; involved in the creation of standardized formative assessments;

involved in management of materials for the schools; developed a more effective materials delivery system; assisted in the Professional Development both planning and implementation occasionally taught lessons but mostly assisted the teacher both in content and in labs; formed working relationships with both teachers and students

Teaching Assistant

General Chemistry II (Fall 2006)
University of Illinois at Chicago
Dr. Paul Young
Organized and independently taught a discussion section, taught lab, tutored students, graded assignments and exams.

Teaching Assistant

ASCEND (Summer 2006)
University of Illinois at Chicago
Dr. Donald Wink
Taught a CASPiE module to incoming freshmen students who participated in the summer ASCEND program; was responsible for guiding approximately 12 students through different term-long projects at different stages of progress; provided feedback on student lab reports on a daily basis.

Fellow

National Science Foundation GK-12 Fellow (Fall 2004-Spring 2006)
University of Illinois at Chicago
Acted as a content specialist for both middle school and high school teachers in the subject areas of Environmental Science, Life Science, and Chemistry; created several labs and taught some of the activities independently; formed working relationships with both teachers and students; taught students proper presentation skills

Teaching Assistant

General Chemistry for Nurses (Fall 2003)
Saint Mary's College
Dr. Kathleen Antol
Taught three nursing chemistry lab courses and supervised 15-25 students per class; found similar topics in nursing for comparison, and tutored students; created quizzes and graded lab reports; collaborated with professor regarding lab and lecture course needs

Tutor

General and Organic Chemistry (2003-2004)
Saint Mary's College
Provided tutoring services through SMAACS for both general and organic chemistry

Counselor — Leadership and Community Development Academy (Summer 2001)
Saint Mary's College
Worked in the team effort to plan LCDA under the Leaders of a New Indiana program involving 45 high school girls; planned workshops, gave presentations, and facilitated group discussion.

Swim Instructor — YMCA and Dunkirk Community Pool (Spring 1997-Summer 2001)
Taught swim lessons to children ages 4-10 including special needs students; gained trust of children to help them learn how to swim

RESEARCH EXPERIENCE

2011-Present Simulations & Interactive Modeling in Science (SIMS)
My Role: Post-doctoral Researcher
PI: Dr. Mike Stieff

2010 – 2011 Simulations & Interactive Modeling in Science (SIMS)
My Role: Graduate Research Assistant
PI: Dr. Mike Stieff

2009 – Present Student Ratio Use and Understanding of Molarity Concepts Within Solutions Chemistry
Dissertation Work
My Role: Principle Investigator
Committee: Donald Wink, Ph.D. (chair), Susan Goldman, Ph.D., Marcy Towns, Ph.D., Mara Martinez, Ph.D., and Tom Moher, Ph.D.

2006 – Present Student Understandings of Solutions
My Role: Graduate Research Assistant
PI: Dr. Donald Wink, Dr. Susan Goldman, and Dr. Jim Pellegrino

2007-2008 Motivation and self-efficacy of high school chemistry students
My Role: Co-Lead Investigator for course project

2006 – 2007 High School Transformation Instructional Development System.
My Role: Graduate Research Assistant

2004 –2007 Masters' Thesis
Rapid separation of amino acids using capillary electrophoresis with laser induced fluorescence
Department of Chemistry, University of Illinois at Chicago
Committee: Scott Shippy, Ph.D. (Chair), Leslie Fung, Ph.D.; Richard Kassner, Ph.D.

2003-2004	Senior Thesis Effects of Lake Breezes on the Levels of Smog Compounds in Chicago Department of Chemistry, Saint Mary's College Advisor: Christopher Dunlap, Ph.D
2003-2003	Characterization of Airborne Particulate Matter During Lake Breeze Events in Chicago Department of Chemistry, Loyola University Chicago Advisor: Martina Schmeling, Ph.D. My Role: REU/NSF Intern

MENTORING EXPERIENCES

Fall 2011	Presentation to the LSSA Students about the Dissertation process
Fall 2009-present	Training of Graduate Student Research Assistants Project: Student Understandings of Solutions
Fall 2008-Spring 2009	Supervision and Training of Undergraduate Research Assistant Project: Student Understandings of Solutions

ORGANIZING AND PRESIDING

Summer, 2012	Co-Organizer of "Theoretical Frameworks: What are they, why should I use them, and, which one(s) should I use" at the 22nd Biennial Conference on Chemical Education in 2012, Penn State University (submitted)
Summer, 2010	Co-Organizer and presider of "Interviews as a Data Collection Method" symposium at 21st Biennial Conference on Chemical Education in 2010, Denton, TX
Fall, 2009	Co-Organizer of Activities Committee for the preparation for the 9th International Conference of the Learning Sciences in 2010, Chicago, IL

WORKSHOPS

Summer 2012	Organizer and Presenter for "The Connected Chemistry Curriculum" at the 22nd Biennial Conference on Chemical Education in 2012, Penn State University (in submission)
Spring, 2011	Co-Organizer and Presenter for "Developing and implementing inquiry activities for teaching science with visualizations" at the second session of the Thirteenth Annual Symposium Series on Excellence in Teaching Mathematics and Science: Research and Practice (Chicago State University)

Spring, 2010	Co-Organizer and Presenter for "Lab 101/102: An After School Matters science37 Program" at the third session of the Twelfth Annual Symposium Series on Excellence in Teaching Mathematics and Science: Research and Practice (Depaul University)

HONORS AND AWARDS:

Gordon Research Conference Chair Fund Award for Visualization in Science and Education, 2009, 2011
Learning Sciences Travel Grant Award SU2011
Women in Science and Engineering Travel Grant Award SU2007, F2007, SP2008, SU 2009, SU 2010
Graduate College Travel Award 2007, 2008, 2009, 2010. 2011
Graduate Student Council Travel Award 2007, 2008, 2009, 2010, 2011
Gordon Research Conference Chair Fund Award for Chemistry Education Research and Practice, 2007
Saint Mary's College Presidential Scholarship 2000-2004
National Dean's List 2000-2001

PROFESSIONAL ASSOCIATIONS

International Society of the Learning Sciences (ISLS)
American Educational Research Association (AERA)
American Chemical Society (ACS)
National Science Teachers Association (NSTA)
Association for Women in Science (AWIS)
Association for Supervision and Curriculum Development (ASCD)

APPENDIX A: ADDITIONAL MATERIALS RELATED TO R1

i. Student initially represents concentration of paint as an intensive quantity for both the SHDC and DHSC tasks

As shown in Figure A1, Student 23-4 used the same total number of cubes for each block with varying amounts of red and white to achieve the different shades of color. She indicated that:

> [00:53:29.06] **Student 23-4:** Oh it's it's like uh like half a cup of paint to paint this one or whatever and then I'll use your cup of paint for this one. And then you'd need a fourth cup it's of paint and then three fourths of white so you'll need a cup.

She used a cup analogy with the cubes to explain that a person would need one cup total to paint each block with varying amounts of red and white paint because each block was the same length. She was able to reason that the white block has no red cubes and that the red block had no white cubes.

Figure A1: Student 23-4's construction for the SHDC task

In the DHSC task (Figure A2), Student 23-4 was able to recognize that concentration was an intensive property and different heights required different amounts of paint to cover the block.

She also recognized that a ratio of red to white paint was necessary to maintain the same color across the blocks. She described her construction as the following:

> [01:00:49.23] **Student 23-4:** Ok alright I was just showing that you're still going to use like half and half like the same paint. Like no matter how much you use but I'm also showing that you need more amount of paint to actually paint the blocks.

This type of intensive property view of concentration and reasoning would enable a student to solve molarity problems. However, as will be shown in Chapter V, her mental model for molarity was inconsistent with this type of reasoning. Her "M is moles" mental model was an extensive view of molarity and therefore incompatible with the intensive view of concentration that she held in the structurally similar tasks.

Figure A2: Student 23-4's construction for the DHSC task

Figure A3: Student 22-3's constructions for the SHDC and DHSC tasks

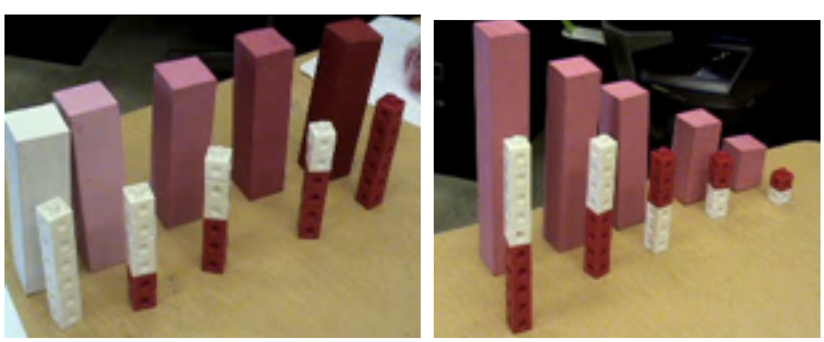


Figure A4: Student 2-1's constructions for the SHDC and DHSC tasks

Regarding the SHDC task, Student 22-3 described her construction as:

> [01:18:26.17] **Student 22-3:**Ok um I did for the completely red one I made a stack of all red, six red. No particular reason why I used six except for to show the mixture a little better over here. And then I have the completely white one. For this block right here. And this block looks a little more red but it's not as red as this one cause I mean there's some white in there. So I made it more red with four reds and two whites. And this one looks completely even between red and white so I have three reds and three whites here. And then this one looks more white than it does red so it's two reds and four whites.

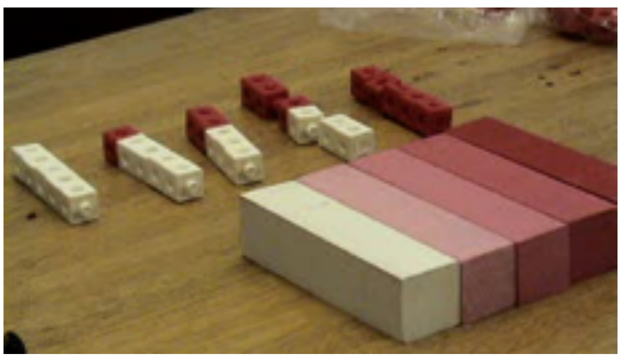
Within the SHDC task, she attended to the variation of colors while the length of the blocks was kept the same time.

Student 24-3 also belongs in this subcategory of students, but she represented her DHSC task models in a slightly different way. As shown in Figure A5, Student 24-3 constructed her cube model in the same way as the other students in this subcategory. However, in her DHSC task model, she started with eight white blocks and eight red blocks and halved each down the line to maintain a ratio across the blocks with decreasing amounts as the height decreases. When asked why she halved the numbers she said the following:

> [01:06:54.11] **Student 24-3:** Because why you, half is always good for me. Like why use all this amount just to paint like let's say this one or this one. Because I just made it shorter cause I don't know if half is to cause I could say like the secure like more or less so it's just to make sure but you still have to make sure the same amount of white and same amount of red.

She indicated that “half is always good” for her and it is inferred that this type of reasoning is easier for her to solve the problem. Therefore, it is believed that she uses this “halving” model out of ease rather than because she believed they were that different in size.

Figure A5: Student 24-3’s cube models for the SHDC and DHSC tasks



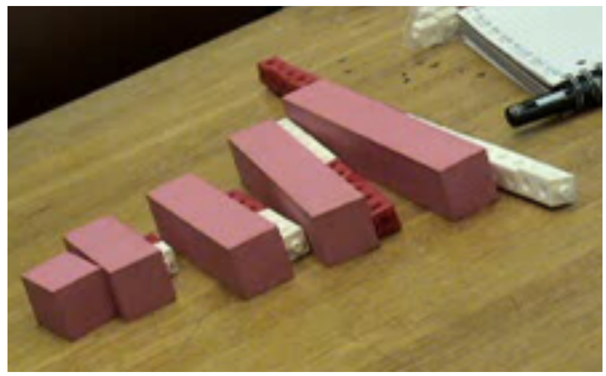


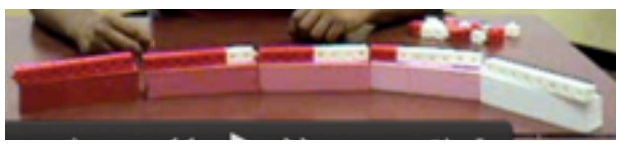
Figure A6: Student 13-4’s cube models for the SHDC and the DHSC tasks

During the structurally similar SHDC task, Student 3-4 was able to reason that two separate colors were needed in different amounts to make up each color but with the same total amounts of paint because the blocks were the same height. Figure A7 shows a screenshot from Student 3-4’s video and he described his construction as:

> [00:55:39.04] **Student 3-4:** Cause I want to even it out. I just want to do the same. Hopefully I can have enough for all of them. I'm gonna do the white one. So we've got both sides. Now I want to do, I want to do the middle one and that one's fifty-fifty so far I've used eight for each one. So I'm gonna use four of the white ones and then four of the red ones just to show that you'd used half and half. This is the middle so now let's say it’s like like six-eighths, like three-fourths. Three-fourths of it’s red and a fourth of it’s like white. In my head, and then same goes for this one. Once again I'm just a math person. I like numbers so I'm just thinking everything in numbers right now.

The blocks in the SHDC task were obviously a different color indicating to him that while the height of the block stays the same there has to be different combinations of colors to create each shade of paint. His cube model reflects this.

Figure A7: Student 3-4’s cube construction to represent the color of the blocks in the SHDC task.



In the DHSC task, Student 3-4 constructed a cube model with a constant ratio of red to white blocks with increasing relative amounts of red and white paint with an increase in height as shown in Figure A8.

Figure A8: Student 3-4's cube construction for the DHSC task

Student 15-1 had a similar model to the other students for the SHDC task shown in Figure A9. She used an even amount of total cubes (six) and varied the amounts of red an white paint for each block to show color variation. When presented with the structurally similar task DHSC task, Student 15-1 constructed the cube towers as shown in Figure A9. The following discussion between the researcher and the student describes her reasoning for her construction of a cube model:

> [01:19:31.21] **Researcher:** Ok is there, can you describe to me what you did?
> [01:19:35.15] **Student 15-1:** Um the, I just kept on adding one red and one white as, like as the block got bigger.
> [01:19:46.27] **Researcher:** Ok and then there's the same amount of red as there is white in each block. Ok? Um so how did you decide to do that?
> [01:20:01.14] **Student 15-1:** Um well the colors the same so if I choose for there to be one white for each red, then that should be constant throughout whether there's two whites and two reds or five whites and five reds. Should be constant cause it's the same color.
> [01:20:19.29] **Researcher:** Ok and, go ahead
> [01:20:22.22] **Student 15-1:** There's more color here because it's a bigger block here.

The student was able to recognize that although the blocks were the same color, it would require more paint to paint a larger block than a smaller block. She further recognized that she could not simply add one color to make up for the height difference but that she needed to keep a constant ratio of red to white blocks across the blocks to maintain the same color.

Figure A9: Student 15-1's construction for the DHSC task using colored cubes

Student 12-1 is shown in Figure A10 and Student 7-4 is shown in Figure A11. In the SHDC task, both students indicated that more total cubes were necessary for larger blocks while attending to the ratio of red to white cubes across the blocks to maintain the same shade of color. Student 12-1 varied the number of white cubes in each block stepping up by one white cube while simultaneously subtracting one red cube. Student 7-4 constructed something similar but note that she started with no white cubes for a red block and jumped to two white cubes and then added one white cube for each successive color while simultaneously removing red cubes to maintain a constant total number of cubes. As shown in the figures for the DHSC task, both students indicated that more cubes were necessary in total for larger blocks while attending to the ratio of red to white cubes across the blocks to maintain the same shade of color. Student 12-1 described her construction in the following snippets of the transcript:

> [01:37:01.15] **Student 12-1:** Ok. So um this one since it's the smallest, it will be this much. I mean it'll still be half but you'll still need less. And then, I'll just sort these.
> [01:39:03.19] **Student 12-1:** So like I said earlier they would um I'd use half and half. Half red and half white and depending on like the um depending on the size of each block, I use like different like numbers of each.

She indicated that there would still be half red and half white but there would be different numbers of each color depending on the length of the block. Had she used this mental model for the DVSM task, she would've gotten it correct.

Student 7-4 had a similar discussion as follows:

> [01:50:15.20] **Student 7-4:** Um I think, I don't really think, cause it feels like all the same color. I can tell they're all the same color but because one is tall requires more paint. to paint one than it does. It takes more paint to paint this than it does to paint this one.

As shown in Figure 45, Student 7-4 did not use a 1:1 ratio for the red and white blocks but rather she used a 1:2 ratio in favor of red. Student 7-4's ratio for the DHSC task showed that to maintain the concentration of color, the number of red and white paint cubes increased as the height of the block increased.

Figure A10: Student 12-1's cube constructions for the SHDC and DHSC tasks

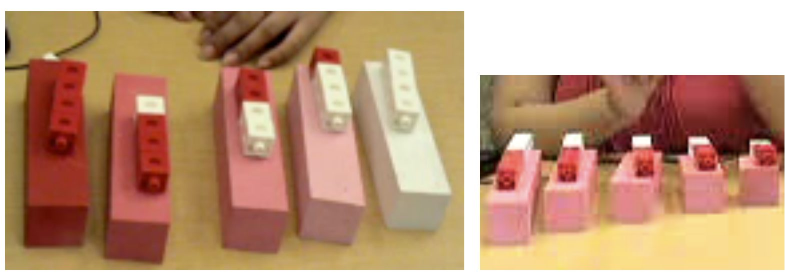
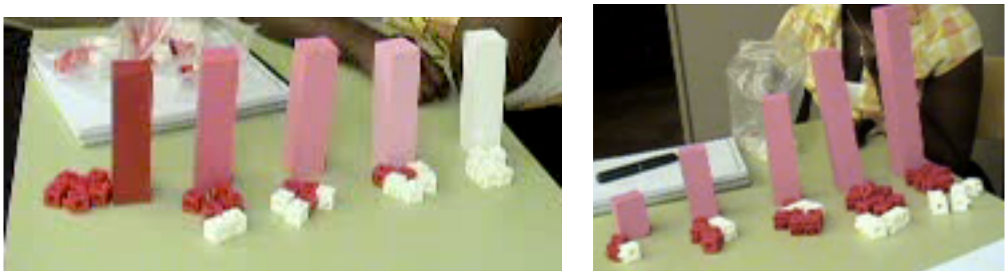


Figure A11: Student 7-4's construction using cubes for the SHDC and DHSC tasks



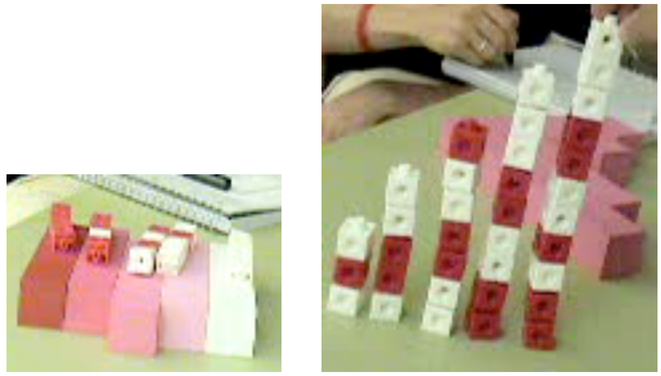
Several students used a 1:1 ratio for the middle pink block in the SHDC task but then used a different ratio for the DHSC task. When the block was not in a graduated color scheme like it was in the SHDC task, it was not obvious to the students that it would be half white and half red in the DHSC task. Occasionally, the researcher would show the middle block for reference, but, which ratio they used is not the important feature of the task but rather whether they used a ratio at all.

Student 1-3 used four total cubes with varying amounts of red and white cubes to represent the paint while attending to the height of the blocks in the SHDC task (Figure A12). When presented with the DHSC task, he constructed cube towers with increasing amounts of red and white cubes as the height increased. His first tower had a 2:1 ratio and he did not maintain the total ratio but rather added one red and one white cube to each tower. He indicated that:

> [01:34:07.23] **Student 1-3:** Uh red's a darker color and if you're adding light colors to darker colors, you'll need more to alter the darker color.
> UNRELATED DISCUSSION REMOVED FROM TRANSCRIPT
> [01:36:32.05] **Student 1-3:** Um well the best way I think, well since this is a larger block of wood I think I figured you would use more paint so I used more blocks. And then I, well that should be another white really. Because you need more white than red in order to make that color.

He believed that to obtain the color it had to have more white paint, which is likely reminiscent of a personal experience with painting.

Figure A12: Student 1-3's cube constructions for the SHDC and DHSC tasks



ii. Student initially represents concentration as an intensive property for SHDC but requires a prompt to represent concentration as an intensive property for the DHSC task

Student 4-2's responses to the SHDC and DHSC tasks are shown in Figure A13. For the SHDC task, she indicated that she constructed "a representation of like if you were making the colors like the parts red and the parts white you would use." For the DHSC task, she immediately noticed that the blocks "are pretty proportional to each other." When asked to construct a model using colored cubes, she initially represented only the length using red cubes. The following snippet was an exchange that occurred between the researcher and the student to prompt the student to consider the length in the model. She had previously discussed the blocks having equal amounts of red and white and only chose to represent the length with the cubes.

> [01:05:46.10] **Student 4-2:** I am representing the size, cuz like they said this one is um they're all proportional to each other and this one and so I don't know how big it is. But it's like one inch this ones. two inches this ones three inches this ones four inches this ones five inches.
> [01:06:08.11] **Researcher:** Okay so is there any way to represent both the length and the color.
> [01:06:13.16] **Student 4-2:** Yeah.
> [01:06:17.25] **Researcher:** Could you do that. For these too. I'm just curious why did you use red instead of white. When you just did the length. You just chose?
> [01:06:28.23] **Student 4-2:** I like red.
> [01:07:24.23] **Researcher:** Okay okay um now could you describe for me what you constructed.

[01:07:28.07] **Student 4-2:** Okay so to show the there's like equal amount so white and red. I have equal amounts of white and red. In each one.

The student again referenced the proportionality of the blocks' sizes and because length was the salient feature that changed, it was all she considered. When prompted to consider the length and color, she immediately used a ratio of red and white.

Figure A13: Student 4-2's constructions for the SHDC and DHSC tasks

Student 16-2 has a response similar to the rest of the students in this subcategory on his SHDC task (shown in Figure A14). However, he differed from Student 4-2 in that he initially only considered the color of the blocks in his cube models. In the DHSC task, he indicated that each block would be represented the same way: with one red cube and one white cube because they are the same color. This is shown in Figure A15. He described his model as:

[01:36:31.10] **Student 16-2:** I just put, uh, I put the cubes down of red and white and since all of them are the same mixture I just made them all just the even two cause there's no. I don't need to show like any difference and like, yeah.
[01:36:49.26] **Researcher:** So you only used two cubes cause you didn't need to show
[01:36:52.09] **Student 16-2:** Yeah I didn't need to show the different amount of like white and red that needed to be used.

The student suggested that he only used one block of each color because he wasn't showing a difference in color. This gives insight into why several students chose to represent the color in the DHSC task as a single red cube and a single white cube. Some students who were keeping height constant in the SHDC task may have been doing this as well. By keeping the total length constant, they were able to show the variations in color.

Figure A14: Student 16-2's cube model for the SHDC task

Figure A15: Student 16-2's initial cube model for the DHSC task

The researcher then asked the student if each block would need the same amount of paint to cover it completely. The transcript occurred as follows:

> [01:37:12.14] **Student 16-2:** You want a, I was just saying a gallon. You probably wouldn't need a gallon of paint but
> [01:37:18.03] **Researcher:** Let's pretend these are houses.
> [01:37:18.03] **Student 16-2:** Yeah, ok. So, you just need a gallon, like a gallon the same. Each cube's a gallon. So a gallon of red and a gallon of white would give you a mixture of some type of pink shade and this would be the pink shade of it.
> [01:37:33.16] **Researcher:** Ok, so my question then is, I see what you're saying and I get it. I'm trying to take it a step further. So let's say we have the suburbs. There are five houses on the block. There are the five houses. They are all different

[01:37:50.08] **Student 16-2:** Oh, you are trying to. I know what you. The different lengths. I'm just going with the shade of the color. But if you are doing different lengths you'll need more cubes.
[01:38:02.06] **Researcher:** Let me get a picture of what you had first, the solution and stuff.
[01:38:08.16] **Student 16-2:** Just have to think about it more. uh, it'd be the same ratio of red to white to keep the same color, just a different length. So, I should have thought about that, I didn't know I was being tested on that.

After the researcher suggested a larger scale, the student immediately began to discuss length. This could also give insight into student reasoning during the task due to the effects of small scale. It may have been difficult for some students to represent gallons of paint with a cube when all of the blocks wouldn't need gallons of paint to paint them in total. A single gallon of paint would cover all of the wooden blocks with a great deal to spare. His final cube representation represents both length and color and can be found in Figure A16.

Figure A16: Student 16-2's final cube model for the DHSC task

Student 19-2's responses to the SHDC and DHSC tasks matched Student 16-2's responses. Her SHDC response is shown in Figure A17 and was similar to many of the other students' responses showing color variation with the same length.

Figure A17: Student 19-2's cube model for the SHDC task

When asked to represent DHSC, she placed one white cube and one red cube in front of the painted blocks to show that they would all have the same model (Shown in Figure A18). The researcher asked the student to consider length in the following exchange:

> [01:15:03.10] **Researcher:** So let's say I have one block and I have to paint it. And I have another block and it's a different length and I have to paint it. But I don't have the same can of paint to use. I have to start from scratch every time.
> [01:15:17.25] **Student 19-2:** Ok.
> [01:15:18.29] **Researcher:** So how would you tell me how much red and white paint I would need?
> [01:15:23.08] **Student 19-2:** Um, I would say like, let's see. I would find like a common, um, like denominator, like I did before. Like this is the full block of wood. Um and none of these are like, let me see, are really like proportionate to half. This one kind of is. Um, it's more like two-thirds. Um, so then I would say like for the big block you need like three blobs of red and three blobs of white. And this looks like one, two looks like it's in fifths. And this one looks like it's in thirds. Hm. Well there's five blocks of wood so I could say you needed five blobs of red and two blobs of white. And then for this one I could say you needed four blobs of red and four blobs of white.

Figure A18: Student 19-2's initial cube model for the DHSC task

Student 19-2 immediately switched from representing solely color to accounting for the length as well as shown in Figure A19. To contrast, in the chemical context she mentioned water but did not factor it into M and held an "M is moles" mental model for both the SVDM and DVSM tasks. She did not view M as an intensive number.

Figure A19: Student 19-2's final cube model for the DHSC task

APPENDIX B: ADDITIONAL MATERIALS RELATED TO R2

i. Student has an "M is moles" mental model and believes that the amount of moles stays the same and the amount of solvent changes

Student 5-3 made a connection to one of the recipe questions and stated:

> [01:02:43.05] **Student 5-3:** Ok so um like in the cooking question that were on the thing, um, like when you're doing it for eight people six people four people, it's all the same difference between each ingredient but it's like still the same meal.
> [01:02:58.25] **Researcher:** Ok so if you could draw for me on this one how I made these three. Um
> [01:03:10.23] **Student 5-3:** Um I don't want to try one cause I don't know how to draw.
> [01:03:13.08] **Researcher:** Ok
> [01:03:14.11] **Student 5-3:** Like I can say it but I don't know how to write it.
> [01:03:16.04] **Researcher:** Ok go ahead and say it.
> [01:03:17.04] **Student 5-3:** Um so how you made it?
> [01:03:20.16] **Researcher:** Yeah so pick some sort of number for one and go through all three.
> [01:03:26.18] **Student 5-3:** Ok so we'll go, what are the differences? We'll go with like a third. So like there's like a third no I don't like that. No a third, two thirds and three thirds of each how it um no I don't like that. Um hm like however much is like if the CaCl is two and the M is one and then for this one it took a little more.
> [01:04:25.21] **Researcher:** Of both?
> [01:04:28.10] **Student 5-3:** Yeah because it'd be the same amount like this that's about doubled so if this used hm. If this had like it used one M and two CaCl twos in this one you used two M and four CaCl twos

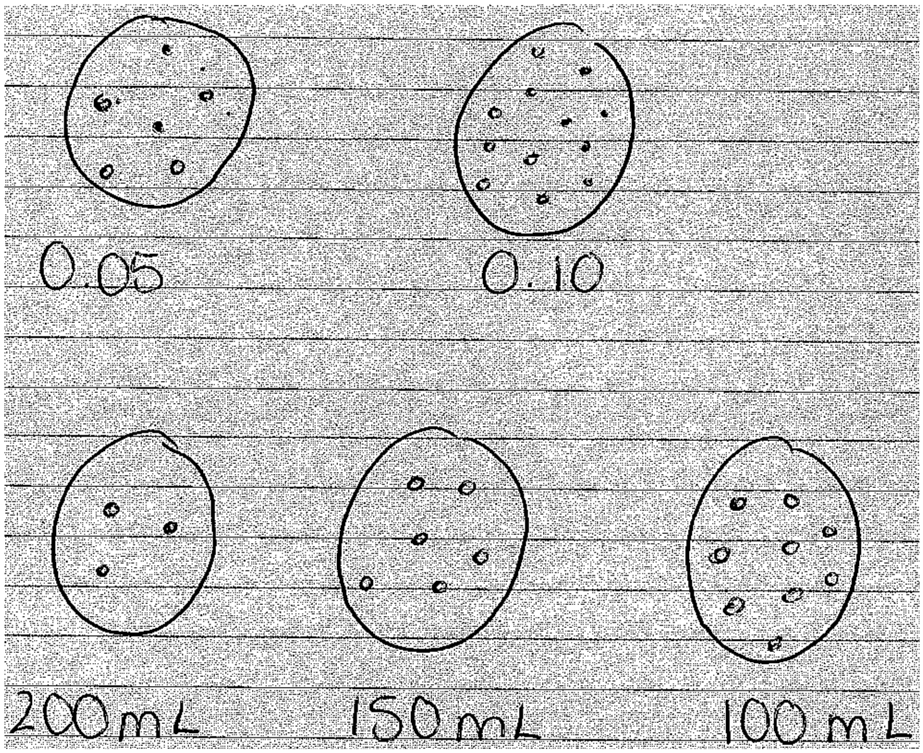
Student 5-3, who previously had been confused about decimals, changed his response to involve proportionality between the jars and increasing amounts of calcium chloride but also with increasing amounts of "M". The student did not exhibit the decimal confusion in this task, which may be due to the nature of the problem in that the decimal number remained constant across the jars.

Student 7-3 also changed her description during the DVSM task but to another incorrect mental model. She switched to an “M is moles” type of reasoning in that the amount of calcium stays the same between the jars and the amount of water changes. She described this change as follows:

> [01:43:10.01] **Student 7-4**: Well I'd say I just put the particles in there but they would all be at the bottom unless it’s spread through it. I'd say all through depending on how wide how far the water goes, thats how far the particles has to spread.

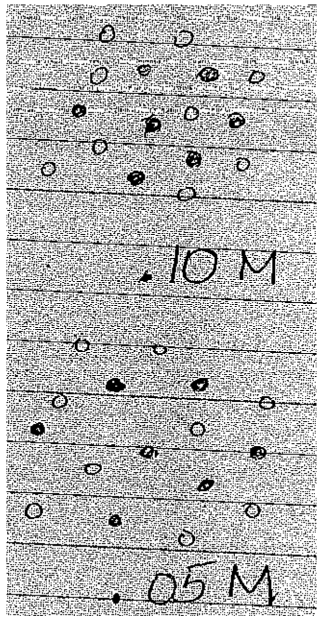
Another example of a student using the “M is moles” mental model can be found in Student 23-4’s interview. As shown in Figure B1, she was able to reason through the change in substance amount for the SVDM task by indicating that there would be twice as many circles (12 circles) in the 0.10M jar than the 0.05M jar (6 circles). The larger circle indicated a “zoomed-in” picture. When presented with a change in volume in the DVSM task, her drawing indicated that the she too held the belief that the smaller volume was more concentrated because she believed all three volumes had the same amount of calcium chloride (Figure B1 bottom).

Figure B1: Student 23-4's drawings for the SVDM and DVSM tasks



Student 2-1 began by drawing the same amount of calcium chloride by using separate circles for calcium and chloride "elements" in the 0.05M and 0.10M jars stating that "I think they look the same because they're both calcium chloride it's just different of M, which I can't remember what that stands for" (Figure B2).

Figure B2: Student 2-1:
Initial drawing for SVDM task

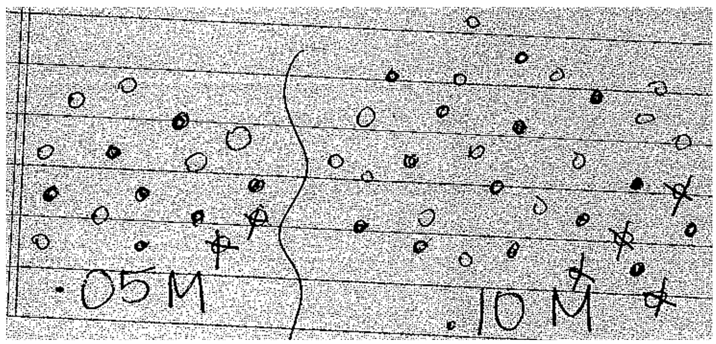


Through the following discussion, she changed her answer to involve molarity:

> [00:57:13.09] Researcher: So what does M have to do how much calcium chloride is in there
> [00:57:21.24] Student 2-1: Is it like the molarity or something.
> [00:57:26.00] Researcher: What's molarity?
> [00:57:27.14] Student 2-1: (laugh) I don't remember.
> [00:57:29.09] Researcher: If it was molarity.
> [00:57:30.16] Student 2-1: Mmhmm
> [00:57:33.02] Researcher: Lets say M equals molarity what would that mean about how much calcium chloride is in each of those.
> [00:57:44.14] Student 2-1: How strong.
> [00:57:47.21] Researcher: Okay.
> [00:57:47.27] Student 2-1: It is.
> [00:57:47.27] Researcher: Okay how strong it is so would point one five M have more the same or less than calcium chloride of point one zero M
> [00:58:05.18] Student 2-1: It would have more

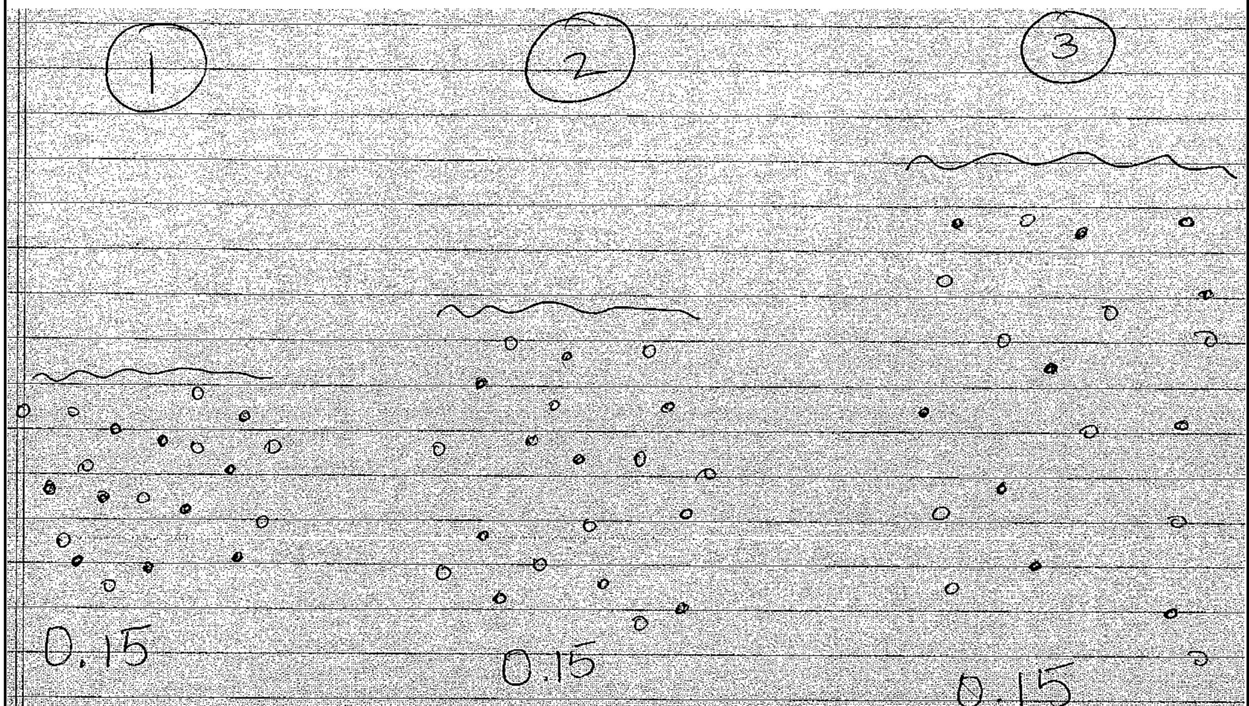
She committed to M standing for molarity and changed her drawing to reflect a change in substance amount (Figure B3). The X's through particles are from a discussion regarding the number of circles drawn. The student stated that she had drawn them "randomly" and if they weren't random "they would be the same" in reference to the amount of calcium circles relative to the amount of chloride circles. While she correctly draws an increase in calcium chloride between two jars, her relative amount of calcium to chloride is incorrect as it should be a 2:1 relationship favoring calcium.

Figure B3: Student 2-1's final drawing for the SVDM task



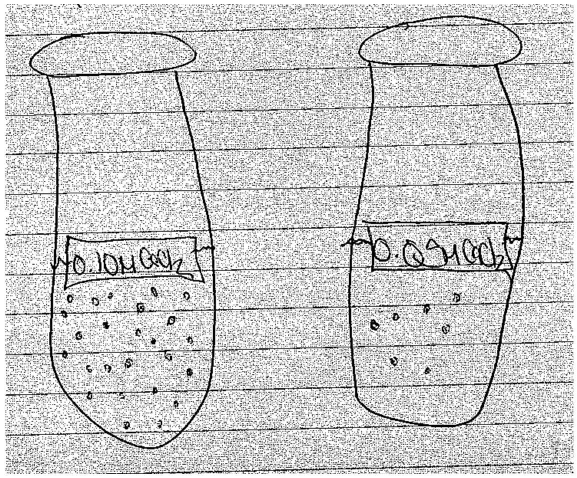
While this student believed that M stood for molarity, her understanding was categorized as "M is moles" because of her interactions with the DVSM task. When asked what the labels on the jar stood for, she reverted back to a "whatever M stands for" stance. When asked how all the jars could have the same M and different "heights" (as the student phrased it), the student stated, "each of them could have been filled with different height of water but then the same amount of calcium chloride could be I guess put into it". Her drawing, shown in Figure 77, is consistent with an "M is moles" mental model. This mental model encompasses students who simply refer to their drawings as amount of calcium chloride as well as students who state that M stands for moles. In the SVDM task, she mentioned that water is "in between them" referring to the calcium and the chloride circles, but does not factor it into her reasoning with number. In the DVSM task, she referred explicitly to water as the reason the volume changes but did not indicate that it had anything to do with the number before M.

Figure 77: Student 2-1 drawing for the DVSM task



Student 8-4 was unsure what the M on the label stood for and suggested milliliters, then milligrams, and then "how diluted" the chemical was as alternative interpretations for the number before M. She also viewed the number as a percentage involving dilution. In her drawing for the SVDM task, shown in Figure B4, she described it as "point zero five be closer to becoming pure. I don't want to say pure water because I don't know what that is but yeah. So it'd be closer to becoming pure water so this one is a little bit more diluted [0.05M]."

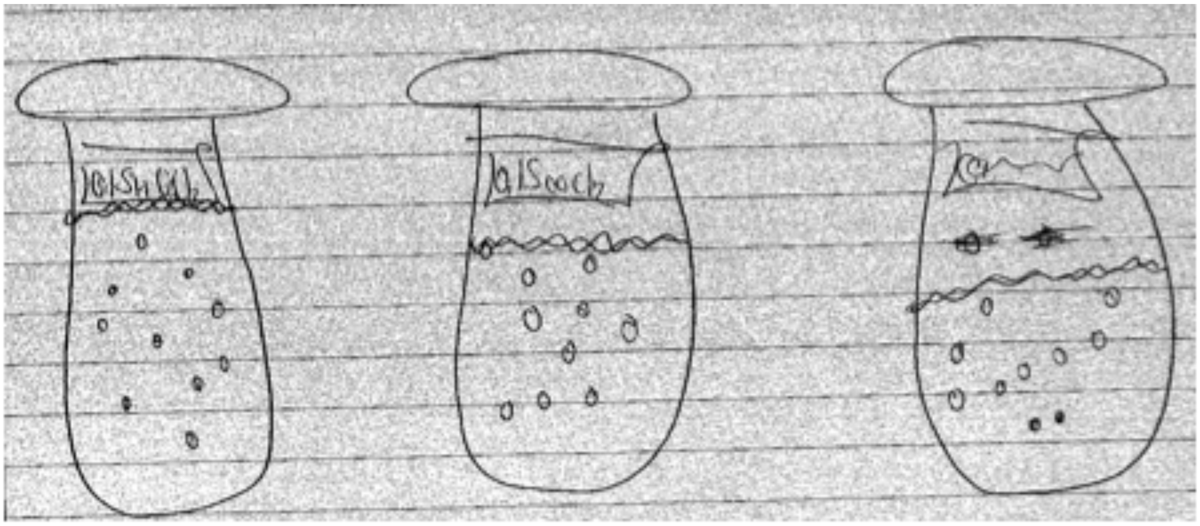
Figure B4: Student 8-4’s drawings for the SVDM task



Her drawing did not distinguish between calciums and chlorides and though she mentioned water, she only drew a wavy line to represent it. The number of circles in the 0.10M jar simply had “more” [calcium chloride] but not double the amount chosen for the 0.05M jar.

In her drawing to represent the DVSM task shown in Figure B5, she did attend to the number of circles in each jar so that each jar would have the same total number of calcium chloride. The jar on the left represented the 200 mL jar and decreases in volume moving to the right. While drawing she stated, “I don't know if it'd be the same” and then “It looks like yeah so I guess it would be all of them have the same but it just be like different amounts of water.” She was unsure before her drawing and through the process of drawing, she verified her incorrect suggestion that they would all have the same amount of calcium chloride in each jar. She did make mention of water making up for the volume difference but did not indicate that it was related to the number in front of M.

Figure B5: Student 8-4's drawing for the DVSM task



ii. Student switches from an "M is moles" mental model to a belief that the size of the molecules change to make up for the volume

The case of Student 6-3 is fits into the size category because of his description of the SVDM task:

> [01:13:40.12] **Researcher:** Ok, um, let me think of the questions I have. Um ok so, which bottle has the most calcium chloride in it or do they all have the same amount?
> [01:13:54.05] **Student 6-3:** I believe that they have the same amount. I think so, Yeah I think they have the same amount of calcium chloride.
> [01:14:02.04] **Researcher:** Ok so in your drawing you have less molecules in one than the other so do they still have the same amount of calcium chloride?
> [01:14:13.01] **Student 6-3:** I believe so
> [01:14:13.20] **Researcher:** Ok how is that possible?
> [01:14:16.15] **Student 6-3:** Um They're the same size but they have different molarity. Hm that's a hard one I'd have to say.
> [01:14:58.28] **Researcher:** Could you double check the battery on your battery pack?
> [01:15:01.09] **Student 6-3:** Yeah it's still working. Um how's it possible? Its possible somehow. Just gotta think about it

[01:15:10.24] **Interview:** Ok, time your time or you can you're done do if you don't want to do it.
[01:15:20.00] **Student 6-3:** Ok let's see. CaCl2 and point ten and point five. (static) I don't know. Maybe how compressed the molecules are. I'm not sure. That doesn't sound right but that's just a wild stab at it.

Student 6-3 initially drew fewer molecules in the the 0.05M drawing than the 0.10M drawing. However, his description morphed to suggest that molecules could have a molarity. He did not believe that the molecules would change size in this task but did indicate that for the number in front of M to change something else had to change. Because he believed that there was the same amount of calcium chloride in each jar, he reasoned that the molarity of the molecules must change and were probably more compressed. This is an extensive view of molarity.

In the DVSM task, Student 6-3 initially expressed an intensive view of molarity, but between the amount of calcium and the amount of chlorine. He indicated that the "200" would have the most calcium chloride, which is a correct answer. Upon further questioning by the researcher, Student 6-3 changed his response and seemed to grow confused.

[01:34:49.06] **Student 6-3:** Well usually if it's CaCl2 or something like that you know there's two times like, two Cl you knows there's two times the chloride as there is the calcium. So like I would say that um there's as much calcium as there is chloride in here to mix it. So in this, this is bigger than the rest of these. It still goes back to the same basis of it being bigger.
[01:35:21.21] **Researcher:** Ok. so what if I told you when I made this I had solid calcium chloride and I added it to water for each one. So this zero point one five, I made it. For this one, I made it inside this bottle and inside this bottle. Does it change how you're answering the question? Cause a moment ago you said that I'm adding solution of chlorine and solution of calcium.
[01:35:52.02] **Student 6-3:** Well, if it's not uh liquid solution, liquid uh then, your question is would it change?
[01:36:04.24] **Researcher:** Your answer?
[01:36:09.01] **Student 6-3:** Um it could possibly. Because you could add the same amount of calcium to all of these and I don't think like it, it's dissolves and powder form

once it dissolves, I don't think it would increase the size of the chlorine that was already in there.
[01:36:34.28] **Researcher:** The chlorine that was already in there? What do you mean by that?
[01:36:36.20] **Student 6-3:** Yeah like oh no. I mean the water cause it's like when something dissolves in solvent it doesn't really like add um to the solvent. It adds to the solvent but it doesn't like make it grow exponentially. Like
[01:36:56.07] **Researcher:** Ok so you're saying when I added it to this one it wouldn't grow into this one?
[01:37:00.16] **Student 6-3:** No it wouldn't.

Student 6-3 understood that the size of the chlorine wouldn't increase exponentially to yield the different volumes. This understanding that separates the calcium and the chlorine as a solid and liquid respectively broke apart in his explanation involving the amount of calcium chloride. He believed that the larger one should have more calcium chloride because it had more volume within the jar and this is a correct response, but his mental model was inconsistent and he was unable to use it to describe his answer.

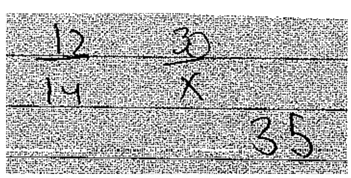
APPENDIX C: ADDITIONAL MATERIALS RELATED TO R3

ii. The "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy

Student 11-1 used the apples$_1$:oranges$_1$::apples$_2$:oranges$_2$ strategy for the dictionary question (P12), shown in Figure C1. The student placed the number of dictionaries created in twelve minutes on the left side of the equation and thirty minutes over the unknown amount of dictionaries on the right side of the problem. This yielded the correct response of thirty five dictionaries. Like many other students in this category, he did not write down his units and he used X to represent the unknown variable. He did not use a unit on his final response on paper either. However, in his discussion of how he solved the problem, he reveals that he thought about the units but did not share them on paper.

> [00:19:48.10] **Student 11-1:** It'd be thirty five. So it'd print thirty-five dictionaries in thirty minutes.

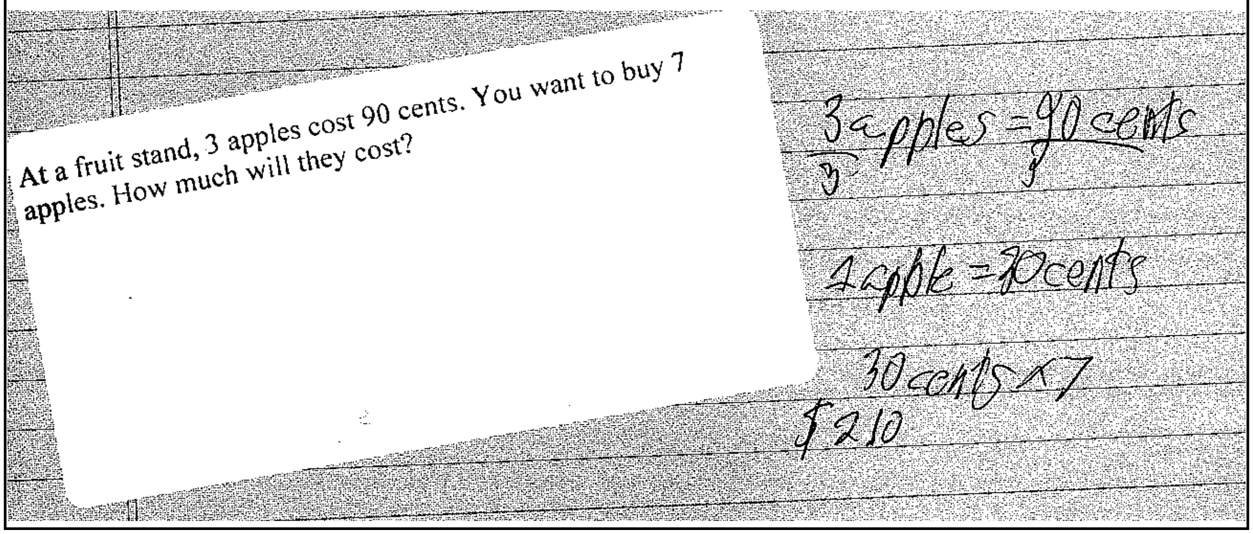
Figure C1: An example of the "apples$_1$:oranges$_1$::apples$_2$:oranges$_2$" strategy used by Student 11-1 on P12



iii. The "solve for one and multiply" strategy

Another example of a student using the "solve for one and multiply" strategy can be seen in Figure C2. Student 14-1 used this strategy to solve P5 on the PR diagnostic. He calculated the cost of one apple and multiplied it by seven to correctly solve the problem. Like Student 15-1, he wrote down his units for each step and also converted between cents and dollars to give his answer in a conventional unit.

Figure C2: An example of the "solve for one and multiply" strategy used by Student 14-1 on P5



Theoretical Statement 7: Students with a robust understanding of molarity also do well on chemistry calculation problems.

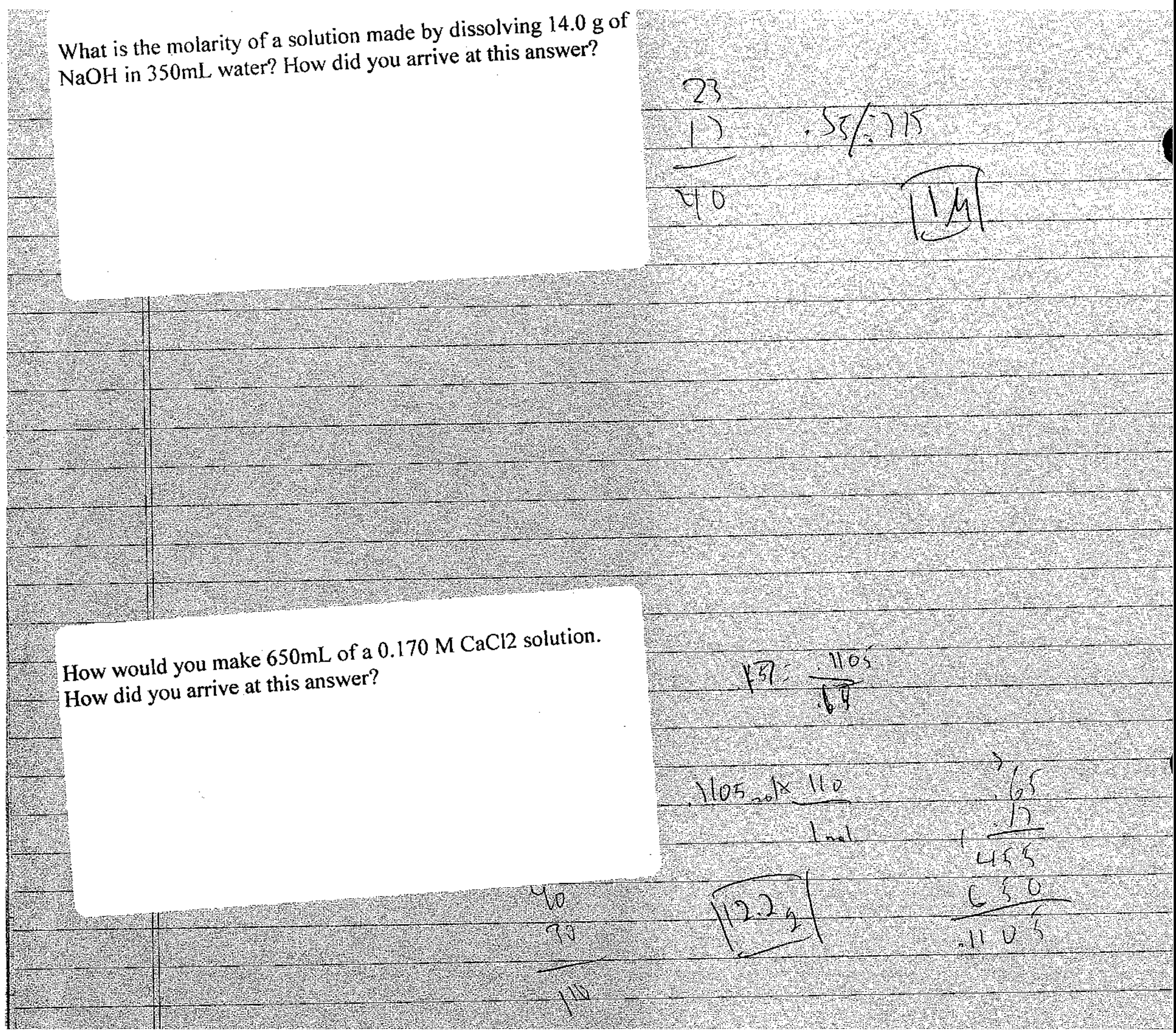
Student 10-1 was one of two students who was deemed to have a "robust understanding" of molarity. Student 10-1 also correctly solves questions 4 and 5 on the CD. His drawings are shown in Figure C3. For question 4, he calculated the molar mass of NaOH and reminded himself out loud that molarity was in liters:

> [00:12:56.13] **Student 10-1**: Oh, yeah, it's twenty-three. Three plus sixteen, seventeen, which is forty I believe, yeah. Ok. So molar mass is moles, gotta divide four have to divide fourteen by forty. Ok, so, fourteen divided by forty, enter. Point three five and then you gotta divide that by, molarity's in liters so it's point three five Well it's just one.

This attention to the units and understanding that molarity is an intensive property consisting of two extensive properties allows him to successfully solve the problem.

Student 10-1 correctly attended to the subscript in the formula for calcium chloride and found the correct molar mass. He set the molarity equal to the moles over the liters to find the number of moles and calculated the number of grams necessary. This indicated that he was able to reason that molarity is an intensive property consisting of two extensive properties.

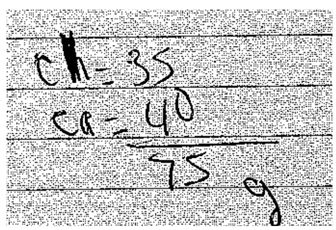
Figure C3: Student 10-1's responses to Questions 4 and 5 on the CD



Theoretical Statement 9: Ability to calculate molar mass does not indicate ability to calculate the number of moles

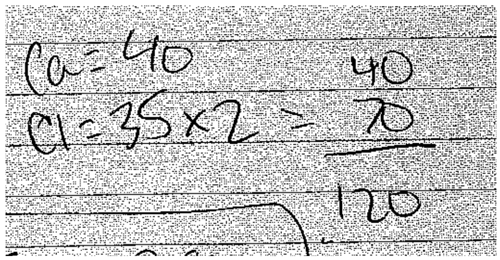
Student 7-4 attempted to calculate molar mass in M6 in the molarity problems, where students are asked to find grams of calcium chloride when given the molarity and volume of the solution. He was able to add the different masses together but did not attend to the subscript in the $CaCl_2$ formula. His work is shown in Figure C4. This indicates that he has a partial understanding of molar mass, one that is missing knowledge of subscripts and their effect on the calculation. He was able to add use mass in this case, but did not find it necessary to do so in Question 1 on the CD. This indicates that he is able to partially calculate molar mass but does not understand the purpose of doing so.

Figure C4: Student 7-4's response to M6 on the molarity problems task



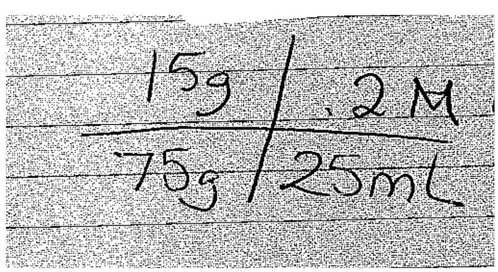
Student 15-1 also attempted to calculate molar mass but in M3, the problem that asks students to calculate the molarity of a solution given the number of grams of calcium chloride and the volume of water provided. Her calculations are shown in figure C5. She correctly sets up the problem such that the subscript is account for and adds the two values together. In her addition, she has an error yielding 120 as an answer instead of the correct 110. This would be categorized as having an understanding of how to calculate molar mass because she set up the problem correctly and just had an addition error. Like Student 7-4, Student 15-1 knows how to calculate molar mass, but does not seem to understand the application of doing so in other problems, such as Question 1 on the CD.

Figure C5: Student 15-1's response to M3 on the molarity problems task



Student 22-3 set up what will later be discussed in this section as a random ratio using a grid as shown in Figure C6 for her response to M3 in the molarity problems task. The 75g that appears in the bottom left quadrant is a molar mass calculation that does not factor in the subscript in $CaCl_2$. Rather, it is the molar mass of CaCl. This is similar to Student 7-4's mistake mentioned earlier. These students had a partial understanding of molar mass, one that did not attend to the subscript. Student 22-3 used her incorrect molar mass for calcium chloride and set up a grid for some sort of canceling of units purpose and knows that calculating the molar mass is necessary. However, due to the random nature of the grid, it is clear that the student was unaware of how to use molar mass she had just calculated.

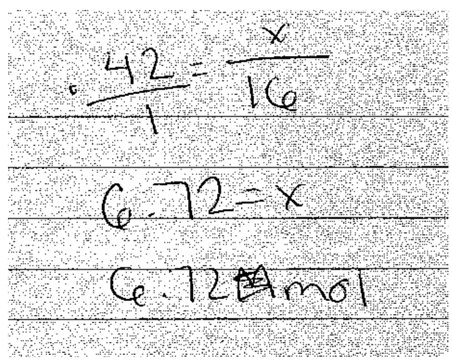
Figure C6: Student 22-3's response to M3 in the molarity problems



(a) The "apples1:oranges2::apples2:oranges1" strategy

Student 19-2 also uses this strategy when solving the same problem (Figure C7). Unlike student Student 9-2, this student provides the wrong units at the end of the problem. Like Student 9-2, Student 19-2 also did not provide any units within the problem and this likely contributed to the student's response with the wrong units. The student could have also merely put down the three numbers at random and gotten the correct answer.

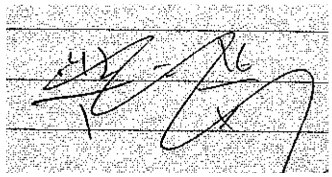
Figure C7: Student 19-2's work for M4 in the molarity problems task



(c) **the "apples$_1$:apples$_2$::oranges$_1$:oranges$_2$" strategy**

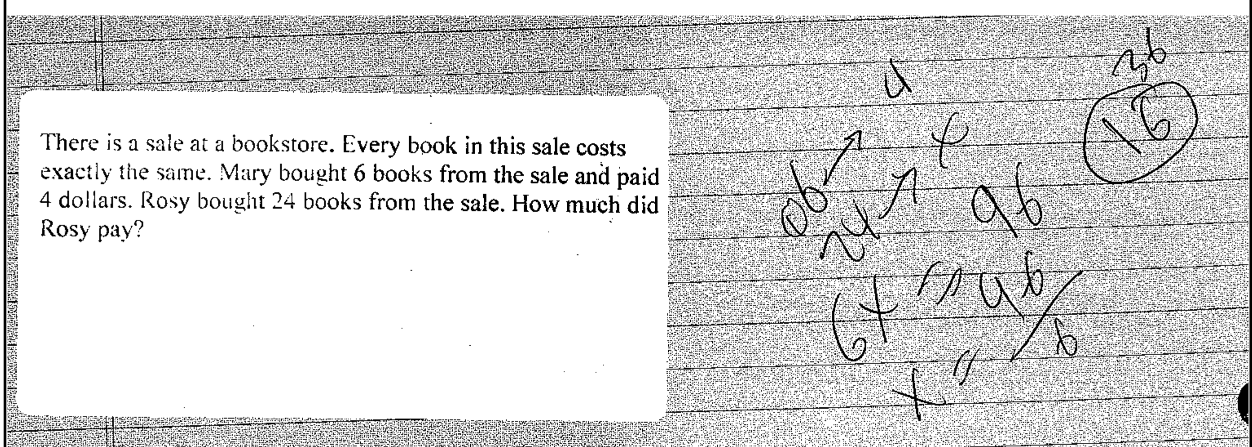
He applied this strategy by setting up a proportion with the molarity of desired solution divided by the molarity of the stock solution equal to the volume of the desired solution divided by the unknown volume of the original solution. This is shown in Figure C8 and he scribbled out the response because he knew it was wrong but was unable to come up with another solution to the problem. Moments earlier, this strategy had been successful for him in the proportional reasoning tasks.

Figure C8: Student 10-1's response to M4 incorrectly applying the "apples$_1$:apples$_2$::oranges$_1$:oranges$_2$" strategy



Student 20-2 also used this strategy successfully in the proportional reasoning diagnostic (shown in Figure C9) and unsuccessfully in the chemistry context (Figures C10 and C11). In Figure 133, the student put the time on the left side and the number of books on the right side to solve for the unknown amount of books. This yielded the correct answer in this generic direct proportion context.

Figure C9: Student 20-2's response to the Dictionary question correctly applying the "apples$_1$:apples$_2$::oranges$_1$:oranges$_2$" strategy in the proportional reasoning diagnostic



However, in the the chemistry context, she used this strategy for both M2 and M4 as an incorrect application. In M2, the student was asked to determine the volume of a given molarity and number of moles. Student 20-2 put the "moles" on the left side, indicating that the 1M was 1 mole and the volumes on the right side. Because she was not given any volumes, this strategy could not go any further. This is further evidence that confusion about M and an attempt to use it as moles leads to failure. In M4, Student 20-2 put the volumes of the solutions on the left and the molarities on the right, which would had yielded an incorrect answer if finished.

Figure C10: Student 20-2's response M2 incorrectly applying the "$apples_1$:$apples_2$::$oranges_1$:$oranges_2$" strategy in the molarity problems task

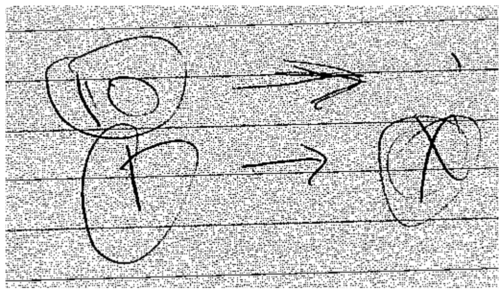
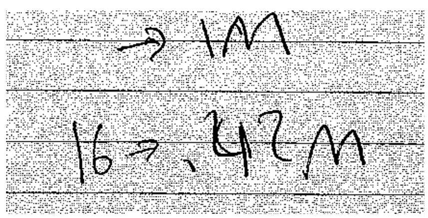


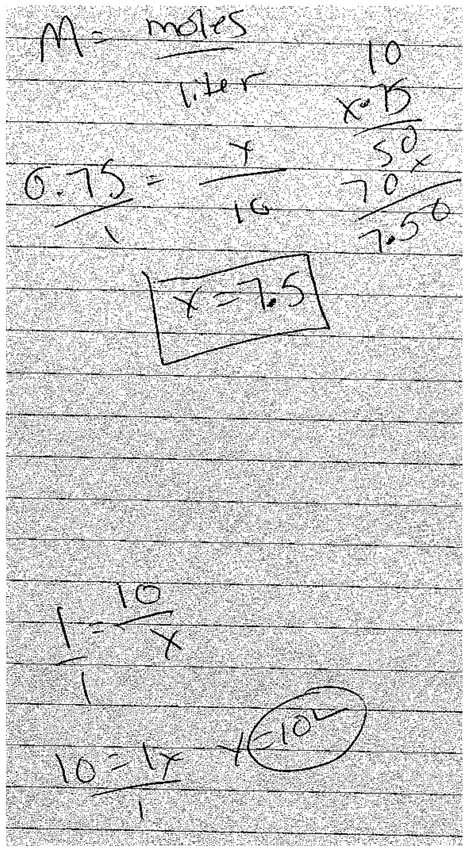
Figure C11: Student 20-2's response M4 incorrectly applying the "$apples_1$:$apples_2$::$oranges_1$:$oranges_2$" strategy in the molarity problems task



(d) The "unit-less one" strategy

Student 19-2 used the "unit-less one" strategy on 4 of the 6 molarity problems: M1 after hint, M2 after hint, M5 after hint, and M6 after hint. This student used the algorithm of M=moles/L for all of the problems and setting M over one to cross multiply. This is evidenced by Figure C12. In M1 (top) the student did not convert the milliliters to liters and was off by a factor of 10. To cross-multiply, she added a 1 underneath the 0.75 molarity. She did the same in M2 to find the liters of solution, in this case with the correct unit because it is the variable she was seeking and the correct value because there was no need to convert mL to L.

Figure C12: Examples of the "unit-less one" strategy made by Student 19-2 on M1 and M2 after a hint on the molarity problems task

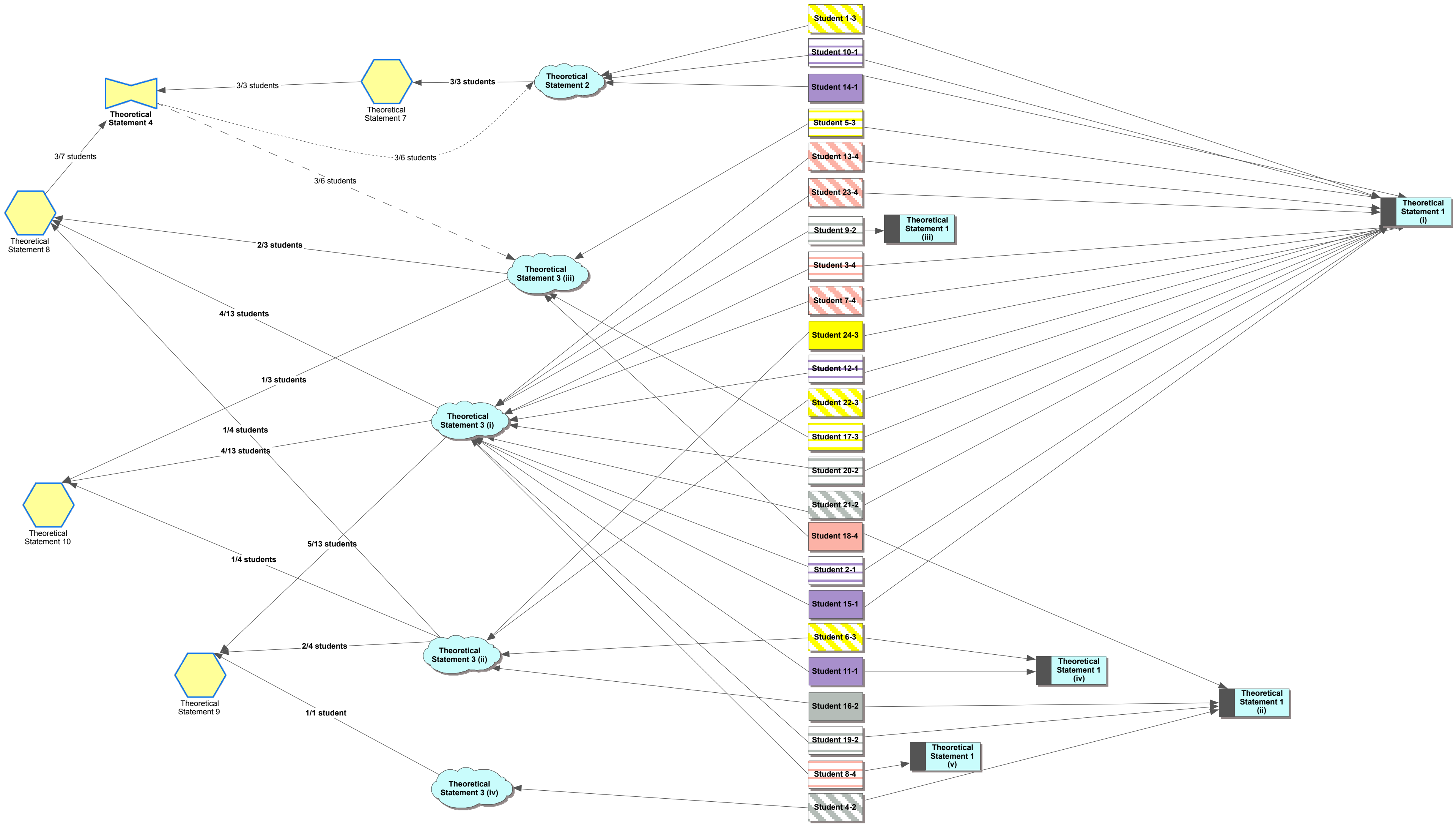


Student 24-3 also used the "unit-less one" strategy for cross-multiplication on three of the problems in the molarity problems task: M1 after hint, M2 after hint and M5 after hint. She similarly used the M=moles/liter formula and divided by 1 for easier cross-multiplication.

APPENDIX D: MAPPING OF THEORETICAL STATEMENTS BY STUDENT

The Figure YYY on page D-314 is a cluster mapping of Theoretical Statements by student showing connections between subcategories and sampling categories. Each of the four sampling categories (-1, -2, -3, -4) has a different color associated with it: purple, grey, yellow, and orange respectively. Each of the students are then coded by their performance on the PR diagnostic (13/13, 11-12/13, and under 11/13) with different patterns: Full color, horizontal lines, and diagonal hashes respectively. Lines connect students to the various Theoretical Statements with which they belong. The subcategories of Theoretical Statement 3 are shown with how many students from that category responded to Theoretical Statements 7-10 (related to calculation performance in chemistry).

Beginning at the top of the map, it is shown that the students who make up Theoretical Statement 2 are the only students who make up Theoretical Statement 7. If a student views molarity as an intensive quantity, it is a good predictor that they will also be able to calculate molar mass and the mole given molar mass. These students are also a subset of students who make up Theoretical Statement 4, which is related to the mole and therefore makes sense that they would be in that category. It is important to note that this group of students shows quite nicely that the PR diagnostic is not a good predictor of performance on chemistry problems. The three students who viewed molarity as an intensive quantity all had different levels of performance on the PR diagnostic. The only similarities among students within a group are that Group 1 students all scored between 11-13 with Groups 2-4 with a variety of scores and students scoring under 11. Group 3 had the most students (3/6) scoring under 11. The sampling categories do not seem be a predictor of performance other than there being more variability in performance on the chemistry problems for Groups 2 and 3. For Theoretical Statement 3, students from Group 3 responded in all four subcategories and students from Group 2 responded in three of the four categories. To contrast, students from Group 1 responded to either Theoretical Statement 2 (intensive) or one other subcategory in Theoretical Statement 3 (extensive). Students from Group 4 all responded to one subcategory within Theoretical Statement 3. These similarities of group performance do not extend to Theoretical Statements 7-10. Students have different levels of solving abilities for the chemistry problems that cannot be predicted by their responses in Theoretical Statement 3, only that they will not respond to Theoretical Statement 7.


Student 1-3
Student 10-1
Student 14-1
Student 5-3
Student 13-4
Student 23-4
Student 9-2
Student 3-4
Student 7-4
Student 24-3
Student 12-1
Student 22-3
Student 17-3
Student 20-2
Student 21-2
Student 18-4
Student 2-1
Student 15-1
Student 6-3
Student 11-1
Student 16-2
Student 19-2
Student 8-4
Student 4-2
Theoretical Statement 1 (i)
Theoretical Statement 1 (ii)
Theoretical Statement 1 (iii)
Theoretical Statement 1 (iv)
Theoretical Statement 1 (v)
Theoretical Statement 2
Theoretical Statement 3 (i)
Theoretical Statement 3 (ii)
Theoretical Statement 3 (iii)
Theoretical Statement 3 (iv)
Theoretical Statement 4
Theoretical Statement 7
Theoretical Statement 8
Theoretical Statement 9
Theoretical Statement 10
3/3 students
3/3 students
3/7 students
3/6 students
3/6 students
2/3 students
4/13 students
1/3 students
1/4 students
4/13 students
5/13 students
1/4 students
2/4 students
1/1 student

APPENDIX E: MAPPING OF OPEN TO AXIAL CODES FOR R1, R2 AND R3

R1: Do students' understandings of ratio vary from domain specific tasks to structurally isomorphic tasks?

Figure III

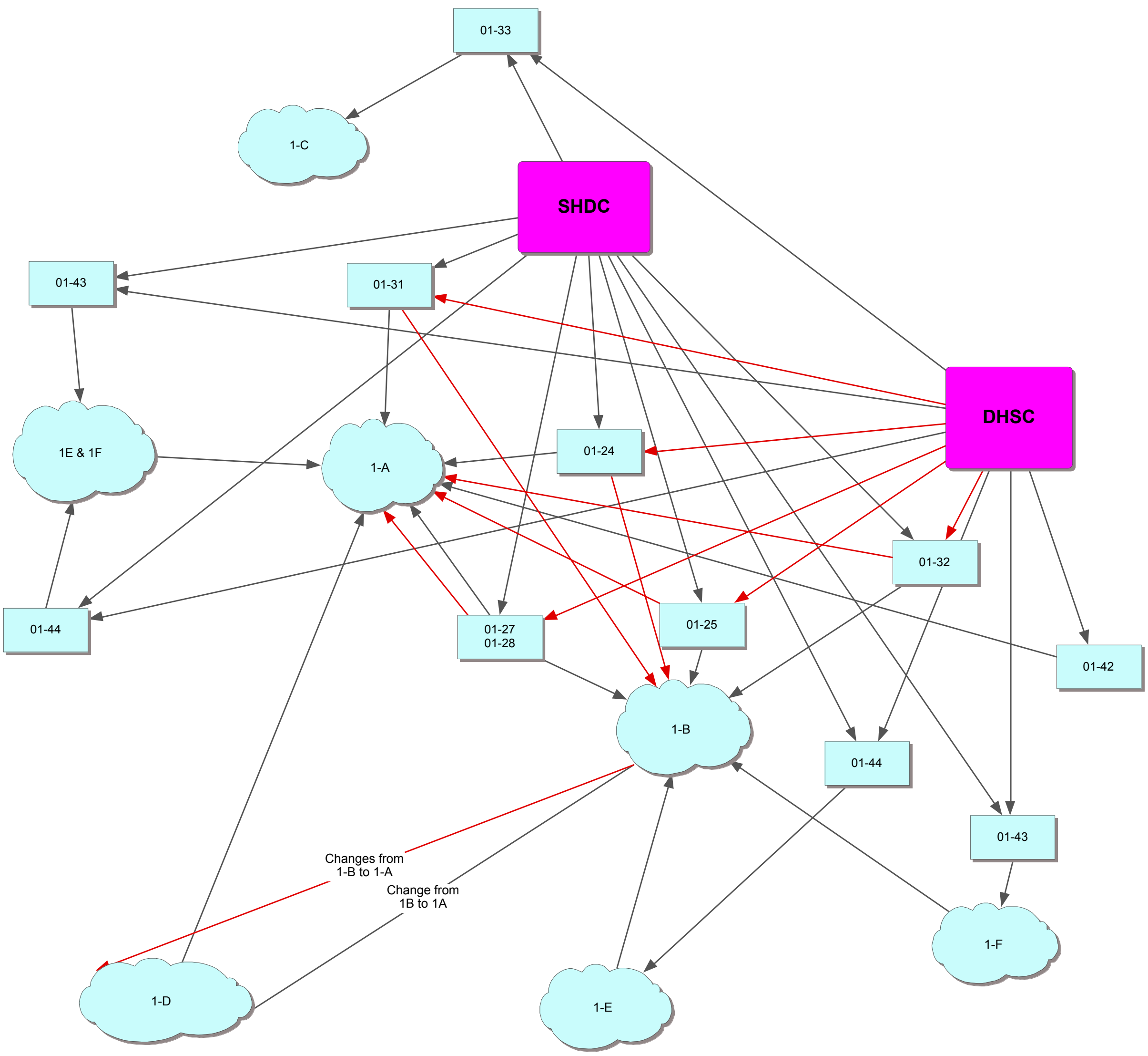


Figure III shows the mapping of open codes as they relate to axial codes for R1. Because there were two tasks related to these codes, the SHDC and DHSC tasks, there is some overlap in the coding scheme. In those cases, the arrows go from SHDC and DHSC tasks to the same open code. However, because the SHDC and DHSC tasks are contrasting cases, a code may correct in one case and incorrect in another case. For example, Code 01-32 (Student uses a different total amount of cubes to represent each block) is an incorrect strategy for the SHDC task, but in the DHSC task it is the correct strategy. Situations such as these are are highlighted by using a red arrow. The specific open codes that were used to create the axial codes are then connected by arrows.

Figure III shows the open codes as they relate to the axial codes for R2. Three open codes have some overlap within the axial codes: 02-48 (Student indicates that there are varying amounts of calcium chloride in each jar), 02-46 (Student indicates that there is the same amount of calcium chloride in each jar), and 02-76 (Student indicates that there are varying amounts of "something else" in the jar). These three open codes can relate to multiple axial codes depending on the context and task.

Figure LLL shows the open codes related to the PR diagnostic as they relate to the PR axial codes. Most of the pertinent open codes map directly onto the axial codes with the exception of PR-E, which involves multiples, which had several varieties within the open codes.

FIGURE III: Mapping of open to axial codes for R2

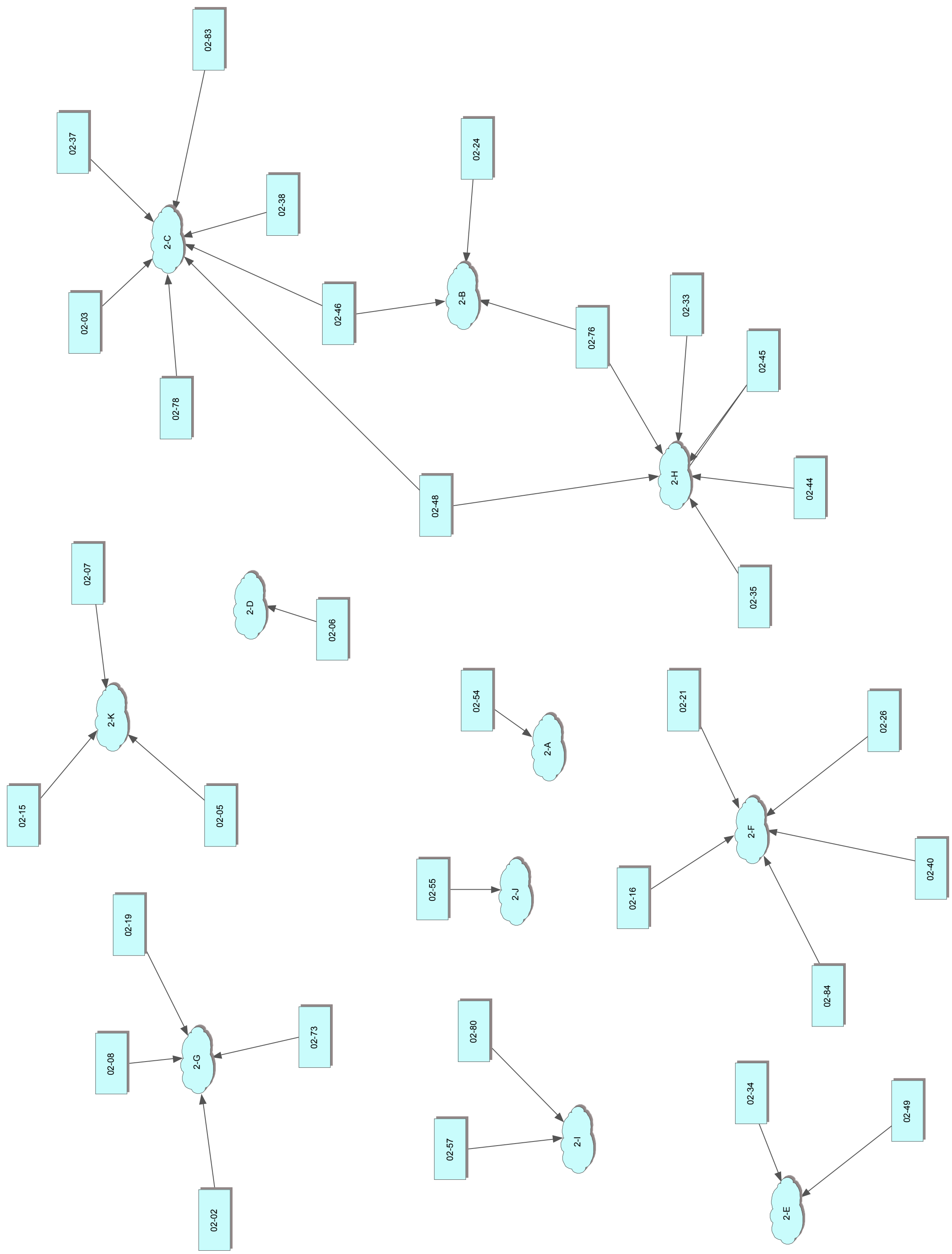
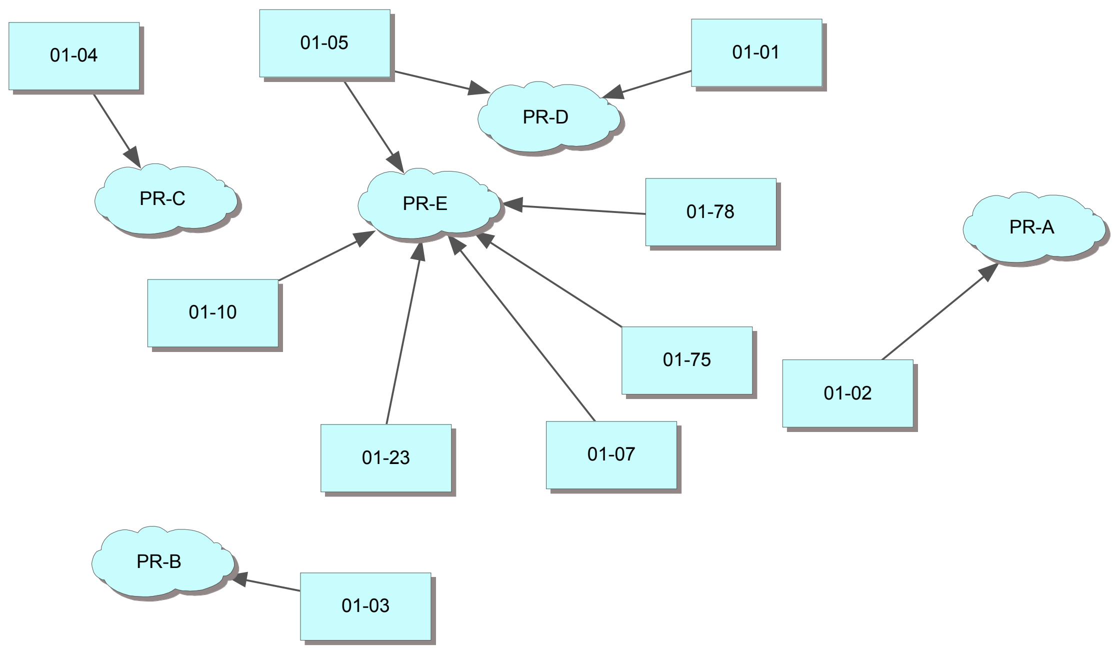
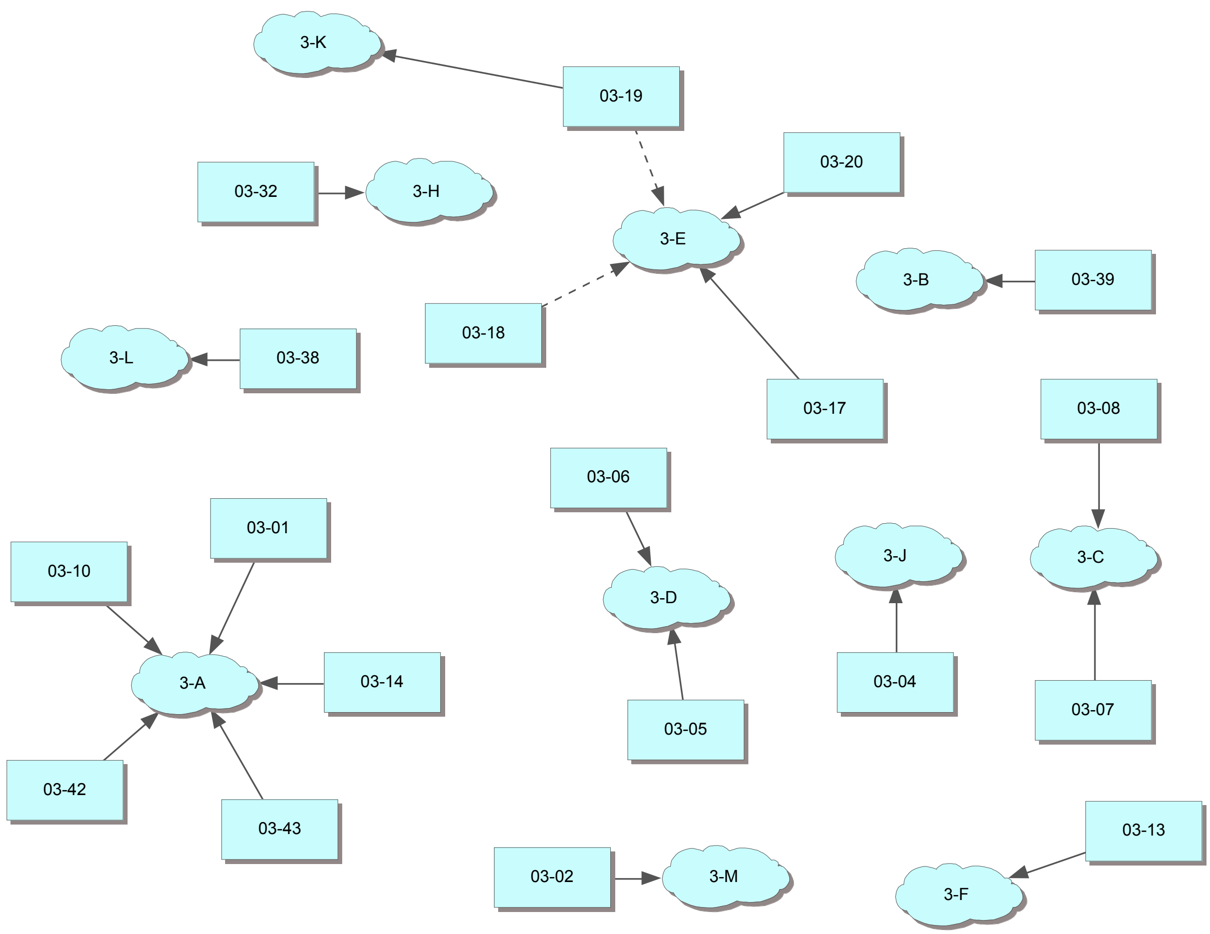


Figure LLL: Mapping of open to axial codes for R3 Proportional Reasoning Questions



Finally, the open codes for R3 chemistry solving strategies are shown as they relate to the axial codes in Figure MMM. The dashed lines for 03-18 and 03-19 to 3-E indicate the absence of those codes.

Figure MMM: Open codes as they relate to axial codes for R3



APPENDIX F: MAPPING OF AXIAL CODES TO THEORETICAL STATEMENTS FOR R1, R2, AND R3

Figure HHH: Mapping of Axial Codes to Theoretical Statements for R1

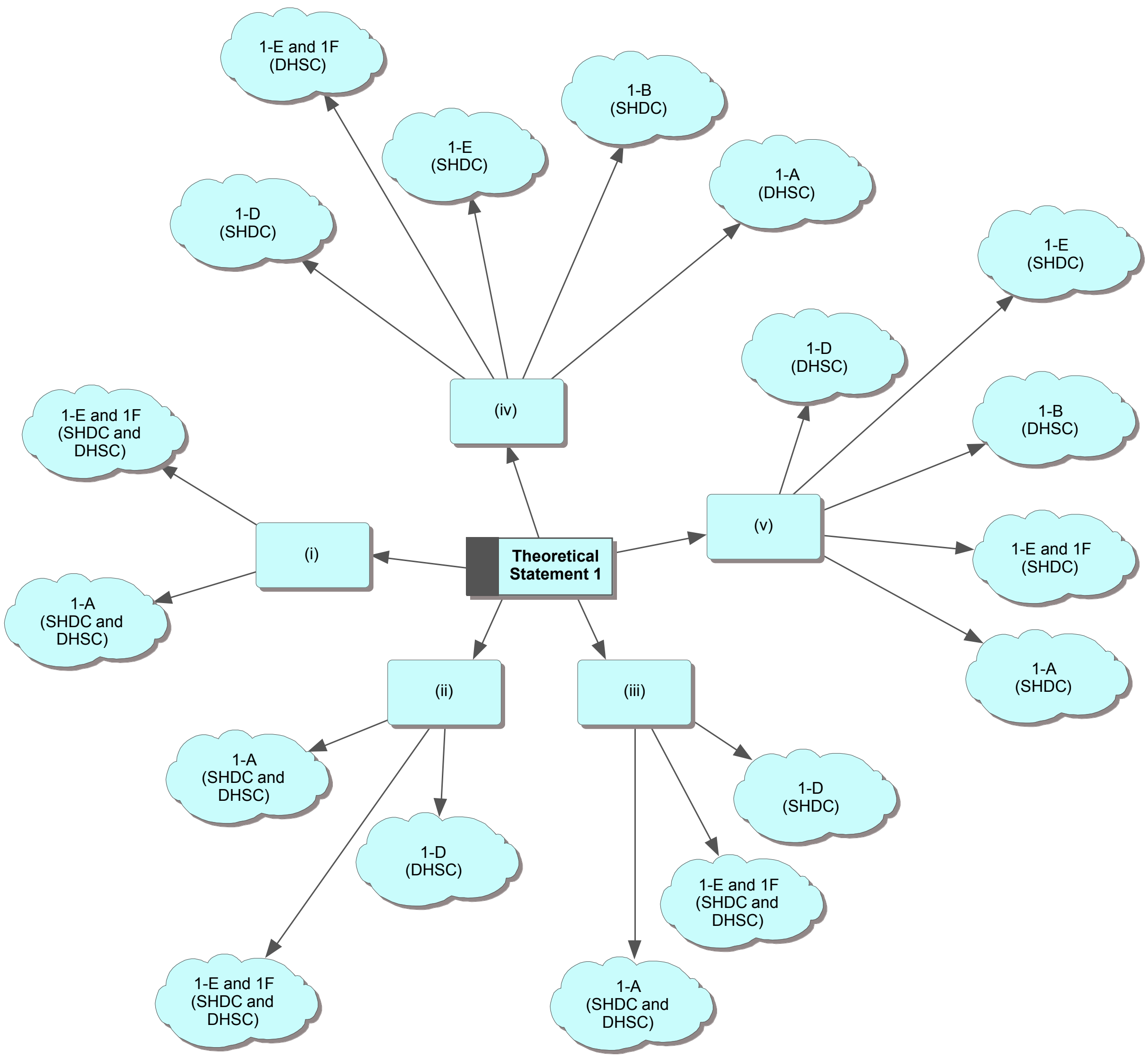


Figure BBB: Mapping of axial codes to Theoretical Statements for R2

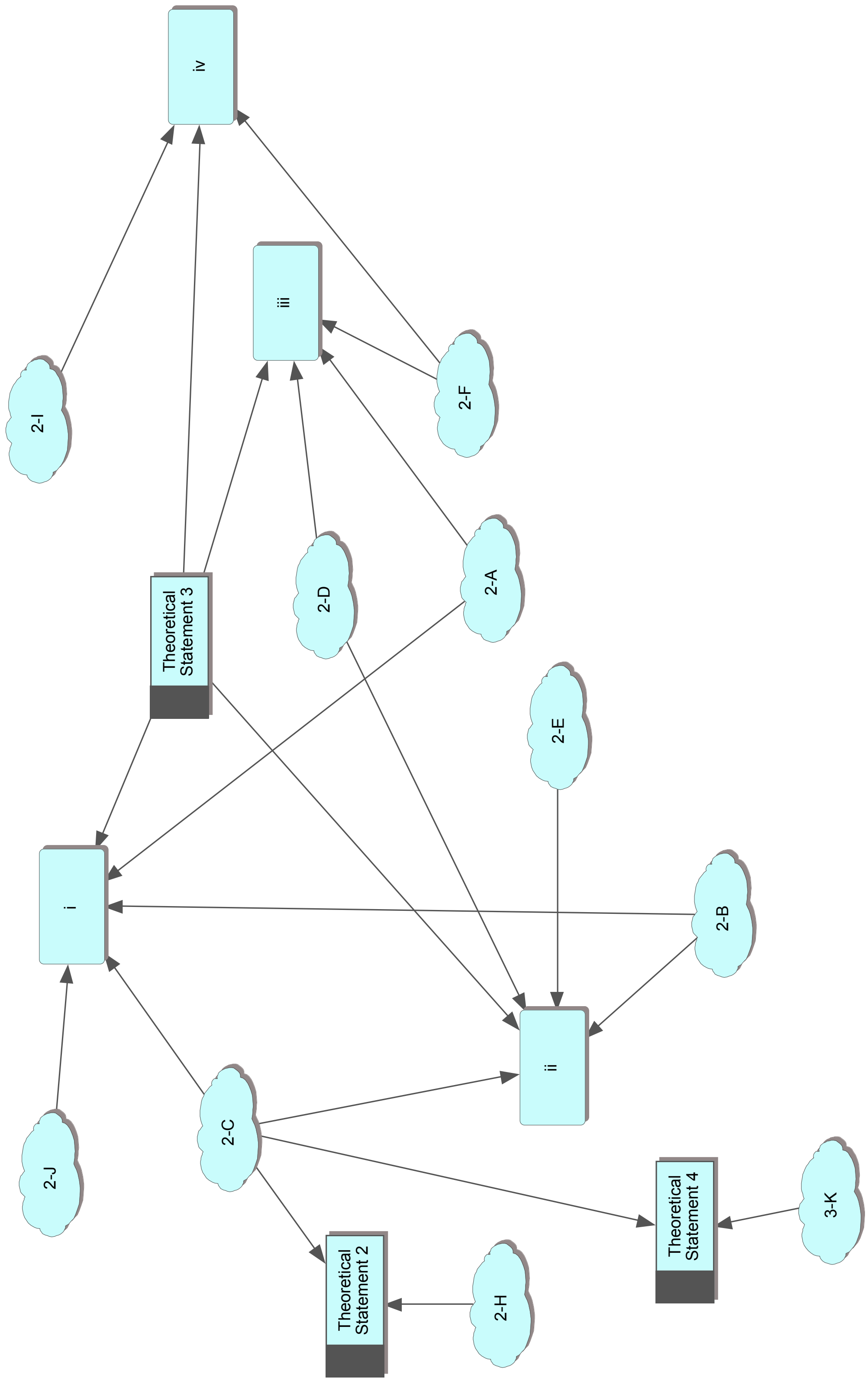
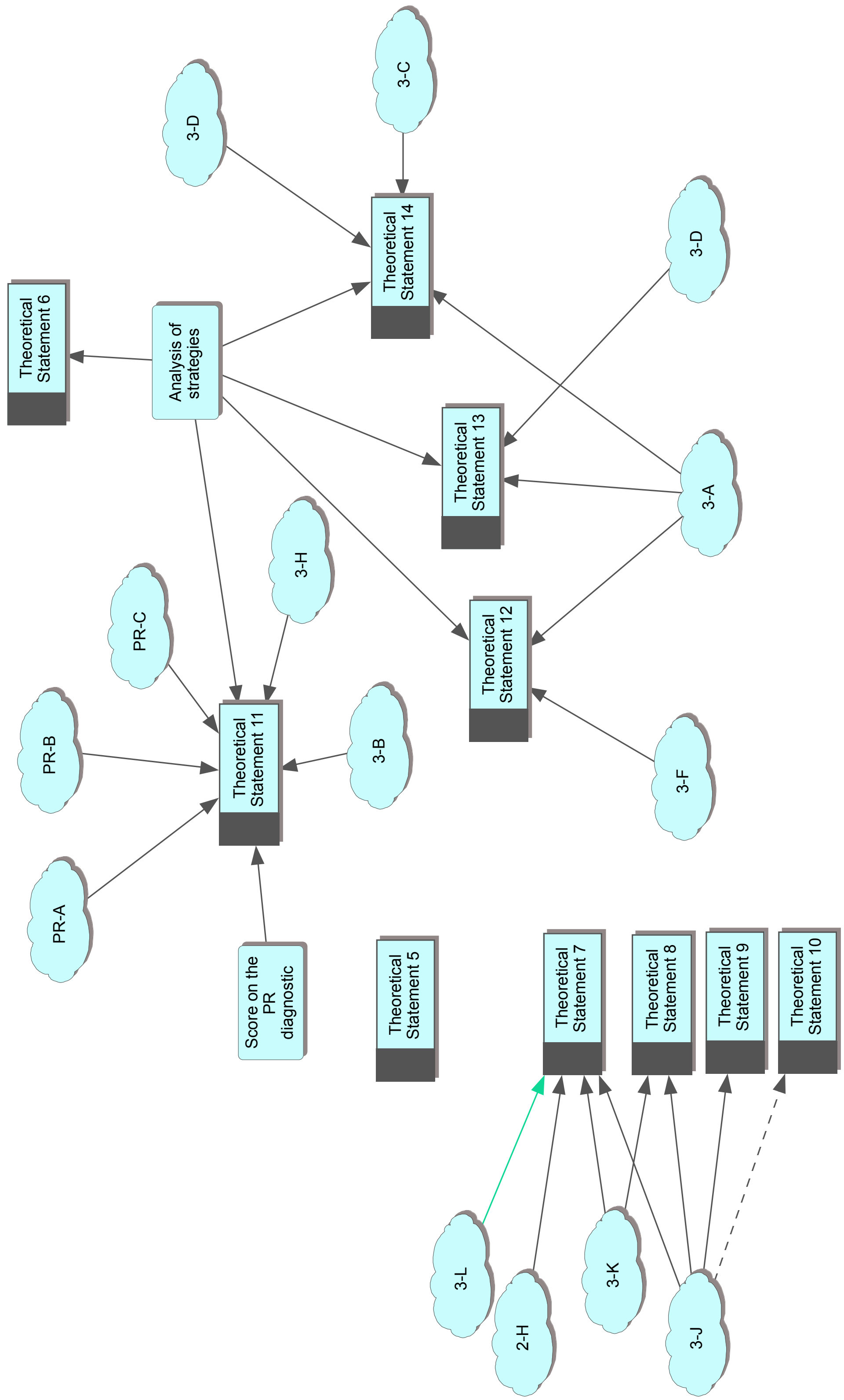


Figure AAA: Mapping of axial codes to Theoretical Statements for R3



APPENDIX G: SCORING

The following pages show how students were scored in the axial coding scheme and in the analysis of solving strategies.

Student	M1 Moles from molarity	M1 after hint	M2 Volume from Molarity	M2 after hint	M3 Molarity from grams	M3 after hint	M4 molarity stock solution	M4 after hint	M5 Dilution	M5 after hint	M6 grams from molarity	M6 after hint
1-3	No- M=g/mL M is moles so .75 mol	Not attempted	No- molar mass; M=g/mL M=moles	Not attempted	No- doesn't convert grams to moles or mL to L	Not attempted	Not attempted	not attempted	Yes- cut in half	not attempted	No- M=g/mL and fills in algorithmic no conversion mL to L or gram to mole	not attempted
2-1	No- regonizes needs conversion and writes mL to g to mol	Not attempted	Not attempted	Not attempted	Not attempted	Not attempted	Not attempted	Not attempted	No- adds mL together	Not attempted	No- recognizes needs conversion writes mL to mole to g	Not attempted
3-4	not attempted	not attempted	not attempted	not attempted	not attempted	not attempted	not attempted	not attempted	not attempted	No- set up proportion mL/ mL=M/M gets double answer	not attempted	not attempted
4-2	not attempted	not attempted	Not attempted	Not attempted	Not attempted	Not attempted	Not attempted	not attempted	No- Wrote 200mL but not sure what it means	not attempted	not attempted	not attempted
5-3	No- M=molesxV no volume conversion	not attempted	not attempted	No- doesn't know how to do it	not attempted	no- molar mass. convert grams to moles divided by wrong decimal mL to L conversion off	not attempted	no- doesn't know how uses molar mass but calls it moles instead of grams	not attempted	not attempted	not attempted	not attempted
6-3	not attempted	no- sometime found molar mass but called it moles	not attempted	yes- molesxM=answer	not attempted	no- gx X=mL no gram to mole conversion no mL to L conversion. algorithm	no- subtracted 1 molarity from other	not attempted	no- guesses it will stay the same	no- misconversion mL to L. algorithm mL=Mxg no gram to mole conversion. attempts mL to L conversion	not attempted	no- set up algorithm mL=MxX no mL to L conversion; no g to mole conversion
7-4	no- volume/ molarity no mL to L conversion	no- used grid but shows same math as before	no- writes an algorithm gets 10 moles/M	not attempted	no- g/mL no g to mole conversion no mL to L conversion	not attempted	not attempted	not attempted	yes- of means multiply drew pic weakened by half. gets M=moles though	no- sets up grid to cancel mL doesn't get answer. Cannot apply new info from hint	no- molar mass didn't have two Cls. took volume/grams. No mL to L conversion. No g to mole conversion.	not attempted
8-4	no- draws bottle	no- writes algorithm but no answer	no- draws bottle	yes- writes algorithm fills in but doesn't answer. but verbally says 10L or mL	no- draws bottle	no- writes algorithm. no mL to L conversion tries to convert g to mole	not attempted	no- writes algorithm M=M/ mL no mL to L converson	not attempted	NO-sets up algorithm partially but unable to finish	no- writes M to g formula but no answer	no- sets up algorithm without converting mL to L doesn't solve
9-2	not attempted	no- first takes M divides by L too big then sets up algorithm correctly but divides wrong	not attempted	no- calculates molar mass and divides M by it sets up algorithm	no- g/ML using fractions to decimal	not attempted	yes- set up proportion	not attempted	yes-proposes 2 ideas concentration will decrease m=moles and stays the same	not attempted	not attempted	not attempted
10-1	no- set algorithm correctly- multiplication error	not attempted	no- set up algorithm correctly but convert mL to L wrog	not attempted	yes- algorithm found moles/L	not attempted	no- set up proportion incorrect	not attempted	yes- just said "half that"	not attempted	yes- setup algorithm to find moles and convert to grams	not attempted
11-1	not attempted	yes- sets up proportion with algorithm x	not attempted	no- set up algorithm but divided wrong	not attempted	no- fond molar mass using proportion. Using # of atoms as moles. Had wrong conversion mL to L	not attempted	No- set up algorithm and filled in answers- wrong algorithm M=M/mL	not attempted	no- set up a proportion and go double answer	not attempted	no- set up proportion to find grams but set it equal to mL. wrong gram to mole conversion.
12-1	no- wrote down same answer M is moles	yes- algorithm mol/L	not attempted	yes- algorithm mol/L	no- molar mass M=moles	no- kept same answer molar mass M=moles	not attempted	no- M/L but stopped	no- molar mass x molarity=molarity	yes-algorithm mol/L	no- molar mass=M	not attempted
13-4	no- 12.01 x M/V no mL to L conversion. M=moles	no- molarity/ volume formula no mL to L conversion M=moles	not attempted	not attempted	no- g/mL x 12.01 no conversion M=moles	not attempted	not attempted	not attempted	not attempted	no- formula molarity/ volume no conversion m=moles	not attempted	not attempted
14-1	not attempted	not attempted	not attempted	not attempted	not attempted	not attempted	not attempted	not attempted	yes- converts mL to L sets up proporiton	not attempted	not attempted	not attempted
15-1	no- M is moles	no- converts mL to L molarity volume M/L	no- sets up grid to cancel moles no answer 10mol/ 1mol no answer	yes- algorithm molarity=moles/X	no- adds molar mass wrong- essentially finds moles right process wrong number; converts g to mole; moles=molarity didn't use volume	no- M-Mass/ volume no g to mole conversion converts mL to L	not attempted	yes- writes algorithm for molarity converts mL to L multiplies M x volume	no- concentration is less verbally but no answer	no- algorithm; molarity of 1st divided by total volume of second; molarity=6/0.2 L did mL to L conversion	no- essentially right process wrong umbers; grid to cancel moles; wrong molar mass; conversion mole to g M=moles	no- algorithm converted mL to L uses wrong molar mass; uses grid to convert mole to g; essentially right process wrong numbers
16-2	not attempted	no- draws picture; molarity/volume no mL to L conversion	yes- 10:1 ratio just wrong unit	yes- algorithm M=mol/L	no- volume/ mass; no g to mole conversion. no mL to L conversion	not attempted	no- knew which pieces to use just not how to use them	not attempted	no- same answer doesn't affect	no- algorithm; molarity/ volume no mL to L conversion	not attempted	no- just rewrote problem
17-3	not attempted	no- algorithm; molarity=x(mole)/ volume no mL to L conversion; no answer	not attempted	no- algorithm; M=moles/ atomic mass. incorrect atomic mass CaCl	no- wrote CaCl2 +H2O--> no response	no- g=some ratio; mentions converting but doesn't know how	not attempted	no- no response/ rewriting data	no- same just more volume	not attempted	not attempted	no- algorithm M= / volume no mL to L conversion assumes moles
18-4	no- Ca and Cl are one mole together	no- algorithm; didn't convert mL to L. proportion for cross multiply units aren't the same	no- no response verbal M is something else that mole against grams	no- algorithm; but doesn't finish. M=mole/L right if had finished	no- calculates molar mass converts g to mole backwards; no volume in work mole=molarity	no- algorithm molarity=g/mL no g to mole conversion; no mL to L conversion. Added zero after and lost decimal	not attempted	no- chooses 0.42M from answer starts proportion 1M= / 16mL looks like trying to find moles	yes- add water divide	yes- begins algorithm and says they were right. M= / 200mL but doesn't solve	no- uses molar mass.	no- keeps answer
19-2	not attempted	no- algorithm; proportion for cross multiply with wrong units. no mL to L conversion	not attempted	yes- algorithm	not attempted	no- attempts g to mole conversion with wrong molar mass; reduces fractions; fraction over volume no mL to L conversion fraction to decimal	not attempted	yes- proportion with X wrong units M first switched to mole	yes- divided by 2	no- M=x/100 no mL to L conversion	not attempted	no- algorithm M=x/ volume no mL to L conversion. attempts to convert g to mole wrong molar mass.
20-2	no- no response- knows needs M to mL conversion knows needs M to g conversion.	no- algorithm; mL to L conversion wrong	no- no response but sets up a kind of proportion uses X	yes- algorithm	not attempted	no- molar mass g to mol conversion backwards. mL to L conversion wrong algorithm	no- no response but sets up a kind of proportion	yes- wrong unit right number MxV	not attempted	no- stays same	not attempted	no- algorithm mL to L conversion wrong. attempted g to mole conversion
21-2	not attempted	no- writes grid volume/molarity no response	not attempted	not attempted	not attempted	not attempted	yes- multiply MxV	yes- keeps answer	not attempted	yes- cut in half	no- finds molar mass	no- keeps molar mass answer
22-3	no- M=moles so same	not attempted	not attempted	no- algorithm sets up doesn't finish. M=mole/L fill in moles	No- CaCl grid wrong molar mass calculation g/ gM/mL random ratio	not attempted	no- M=moles uses 0.42 from problem	no- algorithm- molarity=molarity /volume no mL to L conversion	no- no answer unclear if 0.6M also M=moles	no- keeps same answer- unclear if 0.6M? unclear if volume changes	no- CaCl molar mass incorrect. Random ratio to cancel M - grid- molarity/ volume g/ molarity no mL to L conversion	no- molarity/molar mass calculation incorrect CaCl
23-4	no- M=molesxV no volume conversion	no- breaks out unit of moles/liter no answer	no- uses answer of 0.75M M=moles for answer; 10mL=a complete mole	yes- algorithm uses moles/L to answer verbally; using a proportion 1 mole per liter 10 liters 10 mole	no- grams/mL no conversions M=mole	no- g=mole/L no answer	yes- MxV wrong unit	no- writes out units ; percent discussion no answer need . 42% so helps with numbers	yes- cuts in half because moles/ mL	no- mL to L wrong conversion gets 0.6M	no- mLxM=g no mL to L conversion; answer is in grams	no- 0.85/L molarity/L
24-3	not attempted	no- algorithm M=X/volume mL to L conversion wrong uses proportion for cross-multiply	not attempted	yes- algorithm uses proportion for cross multiply	no- g/L wrong but attempts a mL to L conversion	no- found molar mass; algroithm; M=x/25 began mL to L conversion. g to mole conversion calc verbal but on paper has trouble	not attempted	not attempted	not attempted	no- sets up a sort of proporiton M= ()mL: no mL to L conversion; can't decide between go down or up says double and proportions in answer	not attempted	not attempted

Student	C1	C2	C3	C4	C5	C6	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	M1	M2	M3	M4	M5	M6
1	1	0	0	0	0	0	1	0	0	0	1	1	1	0	1	1	1	1	1	0	0	0	0	1	0
2	0	0	0	0	0	0	1	1	1	0	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0
3	0	0	0	0	0	0	1	1	1	0	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0
4	0	0	0	0	0	0	1	1	1	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0
6	0	0	0	0	0	0	1	1	0	1	1	0	1	1	0	1	0	1	0	0	1*	0	0	0	0
7	0	0	0	0	0	0	1	1	0	1	1	0	1	1	1	1	1	0	1	0	0	0	0	1	0
8	0	0	0	0	0	0	1	1	0	1	1	1	1	1	1	1	1	0	1	0	1*	0	0	0	0
9	1	0	0	0	0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0	0	1	1	0
10	1	0	0	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1
11	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1*	0	0	0	0	0
12	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1*	1*	0	0	1*	0
13	1	0	0	0	0	1	1	1	1	1	1	0	0	0	1	1	0	1	1	1*	0	0	0	0	0
14	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1					1	
15	0	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1*	0	1*	0	0
16	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0
17	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0
18	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	0
19	0	0	0	0	0	0	1	1	1	0	1	1	1	1	1	1	0	1	1	0	1*	0	1*	1	0
20	0	0	0	0	0	0	1	1	1	0	1	1	1	0	1	1	1	1	1	0	1*	0	1*	0	0
21	0	0	0	0	0	0	1	1	1	0	1	1	1	1	0	1	1	1	0	0	0	0	1	1*	0
22	0	0	0	0	0	0	1	1	1	1	1	0	1	1	0	1	0	0	1	0	0	0	0	0	0
*23	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1	1	1	1	1	0	1*	0	**2**	**2**	0
24	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0

*23-4 switched to wrong answers after intially having them correct.

	Procedural	Procedural notes	Change in Substance	Change in Sub Notes	Change in Volume	Change in Volume notes	Same Size Different Conc 2 components	Same Size Diff Conc notes	Different Size Same Conc 2 components	Diff Size Same Conc notes
13-M	yes-c		yes		yes		no	Discusses water but doesn't factor into number; M=moles	no	Discusses water but doesn't factor into number; M=moles
16-M	no		yes		yes		no	Discusses water but doesn't factor into number; M is %of CaCl2	no	Discusses water but does not factor into number; M is strength; Size
17-M	yes		no	M is an element	yes		no	M is an element	no	M is an element; M is liquid; M is related to CaCl2; student sees a relationship between two things but neither is water
18-M	yes-c		no	M is acidity or basicity; size	yes	acidity	no	M is acidity or basicity; size	no	acidity basicity; does find relationship with water and CaCl2 just wrong.
19-M	no		no	Same number of molecules just tighter; M=moles; Drawing switches answer from substance change to same amount.	yes		no	Discusses water but doesn't factor into number; M=moles	no	says label doesn't account for water; M=moles; density
2-M	No		No	Changes answer through discussion to yes	Yes		No	M=strength; Not sure what M stands for	No	M=moles=same; mentions different water
20-M	no		yes	M=moles	yes		no	Doesn't address water; M=moles	no	Discusses water but does not factor into number; M=moles
21-M	no		yes	M somethin g to do with liquid	yes		no	discusses water but doesn't factor into number; M is something to do with liquid; M is quantity of CaCl2	no	Mentions water but does not factor into number; M is quantity of CaCl2; Spreads
22-M	no		no	During drawing changes from substance change to substance same	yes		no	doesn't mention water at all; size of atoms; M=moles; moles nothing to do with molecules	no	doesn't mention water at all; packing; size of atoms; M=moles; moles nothing to do with molecules.

	Procedural	Procedural notes	Change in Substance	Change in Sub Notes	Change in Volume	Change in Volume notes	Same Size Different Conc 2 components	Same Size Diff Conc notes	Different Size Same Conc 2 components	Diff Size Same Conc notes
23-M	no		yes	M=moles	yes		no	discusses water but doesn't factor into number; M=moles	no	discusses water but does not factor into number; M=moles
24-M	yes-c		no	Drawing changes substance different substance same amount	yes		no	doesn't mention water; size; M=moles	no	mentions water but not important part; M=moles; student example of a classroom is wrong siilar to M is Moles
4-M	no		no	attributes number to volume; later changes to number of CaCl2 molecules	yes		no	doesn't address water; M=molecules of element	no	unsure change in volume; same amount of CaCl2; No idea M means
5-M	no		yes	M is element	yes		no	M is element or something else	no	Understood difference in CaCl2; Used recipe; no mention of water
6-M	no		no	Same CaCl2 verbal; draws different amount	yes		no	uses concentration language; different amount of molecules but same amount of CaCl2; compact compressed; no mention of water	no	calcium mixed with chloride; but more calcium with more volume; amount of moles; then same Ca but different water after hint.
9-M	yes		yes		yes		no	M=moles; but says something makes up for volume	no	talks about it right; but M is moles; draws moles the same; discusses water; proportion but changes; proposes both answers and ends on M=moles.
11-C	yes		yes		yes		no	Different lengths	yes	
3-M	No		yes		yes		no	but thinks "whole zero" backwards decimals; can't explain concentration but uses it	yes	uses concentration but can't explain it.
11-M	yes-c		yes		yes		yes	M is amount of CaCl2	no	M=moles; x=more quantity
12-M	yes-c		yes		yes		yes	M=moles	no	uses water; same amount of CaCl2; "part of a mole"

	Procedural	Procedural notes	Change in Substance	Change in Sub Notes	Change in Volume	Change in Volume notes	Same Size Different Conc 2 components	Same Size Diff Conc notes	Different Size Same Conc 2 components	Diff Size Same Conc notes
15-M	no		yes		yes	M=moles	yes		no	Mentions water but does not factor into number; spacing; M=moles
7-M	no		yes	Backwards decimals	yes		yes	strength; M=moles; backwards decimals; label tells us nothing	no	but all calcium is the same; mentions water
8-C	yes		yes		yes		yes		no	no length
8-M	no		yes		yes		yes	M=mL?; talks about percentage	no	talks about water; but all same CaCl2
1-C	Yes-c		yes		yes		yes		yes	But says needs more white to balance color
1-M	Yes-C	estimation	Yes		Yes		Yes	M=g/mL; M=moles; liquids all look the same.	Yes	M=moles; but moles are not equal to substance amount.
10-C	yes		yes		yes		yes		yes	
10-M	yes-c		yes		yes		yes		yes	
12-C	yes-c		yes		yes		yes		yes	
13-C	no		yes		yes		yes		yes	
14-C	yes		yes		yes		yes		yes	
14-M	yes-c		yes		yes		yes		yes	
15-C	yes		yes		yes		yes		yes	
16-C	yes-c		yes		no	Represented color only until house example	yes		yes	
17-C	yes		yes		yes		yes		yes	
18-C	yes		yes		no	Represented color only until house example	yes		yes	
19-C	yes		yes		no	until explicity asked about length then yes	yes		yes	
2-C	yes		yes		yes		yes		yes	
20-C	yes		yes		yes		yes		yes	
21-C	yes		yes		yes		yes		yes	
22-C	no		yes		yes		yes		yes	
23-C	yes		yes		yes		yes		yes	
24-C	yes		yes		yes		yes		yes	
3-C	yes		yes		yes		yes		yes	
4-C	no		yes		yes		yes		yes	
5-C	yes		yes		yes		yes		yes	
6-C	no		yes		yes		yes		yes	
7-C	yes		yes		yes		yes		yes	
9-C	yes		yes		yes		yes		yes	but uses same amount of white-think ratio?