Interpreting Functions of One-Dimensional Kinematics

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THESIS

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This dissertation is dedicated to complexity, the seasoning of life and matter, and to the living mind that encouraged me to embrace it; Mr. Cary Parker, Jr.

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LIST OF ABBREVIATIONS

DSR	DERIVATIONAL STRUCTURE OF REPRESENTATIONS
DSH	DERIVATIONAL STRUCTURE HYPOTHESIS
RT	RESPONSE TIME

SUMMARY

Informationally equivalent external representations can vary in the way that they preserve information and this, in turn, can have consequences for cognitive processing (Larkin & Simon, 1987; Zhang & Norman, 1994). Across four experiments, 64 individuals were tested using a modified sentence-picture verification task in which four major factors were manipulated: Graph Type (position-time, velocity-time), Curve Morphology (linear, non-linear), Judgment Class (general motion, velocity change) and Motion Description (nested within judgment class; e.g., *The object is moving; The object is accelerating*). Manipulating these variables made it possible to test several hypotheses with the express aim of elucidating the nature of graph interpretation difficulty in terms of whether the information could be directly read off the graph or needed additional processing beyond what was directly visible. Experiments 1 and 2 tested the hypothesis that velocity-time graphs are generally more difficult to interpret than position-time graphs (Brasell, 1987). Experiment 2 also tested an alternative hypothesis based on the *derivational structure of representations* (DSR: Palmer, 1978). The derivational structure hypothesis predicts higher judgment accuracy and shorter judgment latency when decisions about object movement are made directly from information in the visual display as compared to requiring an inference from that information if the inference demands are met. This hypothesis was also tested in Experiment 3 for position-time graphs. In brief, the patterns of results across Experiments 1 and 2 indicated no support for the graph difficulty hypothesis and no support for the derivational structure hypothesis as a processing account. The accuracy data suggested that participants simply did not go beyond direct processing of the visual display even when needed for accurate decisions. Rather, a post hoc hypothesis, direct read-off, was proposed and shown to account for response patterns in Experiment 2. Experiments 3 and 4 confirmed that subjects used direct read-off strategies related to the curve such as *up means faster* even when more elaborate processing was required for accurate judgments. Experiments 2 – 4 highlighted the influence of height, direction of slope, and curvature on interpretation. Their relative impacts depended on the type of graph and the concept being interpreted. The results show that slope has at least two properties – direction and curvature – that can independently impact processing. Future work focused on fostering graph reasoning skills should build on natural spatial-conceptual correspondences (e.g., Gattis & Holyoak, 1996; Tversky, 2011) that often bias graph-based judgment. Instructional design can promote direct read-off as a viable interpretation strategy but such knowledge should be accompanied by additional knowledge of when to use those strategies versus strategies that require more cognitive work.

I. INTRODUCTION

Interpreting functions is an important skill for understanding physical data. Educational studies have shown that people reliably exhibit difficulties when interpreting functions in graphs that describe objects in motion (e.g., Beichner, 1994; Bell & Janvier, 1981; McDermott, Rosenquist, & van Zee, 1987). These *kinematic* functions are fundamental components of high school Advanced Placement (Cain, 2006; Gende, 2006) and college-level introductory physics courses (Browne, 1999; Forster, 2004; Knight, 2004; Knight, Jones, & Field, 2007; Saxon, 1993; Young, Freedman, & Ford, 2007). Kinematic functions describe the relationship among three variables in relation to time: distance (i.e., position or displacement), rate (i.e., speed or velocity) and rate of rate change (i.e., acceleration). Three canonical graph types are used to represent this relationship with either position (position-time graph), velocity (velocity-time graph), or acceleration (acceleration-time graph) on the ordinate and time always on the abscissa. Thus, there are three isomorphic graph type representations for a given set of relationships among the three variables: distance, rate, and rate of change. Although the "same" information may be represented in the three graph types, the shape of the function (curve morphology) can vary (e.g., direction of slope; curvature).

The interesting issue from a cognitive perspective is how people process the information represented in the three types of depictions of kinematic functions. For example, is it equally difficult or easy to determine whether an object is moving or not from the three types of graphs? Is rate of movement equally difficult to determine in all three types of graphs? The general expectation is that the accuracy and latency of determining such features of object movement from graphs should depend on the ease of extracting information relevant to that feature from the curve depicted in the graph. In particular, in moving from one graph type to another for some particular aspect of motion, curve morphology would change. Depending on the aspect(s) that needs to be extracted to answer the question accurately, some combinations of graph type and curve morphology may be easier than others. For example, if asked to decide whether an object is accelerating, change in the rate of motion must be determined. Such information is directly represented in the velocity-time graph shown in Figure 1a: the linear curve shows an object with increasing velocity (positive slope). In contrast, for the position-time graph in 1b, what is depicted is a linear change in the location of an object: the amount of change per unit time is constant and therefore the object is neither accelerating nor decelerating rather; it is changing position (i.e., moving) at a constant rate.

Figure 1. (a) Velocity-time graph depicting an object increasing in velocity. (b) Position-time graph depicting an object changing position at a constant rate.

A key aim of cognitive psychology is to elucidate the nature of reasoning in terms of the relationships between internal and external representations and the mechanisms and processes that underlie their coordination (Palmer, 1978; Zhang, 1997; Zhang & Norman, 1994). The relationships between internal and external representations are non-arbitrary (Hegarty, 2004) and, in fact, are known to account for the *representational effect*, the phenomenon whereby informationally equivalent representations are predictive of systematic differences in cognitive behavior (Nickerson, 1988; Zhang, 1997; Zhang & Norman, 1994, 1995). It is therefore natural for the cognitive researcher bent on understanding reasoning as representation to ask how graph type, curve morphology, and the motion concept affect the interpretation of kinematic function graphs.

In physics, interpretation tasks designed to foster conceptual learning or assess knowledge about kinematics functions often demand that a person draw correspondences between verbal statements and accurate or inaccurate graphical representations of a kinematic variable. The following sections define kinematics functions, describe the graph features that interact to afford their accurate representation, and discuss the derivational structure of representations and how it may influence graph interpretation. In addition, there is a review of the difficulties that high school and college students have demonstrated when interpreting one-dimensional kinematics functions in graphs. This literature motivates the proposed experimental work to investigate the effects of the way a kinematic function is represented (graph type and curve morphology) on decisions related to object movement. As discussed below, the manipulations are associated with issues of informational equivalence and computational efficiency.

A. Kinematics Functions: "The Represented World"

The behavior of an object in motion can be described as a kinematics function. A kinematics function is defined by the relationships among three key variables: *displacement*, *velocity*, and *acceleration*. These are formal ideas in physics that represent the extent that an object changes position (displacement) with the passage of time, the rate at which a change in position occurs (velocity), and the change in the rate at which the position of an object changes (acceleration). Each concept is defined in relation to time. Descriptions of the graph types and the curves follow.

B. Graph Types and Curves: "The Representing World"

To interpret representations of kinematics behavior plotted in graphs the interpreter must coordinate information among the axes and the curves. In the current context the variable, *time*, is always represented on the abscissa. Graph type is therefore solely defined by the ordinate label. The behavior of an object in motion is plotted as a curve (including a straight line) within a two-dimensional Cartesian coordinate system. The displacement, velocity, and acceleration of an object are simultaneously represented by the plot. The same information about an aspect of motion is represented differently as a function of graph type. The question is what are the interpretive demands of these different representations?

1. Position-Time and Velocity-Time Graphs

As previously mentioned, position-time graphs and velocity-time graphs are physically distinguished by their ordinate labels. Accordingly, the former has an ordinate labeled, *position*, whereas the latter is marked by an ordinate labeled, *velocity*. What follows are discussions about what the ordinate in each graph *represents* and the different ways in which the ordinate may be interpreted.

In a position-time graph, the ordinate represents a change in the position of an object in relation to time. More specifically, it is used to measure how much an object has changed position in relation to its original position at some given point in time. A basic use of the ordinate in a position-time graph is simply to identify whether an object is moving (i.e., changing position) or is stationary (i.e., not changing position); i.e., *general motion*. If moving, then the height of the curve will change in relation to the ordinate. If stationary, then the height of the curve will remain constant in relation to the ordinate. Thus, as shown in Figure 2, in the position-time graph, change in the *height* of a curve in relation to the ordinate corresponds with a change in the position of an object (i.e., motion).

Given the graph in Figure 2, hypothetically, two different mathematical reasoning strategies can be employed to yield the same information about the general motion of an object. The first involves the subject "drawing" horizontal lines parallel to the abscissa between different points on the curve and the ordinate (see Fig. 2a). If the length of these lines changes then the interpreter can conclude that the object is moving. This is referred to as the "fixed ordinate" strategy. The second strategy involves imagining a sliding ordinate in which the ordinate slides across the abscissa like a rolling library ladder (see Fig. 2b). If the height of the ordinate changes as it slides the length of the curve, then it can be reasoned that the object is moving. This is the "slide ordinate" strategy for recognizing general motion in the Position-Time graph. Of course, more informal reasoning strategies are possible and probably likely for most people. These more informal strategies might verbally describe what the curve represents, as in, "The curve is going up fairly steeply at first; then it starts to flatten out. So as it flattens out it is moving less per unit time so its rate of changing position is getting slower" (decelerating). Whether done through formal mathematics or through more informal mathematics reasoning, the graphical depiction of a curvilinear function directly conveys motion and therefore, minimal visualization or inference are needed; but more processing work is necessary to determine whether the rate of an object's motion is constant or changing.

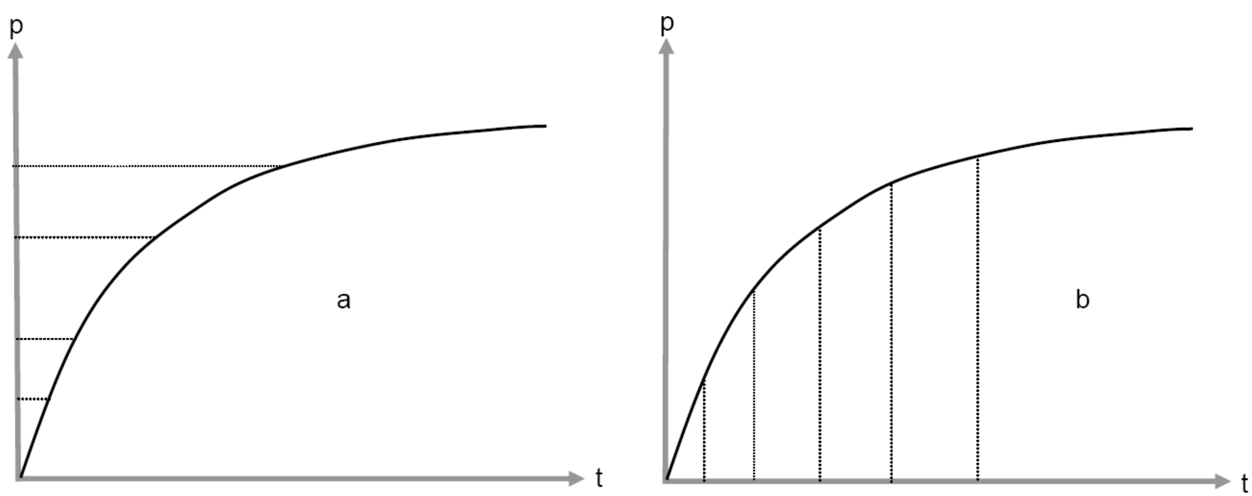


Figure 2. Ordinate strategies for recognizing general motion. In the "fixed ordinate" strategy (a), the different line lengths drawn between the curve and the ordinate indicate motion. In the "slide ordinate" strategy (b), change in vertical line lengths between the curve and the abscissa indicate general motion.

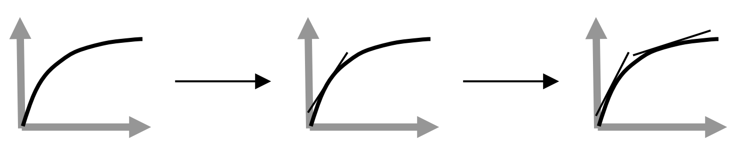
Thus, in a position-time graph the ordinate represents the location of an object and change in position and rate of change in position is interpreted through the curve by relating time (abscissa) to location (ordinate). In contrast, in a velocity-time graph the ordinate represents change in the rate at which an object changes position, i.e., it shows the *velocity* of an object. Therefore, if velocity is any value other than zero, the object is moving. Thus, the ordinate can be used to determine whether the velocity of an object is constant or changing. If constant, then the height of the curve does not change in relation to the ordinate (i.e., the curve is parallel to the abscissa). If changing, then the height of the curve changes in relation to the ordinate. Thus, in the Velocity-Time graph, a change in the height of a curve relative to the ordinate corresponds with a change in the rate at which an object is moving (i.e., changes position). The same ordinate strategies that are available for use with the position-time graph (i.e., fixed ordinate and slide ordinate) are also applicable for use with the velocity-time graph. However, those visualization strategies yield different information about movement, with velocity information being a direct "read out" of the application of either ordinate strategy.

2. Curve Morphology

The ordinate provides one source of information about the motion of an object. However, another important information source that needs to be understood is the curve. Much of the details about the kinematics behavior of an object are encoded in the slope(s) of the curve plotted within the axes of the graph. Let us consider what curves in position-time and velocity-time graphs represent and the strategies that are conventionally employed to interpret them.

In the position-time graph the slope represents a rate of change in position or, *velocity*. The greater the slope, the greater the speed at which the object is changing position. There are several strategies for interpreting the slope in a position-time graph, depending on whether the plotted curve represents a linear or a curvilinear function. When the curve is linear the object has a constant slope and therefore velocity information is represented directly in the curve. Because the slope is constant the velocity is constant and the graph reflects an object moving at a constant rate. When a linear function is horizontal, the slope is zero and therefore velocity equals zero, i.e., the object is not changing its position.

When the function is curvilinear as in Figure 3, velocity is determined by comparing the slopes of tangents to the curve at multiple points along the curve. In the example in Figure 3, tangents are drawn at two different points on the curve (3b and 3c). A comparison of the slopes of the tangents shows that they decrease over time. Because the slopes represent velocity, the object is decelerating (i.e., moving at a slower rate) as time passes.



a b c

Figure 3. A partial depiction of the determination of velocity for a curvilinear function in a position-time graph (a). 3b shows a tangent to the curve for which slope is calculated. 3c shows a second tangent to the curve for which slope is calculated. As is visible from the tangents in 3b and 3c, the slope of tangent 3c is less than the slope of tangent 3b. Therefore the slopes are decreasing and the velocity is decreasing.

Interpreting the slopes of curves in the velocity-time graph follows a similar logic. For a linear function, a positive slope represents a steady increase in velocity, i.e., acceleration; a negative slope represents a steady decrease in velocity, i.e., deceleration. When the slope equals zero, velocity is constant if the y-intercept does not equal zero. This information is directly available in the linear curve. Similarly, when the plotted function is curvilinear, any curve with positive slope indicates that velocity is increasing and any curve with negative slope indicates that velocity is decreasing. Unlike the position-time graph, one does not need to use a change in the slope of a curve to determine acceleration or deceleration; velocity can be determined directly from the direction of the slope. However, changes in the slopes of velocity-time graphs are important for determining whether the rate of acceleration or deceleration is increasing or decreasing. Using the same procedure of comparing the slopes of tangents to the curve provides the necessary information about increase or decrease. If the comparison is positive, the rate of acceleration or deceleration is increasing; if the comparison is negative, the rate of acceleration or deceleration is decreasing.

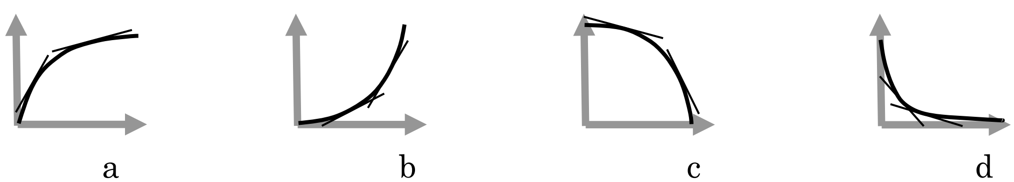


Figure 4. Four types of acceleration and deceleration in a velocity-time graph: (a) decreasing acceleration, (b) increasing acceleration, (c) increasing deceleration, (d) and decreasing deceleration.

The implications for processing are that linear functions (i.e., slope does not change) should be easier to process than nonlinear functions (i.e., slope that does change). This is independent of whether it is a position-time graph or a velocity-time graph.

C. Derivational Structure of Representations

Position-time and velocity-time graphs represent the same information about the motion of objects – whether objects are moving or not and the rate at which they are moving. The graphs are therefore characterized as being *informationally equivalent* (Larkin & Simon, 1987; Palmer, 1978; Zhang & Norman, 1994). As illustrated in the previous two sections, informational equivalence is not synonymous with equivalence of processing the information in the representations. Palmer (1978) captured these differences in the construct he termed *derivational structure of representations*. He recognized that two informationally equivalent representations could vary on the basis of whether they afforded direct access to represented information or whether that information had to be derived inferentially through operations, computations, or the kind of informal mathematical reasoning illustrated earlier. By Palmer's account, a representation is *direct* if it can be directly interpreted; otherwise the represented entity or relation must be *derived*. According to some researchers, there are processing implications of these derivational structures in that direct representations can be directly "read off" (Larkin & Simon, 1987) and lend themselves to perceptual processes (e.g., Trickett & Trafton, 2006; Zhang & Norman, 1994) whereas derived representations require additional cognitive operations to access the desired information including inference (Larkin & Simon, 1987), visualization (e.g., Trickett & Trafton, 2006), and externalization (Cox, 1999).

Specifically, the derivational structure can impact the computational efficiency of a graph because derived representations often require more search and computation time to interpret than direct representations (Larkin & Simon, 1987). Figure 5 shows three graphs that vary by graph type (i.e., Figs. 5*a* versus Fig. 5*b*), curve morphology (i.e., Fig. 5*a* versus Fig. 5*c*), and graph type and curve morphology (i.e., Fig. 5*b* versus Fig. 5*c*). For example, for all three graphs, it can be directly determined that the object is moving: All three graphs have non-zero slopes, thus the object is moving. To determine whether velocity is changing also can be directly determined for the velocity – time graph, *5b*: the curvilinear function is increasing thus acceleration is increasing. For position-time graphs, what is graphed is the rate of change of position of the object over time. Whether the object is accelerating or decelerating needs to be inferred from the rate function. Thus, in graph 5a, the position is changing at a faster rate over time and therefore, accelerating. For 5c, the position is also changing at a faster rate; thus it too shows that the object is accelerating. These examples illustrate what is meant by direct versus derived judgments.

Based on these rational task analyses of the processing required to determine motion and rate of motion from the different graph types and curves, a general hypothesis can be posited from the derivational structure of representations. The derivational structure hypothesis is that direct representations will be associated with faster recognition of a concept than derived representations, assuming equal accuracy. Accuracy for direct and derived representations is hypothesized to be equivalent if people do the extra work demanded by derived representations. If they do not, direct representations would be expected to lead to more accurate responding than derived representations. Underlying this hypothesis is the assumption that individuals know how to process information in the types of graphs under consideration in any given task; else no advantage of a direct over derived representation would be expected to occur (Larkin & Simon, 1987). Prior research on graph interpretation difficulty indicates that this underlying assumption may not be true, especially for certain graph and curve types.

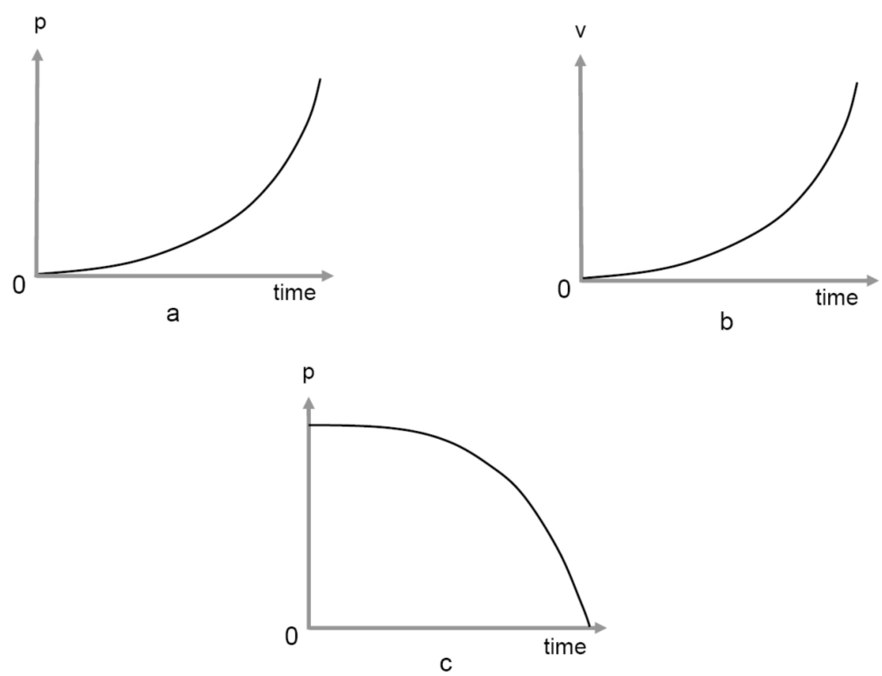


Figure 5. Three graphs used to exemplify variation in derivational structure based on differences in graph type (Fig. 5*a* vs. Fig. 5*b*), curve morphology (Fig. 5*a* vs. Fig. 5*c*), and graph type and curve morphology (Fig. 5*b* vs. Fig. 5*c*).

1. Difficulties Interpreting Kinematics Functions in Graphs

Much of the work on interpreting kinematics graphs has focused on common errors people make when extracting information from them. For example, McDermott, Rosenquist, and van Zee (1987) carefully analyzed the students' patterns of responses on multiple choice items and identified five common graph interpretation errors: *discriminating between slope and height*, *interpreting changes in slope and height*, *interpreting the area under the curve, relating one graph type to another*, and *matching narrative information with features of a graph*. Beichner (1994) identified six errors that overlap with those revealed by McDermott et al. (1987): *graphs as pictures*, *slope/height confusion*, *variable confusion*, *nonorigin slope errors*, *area ignorance*, and *area/slope/height confusion*. The two general classes of error type relevant to this work are related to *relating graph types* and *height and slope confusion*. The other errors are not reviewed because investigating them requires different task types than are used in the present study.

2. Difficulty Related to Graph Type

One purported source of graph interpretation difficulty is graph type. For example, Beichner (1994) administered the Test of Understanding Graphs in Kinematics—a multiple-choice assessment designed to evaluate kinematics graph interpretation skills—to 524 high school and college students across the United States. The test consisted of 21 items constructed to measure seven graph interpretation "skills," each assessed with three items. The seven skills are the following: (a) interpret velocity with position-time graphs; (b) interpret acceleration with velocity-time graphs; (c) interpret displacement (i.e., change in position) with velocity-time graphs; (d) interpret change in velocity with acceleration-time graphs; (e) translate the motion of an object from (i) position-time to velocity-time graphs,

(ii) velocity-time to acceleration-time graphs, and (iii) acceleration-time to velocity-time graphs; (f) map between textual and graphical descriptions of motion (much like the description-graph pairs used in the present study), and (g) discriminate the correct graphical representation of motion from among five graphs.

According to the pattern of item accuracy reported by Beichner (1994), velocity-time graph items were more difficult to interpret than position-time graph items. The students were 71.43% and 50.00% inaccurate on velocity-time and position-time graph items, respectively. There are two plausible explanations for the greater difficulty of velocity-time graphs but Beichner's data could not differentiate between them. One conjecture is that people tend to reflect a belief that it is okay to readily switch axis labels from one variable to another (Beichner, 1994; Leinhardt, Zaslavsky, & Stein, 1990). A second is that people do not know from which features of the graph (e.g., ordinate, slope, tangent) to extract the critical information (McDermott et al., 1987) so they ignore the ordinate label. No previous work has systematically compared performance in relation to the y-axis label; the aforementioned conjectures stem from post hoc observation. Therefore, a major goal of the present research was to see whether and to what extent graph type influences interpretation accuracy and latency.

3. Difficulties Related to Height and Slope

Another source of difficulty is discriminating between the height and the slope of a line. McDermott et al. (1987) showed students a position-time graph that depicted two intersecting curves of varying slope labeled *A* and *B*. The students were instructed to determine whether the speed of an object reflected by one curve was greater than, less than, or equal to the speed of an object reflected by the other curve. The pattern of responses indicated that students tried to extract information about speed (or the magnitude of velocity) from the height of the curve rather than the slope (note that the curves were linear so the slope was constant across time). In a second question students were asked whether the objects ever move at the same speed and if so, when. Incorrect responses emerged mostly from a failure to realize that objects would never move at the same speed because the slopes of their curves were never the same. The researchers reasoned that these types of errors arise in the absence of knowledge about correspondences between graph features (e.g., point coordinates, differences in point coordinates or slope) and particular aspects of motion targeted for interpretation.

In addition to difficulty differentiating when to process height versus slope, McDermott et al. (1987) also found that people had problems determining when to process *change* in height versus *change* in slope. Presented with a position-time graph (see Figure 6) that depicted a curve labeled with seven letters, A-G, students had to designate at which of the lettered points the motion of an object was (a) slowest, (b) speeding up, (c) slowing down, and (d) turning around. The answers had to be determined by evaluating the heights and slopes of the curves and by considering the direction and magnitude of these factors in the graph. In Figure 6, the motion of the object is slowest at point B, where the slope magnitude is smallest (i.e., equal to 0). Selection of point C reflects the common error that speed is slowest when the height not the slope is zero. The increasing slope magnitude at points B and G represent points when the object is speeding up. A predictable error selects point A because it appears to represent an increasing function and hence, a speed up, when in actuality the function decreases at this point and therefore reflects a point when the motion of the object slows down. The error suggests a greater focus on a change in height than on a change in slope. At points A, C, D, and E, the object is slowing down. A common error would be the inclusion of points F and G because the slopes are negative at those points. Thus, "slowing down" is often mistakenly interpreted through the direction (e.g., "downward"), rather than the magnitude (i.e., "decreasing"), of the slope. Lastly, the change in slope direction at point B from positive to negative signifies a reversal in the direction the object is moving. A typical interpretation error is that the object turns around at C, the point when curve height, not slope, changes from positive to negative.

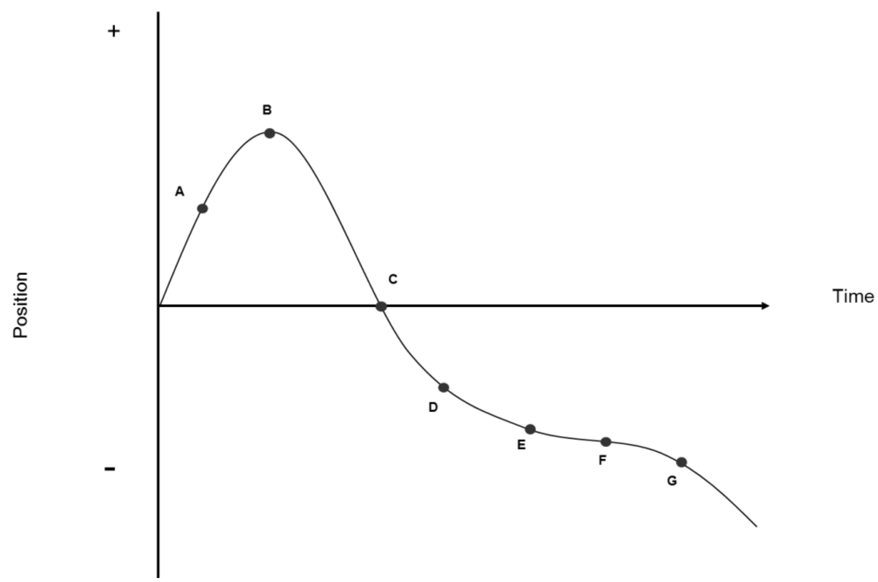


Figure 6. Position-time graph. To address questions about velocity the student must determine whether the targeted physical concepts are encoded in the height, the slope, or changes in the direction and magnitude of these factors. (Item from the Graphing Skills questionnaire, adapted from McDermott et al., 1987).

4. Difficulty Related to Curvature

Curvature provides crucial information about the rate of change. By evaluating student performance across two questions (see Items 1 and 5 in Appendix A), McDermott et al. (1987) noticed that when interpreting motion in nonlinear functions, “students often reveal[ed] vestiges of the slope-height difficulties encountered with straight-line graphs” (p. 505). The post hoc observations of student errors across linear and nonlinear task items suggest that students may have processed nonlinear functions as if they were linear. Failure to differentiate nonlinear from linear functions would generate relatively poor interpretation outcomes for nonlinear functions.

The position that people do not discriminate between linear and nonlinear functions stands counter to the conclusions of Best, Smith, and Stubbs (2007) who found that the curvature feature is typically processed during graph interpretation. Graphs of linear functions do not have curvature and therefore, interpreting them with respect to velocity change should require less processing than curves that reflect nonlinear functions because nonlinear functions demand interpreting additional information in the curve and therefore should take longer to process.

II. OVERVIEW OF THE PRESENT RESEARCH

The present research set out to examine the degree to which interpretations of the motion of objects represented in graphs could be accounted for by considering whether judgments could be made from information directly available in the graph or required additional processing of the directly available information. This perspective on graph interpretation stems from Palmer's (1978) discussion of the derivational structure of representations (DSR). According to Palmer, the DSR posits that in some cases the information in a graph that is directly available through perceptual processes is sufficient to address particular tasks but in other cases, further processing of the information directly available is necessary. Palmer distinguished between these two cases in terms of *direct* as compared to *derived* processing situations. The manipulation of four key variables known to influence graph interpretation difficulty – i.e., Graph Type, Curve Morphology, Judgment Class and Motion Statements nested within Judgment Class – made it possible to test several hypotheses across four experiments with the express aim of elucidating the nature of graph interpretation difficulty in terms of whether the information could be directly read off the graph or needed additional processing based on what was visible in the graph.

The two graph types used in the present research are the same as those typically found in physics classrooms (Beichner et al., 1999; Thornton & Sokoloff, 1998), textbooks (e.g., Saxon, 1993; Young et al., 2007), and assessments (e.g., Beichner, 1994; Savinainen & Scott, 2002). Position-time graphs consist of a y-axis labeled, “position,” and correspond to changes in the position of an object relative to time. Velocity-time graphs consist of a y-axis labeled, “velocity,” and correspond to changes in the “speed” or rate of an object’s movement relative to time.

Eight different functions were used to examine the effect of curve morphology on graph interpretation. McDermott et al. (1987) et al. noted that students find it more difficult to interpret curvilinear functions in graphs than they do linear functions. The eight levels of Curve Morphology used in the present study allowed for a systematic test of curvature when needed information was either direct (velocity-time graphs for motion and velocity-change judgments; position time graphs for motion judgments) or derived (position-time graphs for velocity-change judgments). Additionally, two curve morphologies reflected linear functions where the slope was zero allowing for comparisons that yield results about the extent that *height* and *slope* information were processed in the curves.In addition to the two graph types, the effects of two judgment classes on interpretation were investigated. The first were judgments about general movement; the second were judgments about change in velocity. G*eneral motion*, simply refers to whether an object is moving (i.e., *The object is moving*) or not (i.e., *The object is stationary*). Both position-time and velocity-time graphs were investigated and, as illustrated earlier, judgments about whether an object is moving or not can be directly determined by a single relation in both graph types.

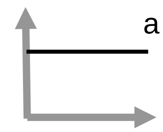
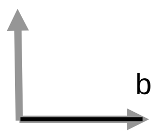
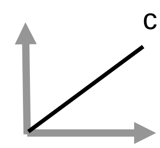
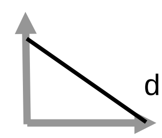
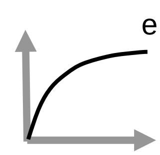
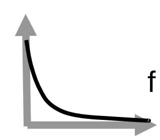
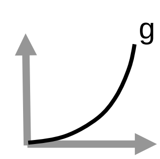
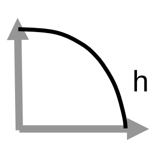
The second judgment class, *velocity change*, refers to decisions about whether object movement is accelerating or decelerating. In life, there are many situations in which a person has to be able to discriminate among objects that are travelling at a constant velocity, that are accelerating, and that are decelerating (think of a bike messenger racing through the streets of New York or a bike commuter riding up Halsted Street in Chicago rush-hour traffic). Conceptually, velocity change is at the core of graph interpretation difficulty in physics (Brasell, 1987; Forster, 2004; McDermott et al., 1987). As discussed earlier, information about velocity change is directly represented in velocity-time graphs but must be derived in position-time graphs.

As indicated earlier, derivational structure refers to the manner in which information is available for processing. A basic question is how well students are able to recognize motion and the absence of motion in graphs that vary as a function of graph type and curve morphology. There is little doubt that the typical high school or college student would be able to classify an object as stationary or moving in the physical world. However, how quickly and accurately they can do so when aspects of movement are represented graphically may well be related to the type of graph and the curve morphology in the representation. When Graph Type is consistent with the class of information that one seeks to interpret (in the present context either information about general motion or information about velocity change), then the derivational structure of the representation is said to be *direct* and is hypothesized to support direct read-off from the graph. That is, direct read-off strategies should facilitate faster and more accurate verification than when the derivational structure of the representation is *derived*; in which case, the desired information must be inferred due to a mismatch between graph type and judgment class.

The general paradigm used in the four experiments conducted was the same. Verbal descriptions (e.g., *The object is moving* or *The object is stationary*) were simultaneously displayed with different graphs (i.e., the stimuli consisted of description-graph pairs). The graphs represented eight different kinematics functions that were plotted against position-time axes and/or velocity-time axes. Upon presentation of a description-graph pair, participants made a TRUE or FALSE judgment to indicate whether the verbal description and the graph corresponded (i.e., the verbal statement was either True or False relative to the motion represented in the graph). A conditional prediction was tested. Specifically, accuracy will be the same for judgments that involve direct and derived structures if and only if subjects actually derive the critical information when the interpretive situation demands it. Otherwise, accuracy will be greater for judgments associated with direct structure compared to derived. If the additional processes are employed when interpreting representations whose structures are derived, then those processes will translate into longer latencies than for direct structures.

As Table 1 shows, the degree to which information for making judgments for the different curve morphologies was directly available varied as a function of Graph Type as well as Judgment Class. For General Motion judgments, six of the eight curve morphologies are direct for both graph types. For curves **a** and **b**, the judgments are also direct for position-time graphs but for velocity-time graphs the situation is less clear. The *a priori* assignment of derivational structure for this particular representation was difficult because graph type and curve morphology

Table I
DERIVATIONAL STRUCTURES OF THE REPRESENTATIONS AS A FUNCTION OF GRAPH TYPE, CURVE MORPHOLOGY, AND JUDGMENT CLASS

	General Motion		Velocity Change	
Curve Morphology	Position-Time	Velocity-Time	Position-Time	Velocity-Time
 a	direct	direct/derived?	derived	direct
 b	direct	direct/derived?	derived	direct
 c	direct	direct	derived	direct
 d	direct	direct	derived	direct
 e	direct	direct	derived	direct
 f	direct	direct	derived	direct
 g	direct	direct	derived	direct
 h	direct	direct	derived	direct

interacted in such a way that direct or derived solution strategies seemed equally plausible. Accordingly, this ambiguity is reflected in Table 1.

A. Initial Hypotheses, Predictions, and Tests by Experiment

Across four experiments several hypotheses were tested, as shown in Table 2.

Table II
HYPOTHESES TESTED ACROSS THE FOUR EXPERIMENTS IN THE STUDY. P-T = POSITION-TIME; V-T = VELOCITY-TIME; GM = GENERAL MOTION; VC = VELOCITY CHANGE; IFF = IF AND ONLY IF

Hypothesis	Experiment: Exp 1	Exp 2	Exp 3	Exp 4
Graph Difficulty	Accuracy: P-T > V-T Latency: P-T < V-T Velocity conceptually more difficult than position	Accuracy: P-T > V-T Latency: P-T < V-T Velocity conceptually more difficult than position		
Derivational Structure		Accuracy: P-T = V-T IFF Ss derive info in P-T conditions Latency: P-T > VT IFF Ss derive info in P-T conditions	Accuracy: GM = VC IFF Ss derive info in VC conditions Latency: GM < VC IFF Ss derive info in VC conditions	
Direct Read-Off			Accuracy: GM > chance for all judgments (consistent with direct read-off rules); VC > chance for some judgments, VC < chance for some judgments (inconsistent with direct read-off rules) Latency: GM = VC	Accuracy: GM > chance for all judgments except curve a where read-off rule does not apply; VC > chance for all judgments (consistent with direct read-off rules) Latency: GM = VC

Experiments 1 and 2 tested the hypothesis that velocity-time graphs are more difficult to interpret than position-time graphs. Experiment 1 tested this prediction for only judgments of general motion, a judgment that, for both graph types, could be made directly from the graphs, with the possible exception noted above of curve morphologies **a** and **b** in the velocity-time graphs. Based on the prior literature, judgments for velocity-time graphs were expected to be less accurate than those for position-time graphs. For accurate judgments, response time was expected to be longer for velocity-time graphs than for position-time graphs. However, based on the derivational structure analysis indicating that all judgments are direct, no differences in accuracy between the two graph types were expected, with the possible exception for curves **a** and **b** in velocity-time graphs. Neither the prior literature nor the derivational structure analysis predicts more accurate and faster judgment with velocity-time graphs than position-time graphs.

Experiment 2 was designed to test graph difficulty for only judgments of velocity-change. According to the literature, velocity-time graphs were expected to be more difficult than position-time graphs. This would be reflected as lower accuracy rates compared to position-time graphs. However, as described in Table 1, the DSH predicts the opposite based on whether judgments are direct or derived. In particular, velocity-time graphs should be easier than position-time graphs for velocity change judgments. The prediction is higher accuracy rates for velocity-time graphs than position-time graphs, and for accurate judgments, longer response times for position time graphs.

One issue in both Experiments 1 and 2 concerns the fact that subjects are asked to deal with two types of graph within the same experimental session. It could be argued that having to switch between judgments about the two graph types would maximize chances of subjects getting confused about which graph type they were working with at any point in time. Although a blocked presentation strategy was used to minimize such potential confusion due to changes in whether position or velocity was on the *y* axis, it was still possible that switching between the two might be responsible for the results. Accordingly, two additional experiments were designed to examine judgment class and curve morphology for a single graph-type at a time.

Experiment 3 was designed to examine the two judgment classes across the eight curve morphologies for only position-time graphs. The judgment-type comparison also reflects a difference between direct (general motion) versus derived (velocity change) derivational structure for this type of graph (see Table 1). The predictions of the DSH were that direct judgments would be more accurate than derived, and for accurate judgments, direct would be faster to make than derived. Experiment 4 was designed to examine the two judgment classes across the eight curve morphologies for only velocity-time graphs. In all but two cases (curves **a** and **b** for general motion judgments), the judgments can be made directly from the graphs and the DSH predicts no differences in accuracy or response time. However, since all linear and curvilinear judgments are direct, this experiment allows a test of the relative difficulty of linear versus nonlinear function curves. The prediction was that linear would be easier than curvilinear: accuracy would be higher for linear and, for accurate judgments, linear would be faster than curvilinear.

III. EXPERIMENT 1.

MOTION IN POSITION-TIME VERSUS VELOCITY-TIME GRAPHS: A TEST OF GENERAL GRAPH DIFFICULTY

Experiment 1 examined the effects of graph type, curve morphology, and motion description statement on verification accuracy and response latency for general motion judgments. The goal was to examine the extent that velocity-time graphs are more difficult to interpret than position-time graphs. The derivational structure of the representations in the task were direct across all conditions with the reasoned exceptions of curves **a** and **b** in relation to velocity-time graphs (see Table 1).

A simple sentence-picture verification task was used to test several claims about difficulties interpreting motion with position-time versus velocity-time graphs. Each subject was presented with a position-time or a velocity-time graph paired with a sentence that described the motion (*The object is moving*) or the absence of motion (*The object is stationary*) of an object. The subject had to verify whether the sentence reflected the state of general motion depicted in the graph. Thus, the task demands were to represent the sentence and the graph, compare those representations, and then indicate whether the two representations were equivalent (TRUE) or not (FALSE) (e.g., Carpenter & Just, 1975; Clark & Chase, 1972, 1974; Glushko & Cooper, 1978). Brasell (1987), who like Beichner (1994) and McDermott et al. (1987) observed a greater proportion of errors made with velocity-time graphs than with position-time graphs, concluded,

> …velocity is conceptually more difficult than is distance [because]…velocity is a more abstract property, being derived as the rate of change of distance" (p. 393)

If velocity is conceptually more difficult to think about than position (or, "distance"), then an interesting question to investigate is whether velocity-time *graphs* are more difficult to interpret than position-time *graphs*. If so, then judgments made with the position-time graphs should be more accurate and faster than when made with velocity-time graphs.

Additionally, if a main effect of curve morphology on response time (RT) is detected, a specific set of contrasts would be conducted to examine the effects of height (a vs. b), slope (ab vs. cd), and curvature (c vs. eg & d vs. fh) on graph interpretation.

A. Method

1. Participants

Sixteen University of Illinois at Chicago undergraduates were recruited from the general UIC population (n = 6) and the Department of Psychology Subject Pool (n = 10). Based on a questionnaire that asked for demographic information, including math and science background, and that included an assessment of graphing skills (described in Materials section below) the following information describes the participants. Nine were female and seven were male. The median age range was 18.0 months, range 17 – 24. Based on a questionnaire subjects completed prior to beginning the experiment, 11 of the 16 subjects were in their first year of college.

However, two, two, and one subjects were, respectively, in their second, third, and fifth year of college. Subjects reported their previous five math courses. The "highest" level of math reported by each subject was calculus (n = 7), pre-calculus (n = 5), trigonometry (n = 2), and algebra II/geometry (n = 2). The study was limited to subjects who had taken at least one year of physics. However in addition to physics, subjects reported taking at least one course in each of the following areas: Anatomy and Physiology (n = 2 subjects), Biology (n = 16 subjects), Chemistry (n = 13 subjects), Earth or Environmental Sciences (n = 7 subjects), Psychology (n = 2 subjects), and Zoology (n = 1 subject). Biology included Introductory, AP, Accelerated, and advanced courses. Chemistry included introductory, general, and organic. One student had taken a second semester of Physics. All students participated in a single session. Students recruited from the general UIC population were paid at the rate of $20.00 per session. Students recruited from the subject pool earned course credit. Non-subject pool participants were screened to ensure that they were comparable to Subject Pool participants in terms of their course experience. Accuracy across the nine response opportunities on the graphing skills assessment ranged from 0.00 to 5.00 out of a possible 9.00, the Modal frequency of correct responses was 1 and the Mean was 1.69 (*SD* = 1.35). The implications of this level of performance for interpreting performance on the experimental task are discussed further in the General Discussion.

2. Design

A three-factor (2 x 8 x 2) within-subjects design was used to examine the effects of the three key independent variables on verification accuracy and mean response latency for accurate judgments. The independent variables were: Graph Type (Position-Time, Velocity-Time); Curve Morphology [Zero-Slope $y > 0$ (Curve a), Zero-Slope $y = 0$ (Curve b), Linear Increasing (Curve c), Linear Decreasing (Curve d), Positive Decreasing (Curve e), Negative Decreasing (Curve f), Positive Increasing (Curve g), Negative Increasing (Curve h)]; and Motion Description (object is stationary, object is moving). The dependent variables were accuracy and response time to make the decision.

3. Stimuli and Materials

Each stimulus was composed of a verbal description of general motion (*The object is stationary* or *The object is moving*) that was presented at the center of the screen directly above a kinematic function situated in either a position-time or velocity-time graph (see example in Figure 7). The task required the participant to indicate whether the aspect of motion represented in the statement and the graph were conceptually congruent or not. If congruent, the participant had to press a key on the keyboard labeled, *TRUE*. If the relationship was incongruent the correct response was, *FALSE*. Eight different curves were plotted in each of the two graphs along with one of the two motion descriptions resulting in a total of 32 unique displays.

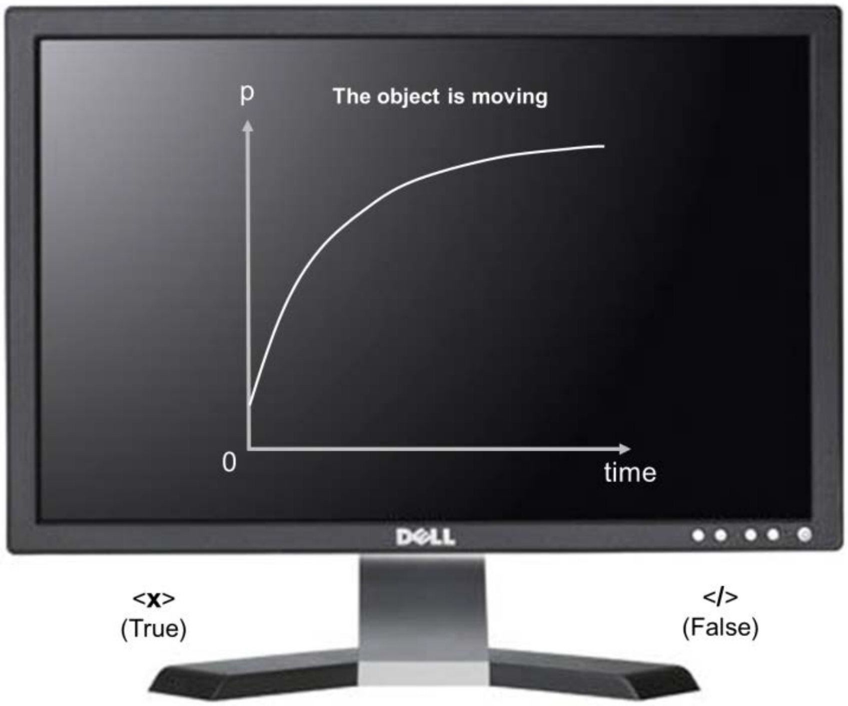
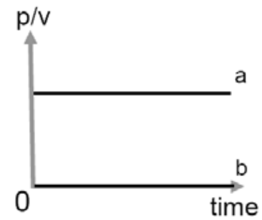
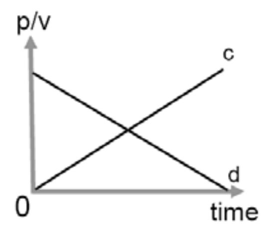
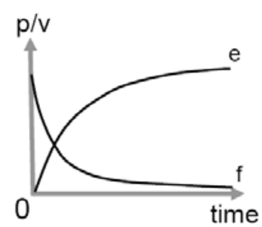
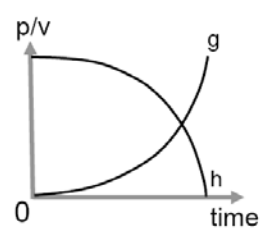


Figure 7. Example stimulus display depicting a motion description (i.e., *The object is moving*), the graph type (i.e., position-time), curve type **e** of the eight canonical kinematic curves (i.e., positive, decreasing), and the response keys.

The graphs represented kinematic functions that are often used in physics instruction and assessment. Their general descriptions are presented in Table 3 that emphasizes the semantic similarities and differences related to a curve for each graph type. For example, curve **e** is positively decreasing. When the curve is appropriately interpreted on a position-time graph, the object is slowing down in relation to time (i.e., decelerating) whereas the same positively decreasing function interpreted on a velocity- time graph indicates that the object is speeding up (i.e., accelerating, albeit at a decreasing rate).

Table III
THE EIGHT KINEMATICS CURVES USED IN THE STUDY, THEIR GENERAL CURVE DESCRIPTIONS AND KINEMATIC MEANING BY GRAPH TYPE.[a]

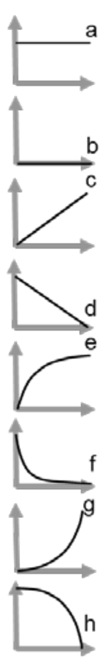
General Description	Curve	Kinematic Description: Position-Time	Kinematic Description: Velocity-Time
a. horizontal, y ≠ 0 b. horizontal, y = 0		a1. stationary a2. constant speed b1. stationary b2. constant speed	a1. moving a2. constant speed b1. stationary b2. constant speed
c. linear, increasing d. linear, decreasing		c1. moving c2. constant speed d1. moving d2. constant speed	c1. moving c2. accelerating d1. moving d2. decelerating
e. positive, decreasing f. negative, decreasing		e1. moving e2. decelerating f1. moving f2. decelerating	e1. moving e2. accelerating f1. moving f2. decelerating
g. positive, increasing h. negative, increasing		g1. moving g2. accelerating h1. moving h2. accelerating	g1. moving g2. accelerating h1. moving h2. decelerating

[a]Semantic value assigned to judgments of general motion are indicated by 1 and velocity change are indicated by 2 for each graph type. For example, e1 for position-time graphs = "the object is moving"; e2 at position-time = "the object is decelerating".

Crossing the two motion descriptions with the eight curve morphologies across both graph types resulted in several unbalanced true-false response distributions for key planned comparisons (see Table 4). In most cases, a perfect balance could not be achieved given the nature of the truth value for various description-graph pairings. The best resolution was to create variants of certain graphs (i.e., the zero-slope curves, **a** and **b**) to arrive at an overall true-false response distribution ratio of roughly 2:3. This increased the number of trials per block by four yielding four observations per description-graph pair.

Table IV
TRUTH TABLE FOR CORRECT RESPONSES (T=TRUE; F=FALSE) FOR EACH GRAPH TYPE, CURVE MORPHOLOGY (A – H), AND VERBAL DESCRIPTION. DERIVED INFORMATION BASED ON DERIVATIONAL STRUCTURE HYPOTHESIS = GREY CELLS; AMBIGUOUS INFORMATION = ?

General Motion



	Position-Time		Velocity-Time	
	Stationary	Moving	Stationary	Moving
a	T	F	F (?)	T (?)
b	T	F	T (?)	F (?)
c	F	T	F	T
d	F	T	F	T
e	F	T	F	T
f	F	T	F	T
g	F	T	F	T
h	F	T	F	T

The full experiment consisted of 160 description-graph pairs. The 160 stimulus pairings were divided into four blocks by graph type. Each block was further divided into two sets of 20-trials. Each block carried one of the two graph types crossed with the eight curves for both motion descriptions (stationary vs. moving). These were the eight critical trials of interest and they were cycled twice per block. Four additional filler trials completed the block that resulted in blocks of 20 trials. The filler trials were added to better equalize the ratio of true to false judgments. Four blocks were associated with position-time graphs (80 trials) and four were associated with velocity-time graphs (80 trials) resulting in a total of 160 trials. The blocks were designated to a particular graph type and the presentation order was counterbalanced across participants.

A background questionnaire was designed to elicit information about participants' math and science backgrounds as well as their graphing skill with respect to height/slope, change-in-height/change-in-slope, and variable confusion associated primarily with position-time graphs (see Appendix A). The graphing skills items were selected from McDermott et al. (1987) and from the revised TUG-K that was acquired from Beichner through personal communication (March 21, 2011). The revised graphing skills assessment contained six multipart items that tapped the following skills: extract information about velocity from height and slope in a position-time graph (Item 1: McDermott et al., 1987)); interpret velocity in a position-time graph, (Item 2: Beichner, 1994); interpret states of position in a position-time graph (Item 3: Beichner, 1994); discriminate the correct graphical representation of velocity and velocity change from among five graphs (Item 4: Beichner, 1994); interpret velocity and velocity change in position-time graphs (Item 5: McDermott et al., 1987); and interpret acceleration in velocity-time graphs (Item 6: Beichner, 1994).

4. Procedure

Each participant individually engaged the experimental task in groups of 12 – 16. The task was to determine, as quickly and as accurately as possible, whether a verbal description of motion and a graphical representation of motion were congruent. The participant pressed one of two keys on the keyboard to indicate that the representational equivalence of the description-graph pair was either *TRUE* ("x") or *FALSE* ("/"). The participant had an unlimited amount of time in each trial to evaluate the display.

The general experimental procedure is illustrated in Figure 8. Prior to the experiment, the participants completed the background questionnaire. After completing the background questionnaire the participants were presented with a brief review of position-time and velocity-time graphs (*Introduction: Graph Types*) followed by a display that discussed stationary and moving objects (*Introduction: General Motion*) (see Appendices B and C).

Before entering the test trials, the participants underwent a series of four practice trials with motion descriptions not used in the present study (*The motion of the object is accelerating* and *The motion of the object is decelerating*) to help orient them to the demands of the task. They were told,

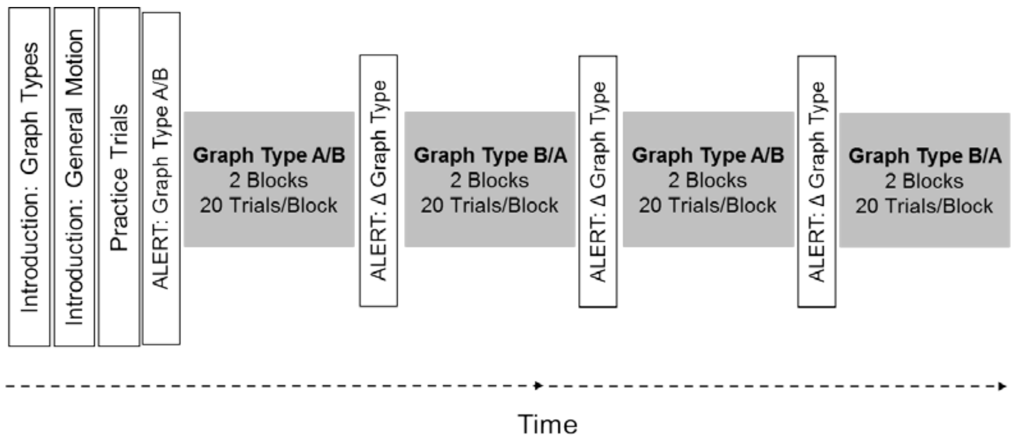


Figure 8. General experimental procedure. Prior to entering the test phase, participants were presented with information about graph types and general motion judgments and then they received four practice trials with feedback from the interviewer. The graph types alternate between blocks according to two block orders: ABAB or BABA.

Ok, let's walk through a few displays together.

The first display was presented and then the experimenter said,

Here is a graph and a sentence.

The sentence may or may not describe the function shown in the graph.

Your task is to determine if the sentence is an accurate description of the function shown in the graph.

If it is, press <TRUE>*; if not, press* <FALSE>.

Go ahead and indicate whether the relationship between the sentence and the graph is true or false.

After the participants made their decision the experimenter asked one of the participants in the room to explain his decision.

Was it true or false? <PARTICIPANT RESPONDS>

So, tell me, why did you say that the relationship was [TRUE] [FALSE]?

The experimenter then asked,

Did anyone else respond the same way?

Then the experimenter asks if anyone responded with the alternative?

Did anyone respond differently?

The experimenter never revealed the correct answer because the purpose was to be sure that the participants understood the task.

Ok, that makes sense.

Following the practice trials, the participant saw a screen that described the focus of the forthcoming test trials with respect to either of the two block orders to which the participant was assigned.

From this point forward, once you press the spacebar, you will enter the test trials.

On each screen you will see a sentence and a [Position-Time] [Velocity-Time] *graph.*

Your task is to determine if the sentence is an accurate description of the function shown in the graph just as you did in the four practice trials.

If the description is accurate, press <TRUE>*; if not, press* <FALSE>.

Again, make your decision as quickly and accurately as possible.

Participants were given a 15 second break between each set of 20 trials. A display was presented that read, *Relax…the next set of trials will continue shortly*. A *Ready* screen prompted them to prepare for the next set of trials. The trials by graph type were presented in alternating blocks. Thus, if the participant evaluated description-graph displays that involved position-time graphs in the first two blocks, then she evaluated description-graph displays that involved velocity-time graphs in the second two blocks and vice-versa. The participant was alerted about the upcoming change in graph type following Blocks 1, 2, and 3 (i.e., *You will now make decisions with* [position-time] [velocity-time] *graphs. Press the space bar to continue.*). Following the last trial, a *Thank You* display was presented to inform the participant that the study was over.

B. Results

The results of the analysis are reported in two main parts. The first part examines the effects of Graph Type, Curve Morphology, and Motion Description on verification accuracy; the second part looks at the effects of those variables on the mean response times for accurate judgments. Estimates of effect sizes are reported using partial eta squared (η_p^2), with the "importance" of the effect corresponding to *small* (0.00 - 0.20), *moderate* (.21 – 0.79), and *large* (0.80 and greater) (Cohen, 1988[1]).

1. Judgment Accuracy

Mean judgment accuracy for all trials was analyzed using a 2 (**Graph Type**: Position-Time, Velocity-Time) x 8 [**Curve Morphology**: Zero-Slope y > 0 (Curve a), Linear Increasing (Curve c), Linear Decreasing (Curve d), Positive Decreasing (Curve e), Negative Decreasing (Curve f), Positive Increasing (Curve g), Negative

[1] According to Richardson (2011), partial-eta squared can be benchmarked against Cohen's criteria for small, moderate, and large effects for *d*, effect size for between-subjects comparisons. Thus, these designations of effect sizes for partial eta squared are conservative.

Increasing (Curve h)] x 2 (**Motion Statement**: The object is stationary, The object is moving) repeated measures analysis of variance (ANOVA).

The judgment accuracy means are presented in Figure 9. The ANOVA on the accuracy data indicated main effects for Graph Type, F (1, 15) = 4.74, p = .05, η_p^2 = .24; Curve Morphology, F (7, 105) = 16.14, p < .001, η_p^2 = .52; and Motion Description, F (1, 15) = 20.53, p < .001, η_p^2 = .58. The main effects of Graph Type and Curve Morphology were qualified by a significant Graph Type x Curve Morphology interaction F (7, 105) = 7.70, p < .001, η_p^2 = .34, but no interactions involving Motion Description.

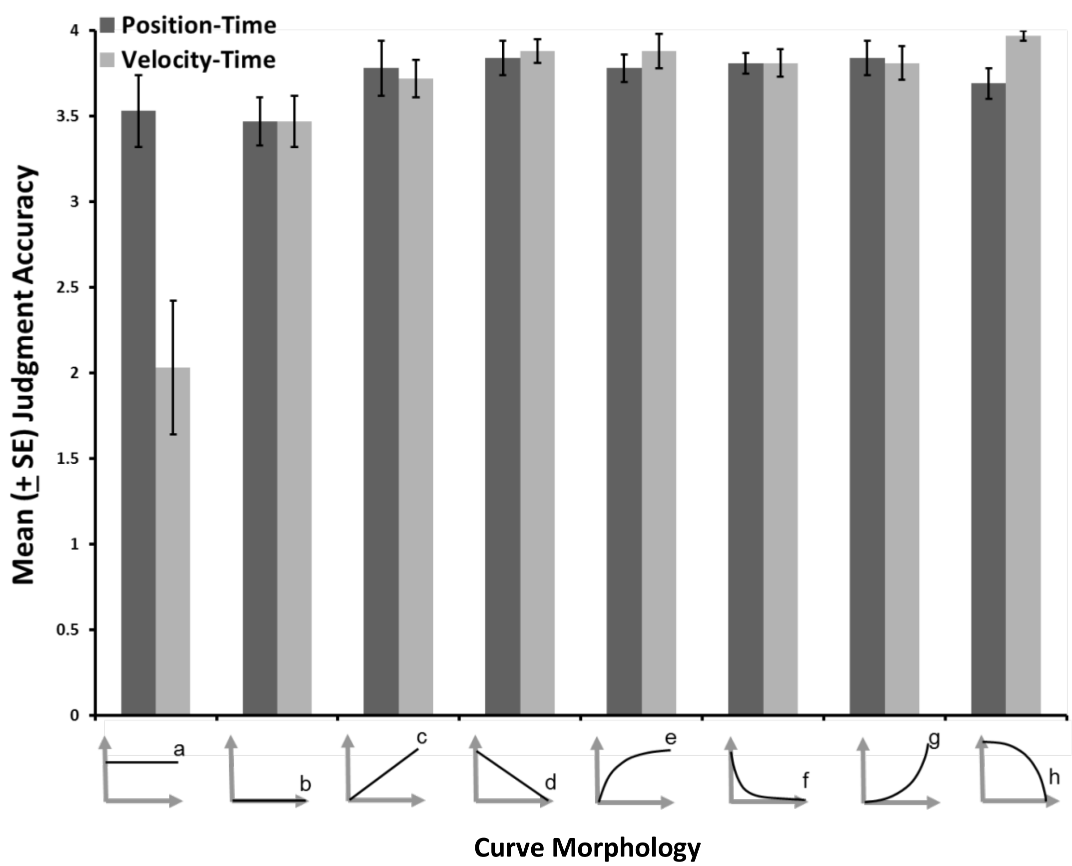


Figure 9. Mean (+SE) verification accuracy on position-time and velocity-time graphs as a function of curve morphology.

The main effect of Motion Description indicated that judgments were more accurate when the statement to be verified was "the object is moving" ($M = 3.77$, $SD = .69$) than when it was "the object is stationary" ($M = 3.52$, $SD = .92$). The motion description effect may be due to the overall differences in the frequency with which the graphs actually depicted objects in motion versus stationary, as shown earlier in Table 4.

The Graph Type by Curve Morphology interaction was examined using simple-effects tests. Whereas accuracy did not vary across the curves at the level of position-time graph, $F\,(1, 105) = .02$, $p = .88$, $\eta_p^2 = .00$, it did at the level of velocity-time graph, $F\,(1, 105) = 7.01$, $p < .01$, $\eta_p^2 = .06$, although the effect was small. The means shown in Figure 9 suggest that the curve morphology effect in the velocity-time graph conditions was largely explained by lower judgment accuracy at curves **a** ($M = 2.03$, $SD = 1.60$) and **b** ($M = 3.47$, $SD = .92$) than at curves **c** – **h** ($M = 3.84$, $SD = .42$). This pattern was confirmed in a series of contrasts. Accuracy on velocity-time graphs differed significantly between curves **a** and **c** – **h**, $F\,(1, 105) = 281.90$, $p < .001$, $\eta_p^2 = .57$; between curves **b** and **c** – **h**, $F\,(1, 105) = 5.90$, $p < .05$, $\eta_p^2 = .05$; and between curves **a** and **b**, $F\,(1, 105) = 51.34$, $p < .001$, $\eta_p^2 = .33$. Curves **a** and **b** are precisely those where the derivational structure was considered to be ambiguous with respect to a general motion judgment (see Table 4).

Further examination of the Graph Type by Curve Morphology interaction looked at simple effects of curve morphology across graph type and indicated that the only significant difference in accuracy was for curve **a**, $F\,(1, 105) = 55.90$, $p <$

.001, $\eta_p^2 = .35$; none of the other curves differed significantly across graph type, Fs < 1.97, ps > .16, η_p^2s < .02.

These results did not support the hypothesis that difficulty varies by y-axis label (i.e., Position-Time graph vs. Velocity-Time graph). In general, judgments made with position-time graphs were no more accurate than judgments made with velocity-time graphs. The one exception was curve **a** where the information related to a general motion judgment was expected to be accessed directly from a position-time graph but the same information about general motion in velocity-time graphs was hypothesized to require derivation through additional processing of the meaning of a key graph feature, namely the intercept of the curve. In the case of curve **a**, judgments with position-time graphs were more accurate than judgments made with velocity-time graphs.

The overall pattern of results is consistent with hypotheses about informational equivalence associated with the derivational structure of representations: Accuracy was consistently high across curves in position-time graphs and velocity time graphs for all but two cases (curve **a**, curve **b**) and those are the cases that were deemed ambiguous with respect to whether the information needed for general motion judgments was directly available or needed to be derived. For all other cases the information was assumed to be directly available and the graphs did not differ in accuracy levels. The response time data examine the question of whether accurate judgments are made faster with position-time graphs than velocity–time graphs.

2. Response Time

Performance was generally very accurate across all conditions with the exception of graph **a** for velocity-time graphs. If the analysis goal is to compare across graph types using all 8 curve morphologies and the 2 motion description statements then data based on accurate responses would be useable from only 11 of the 16 subjects tested. If, however, the goal is to use as much subject data as possible, then an alternative plan that compares across graph type and 7 of the 8 curve morphologies (curves **b** - **h**) and the 2 motion descriptions allows for use of accurate response data from 15 of the 16 subjects tested. The latter plan was chosen to maximize the use of correct judgment response times from the subjects providing data in this experiment. Accordingly, a 2 (Graph Type) x 7 (Curve Morphology) x 2 (Motion Description) repeated measures design was used to analyze the RT for correct responses.

The mean judgment RT for each condition is provided in Figure 10. On average, participants made their judgments in 2000.59 (SD = 900.51) msec. RT varied significantly with Motion Description, F (1, 14) = 16.12, $p < .01$, $\eta_p^2 = .54$. Statements that the object was moving (M = 1838.19, SD = 829.29) – true for all curves except **b** – were verified faster than statements that the object was stationary (M = 2163.00, SD = 940.74). Neither Graph Type, F (1, 14) = 2.88, p = .82, $\eta_p^2 = .03$, nor Curve Morphology, F (6, 84) = .16, p = .69, $\eta_p^2 = .05$, significantly impacted judgment time. None of two-way (Fs < 1.69, ps > .21, $\eta_p^2 < .11$) or three-way, F (6, 84) = 1.01, p = .42, $\eta_p^2 = .07$, interaction effects on RT were significant.

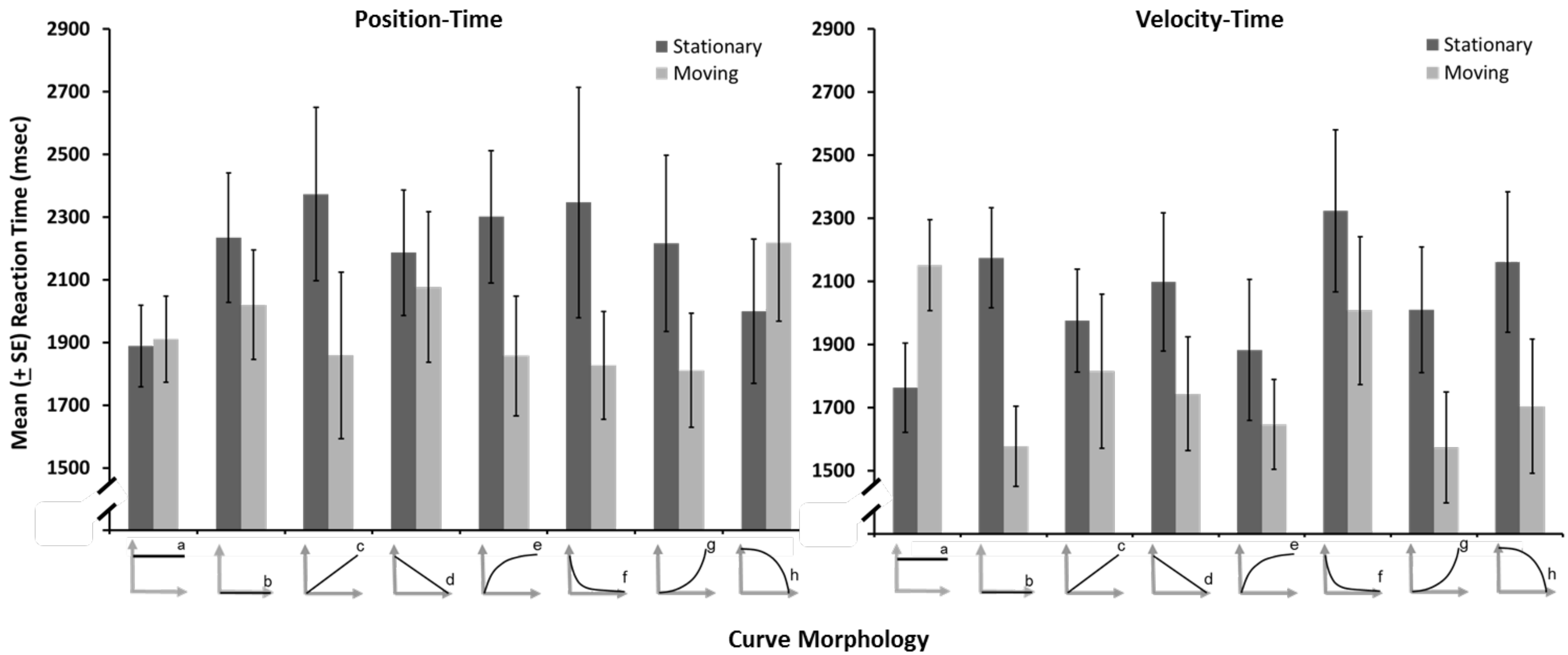


Figure 10. Mean (±SE) response time for judgment across the eight curve morphologies by Graph Type (Position-Time, Velocity-Time) and Motion Description (Stationary, Moving) in Experiment 1.

C. Discussion

Earlier work contended that velocity is conceptually more difficult than position because the former is derived from the latter (Brasell, 1987) and because subjects switch the y-axis labels from “velocity” to “position” during interpretation due to their inability to distinguish the two concepts from one another (Beichner, 1994). Both hypotheses predicted that accuracy for judgments with position-time graphs would be greater than with velocity-time graphs. However, the former hypothesis predicted that latency would be faster with position-time than velocity-time graphs whereas the latter did not predict RT differences related to Graph Type.

To systematically test such a “general difficulty” hypothesis, graph type (position-time vs. velocity-time) was varied while the derivational structure of the graphical information was held constant, except in the case of curve **a** and **b** which were ambiguous. Equivalence in the derivational structure was made possible by restricting the judgment to very general aspects of motion – an object is stationary or it is moving. As noted earlier, the information to make such a judgment is directly available in the graph independent of graph type – position-time or velocity time.

The rate of judgment accuracy did not vary as a function of graph type so long as the derivational structure of the necessary information was direct and held constant. Indeed, judgment outcomes were statistically “flat”; a finding in direct accord with the derivational structure of the representations within the general motion interpretive situation. The only case in which judgment accuracy supported a graph difficulty prediction was when the derivational structure of representations was found to be non-equivalent (i.e., at curve **a** where the information was *direct* for the position-time graph but needed further derivation for the velocity-time graph). In that situation, the direction of accuracy was consistent with expectations related to the derivational structure of representations, i.e., significantly greater accuracy when judgment information was directly available than when it needed to be derived based on further processing of information than was directly available in the graph.

Analysis of RT (for all curves except **a**) also failed to meet the expectation that processing graphical information in velocity-time graphs would be more difficult than processing such information in position-time graphs; there was no Graph Type effect. If processing general motion information in velocity-time graphs was generally more difficult than processing that information in position-time graphs then response time in the former should have been longer than in the latter but this was not observed in the data. Rather, the analysis revealed only a main effect of the Motion Description statement such that verification times were faster for statements that "the object is moving" than for statements that "the object is stationary."

The results of Experiment 1 were inconsistent with the hypothesis that velocity-time graphs are inherently more difficult to interpret than position-time graphs. The results of the experiment also suggest that the derivational structure of representations may be a useful framework for determining how judgment-relevant information needs to be processed in order to achieve optimal accuracy. However, since one cannot make a claim about the adequacy of the derivational structure hypothesis based on accepting the null hypothesis of no predicted accuracy and/or latency difference between position-time and velocity-time graphs, the experiment did not provide evidence in support of the DSH. Experiment 2 provides a further test of the general graph difficulty hypothesis and a direct examination of predictions based upon application of the derivational structure of representations as a task analysis framework.

IV. EXPERIMENT 2.

VELOCITY CHANGE JUDGMENTS:

EFFECTS OF GRAPH TYPE AND DERIVATIONAL STRUCTURE

Experiment 2 was designed to test a general graph difficulty prediction against a competing prediction based on the derivational structure of representations by examining judgments about velocity change instead of general motion. The general graph difficulty hypothesis predicts that judgments of velocity change will be harder to make in velocity-time graphs than in position-time graphs. The derivational structure hypothesis (DSH) predicts the opposite: judgments of velocity change in position-time graphs should be harder than velocity-time graphs, based on the derived versus direct nature of the processing. Cognitive task analysis based on the derivational structure of representations (see Table 1, columns 4 and 5) characterizes velocity change judgments as possible through processing visual information directly in velocity-time graphs but through more intensive processing in position-time graphs. However, the DSH prediction pertains to *accurate* judgments: To be highly accurate subjects would be expected to engage in more elaborate processing when interpreting change in motion for position-time graphs compared to velocity-time graphs because in the velocity-time graphs the information required for those judgments can be accessed directly from the graphs. This difference between velocity-time graphs and position-time graphs pertains to all eight curves for velocity change judgments in the present experiment, unlike the case for general motion judgments examined in Experiment 1.

Thus, according to the general graph difficulty prediction, judgments should be more accurate and faster when made with position-time than velocity-time graphs because the former are conceptually easier to interpret than the latter. Alternatively, the DSH suggests that judgment accuracy will be high in both the velocity-time and position-time graphs *if* the subjects use appropriate processing strategies, i.e., more direct processes for judgments in the former and more elaborate processing in the latter. Furthermore, although accuracy will be high in both conditions, if different but appropriate forms of processing are used in the respective graph conditions, judgments should be slower in the position-time graphs than they should be in the velocity-time graphs because accurate judgments for position-time graphs require more processing.

In the presence of a significant main effect of curve morphology on RT, several contrasts were planned for purposes of examining the extent to which judgment was related to height, slope, and curvature as suggested by prior graph interpretation research (e.g., Beichner et al., 1996; Leinhardt, Zaslavsky, & Stein, 1990; McDermott et al., 1987). Whereas earlier work revealed that interpretation difficulty depended on whether a curve was linear or nonlinear, Best, Smith, & Stubbs (1997) found that subjects in their study processed curvature as a meaningful feature. Therefore, the contrasts in this experiment were set up to analyze slope as a global feature and as a composite structure partitioned into two component features that are potentially available for processing: *directionality* (i.e., zero vs positive and negative; positive vs negative) and *curvature* (i.e., linear vs.

nonlinear; nonlinear increasing rate of change vs nonlinear decreasing rate of change).

A. Testing for a Height Effect

Two curves, **a** and **b**, differed in height on the y-axis but not any additional features. Curve **a** portrays a zero-slope function (horizontal line) that intercepts the y-axis at a point above zero. Curve **b** portrays a zero-slope function (horizontal line) that intercepts the y-axis at zero. The effect was tested by contrasting RTs related to curve **a** against those related to curve **b**.

B. Testing for Effects of Slope

Slope has two primary properties one of which is directionality (i.e., zero, positive, and negative) and the second is curvature.

1. Directionality of slope. It was possible to test the effect of a zero slope (horizontal line) against linear curves having positive or negative slope by contrasting the mean RTs for performance at curves where slope is constant but equal to zero (**a** and **b)**, against the mean RTs for curves with nonzero slopes, namely **c** (positive linear function, slope constant but greater than zero) and **d** (negative linear function, slope constant but less than zero). The comparison allows a general test of the impact of slope directionality relative to a horizontal function.

Non-horizontal functions can have positive or negative slopes. Whether there is a differential impact on processing of these was tested by comparing RTs associated with performance on curves with a positive (curve **c)** and negative (curve **d**) slope.

2. Curvature. To examine the extent that curvature was a factor in subjects' response times, three comparisons were examined. The first comparison contrasted RTs at curve **c** versus curves **e** and **g**; the second contrasted RTs at curve **d** versus those at curves **f** and **h**; the third contrasted RTs at curves **e** and **f** against curves **g** and **h.** In the first contrast, the direction of the curves is positive (i.e., *upward*) and **c** is linear whereas **e** and **g** are nonlinear. In the second contrast, the direction of the curves is negative (i.e., *downward*) and **d** is linear whereas **f** and **h** are nonlinear. The third contrast tests for a curvature effect when the function expresses more specific information about increasing versus decreasing rates of change.

C. Method

1. Participants

Sixteen University of Illinois at Chicago undergraduates were recruited from the general UIC population (n = 4) and the Department of Psychology Subject Pool (n = 12). Twelve were female and four were male. The median age was 18.5 years, range = 18 – 29. Most subjects were in their first (n = 9) or second (n = 6) year of college and one was in her fifth. All participants were instructed to complete the same questionnaire described in Experiment 1. The questionnaire was designed to elicit information about math and science background as well as graph interpretation skill. Subjects reported their previous five math courses. The "highest" level of math reported was calculus (n = 7 subjects), pre-calculus (n = 6 subjects), trigonometry (n = 2 subjects), algebra II/geometry (n = 1 subject). The study was limited to subjects who had taken at least one year of physics. However in addition to physics, subjects reported taking at least one course in each of the following areas: Anatomy and Physiology (N = 1 subject), Biology (N = 15 subjects), Chemistry (N = 15 subjects), Earth or Environmental Sciences (N = 4 subjects), and Psychology (N = 3 subjects). Biology courses reported included Introductory, AP, Accelerated, or advanced courses such as genetics, homeostasis, and ecology/evolution. Chemistry courses reported included introductory, general, and organic. One student had taken a second semester of Physics. Students recruited from the general UIC population were paid to participate in a single session at the rate of $20.00 per session. Students recruited from the subject pool earned course credit for their participation. Non-subject pool participants were screened to ensure that they were comparable to Subject Pool participants in terms of their prior course experience. Accuracy across the nine responses opportunities in the questionnaire ranged from 0.00 to 5.00. The Modal frequency of correct responses was 1 and the Mean was 2.06 (*SD* = 1.77). The implications of this level of performance for interpreting performance on the experimental task are discussed further in the General Discussion.

<u>2. Design</u>

A three-factor (2 x 8 x 2) within-subjects design was used to examine the effects of Graph Type, Curve Morphology, and Motion Description on verification accuracy and mean response latency for accurate responses. The graph types and curve morphologies were the same as those used in Experiment 1. However, in

Experiment 2, each participant was asked to make judgments about *Velocity Change* (Accelerating, Decelerating) statements instead of *General Motion* (i.e., Stationary, Moving).

3. Materials

The materials consisted of the same graph types and curve morphologies used in the previous experiment except in Experiment 2 the graphs were paired with one of two Velocity Change statements: *The object's movement is accelerating* and *The object's movement is decelerating*. The crossing of the graph types with each of the motion descriptions resulted in a set of 32 unique description-graph displays.

Each participant was tested on a total of 128 description-graph pairs. The 128 stimulus pairings were divided into four blocks of 32 trials according to graph type. Each 32 trial block was further divided into two sets of 16 trials. Each set of 16 trials represented one of the two graph types crossed with the eight kinematic curve types and with the two velocity change descriptions (accelerating, decelerating). These constituted the 16 critical trials of interest. The trials in each critical set of 16 trials within a 32 trial block were randomized. Across the four blocks of 32 trials, position-time graphs (64 trials) and velocity-time graphs (64 trials) were equally represented and the graph types alternated across blocks. Finally, graph type presentation order (Position-Time then Velocity-Time or Velocity-Time then Position-Time) was counterbalanced across participants.

4. Procedure

The experimental procedure was nearly identical to that described in Experiment 1 except that participants were instructed to make judgments about velocity change. That is, each participant was presented with information about objects that change speed and go faster (acceleration), or change speed and go slower (deceleration). The stimuli were presented according to the procedure shown in Figure 11.

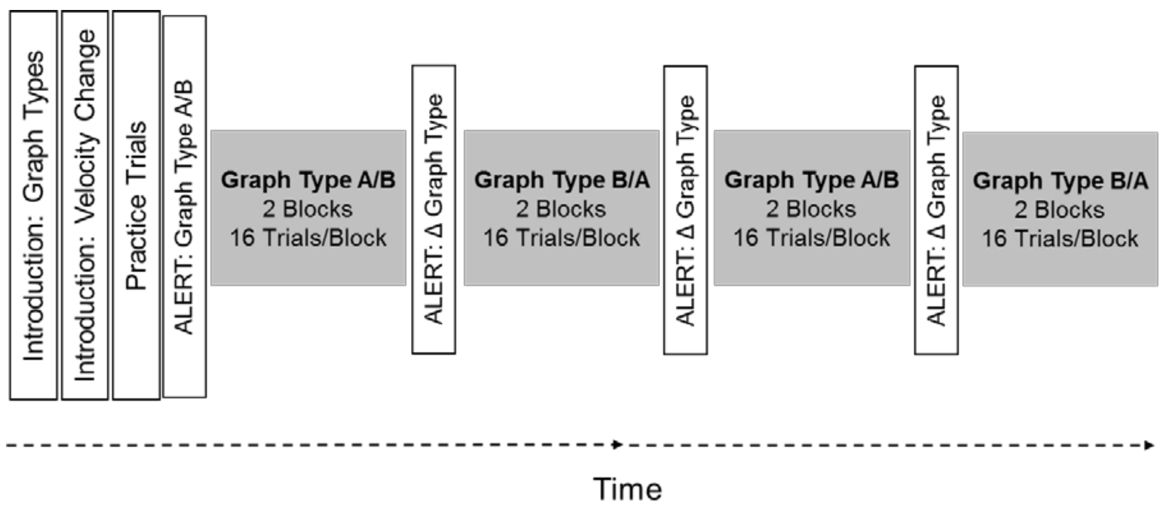
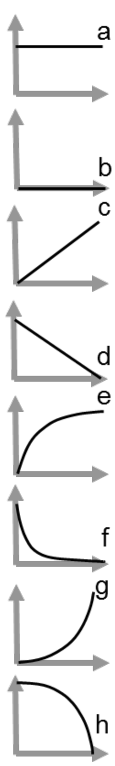


Figure 11. General procedure for Experiment 2. Prior to entering the test phase, participants were presented with information about graph types and velocity change and then received five practice trials with feedback from the interviewer (each trial reflects one of the two verbal descriptions). The graph types alternate between blocks according to two block orders: ABAB or BABA.

Accurate judgments for accelerating/decelerating motion descriptions for the two graph types and eight curve morphologies are shown in Table 5. In the position-time graphs, curves **a** and **b** indicate that the object is not changing position so it is neither accelerating nor decelerating and the accurate response is false. In position-time graphs, curves **c** and **d** indicate that the object is moving but at a constant rate so again accurate judgment requires a false response to both accelerating and decelerating sentences. For graphs **e** and **f**, the rate at which the object is changing position is decreasing so the correct responses are false for acceleration and true for deceleration. The reverse is true for graphs **g** and **h** where the rate at which the object is changing position is increasing, so correct responses are true for accelerating and false for decelerating.

Table V
TRUTH TABLE OF CORRECT RESPONSES (T=TRUE; F=FALSE) FOR EACH GRAPH TYPE, CURVE MORPHOLOGY (A – H), AND VERBAL DESCRIPTION. DERIVED INFORMATION BASED ON DERIVATIONAL STRUCTURE HYPOTHESIS = GREY CELLS.



	Velocity Change			
	Position-Time		Velocity-Time	
	Accelerating	Decelerating	Accelerating	Decelerating
a	F	F	F	F
b	F	F	F	F
c	F	F	T	F
d	F	F	F	T
e	F	T	T	F
f	F	T	F	T
g	T	F	T	F
h	T	F	F	T

The pattern of true and false judgments – and the reasoning that underlies them –is different across the 16 conditions for the velocity-time graphs. For the velocity-time graphs, curve **a** shows an object moving but at a constant rate so accurate responses are false to either statement – accelerating or decelerating. For graph **b**, the object is not moving so again both statements about velocity change are false. Graphs **c**, **e**, and **g**, all show an increasing slope so statements about accelerating objects are true and statements about decelerating objects are false. The opposite is the case for graphs **d**, **f**, and **h** that have a decreasing slope so statements about decelerating objects are true whereas statements about accelerating objects are false.

D. Results

1. Judgment Accuracy

Judgment accuracy was analyzed using a 2 (Graph Type: Position-Time, Velocity-Time) x 8 [Curve Morphology: Zero-Slope y > 0 (Curve a), Slope = 0 (Curve b), Linear Increasing (Curve c), Linear Decreasing (Curve d), Positive Decreasing (Curve e), Negative Decreasing (Curve f), Positive Increasing (Curve g), Negative Increasing (Curve h)] x 2 (Motion Description: The object's motion is accelerating, The object's motion is decelerating) repeated measures analysis of variance (ANOVA). Partial eta square was used to evaluate effect size.

The ANOVA revealed significant effects of Graph Type, $F\,(1, 15) = 14.71$, $p < .01$, $\eta_p^2 = .50$, and Curve Morphology, $F\,(7, 105) = 22.03$, $p < .001$, $\eta_p^2 = .60$, on judgment accuracy but no main effect of Motion Description, $F\,(1, 15) = .12$, $p = .73$,

$\eta_p^2 = .01$. However, each main effect was qualified by two-way interactions of Graph Type x Curve Morphology, $F(7, 105) = 7.39$, $p < .001$, $\eta_p^2 = .35$; Graph Type x Motion Description, $F(1, 15) = 12.08$, $p < .01$, $\eta_p^2 = .45$; and Curve Morphology x Motion Description, $F(7, 105) = 7.60$, $p < .001$, $\eta_p^2 = .43$. These two-way interactions, in turn, were qualified by the three-way interaction of Graph Type x Curve Morphology x Motion Description, $F(7, 105) = 3.77$, $p < .001$, $\eta_p^2 = .34$. The means for the three-way interaction are shown in Figure 12.

Descriptively, the patterns of means in Figure 12 shows a higher degree of variability in judgment accuracy as a function of both curve morphology and motion description for the position-time graphs than for the velocity-time graphs. To examine these effects, statistical comparisons were conducted on the interaction between Curve Morphology and Motion Description at each level of Graph Type. Curve morphology and Motion Description significantly interacted in position-time graphs, $F(7, 105) = 21.41$, $p < .001$, $\eta_p^2 = .24$, but this interaction was not significant in the velocity-time graphs, $F(7, 105) = 3.14$, $p = .08$, $\eta_p^2 = .04$. In other words, whether the judgment was about acceleration or deceleration differentially impacted accuracy depending on curve types for position-time graphs but the motion descriptions behaved similarly for each curve type in the velocity-time graphs. Furthermore, the interaction within position-time graphs reflected significant differences in judgment accuracy as a function of motion description, with the direction of these effects depending on the shape of the curve. Accuracy was higher for judgments about accelerating than decelerating objects for graphs

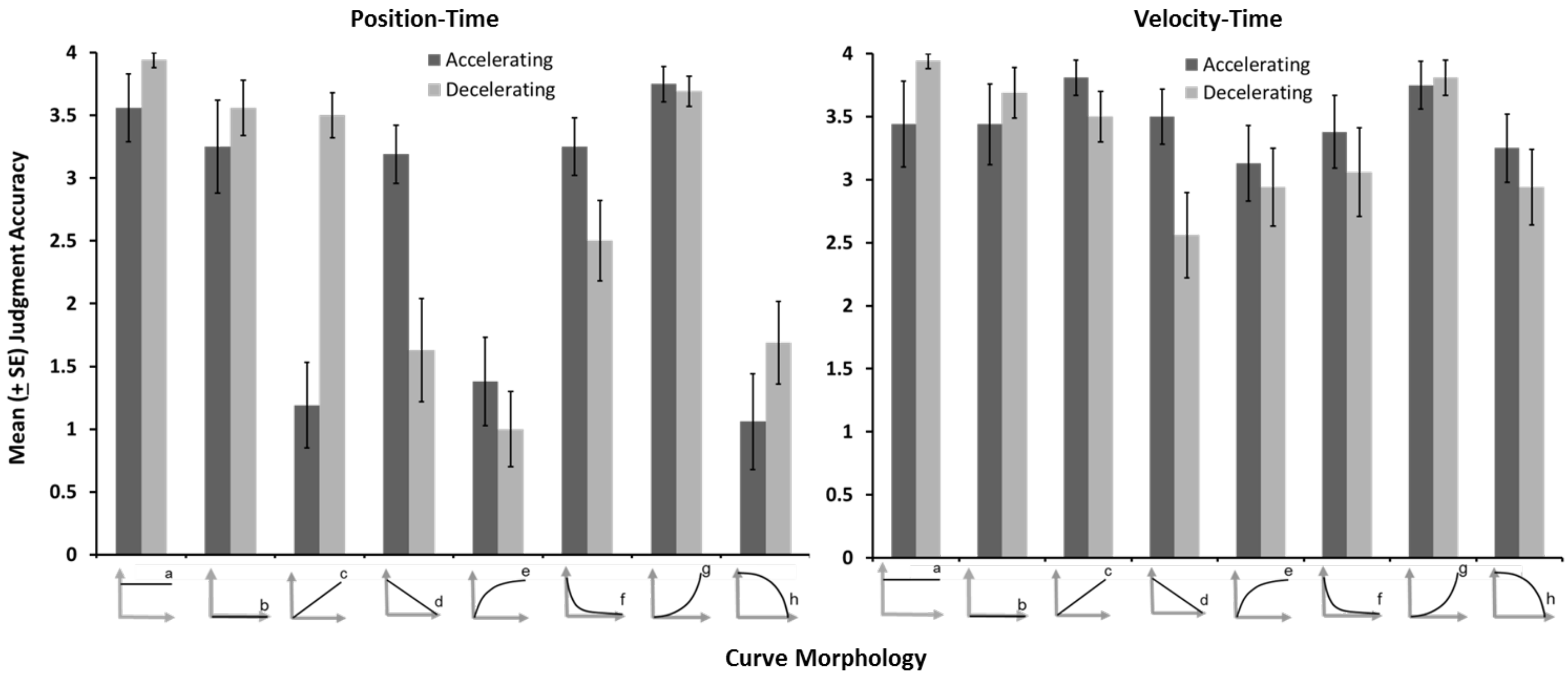


Figure 12. Mean (±SE) judgment accuracy as a function of Curve Morphology and Motion Description at each level of Graph Type (Experiment 2).

with curve **c**, $F(1, 105) = 56.04$, $p < .001$, $\eta_p^2 = .04$, and curve **h**, $F(1, 105) = 5.29$, $p < .05$, $\eta_p^2 = .05$ but lower for graphs that involved curve **d**, $F(1, 105) = 25.59$, $p < .001$, $\eta_p^2 = .24$, and curve **f**, $F(1, 105) = 7.61$, $p < .01$, $\eta_p^2 = .07$. Judgment accuracy did not differ by Motion Description for curves **a**, **b**, **e**, and **g,** Fs < 1.90, ps > .17, $\eta_p^2 < .02$. Such a pattern of results fails to provide support for either the general graph difficulty hypothesis or for the alternative, the derivational structure hypothesis (DSH). Indeed, the varying levels of accuracy for several of the curve morphology x motion description conditions within position time graphs indicate that participants did not engage in the more elaborate processing that the DSH suggests is needed for consistently accurate decisions about velocity change in such graphs. Further consideration of these results will be presented subsequently.

<u>2. Response Time</u>

Predictions regarding response time were predicated on high levels of accurate performance across all conditions. Although performance was generally very accurate across all conditions within the velocity-time graphs, there were significant differences in accuracy for the position-time graph judgments, with some cells showing average accuracy below chance. Thus, the original plan for analyses of the response time (RT) data was modified. Specifically, the original analysis plan of comparing across graph types using all eight curve morphologies and the two velocity-change statements was rendered impractical since it would have been possible to use data from only six of the 16 subjects tested. A modified goal was adopted of using as much subject data as possible to make meaningful RT comparisons by examining RT only within velocity-time graphs across the 8 curve morphologies and the two motion descriptions. This plan allowed for use of data from 14 of the 16 subjects tested. Two subjects were removed from the analysis due to accuracy levels below chance in at least one of the 16 cells. Under this plan, the response time analyses are informative with respect to differential difficulty among the 8 curve morphologies and no longer speak to the general graph difficulty hypothesis or the alternative DSH. RT was analyzed and reported in milliseconds (msec).

a) RT for Velocity Change Judgments in Velocity-Time Graphs

On average, subjects took 2761.10 msec (SD = 1740.84) to make velocity change judgments with velocity-time graphs. Mean RT ($\pm$ SE) as a function of Curve Morphology and Motion Description are displayed in Figure 13. An 8 (Curve Morphology) x 2 (Motion Description) repeated measures ANOVA revealed a main effect of curve morphology on response time, $F\,(7, 91) = 5.97$, $p < .001$, $\eta_p^2 = .31$, no main effect for type of motion description statement, $F\,(1, 13) = 1.47$, $p = .25$, $\eta_p^2 = .10$, and no significant interaction, $F\,(7, 91) = 1.17$, $p = .33$, $\eta_p^2 = .08$.

The main effect of curve morphology on RT was examined further through a series of contrasts that were driven by specific properties of curves presumed to impact difficulty of graph interpretation, as discussed in the prior literature review: height, direction of slope (zero versus positive and negative), and curvature as previously defined. Each was examined to evaluate the extent to which they accounted for variations in processing time underlying the curve morphology effect for velocity-time graphs.

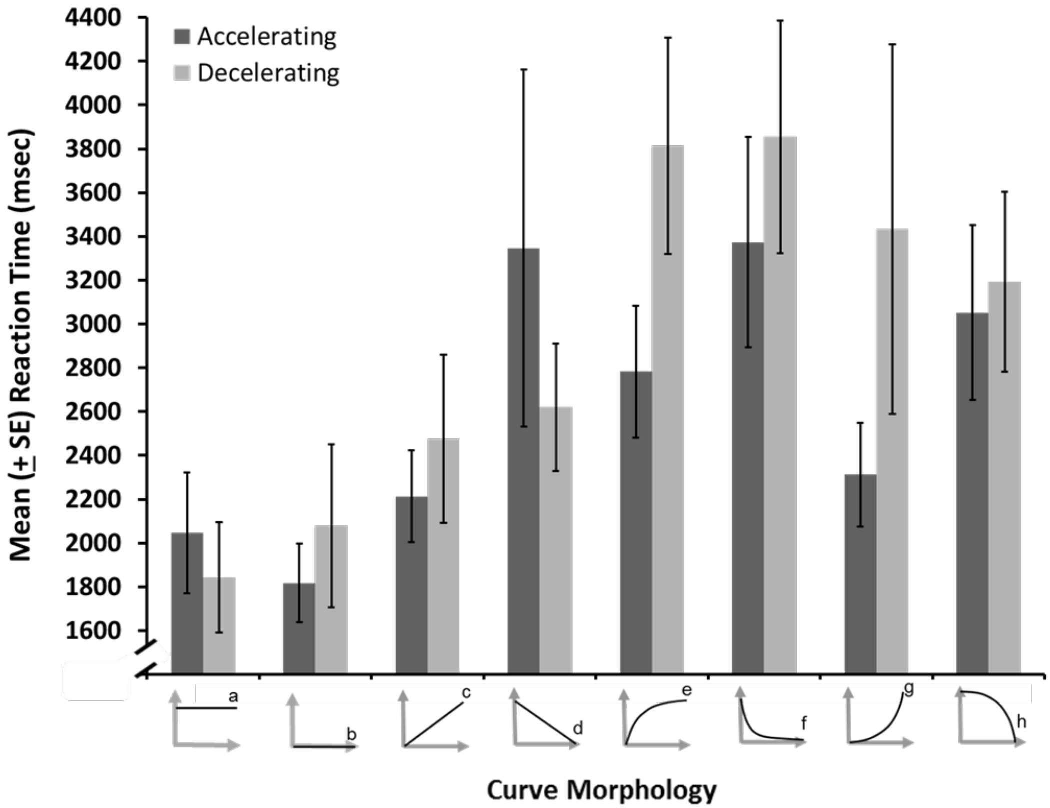


Figure 13. Mean (±SE) response time for accurate judgments as a function of curve morphology and motion description at the level of velocity-time graph.

i. Height Effects. The mean RTs for both curves indicated that judgments that involved curve **b** (M = 1948.33, SD = 1080.42) did not differ significantly from judgments that involved curve **a** (M = 1944.25, SD = 974.75), F (1, 91) = .00, p = .99, η_p^2 = .00.

ii. Direction of Slope Effect. The comparison indicated that subjects took more time to make a judgment when the curve was oriented upward or downward (**c** & **d**: M = 2663.70, SD = 1710.73) than when it was horizontal (**a** & **b**: M = 1946.29, SD = 1027.59), F (1, 91) = 8.19, p < .01, η_p^2 = .11. An additional contrast of

RT on curve **c** against RT for curve **d** indicated that there was no difference in the amount of time that it took to process information about direction in curves with an upward (**c**: $M = 2344.27$, $SD = 1141.88$) versus a downward direction (**d**: $M = 2983.13$, $SD = 2279.57$), $F(1, 91) = 3.44$, $p = .07$, $\eta_p^2 = .04$.

iii. Curvature Effects. The results of the first contrast revealed a statistically significant effect of curvature on RT. Judgment was faster when the curve was linear (**c**: $M = 2344.27$, $SD = 1141.88$) than when it was nonlinear (**e** & **g**: $M = 3085.57$, $SD = 1970.12$), $F(1, 91) = 5.83$, $p < .05$, $\eta_p^2 = .08$. However, the contrast of linear curve **d** ($M = 2983.13$, $SD = 2279.57$) versus nonlinear curves **f** and **h** ($M = 3363.91$, $SD = 1665.50$) revealed a non-significant effect of curvature on RT, $F(1, 91) = 1.53$, $p = .22$, $\eta_p^2 = .02$. The results of the third contrast indicated that processing did not depend on whether the curve represented an increasing (**g** & **h**: $M = 2997.52$, $SD = 1919.78$) or decreasing rate of change (**e** & **f**: $M = 3451.96$, $SD = 1715.84$), $F(1, 91) = 2.06$, $p = .15$, $\eta_p^2 = .01$. Given the small effect sizes regardless of the significance levels, curvature appears to have a relatively minor impact on judgments of velocity change in velocity-time graphs.

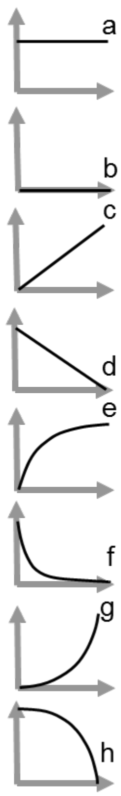
3. Summary of Initial Accuracy and Response Time Analyses

Neither the general graph difficulty hypothesis nor the DSH were supported in the accuracy and RT analyses. Contrary to predictions of the general graph difficulty hypothesis, judgment accuracy was greater with velocity-time graphs than it was with position-time graphs. The DSH predicts that accuracy should be greater than chance in all conditions and any variability related to difficulty deriving the appropriate information should show up in RT but that is not what the results indicate. The question is, if subjects are not processing the information according to expectations derived from the DSH, then what is happening? One reasonable hypothesis is that they processed information in the position-time graphs using the same direct read-off strategy that was appropriate for highly accurate judgments with the velocity-time graphs but was insufficient for accurate judgments with all position-time graphs. This direct read-off hypothesis was explored further as it predicts very specific patterns of accuracy and RT within and across graph types.

4. Direct Read-Off: Judgment Accuracy

The RT analysis in the previous section indicated that judgment of velocity change was sensitive to the directionality in the slope and interpretation was not dependent on whether that direction was positive and negative. Thus, if one is using a direct read-off strategy based on curve directionality, then three simple rules can be applied to make judgments about velocity change: (1) horizontal means no velocity change, i.e., object is neither accelerating nor decelerating (2) up means increase (going faster), i.e., object is accelerating over time, and (3) down means decrease (going slower), i.e., object is decelerating. To determine if use of these rules provided a better account of performance for velocity change judgments than the two hypotheses that the experiment was designed to test (general graph difficulty and DSH), predictions were derived about accuracy of velocity change judgments in the two graph types for all eight curves as shown in Table 6.

Table VI
RESPONSES PREDICTED BY USE OF THE THREE DIRECT READ-OFF RULES FOR VELOCITY CHANGE JUDGMENTS. DERIVED INFORMATION BASED ON DERIVATIONAL STRUCTURE HYPOTHESIS = GREYED CELLS.



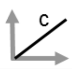
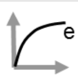
		Velocity Change			
		Position-Time		Velocity-Time	
	Direct Read-Off Rule	Accelerating	Decelerating	Accelerating	Decelerating
a	1. Horizontal, no change	False	False	False	False
b	1. Horizontal, no change	False	False	False	False
c	2. Up, increase, faster	True*	False	True	False
d	3. Down, decrease, slower	False	True*	False	True
e	2. Up, increase, faster	True*	False*	True	False
f	3. Down, decrease, slower	False	True	False	True
g	2. Up, increase, faster	True	False	True	False
h	3. Down, decrease, slower	False*	True*	False	True

The asterisks in Table 6 indicate where use of the directionality rules is expected to produce incorrect judgments and therefore low accuracy. Notice that this variability in accuracy is only predicted to happen with position-time graphs and in selected cases. For velocity-time graphs application of the directionality rules should always produce accurate judgments, consistent with the argument that the velocity change information needed for accurate judgments is directly available in the graphs. The next section examines the relative "success" of the read-off rules for predicting observed accuracy performance for conditions in each graph type.

a) Direct Read-Off from Position-Time Graphs

Table 7 shows each curve type (column 1) and the predicted accuracy (columns 2 and 5) for all position-time graph conditions if judgment was generated by direct read-off. Observed accuracy (columns 3 and 6) reflects the actual number of judgments correct (proportion) out of a possible 64 judgments (i.e., one judgment per trial, four trials per subject, summed over 16 subjects). Observed accuracy was tested against chance performance. Because subjects are making repeated judgments each judgment is not independent of the others. Thus, it is necessary to look at the likelihood distribution across the four trials per condition. there are 16 possible patterns (permutations) of True/False outcomes across the four trials, 1 of which reflects four correct judgments, four of which reflect three correct judgments, six of which reflect two correct judgments, four of which reflect one correct judgment, and one of which reflects no correct judgments. Thus, the probabilities of a subject being correct on "4", "3", "2", "1", and "0" judgments per condition are .0625, .2500, .3750, .2500, and .0625, respectively, if they are operating at a chance or "guess" level. Given that there were 16 subjects in the sample, the expected values for the distribution of subjects with "4", "3", "2", "1", and "0" correct judgments is, 1 = .0625 x 16, 4 =.2500 x 16; 6 =.3750 x 16), 4 = .2500 x 16; and 1=.0625 x 16, respectively. Thus, it was possible to test the distribution of observed subject outcomes against the chance distribution with respect to the number of subjects correct on “4”, “3”, “2”, “1”, and “0” trials per condition. (See Appendix G for a further description of the calculation of the chance distribution).

Table VII
PREDICTED ACCURACY LEVELS AND OBSERVED[A] JUDGMENT ACCURACY FOR POSITION-TIME GRAPHS RELATIVE TO DIRECT READ-OFF OF SLOPE DIRECTION FOR VELOCITY CHANGE JUDGMENTS

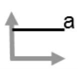
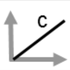
Predicted Chance Distribution															
0 correct = 1 Ss, 1 correct = 4 Ss, 2 correct = 6 Ss, 3 correct = 4 Ss, 4 correct = 1 Ss															
		Observed Distribution (Accelerating)							Observed Distribution (Decelerating)						
Curve	**Read-Off Prediction**	**0**	**1**	**2**	**3**	**4**	**$X^2(df = 4)$**	**Read-Off Prediction**	**0**	**1**	**2**	**3**	**4**	**$X^2(df = 4)$**
	High Accuracy	1	0	1	1	13	154.42^{***}	High Accuracy	0	0	0	1	15	209.25^{***}
	High Accuracy	2	1	0	1	12	132.50^{***}	High Accuracy	0	1	1	2	12	129.42^{***}
	Low Accuracy	6	6	1	1	2	33.42^{***}	High Accuracy	0	0	2	4	10	88.67^{***}
	High Accuracy	0	0	5	3	8	54.42^{***}	Low Accuracy	6	2	4	0	4	39.67^{***}
	Low Accuracy	6	3	4	1	2	29.17^{***}	Low Accuracy	8	2	5	0	1	54.17^{**}
	High Accuracy	0	1	2	5	8	55.17^{***}	High Accuracy	2	1	3	7	3	11.00^{*}
	High Accuracy	0	0	1	2	13	154.17^{***}	High Accuracy	0	0	0	5	11	111.25^{***}
	Low Accuracy	10	0	3	1	2	89.75^{***}	Low Accuracy	3	5	4	2	2	6.92

[a] Frequency (proportion) of correct judgments out of a possible 64.
* $p < .05$. *** $p < .001$.

i. Test of Observed Accuracy Distribution against Chance. As reflected by the columns of X^2 values (see Table 7), the actual accuracy patterns exhibited by subjects were consistent with the predictions of the direct read-off hypothesis and the distribution of correct judgments was either greater or less than chance in 15 of the 16 conditions. In the one case where the test against chance was not significant the response frequencies and subject score patterns were in the predicted direction.

<u>b) Direct Read-Off from Velocity-Time Graphs.</u> Table 8 shows each curve type (column 1) and the predicted accuracy (columns 2 and 5) for all velocity-time

Table VIII
PREDICTED ACCURACY LEVELS AND OBSERVED[A] JUDGMENT ACCURACY FOR VELOCITY-TIME GRAPHS RELATIVE TO DIRECT READ-OFF OF SLOPE DIRECTION FOR VELOCITY CHANGE JUDGMENTS

Predicted Chance Distribution														
0 correct = 1 Ss, 1 correct = 4 Ss, 2 correct = 6 Ss, 3 correct = 4 Ss, 4 correct = 1 Ss														
		Observed Distribution (Accelerating)							Observed Distribution (Decelerating)					
Curve	Read-Off Prediction	0	1	2	3	4	$X^2(df = 4)$	Read-Off Prediction	0	1	2	3	4	$X^2(df = 4)$
 a	High Accuracy	2	0	0	1	13	157.25***	High Accuracy	0	0	0	1	15	209.25***
 b	High Accuracy	1	1	1	0	13	154.42***	High Accuracy	0	1	0	2	13	154.25***
 c	High Accuracy	0	0	1	1	14	180.42***	High Accuracy	0	0	3	2	11	107.50***
 d	High Accuracy	0	1	1	3	11	107.62***	High Accuracy	1	4	1	5	5	20.42***
 e	High Accuracy	0	3	1	3	9	69.67***	High Accuracy	1	1	3	4	7	39.75***
 f	High Accuracy	1	0	2	2	11	107.67***	High Accuracy	2	0	2	3	9	71.92***
 g	High Accuracy	0	1	0	1	14	180.50***	High Accuracy	0	0	1	1	14	180.42***
 h	High Accuracy	1	0	1	6	8	58.17***	High Accuracy	1	1	2	6	6	30.92***

[a] Frequency (proportion) correct judgments out of possible 64. *** $p < .001$.

graph conditions based on direct read off from the graphs. Observed accuracy reflects actual number of correct judgments, maximum = 64. The observed distribution of correct judgments was consistent with the expected distribution in

16 of 16 conditions. Thus, considering the results for both position-time and velocity-time graph conditions, the direct read-off hypothesis predicted 31 of 32 (96.88%) of observed subject response patterns with respect to accuracy.

5. Direct Read-Off: Judgment Response Time

Given the judgment accuracy pattern exhibited in this experiment and its interpretation in terms of a direct read-off of features for both the Velocity-Time and Position-Time graphs, it was deemed reasonable to look at RT data as a further test of the hypothesized direct read-off strategy. Thus, both correct and incorrect judgment RTs were pooled together for each subject to provide an estimate of overall RT within each condition for purposes of comparing graph type, curve morphology, and motion description. The assumption justifying such a collapsing of correct and incorrect judgment RTs is that subjects were responding under the assumption that their judgments were accurate despite the fact that the graph features guiding their judgments led to predictably incorrect responses in certain conditions and predictably correct responses in other conditions.

If judgments are in fact being made on the basis of the aforementioned direct read-off strategy, then the amount of time that it takes to make a judgment should not be affected by graph type. Instead, it should only be affected by information in the curve, specifically directionality as shown in the analysis of RTs for correct judgments with velocity-time graphs. Further, since the appearance of a curve oriented “upward” or “downward” should be equally effective at cueing judgments related to increases and decreases in the speed of an object, RTs should not vary based on whether the slope is positive or negative.

RT data were submitted to a 2 (Graph Type) x 8 (Curve Morphology) x 2 (Motion Description) repeated measures ANOVA. On average, subjects took 2744.43 msec (SD = 1699.74) to make a judgment across all conditions. The only significant main effect on RT was Curve Morphology, F (7, 105) = 10.22, $p < .001$, $\eta_p^2 = .41$. Neither Graph Type, F (1, 15) = .00, p = .95, η_p^2 = .00, nor Motion Description, F (1, 15) = .83, p = .38, η_p^2 = .05 were significant. Neither the two-way (Fs < 1.12, ps > .36, $\eta_p^2 < .07$) nor three-way [F (7, 105) = .97, p = .46, η_p^2 = .06] interaction effects were significant. Mean RT for judgments at each curve are shown in Figure 14.

First, the main effect of Curve Morphology on RT was followed by several single-*df* contrasts to test whether features of the curves germane to the three "direct read-off" rules had an impact of response time. For linear functions, RT was faster for horizontal curves **a** and **b** (M = 2033.62, SD = 1219.51) than for the curves with positive and negative slope, **c** and **d** (M = 2585.34, SD = 1501.12), F (1, 105) = 7.28, $p < .01$, η_p^2 = .06. RT was not affected by differences in the direction of the linear curves with positive and negative slope—judgments of curve **c** (M = 2398.02, SD = 1224.26) did not differ from the time it took for judgments of curve **d** (M = 2772.65, SD = 1777.99), F (1, 105) = 3.34, p = .07, η_p^2 = .03.

Second, an additional set of curve contrasts focused on the effects of height and curvature on RT. RT for judgments made with curve **a** (M = 1992.13, SD =

1282.14) were statistically on par with those made with curve **b** (*M* = 2075.11, *SD* = 1156.87), *F* (1, 105) = .16, *p* = .69, η_p^2 = .00, indicating that height had no bearing on interpreting graphical representations of velocity change. Regarding curvature, judgments of linear curves were faster than nonlinear for positive-trending curves: curve **c** (*M* = 2362.04, *SD* = 1176.45) was significantly faster than curves **e** & **g** (*M* = 3145.80, *SD* = 2017.34), *F* (1, 105) = 4.93, *p* < .05, η_p^2 = .04. In contrast, there was no effect of linear versus nonlinear for downward-oriented curves: linear curve **d** response times (*M* = 2753.04, *SD* = 1817.74) were statistically equivalent to nonlinear curves **f** & **h** (*M* = 3353.62, *SD* = 1686.26), *F* (1, 105) = .82, *p* = .37, η_p^2 = .01. Nor were there any significant differences between curves that represented increasing (**e** & **f**: *M* = 3277.13, *SD* = 1587.41) and decreasing (**g** & **h**: *M* = 3081.65, *SD* = 2069.79) changes in velocity, *F* (1, 105) = 2.04, *p* = .16, η_p^2 = .02.

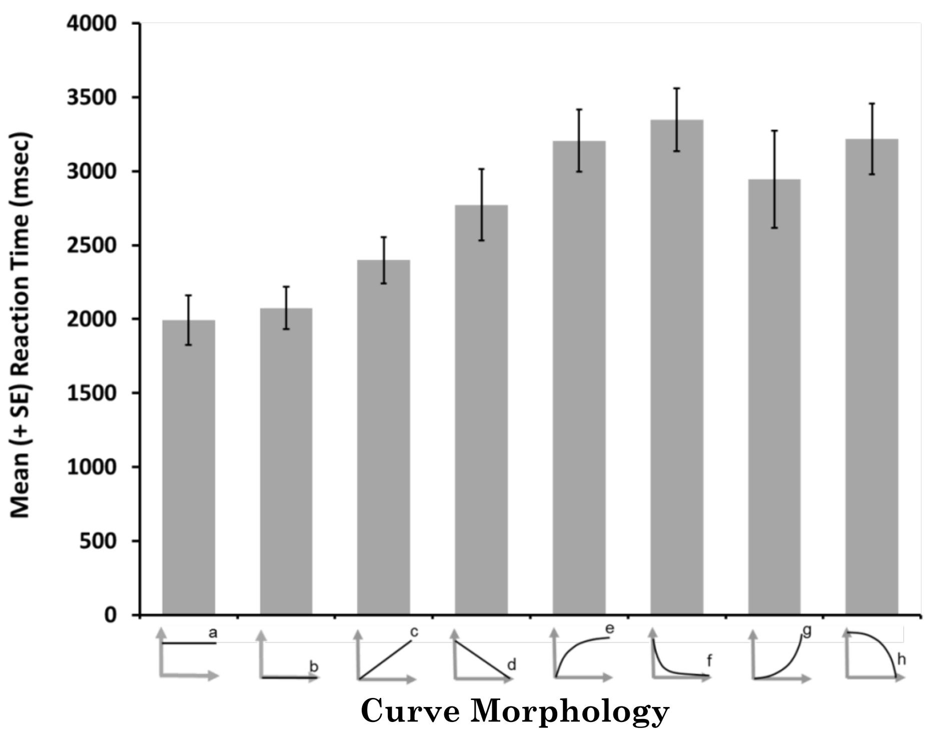


Figure 14. Mean (±SE) response time as a function of curve morphology collapsed across graph type and motion description on correct and incorrect trials.

E. Discussion

The results of Experiment 2 failed to support either of the two hypotheses it was designed to test. Contrary to predictions of the graph difficulty hypothesis, velocity-time graphs were not more difficult to interpret than position-time graphs when making velocity change judgments. In fact, it was just the opposite, with performance on position-time graph motion description judgments reflecting highly variable accuracy. The analyses of response time to make accurate judgments for velocity-time graphs provided evidence for the impact of one key feature of the graphs on time to make motion description judgments: directionality of slope, with zero-slope functions responded to more quickly than positive and negative slope functions. The results suggested faster judgments for linear as opposed to curvilinear functions but the magnitude and consistency of the effect of the curvature feature was low.

1. An Alternative Account: Direct Read-Off

The results of the initial accuracy analysis revealed that subjects did not derive information as required when interpreting position-time graphs. In some conditions, accuracy was relatively high as predicted but in other conditions accuracy was relatively low. Failure to engage in the more elaborate processes that were necessary to be highly accurate could account for patterns of low performance but why then was accuracy so low for a select set of conditions and yet relatively high in others? A third account of processing was offered as a post-hoc alternative to the two original hypotheses tested in Experiment 2: direct read-off using three curve directionality rules for making judgments: (1) horizontal means no velocity change, i.e., object is neither accelerating nor decelerating (2) up means increase (going faster), i.e., object is accelerating over time, and (3) down means decrease (going slower), i.e., object is decelerating. Processing consistent with these rules was also suggested by the analyses of the RT data for accurate judgments made with velocity-time graphs indicating the importance of slope direction. Previous research has shown that when reasoning with line graphs, subjects were susceptible to directional biases such as *up means more* and *down means less* (e.g., Gattis & Holyoak, 1996). Such biases are likely evoked by simple visual rather than cognitive processes. Shah and Freedman (2009) noted that subjects are likely to "rely on bottom-up characteristics of graphs" (p. 571) when they are unfamiliar with information in the graph. This may also be true when the judgments are conceptually difficult as may have been the case for judgments about velocity change, but only with position-time graphs, not in general as proposed by Brasell (1987).

The direct read-off rules generated a set of predictions regarding accuracy that were tested against expectations based on chance responding. Accuracy performance differed from chance in the predicted direction in 15 of 16 cases for position-time graphs and in all 16 cases for the velocity-time graphs, providing evidence that subjects were responding to both types of graphs similarly. Likewise, response time analyses on all judgments (those that were accurate and those that were not) mirrored the pattern of results for velocity-time graphs, highlighting the importance of slope directionality in the curve. The lack of a graph type main effect or any interactions involving graph type for the pooled RT data suggest that subjects are approaching both graphs with a direct read-off strategy. Experiments 3 and 4 examine the direct read-off strategy more directly by comparing two judgment types: general motion and velocity change for each of the two graph types: position-time graphs (Experiment 3) and velocity-time graphs (Experiment 4).

V. EXPERIMENT 3.

DERIVATIONAL STRUCTURE VERSUS DIRECT READ-OFF

As indicated in the overview of the four experiments, Experiments 2 and 3 were originally proposed as tests of predictions accompanying the derivational structure hypothesis. In Experiment 2 such a test was based on comparing the two graph types with respect to the accuracy and latency of velocity change judgments. However, Experiment 2 provided no supporting evidence for the derivational structure hypothesis. The accuracy patterns obtained in Experiment 2 suggested a post-hoc alternative account of processing namely, direct-read-off. Analysis of performance patterns relative to various attributes of the curve suggested that subjects were reading from direction of slope information in the curve to inform their judgments. The use of this information was constrained by three "rules" (horizontal means *not moving*, up means *accelerate*, down means *decelerate*) that yielded a specific set of predictions for accuracy related to velocity change judgments made with both position-time and velocity-time graphs. The predicted patterns of judgment accuracy aligned with the observed patterns in 31 of 32 judgment conditions representing the eight curve morphologies crossed with the two motion descriptions (accelerating, decelerating) and the two graph types.

There are at least two possible explanations for the observed performance patterns. One is that subjects do not engage in the levels of processing required by the derivational structure hypothesis to make accurate judgments regarding velocity change in position-time graphs. Instead, they use a direct read off strategy based on curve features directly available in position time graphs. Such a direct read-off of features is appropriate for making velocity change judgments in velocity-time graphs. However, an alternative explanation of the results obtained in Experiment 2 is that the presence of two graph types, one of which allows for direct read off, caused subjects to adopt this strategy for both graph types. Thus, a question exists about the generalizability of the interpretation of performance in Experiment 2 with respect to use of the direct read off strategy with position-time graphs. In Experiment 3 only one graph type was examined – position-time, removing the issue of switching between graph types.

Experiment 3 allows for a further examination of the applicability of the derivational structure hypothesis within position-time graphs by varying the type of motion judgment – general motion vs velocity change. Velocity change judgments in position-time graphs are predicted to be derived (see analysis, Experiment 2) whereas general motion judgments are predicted to be direct (see analysis, Experiment 1). Experiment 3 also allows for further examination of the direct-read off hypothesis since, as will be presented later in further detail, accurate general motion judgments in position-time graphs can be made using just the first rule – horizontal = no slope --> stationary – for all curve morphologies. As previously discussed, direct read-off as a processing account leads to inaccurate judgments in several cells for velocity change judgments in position-time graphs. This is in contrast to the more elaborate processing that is assumed necessary under the derivational structure hypothesis in order for there to be accurate judgments in all cells for velocity change judgments. In addition, the more elaborate processing required by the derivational structure hypothesis should lead to longer response times for accurate velocity change judgments than general motion judgments because of the greater interpretation demands but the prediction only applies *if* the required information is derived.

In summary, Experiment 3 allowed for a replication of the accuracy patterns found in Experiments 1 and 2 with position-time graphs. By holding graph type constant and varying judgment class it was also possible to examine the relative difficulty and processing speed for General Motion versus Velocity Change judgments in light of predictions from two competing hypotheses: Derivational Structure versus Direct Read-Off.

A. Method

1. Participants

Sixteen University of Illinois at Chicago undergraduates were recruited from the Department of Psychology Subject Pool. Thirteen were female and three were male. The median age was 18.5 years, range = 18 – 20. Most subjects were in their first (n = 10) year of college; four were in their second year and two were in their third year. All participants completed the same questionnaire used in the previous experiment that was designed to elicit information about math and science background as well as graph interpretation skill with respect to height/slope, change-in-height/change-in-slope, variable confusion (see Appendix A). Subjects reported their previous five math courses. The “highest” level of math reported was calculus (n = 6 subjects), pre-calculus (n = 7 subjects), trigonometry (n = 1 subjects), algebra II/geometry (n = 2 subject). The study was limited to subjects who had taken at least one year of physics. However in addition to physics, subjects reported taking at least one course in each of the following areas: Anatomy and Physiology (N = 4 subjects), Biology (N = 14), Chemistry (N = 16 subjects), Earth/Environmental Sciences (N = 3 subjects), and Psychology (N = 0 subjects). Biology courses reported included Introductory, AP, Accelerated, and genetics. Chemistry courses reported included introductory, general, and organic. One student had taken a second semester of Physics. Accuracy across the nine response opportunities ranged from 0.00 to 6.00. The modal frequency of correct responses was 1 and the mean was 2.13 (SD = 1.67) out of a possible nine correct responses. Further implications of this level of performance for interpreting performance on the experimental task are discussed in the General Discussion.

2. Design

A three-factor 2 (Judgment Class) x 8 (Curve Morphology) x 2 (Motion Description) within-subjects design with motion description nested within judgment class was used to examine the effects of three independent variables on verification accuracy and mean response latency for accurate verification judgments. The independent variables were: Curve Morphology [Zero-Slope y > 0 (Curve a), Zero-Slope y = 0 (Curve b), Linear Increasing (Curve c), Linear Decreasing (Curve d), Positive Decreasing (Curve e), Negative Decreasing (Curve f), Positive Increasing (Curve g), and Negative Increasing (Curve h)]; Judgment class (General Motion, Velocity Change); and Motion Description Nested within Judgment Class (Stationary vs Moving within General Motion; Accelerating vs Decelerating within Velocity Change). The dependent variables were accuracy and response time.

3. Materials

The materials consisted of the same curve morphologies, judgment classes, and motion descriptions used in the prior two experiments. Each graph was paired with the four motion descriptions. Two motion descriptions reflected general motion (*The object is stationary, The object is moving*) and two reflected velocity change (*The object's movement is accelerating*, *The object's movement is decelerating*). The crossing of the graph types with each of the motion descriptions resulted in a set of 32 unique description-graph displays.

Each verbal description was simultaneously displayed with each of the eight curve morphologies. Each participant was tested on a total of 144 description-graph pairs. The 144 stimulus pairings were divided into four blocks, two per judgment class. The two general motion blocks were further divided into two 20-trial sets. The two velocity change blocks were divided into two 16-trial sets. Each general motion block represented the two descriptions of general motion (stationary vs. moving) crossed with the eight kinematic curve types. Each velocity change block represents the two velocity change descriptions (accelerating vs. decelerating) crossed with the eight curves. The result was the eight critical trials of interest. Four additional filler trials completed each set of 20 trials in the general motion blocks. The filler trials helped equalize the ratio of same to different judgments.

Two blocks were associated with general motion (80 trials) and two were associated with velocity change (64 trials) yielding a total of 144 trials that required general motion and velocity change interpretations in position-time graphs. The blocks that reflected a particular class of judgment were blocked together and presented in one of two counterbalanced orders (i.e., ABAB or BABA).

4. Procedure

The experimental procedure was nearly identical to that described in the previous two experiments with the main exception being that the emphasis was on judgments made about different classes of motion which was achieved by holding graph type (Position-Time) constant. Therefore, each participant was instructed to verify the accuracy of description-graph pairs that represented the presence (moving) or absence (stationary) of motion and the extent that objects change speed and go faster (acceleration), or change speed and go slower (deceleration). The description-graph stimuli were presented according to the procedure illustrated in Figure 15.

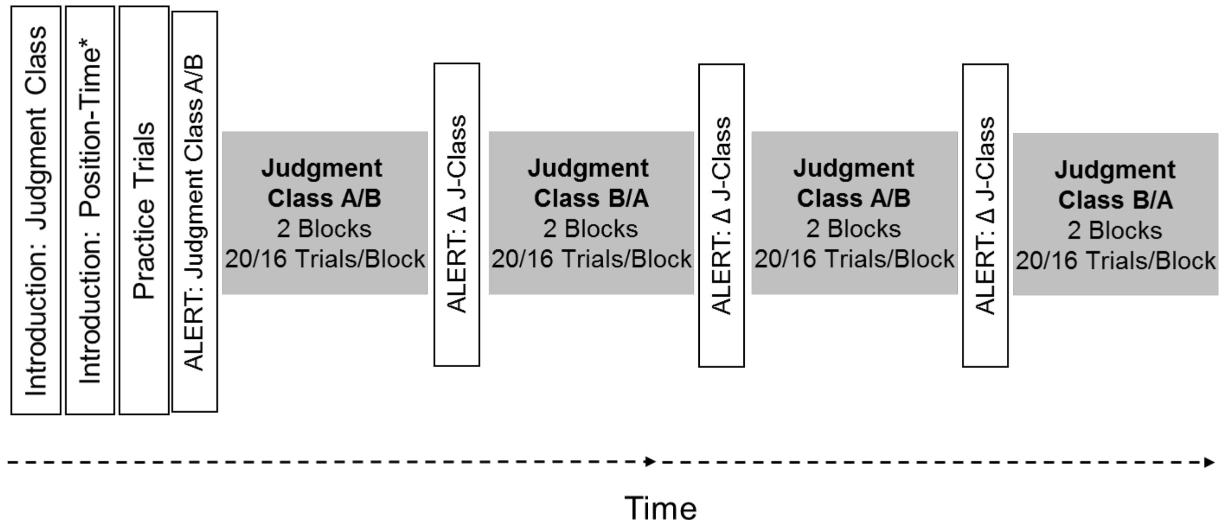


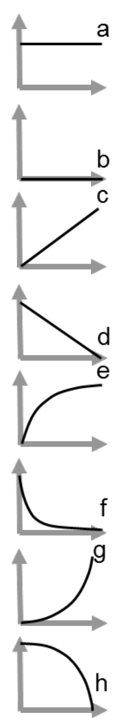
Figure 15. General experimental procedure for Experiment 3 (w/position-time graphs). The class of judgment alternates across blocks in one of two block orders: ABAB or BABA. A display between blocks was designed to alert the subject to the change in judgment class (represented above as "Δ J-Class").

Accurate judgments for the motion descriptions nested within the General Motion (stationary/moving) and Velocity Change (accelerating/decelerating) judgment classes and the eight curve morphologies are presented in Table 9. For general motion judgments, curves **a** and **b** indicate that the object is not changing position so the accurate response is true for stationary and false for moving statements. For general motion judgments, curves **c** – **h** indicate that the object is moving at either constant (**c** and **d**) or varying (**e** – **h**) rates so accurate judgment is false for stationary and true for moving sentences.

As shown in Table 9, there is a different pattern of true and false judgments over the 16 velocity change judgment conditions. Curves **a** and **b** show a stationary object so accurate responses are false to both accelerating and decelerating statements. For graphs **c** and **d**, the object is moving but at a constant rate so again the accurate responses for both velocity change statements are false. Graphs **e** and **f**, show slopes with curvature that reflect a moving object that is changing position at a decreasing rate over time so statements about accelerating objects are false and statements about decelerating objects are true. The opposite is the case for graphs **g** and **h** that have slopes with curvature that reflects a moving object changing position at an increasing rate over time; statements about decelerating objects are false whereas statements about accelerating objects are true.

Table IX
TRUTH TABLE OF CORRECT RESPONSES (T=TRUE; F=FALSE) FOR EACH JUDGMENT CLASS, MOTION DESCRIPTION-NESTED-WITHIN-JUDGMENT-CLASS AND CURVE MORPHOLOGY (A - H).DERIVED INFORMATION BASED ON DERIVATIONAL STRUCTURE HYPOTHESIS = GREY CELLS.

Position-Time



	General Motion		Velocity Change	
	Stationary	Moving	Accelerating	Decelerating
a	T	F	F	F
b	T	F	F	F
c	F	T	F	F
d	F	T	F	F
e	F	T	F	T
f	F	T	F	T
g	F	T	T	F
h	F	T	T	F

B. Results

Judgment accuracy results are presented first, followed by the response time analyses, and tests for the use of direct read-off strategies.

1. Judgment Accuracy

Accuracy of judgment was submitted to a 2 (Judgment Class) x 8 (Curve Morphology) x 2 (Motion Description) repeated measures ANOVA using Motion Description nested within the Judgment Class variable. The mean rate of judgment accuracy in relation to Judgment Class, Curve Morphology, and Motion Description is presented in Figure 16. ANOVA revealed a main effect of Judgment Class, F (1, 15) = 71.84, $p < .001$, $\eta_p^2 = .83$, indicating that General Motion ($M = 3.56$, $SD = .95$) judgments were significantly more accurate than Velocity Change judgments ($M = 2.66$, $SD = 1.65$). The main effects of curve morphology, F (7, 105) = 37.62, $p < .001$, $\eta_p^2 = .72$, and Motion Description-within-Judgment Class, F (2, 30) = 13.68, $p < .001$, $\eta_p^2 = .48$, were also significant.

The effect of motion description nested within judgment class indicates that the contents of the statement differentially affected accuracy within its respective judgment classes. In particular, within the General Motion class of judgment, accuracy regarding statements that an object was moving ($M = 3.71$, $SD = .80$) was significantly greater than accuracy regarding statements that an object was stationary ($M = 3.39$, $SD = 1.06$), F (1, 15) = 7.49, $p < .01$, $\eta_p^2 = .67$. Within the Velocity Change class, judgment accuracy regarding statements that an object was accelerating ($M = 2.64$, $SD = 1.65$) did not differ from accuracy regarding statements

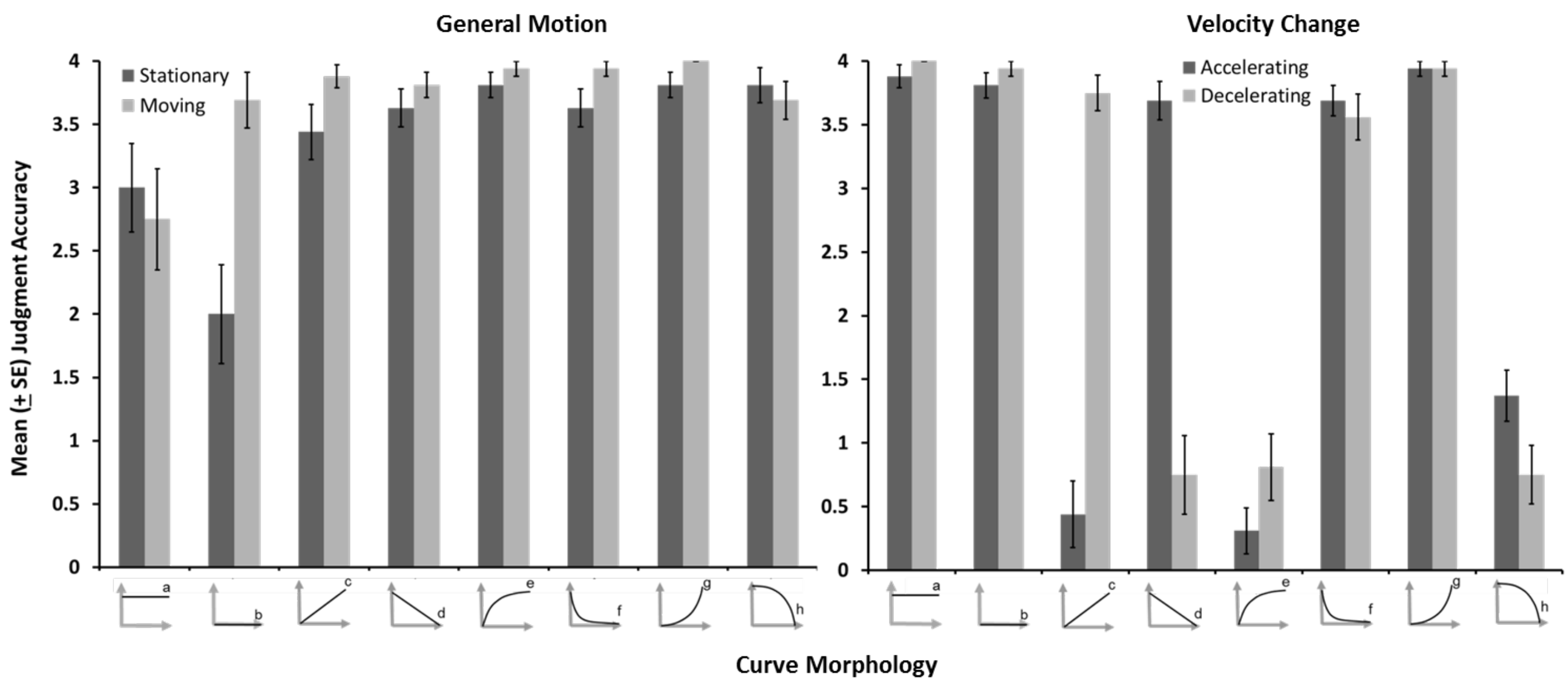


Figure 16. Mean (±SE) verification accuracy in Experiment 3 for each curve as a function of motion description nested within judgment class.

that an object was decelerating (*M* = 2.69, *SD* = 1.65), $F\,(1, 15) = .58$, $p = .82$, $\eta_p^2 = .02$ (but see Figure 11 and the discussion below for a further complication of this conclusion for Velocity Change judgment involving the interaction between curve morphology and motion description statement).

Both main effects of judgment class and curve morphology were qualified by significant two-way interactions of Judgment Class by Curve Morphology, $F\,(7, 105) = 55.70$, $p < .001$, $\eta_p^2 = .79$, and Curve Morphology by Motion Description-within-Judgment Class, $F\,(14, 210) = 30.53$, $p < .001$, $\eta_p^2 = .67$. The interaction of Judgment Class and Curve Morphology suggested that there were reliable differences in the impact that curve morphology had on judgment accuracy in the two judgment classes. The shape of the curve had a much larger impact on judgments that involved velocity change, $F\,(7, 210) = 77.80$, $p < .001$, $\eta_p^2 = .84$, than it did on judgments that involved general motion, $F\,(7, 210) = 7.82$, $p < .01$, $\eta_p^2 = .34$. As was the case in Experiment 1, the effect of curve morphology on judgments about general motion was related to variability in accurate performance for the zero slope curves (**a** and **b**: *M* = 2.86, *SD* = 1.50) and non-zero slope curves (**c** - **h**: *M* = 3.78, *SD* = .37), $F\,(1, 210) = 52.84$, $p < .001$, $\eta_p^2 = .33$. On the other hand, variability in accurate judgment across the curves in the velocity change condition was clearly more complicated and was associated with a higher amount of variation in judgment accuracy in relation to both the shape of the curve and the particular aspect of motion referenced by the statement. To validate this observation, the significant interaction of Curve Morphology and Motion Description-within-Judgment Class was examined further.

The patterns presented in Figure 11 along with the significant interaction of Curve Morphology and Motion Description-within-Judgment Class indicated that the effect of motion description on accuracy was differentially influenced by the shape of the curve and that the nature and magnitude of that effect depended on judgment class. Judgment accuracy associated with general motion varied by motion description across the different curves, F (7, 210) = 6.65, $p < .001$, $\eta_p^2 = .18$, but not to the extent that it did in relation to velocity change, F (7, 210) = 29.81, $p < .001$, $\eta_p^2 = .65$. In the Curve **b** condition within the general motion class, subjects had greater difficulty making correct judgments regarding statements that indicated an object was stationary (M = 2.00, SD = 1.55) versus those that indicated that an object was moving (M = 3.69, SD = .87), F (1, 210) = 29.51, $p < .001$, $\eta_p^2 = .20$. Accuracy was not affected by motion description in the other curve conditions (Fs <1.17, $p > .13$, η_p^2s < .01).

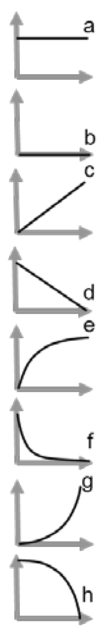
In contrast, velocity change judgments varied significantly as a function of motion description at three of the eight curve conditions and the variation in performance depended on the curve. At curve **c**, accuracy was greater for statements about deceleration (M = 3.75, SD = .58) than it was for statements about acceleration (M = .44, SD = 1.03), F (1, 210) = 113.71, $p < .001$, $\eta_p^2 = .50$. At curve **d**, accuracy was greater for statements about acceleration (M = 3.69, SD = .60) than it was for statements about deceleration (M = .75, SD = 1.24) and the magnitude of the effect was rather substantial, F (1, 210) = 89.42, $p < .001$, $\eta_p^2 = .44$. A third smaller motion description effect was found at curve **h**, F (1, 210) = 4.05, $p < .05$, $\eta_p^2 = .03$. That effect was explained by slightly more accurate judgment for statements that referenced acceleration (M = 1.38, SD = .81) versus statements about deceleration (M = .75, SD = .93). Finally, a contrast revealed a marginal difference in accuracy at curve **e** such that subjects were less accurate for statements about deceleration (M = .31, SD = .70) versus acceleration (M = .81, SD = 1.05) objects, F (1, 210) = 4.69, $p < .05$, $\eta_p^2 = .02$. The differences in judgment accuracy as a function of motion description at each of the other four conditions were not statistically significant, (Fs < .26, ps > .61, η_p^2s < .01).

The accuracy patterns observed in relation to general motion judgments in this experiment replicate patterns in the position-time graph conditions that were obtained in Experiment 1. The patterns of erratic accuracy associated with velocity change judgments replicate the pattern of variability for velocity change judgments with position-time graphs obtained in Experiment 2. The effects and patterns seem consistent with the idea that subjects are engaged in the use of information directly represented in the curve. However, none of the previous analyses of data in the current experiment test for the prediction that accuracy will be significantly *above* or *below* chance for certain conditions based on an hypothesized direct read off strategy. To do so, the procedure used in Experiment 2 to test the observed distribution of correct judgments against chance was applied.

a) Direct Read-Off Hypothesis: Predictions for Accuracy

The direct read-off hypothesis predicts a distinct pattern of accuracy across and within each judgment class. In particular, in conditions in which the information should be accessed directly from the curves (i.e., General Motion), accuracy should be fairly stable and greater than chance for each of the 16 judgment outcomes. However, in conditions in which information should be derived (Velocity Change), accuracy is expected to be more erratic but predictably above and below chance for particular judgment outcomes. Predicted responses with respect to curve morphology and motion description are displayed in Table 10.

Table X
RESPONSES PREDICTED BY USE OF THE THREE DIRECT READ-OFF RULES FOR GENERAL MOTION AND VELOCITY CHANGE JUDGMENTS WITH POSITION-TIME GRAPHS

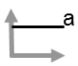
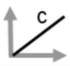
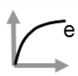


		Position-Time Graphs			
		General Motion		Velocity Change	
	Direct Read-Off Rule	Stationary	Moving	Accelerating	Decelerating
a	1. Horizontal, no change	True	False	False	False
b	1. Horizontal, no change	True	False	False	False
c	2. Up, increase, faster	False	True	True*	False
d	3. Down, decrease, slower	False	True	False	True*
e	2. Up, increase, faster	False	True	True*	False*
f	3. Down, decrease, slower	False	True	False	True
g	2. Up, increase, faster	False	True	True	False
h	3. Down, decrease, slower	False	True	False*	True*

* Judgment accuracy expected to be less than chance.

i. Test of Observed Accuracy against Chance (General Motion). As shown in the columns of X^2 values (see Table 11), the observed patterns of correct responses generated by subjects were consistent with the predictions from the direct read off strategy introduced in Experiment 2. Specifically, the distribution of correct judgments was greater than chance in 15 of the 16 conditions in which it was expected to exceed chance using the rules shown in Table 10. The anomalous case is for judgments involving stationary motion statements for curve **b.** Recall that the pattern of predictions that performance is tested against is the following: 1 subject with 4 correct, 4 with 3 correct, 6 with 2 correct, 4 with 1 correct and 1 with 0 correct. However, the pattern that was observed produced 50% accuracy overall, neither high nor low average accuracy. This resulted from 4 subjects with 4 correct, 2 with 3 correct, 4 with 2 correct, 2 with 1 correct and 4 with 0 correct. In effect, there is a “tri-modal” distribution of accuracy performance on this curve type. These results suggest that there was a mixture of processing strategies being used by the subjects with respect to this one curve when making judgments about whether the depicted object was stationary. All of the other conditions shown in Table 11 reveal differences from chance in the predicted direction.

Table XI
PREDICTED ACCURACY LEVELS AND OBSERVED[A] JUDGMENT ACCURACY RELATIVE TO DIRECT READ-OFF OF SLOPE DIRECTION FOR GENERAL MOTION JUDGMENTS WITH POSITION-TIME GRAPHS

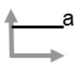
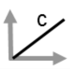
Predicted Chance Distribution														
0 correct = 1 Ss, 1 correct = 4 Ss, 2 correct = 6 Ss, 3 correct = 4 Ss, 4 correct = 1 Ss														
	Observed Distribution (Stationary)							Observed Distribution (Moving)						
Curve	**Read-Off Prediction**	**0**	**1**	**2**	**3**	**4**	$X^2(df = 4)$	**Read-Off Prediction**	**0**	**1**	**2**	**3**	**4**	$X^2(df = 4)$
	High Accuracy	2	0	2	3	9	71.50***	High Accuracy	3	1	1	3	8	59.67***
	High Accuracy	4	2	4	2	4	20.67***	High Accuracy	0	1	1	0	14	180.42***
	High Accuracy	0	1	1	4	10	88.42***	High Accuracy	0	0	0	2	14	181.00***
	High Accuracy	0	0	1	4	11	109.17***	High Accuracy	0	0	0	3	13	155.25***
	High Accuracy	0	0	0	3	13	155.25***	High Accuracy	0	0	0	1	15	209.25***
	High Accuracy	0	0	1	4	11	109.17***	High Accuracy	0	0	0	1	15	209.25***
	High Accuracy	0	0	0	3	13	155.25***	High Accuracy	0	0	0	0	16	240.00***
	High Accuracy	0	0	1	1	14	180.42***	High Accuracy	0	0	1	3	12	130.42***

[a] Frequency (proportion) of correct judgments out of a possible 64. *** $p < .001$.

<u>ii. Test of Observed Accuracy against Chance (Velocity Change).</u> Table 12 shows each curve type (column 1) along with predicted (columns 2 and 5) and observed (columns 3 and 6) accuracy for the velocity change conditions. Observed accuracy reflects the actual number of correct judgments, maximum = 64. The observed distribution of correct judgments was consistent with the predicted distribution in 16 of 16 conditions. Thus, considering the results for both general motion and velocity change judgment conditions, the direct read-off hypothesis

accounted for 31 of 32 (96.88%) of the predicted response patterns with respect to judgment accuracy.

Table XII
PREDICTED ACCURACY LEVELS AND OBSERVED[A] JUDGMENT ACCURACY RELATIVE TO DIRECT READ-OFF OF SLOPE DIRECTION FOR VELOCITY CHANGE JUDGMENTS WITH POSITION-TIME GRAPHS

Predicted Chance Distribution														
0 correct = 1 Ss, 1 correct = 4 Ss, 2 correct = 6 Ss, 3 correct = 4 Ss, 4 correct = 1 Ss														
	Observed Distribution (Accelerating)							Observed Distribution (Decelerating)						
Curve	Read-Off Prediction	0	1	2	3	4	$X^2(df = 4)$	Read-Off Prediction	0	1	2	3	4	$X^2(df = 4)$
 a	High Accuracy	0	0	0	2	14	181.00***	High Accuracy	0	0	0	0	16	240.00***
 b	High Accuracy	0	0	0	3	13	155.25***	High Accuracy	0	0	0	1	15	209.25***
 c	Low Accuracy	12	3	0	0	1	131.25***	High Accuracy	0	0	1	2	13	154.17***
 d	High Accuracy	0	0	1	3	12	130.42***	Low Accuracy	10	3	1	1	1	87.67***
 e	Low Accuracy	13	1	2	0	0	153.92***	Low Accuracy	8	5	1	2	0	55.42***
 f	High Accuracy	0	0	0	5	11	111.25***	High Accuracy	0	0	2	3	11	107.92***
 g	High Accuracy	0	0	0	1	15	209.25***	High Accuracy	0	0	0	1	15	209.25***
 h	Low Accuracy	1	10	3	2	0	12.50*	Low Accuracy	8	5	2	1	0	55.17***

[a] Frequency (proportion) correct judgments out of possible 64. * $p < .05$. *** $p < .001$.

<u>b) Direct Read-Off: Summary of Accuracy Analysis</u>

The results demonstrate that the pattern of accuracy observed in Experiment 2 was not unique to making judgments of velocity change with two different graph types. The same patterns observed in that experiment were found in the present experiment that held graph type constant while judgment class was allowed to vary.

Further, the erratic accuracy pattern related to velocity change reflects another failure to find evidence for the derivational structure hypothesis that only accounted for accuracy data in 10 of 16 (62.50%) velocity change conditions. In contrast, the direct read-hypothesis accounted for accuracy in 15 of 16 (93.75%). The following section examines the direct read-off hypothesis as a process account for graph-based judgments by analyzing response times for accurate as well as pooled judgments.

2. Response Time

As indicated above, performance was generally very accurate across all conditions for General Motion judgments whereas it was highly variable across conditions for Velocity Change judgments, with some cells showing average accuracy below chance. An analysis goal of comparing across Motion Class using all 8 curve morphologies and the 2 motion description statements nested within each Motion Class was therefore rendered impossible since response time data for accurate judgments would be useable from 3 of the 16 subjects tested. If, however, the goal is to use as much subject data as possible to make meaningful RT comparisons for accurate judgments, then an alternative plan that examines RT only within General Motion judgments across the 8 curve morphologies and the two motion description statements allows for use of data from 14 of the 16 subjects tested. The latter plan was chosen to conduct a meaningful RT analysis and maximize the use of correct judgment response times from the subjects providing data in this experiment. RT was reported in milliseconds (msec).

Mean RT as a function of Curve Morphology and Motion Description at the General Motion level of Judgment Class is presented in Figure 17. On average, participants took about 1463.41 msec (SD = 482.42) to correctly verify the relationship between the graphs and the corresponding motion statements. A 8 (Curve Morphology) x 2 (Motion Description) repeated measures ANOVA indicated that the amount of time it took to make accurate judgments did not vary by Curve Morphology, $F\,(7, 91) = .52$, $p = .82$, $\eta_p^2 = .04$, Motion Description, $F\,(1, 13) = .09$, $p = .77$, $\eta_p^2 = .01$, nor the interaction of those variables, $F\,(7, 91) = 1.06$, $p = .40$, $\eta_p^2 = .08$.

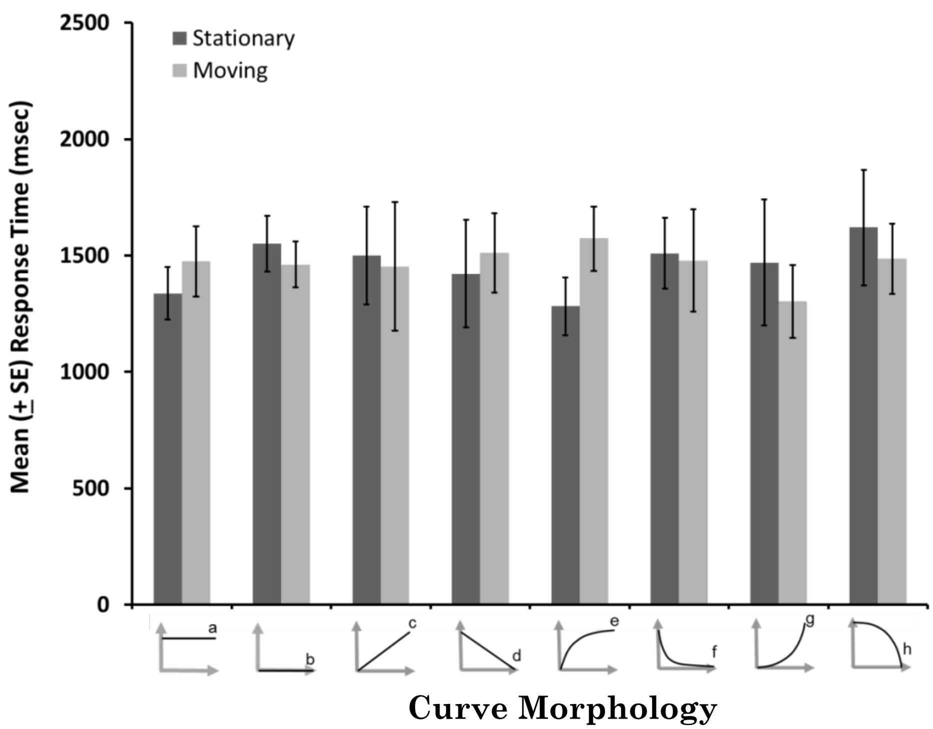


Figure 17. Mean (±SE) response time as a function of curve morphology and motion description at the level of general motion (Judgment Class).

The results suggest that the interpretive situations involved similar processing. Judgment in these conditions was made with either the same strategy across all conditions or else with multiple strategies that are similar in computational efficacy and efficiency. The failure to find a curve morphology effect is consistent with a direct read-out prediction that only a single graph feature, zero vs non-zero slope, is needed to make general motion judgments. The RT results also replicate the findings for general motion judgments observed in Experiment 1.

a) Direct Read-Off: Judgment Response Time (Correct and Incorrect)

Given the judgment accuracy pattern exhibited in this Experiment and its interpretation in terms of a direct read-off of features for both the General Motion and Velocity Change judgments, it was deemed reasonable to look at all RT data as a further test of the hypothesized direct read-off strategy. Thus, both correct and incorrect judgment RTs were pooled for each subject to provide an estimate of overall RT within each condition for purposes of comparing motion class, curve morphology, and motion description statement nested within motion class. The assumption justifying such a collapsing of correct and incorrect judgment RTs is the same as that used in Experiment 2, namely that subjects were responding under the assumption that their judgments were accurate despite the fact that the graph features guiding their judgments led to predictably incorrect responses in certain conditions and predictably correct responses in other conditions.

The direct read off hypothesis predicts that RT will be influenced by information in the curve. Contrasts focused on the effects of curve attributes (i.e., *height*, *directionality*, and *curvature*) should only reveal a direction of slope effect on RT for velocity change judgments because direction corresponds with acceleration and deceleration. Thus, in those conditions, RT will be faster for the average of curves **a** and **b** than the average of curves **c** and **d** and the contrasts for *height* (curve **a** vs **b**) and *curvature* [(**c** vs **e** & **g**), (**d** vs **f** & **h**), and (**e** & **g** vs **f** & **h**)] should not be significant, because these features are not expected to factor into the three-rule direct read-off strategy applied in these tasks. However, judgments related to general motion do not require information about directionality. Rather, they can be made based on breaks in the horizontal plane. Thus, the direct read-off hypothesis predicts an interaction in the current experiment specifically, curve morphology will impact RT related to velocity change but not general motion.

RT for correct and incorrect responses was submitted to a 2 (Judgment Class) x 8 (Curve Morphology) x 2 (Motion Description) repeated measures ANOVA using Motion Description nested within the Judgment Class variable. The mean RT ($\pm$ SE) is presented in Figure 18. The overall mean RT for judgment collapsed across all interpretive situations in the experiment was 1569.05 (SD = 619.06). The results of the ANOVA revealed a main effect of Judgment Class on RT, F (1, 13) = 5.30, p < .05, η_p^2 = .29, indicating that judgments about general motion (stationary or moving objects) (M = 1584.55, SD = 632.85) were significantly faster than judgments about velocity change (accelerating or decelerating objects) (M = 1834.88, SD = 813.00). RT was also impacted by the shape of the curve, F (7, 91) = 2.29, p <.05, η_p^2 = .15, and Motion Description, F (2, 26) = 3.62, p <.05, η_p^2 = .22, but these results were compromised by significant interactions of judgment class by curve

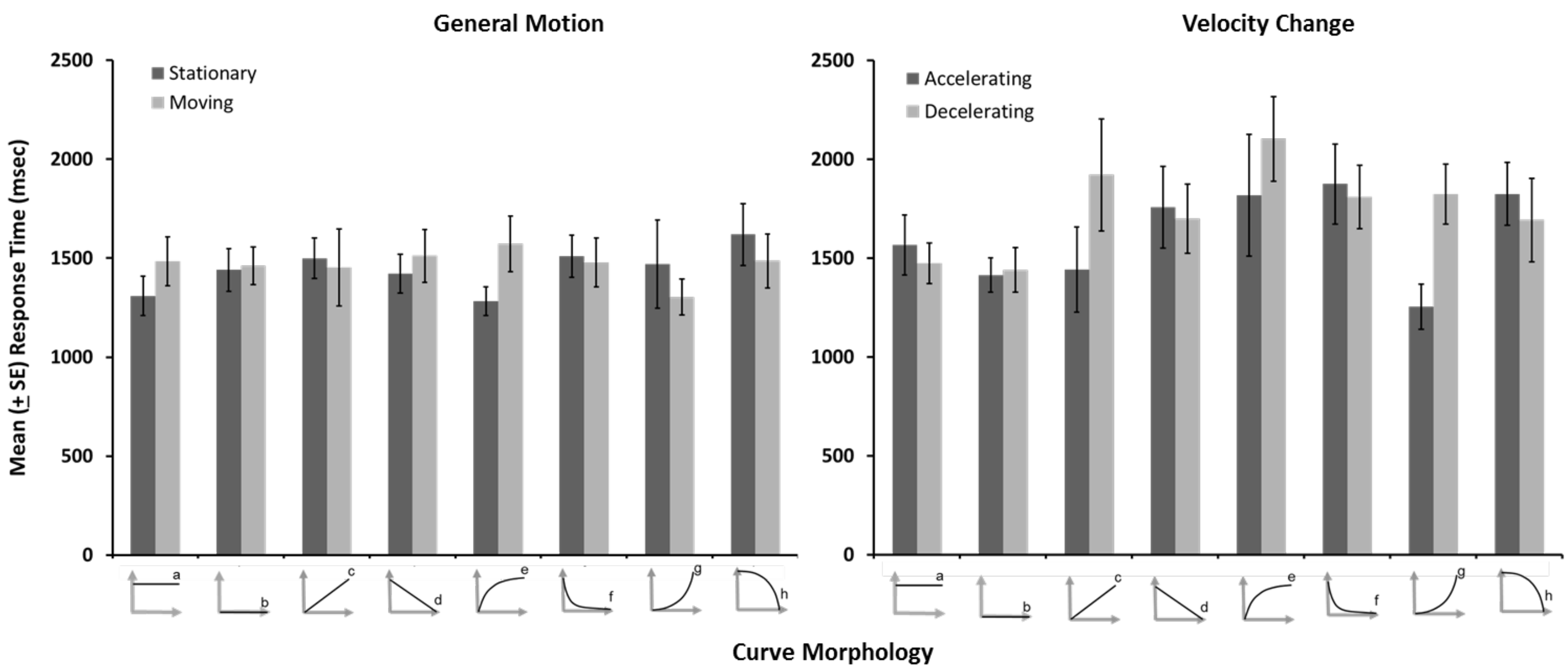


Figure 18. Mean (+SE) response time as a function of Judgment Class, Curve Morphology, and Motion Description (nested-within-Judgment Class) across correct and incorrect responses included in model.

morphology, $F(7, 91) = 2.37$, $p < .05$, $\eta_p^2 = .15$, and curve morphology by motion description, $F(14, 182) = 2.00$, $p < .05$, $\eta_p^2 = .13$.

The Judgment Class by Curve Morphology interaction suggested that the extent that RT was influenced by the shape of the curve depended on the contents of the statement. A pair of simple interaction contrasts examined the effect of Curve Morphology at each level of the Judgment Class variable. As the patterns in Figure 18 suggest, RT associated with judgments for general motion statements did not vary as a function of curve morphology, $F(7, 91) = .51$, $p = .82$, $\eta_p^2 = .03$, whereas RT related to judgments for velocity change statements did, $F(7, 91) = 5.53$, $p < .001$, $\eta_p^2 = .30$.

The effect of curve morphology for velocity change was further examined by several contrasts aimed at elucidating the effect in terms of specific curve features namely, *height* (**a** vs **b**), *slope direction* [(**a** & **b** vs **c** & **d**), (**c** vs **d**)], and *curvature* [(**c** vs **e** & **g**), (**d** vs **f** & **h**), (**e** & **f** vs **g** & **h**)]. The results indicated that the interactive effect of judgment class and curve morphology on RT was mainly explained by variability in RT performance across the curves and that the direction of slope was a major factor, $F(1, 91) = 9.35$, $p < .01$, $\eta_p^2 = .09$; judgments related to curves **a** and **b** ($M = 1473.58$, $SD = 424.32$) were made faster than judgments associated with curves **c** and **d** ($M = 1704.96$, $SD = 830.88$) and whether the curve was oriented upwards (curve **c**) or downwards (curve **d**) had no statistical consequence, $F(1, 91) = .19$, $p = .66$, $\eta_p^2 = .00$. The tests for height and curvature effects on RT revealed that the differences in RT among the curves implicated in those comparisons were not significant, *F*s (1, 91) < .76, *p*s > .39, $\eta_p^2 < .02$, with the exception that RTs related to slopes that reflected increasing rates of change (**g** & **h**: *M* = 1647.13, *SD* = 630.11) were significantly faster than those related to slopes that reflected decreasing rates of change (**e** & **f**: *M* = 1901.04, *SD* = 825.39), $F(1, 91) = 11.26$, $p < .01$, $\eta_p^2 = .11$. Thus, curvature was another factor specifically for the feature that represented increasing and decreasing rates of change.

Since motion description was nested within Judgment Class, the Curve Morphology by Motion Description interaction effect on RT suggested that processing time within each judgment class may have been differentially influenced by the respective motion descriptions. The effect was followed by tests of two 8 (Curve Morphology) x 2 (Motion Description) simple interactions, one at each level of Judgment Class. At the level of general motion, the effects of Motion Description, $F(1, 91) = .19$, $p = .66$, $\eta_p^2 = .00$, Curve Morphology, $F(7, 91) = .47$, $p = .86$, $\eta_p^2 = .00$, and Curve Morphology x Motion Description interaction, $F(7, 91) = .95$, $p = .47$, $\eta_p^2 = .02$, were not significant. At the level of velocity change, RT significantly varied as a function of Motion Description such that judgments for acceleration statements (*M* = 1617.72, *SD* = 727.22) were faster than judgments for deceleration statements (*M* = 1745.63, *SD* = 699.74) but that effect was small, $F(1, 91) = 5.22$, $p < .05$, $\eta_p^2 = .03$. Curve Morphology made a significant impact on RT, $F(7, 91) = 5.05$, $p < .001$, $\eta_p^2 = .16$, and it also interacted with the motion description variable, $F(7, 91) = 3.06$, $p < .01$, $\eta_p^2 = .11$. The interaction helps account for the small motion description effect as RT was only influenced by velocity change statements (accelerating or decelerating) when judgments were made with curves **c,** $F(1, 91) = 10.02$, $p < .001$, $\eta_p^2 = .05$, and **g,** $F(1, 91) = 12.95$, $p < .001$, $\eta_p^2 = .07$. In both cases, judgments for accelerating statements were faster than for decelerating statements.

C. Discussion

Experiment 3 provided additional support for direct read-off as a process underlying graph-based judgment and again failed to find evidence in support of the derivational structure hypothesis. As predicted by the direct read-off hypothesis, accuracy was consistently above chance when the judgment demands were limited to simply extracting information from visual features in the graph and predictably above and below chance when judgment required more elaborate forms of processing such as visualization or inference. In the present experiment, the direct read-off hypothesis predicted 96.88% (31 of 32) of the pattern of correct responses in the data, a full replication of the match between predictions and results observed in Experiment 2.

Further, the results of the RT analyses revealed that the extent to which curve features figure prominently in processing was dependent on the aspects of motion referenced by the verbal statement. In the present experiment the effect of curve morphology on RT varied according to judgment class and motion description. For general motion, RT was consistent across the eight curves whereas for velocity change it varied. One speculation for this lack of variability in RT across the curves associated with general motion judgments is that they only required information about a single feature, namely, the direction of slope. Subjects never had to search for additional information in either the graph or in memory. All judgments in those conditions could be made based on whether the slope was flat ("stationary") or not ("not stationary"). If so, RT would not be expected to vary because subjects had no need to process additional features in the curve, visible or inferred, beyond a mere break in the horizontal plane. Although this conjecture was not examined in the present experiment, the results of Experiments 1 and 3 suggest that with position-time graphs RT for judgments about general motion statements did not depend on the differential use of slope direction info in the curves because there was no difference in RTs related to curves **a** and **b** vs those related to curves **c** and **d** as was the case for judgments made with velocity change statements. Rather, read-off for general motion in position-time graphs was likely driven by a single rule: *flat = no motion*; *non-flat = motion*. Indeed, this single direct read-off rule provides a more parsimonious account of the accuracy patterns observed for this judgment class (see Table 11) especially in light of the corresponding RT patterns for those judgments as corroborated by the RT and accuracy patterns observed in the velocity change conditions.

Consider the RT patterns for velocity change. Variability across the curves in the velocity change condition was clearly more complicated and was associated with a higher degree of variation in judgment accuracy relative to the shape of the curve and the particular aspect of motion referenced by the statement. RTs in those conditions were revealing in the sense that the data and results indicated that interpreting velocity change in position-time graphs was impacted by (a) slope direction (i.e., **a** & **b** versus **c** & **d**) but not the specific direction of slope (i.e., **c** vs **d)** and (b) curvature but only for functions that expressed information about increasing (**e** & **g**) and decreasing (**f** & **h**) rates of change.

One plausible mechanism accounting for the variation in RT judgments for general motion versus velocity change statements is the number of curve attributes that need to be processed. Whereas general motion judgments require processing one slope direction feature (i.e., "flatness"), velocity change demands at least three slope direction features (flat, upwards, downwards) to achieve accuracy in all conditions. However, processing five attributes (flat, upwards, downwards, increasing, decreasing) is necessary for optimal accuracy in selected conditions where information about velocity change depends on an interpretation of the nature of curvature in the slope. As the results of the RT analyses and the means plotted in Figure 18 suggest, performance variability increased positively as the information-processing demands increased from one (general motion and velocity change at curves **a** and **b**) to three (velocity change at curves **c** and **d**) to five (velocity change at curves **e** – **h**) features. However, as the accuracy patterns indicate, although subjects processed curvature as they should when interpreting graphs about velocity change, the low accuracy performance in certain conditions confirms that most subjects did so without extracting much meaning from that information.

In conclusion, the results of Experiment 3 failed to provide evidence for the derivational structure hypothesis. There was however, further support for the direct read-off hypothesis indicating that much of the graph-based judgments taking place in the experiment were driven by the number of curve attributes that need to be processed to achieve reliably accurate judgment. Still, it is possible that the performance observed in Experiments 2 and 3 for position-time graphs was related to differences in accuracy rates between the two judgment classes. Experiment 4, in addition to providing an examination of curve morphology in velocity-time graphs, also examines processing under conditions in which accuracy is expected to be relatively high for both general motion and velocity change judgments. Thus, Experiment 4 provides an investigation of the direct read-off of information in the various types of curves unconfounded with possible accuracy differences.

VI. EXPERIMENT 4.

JUDGMENT CLASS EFFECTS WITH VELOCITY-TIME GRAPHS: ASPECTS OF MOTION AND DIRECT READ-OFF

Experiment 4 was originally designed to test the effect of judgment class for velocity -time graphs, a situation in which all judgments were predicted to be made directly, except for the case of curve **a** for general motion judgments (Experiment 1), where a priori it was ambiguous as to whether or not there was a need for additional processing. The accuracy results of Experiment 1 suggested that, indeed, curve **a** behaved differently than the other 30 cells in the design in that subjects were less accurate on curve **a** in velocity-time graphs than all of the other curves in velocity-time graphs, and performance on curve **a** for velocity–time graphs was lower than for curve **a** in position-time graphs. In Experiment 2 where the judgment for each motion description was predicted to be direct for all curves in velocity time graphs, performance on curve **a** was highly accurate and did not differ from the other curve types in velocity-time graphs; nor did accuracy on curve **a** differ as a function of graph type. Thus, for accelerating and decelerating motion statements, curve **a** accuracy was consistent with a direct processing approach.

One explanation for the lower performance on curve **a** in Experiment 1 is that subjects were dealing with two graph types and there was some switching cost and/or uncertainty. However, there appeared to be no switch cost in Experiment 2 and it seems rather unlikely that switch cost would affect *only* general motion judgments. Nevertheless, one original purpose of Experiment 4 was to vary judgment class while holding graph type constant and obtain a second set of observations for general motion judgments in velocity-time graphs. Another purpose was the comparison of linear and curvilinear morphologies for both motion judgment classes.

An additional purpose of Experiment 4 emerged from the results of Experiment 2 regarding the direct-read off hypothesis. As noted in discussing Experiment 3, although the RT results produced support for the direct-read off hypothesis in processing position-time graphs there were differential accuracy rates across the two judgment classes (general motion and velocity change). In Experiment 4, accuracy was predicted to be greater than chance in all conditions (except curve **a**) allowing a more sensitive test of the curve morphology effect in terms of features that need to be taken into account to make general motion as compared to velocity change judgments.

A. Method

1. Participants

Sixteen University of Illinois at Chicago undergraduates were recruited from the Department of Psychology Subject Pool. Five were female and 11 were male. The median age was 19.0 years, range = 18 – 24. Most subjects were in their first (n = 6) and third (n = 4) years of college; three, two, and one subjects were in their second, fourth, and fifth years, respectively. All participants completed the same questionnaire used in the previous experiments to assess math and science background as well as graph interpretation skill (see Appendix A). Subjects reported their previous five math courses. The "highest" level of math reported was calculus (n = 12 subjects), pre-calculus (n = 1 subjects), trigonometry (n = 1 subjects), algebra II/geometry (n = 1 subject), and discrete math (n = 1 subject). The study was limited to subjects who had taken at least one year of physics. However in addition to physics, subjects reported taking at least one course in each of the following areas: Anatomy and Physiology (N = 3 subjects), Biology (N = 16 subjects), Chemistry (N = 13 subjects), Earth or Environmental Sciences (N = 1 subject), and Anthropology (N = 2 subjects). Biology included Introductory, AP, Accelerated, cellular, microbiology, ecology, genetics, homeostasis, and zoology. Chemistry courses reported included introductory, general, and organic. One student had taken a second semester of Physics. Accuracy across the nine response opportunities ranged from 0.00 to 7.00. Unlike the previous experiments, the distribution of correct responses was multimodal and the Mean frequency of correct response was 3.00 (*SD* = 1.90). The implications of this level of performance for interpreting performance on the experimental task are discussed later in the General Discussion.

2. Design

A three-factor 2 (Judgment Class) x 8 (Curve Morphology) x 2 (Motion Description) within-subjects design with motion description nested within judgment class was used to examine the effects of three independent variables on verification accuracy and mean response latency for accurate verification judgments. The independent variables were: **Curve Morphology** [Zero-Slope y > 0 (Curve a), Zero-

Slope y = 0 (Curve b), Linear Increasing (Curve c), Linear Decreasing (Curve d), Positive Decreasing (Curve e), Negative Decreasing (Curve f), Positive Increasing (Curve g), and Negative Increasing (Curve h)]; **Judgment class** (General Motion, Velocity Change); and **Motion Description Nested within Judgment Class** (Stationary versus Moving within General Motion; Accelerating versus Decelerating within Velocity Change). The dependent variables were accuracy and response time.

3. Materials

The materials consisted of visual displays with the same curve morphologies, judgment classes, and motion descriptions used in Experiment 3 except that the curves were plotted on velocity-time graphs.

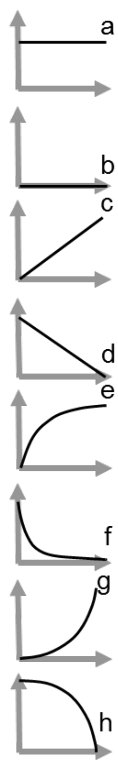
4. Procedure

The procedure was exactly the same as that used in Experiment 3 except that Experiment 4 required judgments in the context of velocity-time graphs as opposed to position-time graphs (see Figure 10).

Accurate judgments for the motion descriptions nested within the General Motion (stationary, moving) and Velocity Change (accelerating, decelerating) judgment classes and the eight curve morphologies are presented in Table 13. For general motion judgments, the slope is non-zero so the object is moving for all curves except **a** and **b**. For curve **b**, the slope is zero and so is the intercept so the object is stationary. For curve **a**, the slope is zero but the intercept is positive so the object is moving but at a constant rate. These two cells for curve **a** are depicted as possibly requiring more than direct processing of the information available in the graph. For velocity-change judgments, curves **a** and **b** have zero slope so motion represented by them is neither accelerating nor decelerating. Curves **c** – **h** show either increasing (**c**, **e**, **g**) or decreasing (**d**, **f**, **h**) slope and the True/False patterns are in accord with accelerating and decelerating responses. As noted in Experiment 1 for General Motion and in Experiment 2 for Velocity Change statements, among velocity-time graphs the only cells in which the processing demand is predicted to be more than direct are curves **a** and **b**. However, based on the results of Experiments 1 and 2 only curve **a** is still considered to be ambiguous with respect to the need for additional processing.

Table XIII
TRUTH TABLE OF CORRECT RESPONSES (T=TRUE; F=FALSE) FOR EACH JUDGMENT CLASS, MOTION DESCRIPTION-NESTED-WITHIN-JUDGMENT-CLASS AND CURVE MORPHOLOGY (A – H) FOR VELOCITY-TIME GRAPHS. DERIVED INFORMATION BASED ON DERIVATIONAL STRUCTURE HYPOTHESIS = GREY CELLS.

Velocity-Time Graphs



	General Motion		Velocity Change	
	Stationary	Moving	Accelerating	Decelerating
a	F(?)	T(?)	F	F
b	T	F	F	F
c	F	T	T	F
d	F	T	F	T
e	F	T	T	F
f	F	T	F	T
g	F	T	T	F
h	F	T	F	T

B. Results

The results are presented in three parts. The first set of results examines whether the accuracy effects for general motion found in Experiment 1 and those for velocity change found in Experiment 2 are replicated when subjects make judgments with only the velocity-time graphs. The second set of results then looks at the fit of a Direct Read-Off explanation for the accuracy results. The third set of results then considers response time effects.

1. Judgment Accuracy

Judgment accuracy was submitted to a 2 (Judgment Class) x 8 (Curve Morphology) x 2 (Motion Description) repeated measures ANOVA using Motion Description nested within the Judgment Class variable. The mean rates of accuracy in relation to Judgment Class, Curve Morphology, and Motion Description are shown in Figure 19. There were main effects of Curve Morphology, F (7, 105) = 2.76, p <.05, η_p^2 = .16, and Motion Description, F (2, 30) = 6.01, p < .05, η_p^2 = .29, but no main effect of judgment class, F (1, 15) = .05, p = .82, η_p^2 = .00): accuracy for general motion judgment (M = 3.56, SD = .90) did not differ from accuracy for velocity change judgment (M = 3.54, SD = .92). Within the General Motion judgment class subjects were slightly more accurate in judgment when the statement was that the object was moving (M = 3.68, SD = .80) as contrasted with the statement that the object was stationary (M = 3.45, SD = .98), F (1, 30) = 8.06, p < .01, η_p^2 = .21, and more accurate within the Velocity Change judgment class when the statement was that the object was accelerating (M = 3.62, SD = .82) than when the statement was that the object was decelerating (M = 3.45, SD = 1.00) objects, F (1, 30) = 3.95, p = .06, η_p^2 = .12, although the latter exceeds the .05 level. There were also significant interactions of Judgment class and Curve Morphology, F (7, 105) = 8.42, p < .001, η_p^2 = .36, and between Curve Morphology and Motion

Description, nested within Judgment class, F (14, 210) = 2.02, $p < .05$, $\eta_p^2 = .12$. Each of these interactions was further analyzed to ascertain the locus of effect.

a) Judgment Class x Curve Morphology

Simple effects tests were used to examine the Curve Morphology effect within each Judgment Class. To preview the results of these tests, the curve morphology effect was significant within each judgment class although the patterns of significance within each differed.

For General Motion judgments, there was a significant curve morphology effect, F (7, 210) = 7.51, $p < .001$, $\eta_p^2 = .33$. Contrasts among the curves focused on the same features that were examined in the RT analyses of Experiments 2 and 3: height (**a** vs **b**), direction of slope [(**a** & **b** vs **c** & **d**), (c vs **d**)], and curvature [(**c** vs **e** & **g**), (**d** vs **f** & **h**), (**e** & **f** vs **g** & **h**)]. Specifically, judgments were significantly less accurate for curve **a** (M = 2.53, SD = 1.61) than curve **b** (M = 3.34, SD = 1.07), F (1, 105) = 12.37, $p < .001$, $\eta_p^2 = .11$. Judgments were also less accurate for linear curves with a zero slope (**a** & **b**: M = 2.94, SD = 1.3) than linear curves with a non-zero slope (**c** & **d**: M = 3.86, SD = .40), F (1, 105) = 31.84, $p < .001$, $\eta_p^2 = .23$. Both of those effects were related to hypothesized differences in interpretation demands such that curve **a** is presumed to require additional processing relative to the other curves when the goal is to interpret general motion. A task analysis based on the derivational structure of representations reveals that a general motion judgment in the curve **a** condition is indirect (i.e., *derived*). The significantly less accurate performance on **curve a** replicates the same pattern of performance observed in

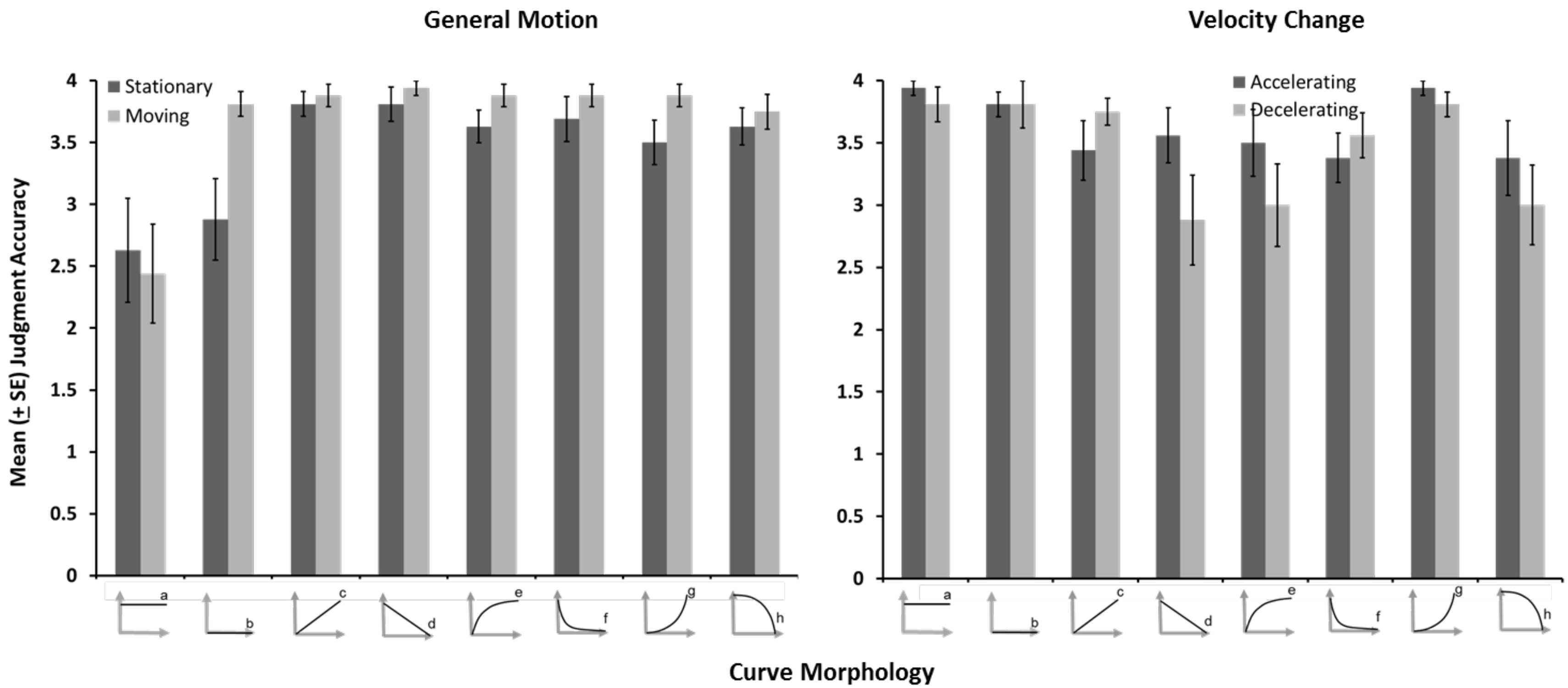


Figure 19. Mean (±SE) verification accuracy in Experiment 4 for each curve as a function of motion description nested within judgment class (velocity-time graphs).

Experiment 1 and implies that subjects did not engage in the extra processing needed to be highly accurate on curve **a.** Further, the specific direction of the curve, i.e., whether the slope was positive (**c**: $M = 3.84$, $SD = .37$) or negative (**d**: $M = 3.88$, $SD = .42$), had no significant effect on judgment accuracy, $F(1, 105) = .01$, $p = .89$, $\eta_p^2 = .00$.

Accuracy was unrelated to curvature. Accuracy for positive trending linear functions (**c**: $M = 3.84$, $SD = .37$) was the same as it was for negative nonlinear functions (**e** & **g**: $M = 3.72$, $SD = .52$), $F(1, 105) = .39$, $p = .53$, $\eta_p^2 = .00$. Likewise, accuracy for negative trending linear functions (**d**: $M = 3.88$, $SD = .42$) did not differ from downwards trending nonlinear functions (**f** & **h**: $M = 3.73$, $SD = .52$), $F(1, 105) = .49$, $p = .48$, $\eta_p^2 = .00$. Accuracy did not depend on whether the curve reflected an increasing (**g** & **h**: $M = 3.69$, $SD = .59$) or decreasing (**e** & **f**: $M = 3.77$, $SD = .50$) rate of change, $F(1, 105) = .49$, $p = .48$, $\eta_p^2 = .00$.

For Velocity-Change judgments, there was also a significant curve morphology effect, $F(7, 105) = 3.31$, $p < .01$, $\eta_p^2 = .18$. Contrasts among the curves examined the same set of features as were examined for judgments within the General Motion class. As the patterns of means shown in Figure 19 suggest, the variation in accuracy across the curves was different for velocity change judgments than it was for general motion judgments. Unlike the case of general motion, there was no significant height effect: curve **a** ($M = 3.88$, $SD = .42$) did not differ from accuracy related to curve **b** ($M = 3.81$, $SD = .59$) $F(1, 105) = .07$, $p = .79$, $\eta_p^2 = .00$. Also unlike General Motion, direction of slope had a significant effect on accuracy which was greater for linear curves with a zero slope (**a** & **b**: $M = 3.84$, $SD = .51$) than those with a positive and negative slope (**c** & **d**: $M = 3.41$, $SD = 1.00$), F (1, 105) = 7.17, $p < .01$, $\eta_p^2 = .06$. However, whether slope direction was positive (**c**: $M = 3.59$, $SD = .76$) or negative (**d**: $M = 3.22$, $SD = 1.24$) had no consequence on judgment accuracy, F (1, 105) = 2.63, $p = .11$, $\eta_p^2 = .02$. It is worth noting that the direction of slope effect (i.e., **a** & **b** > **c** & **d**) related to velocity change judgments ran converse to the slope effect associated with general motion judgments but the effect was relatively small.

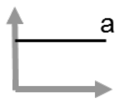
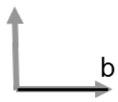
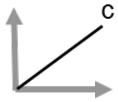
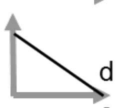
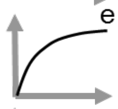
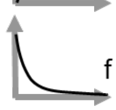
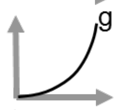
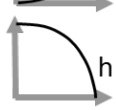
There was no curvature effect associated with the velocity change judgments. Accuracy for positive-trending linear functions (**c**: $M = 3.59$, $SD = .76$) was the same as accuracy for positive-trending nonlinear functions (**e** & **g**: $M = 3.56$, $SD = .78$), F (1, 105) = .02, $p = .88$, $\eta_p^2 = .00$. Similarly, accuracy for negative-trending linear functions (**d**: $M = 3.22$, $SD = 1.24$) did not differ from negative-trending nonlinear functions (**f** & **h**: $M = 3.33$, $SD = 1.00$), F (1, 105) = .30, $p = .59$, $\eta_p^2 = .00$. Moreover, whether the curve reflected increasing (**g** & **h**: $M = 3.53$, $SD = .78$) or decreasing (**e** & **f**: $M = 3.36$, $SD = .99$) rates of change, accuracy was unaffected, F (1, 105) = .30, $p = .59$, $\eta_p^2 = .00$.

In summary, the statistical tests of the curve morphology effect on accuracy within each judgment class replicate the finding from Experiment 1 that general motion judgments for **curve a** are significantly less accurate than judgments on all other curves including **b** (as reported earlier) and **c** – **h** ($M = 3.77$, $SD = .49$), F (1, 105) = 49.35, $p < .05$, $\eta_p^2 = .32$. None of the other curves negatively impacted judgment accuracy for general motion judgments. The findings within velocity change judgments indicated that performance on curve **a** was no different than on curve **b** although both curves with zero slope were more accurate than linear graphs with positive and negative direction of slope. Overall, where direct processing was expected, accuracy was relatively high. Relatively lower accuracy was observed only for the curve type that was hypothesized to require more than direct processing, curve **a** within general motion judgments.

b) Curve Morphology x Motion Description

The interaction of curve morphology and motion description was examined further with separate analyses for general motion and velocity change statements in which mean judgment accuracy for the two statements was compared on each curve. The means (*SD*), the ANOVA *F*-statistic, and the effect size for each single-*df* contrast are provided in Table 14. As the table shows, within General Motion, accuracy was greater for statements about moving objects than stationary objects for judgments made with curve **b**. Accuracy relative to stationary and moving statements did not vary for judgments made with the other curves. Within Velocity Change, accuracy was significantly greater for statements about accelerating motion than it was for decelerating motion particularly in relation to judgments made with curves **d** and **e**. Thus, the interaction was due to different curves having significant differences between the two motion descriptions for general motion as compared to velocity change.

Table XIV
MEAN (SD) ACCURACY, ANOVA *F*-STATISTIC, AND EFFECT SIZE (PARTIAL ETA SQUARE) FOR JUDGMENTS MADE ABOUT MOTION STATEMENTS WITHIN THE TWO JUDGMENT CLASSES

	General Motion				Velocity Change			
	Stat	Mov	F (1, 210)	η_p^2	Acc	Dec	F (1, 210)	η_p^2
 a	2.63 (1.67)	2.44 (1.59	.64	.00	3.94 (.25)	3.81 (.54)	.28	.00
 b	2.88 (1.31)	3.81 (.40)	15.94***	.07	3.81 (.40)	3.81 (.75)	.60	.00
 c	3.81 (.40)	3.88 (.34)	.07	.00	3.44 (.96)	3.75 (.45)	1.77	.01
 d	3.81 (.54)	3.94 (.25)	.28	.00	3.56 (.89)	2.88 (1.45)	8.57**	.04
 e	3.63 (.50)	3.88 (.34)	1.13	.01	3.50 (1.10)	3.00 (1.32)	4.54*	.02
 f	3.69 (.70)	3.88 (.34)	.64	.00	3.38 (.81)	3.56 (.73)	.64	.00
 g	3.50 (.73)	3.88 (.34)	2.55	.01	3.94 (.25)	3.81 (.40)	.28	.00
 h	3.63 (.62)	3.75 (.58)	.28	.00	3.38 (1.20)	3.19 (1.26)	2.55	.01

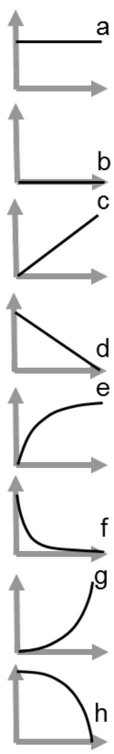
* $p < .05$. ** $p < .01$. *** $p < .001$.

Judgment accuracy was also examined with respect to predictions of the Direct Read-Off Hypothesis to examine whether the patterns of accuracy could be predicted by assuming that subjects were using the same three curve directionality rules first discussed in Experiment 2 and that were also evaluated in Experiment 3 where they were shown to accounts for almost all of the accuracy data in that experiment.

c) Direct Read-Off Hypothesis: Predictions for Accuracy

A task analysis determined that, with the exception of judgments for general motion with curve **a**, highly accurate judgment about both general motion and velocity change can be achieved by simply processing information directly from the curve, particularly direction of slope (zero, positive, negative). Therefore, if subjects engage in direct read-off then judgment accuracy should be greater-than-chance for 30 of the 32 interpretive conditions in the experiment. The two remaining conditions correspond to the two motion description statements at curve **a** in the general motion trials. The results of Experiment 1 and this experiment suggested that subjects process information in that curve differently than they process information in the other curves. Thus, direct read read-out of information in curve **a** for general motion was expected to generate accuracy below chance. As Table 15 shows, although information can be accessed directly from the curve for both judgment classes (General Motion or Velocity Change), the predicted responses for each judgment are expected to vary as a function of the curve, the judgment class, and the motion descriptions nested within each judgment class.

Table XV
RESPONSES PREDICTED BY USE OF THE THREE DIRECT READ-OFF RULES FOR GENERAL MOTION AND VELOCITY CHANGE JUDGMENTS WITH VELOCITY-TIME GRAPHS



		Velocity-Time Graphs			
		General Motion		Velocity Change	
	Direct Read-Off Rule	Stationary	Moving	Accelerating	Decelerating
a	1. Horizontal, no change	True*	False*	False	False
b	1. Horizontal, no change	True	False	False	False
c	2. Up, increase, faster	False	True	True	False
d	3. Down, decrease, slower	False	True	False	True
e	2. Up, increase, faster	False	True	True	False
f	3. Down, decrease, slower	False	True	False	True
g	2. Up, increase, faster	False	True	True	False
h	3. Down, decrease, slower	False	True	False	True

* Direct read-off makes the wrong prediction therefore, accuracy should be less than chance.

<u>i. Direct Read-Off Accuracy: General Motion from Velocity-Time Graphs.</u> As shown in the columns of X^2 values (see Table 16), the observed patterns of correct responses generated by subjects were mostly consistent with predictions from the direct read-off rules introduced in Experiment 2. The distribution of correct judgments was greater than chance in 14 of the 14 conditions in which it was

expected to be greater than chance, curves **c – h**. However, the distribution of correct responses for the two curve **a** judgments did not behave according to prediction. Recall that the chance pattern of predictions that performance is tested against is the following: 1 subject with 4 correct, 4 with 3 correct, 6 with 2 correct, 4 with 1 correct and 1 with 0 correct. However, the performance patterns across

Table XVI
PREDICTED AND OBSERVED[A] JUDGMENT ACCURACY RELATIVE TO DIRECT READ-OFF OF SLOPE DIRECTION FOR GENERAL MOTION JUDGMENTS WITH VELOCITY-TIME GRAPHS

Predicted Chance Distribution														
0 correct = 1 Ss, 1 correct = 4 Ss, 2 correct = 6 Ss, 3 correct = 4 Ss, 4 correct = 1 Ss														
	Observed Distribution (Stationary)							Observed Distribution (Moving)						
Curve	Read-Off Prediction	0	1	2	3	4	$X^2(df = 4)$	Read-Off Prediction	0	1	2	3	4	$X^2(df = 4)$
a	High Accuracy	4	0	1	4	7	53.17***	High Accuracy	3	2	2	3	6	32.92***
b	High Accuracy	1	2	2	4	7	39.67***	High Accuracy	0	0	0	3	13	155.25***
c	High Accuracy	0	0	0	3	13	155.25***	High Accuracy	0	0	0	2	14	181.00***
d	High Accuracy	0	0	1	1	14	180.42***	High Accuracy	0	0	0	1	15	209.25***
e	High Accuracy	0	0	0	6	10	93.00***	High Accuracy	0	0	0	2	14	181.00***
f	High Accuracy	0	0	2	1	13	153.92***	High Accuracy	0	0	0	2	14	181.00***
g	High Accuracy	0	0	2	4	10	88.67***	High Accuracy	0	0	0	2	14	181.00***
h	High Accuracy	0	0	1	4	11	109.17***	High Accuracy	0	0	1	2	13	154.17***

[a] Frequency (proportion) of correct judgments out of a possible 64. *** $p < .001$.

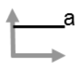
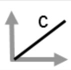
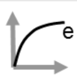
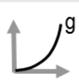
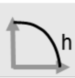
subjects for stationary statements at curve **a** reflect a bimodal distribution with the peaks at either largely correct or largely incorrect decisions. That is 11 subjects got

3 or 4 correct and 5 subjects got 0, 1, or 2 correct. For "moving" statements the distribution was again bimodal with peaks at either largely correct (9 subjects with 3 or 4 correct) or incorrect (7 subjects with 0, 1, or 2 correct). Discussion of this finding appears below.

ii. Direct Read-Off Accuracy: Velocity Change from Velocity-Time Graphs. Table 17 shows predicted and observed judgment accuracy for the velocity change conditions relative to each curve morphology and motion aspect in the statement (accelerating or decelerating). The maximum number of correct responses reflected by observed accuracy is 64. The observed distribution of correct judgments was consistent with the predicted distribution in 16 of 16 conditions. Thus, across both judgment classes, the direct read-off hypothesis accounted for 30 of the 32 (93.75%) predicted response patterns across conditions.

The results largely support the direct read-off hypothesis. The relatively high frequency of correct responses related to statements about stationary (42.00) and moving (39.00) objects for curve **a** was somewhat surprising since it was predicted that the frequency of correct responses for those judgments would be relatively low if subjects used one of the three slope/directionality rules. One explanation for these results is that a larger proportion of the sample than expected engaged in the in-depth processing required to be highly accurate. However, there is an alternative direct read-off strategy that distinguishes horizontal curves according to *height* of the plotted function: *horizontal = 0, stationary* and *horizontal > 0, moving.* With this more differentiated direct read-off rule, judgments of

Table XVII
PREDICTED AND OBSERVED[A] JUDGMENT ACCURACY RELATIVE TO DIRECT READ-OFF OF SLOPE DIRECTION FOR VELOCITY CHANGE JUDGMENTS WITH VELOCITY-TIME GRAPHS

Predicted Chance Distribution														
0 correct = 1 Ss, 1 correct = 4 Ss, 2 correct = 6 Ss, 3 correct = 4 Ss, 4 correct = 1 Ss														
	Observed Distribution (Stationary)							Observed Distribution (Moving)						
Curve	Read-Off Prediction	0	1	2	3	4	$X^2(df = 4)$	Read-Off Prediction	0	1	2	3	4	$X^2(df = 4)$
 a	High Accuracy	0	0	0	1	15	209.25***	High Accuracy	0	0	1	1	14	180.42***
 b	High Accuracy	0	0	0	3	13	155.25***	High Accuracy	0	0	0	1	15	209.25***
 c	High Accuracy	0	1	2	2	11	106.92***	High Accuracy	0	0	0	4	12	132.00***
 d	High Accuracy	0	1	1	2	12	129.42***	High Accuracy	1	3	2	1	9	69.17***
 e	High Accuracy	1	0	1	2	12	130.17***	High Accuracy	2	0	1	6	7	46.17***
 f	High Accuracy	0	0	3	4	9	70.50***	High Accuracy	0	0	2	3	11	107.92***
 g	High Accuracy	0	0	0	1	15	209.25***	High Accuracy	0	0	0	3	13	155.25***
 h	High Accuracy	1	1	0	3	11	108.50***	High Accuracy	1	1	3	3	8	53.00***

[a] Frequency (proportion) correct judgments out of possible 64. *** $p < .001$.

stationary for curve **a** would be predicted to be False, an accurate response, and judgments of moving would be predicted to be True, also an accurate response. If the height rule is used to interpret statements that reference stationary objects with curve **b**, the predicted response would be True and correct because the y-intercept equals zero and therefore, the motion of the object is stationary. But if the height rule is used to interpret statements that the object is moving the correct response would be False because the y-intercept equals zero. If some of the subjects are using this *height* read-off strategy or some of the subjects are using the strategy for some trials and the *direction of slope* read-off strategies for others, it would explain the pattern of correct response distributions at curves **a** and **b**. Further, these differences in direct strategy use may manifest themselves as RT differences between judgments with curve **a** versus curve **b** and between judgments with curve **a** versus curves **c** – **h**.

2. Response Time

As indicated above, performance was generally very accurate across all conditions with the exception of graph **a** for General Motion judgments. If the analysis goal is to compare across Judgment Class using all 8 curve morphologies and the 2 motion description statements nested within each Judgment Class then data would be useable from only 11 of the 16 subjects tested. If, however, the goal is to use as much subject data as possible, then an alternative plan that compares across Judgment Class and 7 of the 8 curve morphologies (curves **b** – **h**) and the 2 motion descriptions statements nested within each Motion Class allows for use of data from 14 of the 16 subjects tested. The latter plan was chosen to maximize the use of correct judgment response times from the subjects providing data in this experiment.

Response time was submitted to a 2 (Judgment Class) x 7 (Curve Morphology) x 2 (Motion Description) repeated measures ANOVA using Motion Description nested within the Judgment Class variable. The mean RTs are plotted in Figure 20. The average amount of time to make an accurate judgment was

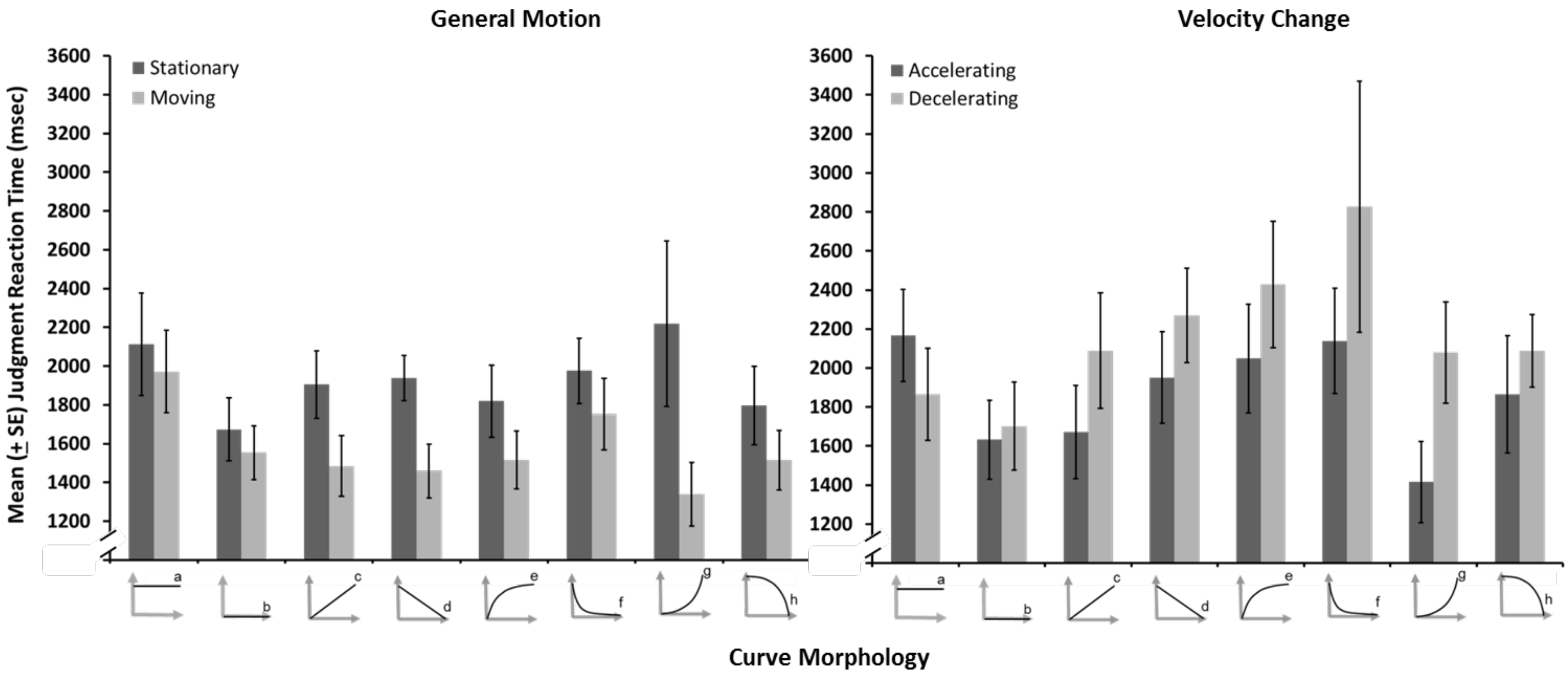


Figure 20. Mean (±SE) response time for correct judgments across the eight curve morphologies by Judgment Class (General Motion, Velocity Change) and Motion Description nested within judgment class (General Motion: Stationary, Moving; Velocity Change: Accelerating, Decelerating) in Experiment 4.

1862.61msec (*SD* = 970.33). Judgments were faster for statements about general motion (*M* = 1710.60, *SD* = 741.14) than velocity change (*M* = 2014.61, *SD* = 1136.74), but this difference was not statistically significant, $F(1, 13) = 2.84$, $p = .12$, $\eta_p^2 = .18$. RT was significantly impacted by Curve Morphology, $F(6, 78) = 3.60$, $p < .01$, $\eta_p^2 = .22$, and Motion Description, $F(2, 26) = 27.69$, $p < .001$, $\eta_p^2 = .68$. Neither the Judgment Class by Curve Morphology, $F(6, 78) = 1.24$, $p = .30$, $\eta_p^2 = .09$, nor the Curve Morphology by Motion Description, $F(12, 156) = .70$, $p = .75$, $\eta_p^2 = .05$, interactions were significant.

The Motion Description main effect revealed a pattern that was similar to that observed in the accuracy data. Within the General Motion judgment class, RTs for statements that the object is moving (*M* = 1511.30, *SD* = 588.56) were significantly faster than RTs for statements that the object is stationary (*M* = 1942.22, *SD* = 863.00), $F(1, 13) = 19.70$, $p < .001$, $\eta_p^2 = .43$, a finding that was consistent with the accuracy data. Within the Velocity Change judgment class, RTs for statements that the object is accelerating (*M* = 1848.29, *SD* = 963.20) were significantly faster than those that the object is decelerating (*M* = 2296.78, *SD* = 1325.05), $F(1, 13) = 21.34$, $p < .001$, $\eta_p^2 = .45$.

The means and SDs for each curve collapsed over judgment class and motion description are shown in Table 18. The curve morphology effect was examined in terms of specific curve features that were required to make accurate judgments. The contrast of curve **b** versus curves **c** and **d** revealed a difference in the effect of those curves on RT indicating that judgment was marginally influenced by slope, *F*

(1, 78) = 3.49, p = .06, η_p^2 = .04. Tests for the effects of other curve features revealed that neither the specific direction of the slope (**c** vs **d**) nor curvature related to positive (**c** versus **e** & **g**) or negative (**d** vs **f** & **h**) functions made significant impacts on interpretation, Fs < .85, ps > .36, η_p^2s < .02. However, curvature linked to increasing (**e** & **f**) versus decreasing (**g** & **h**) rates of change did, F (1, 78) = 9.29, p < .01, η_p^2 = .11, indicating that that these attributes of curvature in the function were processed differently. In particular, judgments associated with curvature that reflected an increasing rate of change was processed faster than curvature that reflected a decreasing rate of change.

Table XVIII
MEAN (SD) RT FOR EACH CURVE COLLAPSED ACROSS JUDGMENT CLASS AND MOTION DESCRIPTION

a	b	c	d	e	f	g	h
1979.67 (842.88)	1640.68 (684.89)	1787.08 (844.88)	1897.14 (761.25)	1921.99 (930.22)	2173.75 (1408.35)	1763.70 (1102.42)	1816.44 (818.04)

a) Direct Read-Off: Testing for a Height Effect in Processing

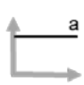
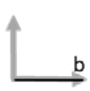
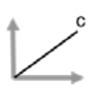
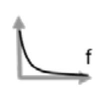
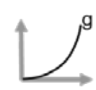
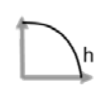
As discussed earlier, the accuracy analysis indicated that accuracy related to judgments made with curve **a** was not below chance as expected had judgment been based on the use of direction of slope information in the curve (i.e., zero, positive, negative) which would have corresponded with the three direct read-off strategies: *flat means no motion*, *upwards means going faster*, *downward means going slower*. An alternative direct read-off strategy based on the use of *height* information was proposed for judgments at curve **a**. Because judgment accuracy with that curve was not as low as expected a decision was made to reanalyze the RT data for all responses regardless of accuracy with all eight curve morphologies for the 14 subjects with the aim of the analysis being to investigate the possibility that judgment was influenced by direct read-off of height information in the graph in addition to slope.

Response time was submitted to a 2 (Judgment Class) x 8 (Curve Morphology) x 2 (Motion Description) repeated measures ANOVA using Motion Description nested within the Judgment Class variable. The pattern of effects was nearly identical to the prior analysis of only correct judgment RTs. The average amount of time to make a judgment was 1883.44msec (SD = 959.38). Judgments were faster for statements about general motion (M = 1752.09, SD = 766.07) than velocity change (M = 2014.80, SD = 1106.08), but again, this difference was not statistically significant, $F(1, 13) = 2.50$, $p = .14$, $\eta_p^2 = .16$. RT was significantly impacted by Curve Morphology, $F(7, 91) = 3.64$, $p < .01$, $\eta_p^2 = .22$, and Motion Description, $F(2, 26) = 23.63$, $p < .001$, $\eta_p^2 = .65$, but not by the interactions of Judgment Class and Curve Morphology, $F(7, 91) = 1.42$, $p = .21$, $\eta_p^2 = .10$, or Curve Morphology and Motion Description, $F(14, 182) = 1.00$, $p = .45$, $\eta_p^2 = .07$.

The patterns underlying the motion description effect were the same as the earlier RT analysis. Within General Motion, RTs for movement statements (M = 1574.17, SD = 852.69) were faster than RTs for stationary statements (M = 1930.01, SD = 622.95), $F(1, 13) = 27.05$, $p < .001$, $\eta_p^2 = .51$. Within Velocity Change, RTs were faster for acceleration statements (M = 1861.03, SD = 933.13) than deceleration statements (M = 2168.56, SD = 1240.73), F (1, 13) = 20.20, p < .001, η_p^2 = .44.

The means and SDs for all responses regardless of accuracy across the eight curves are presented in Table 19. The results of tests for curve feature effects indicated that RT was affected by height as evidenced by the significant contrast between curves **a** and **b**, F (1, 91) = 9.74, p < .01, η_p^2 = .10. The results of tests for slope effects [(**a** & **b** vs **c** & **d**), (**c** vs **d**)] indicated that neither of those factors were significant in processing general motion and velocity change concepts with velocity-time graphs, Fs < .89, ps > .34, η_p^2s = .00. Finally, rather than process curvature in nonlinear functions on the basis of whether that feature directs the curve to go up (positive) or down (negative) [Fs < .72, ps > .50, η_p^2s = .00], the results suggest that subjects were indeed sensitive to curvature but only in terms of processing increasing (**g** & **h**) versus decreasing (**e** & **h**) rates of change and those features were processed discriminately, F (1, 91) = 9.61, p < .01, η_p^2 = .10.

Table XIX
MEAN (SD) RT FOR ALL JUDGMENTS AS A FUNCTION CURVE MORPHOLOGY

							
2029.21 (873.03)	1640.44 (678.64)	1787.08 (844.88)	1904.13 (756.11)	1952.77 (950.08)	2173.75 (1408.35)	1763.70 (1102.42)	1816.44 (818.04)

C. Discussion

Experiment 4 provided additional support for the direct read-off hypothesis and introduced evidence that the use of direct read-off strategies is nuanced by the aspect of motion being interpreted. Although all judgments in the experiment demanded extraction of meaning from visual information in the graph, the results revealed that the particular kinematics concept referenced by the motion description (i.e., stationary versus moving or accelerating versus decelerating) had a significant influence on the amount of time that it took to make a judgment. Since, within a judgment class, judgments were made on the same graphs, these differences suggest that categorically related knowledge can vary by the degree that it is either (a) accessed in memory through graph-induced computation and inference, or (b) cued directly by curve features that trigger associative spatial-conceptual correspondences.

The patterns of correct responses across the cells in the two judgment classes replicated the patterns observed in Experiments 1 (General Motion) and 2 (Velocity Change) with velocity-time graphs. The *a priori* predictions that accuracy would be relatively high in 30 of 32 cells and relatively low in two of 32 cells accounted for 93.75% of the observed pattern of correct responses in the data. The unaccounted 6.25% was due to accuracy being observed at *greater*-than-chance levels for general motion judgments with curve **a** which ran counter to the expectations that those judgments would be *less* than chance if subjects used slope information. The observed accuracy patterns suggested that subjects may have used an alternative read-out strategy based on the height of the curve in terms of the y-intercept. The RT results confirmed this speculation; judgment was significantly faster when the y-intercept was greater than zero (curve **a**) than when it was equal to zero (curve **b**). Consequently, if height information in the graph is factored into the prediction model, then the direct read-off hypothesis explained 100.00% of performance across the 32 cells.

The amount of time used to judge the correspondence between the graph and the sentence was influenced by the value of the motion description statements, specifically stationary vs moving or accelerating vs decelerating. The current experiment cannot differentiate between two potential explanations of these effects. One is related to linguistic characteristics of the terms, wherein one of the terms is responded to more quickly than the other, consistent with a linguistic marking construct. This has been observed in other sentence-picture verification research in that *above* is responded to faster than *below* irrespective of truth or falsity of the judgment (Clark & Chase, 1974; Carpenter & Just, 1975). The other explanation is related to actual properties of motion.

VII. GENERAL DISCUSSION

This dissertation examined several accounts of graph interpretation difficulty each of which is discussed below relative to evidence in support of particular assumptions and hypotheses that could be derived from the literature discussed in the introduction to this thesis.

A. Graph Type Not a Factor in Graph-Based Judgment

The study started with a test of the general hypothesis that velocity-time graphs were more difficult to interpret than position-time graphs. One account of this hypothesis was that velocity is conceptually more difficult than position (i.e., "distance") and that this difficulty should extend to the corresponding graph types. A second account was that people believe it is acceptable to switch the y-axis label from say, "velocity" to "position", which leads them to process velocity-time graphs as if they are position-time graphs and thus, generating incorrect responses. The results of Experiments 1 and 2 did not yield supporting evidence for either account of general graph difficulty. In Experiment 1, neither accuracy nor RT depended on Graph Type and in Experiment 2 accuracy was greater for judgments with velocity-time than position-time graphs; the opposite effect predicted by the hypothesis. If velocity is more conceptually difficult than position as Brasell (1987) argues, these results indicate that it is independent of the y-axis label. Further, while the results of Experiment 2 do not completely rule out the possibility that subjects switched y-axis labels to generate judgments, it is highly unlikely because doing so would have required a switch from the "easy" (position-time) to the "difficult" graph type (velocity-time). Thus, the results established that Graph Type (i.e., the y-axis label) was not a significant factor in graph-based judgment, at least not by the accounts considered in this study.

B. No Evidence for Derivational Structure Hypothesis

Another possible account of graph-based judgment was examined in Experiments 2 and 3 based on the derivational structure of representations (Palmer, 1978). The derivational structure hypothesis (DSH) posits that information can be graphically represented such that it is accessible either directly through visual features in the graph (e.g., Ratwani, Trafton, & Boehm-Davis, 2008) or through more elaborate retrieval-based processes such as visualization (Trickettt & Trafton, 2006), externalization (Cox, 1999), or inference (Larkin & Simon, 1987). Regardless of whether information about motion is gathered from *direct* or *derived* processes, the DSH does not predict differences in accuracy but it does predict faster judgment when the information is accessed directly than when it is derived through indirect processes but this presumes that the representations in the graphs are accessed according to the demands stipulated by their respective derivational structures.

Experiments 2 and 3 were designed to allow for direct contrast of accuracy and RT relative to the two derivational structures. According to rational task analyses based on the derivational structure of representations, in Experiment 2, velocity change could be interpreted directly from information in the curve when judgment was made with velocity-time graphs but it had to be interpreted through more elaborate processes when judgment involved position-time graphs. However, in Experiment 3, all judgments were made with position-time graphs and judgments made for general motion statements (stationary, moving) demanded direct processing whereas judgments made for velocity change (accelerating, decelerating) demanded elaborate processing. Although the DSH predicted greater-than-chance accuracy for all judgments, it was only supported for conditions in which the derivational structure was hypothesized as *direct*. Accuracy among conditions that demanded more than direct processing was erratic but varied systematically above *and* below chance. The erratic accuracy performance in the *derived* conditions made it impossible to test the DSH and thus, the experiments provided no support for the DSH.

C. Direct Read-Off Strategies in Graph-Based Judgment

A third hypothesis emerged from the unexpectedly erratic pattern of accuracy associated with velocity change judgments with position-time graphs in Experiment 2. A task analysis based on the derivational structure of representations in those conditions revealed that velocity change judgments with position-time graphs required in-depth processing in order to achieve greater-than-chance accuracy across all conditions. But the pattern of results suggested that the scope of information being evaluated for judgment was limited to direct read-off of features in the curve which, consequently, generated the distinct erratic pattern of accuracy observed among the conditions in which the derivational structure of representations was *derived*. It was hypothesized that regardless of derivational structure, judgments were generated primarily by indiscriminant read-off of information in the curve even within conditions that required processing beyond information that was directly available for visual-based processing. Models of expected patterns of judgment accuracy if subjects used direct read-off strategies were proposed, tested, and confirmed by the results of chi-square analyses in Experiments 2 – 4 which indicated that the predicted distributions of correct responses for each judgment "fit" the observed patterns of correct responses in the data. Thus, these experiments yielded strong evidence that judgments were generated from direct read-off strategies.

1. Specific Curve Features

The tests of direct read-off yielded support for the hypothesis that subjects used information in the curve to make their judgments but these tests could not specify what aspects of the curve were important for the judgment. According to previous research (e.g., Beichner, 1994; Clement, 1985; Leinhardt et al., 1990; McDermott et al., 1987), students struggle to discriminate cases in which it is appropriate to interpret height from cases when it is appropriate to interpret slope. By comparing RTs between certain curves it was possible to localize these effects in three features: *height* (Experiment 4), *direction of slope* (Experiments 2 & 3), and *curvature* (Experiments 2 – 4).

Planned contrasts in Experiment 4 revealed *height* to be a significant factor when velocity-time graphs were used to make judgments about general motion and velocity change. One plausible interpretation for the use of the height feature is its empirical basis, e.g., “the object is off the ground” (i.e., the x-axis at y-intercept = 0) therefore, the object is not stationary it is “in motion”.

A pair of contrasts in Experiments 2 and 3 found evidence that the *direction of slope* was an important curve feature underlying graph-based judgment about kinematics functions. Different accuracy and RT patterns were observed between conditions in which the slope of the function was zero (curves **a** & **b**) versus when it was positive and negative (curves **c** & **d**). Further, whether the slope was positive or negative (curve **c** vs curve **d**) had no effect on the processes underlying judgment. The patterns supported the direct read-off hypothesis which posits that judgment in those experiments – especially about velocity change – was primarily based on the use of three feature-concept associations: (1) “flat” means no velocity change, i.e., object is neither accelerating nor decelerating (2) “up” means increase (going faster), i.e., object is accelerating over time, and (3) “down” means decrease (going slower), i.e., object is decelerating. When the curve was oriented upwards or downwards, the *direction of the slope* respectively cued increases and decreases in speed which always generated correct judgments with velocity-time graphs but only sometimes with position-time graphs. When judgments were focused on general motion, the use of these rules was simpler as accurate judgments could be made around the “flatness” in the curve; i.e., visually detecting a break in the horizontal plane, especially for statements about moving objects.

In addition to *height* and *direction of slope* effects, Experiments 2 – 4 provided evidence that judgment was influenced by *curvature* in the slope. This is not the same as saying that nonlinear functions are more difficult to interpret than linear functions; i.e., *linearity*. Indeed, two contrasts [(**c** vs **e** & **g**) and (**d** vs **f** & **h**)] performed across the four experiments repeatedly showed linearity was a nonfactor in judgment. Rather, according to the RT analyses, judgment was particularly sensitive to information in the curve that reflected increasing and decreasing rates of change. This result is consistent with that of Best, Smith, and Stubbs (1997) who identified curvature as a key feature among a small (n = 5) sample of two faculty and three graduate students who were observed interpreting linear and nonlinear trends. The major difference between the studies being that subjects in their sample had more graph-interpretation skill than the subjects in the present study. In fact, the pattern of accuracy results in coordination with the RT results indicate that although the subjects in this study processed curvature information, they lacked the knowledge – conceptual or procedural – to assign meaning to those features.

2. Spatial-Conceptual Relations

The aforementioned feature effects and related direct read-off rules are consistent with the "natural" spatial – conceptual correspondences characterized by Gattis (2002, 2003, 2004; Gattis & Holyoak, 1996) and discussed to great depth by Tversky (2011). The impact of these correspondences on reasoning with external representations such as visualizations and graphs is well-demonstrated and they are powerful enough to override domain knowledge. Thus, these results do not indicate that subjects were in any way "confused" about what feature to process as concluded by Clement (1985) and Beichner (1994). Rather, judgment appeared to be rationally guided by coherent patterns of strongly reinforced spatial-conceptual associations either between the y-intercept and the aspect of motion referenced by the statement, or else by the direction of the curve (i.e., zero, positive, or negative) and the *type* of curvature (i.e., one reflecting increasing or decreasing rates of change).

In a series of experiments, Gattis and Holyoak (1996) had science and non-science majors make one of two types of rate-judgments when the correspondence between the spatial configuration of a graph's axes and the concepts of altitude and rate of change either matched or were in conflict. In one task, subjects saw a graph with a single solid line accompanied by a verbal description of a data set that reflected a slow change in temperature relative to altitude and they were instructed to draw a line on the graph that represented the data set. In the second task a dotted line was presented with the solid line used in the other task and the subjects were asked to indicate whether the dotted line reflected a faster or slower rate of change in temperature than the solid line. For each condition, half of the subjects saw a graph in which altitude was on the y-axis whereas the other half were presented with a graph in which it was located on the x-axis. When altitude was located on the y-axis, its spatial-conceptual correspondence was preserved (i.e., *up* = *up*) but the correspondence for slope (i.e., *steeper means faster*) was conflicted. The properties were reversed when altitude was placed on the x-axis. Accuracy was greater when the higher-order correspondence (i.e., slope or "rate of change") was preserved because, according to the researchers, the natural correspondence, *steeper means faster*, was visually transparent. To interpret those findings in light of the present research, domain knowledge was not necessary so long as the information to be interpreted could be accessed directly from information in the curve.

The present study shows that spatial-conceptual correspondences can function as powerful cues that bias graph-based judgment. It builds on the earlier work of Gattis and Holyoak (1996) as well as top-down bottom-up graph interpretation models (e.g., Shah & Freedman, 2009; Trickettt & Trafton, 2006) and two-system theories of judgment and decision-making (Morewedge & Kahneman, 2010; Stanovich & West, 2000) in the sense that it specifies when those biases will occur (i.e., when the mind has formed very simple and effective associations to match the demands of the environment) and how they are likely to impact judgment (fast and effective when the correspondences match or fast and ineffective when they are in conflict).

D. Future Research

One goal of the present work was to account for graph interpretation in terms of the derivational structure of representations (Palmer, 1978). However, these experiments provide overwhelming evidence that subjects responded on the basis of what they saw in the graphs with little further processing of that information. This led to erratic but predictable patterns of judgment accuracy in conditions in which the interpretation demands required more elaborate processing. This made it impossible to directly test the DSR-based hypothesis and thus, there was no evidence to confirm or disconfirm the hypothesis. It is particularly striking that this pattern was observed given that the subjects in this study had all completed physics, in addition to other science and mathematics courses. However, this performance was also reflected in their accuracy on the graph interpretation assessment, suggesting that although subjects may have had the conceptual knowledge of the concepts, that knowledge is distinct from knowledge of how to use graphs.

Indeed, Shah and Freedman (2009) were recently able to distinguish content knowledge from knowledge of how to use graphs and found that the two forms of knowledge have different implications for interpretation processes. Whereas prior knowledge of content influenced understanding of the interpretation goals (i.e., "I know that I am looking for the representation of acceleration and I know what acceleration means"), knowledge of graph use would have supported the use of inferences that were specific to a particular graph type and/or function (e.g., "I could read off the value on the y-axis relative to a point on the curve" or "I could look at changes in the slope tangent to the curve"). What that work does not show – and what this work suggests – is that in addition to knowing how to draw inferences from a particular graph one probably needs schema to activate graph-specific inferences in order to determine *when* those inferences are appropriate (e.g., "given that this is a position-time graph, I need to evaluate what's happening with the slope tangent to the curve"). Uesaka and Manalo (2006) refer to this as, "abstract conditional knowledge", which was empirically demonstrated by Novick, Hurley, and Francis (1999) and is useful for choosing task-appropriate representations.

The subjects in this study all reported solid math and science backgrounds, including at least one course in physics which was expected to provide the content knowledge that would help them interpret the graphs more deeply when necessary. However, as Shah and Freedman (2009) suggest, interpreting graphs effectively also requires knowledge of how to use different graph types. The present experiment screened all subjects primarily for knowledge of how to use position-time graphs which were thought to be the least difficult of the three graph types that are central to reasoning about one-dimensional kinematics. Performance on those items was poor in all four experiments (see Appendix B). Much of the observed performance could be accounted for in terms of a powerful set of rules that associate everyday spatial orientations (e.g., straight, curved, up, down, high, low) to states of magnitude (e.g., constant, changing, increasing, decreasing, more, less). This knowledge was evoked when subjects were presented with graphs of linear and nonlinear functions oriented upwards, downwards and horizontally and associated with various states of objects in motion, specifically stationary, moving, accelerating, and decelerating. These associations appear to be so strong that, given a particular context such as interpreting graphs, a subject might read a statement about a state of motion – say, *accelerating* – and "automatically" activate an image of a line that slopes upward. Or conversely, the sight of a line sloping upward might activate the "object's speed is going faster."

Equipped with weak knowledge of how to use the graphs, subjects readily used the direct read-off rules with relatively high success under certain conditions but lower success in others. Further, the use of these associations was not limited to the relatively “simple” judgment tasks employed in the experimental trials. As demonstrated in Appendix B, the analysis of performance on the more complex graph interpretation skills items of the background questionnaire (Appendix A) suggested that direct read-off could account for much of the response selection patterns elicited by subjects in that assessment. Future studies should aim to better understand this deep-seated knowledge and how it may inhibit learning, explain performance on high-stakes assessments, and interact with other forms of knowledge at various stages of graph interpretation. A major question to be addressed is how other forms of knowledge may constrain the automatic activation of the direct read-off strategies that seemed to drive judgments in this study. One promising approach is to promote meaningful reasoning about the relationship between visual features in the graph, *the influence of such features on decisions*, and the goals of interpretation.

Recently, Fendley & Narayanan (2012) demonstrated that it is possible to reduce the impulse to immediately respond to visual information through the use of a decision support system designed to enhance cognitive decision-making capabilities and thus prevent the interpreter from falling victim to biased judgment. A similar system for controlling the impulse to react to directional read-off cues in graphs could be designed and validated. A good place to start is with a system that promotes comparisons among other *judgments* including those that reflect tendencies to rely on a repository of read-off "rules" for this domain. Similar efforts in the areas of computational estimation (Star & Rittle-Johnson, 2009) and solving equations (Rittle-Johnson, Star, & Durkin, 2009) has proven successful but to date no work has evaluated the effectiveness of such an intervention as a strategy for promoting reasoning from graphical representations.

VIII. REFERENCES

Ali, N., & Peebles, D. (2013). The effect of gestalt laws of perceptual organization on the comprehension of three-variable bar and line graphs. *Human Factors: The Journal of the Human Factors and Ergonomics Society*, 55 (1), 183 – 203.

Araujo, I. S., Veit, E. A., & Moreira, M. A. (2008). Physics students' performance using computational modeling activities to improve kinematics graphs interpretation. *Computers & Education*, 50 (4), 1128 – 1140.

Bauer, M. I., & Johnson-Laird, P. N. (1993). How diagrams can improve reasoning. *Psychological Science*, 4, 372 – 378.

Beichner, R. J. (1994). Testing student interpretation of kinematic graphs. *American Journal of Physics*, 62, 750 – 762.

Beichner, R., Bernold, L., Burniston, E., Dail, P., Felder, R., Gastineau, J., Gjertsen, M., & Risley, J. (1999). Case study of the physics component of an integrated curriculum. *American Journal of Physics*, 67(S1), S16 – S24.

Bell, A., & Janvier, C. (1981). The interpretation of graphs representing situations. *For the Learning of Mathematics*, 2 (1), 34 – 42.

Brasell, H. (1987). The effect of real-time laboratory graphing on learning graphic representations of distance and velocity. *Journal of Research in Science Teaching*, 24 (4), 385 – 395.

Browne, M. E. (1999). Schaum's outline of theory and problems of physics for engineering and science. New York: McGraw-Hill.

Cain, L. S. (2006). Graphical analysis for physics: Introduction. In D. Gende (Ed.), *Advanced Placement Program AP Physics Professional Development Materials (Special Focus: Graphical Analysis)*, pp. 4 – 8. New York: The College Board.

Cheng, P. C.-H., Lowe, R. K., & Scaife, M. (2001). Cognitive science approached to understanding diagrammatic representations. *Artificial Intelligence Review*, 15, 79 – 94.

Clark, H. H., & Chase, W. G. (1972). On the process of comparing sentences against pictures. *Cognitive Psychology*, 3 (3), 472 – 517.

Clark, H. H., & Chase, W. G. (1974). Perceptual coding strategies in the formation and verification of descriptions. *Memory and Cognition*, 2 (1A), 101 – 111.

Clement, J., (1985). Misconceptions in graphing. Proceedings of the Ninth Conference of the International Group for the Psychology of Mathematics Education, pp. 1 -7. Noordwijkerhout: The Netherlands.

Cohen, J. (1988). Statistical power analysis for the behavioral sciences (2nd Edition). Hillsdale, NJ: Erlbaum

Cook, M. B., & Smallman, H. S. (2008). Human factors of the confirmation bias in intelligence analysis: Decision support from graphical evidence landscapes. *Human Factors: The Journal of the Human Factors and Ergonomics Society*, 50, 745 – 754.

Cox, R. (1999). Representation construction, externalised cognition and individual differences. *Learning and Instruction*, 9, 343 – 363.

Ericsson, K. A., & Simon, H. A. (1993). Protocol analysis; Verbal reports as data (revisededition). Cambridge, MA: Bradford books/MIT Press.

Fendley,M., & Narayanan, S. (2012). Decision aiding to overcome biased in object identification. *Advances in Human-Computer Interaction*, 12, 1 – 7.

Forster, P. A. (2004). Graphing in physics: Processes and sources of error in tertiary entrance examinations in Western Australia. *Research in Science Education*, 34, 239 – 265.

Fritz, C. O., Morris, P. E., & Richler, J. J. (2012). Effect size estimates: Current use, calculations, and interpretation. *Journal of Experimental Psychology: General*, 141 (1), 2 – 18.

Gattis, M. (2002). Structure mapping in spatial reasoning. *Cognitive Development*, 17, 1157 – 1183.

Gattis, M. (2003). How similarity shapes diagrams. *Lecture Notes in Artificial Intelligence*, 2685, 249 – 262.

Gattis, M. (2004). Mapping relational structure in spatial reasoning. *Cognitive Science*, 28, 589 – 610.

Gattis, M., & Holyoak, K. J. (1996). Mapping conceptual to spatial relations in visual reasoning. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 22 (1), 231 – 239.

Gende, D. (2006). Graphical analysis of motion: Kinematics. In D. Gende (Ed.), *Advanced Placement Program AP Physics Professional Development*

Materials (Special Focus: Graphical Analysis), pp. 9 – 25. New York: The College Board.

Glushko, R. J., & Cooper, L. A. (1978). Spatial comprehension and comparison processes in verification tasks. *Cognitive Psychology*, 10 (4), 391 – 421.

Hale, P. (2000). Kinematics and graphs: Students' difficulties and CBLs. *The Mathematics Teacher*, 93(5), 414 – 417.

Hegarty, M. (2004). Diagrams in the mind and in the world: Relations between internal and external visualizations. In A. F. Blackwell, K. Marriott, A. Shimojima (Eds.), Diagrammatic Representation and Inference, Third International Conference, Diagrams 2004 (LNAI 2980), pp. 1 – 13. Berlin: Springer-Verlag.

Hostetter, A. B., & Alibali, M. W. (2004). On the tip of the mind: Gesture as a key to conceptualization. *Proceedings of the 26th Annual Meeting of the Cognitive Science Society*. Chicago, IL.

Knight, R. (2004). Instructor's guide for Physics for scientists and engineers. Prentice Hall.

Knight, R., Jones, B., & Field, S. (2007). College physics: A strategic approach. New York: Addison-Wesley.

Kozhevnikov, M., Hegarty, M., & Mayer, R. E. (2002). Revising the visualizer-verbalizer dimension: Evidence for two types of visualizers. *Cognition and Instruction*, 20(1), 47 – 77.

Lapp, D. A., & Cyrus, V. F. (2000). Using data-collection devices to enhance students' understanding. *The Mathematics Teacher*, 93(6), 504 – 510.

Larkin, J. & Simon, H. (1987) Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science*, 11, 65 – 99.

Leinhardt, G., Zaslavsky, O, & Stein, M. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research*, 60 (1), 37 – 42.

McDermott, L. C., Rosenquist, M. L., & van Zee, E. H. (1987). Student difficulties in connecting graphs and physics: Examples from kinematics. *American Journal of Physics*, 55 (6), 503 – 513.

McNeil, D. (1992). Hand and mind. University of Chicago Press.

Mokros, J., & Tinker, R. (1987). The impact of microcomputer-based labs on children's ability to interpret graphs. *Journal of Research in Science Teaching*, 24, 369 – 383.

Morewedge, C. K., & Kahneman, D. (2010). Associative processes in intuitive judgment. *Trends in Cognitive Sciences*, 14 (10), 435 – 440.

Nickerson, R. (1988). Counting, computing, and the representation of numbers. *Human Factors*, 30, 181 – 199.

Novick, L. R., Hurley, S. M., & Francis, M. (1999). Evidence for abstract, schematic knowledge of three spatial diagram representations. *Memory & Cognition*, 27 (2), 288 – 308.

Palmer, S. E. (1978). Fundamental aspects of cognitive representation. In E. Rosch and B. B. Lloyd (Eds.), *Cognition and Categorization*, pp. 259 – 303. Hillsdale, NJ: Erlbaum.

Ratwani, R. M., & Trafton, J. G. (2008). Shedding light on the graph schema: Perceptual features versus invariant structure. *Psychonomic Bulleting & Review*, 15 (4), 757 – 762.

Ratwani, R. M., Trafton, J. G., & Boehm-Davis, D. A. (2008). Thinking graphically: Connecting vision and cognition during graph gomprehension. *Journal of Experimental Psychology: Applied*, 14 (1), 36 – 49.

Rittle-Johnson, B., Star, J. R., & Durkin, K. (2009). The importance of prior knowledge when comparing examples: Influences on conceptual and procedural knowledge of equation solving. *Journal of Educational Psychology*, 101 (4), 836 – 852.

Savinainen, A., & Scott, P. (2002). The force concept inventory: A tool for monitoring student learning. 37 (1), 45 – 52.

Saxon Jr., J. H. (1993). Physics: An incremental development. Chicago: Saxon Publishers.

Shah, P., & Freedman, E. G. (2011). Bar and Line Graph Comprehension: An Interaction of Top-Down and Bottom-Up Processes. *Topics in Cognitive Science*, 3 (3), 560 – 578.

Skalsky, N., & Pastel, R. (2004). Transportable research instrument: A PDA-based laboratory for science experiments. *Proceedings of the 2nd IEEE International Workshop on Wireless and Mobile Technologies in Education.*

Smelcer, J. B., & Carmel, E. (1997). The effectiveness of different representations for managerial problem solving: comparing tables and maps. *Decision Sciences*, 28 (2), 391-420.

Speier, C. (2006). The influence of information presentation formats on complex task decision-making performance. *International Journal of Human-Computer Studies*, 64 (11), 1115 – 1131.

Spence, I., & Krizel, P. (1994). Children's perception of proportions in graphs. *Child Development*, 65 (4), 1193 – 1213.

Stanovich, K. E., & West, R. F. (2000). Individual differences in reasoning: Implications for the rationality debate? *Behavioral and Brain Sciences*, 23 (5), 645 – 665.

Star, J. R., & Rittle-Johnson, B. (2009). It pays to compare: An experimental study on computational estimation. *Journal of Experimental Child Psychology*,102 (4), 408 – 426.

Thornton, R. K., & Sokoloff, D. R. (1998). Assessing student learning of Newton's laws: The force and motion conceptual evaluation and the evaluation of active learning laboratory and lecture curricula. *American Journal of Physics*, 66 (4), 338 – 351.

Trafton, J. G., Marshall, S., Mintz, F., & Trickettt, S. B. (2002). Extracting explicit and implicit information from complex visualizations. Technical Report. Naval Research Laboratory. Washington, DC.

Trafton, J. G., Trickettt, S. B., & Mintz, F. E. (2005). Connecting internal and external representations: Spatial transformations of scientific visualizations. *Foundations of Science*, 10, 89 – 106.

Trickettt, S. B., & Trafton, J. G. (2006). Toward a comprehensive model of graph comprehension: Making the case for spatial cognition. In D. Barker-Plummer, R. Cox, and N. Swoboda (Eds.), *Diagrams 2006, LNAI 4045*, pp. 286 – 300. Berlin, Germany: Springer-Verlag.

Tversky, B. (2011). Visualizing thought. *Topics in Cognitive Science*, 3, 499 – 535.

Uesaka, Y., & Manalo, E. (2006). Active comparison as a means of promoting the development of abstract conditional knowledge and appropriate choice of diagrams in math word problem solving. In D. Barker-Plummer, R. Cox, and N. Swoboda (Eds), *Diagrammatic Representation and Inference*, pp. 181-195. Springer: Berlin Heidelberg.

Vessey, I. (1991). Cognitive fit: A theory-based analysis of the graphs versus tables literature. *Decision Sciences*, 22 (2), 219 – 240.

Young, D. H., Freedman, R. A., & Ford, A. L. (2007). Sears and Zemansky's University Physics with Modern Physics (12th Edition). New York: Addison-Wesley.

Zacks, J., & Tversky, B. (1999). Bars and Line: A study of graphic communication. *Memory and Cognition*, 27 (6), 1093 – 1099.

Zhang, J. (1997). The nature of external representations in problem solving. *Cognitive Science*, 21 (2), 179 – 217.

Zhang, J., & Norman, D. A. (1994). Representations in distributed cognitive tasks. *Cognitive Science*, 18, 87 – 122.

Zhang, J., & Norman, D. A. (1995). A representational analysis of numeration systems. *Cognition*, 57, 271 – 295.

IX. APPENDICES

A. APPENDIX A

Math, Science, and Graphing Skills Background Questionnaire

Sex Male Female (circle one)

Age _____

Year First Second Third Fourth Fifth

Math Background

List the last five math course that you have taken in order from the most recent to the most distant. (include high school courses and statistics courses if applicable)

____

____

____

____

____

Science Background

List the last five science course that you have taken in order from the most recent to the most distant. (include high school courses if applicable)

____

____

____

____

____

APPENDIX A (continued)

1. Figure 21 shows a position versus time graph for the motions of two objects A and B that are moving along the same road.

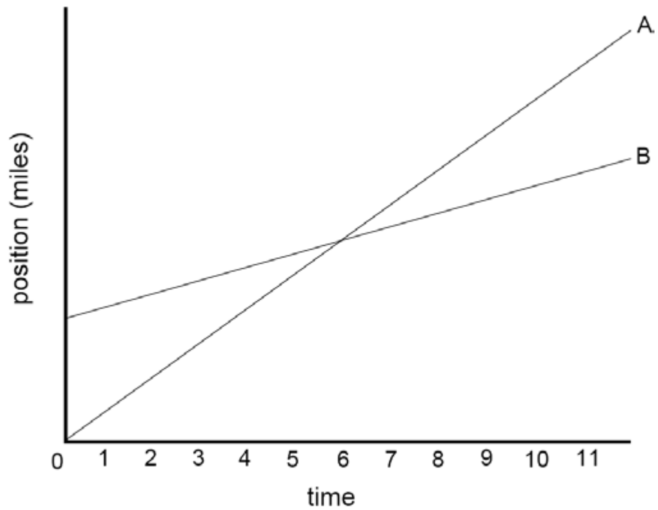


Figure 21. Position-time graph for problem 1.

(a) At the instant *time* = 2, is the speed of the object A greater than, less than, or equal to the speed of the object B? (circle the correct response)

i. Greater than

ii. Less than

iii. Equal to

Explain your reasoning in the space provided.

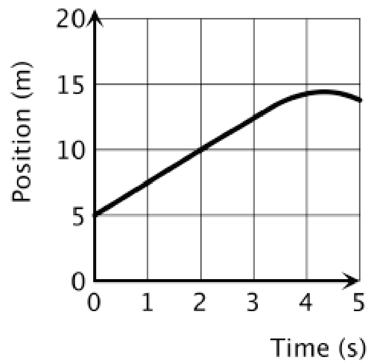
(b) Do objects A and B ever have the same speed? Yes No (circle one)

(c) If so, at what time(s)? ________________________

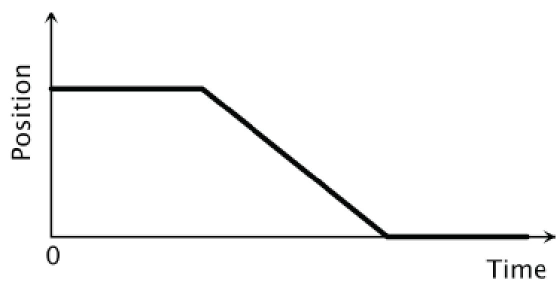
Explain your reasoning in the space provided.

APPENDIX A (continued)

2. To the right is a position versus time graph of an object's motion. The velocity of the object at time = 2 s is? (circle the correct response)
    a. 0.5 m/s
    b. 8.5 m/s
    c. 2.5 m/s
    d. 5.0 m/s
    e. 10.0 m/s



3. The following graph shows the position versus time graph of an object's motion. Which sentence is a correct interpretation?

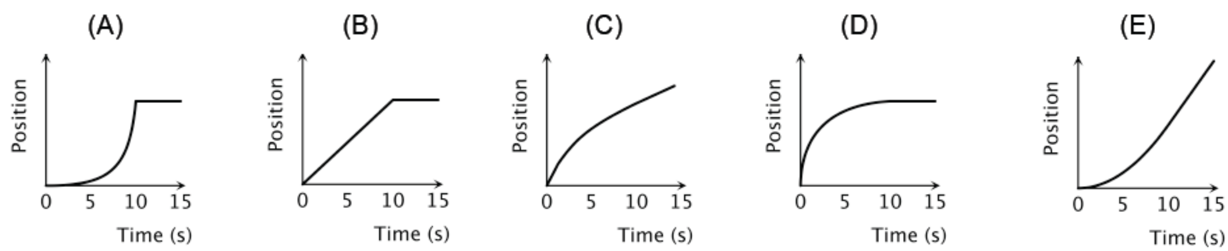


    a. The object rolls along a flat surface. Then it rolls forward down a hill, then finally stops.
    b. The object doesn't move at first. Then it rolls forward down a hill, and finally stops.
    c. The object is moving at a constant velocity. Then it slows down and stops.
    d. The object doesn't move at first. Then it moves backwards, and then finally stops.

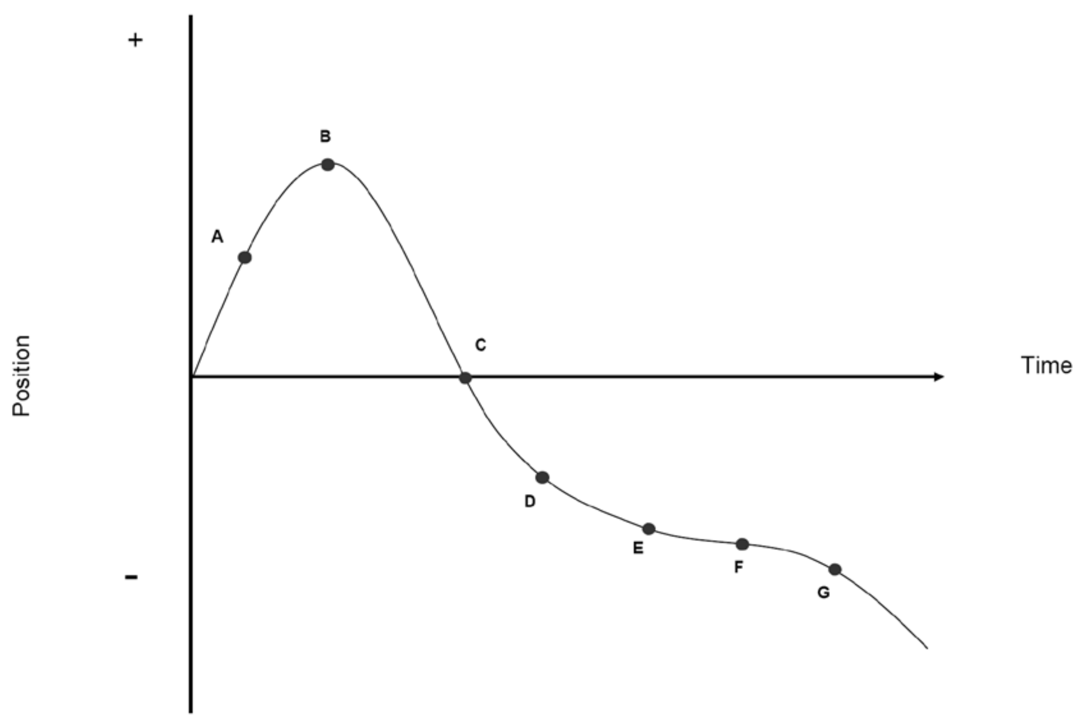
APPENDIX A (continued)

e. The object moves along a flat area, moves backwards down a hill, and then it keeps moving.

4. An object starts from rest and undergoes a positive, constant acceleration for ten seconds, it then continues on with constant velocity. Circle the graph that correctly describes this situation.



5. At which of the lettered points on the graph below is



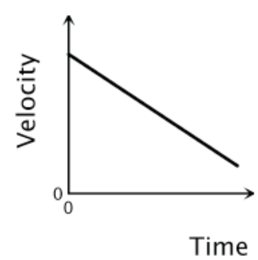
a. the motion slowest? A B C D E F G

b. the object speeding up? A B C D E F G

APPENDIX A (continued)

c. the object slowing down? A B C D E F G

6. The graph below represents the velocity of an object's motion.



Circle the sentence that is the best interpretation of the graph.

a. The object is moving with a constant acceleration.

b. The object is moving with a uniformly decreasing acceleration.

c. The object is moving with a uniformly increasing velocity.

d. The object is moving at a constant velocity.

e. The object does not move.

B. APPENDIX B

Item Analysis of Math, Science, and Graphing Skills Background Questionnaire

Overall accuracy data on the graphing skills assessment were presented in the reporting of results for each experiment. To recap those performance data: Mean correct out of maximum score of 9 for Experiment 1 was 1.69; for Experiment 2 it was 2.06; for Experiment 3 it was 2.13, and for Experiment 4 it was 3.00. A Kruskall-Wallis test on the accuracy data for the four experiments failed to reject the null hypothesis, X^2 (3) = 4.50, p = .21, indicating that performance on the graph interpretation skills assessment was the same across the four experiments. The relatively poor accuracy of the sample as a whole on the instrument suggested that it would be useful to examine the choices that subjects made on each item and, where possible, justification for those choices when asked to do so. The primary goal of this analysis was to ascertain the degree to which subjects' performance in the experiments was consistent with their performance on the graph skills assessment. Specifically, the patterns of correct and incorrect choices on the graph assessment items were examined in terms of whether the direct read-off rules and associated graph features described in the interpretation of performance in the experimental studies also tended to account for performance on the graph interpretation skills items. In what follows, each item is discussed separately.

Item 1

Item 1 was designed to assess the degree that people could extract information about velocity from height and slope features of the curve in a position-

APPENDIX B (continued)

time graph (see Appendix A). The item was adapted from McDermott et al. (1987) and was comprised of three sub-items.

Item 1a. Determining the Relative Speed of Two Functions in a Position-Time Graph

The frequency of subjects who noted that the speed of the object A at "time = 2" in the graph was *greater than*, *less than*, or *equal to* the speed of object B was 39, 23, and 2, respectively. The correct response was that the speed of object A was *greater than* the speed of object B at that point in time on the graph. The results of a Chi-Square analysis indicated that the distribution of selected responses differed significantly from chance, $X^2(2) = 32.38$, $p < .001$.

Two of the 64 subjects did not provide an explanation for their response to sub-item 1a. Across the 64 subjects, five different explanations were observed: *slope*, *acceleration*, *height*, *greater distance traveled*, and *same pace*. The proportion of subjects who based their response on one of these explanatory frameworks is depicted in Figure 22 in relation to each of the three response alternatives. As the graph shows, four explanatory frameworks were used by subjects who indicated that object A is moving faster than object B: *slope* (n = 20), *acceleration* (n = 12), *height* (n = 3), and *greater distance traveled* (n = 4). Subjects who indicated that object A is moving slower than object B used three: *acceleration* (n = 2), *height* (n = 19), and *greater distance traveled* (n = 1). The one subject who indicated that the speed of the two objects was equal offered that the objects were moving at the *same*

APPENDIX B (continued)

pace. Thus, when the slope feature was used, the selected response was always correct.

Most of the time that *height* was used it generated an incorrect response. This occurred when subjects compared the relative heights at time = 2 and saw that curve A was lower than curve B. Among subjects for whom *height* led to a correct response, this was due to misreading the curves at time = 11 at which point curve A was higher than curve B. Neither group of subjects recognized slope as the appropriate feature to extract velocity information from when interpreting position-time graphs and clearly, those who made the correct response failed to read the curves from the proper point on the x-axis.

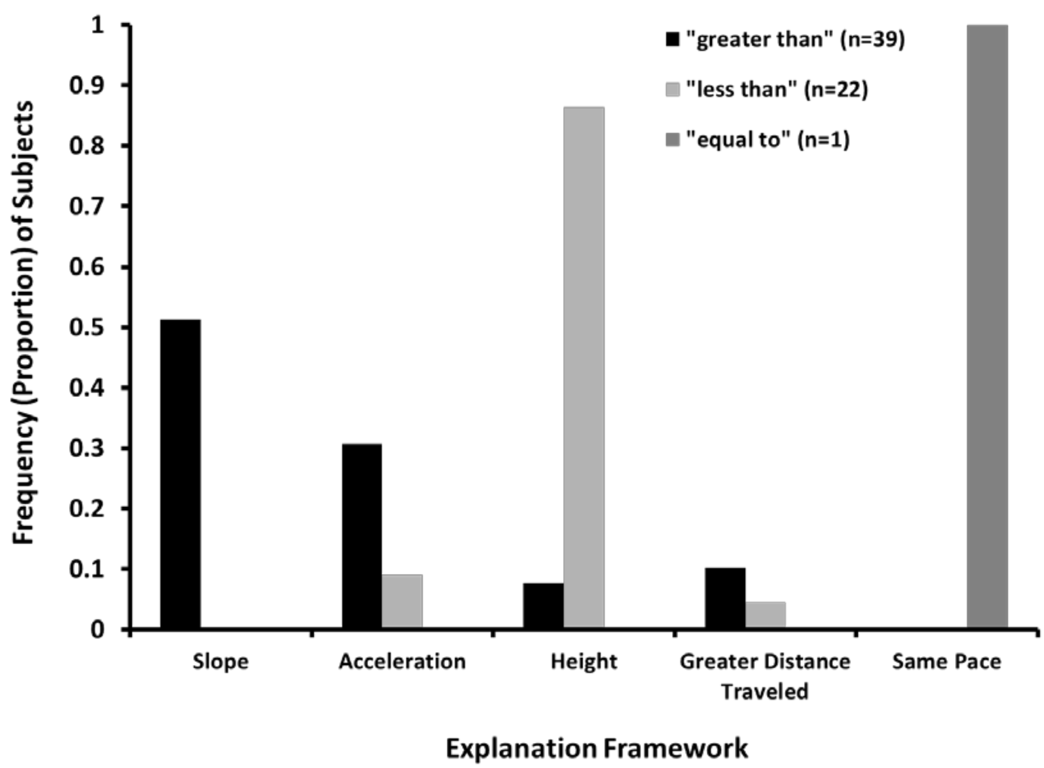


Figure 22. Frequency of subjects who used one of five explanatory frameworks depending on their interpretation of the speed of two objects represented in the graph in Item 1.

APPENDIX B (continued)

Acceleration was another widely used framework among subjects who selected the correct alternative. In actuality, these students used slope information but inappropriately mapped it onto the wrong motion concept. Slope represents velocity in position-time graphs not acceleration. The slope feature represents acceleration in velocity-time not position-time graphs.

Four subjects used the *greater distance* framework. They made the correct interpretation by reasoning that, since object A started farther back than object B but ended up farther ahead at a later point in time on the graph, object A must have been moving faster. Finally, the one subject who indicated that the speed of the objects was equal noted that the objects were moving at the same pace.

Item 1b and 1c. Interpreting Speed in a Position-Time Graph

One would expect that, since a single subject indicated that the speed of the objects was equal then, when asked if the objects ever move at the same speed (sub-item 1c), that only one subject would incorrectly say, “yes,” because the others indicated that the speeds of the objects were different (i.e., either greater or less than) but that was not the case. Instead, 42 subjects circled, “yes,” indicating that they interpreted the objects as moving at the same speed at some point in time whereas 22 circled, “no”. Those subjects who circled “yes” noted that their response was based on the lines intersecting.

Among the 39 subjects who correctly recognized that the speed of object A was greater than the speed of object B (see sub-item 1a), 20 said “no” and 19 said, "yes". Of those who correctly said "no", 14 used the *slope* explanatory framework, four used the *acceleratio*n framework, and two used the *greater distance* framework. Thus, these 18 subjects were consistent in their reasoning across the three sub-items.

These results can be interpreted in light of the results from the experimental trials. Some subjects extracted information about speed (i.e., velocity) directly from the slope feature of the curve. A subgroup of these subjects used slope information but inappropriately mapped that information onto acceleration rather than velocity. Both groups of subjects made use of the directionality rule, *up means going faster*, and probably compared the two curves on the basis of direction magnitude (i.e., the "more" up that the line is, the faster that the object is moving). As mentioned earlier, subjects who based their selection on differences in distance had to derive that information through more complex reasoning processes than was used to get meaning from the slope.

Item 2. Deriving Velocity Information from Nonlinear Curves in Position-Time Graphs

Item 2 assessed the capacity to interpret velocity in a position-time graph. The item is part of the TUG-K (Beichner, 1994). Subjects were instructed to find the velocity of the object at "time = 2". If, as suggested by the results of the experimental trials, subjects only processed information that was available for direct read-off, then it is no surprise that only 10 out of 64 subjects selected the correct alternative, 2.5 m/s. In this case, generating the correct solution required

APPENDIX B (continued)

substantially more computation than was needed to reach the two modal solutions that happened to be incorrect.

Most subjects either indicated that the velocity of the object at time = 2s was 5.0m/s (n = 27) or 10.0m/s (n = 23). There are two plausible reasons for the modal responses. First, subjects who selected "5.0m/s" could have used the formal definition for velocity (i.e., "speed") as position/time (i.e., 10.0m/2.0s) which would have generated the incorrect response of "5m/s". However, since the actual change in position was 5m rather than 10m (i.e., the y-intercept was 5m and after 2s the object had travelled another 5m, which amounts to 2.5m the first second and 2.5m the next second or 5.0m/2s), the strategy was accurate but the problem

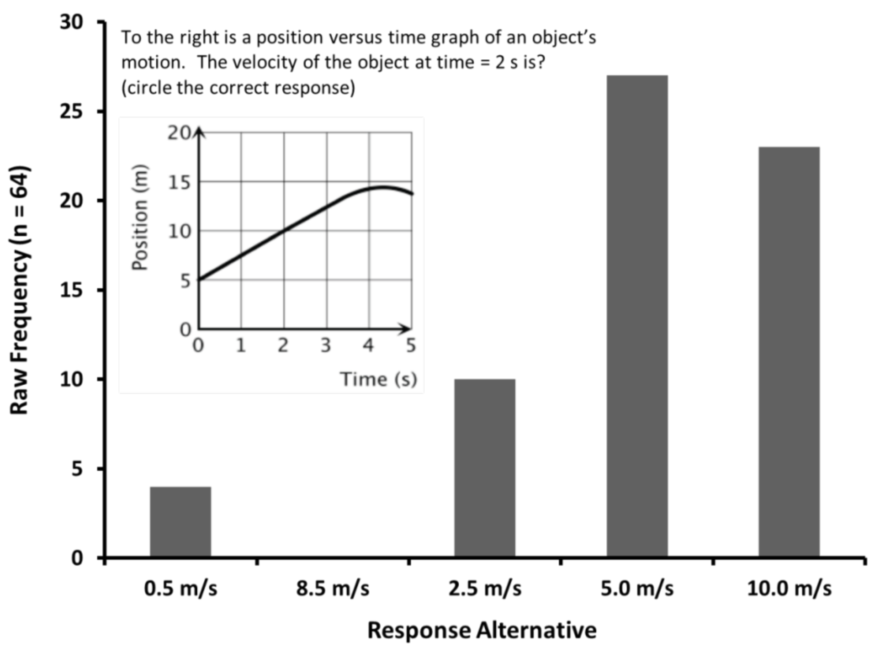


Figure 28. Frequency of subjects who selected each response alternative (correct response is 2.5m/s).

APPENDIX B (continued)

representation was not. According to a direct read-off account, subjects selected this alternative because the 5.0m change in position was not directly available like the 10.0m and the 2.0s; it had to be derived.

Second, subjects probably selected the incorrect alternative, “10m/s”, by using a domain-general cross-coordinate read-off strategy in which 10.0m is located in relation to the intersection of the curve and “2s” on the x-axis. This information was directly extracted from the graph because it was available for visual processing and was consistent with a common graph interpretation strategy that is cultivated in math and science instruction. Unfortunately, direct read-off of that information does not return a value about the velocity of the object; only its position.

Item 3. Interpreting States of Position in Position-Time Graphs

Item 3 evaluated the extent that one could represent changes in the movement of an object, particularly, its direction and whether the object is stationary or moving. The item is part of the TUG-K (Beichner, 1994). Subjects were required to interpret changes in the displacement (i.e., state of position) of an object using a position-time graph. They saw a graph with a curve that reflected three different position-time relationships. The task was to match the graph with one of five sentences that contained three clauses; each clause describing one of the position-time relationships depicted in the graph (see Appendix A).

The distribution of selected responses was multimodal. Seventeen of the 64 subjects made the correct interpretation of D, “The object doesn’t move at first.

APPENDIX B (continued)

Then it moves backwards, and then finally stops." Fourteen subjects incorrectly interpreted the function as B, "The object doesn't move at first. Then it rolls forward down a hill, and finally stops." Subjects related the absence of slope to the absence of movement which was consistent with the *no slope means no motion* rule but did not relate the direction of slope to the direction of the object's movement. Eighteen subjects selected C, "The object is moving at a constant velocity. Then it slows down and stops." According to Beichner (1994) and Leinhardt, et al. (1990), this interpretation reflects an example of a y-axis label switch since the interpretation that an object is moving at a constant velocity would be correct if the curve was plotted in a velocity-time graph. However, the results of the present study suggest that this response selection could also be accounted for by the use of the direct read-off *height* rule such that when the curve lay on the x-axis, there is no motion but when the curve is above the x-axis, it is moving. Eight subjects selected A, "The object rolls along a flat surface. Then it rolls forward down a hill, and then finally stops." And seven subjects selected E, "The object moves along a flat area, moves backwards down a hill, and then it keeps moving." These subjects partially interpreted the plotted function as a picture. Thus, much of the response patterns observed in performance on this item can be accounted for by the rules for direct read-off examined in the experimental trials.

APPENDIX B (continued)

Item 4. Discriminating among Graphical Representations of Velocity and Velocity Change

Item 4 required subjects to *discriminate the correct graphical representation of velocity and velocity change from among five graphs*. The motion description to be interpreted was,

> An object starts from rest and undergoes a positive, constant acceleration for ten seconds, it then continues on with constant velocity. Circle the graph that correctly describes this situation.

Five position-time graphs (see Appendix A) were situated below the description. Nineteen subjects selected the correct graph (E) but the modal selection (n = 35) was graph B which consisted of an upward oriented line that broke into a straight curve that was horizontal with the x-axis. A small number of subjects selected A (n = 5), C (n = 3), or D (n = 2). Since all graphs started with an upward trend (whether linear or nonlinear), it was not possible to determine whether the subjects used the upward direction rule for acceleration or "going faster" but it could be assumed that subjects did associate "constant" (i.e., *no change*) with a straight line because a straight line does not change. In addition to using direct read-off rules for *acceleration* and *constant*, the results of Item 4 suggest that subjects applied the rule related to height. Reading the description would activate these three direct read-off rules onto features of the graph that match particularly, a linear curve in

APPENDIX B (continued)

which the first part is oriented in an upward direction and the second part is parallel to the abscissa accordingly; (a) constant (straight line means no change) acceleration (upward direct means going faster) and (b) constant (straight line means no change) velocity (horizontal curve is greater than zero so the object must be moving); therefore, velocity is not changing. Mapping these direct read-off rules between the description and graphs would lead to a perfect match: alternative B which, although rational, is incorrect.

Item 5. Interpreting Faster, Slower, and Slowest Moving Objects in Position-Time Graphs

Item 5 was adapted from McDermott et al. (1987) and was comprised of three sub-items. Sub-item 5a required the use of the tangent to the curve in order to recognize that the slowest point on the graph was B because at that point, the tangent was horizontal and therefore reflected a velocity of zero. Only 12 of the 64 subjects selected the correct alternative. The distribution of incorrect responses was multimodal with just as many subjects indicating that points G (n = 12), C (n = 10), E (n = 9), and F (n = 9) represented the slowest state of motion. G is the lowest point on the curve so that selection reflects the use of a height rule. Point C is plotted on the x-axis which is a horizontal feature in the graph and therefore may have conveyed a lack of motion. Points E and F each reflect points where the curve appears to be *approaching* a horizontal direction.

APPENDIX B (continued)

Items 5b and 5c were intended to measure the capacity to interpret velocity and velocity change in position-time graphs. Item 5b required the subject to identify the point(s) on the graph that represented "the object…speeding up". Out of 64 college-educated subjects who had taken physics and other science and mathematics courses, not a single one of them recognized G as the correct alternative. Point G was the lowest point on the curve and at that point the slope dives downward. Thus, it was not surprising to find that so few subjects identified the object as getting faster at this point. Interpretation could have well been guided by height, directionality, or both direct read-off strategies. The modal response was A. Thirty-six subjects selected that point using on a read-out rule based on the direction of the curve (i.e., "up means going faster"). Unfortunately, the tangent to the curve reflected a decreasing slope and hence, a decrease rather than an increase in the rate of velocity change. Thirteen subjects selected B, the highest point on the curve. Since the selection would have been appropriate in the context of a velocity-time graph, it is easy to understand how it might be attributed to a switch of the y-axis labels. However, that is pure speculation. The empirical results of the current study suggest that this second group of subjects tried to extract velocity change information from a height feature in the graph based on the use of a direct read-off rule.

Item 5c required recognition of the point in the same graph at which the motion of the object is slowing down. The correct answers are A, C, D, and E. None

APPENDIX B (continued)

of the subjects selected all four alternatives. However, 12, 9, and 14 subjects selected C, D, and E, respectively. Each of these points is situated at a position where the direction of the curve is consistent with the read-off rule, *down means going slower*. Only one of 64 subjects selected point A, another correct response, but one that is visually consistent with the direct read-off rule, *up means going faster*, instead of *down means going slower*. The selection requires processing change in the tangent to the curve in order to extract information about velocity.

Item 6. Velocity Change in Velocity-Time Graphs

Finally, item 6 required the subject to interpret acceleration in a velocity-time graph. Ten out of the 64 subjects correctly recognized that the graph represented a constant rate of change whereas the majority of subjects (n = 49) interpreted the motion of the object as "uniformly decreasing acceleration" which would have been correct had the function been plotted in an acceleration-time graph. Again, the assumption that subjects "switched" the y-axis label from "velocity" to "acceleration" is a reasonable one but lacks evidence. The direct-read-off hypothesis which was supported through the series of experiments conducted in the present study, suggests that subjects were led to "uniformly decreasing acceleration" because the direction of the curve evoked a *down means decrease* (*or*, *going slower*) rule. In addition, the word, "decrease", in the motion statement may have increased the likelihood of that particular response selection. Thus, whereas the nature of the TUG-K (Beichner, 1996) item makes it difficult to tease apart potential explanations, the results of the present study suggest that the performance on the item was related to direct read-off and a failure to process the critical information about acceleration from the slope of the curve.

C. APPENDIX C

Introduction: Graph Types

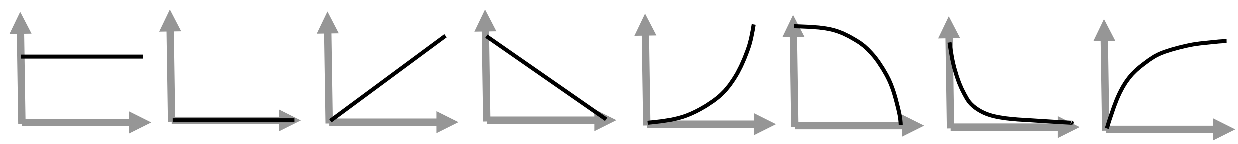
Kinematics, the branch of physics concerned with the motion of objects, relies on graphs to communicate various aspects of motion. Two types of graph in particular, the Position-Time graph and the Velocity-Time graph, are conventions in the domain. Specific patterns of object motion are represented by the shape and slope of curves plotted within the axes of these graphs. Examples of Position-Time and Velocity-Time graphs are shown in the figure below.

D. APPENDIX D

Introduction: General Motion

In this study, you will interpret two types of graph. In one graph, the ordinate or y-axis is labeled, *p*, for "*Position*"; in the other graph the ordinate or y-axis is labeled, *v*, for "*Velocity*". In both graphs the x-axis is labeled, *time*.

Each type of graph will be shown with a variety of curves. Some will be linear and some will be curvilinear.



In this first part of the study, you will see each graph paired with one of two possible statements:

The object is moving or *The object is stationary*.

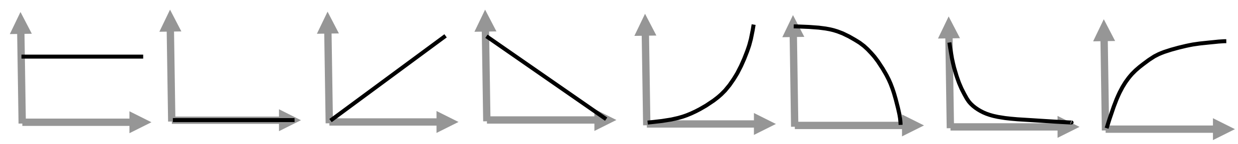
You need to look at the graph and decide whether the statement is an accurate description of what the graph shows (TRUE) or is inaccurate (DIFFERENT).

E. APPENDIX E

Introduction: Velocity Change

In this second part of the study, you will continue to interpret two types of graphs. In one graph, the ordinate or y-axis is labeled, *p*, for "*Position*"; in the other graph the ordinate or y-axis is labeled, *v*, for "*Velocity*". In both graphs the x-axis is labeled, *time*.

As was true in the first part of the experiment, each type of graph will be shown with a variety of curves. Some will be linear and some will be curvilinear.



In this second part of the study, each of the graphs will now be paired with one of three possible statements about motion:

The object's movement is accelerating
The object's movement is decelerating.

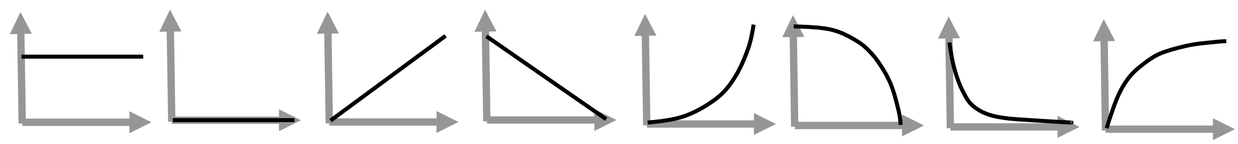
As in the first part of the study, you will need to decide if the statement is an accurate description of what the graph shows. If it is, you will respond *TRUE*; if not, *DIFFERENT*.

F. APPENDIX F

Introduction: Kinematic Behaviors

In this study, you will use [position-time] [velocity-time] graphs to interpret two types of kinematic behavior: *General Motion* and *Velocity Change*.

Each kinematic behavior will be represented as one of several different curves. Some will be linear and some will be curvilinear.



Half of the time, each graph will be paired with one of two statements about *General Motion*:

The object is moving.
The object is stationary.

The other half of the time, each graph will be paired with one of three statements about *Velocity Change*:

The object's movement is accelerating.
The object's movement is decelerating.

You will need to decide if the statement is an accurate description of what the graph shows. If it is, you will respond *TRUE*; if not, *DIFFERENT*.

G. APPENDIX G

In-Depth Rationale for Use of Chi-Square in Direct Read-Off Test

Although a rationale for the chi-square tests is provided on pp. 70 – 71 of the dissertation in the section, "Determination of Chance Performance (Position-Time Graphs)," this appendix provides the reader with a more in-depth explanation for the analysis.

Each of the 16 subjects made four judgments per curve for total number of 64 observed judgments. If we assume that these judgments are independent events, then there is 50/50 chance of being correct over the 64 observations, and 32 would be expected to be correct by chance. However, each judgment by a subject is a non-independent event. For non-independent events we need to look at the number of ways (permutations) a subject could be correct by chance on 0, 1, 2, 3, or 4 trials per condition. This turns out to be 1 way (1/16 = .0625) for 0 correct or 4 correct; 4 ways to get 1 correct (4/16 = .25) and 4 ways to get 3 correct (4/16 = .25), and 6 ways to get 2 correct (6/16 = .375). This is the chance distribution for 1 individual. For 16 subjects we want to know how many subjects we expect to get 0, 1, 2, 3 or 4 trials per condition correct. We determine this by multiplying each odds ratio by 16 subjects. This results in the expected chance distribution of 0 correct = 1 individual; 1 correct = 4 individuals; 2 correct = 6 individuals; 3 correct = 4 individuals; 4 correct = 1 individual. We calculate the chi-square by comparing the observed distribution against the chance distribution. For example, we observe the following in a particular condition:

0 correct – 10 subjects
1 correct – 0 subjects
2 correct – 3 subjects
3 correct – 1 subjects
4 correct – 2 subjects

This distribution is tested against the chance distribution (1, 4, 6, 4, 1), producing a chi square (4) = 89.75, $p < .001$, indicating that the observed distribution is significantly different from the chance distribution.

H. APPENDIX H
IRB Approval: Initial Review

Approval Notice
Initial Review (Response To Modifications)

July 27, 2011

Reality S. Canty, BS
Psychology
1015 BSB
M/C 285
Chicago, IL 60607
Phone: (312) 355-1323

RE: **Protocol # 2011-0459**
"Interpreting Graphical Functions of One-Dimensional Kinematics"

Dear Mr. Canty:

Your Initial Review application (Response To Modifications) was reviewed and approved by the Expedited review process on July 21, 2011. You may now begin your research.

Please note the following information about your approved research protocol:

Please remember that a subject who participates in the research for monetary compensation and is allowed to participate again for Subject Pool credits is also entitled to PEC compensation as well (and vice versa).

Protocol Approval Period: July 21, 2011 - July 19, 2012
Approved Subject Enrollment #: 136
Additional Determinations for Research Involving Minors: The Board determined that this research satisfies 45CFR46.404, research not involving greater than minimal risk. Therefore, in accordance with 45CFR46.408, the IRB determined that only one parent's/legal guardian's permission/signature is needed. Wards of the State may not be enrolled unless the IRB grants specific approval and assures inclusion of additional protections in the research required under 45CFR46.409. If you wish to enroll Wards of the State contact OPRS and refer to the tip sheet.
Performance Site: UIC
Sponsor: None
Research Protocol:

a) Interpreting Functions of One-Dimensional Kinematics, Dissertation Proposal;04/06/2011

Recruitment Materials:

a) Mass Mail Solicitation for Interpreting Graphical Functions Study; Version 2; 07/08/2011

APPENDIX H (continued)

b) Event Calendar Solicitation for Interpreting Graphical Functions Study; Version 2; 07/08/2011
c) Recruitment Flyer for Interpreting Graphical Functions Study; Version 2; 07/08/2011
d) Eligibility Screening Questionnaire for Interpreting Graphical Functions; Version 2; 07/08/2011

Informed Consents:

a) Educational Debriefing for Interpreting Graphical Functions; Version 1; 05/23/2011
b) Agreement to Participate in Interpreting Graphical Functions Study, Subject Pool; Version 3; 07/18/2011
c) Consent for Interpreting Graphical Functions Study, Non-Subject Pool; Version 3; 07/18/2011

Assent:

a) Assent for Interpreting Graphical Kinematics Functions Study; Version 2; 07/08/2011

Parental Permissions:

a) Parental Permission Form for Interpreting Kinematics Functions Study; Version 3; 07/18/2011
b) A waiver of parental permission has been granted under 45 CFR 46.116(d) and 45 CFR 46.408(c); however, as per UIC Psychology Subject Pool policy, as least one parent must sign the Blanket Parental Permission document prior to the minor subject's participation in the UIC Psychology Subject Pool.

Your research meets the criteria for expedited review as defined in 45 CFR 46.110(b)(1) under the following specific category:

(7) Research on individual or group characteristics or behavior (including but not limited to research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

Please note the Review History of this submission:

Receipt Date	Submission Type	Review Process	Review Date	Review Action
06/06/2011	Initial Review	Expedited	06/09/2011	Modifications Required
07/12/2011	Response To Modifications	Expedited	07/14/2011	Modifications Required
07/18/2011	Response To Modifications	Expedited	07/21/2011	Approved

Please remember to:

→ Use your **research protocol number** (2011-0459) on any documents or correspondence with

APPENDIX H (continued)

the IRB concerning your research protocol.

→ Review and comply with all requirements on the enclosure,
"**UIC Investigator Responsibilities, Protection of Human Research Subjects**"

Please note that the UIC IRB has the prerogative and authority to ask further questions, seek additional information, require further modifications, or monitor the conduct of your research and the consent process.

Please be aware that if the scope of work in the grant/project changes, the protocol must be amended and approved by the UIC IRB before the initiation of the change.

We wish you the best as you conduct your research. If you have any questions or need further help, please contact OPRS at (312) 996-1711 or me at (312) 996-2014. Please send any correspondence about this protocol to OPRS at 203 AOB, M/C 672.

Sincerely,

Sandra Costello
Assistant Director, IRB # 2
Office for the Protection of Research
Subjects

Enclosures:

1. **UIC Investigator Responsibilities, Protection of Human Research Subjects**
2. **Informed Consent Documents:**
   a) Educational Debriefing for Interpreting Graphical Functions; Version 1; 05/23/2011
   b) Agreement to Participate in Interpreting Graphical Functions Study, Subject Pool; Version 3; 07/18/2011
   c) Consent for Interpreting Graphical Functions Study, Non-Subject Pool; Version 3; 07/18/2011
3. **Assent Document:**
   a) Assent for Interpreting Graphical Kinematics Functions Study; Version 2; 07/08/2011
4. **Parental Permission:**
   a) Parental Permission Form for Interpreting Kinematics Functions Study; Version 3; 07/18/2011
5. **Recruiting Materials:**
   a) Mass Mail Solicitation for Interpreting Graphical Functions Study; Version 2; 07/08/2011
   b) Event Calendar Solicitation for Interpreting Graphical Functions Study;

APPENDIX H (continued)

Version 2; 07/08/2011

c) Recruitment Flyer for Interpreting Graphical Functions Study; Version 2; 07/08/2011
d) Eligibility Screening Questionnaire for Interpreting Graphical Functions; Version 2; 07/08/2011

6. Data Security Enclosure

cc: Gary E. Raney, Psychology, M/C 285
Susan R. Goldman (faculty advisor), Psychology, M/C 285

I. APPENDIX I
IRB Approval: Continuing Review (Year 1)

Approval Notice
Continuing Review

July 13, 2012

Reality S. Canty, BS
Psychology
1015 BSB
M/C 285
Chicago, IL 60607
Phone: (312) 355-1323

RE: **Protocol # 2011-0459**
"Interpreting Graphical Functions of One-Dimensional Kinematics"

Dear Mr. Canty:

Your Continuing Review was reviewed and approved by the Expedited review process on July 9, 2012. You may now continue your research.

Please note the following information about your approved research protocol:

Protocol Approval Period: July 20, 2012 - July 19, 2013
Approved Subject Enrollment #: 136 (limited to data analysis from 80 subjects)
Additional Determinations for Research Involving Minors: The Board determined that this research satisfies 45CFR46.404, research not involving greater than minimal risk.
Performance Sites: UIC
Sponsor: None
PAF#: - Not Applicable
Research Protocol(s):

b) Interpreting Functions of One-Dimensional Kinematics, Dissertation Proposal;04/06/2011

Recruitment Material(s):

e) N/A: Limited to data analysis only

Informed Consent(s):

d) N/A: Limited to data analysis only

Your research meets the criteria for expedited review as defined in 45 CFR 46.110(b)(1) under the following specific category(ies):

(7) Research on individual or group characteristics or behavior (including but not limited to research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices and social behavior) or research employing survey, interview, oral history,

APPENDIX I (continued)

focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

Please note the Review History of this submission:

Receipt Date	Submission Type	Review Process	Review Date	Review Action
07/06/2012	Continuing Review	Expedited	07/09/2012	Approved

Please remember to:

→ Use your **research protocol number** (2011-0459) on any documents or correspondence with the IRB concerning your research protocol.

→ Review and comply with all requirements on the enclosure,
"**UIC Investigator Responsibilities, Protection of Human Research Subjects**"

Please note that the UIC IRB has the prerogative and authority to ask further questions, seek additional information, require further modifications, or monitor the conduct of your research and the consent process.

Please be aware that if the scope of work in the grant/project changes, the protocol must be amended and approved by the UIC IRB before the initiation of the change.

We wish you the best as you conduct your research. If you have any questions or need further help, please contact OPRS at (312) 996-1711 or me at (312) 355-0816. Please send any correspondence about this protocol to OPRS at 203 AOB, M/C 672.

Sincerely,

Alison Santiago, MSW, MJ
IRB Coordinator, IRB # 2
Office for the Protection of Research
Subjects

Enclosure(s):

7. **UIC Investigator Responsibilities, Protection of Human Research Subjects**
8. **Data Security Enclosure**

cc: Jon D. Kassel, Psychology, M/C 285
Susan R. Goldman (Faculty Sponsor), Psychology, M/C 285

J. APPENDIX J

IRB Approval: Continuing Review (Year 2)

Approval Notice
Continuing Review

July 23, 2013

Reality S. Canty, BS
Psychology
1015 BSB
M/C 285
Chicago, IL 60607
Phone: (312) 355-1323

RE: **Protocol # 2011-0459**
"Interpreting Graphical Functions of One-Dimensional Kinematics"

Dear Mr. Canty:

Your Continuing Review was reviewed and approved by the Expedited review process on July 23, 2013. You may now continue your research.

Please note the following information about your approved research protocol:

Please note that this research did not have Institutional Review Board (IRB) approval beginning on July 19, 2013; approval re-commenced on July 23, 2013. Any research activities conducted during this time were done without IRB approval and were not compliant with UIC's human subject protection policies, *The Belmont Report,* UIC's Assurance awarded by the Office for Human Research Protection (OHRP) at HHS, and with the federal regulations for the protection of human research subjects, 45 CFR 46.

Protocol Approval Period: July 23, 2013 - July 23, 2014
Approved Subject Enrollment #: 136 (Limited to data analysis from 80 subjects)
Additional Determinations for Research Involving Minors: The Board determined that this research satisfies 45CFR46.404, research not involving greater than minimal risk.
Performance Site: UIC
Sponsor: None
Research Protocol:

c) Interpreting Functions of One-Dimensional Kinematics, Dissertation Proposal;04/06/2011

Recruitment Material:

f) N/A- Data analysis only

Informed Consent:

e) N/A – Data analysis only

APPENDIX J (continued)

Your research meets the criteria for expedited review as defined in 45 CFR 46.110(b)(1) under the following specific category:

(7) Research on individual or group characteristics or behavior (including but not limited to research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

Please note the Review History of this submission:

Receipt Date	Submission Type	Review Process	Review Date	Review Action
07/11/2013	Continuing Review	Expedited	07/23/2013	Approved

Please remember to:

→ Use your **research protocol number** (2011-0459) on any documents or correspondence with the IRB concerning your research protocol.

→ Review and comply with all requirements on the enclosure,
"UIC Investigator Responsibilities, Protection of Human Research Subjects"
(http://tigger.uic.edu/depts/ovcr/research/protocolreview/irb/policies/0924.pdf)

Please note that the UIC IRB has the right to seek additional information, require further modifications, or monitor the conduct of your research and the consent process.

Please be aware that if the scope of work in the grant/project changes, the protocol must be amended and approved by the UIC IRB before the initiation of the change.

We wish you the best as you conduct your research. If you have any questions or need further help, please contact OPRS at (312) 996-1711 or me at (312) 355-2764. Please send any correspondence about this protocol to OPRS at 203 AOB, M/C 672.

Sincerely,

Betty Mayberry, B.S.
IRB Coordinator, IRB # 2
Office for the Protection of Research
Subjects

cc: Michael E. Ragozzino, Psychology, M/C 285
Susan R. Goldman, Faculty Sponsor, Psychology, M/C 285

X. CURRICULUM VITA

Reality Sincere Canty
University of Illinois at Chicago
Learning Sciences Research Institute
1240 W. Harrison Street, Suite 1535
Chicago, IL 60607
(312) 996–7706
rcanty1@uic.edu

EDUCATION

University of Illinois at Chicago (UIC)
PhD in Psychology, September 2013
Area: Cognitive Psychology
Minor: Mathematics Education

University of Illinois at Chicago (UIC)
MA in Psychology, July 2007

University of Alaska Anchorage (UAA)
B.S. in Psychology, May 2003
Minor: Education

RESEARCH INTERESTS

Representation and reasoning in math, science, and other complex domains;
Cognitive aspects of validity in everyday mathematics assessments;
Applications of cognitive science principles to design

PUBLICATIONS

Refereed Journal Articles

Marshall, A. M., Castro Superfine, A., & Canty, R. S. (2010). Star students make connections. Teaching Children Mathematics, 17(1), 39 – 47.

Castro Superfine, A., Canty, R. S., & Marshall, A. M. (2009). Translation between external representation systems in mathematics: All-Or-None or skill conglomerate? Journal of Mathematical Behavior, 28, 217 – 236.

Refereed Conference Proceedings

Canty, R. S., Kaduk, C., & Soffer Goldstein, D. (2012). A model for designing cognition-and-instruction-based goal trajectories for research in K-6 math curricula. The Proceeding of the 34th Annual Conference of the PME-NA. Kalamazoo, MI: Western Michigan University.

Canty, R. S., Castro Superfine, A., & Marshall, A. M. (2008). Representing part-whole relations in diagrams. [Abstract]. In B. C. Love, K. McRae, & V. M. Sloutsky (Eds.), Proceedings of the 30th Annual Conference of the Cognitive Science Society (p. 1377). Washington, DC: Cognitive Science Society.

Canty, R. S., & Goldman, S. R. (2006). The effects of base ratio and conceptual structure on accuracy in multiplicative situations. In S. Barab, K. Hay, & D. Hickey (Eds.), Proceedings of the Seventh International Conference of the Learning Sciences, (pp. 898 – 899). Mahwah, NJ: Erlbaum.

In Preparation

Canty, R. S., Pellegrino, J. W., Goldman, S. R., DiBello, L. V., & Dejaresco, T. G. T. (In Preparation). Mapping operations to arithmetic principles: Reasoning with spatial-numeric relations in function tables.

Canty, R. S., Pellegrino, J. W., Goldman, S. R., & DiBello, L. V. (In preparation). Structure-property relations of the unit-whole and their effects on fraction estimation.

PAPER AND POSTER PRESENTATIONS

Du, Y. Y., Canty, R. S., Pellegrino, J. W., & Goldman, S. R. (2013). Empirical halves and mathematical half-nots: Two parts of the whole story in children's representation of fractions. Poster presented at the 2013 Annual Meeting of the Midwestern Psychological Association. Chicago, IL: Midwestern Psychological Association.

Kertayuda, S., Canty, R. S., Pellegrino, J. W., & Goldman, S. R. (2013). Comparing fractions as a function of unit-whole representation, fraction structure, and representation format. Poster presented at the 2013 Annual Meeting of the Midwestern Psychological Association. Chicago, IL: Midwestern Psychological Association.

Thomas, S., Canty, R. S., & Pellegrino, J. W. (2013). Representations of arithmetic principles in algebraic reasoning. Poster presented at the Sigma Xi Undergraduate Research Symposium at University of Illinois at Chicago. Chicago, IL.

Weishaar, T., Canty, R. S., Pellegrino, J. W., & Goldman, S. R. (2013). The role of visual patterns in early algebraic reasoning. Poster presented at the Sigma

Xi Undergraduate Research Symposium at University of Illinois at Chicago. Chicago, IL.

Canty, R. S., Pellegrino, J. W., & Goldman, S. R. (2012). Derivational structure of representations: Effects of accuracy and rate of processing graphical information about objects in motion. Poster presented at The 53rd Annual Meeting of the Psychonomic Society. Minneapolis, MN.

Canty, R. S. and Pak, D. (2012). Relationships between representational flexibility and number concepts: Drawing arrays and (mis)understanding square number properties. Poster presented at The Proceeding of the 34th Annual Conference of the PME-NA. Kalamazoo, MI: Western Michigan University.

Chokshi, A., & Canty, R. S. (2012). The influence of schematic knowledge and arithmetic structure on children's use of function tables. Poster presented at the Sigma Xi Undergraduate Research Symposium at University of Illinois at Chicago. Chicago, IL.

Grano, V., Canty, R. S., & Pellegrino, J. W. (2012). The influence of unit-benchmark knowledge and fraction structure on ordering fractions: Investigating the validity of curriculum-embedded assessments. Poster presented at the Sigma Xi Undergraduate Research Symposium at University of Illinois at Chicago. Chicago, IL.

Canty, R. S. (2012). Interpreting graphical representations of motion. Paper presented at the Cognitive Brownbag. Chicago, IL: University of Illinois at Chicago.

Pak, D., Rodríguez, E., & Canty, R. S. (2010). External representation use in children's multiplicative reasoning: Performance and task difficulty effects. Poster presented at the 83rd Annual Meeting of the Midwestern Psychological Association – Psi Chi Poster Session. Chicago, IL

Jaber, C. A. N., Lambie, A., & Canty, R. S. (2010). The effects of performance and question difficulty on interpretation and planning in multiplicative reasoning tasks. Poster presented at the 83rd Annual Meeting of the Midwestern Psychological Association – Psi Chi Poster Session. Chicago, IL.

Canty, R. S., Castro Superfine, A., & Marshall, A. M. (2008). Representing part-whole relations in diagrams. Poster presented at the 30th Annual Conference of the Cognitive Science Society. Washington, DC: Cognitive Science Society.

Canty, R. S., & Goldman, S. R. (2007). Problem structure and children's use of external representations in mathematical problem solving. Paper presented at Cognitive Brownbag, University of Illinois at Chicago.

Canty, R. S., & Rivette, K. (2007). The development and use of cognitive assessment tools for research, learning, and teaching of math trailblazers. Paper presented at the Eleventh Annual Conference for the Association of Mathematics Teacher Educators, Irvine, CA.

Beal, S., Canty, R. S., & Rivette, K. (2007). Whole number understanding and its assessment in school mathematics. Paper presented at the 39th Annual National Council of Supervisors of Mathematics Conference, Atlanta, GA.

Brown, S., Canty, R. S., Ditto, C., Beal, S., Pitvorec, K., & Kelso, C. R. (2007). Fidelity of Implementation and Student Learning: Making the Connection. Paper presented at the Research Presession of the Annual Meeting of the National Council of Teachers of Mathematics, Atlanta, GA.

Brown, S. A., Kelso, C., Bay-Williams, J., Ditto, C., Canty, R. S., Cramer, K., Wyberg, T., & Flevares, L. (2006). Standards-based curricula: Linking teachers' use and students' learning. Paper presented at the Research Pre-Session of the National Council of Teachers of Mathematics 2006 Annual Meeting and Exposition, Anaheim, California.

Canty, R. S., & Goldman, S. R. (2006). Number size, structural invariance, and accuracy: Towards understanding children’s thinking in multiplicative situations. Poster presented at the 2006 Sigma Xi Science Graduate Student Research Forum. University of Illinois at Chicago.

Beal, S., Canty, R. S., Ditto, C., Pitvorec, K., & Rivette, K. (2006). Investigation of teaching and learning of elementary school mathematics using Math Trailblazers. Paper presented at the Eighth Annual Chicago Symposium Series: Excellence in Teaching Mathematics and Science: Research and Practice. Loyola University, Chicago, Illinois.

Canty, R. S. (2005, April). Assessing student understanding of whole number concepts in Math Trailblazers. In J. Remillard (Chair), How are standards-based elementary school mathematics curricula used in schools? Paper presented at the Research Pre-Session of the National Council of Teachers of Mathematics 2005 Annual Meeting and Exposition, Anaheim, California.

Canty, R. S. (2003). The structure and function of knowledge: Exploring the learning of biochemistry concepts in text. Paper presented at Cognitive Brownbag, University of Illinois at Chicago.

Canty, R. S. (2003). Structural and functional knowledge disparities in a text situated biology problem space. Paper presented at the Committee on Institutional Cooperation Summer Research Symposium, University of Minnesota.

Canty, R. S., Shake, M., Szlemko, W., Smith, J., Madigan, R., & Wesolowski, V., (2003). Detecting threat: The effect of emotional faces in the RSVP. Poster presented at the Western Psychological Association, Vancouver, British Columbia.

Shake, M., Canty, R. S., Szlemko, W., Smith, J., Wesolowski, V., & Madigan, R. (2003). Anxiety, attention, and emotional expression. Poster presented at the Western Psychological Association, Vancouver, British Columbia.

Canty, R. S. (2002). Constructing meaning in biological text environments: A research proposal. Paper presented at the Student Research Opportunities Program, Chicago, Illinois.

Canty, R. S. (2002). Exploring the use of mathematics functions to appraise learning structures and their environments. Paper presented at the Behavioral Sciences Conference of the North, Anchorage, Alaska.

Canty, R. S., Morgan, S., & Shake, M. (2002). See spot run: Investigating noun-verb storage in the lexicon. Poster presented at the Behavioral Sciences Conference of the North, Anchorage, Alaska.

Shake, M., Canty, R. S., & Morgan, S. (2002). Perceptions and fear: Cultural implications of 9/11. Poster presented at the Behavioral Sciences Conference of the North, Anchorage, Alaska.

Morgan, S., Shake, M., & Canty, R. S., (2002). Attention, anxiety, and facial perception. Poster presented at the Behavioral Sciences Conference of the North, Anchorage, Alaska.

Madigan, B., Morgan, S., Canty, R. S., & Shake, M. (2002). Prime suspects: Can nouns trigger verbs? Poster presented at the Western Psychological Association. Irvine, California.

FACILITATED MEETINGS AND WORKSHOPS

Pellegrino, J. W., DiBello, L. V., Goldman, S. R., Canty, R. S., Goldstein, D. S., Kaduk, C., & Li, W. (2013). A multifaceted approach to establishing the validity of assessments embedded in K-5 math curricula. Laptop poster presented at Institute of Education Sciences: Washington, DC.

Canty, R. S. (2012). Assessing what assessments assess: Mathematical minds of children as windows into the cognitive validity of K-6 math activities. Workshop facilitated with Chicago Public School Teachers. University of Illinois at Chicago.

Canty, R. S., Pak, D., & Pellegrino, J. W. (2011). Investigating the cognitive aspects of curriculum-embedded assessments through "online" student performance. Talk presented to a National Science Foundation Advisory Board. University of Illinois at Chicago.

INSTRUCTION AND TEACHING

Assistant Lab Instructor

2001-2003 Learning and Cognition UAA [Prof. Bob Madigan]

Teaching Assistant

Spring 2012	Research Methods	UIC	[Dr. Edward G. Sargis]
Spring 2009	Intro to Psychology	UIC	[Dr. Susan Morriss]
Spring 2000	Stats for Psychology	UAA	Dr. John Petraitis]
Fall 1999	Life Span Development	UAA	[Dr. Rosellen Rosich]
Spring 1999	Life Span Development	UAA	[Dr. Rosellen Rosich]

PROFESSIONAL ACTIVITIES

Research Assistant

Learning Sciences Research Institute, UIC Evaluating the Cognitive, Psychometric, and Instructional Affordances of Curriculum-Embedded Assessments: A Comprehensive Validity-Based Approach (NSF Award No. DRL- 0732090) under the direction of Dr. James Pellegrino, Dr. Lou DiBello, and Dr. Susan R. Goldman. May 2008 – Present

Institute of Mathematics and Science Education / Learning Science Research Institute, UIC Research and Revision of the TIMS/Math Trailblazer Elementary Mathematics Curriculum (NSF Award No. DRL-0242704) under the direction of Dr. Philip Wagreich, Dr. Stacy Brown, Dr. Alison Castro-Superfine, and Cathy Kelso. Jan 2004 – Aug 2007

Center for the Study of Learning, Instruction, and Teacher Development, UIC Understanding in Science (NSF Award No. ROLE-0126265) under the direction of Dr. Susan R. Goldman and Dr. Jennifer Wiley. May 2002 – Aug 2002

Research Team Leader
UIC Summer Research Opportunities Program (SROP)
Graduate College
Allen Bryson, Coordinator
Summer 2009

Reviewer
Cognitive Science Society (CSS)
2009, 2010

International Conference for the Learning Sciences (ICLS)
2006, 2008, 2010

International Group for the Psychology of Mathematics Education – North American Chapter (PME-NA)
2010, 2012, 2013

Faculty Selection Committee Department of Psychology, UAA
Spring 2000

Undergraduate Advisory Committee Department of Psychology, UAA
Fall 1998 – Spring 2000

PROFESSIONAL WORKSHOPS ATTENDED

Jul 2010	Hands-On Introduction to Creating Intelligent Tutoring Systems without Programming Using the Cognitive Tutor Authoring Tools, International Conference of the Learning Sciences, Palmer House Hilton, Chicago
Nov 2008	Purdue Winer Memorial Lectures, New perspectives on human problem solving, Purdue University
Jul 2003	Categorization inside and outside of the lab: Festschrift in Honor of Douglas L. Medin, Chicago Botanical Gardens

UNIVERSITY AND COMMUNITY SERVICE

UIC-Chancellor's Committee on The Status of Blacks
Fall 2009 – Present (Student Subcommittee Co-Chair 2009-2012)

Graduate Educational Opportunities Committee, UIC Psychology
Spring 2009

Psi Chi at UIC [Invited Speaker]
Spring 2009

SROP Transitions Workshop, UIC [Invited Panelist]
Summer 2008

Manley High School, Chicago, IL [Invited Speaker]
Spring 2008

Englewood High School, Chicago, IL [Invited Speaker]
May 2007

Denali Montessori Elementary School, Anchorage, AK [Math Tutor]
Fall 2001

Center for Child Development at Providence Hospital, Anchorage, AK [volunteer assistant]
Spring 2001

AWARDS, GRANTS, AND HONORS

May 2013	Graduate College, UIC UIC Undergraduate Mentoring Award for Graduate Students (Inaugural Year)
May 2011	Department of Psychology, UIC Dissertation Research Grant
Aug 2007	Abraham Lincoln Fellowship Tuition and Stipend, UIC
Mar 2007	Kendall Hunt Publishing Company Travel Grant
Mar 2006	Sigma Xi Graduate Student Research Forum Honorable Mention, UIC
May 2006	Mathematical Sciences Research Institute Lodging Grant
Aug 2003	Diversity Fellowship Tuition and Stipend, UIC
Jun 2003	Summer Research Opportunities Program

	Research Stipend, UIC
Jan 2003	Dean of Students: Academic and Leadership Award Tuition Waiver, UAA
Aug 2002	Office of Undergraduate Research and Scholarship Thesis Grant, UAA
Jun 2002	Summer Research Opportunities Program Research Stipend, UIC
Aug 2001	AHAINA Man of Excellence Tuition Scholarship, UAA
Apr 2000	Distinguished Psychology Student Award Tuition Grant, Department of Psychology, UAA
Jun 1998	UCLA Summer Research Program Research Assistantship and Stipend, UCLA

PROFESSIONAL ASSOCIATION & SOCIETY MEMBERSHIPS

American Association for the Advancement of Science
Cognitive Development Society
Cognitive Science Society
International Society of the Learning Sciences
North America Group for the Psychology of Mathematics Education
Society for Research in Child Development
Psi Chi, National Honor Society in Psychology

DIRECTED RESEARCH AND ADVISED STUDENTS

Ashley Ballard, UIC Psychology (Visiting Research Specialist, Learning Sciences Research Institute at UIC)
Hazel Blackman, Oakwood University, B.A in Psychology (University of Massachusetts at Boston, Family Planning)
Anjlee Chokshi, UIC B.S., Biological Sciences/Honors College (Illinois College of Optometry)
Yan Yan Du, UIC B.A., General Psychology
Elizabeth Garcia, UIC Psychology
Vanessa Grano, UIC Kinesiology
Julia Ivan, UIC B.A. in Psychology/B.S in Mathematics
Camaleigh Ameera Novalla Jaber, UIC Psychology (Honors College)
Nidal Jibawi, UIC Psychology

Marlyse Jonfe Quinto, UIC Psychology
Natalia A. Kaczmarek, UIC Psychology (Adler School of Professional Psychology)
Sagung Kertayuda, UIC B.A., General Psychology
Amanda Lambie, UIC Psychology and Biology
Marty Muloski, UIC Psychology (University of Illinois at Springfield, Biology)
Daniel Pak, UIC Psychology (Visiting Research Specialist, Learning Sciences Research Institute at UIC)
Elizabeth Rodriguez, UIC Psychology
Stephany Thomas, UIC B.S. Biological Sciences, B.S. General Psychology Honors College (Northwestern, PhD Physical Therapy)
Katy Tuchscherer, UIC B.A. in Psychology, B.A. in Chemistry
Karletta White, Grambling University B.A. in Psychology (University of Iowa, Sociology)
Julian Williams, Dennison University (University of Illinois at Urbana-Champaign, Education)
Sandra Zamat, UIC B.A. in Psychology

REFERENCES

Susan R. Goldman[1,3,4]	sgoldman@uic.edu	(312) 996-4462
James W. Pellegrino[1,3,4]	pellegjw@uic.edu	(312) 355-2493
Louis V. DiBello[4]	ldibello@uic.edu	(215) 534-6239
Alison Castro Superfine[2,4]	amcastro@uic.edu	(312) 413-8029

1 Department of Curriculum and Instruction
2 Department of Mathematics, Statistics, and Computer Science
3 Department of Psychology
4 Learning Sciences Research Institute