Consignment Contract with Competition

BY

NANTAPORN RATISOONTORN
B.S., Mahidol University, Thailand, 2002
M.S., National Institute of Development Administration, Thailand, 2004
M.S., University of Chicago, 2007

THESIS

Submitted as partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Industrial Engineering and Operations Research
in the Graduate College of the
University of Illinois at Chicago, 2012

Chicago, Illinois

Defense Committee:

Dr. Elodie Goodman, Chair and Advisor
Dr. Houshang Darabi
Dr. Urmila Diwekar, Bioengineering
Dr. David He
Dr. Tito Homem-de-Mello, Universidad Adolfo Ibanez

This dissertation is dedicated to my parents, *Naruemon* and *Sriwichai*, who have provided encouragement and direction throughout my entire life.

ACKNOWLEDGMENTS

This dissertation would not have been possible without the guidance and the help of many people who, in one way or another, contributed and extended their valuable assistance in the preparation and completion of my work.

First and foremost, I extend my utmost gratitude to my advisor and mentor, Dr. Elodie Goodman for continuous support of my Ph.D. study and research. I have been both blessed and incredibly fortunate to have met and to have Dr. Goodman as an advisor, who provided not only academic guidance, but emotional support during the difficult times. Dr. Goodman has always provided succinct answers to my questions, no matter how elementary or complex, spent hours reading and editing my papers, and provided our great weekly one-on-one meetings. Her patience, motivation, enthusiasm and immense knowledge helped me overcome many crisis situations and complete this dissertation. I hope that, one day, I would become as good an advisor to my students as Dr. Goodman has been to me.

I am truly grateful to my dissertation committee: Dr. Houshang Darabi, Dr. Urmila Diwekar, Dr. David He and and Dr. Tito Homem-de-Mello for their encouragement, insightful comments and constructive criticism of my research work. I am also grateful to Dr. Saeed Manafzadeh for his encouragement, guidance and support throughout the years.

Dr. Stanley Sclove, from the Department of Information and Decision Sciences, is really a kind of person and teacher one would want to emulate. He is thoughtful and caring and I would like to thank him for his friendship.

I am thankful to my Ph.D. fellows, especially Maryam Haji, Silvio Rizzi, Amy David and Tatiana Benavides, for their friendship, good conversation and support in the past years. I could not forget my students, enrolled in IE 472 Operations Research II, Spring 2012, for their patience and assistance in my initiation to the profession of teaching. I really enjoyed the experience, their acquaintance and good humor.

I would like to acknowledge my sponsor, the Ministry of Science and Technology, Thailand, which gave me an opportunity to further my studies in the United States. My appreciation also goes to the Department of Information and Decision Sciences and the Department of Mechanical and Industrial Engineering, University of Illinois at Chicago for teaching and research opportunities during my graduate study.

Most importantly, none of this would have been possible without the love and patience of my family. My parents, to whom this dissertation is dedicated, have been a constant source of love, concern, support and strength all these years. I would like to express my gratitude to my sister and brother, Aksaranan and Supaloek, who have been my best friends and have given me their unwavering support throughout.

Last but no least, I would like to thank my best friend, Martin Chrobak, who has helped me maintain my equilibrium during difficult years. His patience, support and care helped me stay confident and focused on my graduate studies. I greatly value our relationship and I deeply appreciate his belief in me. His presence was a crucial part of this chapter in my life.

ACKNOWLEDGMENTS (Continued)

I have tried to name all those who contributed to various aspects of not only my dissertation, but also my graduate school life. If I have inadvertently neglected to acknowledge anyone, I am sorry and I am really appreciative of all they did for me and my endeavor.

NR

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

CHAPTER PAGE

TABLE OF CONTENTS (Continued)

LIST OF TABLES

LIST OF FIGURES

FIGURE **PAGE**

LIST OF FIGURES (Continued)

FIGURE **PAGE**

SUMMARY

Consignment contracts have recently received increasing attention and have been widely employed in many industries. Under this contract, items are sold at a retailer's but the supplier retains the full ownership of the inventory until purchased by consumers; the supplier collects payment from the retailer based on actual units sold. By building a game-theoretic model, we are able to obtain insightful results from analysis of the supply chain decisions and performance under different consignment arrangements. We extend the research in consignment contracts to more realistic situations with the presence of (1) more than one retailer in the supply chain, (2) more than one supplier in the supply chain and (3) a dual channel supply chain. Incorporating these new features enables us to gain new insights and derive practical implications on the implementation of consignment contracts.

In the first part of the thesis, we investigate how competition between two retailers influences the supply chain decisions and profits under different consignment arrangements with one supplier, namely a consignment price contract and a consignment contract with revenue share. First, we investigate how these two consignment contracts and a price only contract compare from the perspective of each supply chain partner. We find that the retailers benefit more from a consignment price contract than from a consignment contract with revenue share or a price only contract, regardless of the level of retailer differentiation. The supplier's most beneficial contract, however, critically depends upon the level of retailer differentiation: a consignment contract with revenue share is preferable for the supplier if retailer differentiation is strong; otherwise, a consignment price contract is preferable. Second, we study how retailer differentiation affects the profits of all supply chain partners. We find that less retailer differentiation improves the supplier's profit for both types of consignment contract. Moreover, less retailer differentiation improves profits of the retailers in a consignment price contract, but not necessarily in a consignment contract with revenue share.

In the second part of the thesis, we study how the presence of supplier competition affects the decisions and performance of a supply chain with a single retailer and two suppliers. The impact of supplier competition on the retailer's profit critically depends upon the type of consignment contract. Specifically, supplier competition helps improve the retailer's profit in the consignment price contract, but not necessarily in a consignment contract with revenue share. Next, we consider a situation in which one supplier sells products directly to consumers, through a direct channel, in addition to selling through the retailer via consignment. The added level of competition between the supplier's direct channel and the retailer has a negative impact on the retailer's profit but not on the suppliers' profits due to access to a larger consumer base. Higher channel competition between the supplier's direct channel and the retailer does not always increase the total supply chain profit and the efficiency. That is, channel competition improves the total channel profit when competition is not intense.

CHAPTER 1

INTRODUCTION

The global economic slowdown has had a noticeable negative impact on many businesses in virtually all industries. Companies have found it necessary to become more focused and fastidious in their business practices, than ever before. Many creative, and sometimes novel, business solutions have been proposed, explored, and implemented. Retailers have been every bit as interested in discovering a new and mutually beneficial relationship with their suppliers in order to maximize profits. A variety of contractual relationships exists including: the wholesale-price contract, buyback contract, price-discount contract and consignment contract. This research will focus on one of these, namely, the consignment contract.

1.1 Motivations

In the Spring of 2005, The Home Depot, a leading home improvement retailer, launched the Pay-By-Scan (PBS) program across the live plant industry. The goal was to help reduce risk of selling seasonal plant products, while providing a venue for growers to sell their production. The PBS program allows Home Depot to decide the retail price for each of the products, and the growers decide how much inventory is allocated into designated Home Depot retail sites. Ownership of the products does not transfer until they are sold to customers. Once products are sold, Home Depot makes a payment to the suppliers and retains the difference between this payment and the retail price (For more details on Home Depot and PBS, see `http://suppliercenter.homedepot.com/wps/portal`). Similarly, Autozone, one of the biggest automobile parts resellers, adopts Pay-On-Scan contracts with its suppliers. Under such a contract, Autozone selects suppliers and sets the retail price for each product. Autozone pays the supplier only when a product has been scanned and sold, with payment terms of up to 90 days after the sale (Boorstin, 2003; Fahey, 2003).

Several types of consignment contracts with various characteristics are being used, in varying degrees, across all product categories and in many industries. A prime example of a consignment contract in Internet commerce is Amazon Marketplace (Wang et al., 2004). Amazon.com, America's largest online retailer, not only sells products directly to consumers, but also introduced an additional online site called "Amazon Marketplace". Marketplace enables and invites other merchants to sell their products through the Amazon website. The merchants can decide the quantity of products and the selling price. The merchants incur no cost for listing their items, but Amazon receives a commission on products sold by deducting a certain percentage from the final sale price. Amazon has adopted, essentially, a consignment contract with revenue sharing in its Marketplace. The Marketplace strategy has proven to be a successful business strategy for Amazon, because 30% of items sold on Amazon are sold by third parties (Wikinvest, 2012) (For more details on Amazon Marketplace, go to: `http://www.amazon.com`).

Consignment contracts have recently gained popularity in many other industries, such as healthcare (Bendavid et al., 2011) and a variety of retail businesses. Consignment contracts allow the supplier to retain ownership of the items in the healthcare retail facility at no charge until items are actually dispensed. Items with consignment potential include intraocular lenses, orthopedic implants and pulse generators. For example, the University of California San Francisco Medical Center implements consignment contracts with its suppliers of various products (University of California, 2006). Large retailers, such as Wal-mart and Target, and category specialists, including Toys "R" Us, often use consignment with their small or specialized suppliers (Gosman and Kelly, 2002).

1.1.1 Consignment contracts

The APICS Dictionary (Blackstone and Cox, 2005) defines consignment as the process in which the supplier retains the full ownership of the inventory, places items at a retail location (or, virtually, anywhere) with no payment received until the goods are sold to consumers. This process presents to the retailer the advantage of not incur, if any, risk associated with the uncertainty of the demand other than the storage cost and the opportunity cost due to shelf space usage. The supplier bears all risk associated with demand uncertainty. If the merchandise does not sell, no money is exchanged. The supplier gains access to consumers and transfers the responsibility and the cost of storage to the retailer, which could potentially increase sales volume and profits.

One of the most critical features of a consignment contract is the payment mechanism specified in the contract. The specific mechanism for determining the supplier's revenue impacts all parties in the contract. To our knowledge, two consignment contract terms are used in practice, and have been discussed in the literature. On the one hand, in (Wang et al., 2004), a Stackelberg game model is proposed in which the retailer, acting as the leader, offers the supplier a consignment contract which specifies the supplier's revenue share as a percentage of the retailer's revenue for each unit sold. On the other hand, in (Ru and Wang, 2010), the supplier is the leader and selects a fixed consignment price, specifying the amount of payment to the supplier for each unit sold at the retailer. The retailer acts as a follower and, based upon the consignment price selected by the supplier, decides the retail price and order quantity.

Our research focuses on a consignment contract under the two different payment schemes for the supplier previously mentioned: (i) a fixed consignment price per unit sold and (ii) a percentage of the revenue earned by the retailer as the supplier's revenue share.

1.1.2 Dual channel supply chain

Although suppliers can benefit from selling through a large retailer via consignment, it could be more profitable for the supplier to sell its products directly to consumers and retain the margin generally taken by retailers. Instead of solely selling on consignment with a retailer, suppliers also often sell their products directly to end users and bypass a downstream retailer. Realizing the great potential of the Internet to reach consumers, many companies have added direct channel operations while continuing to sell through a retailer (Tsay and Agrawal, 2004b). Examples include *L' Occitane en Provence*, a company that produces body, face, and home products, and that sells its products directly to consumers at its store locations or through its website (`http://usa.loccitane.com`) as well as on Amazon.com. *Aldo Group*, a Canadian company that owns and operates a chain of shoe and accessory stores, has its own online retail store and also sells through Amazon.com.

The existence of a dual channel could imply more shopping options and price savings due to additional retail competition for consumers. The implications for the supplier and the retailer are, however, not straightforward. Adding a direct channel could lead to greater demand due to access to new customers, and hence a higher profit for the supplier; on the other hand, higher competition on the retail market could lower prices and thus profits. This raises the question of how the supplier's dual channel affects the entire supply chain, under a consignment contract.

1.1.3 Competition

The majority of research in consignment contracts has focused on a channel structure consisting of a single supplier and a single retailer (Wang et al., 2004; Li et al., 2009; Ru and Wang, 2010). A large number of researchers have considered horizontal competition in the supply chain and contracts (for example, (Choi, 1996; Dana Jr. and Spier, 2001; Bernstein and Federgruen, 2005; Yao et al., 2008b; Cachon and Kok, 2010)). (Wang, 2006) studied the equilibrium decisions of multiple competing suppliers of complementary products in consignment contracts. (Zhang, 2008) has included competition between two manufacturers of substitutable products under deterministic demand. However, none of these studies has considered consignment contracts.

Our work differs from prior studies in consignment contracts in the following ways: (1) We assume the demand is stochastic and analyze the effect of demand uncertainty on the equilibrium result. (2) We consider horizontal competition at the supplier or retailer level in addition to vertical competition between the retailer and the supplier levels, (3) We relax the model assumption of perfectly complementary products to allow product differentiation. (4) We extend the consignment demand model to a dual channel supply chain. This allows us to study the impact of product differentiation as well as store differentiation. Incorporating these new features in our model enables us to gain new insights that have practical implications for research in consignment contracts.

1.2 Research questions and contributions

The large majority of research on consignment contracts has focused on a channel structure consisting of a single supplier and a single retailer (Wang et al., 2004; Li et al., 2009; Ru and Wang, 2010). Consignment contracts in the presence of retail competition have received little or no attention. A large number of researchers have considered horizontal competition in the supply chain and contracts (for example, (Choi, 1996; Dana Jr. and Spier, 2001; Bernstein and Federgruen, 2005; Yao et al., 2008b; Cachon and Kok, 2010)), but none of them studied consignment contracts. Regarding horizontal competition in the framework of consignment contracts, only the effect of upstream competition among *suppliers* has been marginally discussed (Wang, 2006; Zhang, 2008). Downstream competition among retailers has not. Our contribution is significant in that it fills the gap in the literature by considering consignment in a setting where retailers compete horizontally, thereby extending the research in consignment contracts from a non-competing market to the more realistic setting in which downstream competition exists. The first part of the thesis, thus, introduces horizontal competition at the retailer level in addition to vertical competition between the supplier and the retailers.

Although upstream competition among suppliers has been discussed to some extent, existing work relies upon some restrictive assumptions, such as perfect complementarity of products and deterministic demand (Wang, 2006; Zhang, 2008). Despite the popularity of direct sales channel, no existing consignment study has considered a supply chain in which a supplier can sell directly to consumers through a direct channel, in addition to selling through the retailer via consignment contracts. The second part of the thesis contributes to the literature on consignment contracts and competition by providing insights on how the presence of supplier competition affect the decisions and performance of the supply chain. In addition, we are interested in how the presence of a direct channel affects the channel decisions and profits.

The purpose of this thesis is to investigate the effect of retail competition and supplier competition on the decisions and the channel performance under different scenarios. This includes the case with consignment price contracts, consignment with revenue share contracts and dual channel supply chain.

The results of the study are used to answer the following research questions:

1. How do consignment contracts compare with price-only contracts from the entire system's perspective? Do consignment contracts improve channel performance? Do consignment contracts benefit all supply chain members?

2. Which contract term (e.g., fixed consignment price or revenue sharing) should be used in a consignment contract to provide the greatest benefit to all supply chain members?

3. How does the presence of competition among retailers and the level of retailer differentiation affect decisions such as retail prices and consigned quantity?

4. Does increased retail differentiation improve total channel profits and the profit of each supply chain member? Are these patterns of improvement the same across different supplier payment mechanisms?

5. How does the decentralized channel under consignment contracts perform, compared to the centralized channel with respect to the total channel profits, in the presence of competition among suppliers?
6. How do the members of of the supply chain make their pricing and quantity decisions, in a supply chain under supplier competition? Does supplier competition improve total channel profits and each supply chain member's profit?
7. Under what circumstances, does one supplier benefit from adding a direct channel, in addition to its retail channel? How does this additional direct channel affect other members of the supply chain members?

1.3 Structure of the thesis

One of our contributions to research in consignment contracts is to provide insights on how demand uncertainty and the presence of competition affect the decisions and performance of the supply chain. In order to achieve this, we consider a supply chain under uncertainty, in which competition between retailers and/or competition between suppliers exist.

In the first part of the thesis, we utilize game-theoretic concepts by modeling the problem as a Stackelberg game between a supplier and two competing retailers and a non-cooperative Nash game between the two retailers. We solve a two-period problem via backward induction in order to find the Stackelberg/Nash equilibrium between the supplier and two retailers.

In the second part of the thesis, we build a game-theoretic model in order to analyze consignment contracts between two competing suppliers and one retailer acting as the leader. Moreover, we are interested in how a direct channel for one of the suppliers affects the channel decisions and profits. We solve a two-period problem via backward induction in order to find the Stackelberg/Nash equilibrium between the agents of the supply chain.

We first determine the best response functions of each player. Subsequently, we derive certain conditions of the profit functions, first order or second-order conditions that ensure the existence of a unique equilibrium of the game. The presence of competition and demand uncertainty significantly increase the level of analytical complexity, and in certain cases, closed form solutions for the equilibrium solution are unattainable. In those cases, we use extensive numerical computations to solve the equilibrium and gain useful insights.

The thesis is organized as follows: **Chapter two** provides an overview of related literature. **Chapter three** describes the non-linear demand model that incorporates retail competition as well as demand uncertainty. We examine the effect of retail competition on the equilibrium channel decisions and the profits, under retailer-managed consignment inventory. Then, we derive expressions for the equilibrium retail prices, supplier's price (or revenue share), and stocking quantity under three types of contracts. Finally, we provides a numerical study to investigate the equilibrium solutions for each contract. In **Chapter four**, we propose a linear demand model for two supply chain structures: (1) consignment channel with supplier competition and (2) dual channel supply chain. We first present how product differentiation (supplier competition) impacts the equilibrium decisions and profits for the supply chain members. Moreover, this chapter studies the effect of a supplier's additional direct channel on the decisions and profits of each supply chain member. **Chapter five** concludes and suggests future research directions. An appendix contains the detailed mathematical proofs.

CHAPTER 2

LITERATURE REVIEW

In this chapter, we provide an overview of the literature on topics closely related to different aspects of the thesis. We first review the literature on supply chain coordination via contracts and on consignment contracts. Next, we cover literature related to competition. Finally, we present references that are relevant to a dual channel supply chain.

2.1 Supply chain coordination via contracts

It is well known in the literature on supply chain management that total supply chain profits in a decentralized channel are in general lower than those in an integrated or centralized channel–a characteristic known as double marginalization. In a decentralized chain, supply chain members maximize their own objective and their "selfish" actions result in poor overall channel performance (i.e., lower total channel profits than in a centralized channel). The loss in total supply chain profit in the decentralized chain is referred to as *supply chain inefficiency*. Several types of contracts have been introduced and implemented to coordinate the supply chain. A contract coordinates the supply chain if it provides incentives to all participants so that the total profits in the decentralized channel match those in the centralized channel (Cachon, 2003). A variety of contractual relationships for reducing supply chain inefficiency, their benefits and drawbacks have been discussed in the literature: e.g. the price-only contract (wholesale-price contract), buy-back contract, price-discount contract, revenue sharing contract and consignment contract.

The simplest and the most common contracts are price-only contracts, or wholesale price contracts. As described in (Perakis and Roels, 2007), the price-only contract specifies a constant price per unit purchased from a supplier by a retailer. Despite its popularity, (Cachon, 2003) shows that the price-only contract cannot coordinate the supply chain since the retailer does not order enough inventory to maximize the supply chain total profits. Buy-back contracts allow a retailer to return unsold merchandise up to a specified amount at an agreed-upon price. This contract gives the retailer an incentive to increase her order quantity, resulting in higher product availability and higher profits for both the supplier and the retailer. However, the buy-back contract may result in surplus inventory for the supplier and it may lead to inflated retail orders (Tsay, 1999). (Bernstein and Federgruen, 2005) study a price-discount contract, in both linear and nonlinear forms. Closely related to a buy-back contract, a linear price-discount contract specifies a wholesale price and a buy-back rate. These two terms are linear functions of the chosen retail price. Bernstein and Federgruen conclude that such a contract can coordinate the price-setting newsvendor problem in a supply chain with one supplier and multiple non-competing retailers. Furthermore, they demonstrate that a nonlinear version of the price-discount contract can coordinate the channel in the case of competing retailers.

It is common in the literature for a price-only contract to be used as a benchmark for comparing the channel performance (total profit earned) of several types of contracts. For example, (Cachon, 2004) studies and compares the allocation of inventory risk under several types of contracts (e.g., an advance-purchase contract) against a wholesale price contract. (Özer and Wei, 2006) focus on comparing capacity-reservation and advance-purchase contracts with a price-only contract. (Pasternack, 2002; Gerchak and Wang, 2004; Yao et al., 2008a; Pan et al., 2010; Katok and Wu, 2009) compare the channel performance of the revenue sharing contract with the performance of the price-only contract. (Su and Zhang, 2008) compare buyback, markdown money, and sales rebates contracts with a price-only contract. (Chen, 2011) compares the performance of a return-discount contract and a price-only contract.

(Cachon and Lariviere, 2005) introduce a *revenue sharing contract* with a single retailer and a single supplier. Under such a contract, the supplier charges the retailer a wholesale price for each unit purchased and a percentage of the revenue that the retailer generates. The authors find that revenue sharing is equivalent to buy-backs in the newsvendor case and is equivalent to price discounts in the price-setting newsvendor case. (Yao et al., 2008a) find that the benefits of revenue sharing contracts for each supply chain member vary depending on the price sensitivity and demand variability. (Linh and Hong, 2009) generalize a revenue sharing contract with a single retailer and a single supplier to a two-period newsvendor problem. They find that the optimal revenue sharing ratio for the retailer is linearly increasing in wholesale prices. In every revenue sharing study cited above, the retailer makes decisions on the inventory level and the retail price. (Pan et al., 2010) consider both the case when retailer(s) or supplier(s) make these decisions, and compare a revenue sharing contract with a wholesale price contract under deterministic demand in a supply chain with two different channel structures: (a) two manufacturers and one retailer; (b) one manufacturer and two retailers.

2.2 Consignment contracts

Consignment is different from other types of contracts in terms of inventory ownership and time of payment. (Wang et al., 2004) propose a single product consignment contract with revenue sharing between a supplier and a retailer. The retailer first decides the fraction of the revenue to keep for each unit sold; the supplier then chooses the retail price and the quantity placed at the retailer's. The authors assess the impact of the retailer's share of the channel cost and the demand-price elasticity on channel profits. They conclude that the loss of profit in a decentralized supply chain decreases with the retailer's cost share and increases with the demand-price elasticity. (Wang, 2006) extends consignment contracts to a supply chain with multiple suppliers of complementary products and a single retailer. The suppliers decide on the price and product quantity, either simultaneously or sequentially.

Contrary to (Wang et al., 2004), (Li et al., 2009) utilize a cooperative game approach (Nash bargaining model) to coordinate the decentralized consignment channel with a single supplier and a single retailer. They show that coordination between the two supply chain partners can be achieved. (Zhang et al., 2010) study coordination under a consignment channel with a multi-tier bonus structure and revenue-sharing with side payment (RSSP). With the bonus structure, a supplier earns a bonus i.e., a reduction in the retailer's commission, when its total sales revenue exceeds a certain threshold. With the RSSP structure, a supplier pays a fee (side payment) to a retailer for permission to sell its products and a discount in the retailer's commission. The authors find that consignment contracts with a bonus structure cannot coordinate the channel, whereas consignment contracts with RSSP can coordinate the channel.

(Ru and Wang, 2010) further study a consignment contract in two different settings: a retailer-managed consignment inventory (RMCI) and a vendor-managed consignment inventory (VMCI) settings. They demonstrate that both settings lead to an equal split of the channel profits between the supplier and the retailer. However, the supply chain inefficiency under VMCI is lower than that under RMCI. They thus conclude that both the supplier and the retailer benefit from a supplier-controlled inventory.

The consignment channel with competition has been recently discussed in the literature. (Wang, 2006) extends consignment contracts to a supply chain with multiple suppliers of complementary products and a single retailer. The suppliers make decisions either simultaneously or sequentially. They find that the profits for both the manufacturers (in the direct channel) and the retailer always increase from a change from the simultaneous to the sequential decision structure. However, manufacturers in the indirect channel, do not benefit from this change.

2.3 Competition

(Choi, 1996) describes three major factors that could be used to represent the nature of channel competition: (1) the channel structure describes how products flow from suppliers to retailers, (2) the channel leadership determines whether a supplier or a retailer has the power to exploit the other's reaction function, and (3) the horizontal product and store differentiations specifies competing products and stores. The supply chain with one common retailer selling competing products from multiple suppliers have been widely studied. (Choi, 1991) examines price competition between two manufacturers that produce substitutable products. (Cachon and Lariviere, 2005) study coordination via revenue-sharing contracts. They extend their main results for a single manufacturer and a single retailer setting to the case with one manufacturer and multiple retailers. (Cachon and Kok, 2010) consider two manufacturers who compete to supply a single retailer. They conclude that, in the presence of competition among manufacturers, it is more beneficial for the retailer to adopt the quantity discount and two-part tariff contracts than the wholesale-price contract. This finding is contrary to the results for the case with a single manufacturer

The literature on supply chain coordination with a single manufacturer selling to multiple competing retailers is very rich. (Dana Jr. and Spier, 2001) focus on a revenue sharing contract in the presence of perfectly competitive retailers. They conclude that revenue sharing can coordinate price-setting retailers, while each retailer earns zero profit in equilibrium. (Bernstein and Federgruen, 2005) investigate the equilibrium behavior of a decentralized channel with a monopolistic supplier and competing retailers under demand uncertainty. They employ a combination of wholesale price and buy-back contracts to coordinate the decentralized channel. (Yao et al., 2008b) study a revenue sharing contract for coordinating a supply chain with one manufacturer and two competing retailers facing stochastic demand. They analyze the impact of demand variability, price-sensitivity, and level of competition on decisions such as retail prices, order quantity and profit sharing between manufacturer and retailers. The channel structure in one of the models considered in (Pan et al., 2010) consists in one manufacturer interacting with two retailers. They focus their study on comparing the channel performance of the revenue-sharing contract (without consignment) with the channel performance of the price-only contract. None of these studies, however, considers consignment. We thus analyze the effect of downstream competition between two retailers under consignment contracts in Chapter 3.

A significant amount of literature focuses on supply chain coordination with a single retailer and multiple competing suppliers. Well known examples include (McGuire and Staelin, 1983; Choi, 1991; Pan et al., 2010). (McGuire and Staelin, 1983) examine Nash equilibria in duopoly structures, in which each of two competing manufacturers can distribute its goods through a common and exclusive retailer. They find that product substitutability affects the supply chain's equilibrium decisions and profits. (Choi, 1991) considers a supply chain consisting of two competing manufacturers and one common retailer. He focuses on three different noncooperative games of different power structures in a channel for linear and non-linear demand functions. He investigates the effect of power structures, product differentiation and production cost on equilibrium decisions and profits. Interestingly, he finds that the prices and profits are increasing as products become less differentiated under the linear demand function. The channel structure in one of the models considered in (Pan et al., 2010) consists of two manufacturers and one retailer. In their setting, each manufacturer can select either a price-only contract or a revenue-sharing contract with his retailer. They focus their study on comparing the channel performance of the revenue-sharing contract (without consignment) with the price-only contract, under different channel power structure. They conclude find that it is profitable for either one manufacturer or both manufacturers to adopt a revenue-sharing contract under the manufacturer-dominated scenario, given certain system parameters.

Nevertheless, only a few studies consider the effect of supplier competition in a consignment setting. (Wang, 2006) examines the equilibrium price and stocking factor of multiple suppliers of perfectly complementary products and the revenue share decision of the retailer. He concludes that competition among suppliers lead to higher product prices and lower production quantities. Results are opposite from those in substitutable products settings. (Zhang, 2008) extends the work by (Wang et al., 2004) by including competition between two manufacturers, producing substitutable products under deterministic demand. He finds that higher product substitutability benefits the retailer. Conversely, the suppliers only benefit from higher product substitutability when product substitutability is not too strong. Our work in Chapter 4 is closely related to (Wang, 2006) and (Zhang, 2008), in that we study the channel performance under consignment contracts in a supply chain with supplier competition. However, our work focuses on a more realistic setting in which each supplier competes by selling substitutable, but differentiated, products with uncertain demand. In contrast to perfectly complementary products, substitutable (but differentiated) products do not have to be consumed together, i.e., there can be a different demand of each of the products.

2.4 Dual channel supply chain

The rise of the use of Internet Commerce enables the suppliers to set up their own direct sales channel, in addition to selling through the retailer. Dual channels, the hybrid channels of direct and reseller-intermediated channels, are widely discussed in the marketing and economics literature (see (Tsay and Agrawal, 2004b) and references therein). A substantial number of researchers focus on a dual channel with a single supplier and a single retailer, and its impact on the channel's decisions and profits (for example, (Tsay and Agrawal, 2004a; Dumrongsiri et al., 2008; Cai, 2010; Niu et al., 2012)). However, only a few studies consider a supply chain channel with the existence of supplier competition (Kurata et al., 2007; Liao and Tseng, 2007; Chiang et al., 2003).

(Liao and Tseng, 2007) consider a wholesale price contract under a dual channel supply chain, with two manufacturers and one common retailer. Each manufacturer can decide whether or not to use a direct channel with its consumers. A retailer also can choose to sell its private brand, in addition to selling the manufacturers' products. The authors find that when products (manufacturers' and retailer's private brands) are less differentiated, the retail prices in the direct selling, private brand and retail channels decrease. This, in turn, leads to lower profits for the manufacturers. On the other hand, when a retailer and a manufacturer's direct channel are less differentiated, the retailer's profit decreases. They conclude that despite more intense market competition, manufacturers should implement a direct channel and the retailer should sell its private brand in addition to selling manufacturers' products. (Chiang et al., 2003) consider the effect of adding a direct channel to a conventional retail channel on prices and profits of a vertically integrated firm. The authors find that the presence of a direct channel (in addition to a retail channel) induces the retailer to lower the price, which, in turn, increases the demand as well as the profit in the retail channel. Additionally, they find that the presence of a direct channel increases channel efficiency, not only in a one-manufacturer and one-retailer setting, but also in a one-manufactuerer and oligopolistic retailers setting. (Wang, 2006) explores and compares equilibrium decisions and profits, under two different decision sequences in a supply chain including n manufacturers selling complementary products in a consignment channel. He shows that the difference in the manufacturers' profit between the simultaneous and sequential decision settings, depends critically upon whether the channel involves a retailer. To the best of our knowledge, no existing of consignment studies has considered a dual channel. Thus, we extend the research in consignment contracts by considering the effect of channel competition (i.e., store differentiation) from a supplier's direct channel. In particular, we are interested in how this additional direct channel impacts each of the supply chain members' decisions and profits.

CHAPTER 3

CONSIGNMENT CONTRACTS WITH RETAIL COMPETITION

This chapter focuses on a consignment contract under the two different payment schemes for the supplier previously mentioned: (i) a fixed consignment price per unit sold and (ii) a percentage of the revenue earned by the retailer as the supplier's revenue share. In order to understand the impact of these two types of consignment contracts, a price-only contract (wholesale price contract) is used as a benchmark to evaluate consignment contracts. The study quantifies the benefits to all members of the supply chain under different contract settings and helps determine which contract terms are most beneficial to the entire system as well as to the different parties involved.

3.1 Model assumptions

Consider a supply chain with one supplier and two retailers. We assume the supplier (S) produces one product and sells it through two competing differentiated retailers (R_1 and R_2). The supplier produces at a constant unit cost of $\$c_M$, and retailer i incurs a unit cost of $\$c_{R_i},\ i = 1, 2$ for handling and selling the product to consumers. Define $c = c_S + c_{R_1} + c_{R_2}$ as the total unit cost for the channel, and $\alpha_i = c_{R_i}/c$ as the share of the channel cost that is incurred at retailer i, $i = 1, 2$. Note $\alpha_1 + \alpha_2 < 1$.

We consider a demand for the product at each retailer during a single selling season that is price-dependent and uncertain. We use a multiplicative model to capture the randomness in the demand. A multiplicative demand uncertainty is widely used in the literature (see for example (Karlin and Carr, 1962; Petruzzi and Dada, 1999)). In the context of consignment, (Wang et al., 2004; Wang, 2006; Ru and Wang, 2010) also adopt this model. We thus model the demand for the product at retailer i, denoted by $D_i(\mathbf{p})$ where $\mathbf{p} = (p_1, p_2)$, as:

$$D_i(\mathbf{p}) = y_i(\mathbf{p}) \cdot \epsilon, \quad i = 1, 2, \tag{3.1}$$

where p_i, $i = 1, 2$ is the retail price charged by retailer i to consumers, $y_i(\mathbf{p})$ is the expected demand at retailer i and ϵ is a random scaling factor, representing randomness of the demand, with a mean value of 1, cumulative distribution function $F(\cdot)$ and probability density function $f(\cdot)$ that have support $[A, B] \subset \Re^+$ with $B > A$. Let $h(x) = f(x)/[1 - F(x)]$ denote the failure rate function.

We model the expected demand as an exponential function of both prices. In absence of competition, an exponential demand model (also called log-linear) $y(p) = ae^{-\beta p}$, where β is a price sensitivity parameter and p is the retail price, has been adopted in several studies in the supply chain management literature (Gallego and Van Ryzin, 1994; Petruzzi and Dada, 1999; Huang et al., 2006; Ru and Wang, 2010; Besbes and Maglaras, 2009). This model is also common in the economics literature (Greenhut et al., 1988; Jeuland and Shugan, 1988; Cowan, 2008). In the presence of competition, (Talluri and Van Ryzin, 2005; Simon, 2007) propose a natural extension of this demand model in the form of $y_i(p_i, \mathbf{p_{-i}}) = a_i \exp\left\{\sum_{j=1}^{N} \beta_{ij} p_j\right\}$, where β_{ij} is a price-elasticity coefficient and $\beta_{ii} > 0,\ \beta_{ij} \leq 0$ for $j \neq i$. The main motivation behind such a model is that it follows intuitive monotonicity properties (increasing in the competitor's price and decreasing in the price of the corresponding retailer) and that the logarithm of the demand is linear in prices. We also adopt this type of model in the case of two competing retailers by modeling the expected demand at retailer i as

$$y_i(\mathbf{p}) = ae^{-\beta p_i + \gamma p_{-i}}, \qquad a, \beta, \gamma > 0;\ \beta > \gamma. \tag{3.2}$$

Note that the expected demand at retailer i is a decreasing function of the retailer's own price p_i, and an increasing function of its competitor's price p_{-i}, where $-i = 2$ if $i = 1$ and $-i = 1$ if $i = 2$.

In this formulation, a is the primary demand of each retailer (i.e., demand if both prices were zero), β is each retailer's own price sensitivity of demand, and γ is the price sensitivity of demand with respect to the competitor's price. The larger the value of β (resp. γ), the more a retailer's expected demand is affected by a change in her own (resp. the competitor's) price. The assumption $\beta > \gamma$ indicates that sales at a given retailer are relatively more sensitive to price changes at the same retailer than at the competitor's, which is a standard assumption in economics when sellers are differentiated. Parameter γ is related to the level of retailer differentiation: the larger γ, the less differentiated the two retailers, and the more potential price competition.

Retail competition is incorporated into the model as retailers compete simultaneously on both retail prices and order quantities. Each retailer makes decisions (price and quantity decisions) that maximize its own objective (expected profit). Each retailer's objective depends on the decisions of the competitor via the demand function because the demand for the product at each retailer depends not only on its own price, p_i, but also on its competitor price, p_{-i}. Note that the retailers are not constrained to select the same retail prices. This type of retail competition model via demand dependency on the competitor's price is common to many studies in the operation management literature (McGuire and Staelin, 1983; Jeuland and Shugan, 1988; Choi, 1991; Choi, 1996; Pan et al., 2010; Yao et al., 2008b; Zhang, 2008).

Similarly to (Vives, 1984; Vives, 1985; Petruzzi and Dada, 1999), we impose a mild restriction on the demand distribution known as the increasing failure rate (IFR) condition.

Assumption 3.1.1. *The demand distribution satisfies the IFR property:* $h(x) = f(x)/[1 - F(x)]$ *is increasing in* x.

In practice, it is possible for the supplier to offer different contract parameters to different retailers in the industry. However, (Villas-Boas, 2008) suggests that "if the manufactured products sold through different retailers are the same, then they should be set at the same wholesale price." Another reason to support the common use of this assumption is a policy of banning wholesale price discrimination (Meyer and Fischer, 2004; Villas-Boas, 2008; Hastings, 2009), in which the manufacturer or the supplier is constrained to set a uniform, nondiscriminatory wholesale price for a brand sold at any of the retailers. Moreover, the assumption that the single supplier offers two competing retailers identical contract terms (i.e., the same consignment price or revenue share) is common to many studies in the contract literature (Dana Jr. and Spier, 2001; Yao et al., 2008a). This assumption has also been made by a number of studies of two-part tariff in competitive supply chains (Tsay and Agrawal, 2000; Xiao et al., 2005; Narayanan et al., 2005). As a result, we also make the assumption that the supplier offers the same contract terms to the two competing retailers.

3.2 Retailer managed consignment inventory with retail competition

The decision on the inventory quantity, in consignment contracts, can be made by the upstream supplier or the downstream retailers. The former arrangement is known as retailer managed consignment inventory (RMCI) and the latter arrangement is called vendor managed consignment inventory (VMCI). In this chapter, we focus our study on the RMCI in which the retailer has full control over the inventory quantity. This agreement is commonly used in supply chain in which the retailer is more powerful than the supplier (Gümüs et al., 2008). A prime example, Autozone, Inc., one of the biggest auto parts resellers, operates under a consignment contract (called pay-on-scan agreements) with their suppliers and chooses how much inventory to order.

3.3 Analytical results

We model the decision making of the two-tier supply chain as a Supplier-Stackelberg game. Following the standard newsvendor model (Cachon, 2003), the following sequence of events takes place: (1) the supplier, acting as a leader, offers a contract specifying the terms of payment to him from the retailers upon sale of items to consumers; (2) each retailer, acting as a follower, chooses the quantity Q_i to order from the supplier and the retail price p_i; (3) before the start of selling season, the supplier produces $Q = Q_1 + Q_2$ units of the product and delivers Q_i units to retailer i, $i = 1, 2$; (4) demand realizes; (5) transfer payments are made between supplier and retailers according to the agreed contract.

The supplier and two retailers play, vertically, a Stackelberg game with the supplier as the leader and the two retailers as followers. Horizontally, the two retailers play a Nash game, i.e. they simultaneously decide their prices and stocking quantities. We solve this equilibrium problem to find the Stackelberg/Nash equilibrium. Here, we present equilibrium solutions for three types of contracts and derive their implications.

3.3.1 Price-only (PO) contracts

In this type of contract, the supplier charges each retailer a wholesale price w_p per unit ordered. The time of payment and the ownership of inventory are key differences between price-only and consignment contracts. In price-only contracts, the retailers have full ownership of the inventory ordered and thus bear all the risks for all unsold units. The supplier receives the payment for all the units ordered by retailers, regardless of whether the retailer sells them. However, in a consignment agreement, the supplier retains ownership of merchandise even though items are at retail locations. The supplier receives no payment until the items are sold by retailers. Therefore, the retailers incur no risk for any unsold units. We use the price-only (PO) contract as a benchmark for evaluating the performance of consignment contracts with two different payment schemes.

In this section, we adapt a PO contract in a single retailer situation (Cachon and Lariviere, 2005) to a setting with our demand model and with two competing retailers that act as followers. The sequence of events is as follows: (1) the supplier specifies the wholesale price w_p for each unit ordered; (2) each retailer i simultaneously selects the retail price p_i and order quantity Q_i; (3) demand is realized. We find the equilibrium solution by using backward induction. We first derive each retailer's best response price and inventory quantity to the supplier's wholesale price decision.

3.3.1.1 Retailer i's selling price and stocking factor best response

At the second step of the decision sequence, for a given wholesale price w_p selected by the supplier, retailer i selects the retail price p_i and order quantity Q_i to maximize her own expected profit:

$$\pi_{R_i}(p_i, Q_i|w_p) = p_i E\{\min(D_i, Q_i)\} - (c\alpha_i + w_p)Q_i. \tag{3.3}$$

Similarly to (Petruzzi and Dada, 1999; Wang et al., 2004; Ru and Wang, 2010), we define $z_i = Q_i/y_i(\mathbf{p})$ the *stocking factor* of inventory. The stocking factor is defined as a surrogate for safety factor and is a measure of the deviation of the ordered quantity from the expected demand (see (Petruzzi and Dada, 1999) and the references therein). Using z_i as a decision variable instead of Q_i, we can rewrite retailer i's profit function (Equation 3.3) as

$$\begin{aligned}\pi_{R_i}(p_i, z_i|w_p) &= y_i(\mathbf{p})\{p_i(z_i - \Lambda(z_i)) - (c\alpha_i + w_p)z_i\} \\ &= ae^{-\beta p_i + \gamma p_j}\{p_i(z_i - \Lambda(z_i)) - (c\alpha_i + w)z_i\},\end{aligned}$$

where $\Lambda(z_i) = \int_A^{z_i}(z_i - x)f(x)\,dx$.

We provide two lemmas that will be useful in the remaining of the thesis.

Lemma 3.3.1. *The quantity* $z_i - \Lambda(z_i)$ *is positive for any given stocking factor* $z_i > 0$.

The following lemma is provided in (Ru and Wang, 2010).

Lemma 3.3.2. *Let* $G(z_i) = \frac{1}{1-F(z_i)} - \frac{z_i}{z_i - \Lambda(z_i)}$. *Under Assumption 3.1.1,* $G(z_i)$ *is increasing in* z_i.

To find the best response, denoted by $(\bar{z}_i, \bar{p}_i)$, that maximizes $\pi_{R_i}(p_i, z_i|w_p)$ for a given w_p, we first derive the retailer's best response retail price $\tilde{p_i}(z_i|w_p)$ for a given stocking factor z_i; we then find the best response stocking factor $\bar{z}_i$ that maximizes $\pi_{R_i}(\tilde{p_i}(z_i|w_p), z_i|w_p)$. Note that $\bar{z}_i$ and $\bar{p}_i$ are functions of w_p but we omit to explicitly show the dependency to keep the notation simpler. The results are summarized in the following propositions.

Proposition 3.3.3. *For any given stocking factor* z_i*, wholesale price* $w_p > 0$ *and price* p_{-i} *of retailer* $-i$*, retailer* i*'s unique best response price* $\tilde{p_i}(z_i|w_p)$ *is given by*

$$\tilde{p_i}(z_i|w_p) = \frac{1}{\beta} + \frac{(c\alpha_i + w_p)z_i}{z_i - \Lambda(z_i)}. \tag{3.4}$$

Proposition 3.3.3 implies in particular that each retailer's best response price (for a given z_i and w_p) is independent of the competitor's price decision. A price strategy that is independent of the competitor's is a property that appears in previous literature. (Moorthy, 1988) and (Choi, 1991) found that the class of constant price elasticity (iso-elastic) demand functions, such as a multiplicative function $q_i = ap_i^{-\beta}p_j^{\gamma}$, $i, j = 1, 2$; $i \neq j$, is known to result in price strategies that are independent of the competitor's strategy. Our demand model, $y_i(\tilde{\mathbf{p}}) = ae^{-\beta\tilde{p_i}+\gamma\tilde{p_j}}$ is, in fact, related to an iso-elastic demand function after a change of variable $\tilde{p}_i = log(p_i)$. Thus, this characteristic of the result in Proposition 3.3.3 is consistent with their findings.

According to Proposition 3.3.3, for a given stocking factor z_i and wholesale price w_p, retailer i's best response retail price $\tilde{p}_i(z_i|w_p)$ consists of two components: the first component $1/\beta$ is related to the sensitivity of consumers to price changes, and the second component $(c\alpha_i + w_p)z_i/(z_i - \Lambda(z_i))$ reflects the retailer's costs, that is, the wholesale price paid to the supplier and the holding cost, for each unit ordered. The first component increases in β because as consumers become more sensitive to price changes, the retailer lowers the price. The second component increases proportionately to the total cost per unit. Specifically, the effect of the retailer's costs on the retail price depends upon the ratio $z_i/(z_i - \Lambda(z_i)) = y_i(\mathbf{p})z_i/(y_i(\mathbf{p})(z_i - \Lambda(z_i)))$, representing the ratio of expected demand to the expected quantity sold. If this ratio is high, meaning that the retailer incurs a higher risk of over-ordering merchandise, then the retailer increases the retail price.

Proposition 3.3.4. *The retailer i's best response stocking factor $\bar{z}_i$ that maximizes the retailer i's profit $\pi_{R_i}(\tilde{p}_i(z_i|w_p), z_i|w_p)$ for a given w_p is uniquely determined as the solution of:*

$$\frac{1}{(c\alpha_i + w_p)\beta} + \frac{\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} = \frac{1}{1 - F(\bar{z}_i)}. \tag{3.5}$$

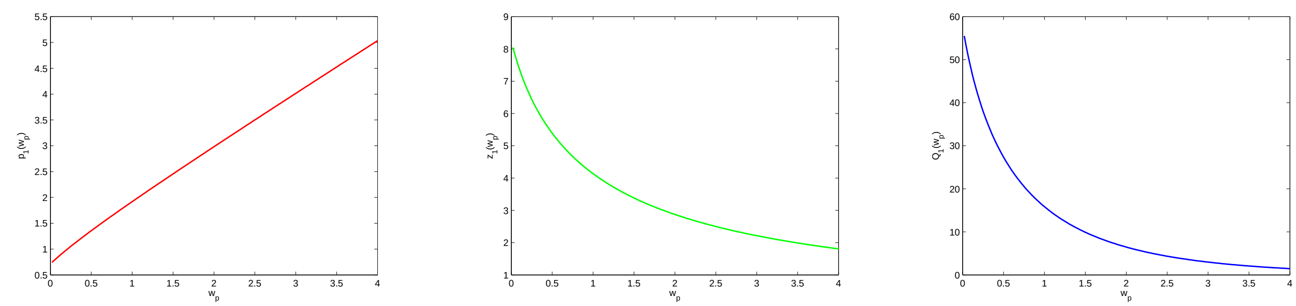
Similarly to the best response price, Proposition 3.3.4 suggests that the retailer's best response stocking factor $\bar{z}_i$ is independent of the competitor's stocking factor. Note that there is no closed form expression for $\bar{z}_i$. However, we are able to prove the following property.

Corollary 3.3.5. *The best response stocking factor* $\bar{z}_i$ *is decreasing in* w_p.

This result means that as the supplier charges the retailer more per item, the retailer orders less compared with the expected demand to lower her overstock risk exposure.

Using (Equation 3.4) and (Equation 3.5), we obtain that the best response retail price to a wholesale price w_p is

$$\bar{p}_i = \tilde{p}_i(\bar{z}_i|w_p) = \frac{1}{\beta} + \frac{(c\alpha_i + w_p)\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)}. \tag{3.6}$$



(a) Best response price $\bar{p}_1$ (b) Best response stocking factor $\bar{z}_1$ (c) Best response quantity $\bar{Q}_1$

Figure 1. Retailer 1's best response price, stocking factor and quantity as a function of the wholesale price w_p when $\beta = 2,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ in the PO contract

Figure Figure 1 illustrates the retailer's best response price, stocking factor and quantity as a function of the supplier's wholesale price. The retailer's best response price increases with w_p. This observation is intuitive because as the supplier's wholesale price increases, the retailer transfers this cost increase to consumers by increasing the retail price. The higher retail price causes the demand to go down, which leads to a lower quantity at each retailer. As a result, both the expected demand and quantity decrease with w_p. However, the order quantity decreases faster than the expected demand. Thus, the stocking factor decreases with the supplier's wholesale price (consistent with Corollary 3.3.5).

3.3.1.2 Supplier's wholesale price decision

At the first step, anticipating the retailers' reaction, the supplier sets the wholesale price w_p to maximize her own expected profit:

$$\begin{aligned}\pi_S(w_p) &= w_p(\bar{Q}_1 + \bar{Q}_2) - c(1-\alpha_1-\alpha_2)(\bar{Q}_1 + \bar{Q}_2) \\ &= [w_p - c(1-\alpha_1-\alpha_2)]\{y_1(\bar{\mathbf{p}})\bar{z}_1 + y_2(\bar{\mathbf{p}})\bar{z}_2\} \\ &= [w_p - c(1-\alpha_1-\alpha_2)]\{ae^{-\beta\bar{p}_1+\gamma\bar{p}_2}\bar{z}_1 + ae^{-\beta\bar{p}_2+\gamma\bar{p}_1}\bar{z}_2\}.\end{aligned}$$

To find the equilibrium solution w_p^*, we seek to maximize $\pi_S(w_p)$ over w_p. Since $\bar{z}_i$ and $\bar{p}_i$ are only known as implicit functions of w_p given by (Equation 3.5) and (Equation 3.6), this problem has no analytical solution. Thus, we find w_p^* numerically; numerical results are discussed in Section 3.4.

3.3.2 Consignment price (CP) contracts

In the consignment setting, the supplier retains full ownership of the inventory that is placed at retailers'. Therefore, the supplier bears all the risk associated with demand uncertainty while the retailers incur only a holding cost for over-ordered merchandise. Two types of consignment contract exist in the literature. In this section, we consider *consignment price* (CP) contracts. We will discuss the second type, *consignment contract with revenue share* (CR) in the next section.

Our model is different from previous studies (Wang, 2006; Zhang, 2008) in that we consider consignment contracts *with retail competition.* Furthermore, our focus differs from (Wang et al., 2004) due to the fact that we focus on retail-managed inventory, meaning that retailers decide the inventory quantity. This agreement is commonly used in supply chains (Gümüs et al., 2008). A prime example of such a setting is seen in the automotive market. For example, AutoZone, Inc., one of the biggest auto parts resellers, operates under a consignment contract (called pay-on-scan agreements) with suppliers.

Under CP contracts, decisions are made in two sequential steps. At the first step, the supplier decides the consignment price w corresponding to the amount of payment to be received from the retailers for each unit sold to consumers. At the second step, given this consignment price, each retailer simultaneously selects the retail price p_i and order quantity Q_i. We find the equilibrium solution by using backward induction. We first derive each retailer's best response price and inventory quantity to the supplier's consignment price decision.

3.3.2.1 Retailer i's selling price and stocking factor decision

At the second step of the decision sequence, for a given consignment price w selected by the supplier, retailer i selects the retail price p_i and order quantity Q_i to maximize her own expected profit:

$$\pi_{R_i}(p_i, Q_i|w) \quad = \quad [p_i - w]E\{min(D_i, Q_i)\} - c\alpha_i Q_i.$$

Notice that, in contrast with (Equation 3.3) for a PO contract, the price w paid to the supplier applies only to sold quantities and not to ordered quantities. Since $z_i = Q_i/y_i(\mathbf{p})$, the profit can be rewritten as

$$\begin{aligned} \pi_{R_i}(p_i, z_i|w) &= y_i(\mathbf{p})\{(p_i - w)(z_i - \Lambda(z_i)) - c\alpha_i z_i\} \\ &= ae^{-\beta p_i + \gamma p_j}\{(p_i - w)(z_i - \Lambda(z_i)) - c\alpha_i z_i\}. \end{aligned}$$

To find the best response, denoted by $(\bar{p}_i, \bar{z}_i)$, that maximizes $\pi_{R_i}(p_i, z_i|w)$ for a given w, we first derive the retailer's best response retail price $\tilde{p}_i(z_i|w)$ for a given stocking factor z_i; we then find the best response stocking factor $\bar{z}_i$ that maximizes $\pi_{R_i}(\tilde{p}_i(z_i|w), z_i|w)$. Note that $\bar{z}_i$ and $\bar{p}_i$ are functions of w but we omit to explicitly show the dependency to keep the notation simpler. The results are summarized in the following propositions.

Proposition 3.3.6. *For any given stocking factor z_i, consignment price $w > 0$ and price p_{-i} of retailer $-i$, retailer i's unique best response price $\tilde{p_i}(z_i|w)$ is given by*

$$\tilde{p_i}(z_i|w) = \frac{1}{\beta} + \frac{c\alpha_i z_i}{z_i - \Lambda(z_i)} + w. \tag{3.7}$$

For a given stocking factor z_i and consignment price w, the retailer's best response price $\tilde{p_i}(z_i|w)$ consists of two components. Similarly to the PO contract, the first component $1/\beta$ is related to the sensitivity of consumers to price changes. The second component $c\alpha_i z_i/(z_i - \Lambda(z_i)) + w$ reflects the retailer's total cost, including the holding cost for each unit ordered and the consignment price paid to the supplier for each unit sold. The effect of the retailer's holding cost on the retail price depends upon the ratio $z_i/(z_i - \Lambda(z_i))$, which is related to the risk of excess inventory. The effect of the consignment price, however, is independent of this ratio. This is because under the CP contract, each retailer only pays the consignment price to the supplier for each unit sold (not for each unit ordered). The retailer incurs no risk of loss associated with unsold merchandise, thus the retail price increases with an increase in the consignment price w, regardless of how the quantity ordered compares with the quantity sold.

Comparing the best response retail price in a CP contract (Equation 3.7) and a PO contract (Equation 3.4), we observe that for a fixed stocking factor z_i and a given supplier's price $w_p = w$, the PO best response retail price is higher than the CP best response retail price in a consignment price contract by $\frac{z_i}{z_i - \Lambda(z_i)}$. This finding reflects the fact that in a PO contract, the retailers incur more risk associated with over-ordered products than in a CP contract, and therefore charge consumers a higher retail price.

Proposition 3.3.7. *The retailer i's best response stocking factor $\bar{z}_i$ that maximizes the retailer i's profit $\pi_{R_i}(\tilde{p}_i(z_i), z_i|w)$ for a given w is uniquely determined as the solution of:*

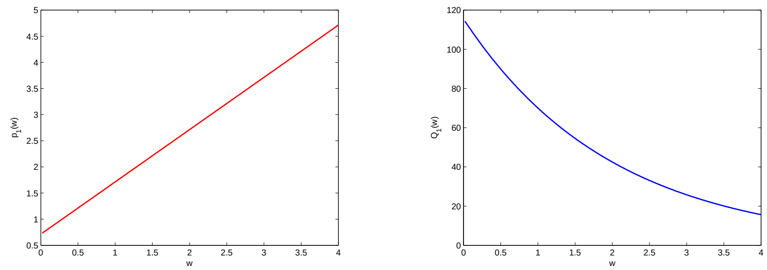
$$\frac{1}{c\alpha_i\beta} + \frac{\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} = \frac{1}{1 - F(\bar{z}_i)}. \tag{3.8}$$

Proposition 3.3.7 implies that $\bar{z}_i$ does not depend on the supplier's consignment price w, thus $\bar{z}_i$ is the retailer i's equilibrium stocking factor z_i^*. In particular, using (Equation 3.7), this implies that $\tilde{p}_i(z_i|w) - w$ is independent of w.

Using (Equation 3.7) and (Equation 3.8), we find that the best response retail price to a consignment price w is

$$\bar{p}_i = \tilde{p}_i(\bar{z}_i|w) = \frac{1}{\beta} + \frac{c\alpha_i\bar{z}_i}{z_i - \Lambda(\bar{z}_i)} + w. \tag{3.9}$$

Figure Figure 2 illustrates the retailer's best response price and quantity as a function of the consignment price. The best response retail price increases with w. Similarly to the PO contract, the retailers transfer any consignment price increase to consumers by increasing their retail prices, which causes the demand to decrease and thus the quantity to decrease.



(a) Best response price $\bar{p_1}$ (b) Best response quantity $\bar{Q_1}$

Figure 2. Retailer 1's best response price and quantity as a function of the consignment price w when $\beta = 2,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ in the CP contract

3.3.2.2 Supplier's consignment price decision

At the first step, anticipating the retailers' reaction to her decision, the supplier sets the consignment price w to maximize her own expected profit $\pi_S(w)$, given by

$$\begin{aligned}
\pi_S(w) &= w[E\{min(D_1, \bar{Q_1})\} + E\{min(D_2, \bar{Q_2})\}] - c(1-\alpha_1-\alpha_2)[\bar{Q_1}+\bar{Q_2}] \\
&= w[y_1(\bar{\mathbf{p}})(\bar{z_1} - \Lambda(\bar{z_1})) + y_2(\bar{\mathbf{p}})(\bar{z_2} - \Lambda(\bar{z_2}))] - c(1-\alpha_1-\alpha_2)[y_1(\bar{\mathbf{p}})\bar{z_1} + y_2(\bar{\mathbf{p}})\bar{z_2}] \\
&= ae^{-\beta\bar{p}_1+\gamma\bar{p}_2}[w(\bar{z}_1 - \Lambda(\bar{z}_1)) - c(1-\alpha_1-\alpha_2)\bar{z}_1] \\
&\quad + ae^{\beta\bar{p}_2+\gamma\bar{p}_1}[w(\bar{z}_2 - \Lambda(\bar{z}_2)) - c(1-\alpha_1-\alpha_2)\bar{z}_2].
\end{aligned}$$

To find the equilibrium solution, denoted by w^*, we maximize $\pi_S(w)$ over w.

Proposition 3.3.8. *The supplier's unique equilibrium consignment price w^* is given by*

$$w^* = \frac{k_1[(\bar{z}_1 - \Lambda(\bar{z}_1)) + (\beta-\gamma)c(1-\alpha_1-\alpha_2)\bar{z}_1] + k_2[(\bar{z}_2 - \Lambda(\bar{z}_2)) + (\beta-\gamma)c(1-\alpha_1-\alpha_2)\bar{z}_2]}{(\beta-\gamma)[k_1(\bar{z}_1 - \Lambda(\bar{z}_1)) + k_2(\bar{z}_2 - \Lambda(\bar{z}_2))]}$$

where $k_i = e^{-c\left(\frac{\beta\alpha_i\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} - \frac{\gamma\alpha_{-i}\bar{z}_{-i}}{\bar{z}_{-i} - \Lambda(\bar{z}_{-i})}\right)}$, $i = 1, 2$.

We interpret the result above in a special case.

3.3.2.3 Special case: Consignment price contracts for symmetric retailers

We now consider the case where the two retailers have symmetric cost structure, i.e., $\alpha_1 = \alpha_2 \equiv \alpha$, where $0 < \alpha < 0.5$. It follows from Proposition 3.3.7 that retailer i's equilibrium stocking factor z_i^* satisfies $z_1^* = z_2^* \equiv z^*$ where

$$\frac{1}{c\alpha\beta} + \frac{z_i^*}{z^* - \Lambda(z^*)} = \frac{1}{1 - F(z^*)}. \tag{3.10}$$

It follows from Proposition 3.3.8 that, since $k_1 = k_2 = e^{-\frac{c(\beta-\gamma)\alpha z^*}{z^* - \Lambda(z^*)}}$, the supplier's unique equilibrium consignment price w^* is given by

$$w^* = \frac{1}{\beta - \gamma} + \frac{c(1-2\alpha)z^*}{z^* - \Lambda(z^*)}. \tag{3.11}$$

Let $p_i^* = \tilde{p}_i(z_i^*|w^*)$; It follows from Proposition 3.3.7 that $p_1^* = p_2^* = p^*$ where

$$p^* = \frac{1}{\beta} + \frac{1}{\beta - \gamma} + \frac{c(1-\alpha)z^*}{z^* - \Lambda(z^*)}. \tag{3.12}$$

Notice that at equilibrium, the retailers' margin

$$p^* - w^* = \frac{1}{\beta} + \frac{c\alpha z^*}{z^* - \Lambda(z^*)}$$

does not depend on the competitor's price sensitivity γ. Therefore, the level of retail competition affects the retail price only through the consignment price selected by the supplier.

We first focus on the impact of the price sensitivity on the supplier's equilibrium consignment price and the retailers' equilibrium retail price.

Proposition 3.3.9.

- *The equilibrium stocking factor z^* decreases in β.*
- *The equilibrium supplier's consignment price w^* decreases in β.*
- *The equilibrium retail price p^* decreases in β.*

Proposition 3.3.9 indicates that the equilibrium stocking factor z^* decreases with the consumers' sensitivity to the retail price. Since the expected demand decreases when consumers become more sensitive to the retail price, retailers reduce their order quantity to reduce the risk of excess inventory. Specifically, the quantity ordered by each retailer Q^* decreases in β faster than the (expected) demand $y^*(\mathbf{p}^*)$. Furthermore, the consignment price w^* and the retail price p^* are decreasing functions of β: as consumers are more sensitive to the retail price, the supplier charges each retailer a lower consignment price so that retailers can lower their retail prices.

We now focus on the impact of retailer differentiation. Since the equilibrium stocking factor z_i^* is independent of the price cross-sensitivity, we study how the supplier's equilibrium consignment price w^*, the retailer's equilibrium selling price p_i^*, the retailer's equilibrium profit, the supplier's equilibrium profit and the total profit of the channel vary with the level of retailer differentiation.

Proposition 3.3.10.

- *The supplier's consignment price at equilibrium w^* increases in γ.*
- *The retail price at equilibrium p^* increases in γ.*
- *The ratio p^*/w^* decreases in γ.*
- *The retailer's order quantity at equilibrium Q^* increases in γ.*
- *The retailer's profit at equilibrium $\pi_R^{d^*}$ increases in γ.*
- *The supplier's profit at equilibrium $\pi_S^{d^*}$ increases in γ.*

Proposition 3.3.10 indicates that the supplier's consignment price increases in the price cross-sensitivity. This suggests that the supplier takes advantage of the increased competitiveness between less differentiated retailers (large γ) by charging a higher consignment price. The retailers transfer this price increase to consumers by increasing their retail price. This result is consistent with several existing studies (Jeuland and Shugan, 1988; Choi, 1991). Moreover, the ratio of the retail price to the consignment price p^*/w^* decreases when retailer differentiation decreases, implying that the retail price does not increase as fast as the consignment price when retailers are less differentiated. Furthermore, Proposition 3.3.10 indicates that the quantity ordered by each retailer increases in γ. The effect of retail differentiation on the order quantity is subject to two opposing effects: a direct effect, and an indirect effect through the retail price. On the one hand, the direct effect of an increase of γ is to *increase* the (expected) demand which could drive the ordered quantity to go up. On the other hand, as γ increases, the retail price increases which tends to make the (expected) demand *decrease* and thus would drive the quantity to go down. Because the direct effect is stronger, overall the order quantity increases when γ increases. Since both the supplier's consignment price, the retailers' selling price and the order quantity increase in γ, the profits for the supplier and the retailers increase as the level of retailer differentiation decreases.

We can quantify the effect of retail competition on the retailers' share of the decentralized channel profits (Lariviere and Porteus, 2001)

$$\frac{\pi_{R_1}^{d^*} + \pi_{R_2}^{d^*}}{\pi_S^{d^*}} = 1 - \frac{\gamma}{\beta}.$$

Note that the retailers' share of the channel profits is linearly decreasing in γ but increasing in β. That is, the retailers jointly earn proportionally less in channel profits when the competition is more intense, as expected. However, the retailers collect a larger share of channel profits when the demand is more sensitive to a change in their own prices. Surprisingly, the retailers' share of channel profits does not depend on the retailers' cost αc.

3.3.3 Consignment contracts with revenue share (CR)

In this section, we consider another type of consignment contract, known as *consignment contract with revenue share* (CR). Under CR contracts, decisions are made in two sequential steps. At the first step, the supplier decides the revenue share r of the retailers' revenue that she will receive for each unit sold to consumers. At the second step, given this revenue share, each retailer simultaneously selects the retail price p_i and order quantity Q_i. We find the equilibrium solution by using backward-induction. We first derive each retailer's best response price and inventory quantity to the supplier's revenue share decision.

3.3.3.1 Retailer i's selling price and stocking factor decision

At the second step, for a given revenue share r selected by the supplier, retailer i selects the retail price p_i and order quantity Q_i to maximize her own expected profit which is given by

$$\pi_{R_i}(p_i, Q_i|r) \quad = \quad (1-r)p_i E\{min(D_i, Q_i)\} - c\alpha_i Q_i.$$

Since $z_i = Q_i/y_i(\mathbf{p})$, the profit can be rewritten as

$$\begin{aligned} \pi_{R_i}(p_i, z_i|r) &= y_i(\mathbf{p})\{(1-r)p_i(z_i - \Lambda(z_i)) - c\alpha_i z_i\} \\ &= ae^{-\beta p_i + \gamma p_{-i}}\{(1-r)p_i(z_i - \Lambda(z_i)) - c\alpha_i z_i\}. \end{aligned} \tag{3.13}$$

To find the best response, denoted by $(\bar{p}_i, \bar{z}_i)$, that maximizes $\pi_{R_i}(p_i, z_i|r)$ for a given r, we first derive the retailer's best response retail price $\tilde{p}_i(z_i|r)$ for a given stocking factor z_i; we then find the best response stocking factor $\bar{z}_i$ that maximizes $\pi_{R_i}(\tilde{p}_i(z_i|r), z_i|r)$. Note that $\bar{z}_i$ and $\bar{p}_i$

are functions of r but we omit to explicitly show the dependency to keep the notation simpler. The results are summarized in the following propositions.

Proposition 3.3.11. *For any given stocking factor z_i, revenue sharing proportion $0 < r < 1$ and price p_{-i} of retailer $-i$, retailer i's unique best response price $\tilde{p_i}(z_i|r)$ is given by*

$$\tilde{p_i}(z_i|r) = \frac{1}{\beta} + \frac{c\alpha_i z_i}{(1-r)(z_i - \Lambda(z_i))}. \tag{3.14}$$

For a given stocking factor z_i and revenue share r, the retailer's best response price $\tilde{p_i}(z_i|r)$ consists of two components. Similarly to the PO and CP contracts, the first component $1/\beta$ is related to the consumers' sensitivity to price changes. The second component $\frac{c\alpha_i z_i}{(1-r)(z_i-\Lambda(z_i))}$ reflects the retailer's cost for each unit ordered. The effect of the retailer's cost on the retail price depends upon the ratios $\frac{z_i}{z_i-\Lambda(z_i)} = \frac{y_i(\mathbf{p})z_i}{y_i(\mathbf{p})(z_i-\Lambda(z_i))}$ and $\frac{1}{1-r}$. The first ratio represents the risk of excess inventory. The second ratio can be considered a "markup" factor due to the revenue share owed to the supplier.

Proposition 3.3.12. *The retailer i's best response stocking factor $\bar{z}_i$ that maximizes $\pi_{R_i}(\tilde{p_i}(z_i|r), z_i|r)$ for a given revenue share r is uniquely determined as the solution of:*

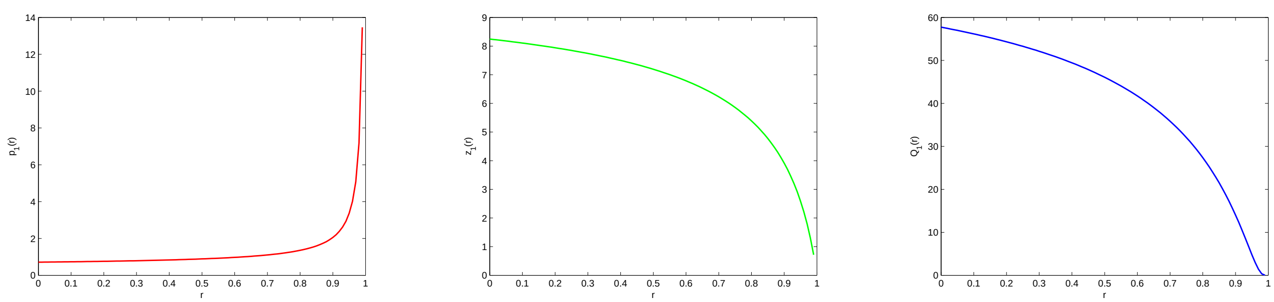
$$\frac{1-r}{c\alpha_i\beta} + \frac{\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} = \frac{1}{1-F(\bar{z}_i)}. \tag{3.15}$$

Proposition 3.3.12 indicates that the retailer's best response stocking factor does depend on the supplier's decision in a CR contract, as opposed to a CP contract (Proposition 3.3.7). One explanation for this difference is that at the best response in a CP contract the retailer's margin is independent of the supplier's decision, as noted in Section 3.3.2.3, while in a CR contract it depends on r.

Using (Equation 3.14) and (Equation 3.15), we find that the best response retail price to a revenue share r is

$$\bar{p_i} = \tilde{p_i}(\bar{z}_i|r) = \frac{1}{\beta} + \frac{c\alpha_i \bar{z}_i}{(1-r)(\bar{z}_i - \Lambda(\bar{z}_i))}. \tag{3.16}$$

Corollary 3.3.13. *The best response stocking factor $\bar{z}_i(r)$ is decreasing in r.*



(a) Best response price $\bar{p_1}$ (b) Best response stocking factor $\bar{z_1}$ (c) Best response quantity $\bar{Q_1}$

Figure 3. Retailer 1's best response price, stocking factor and stocking quantity as a function of the revenue share r where $\beta = 2.0,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ in the CR contract

Figure Figure 3 illustrates that the retailer's best response retail price increases with the supplier's revenue share r: when the supplier keeps a higher share of the retailers' revenue, the retailers transfer this increasing revenue loss to consumers. The higher retail price causes the demand to go down, which leads to a lower quantity at each retailer. While both the expected demand and order quantity decrease with the supplier's revenue share r, the order quantity decreases faster than the expected demand. Therefore, the stocking factor decreases with the supplier's revenue share (consistent with Corollary 3.3.13).

3.3.3.2 Supplier's revenue sharing fraction decision

At the first step, anticipating the retailers' reaction to her decision, the supplier sets the revenue sharing fraction r to maximize her own expected profit $\pi_S(r)$, given by

$$\begin{aligned}
\pi_S(r) &= r\bar{p}_1 E\{\min(D_1, \bar{Q}_1)\} - c(1-\alpha_1-\alpha_2)\bar{Q}_1 + r\bar{p}_2 E\{\min(D_2, \bar{Q}_2)\} - c(1-\alpha_1-\alpha_2)\bar{Q}_2 \\
&= y_1(\bar{\mathbf{p}}))[r\bar{p}_1(\bar{z}_1 - \Lambda(\bar{z}_1)) - c(1-\alpha_1-\alpha_2)\bar{z}_1] + y_2(\bar{\mathbf{p}}))[r\bar{p}_2(\bar{z}_2 - \Lambda(\bar{z}_2)) - c(1-\alpha_1-\alpha_2)\bar{z}_2] \\
&= ae^{-\beta\bar{p}_1+\gamma\bar{p}_2}[r\bar{p}_1(\bar{z}_1 - \Lambda(\bar{z}_1)) - c(1-\alpha_1-\alpha_2)\bar{z}_1] \\
&\quad + ae^{\beta\bar{p}_2+\gamma\bar{p}_1}[r\bar{p}_2(\bar{z}_2 - \Lambda(\bar{z}_2)) - c(1-\alpha_1-\alpha_2)\bar{z}_2]. \qquad (3.17)
\end{aligned}$$

To find the equilibrium solution, denoted by r^*, we would have to maximize $\pi_S(r)$ over r. Obtaining an analytical solution for this maximization problem is intractable, therefore, we use numerical methods as shown in Section 3.4.

3.3.3.3 Special case: Consignment contracts with revenue share under deterministic demand

In order to shed some light on the supplier's equilibrium revenue share r^*, we consider a special, more tractable case. In this section, we assume that the demand function is deterministic, i.e., $\epsilon = 1$ and $D_i(p) = y_i(p)$. Since there is no stochasticity, the quantity ordered by the retailers matches the demand (i.e. the stocking factor equals 1) and the only decision left to retailers is the retail price.

It follows from Proposition 3.3.11 that the best response retail price of retailer i, $\bar{p}_i$, to the supplier's revenue share r is given by

$$\bar{p}_i = \frac{1}{\beta} + \frac{\alpha_i c}{1-r}. \tag{3.18}$$

It follows from (Equation 3.17) that the supplier's profit can be expressed as

$$\pi_S(r) \quad = \quad ae^{-\beta p_1 + \gamma p_2}[rp_1 - (1-\alpha_1-\alpha_2)c] + ae^{-\beta p_2 + \gamma p_1}[rp_2 - (1-\alpha_1-\alpha_2)c]. \tag{3.19}$$

Using (Equation 3.18) into (Equation 3.19), we find

$$\begin{aligned} \pi_S(r) \quad = \quad & ae^{-\beta[\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}]+\gamma[\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}]}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right] \\ & +ae^{\beta[\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}]+\gamma[\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}]}\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]. \end{aligned}$$

Proposition 3.3.14. *If $\frac{\gamma}{\beta} < \frac{\alpha_1}{\alpha_2} < \frac{\beta}{\gamma}$, the supplier's equilibrium revenue share r^* is the solution of*

$$\sum_{i=1}^{2} e^{\frac{-(\beta\alpha_i - \gamma\alpha_{-i})c}{(1-r)}}\left\{\frac{1}{\beta}+\frac{\alpha_i c}{(1-r)}+\frac{\alpha_i cr}{(1-r)^2}-\left[\frac{(\beta\alpha_i-\gamma\alpha_{-i})c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_i c}{(1-r)}\right)-(1-\alpha_i-\alpha_{-i})c\right]\right\} = 0.$$

In the proposition above, the condition on α_1/α_2 guarantees that the supplier's profit function is bounded from above. If $\frac{\alpha_1}{\alpha_2} \notin \left(\frac{\gamma}{\beta}, \frac{\beta}{\gamma}\right)$, then the supplier's profit function is unbounded.

3.4 Numerical results

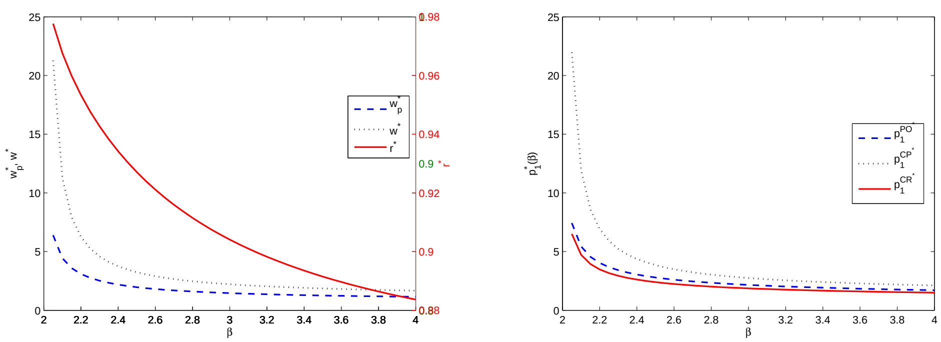
In this section, we obtain numerically the equilibrium quantities that could not be obtained in closed-form in previous sections, and we interpret the findings.

Our numerical study is geared at understanding the impact of price sensentivity parameters β and γ on the performance of consignment contracts in comparison to the price-only contract. It is common in the literature to focus on the impact of such parameters (Yao et al., 2008a; Yao et al., 2008b; Lau and Lau, 2002). We first investigate the effect of the price sensitivity parameter β on the equilibrium decisions and profits. In order to properly evaluate this effect, we need to isolate it from the effect of other parameters (such as the cross-price sensitivity γ and each retailer's share of the channel cost α_i) by keeping all these parameters constant. Likewise, when we next consider the effect of retailer differentiation on the equilibrium decisions and profits through parameter γ, we keep β and α_i constant. In both cases, we choose values of α_1 and α_2 that are equal to avoid introducing a cost difference that could bias the effect of the parameter of interest.

The random perturbation on the demand, ϵ, is assumed to follow the uniform distribution on $[A, B]$. Following previous numerical studies (Choi, 1991; Li et al., 2009; Zhang et al., 2010), we set $a = 10,\ c = 1$, and $\alpha_1 = \alpha_2 = 0.125$. Moreover, we choose $A = 0$ and $B = 2$ in order to ensure that the perturbation on the demand has a mean value of 1.

3.4.1 The effect of the price sensitivity factor

We study the impact of the price sensitivity parameter β on the supplier's equilibrium price (PO and CP contracts), equilibrium revenue share (CR contract), equilibrium retail price, equilibrium quantity and equilibrium profits. The values of the parameters are chosen to ensure that $\beta > \gamma$. The value of γ is fixed at $\gamma = 2$.

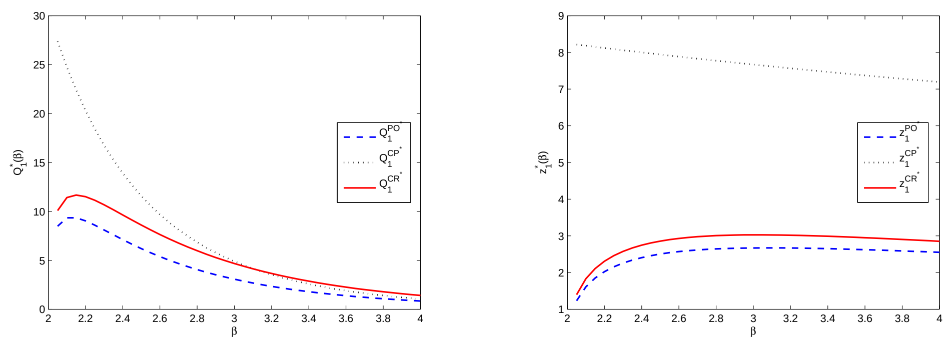


(a) Supplier's wholesale price w_p, consignment price w and revenue share r

(b) Retailer i's selling price p_i^*

Figure 4. Supplier's price (or revenue share) and retailer's selling price as a function of β under three contracts

Figure Figure 4(a) demonstrate that the supplier's wholesale price (PO contract), consignment price (CP contract), and revenue share (CR contract) decrease in β. Indeed, the supplier must decrease the price/ revenue share charged to the retailers when consumers are more sensitive to price changes. As a result, the retail price decreases in β under all types of contracts, as seen in Figure Figure 4(b), because the retailers transfer their decreased loss of revenue to consumers. The fact that the consignment price and the retail price decrease in β under the CP contract is consistent with Proposition 3.3.9.

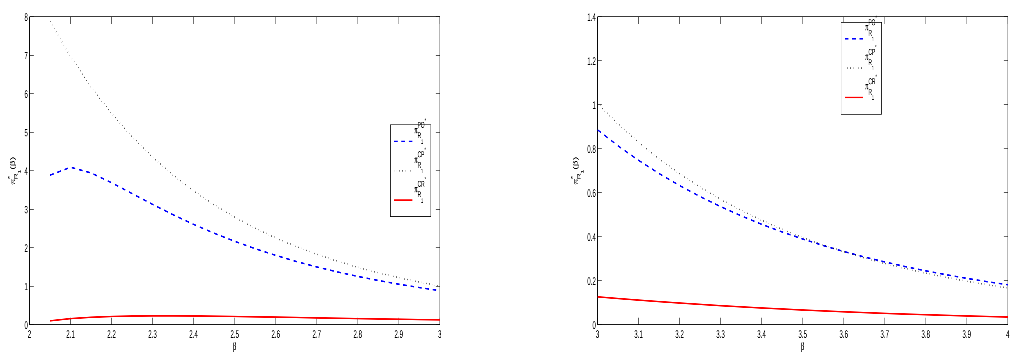


(a) Retailer i's order quantity Q_i^* (b) Retailer i's stocking factor z_i^*

Figure 5. Retailer i's stocking quantity and stocking factor as a function of β under three contracts

The effect of the price sensitivity parameter β on the order quantity Q_i^* depends on the type of contract. An increase of β drives the (expected) demand to decrease, everything being kept constant (direct effect). On the other hand, as β increases, retail prices decrease, which could cause the (expected) demand to increase (indirect effect). The cumulative effect of β is thus a combination of two opposite effects. Figure Figure 5(a) illustrates that in the CP contract, the direct effect is stronger, leading to a decrease of the order quantity in β. However, in the PO and CR contracts the order quantity is not a monotonic function of β: it first increases then decreases. One explanation is that the direct effect of β (which causes the demand to decrease) dominates the the indirect effect of β (which drives the demand to increase) when price sensitivity β is low. However, the indirect effect prevails when price sensitivity β is high.

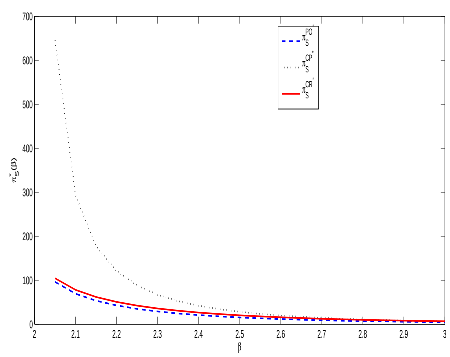
Figure Figure 5(b) shows the effect of the price sensitivity parameter on the stocking factor. The stocking factor in the PO and CR contracts is not monotonic with β, due to the non-monotonicity of the order quantity with β. The stocking factor in the CP contract, however, decreases in β, which is consistent with Proposition 3.3.9.



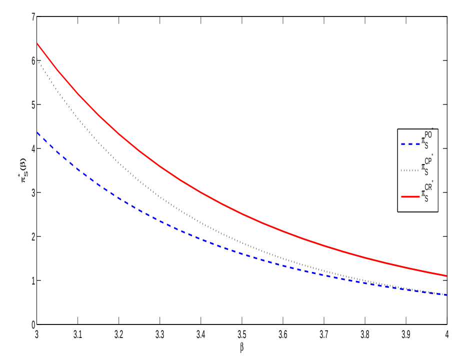
(a) Retailer i's profit $\pi^*_{R_i}$ for low price sensitivity (b) Retailer i's profit $\pi^*_{R_i}$ for high price sensitivity

Figure 6. Retailer i's profit as a function of β under three contracts

Figure Figure 6 depicts the effect of β on the retailers' profits. The retailers' profits decrease in β under the CP contract because the retail price and the order quantity decrease in β. The retailers' profits under the PO and CR contracts, however, do not display a monotonic pattern because the order quantity is not monotonic. No contract yields higher profits to the retailers than the other two contracts for any level of price sensitivity. Specifically, when the price sensitivity is low, the CP contract yields a higher profit to the retailers than both the PO and the CR contracts; when the price sensitivity is high, the PO yields higher retailers' profits than both consignment contracts.



(a) Supplier's profit π_S^* for low price sensitivity



(b) Supplier's profit π_S^* for high price sensitivity

Figure 7. Supplier's profit as a function of β under three contracts

The supplier's profit decreases in β in all contracts since the supplier's price/ revenue share decreases in β, as depicted in Figure Figure 7. Surprisingly, the share of the channel profits for the retailers $\frac{\pi_{R_1}^*+\pi_{R_2}^*}{\pi_S^*}$ increases in β for all contracts (Figure Figure 8). This indicates that the supplier's profit decreases in β at a higher rate than the retailers' profits.

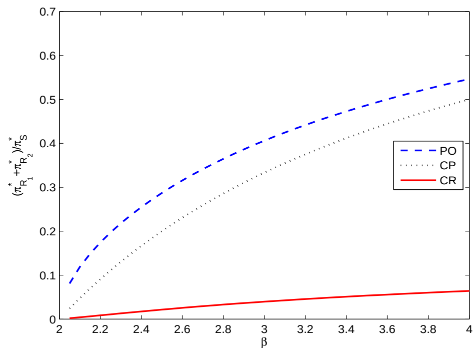


Figure 8. Retailer's share of the total channel profits as a function of β under three contracts $\frac{\pi^*_{R_1}+\pi^*_{R_2}}{\pi^*_S}$

The effect of the price sensitivity parameter on the decisions and profits for each of the supply chain members is summarized in Table Table I.

3.4.2 The effect of retailer differentiation

The major difference between our work and previous studies in consignment contracts (Wang et al., 2004; Wang, 2006; Zhang, 2008; Ru and Wang, 2010) is that we incorporate retailer differentiation into our model. The price cross-sensitivity γ represents the price sensitivity with respect to the competitor's price, and captures retailer differentiation. That is, if the retailers are less differentiated, then γ is larger (closer to β).

The values of the parameters are chosen to ensure that $\beta > \gamma$. The value of β is fixed at $\beta = 4$.

We now examine the effect of retailer differentiation on equilibrium prices, quantities and profits. Figure Figure 9 suggests that the supplier's wholesale price (PO contract), the consign-

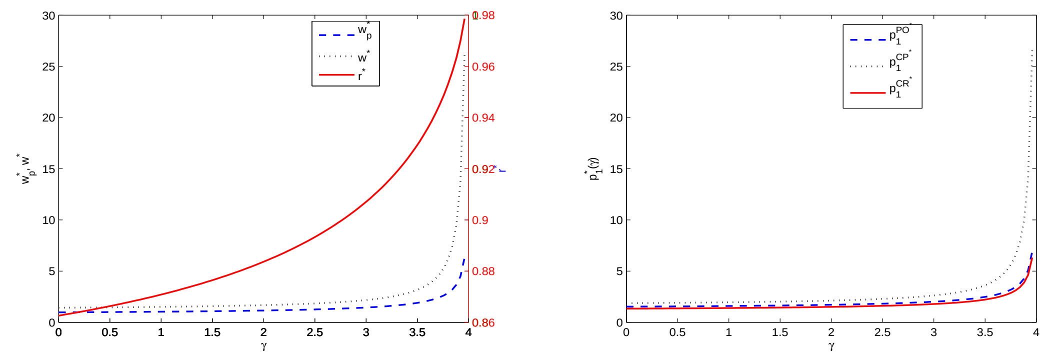
Decisions and profits	PO contract	CP contract	CR contract	Remark
w_p^* or w^* or r^*	decreasing	decreasing	decreasing	$w^* > w_p^*$
p_i^*	decreasing	decreasing	decreasing	$p_i^{CP^*} > p_i^{PO^*} > p_i^{CR^*}$
Q_i^*	not monotonic	decreasing	not monotonic	$Q_i^{CP^*} > Q_i^{CR^*} > Q_i^{PO^*}$ for small β $Q_i^{CR^*} > Q_i^{CP^*} > Q_i^{PO^*}$ for large β
z_i^*	not monotonic	decreasing	not monotonic	$z_i^{CP^*} > z_i^{CR^*} > z_i^{PO^*}$
$\pi_{R_i}^*$	not monotonic	decreasing	not monotonic	$\pi_{R_i}^{CP^*} > \pi_{R_i}^{PO^*} > \pi_{R_i}^{CR^*}$ for small β $\pi_{R_i}^{PO^*} > \pi_{R_i}^{CP^*} > \pi_{R_i}^{CR^*}$ for large β
π_S^*	decreasing	decreasing	decreasing	$\pi_S^{CP^*} > \pi_S^{CR^*} > \pi_S^{PO^*}$ for small β $\pi_S^{CR^*} > \pi_S^{CP^*} > \pi_S^{PO^*}$ for large β
$\frac{\pi_{R_1}^*+\pi_{R_2}^*}{\pi_S^*}$	increasing	increasing	increasing	$PO > CP > CR$

TABLE I

EFFECT OF THE PRICE SENSITIVITY PARAMETER β ON THE EQUILIBRIUM DECISIONS AND PROFITS

ment price (CP contract) and the revenue share (CR contract) increase in γ. This indicates that the supplier takes advantage of lower retailer differentiation (a higher value of γ) by increasing her price or revenue share. The retail price p_i^*, in all three contracts, also increases in γ. This reflects the fact that retailers transfer to consumers their increased supplier costs, and is consistent with the intuition that consumer surplus is lower when retailers are less competitive. Since the consignment price in a CP contract is always higher than the wholesale price in a PO contract, the retail price is also higher in a CP contract than in a PO contract, for any level of retailer differentiation.

Figure Figure 10 depicts the effect of retailer differentiation on the order quantity Q_i^*. This effect depends on the type of contract. An increase of γ drives the (expected) demand to

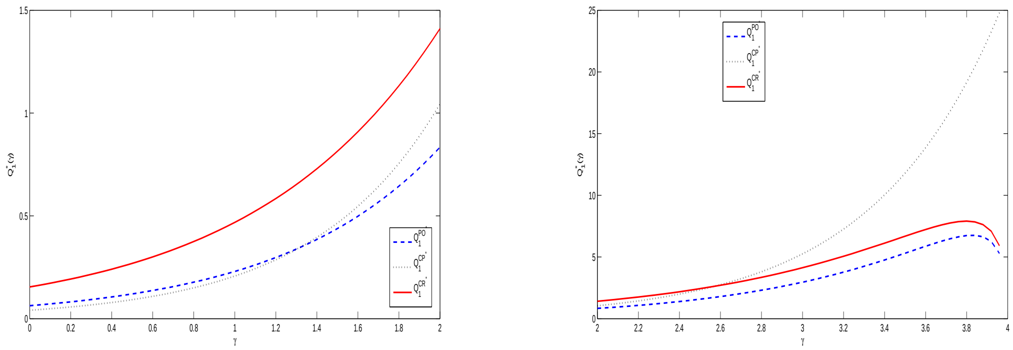


(a) Supplier's wholesale price w_p, consignment price w and revenue share r

(b) Retailer i's selling price p_i^*

Figure 9. Supplier's price (or revenue share) and retailer's selling price as functions of γ under three contracts

increase, everything being kept constant (direct effect). On the other hand, as γ increases, retail prices increase, which could cause the (expected) demand to decrease (indirect effect). The cumulative effect of γ is thus a combination of two opposite effects. Figure Figure 10 illustrates that in the CP contract, the direct effect is stronger, leading to an increase of the order quantity in γ (which is consistent with Proposition 3.3.10). However, in the PO and CR contracts the order quantity is not a monotonic function of γ: it first increases then decreases. One explanation is that the direct effect of γ (which drives the demand to increase) dominates the the indirect effect of γ (which causes the demand to decrease) when the level of retailer differentiation is strong (small values of γ). On the other hand, when retailer differentiation is less intense (larger values of γ), the indirect effect prevails.



(a) Retailer i's order quantity Q_i^* for high retailer differentiation

(b) Retailer i's order quantity Q_i^* for low retailer differentiation

Figure 10. Retailer i's order quantity as a function of γ under three contracts

Figure Figure 11 demonstrates that the effect of retailer differentiation on the stocking factor depends on the contract. The stocking factor in the PO and CR contracts decreases in γ while in the CP contract it is independent of γ. The explanation is that under the PO and the CR contracts, the best response stocking factor depends on the supplier's wholesale price w_p and revenue share r, respectively. However, the best response stocking factor in a CP contract does not depend on the consignment price w. Therefore retailer differentiation affects the equilibrium stocking factor in the PO and the CR contracts through the equilibrium wholesale price and revenue share, respectively, but has no effect on the equilibrium stocking factor in the CP contract. The fact that the stocking factor in the PO and CR contracts decreases in γ means that the order quantity does not increase as fast as the (expected) demand when the level of retailer differentiation decreases.

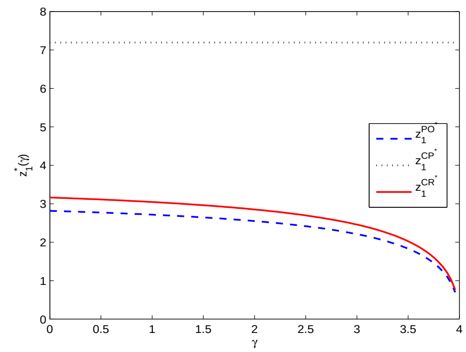
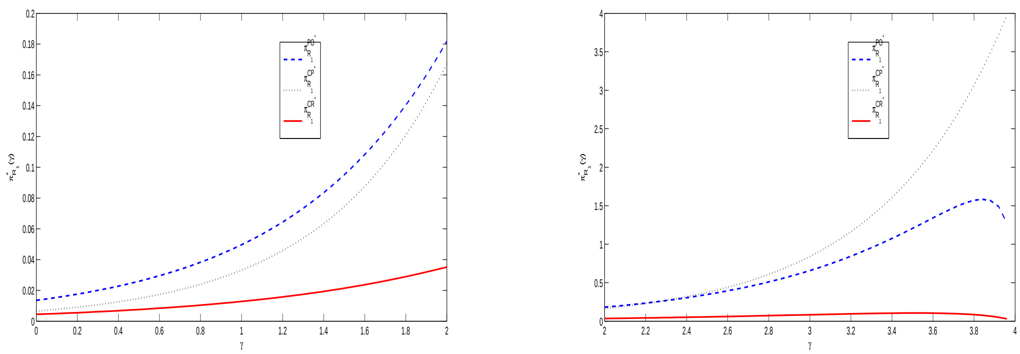


Figure 11. Retailer i's stocking factor as a function of γ under three contracts, where $\beta = 4,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$

Since the retail price and the order quantity in the CP contract increase when the level of retailer differentiation decreases, the retailers' profits, consequently, increases in γ, as illustrated in Figure Figure 12, which is consistent with Proposition 3.3.10. The retailers' profits under the PO and CR contracts, however, are non-monotonic functions of γ because the order quantity is not monotonic. No contract yields higher profits to the retailers than the other two contracts for any level of retailer differentiation. Specifically, when the retailer differentiation is high (small γ), the PO contract yields a higher profit to the retailers than both consignment contracts; when the retailer differentiation is low, the CP yields higher retailers' profits than the PO and CR contracts.

Figure Figure 13 indicates that the supplier's profit under all types of contracts increases in γ, meaning that the supplier profits more when the retailers are less differentiated, which is consistent with Proposition 3.3.10 (in a CP contract with symmetric retailers). No contract



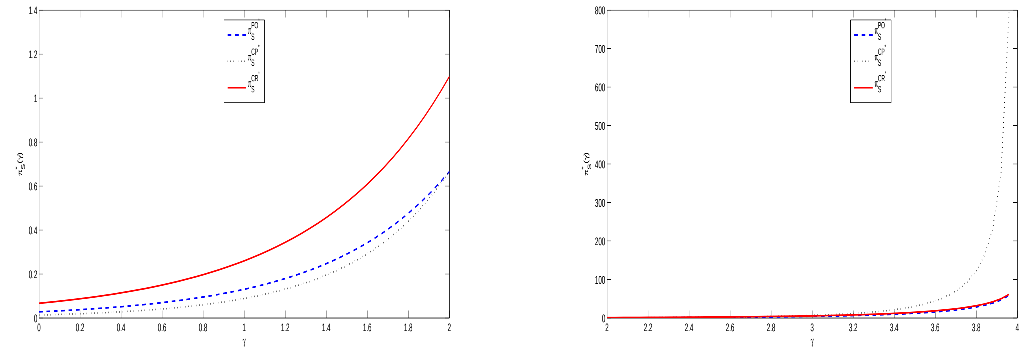
(a) Retailer i's profit $\pi^*_{R_i}$ for high retailer differentiation

(b) Retailer i's profit $\pi^*_{R_i}$ for low retailer differentiation

Figure 12. Retailer i's profit as a function of γ under three contracts

yields higher profits to the supplier than the other two contracts for any level of retailer differentiation. If retailer differentiation is strong, the CR contract is the most beneficial; otherwise the CP contract yields highest supplier profit.

The retailers' share of the total channel profits, $\frac{\pi^*_{R_1}+\pi^*_{R_2}}{\pi^*_S}$, as illustrated in Figure Figure 14, decreases in γ, implying that as retailer differentiation decreases, the retailers' share of the total channel profits decreases under all contracts. In particular, when the retailers are completely differentiated and there is thus no competition among them ($\gamma = 0$), the retailers jointly earn close to 100% of the total channel profits in the PO and CP contracts. As the level of retailer differentiation decreases, the retailers compete more and their profits do not increase as fast as the supplier's profit. Furthermore, under any level of retailer differentiation, the retailers earn a higher share of the total profits under the PO than under either the CP or CR contracts.



(a) Supplier's profit π_S^* for high retailer differentiation

(b) Supplier's profit π_S^* for low retailer differentiation

Figure 13. Supplier's profit as a function of γ under three contracts

The retailers' share of the total channel profits decreases as the level of retailer differentiation decreases, implying that the supplier collects a larger share of the total profits than the retailers as γ increases. Thus, the supplier's profit has a stronger impact on the total channel profits than the retailers'. As a result, the total channel profits exhibit the same trend as the supplier's profit. That is, the total channel profits under all contracts increase in γ (shown in Fig. Figure 15). When the level of retailer differentiation is high, the CR contract yields higher total channel profits than both the PO and the CP contracts. As the level of retailer differentiation decreases, the CP contract prevails over the other contracts.

The effect of retailer differentiation on the decisions and the profits for each of the supply chain members (with a fixed $\beta = 4$) is summarized in Table Table II.

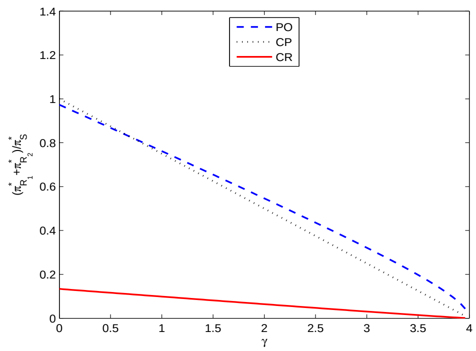
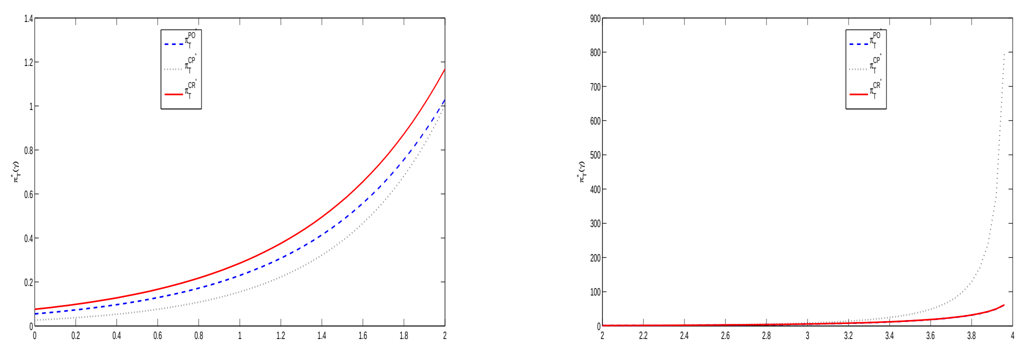


Figure 14. The retailers' share of the total channel profits as a function of γ under three contracts



(a) Total channel profits $\pi^*_T = \pi^*_{R_1} + \pi^*_{R_2} + \pi^*_S$ under low retail competition

(b) Total channel profits $\pi^*_T = \pi^*_{R_1} + \pi^*_{R_2} + \pi^*_S$ under high retail competition

Figure 15. Total channel profits as a function of γ under three contracts

Decisions and profits	PO contract	CP contract	CR contract	Remark
w_p^* or w^* or r^*	increasing	increasing	increasing	$w^* > w_p^*$
p_i^*	increasing	increasing	increasing	$p_i^{CP^*} > p_i^{PO^*} > p_i^{CR^*}$
Q_i^*	not monotonic	increasing	not monotonic	$Q_i^{CR^*} > Q_i^{PO^*} > Q_i^{CP^*}$ for small γ $Q_i^{CP^*} > Q_i^{CR^*} > Q_i^{PO^*}$ for large γ
z_i^*	decreasing	independent	decreasing	$z_i^{CP^*} > z_i^{CR^*} > z_i^{PO^*}$
$\pi_{R_i}^*$	not monotonic	increasing	not monotonic	$\pi_{R_i}^{PO^*} > \pi_{R_i}^{CP^*} > \pi_{R_i}^{CR^*}$ for small γ $\pi_{R_i}^{CP^*} > \pi_{R_i}^{PO^*} > \pi_{R_i}^{CR^*}$ for large γ
π_S^*	increasing	increasing	increasing	$\pi_S^{CR^*} > \pi_S^{PO^*} > \pi_S^{CP^*}$ for small γ $\pi_S^{CP^*} > \pi_S^{CR^*} > \pi_S^{PO^*}$ for large γ
$\frac{\pi_{R_1}^*+\pi_{R_2}^*}{\pi_S^*}$	decreasing	decreasing	decreasing	$PO > CP > CR$

TABLE II

EFFECT OF PARAMETER γ ON THE EQUILIBRIUM DECISIONS AND PROFITS

3.5 Extension: The case of ten competing retailers

Thus far, we have considered three different contracts (the PO contract, the CP contract, and the CR contract) under a supply chain with one supplier and two competing retailers. In the previous section, we solve for the equilibrium solutions numerically and we draw managerial insights on the effect of the price sensitivity factor and the effect of retail differentiation on the equilibrium decisions and profits of the supply chain members. In practice, however, there are often more than two retailers in competition. One question of interest is whether our conclusions remain valid in the case of multiple (more than two) retailers.

In this section, we consider an extension of our results to the case of 10 competing retailers, and we investigate numerically whether our findings still hold. In addition, we also study the effect of an increasing number of retailers (i.e., an increasing level of retail competition) on the profit of each supply chain member.

3.5.1 Analytical results

We extend the analysis to a supply chain with 10 competing retailers. In order to keep the problem tractable, we assume that the cross price-elasticity of demand γ is symmetric for all of the 10 retailers. The generalized demand function can then be written as:

$$D_i(\mathbf{p}) = y_i(\mathbf{p}) \cdot \epsilon, \quad i = 1, \ldots, 10,$$

where

$$y_i(\mathbf{p}) = ae^{-\beta p_i + \gamma \sum\limits_{j=1,\ j \neq i}^{9} p_j}, \qquad a, \beta, \gamma > 0;\ \beta > 9\gamma.$$

Note that in this model the expected demand at retailer i, $y_i(\mathbf{p})$, is a decreasing function of the retailer's own price (p_i), and an increasing function of any of its competitor's price (p_j, where $1 \leq j \leq 10,\ j \neq i$).

We also assume that the 10 retailers have a symmetric cost structure, i.e., $\alpha_1 = \ldots = \alpha_{10} \equiv \alpha$. As a result, each retailer i incurs a cost $\alpha_i c = \alpha c$, to retain the products from the supplier, and the supplier's production cost is $(1 - 10\alpha)c$, where $0 < \alpha < 0.1$.

We compute the best response decisions under the the PO contract and find that the retailer i's unique best response stocking factor $\bar{z}_i$ for a given w_p is given by equation (Equation 3.5) and the retailer i's best response price for any given wholesale price w_p is given by equation (Equation 3.6). This is consistent with the fact that, in the case of two retailers, the retailer's best response to the supplier's decision was independent of the other retailer's price.

Similarly, under the CP contract, we find that retailer i's unique best response stocking factor $\bar{z}_i$ for a given w is given by equation (Equation 3.8) and the best response price for any given consignment price w_p is given by equation (Equation 3.9). Under the CR contract, the retailer i's unique best response stocking factor $\bar{z}_i$ for a given r is given by equation (Equation 3.15) and the retailer i's best response price for any given consignment price r is given by equation (Equation 3.16). Consequently, the retailer's best response price and quantity functions to a given supplier's decision exhibit the same trend as in the case of two retailers.

Since it is intractable to obtain the equilibrium solution in closed-form, including the supplier's price, the retail price, and the quantity for each contract, we solve for the equilibrium quantities numerically.

3.5.2 Numerical results

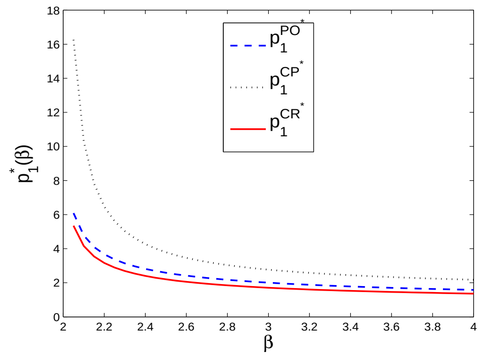
Following our previous numerical study, we set $a = 10,\ c = 1$, and $\alpha_1 = \ldots = \alpha_{10} = 0.025$ and the random perturbation on the demand ϵ is assumed to follow a uniform distribution on $[0, 2]$. For the analysis of the effect of the price sensitivity parameter, we fix $\gamma = \frac{2}{9}$; for the analysis of the effect of retailer differentiation, we fix $\beta = 4$.

3.5.2.1 The effect of price sensitivity

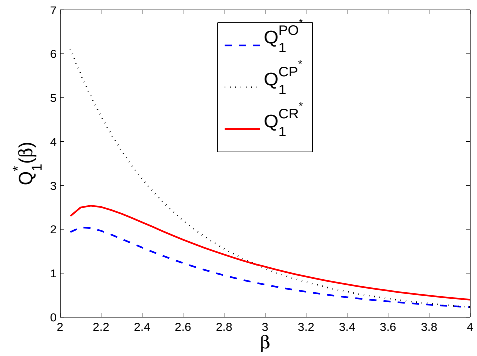
Overall, the conclusions on the effect of the price sensitivity parameter β on the equilibrium decisions and profits in the case of two retailers (as summarized in Table Table I) remain valid for the case of 10 retailers. For example, we still see that the retail price is highest in the CP contract, for any level of price sensitivity (as depicted in Figure Figure 16(a)). Moreover, the effect of β on the order quantity varies, depending on the type of contract. Figure Figure 16(b) illustrates that the order quantity decreases in β, under the CP contract. However, in the PO and CR contracts, the order quantity is not a monotonic function of β.

3.5.2.2 The effect of retailer differentiation

The numerical results on the effect of retailer differentiation on the equilibrium decisions and profits, for a channel of one supplier and 10 retailers under all types of contract, lead us to conclusions similar to those obtained in the case of two retailers (as summarized in Table Table II). For instance, we still observe that the consignment price in the CP contract is always higher than the wholesale price in the PO contract, regardless of level of retailer differentiation. Furthermore, the retailers' share of the total channel profits, $\frac{\pi^*_{R_1}+\pi^*_{R_2}}{\pi^*_S}$, decreases in γ, as



(a) Retailer i's price p_i^*



(b) Retailer i's order quantity Q_i^*

Figure 16. Retailer i's equilibrium price and order quantity as a function of β under three contracts

illustrated in Figure Figure 17. The retailers' share of the total channel profits is highest in the PO contract, for any level of retailer differentiation.

3.5.2.3 The effect of an increasing number of retailers

We consider the effect of an increasing number of retailers (i.e., an increasing level of retail competition) on the profit of each supply chain member (for any given price sensitivity of demand and cross price sensitivity of demand). As expected, the supplier can exploit the increased retailer competition by gaining greater profit, under all types of contracts

3.6 Extension: Consignment price contracts with revenue share (CPR)

In this subsection, we consider a more general consignment contract which combines the consignment price with a revenue share. The study of a combination of the consignment price and the revenue share contracts could potentially bring new insights into the study of con-

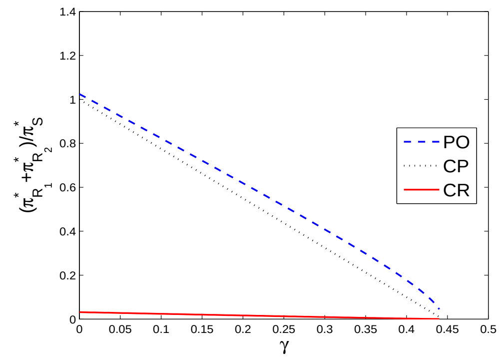


Figure 17. The retailers' share of the total channel profits as a function of γ under three contracts

signment contracts with retail competition. Therefore, we analyze the outcome of this type of contract, which we refer to as *consignment price with revenue share* (CPR) contract.

3.6.1 Analytical results

In this contract, decisions are made in two steps. In the first step, the supplier decides the consignment price w_r and the revenue share $\breve{r}$ to be received from the retailers for each unit sold to consumers. In the second step, given this consignment price and revenue share, each retailer simultaneously chooses the retail price p_i and order quantity Q_i.

3.6.1.1 Retailer i's selling price and stocking factor decision

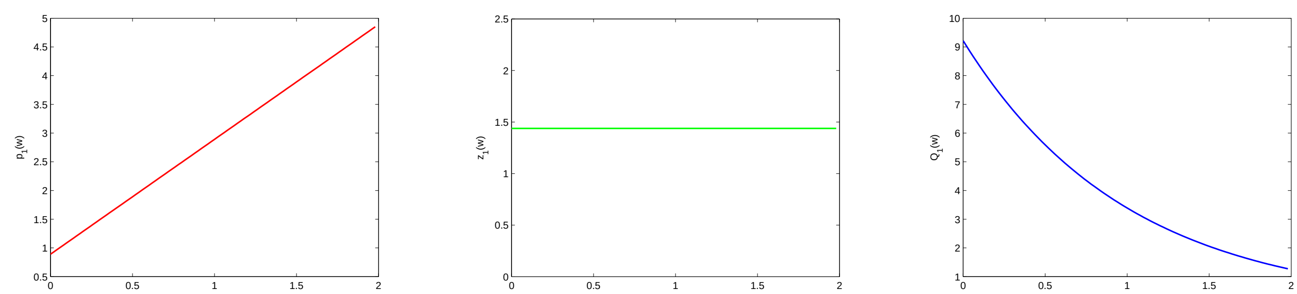
For a given stocking factor z_i, consignment price $w_r > 0$, revenue share $\breve{r}$ and price p_{-i} of retailer $-i$, retailer i's unique best response price $\tilde{p}_i(z_i|w_r, \breve{r})$ is given by

$$\tilde{p}_i(z_i|w_r, \breve{r}) = \frac{1}{\beta} + \frac{c\alpha_i z_i}{(1-\breve{r})(z_i - \Lambda(z_i))} + \frac{w_r}{1-\breve{r}}.$$

Retailer i's best response stocking factor $\bar{z}_i$ that maximizes the profit for a given w_r and a given $\breve{r}$ is uniquely determined as the solution of:

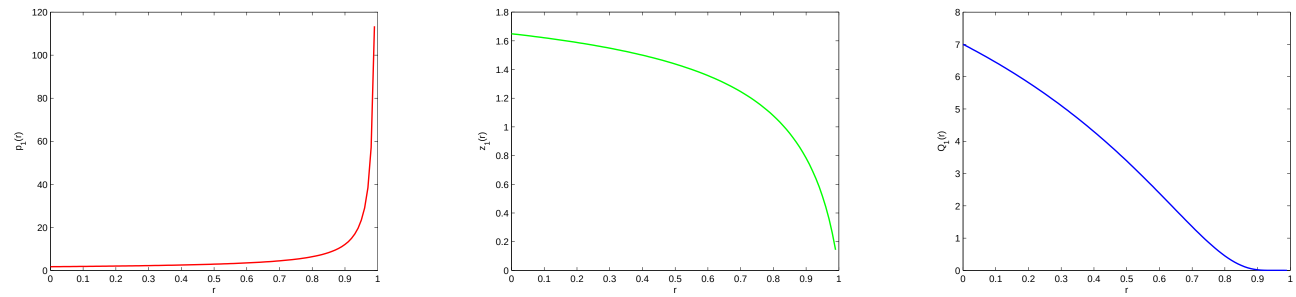
$$\frac{1-\breve{r}}{c\alpha_i\beta} + \frac{\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} = \frac{1}{1-F(\bar{z}_i)}.$$

The CPR best response stocking factor does not depend on the supplier's consignment price w_r, but does depend on the supplier's revenue share $\breve{r}$. This is consistent with the best response stocking factor in both the CP and the CR contracts. By definition, the retailer's best response quantity is $y_i(\mathbf{p})\bar{z}_i$, where $y_i(\mathbf{p}) = ae^{-\beta p_i+\gamma p_{-i}}$.



(a) Best response price $\bar{p_1}$ (b) Best response stocking factor $\bar{z_1}$ (c) Best response quantity $\bar{Q_1}$

Figure 18. Retailer 1's best response price, stocking factor and quantity as a function of the consignment price w_r when $\beta = 2,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ and r is fixed at 0.5 in the CPR contract



(a) Best response price $\bar{p}_1$ (b) Best response stocking factor $\bar{z}_1$ (c) Best response quantity $\bar{Q}_1$

Figure 19. Retailer 1's best response price, stocking factor and quantity as a function of the revenue share $\breve{r}$ when $\beta = 2,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ and w is fixed at 1 in the CPR contract

Figures Figure 18 and Figure 19 depict that the retailer's best response retail price increases with the supplier's consignment price w_r and revenue share $\breve{r}$: when the supplier keeps a higher consignment price and/or a higher share of the retailer's revenue, the retailers transfer the increasing cost to consumers. The higher price causes the demand to decrease, which leads to a lower quantity at each retailer. While both the expected demand and order quantity decrease with the supplier's consignment price and revenue share, the order quantity decreases faster than the expected demand. Therefore, the stocking factor decreases with the supplier's consignment price and revenue share.

3.6.1.2 Supplier's consignment price and revenue share decision

At the first step, anticipating the retailers' reaction to her decision, the supplier sets the consignment price w_r and revenue share $\breve{r}$ to maximize her own expected profit. For any given revenue share $\breve{r}$, the supplier's unique equilibrium consignment price $\tilde{w_r}(\breve{r})$ is given by

$$\tilde{w_r}(\breve{r}) = \frac{k_1 v_1 + k_2 v_2}{(\frac{\beta-\gamma}{1-\breve{r}})(\frac{1}{1-\breve{r}})[k_1(\bar{z}_1 - \Lambda(\bar{z}_1)) + k_2(\bar{z}_2 - \Lambda(\bar{z}_2))]},$$

where $k_i = e^{-\frac{c}{1-\breve{r}}\left(\frac{\beta\alpha_i \bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} - \frac{\gamma\alpha_{-i}\bar{z}_{-i}}{\bar{z}_{-i} - \Lambda(\bar{z}_{-i})}\right)}$, and

$$v_i = \frac{1}{1-\breve{r}}(\bar{z}_i - \Lambda(\bar{z}_i)) - \frac{\beta-\gamma}{1-\breve{r}}\left[\breve{r}(\frac{1}{\beta} + \frac{c\alpha_i \bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)})(\bar{z}_i - \Lambda(\bar{z}_i) - c(1-\alpha_i-\alpha_{-i})\bar{z}_i\right],\ i = 1, 2.$$

To find the equilibrium solution, denoted by $\breve{r}^*$ and $w_r^* = \tilde{w_r}(\breve{r}^*)$, we have to maximize $\pi_S(\tilde{w_r}(\breve{r}), \breve{r})$ over $\breve{r}$. Obtaining an analytical solution for this maximization is intractable, therefore, we use numerical methods.

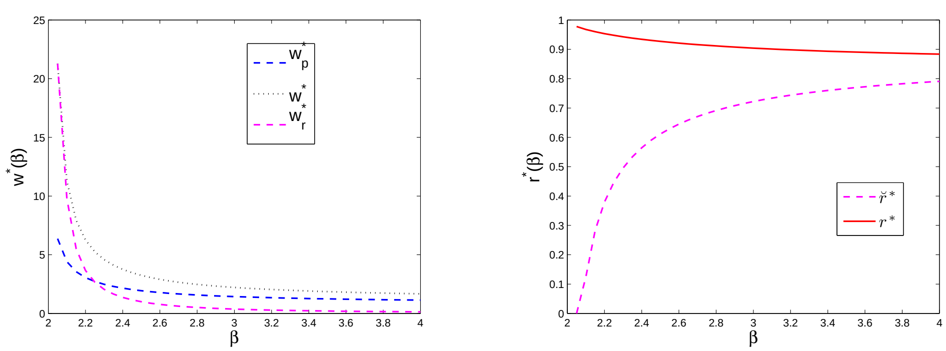
3.6.2 Numerical results

Following our previous numerical study, we set $a = 10$, $c = 1$, and $\alpha_1 = \alpha_2 = 0.125$ and the random perturbation on the demand ϵ is assumed to follow a uniform distribution on $[0, 2]$. For the analysis of the effect of the price sensitivity parameter, we fix $\gamma = 2$; for the analysis of the effect of retailer differentiation, we fix $\beta = 4$.

3.6.2.1 The effect of price sensitivity

Figures Figure 20(a) and (b) depict the effect of the price sensitivity parameter on the supplier's consignment price w_r and revenue share $\breve{r}$, respectively. The supplier's consignment price w_r decreases in β, which is consistent with our finding in the CP contract. Interestingly, the supplier's revenue share $\breve{r}$ increases in β (while the revenue share r in the CR contract decreases in β). One explanation is that as β increases the consignment price w_r decreases, the supplier needs to compensate this loss by increasing the revenue share $\breve{r}$.

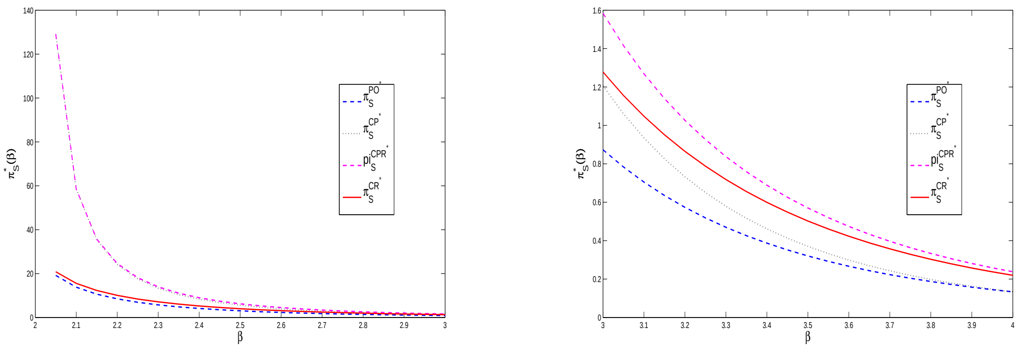
We also find that the effect of the price sensitivity parameter β on the retail price, the order quantity, the stocking factor, the retailers' profits, the supplier profit, and the share of the channel profits for the retailers are consistent with the findings in the CP contract (i.e., they are decreasing functions of β). Figure Figure 21 shows the effect of the price sensitivity parameter on the supplier's profit. Interestingly, the supplier earns the highest profit in the CPR contract, for any level of price sensitivity. The effect of the price sensitivity parameter on the decisions and profits for each of the supply chain members is summarized in Table Table III.



(a) Supplier's prices w_p^* (CP), w^* (CR) and w_r^* (CPR)

(b) Supplier's revenue share $\breve{r}^*$ (CPR) and r^* (CR)

Figure 20. Supplier's price or revenue share at equilibrium as a function of β under four contracts



(a) Supplier's profit π_S^* for low price sensitivity (b) Supplier's profit π_S^* for high price sensitivity

Figure 21. Supplier's profit at equilibrium as a function of β under four contracts

Decisions and profits	PO contract	CP contract	CR contract	CPR contract	Remark
w_p^* or w^* or w_r^*	decreasing	decreasing	-	decreasing	$w^* > w_r^* > w_p^*$ for small β $w^* > w_p^* > w_r^*$ for large β
r^* or $\breve{r}^*$	-	-	decreasing	increasing	$r^* > \breve{r}^*$
p_i^*	decreasing	decreasing	decreasing	decreasing	$p_i^{CP^*} > p_i^{CPR^*} > p_i^{PO^*} > p_i^{CR^*}$ for small β $p_i^{CP^*} > p_i^{PO^*} > p_i^{CPR^*} > p_i^{CR^*}$ for large β
Q_i^*	not monotonic	decreasing	not monotonic	decreasing	$Q_i^{CP^*} > Q_i^{CPR^*} > Q_i^{CR^*} > Q_i^{PO^*}$ for small β $Q_i^{CPR^*} > Q_i^{CR^*} > Q_i^{CP^*} > Q_i^{PO^*}$ for large β
z_i^*	not monotonic	decreasing	not monotonic	decreasing	$z_i^{CP^*} > z_i^{CPR^*} > z_i^{CR^*} > z_i^{PO^*}$
$\pi_{R_i}^*$	not monotonic	decreasing	not monotonic	decreasing	$\pi_{R_i}^{CP^*} \geq \pi_{R_i}^{CPR^*} > \pi_{R_i}^{PO^*} > \pi_{R_i}^{CR^*}$ for small β $\pi_{R_i}^{PO^*} > \pi_{R_i}^{CP^*} > \pi_{R_i}^{CPR^*} > \pi_{R_i}^{CR^*}$ for large β
π_S^*	decreasing	decreasing	decreasing	decreasing	$\pi_S^{CPR^*} > \pi_S^{CP^*} > \pi_S^{CR^*} > \pi_S^{PO^*}$ for small β $\pi_S^{CPR^*} > \pi_S^{CR^*} > \pi_S^{CP^*} > \pi_S^{PO^*}$ for large β
$\frac{\pi_{R_1}^*+\pi_{R_2}^*}{\pi_S^*}$	increasing	increasing	increasing	increasing	$PO > CP > CPR > CR$

TABLE III

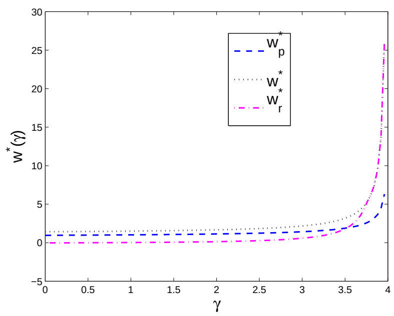
EFFECT OF THE PRICE SENSITIVITY PARAMETER β ON THE EQUILIBRIUM DECISIONS AND PROFITS

3.6.2.2 The effect of retailer differentiation

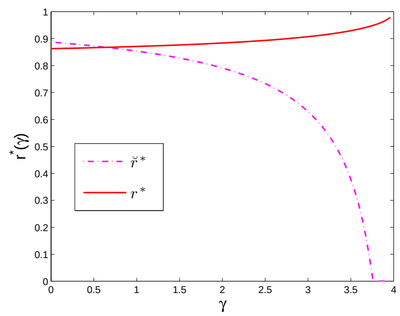
We find that the supplier's revenue share $\breve{r}$ is a non-increasing function of γ while the consignment price w_r is increasing in γ (as depicted in Figure Figure 22). This reflects the fact that the supplier cannot simultaneously increase both the consignment price and the revenue share because this would lead to a very high retail price.

The effect of retailer differentiation on the retail price, the order quantity, the retailers' profits, and the supplier profit are consistent with the findings in the CP contract (i.e., they are increasing in γ). Figure Figure 23 depicts that the retailer suffers the highest profit loss in the CPR contract, when retailers are more differentiated. This loss in profit decreases as retailers are less differentiated (a higher value of γ). On the other hand, figure Figure 24 shows that the supplier earns a highest profit from the CPR contract, when the retailers are more differentiated. This benefit decreases as retailers become less differentiated. The effect of retailer differentiation on the decisions and profits for each of the supply chain members is summarized in Table Table IV.

Our numerical study shows that the CPR contract yields the highest profit to the supplier, as compared with the other types of consignment contracts considered, namely CP and CR contracts, for any level of price sensitivity. Our numerical study also illustrates that the benefit of the CPR contract to the supplier and the retailers depends upon the level of retailer differentiation. When retailer differentiation is strong, the CP contract yields highest profits to the retailers; when it is weak, the CPR contract yields highest profits. Conversely, the supplier



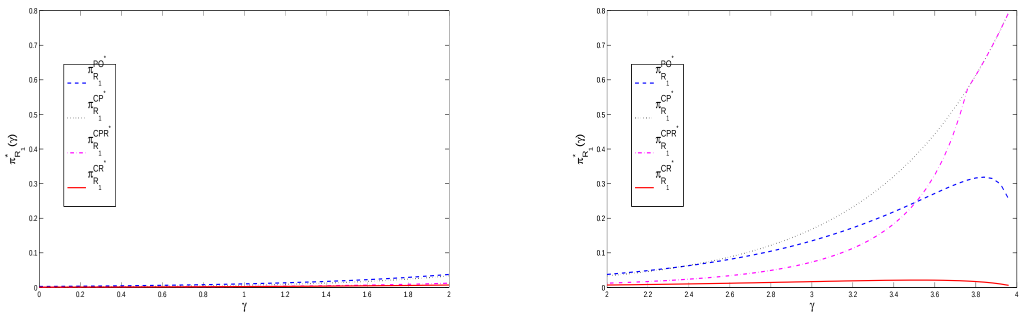
(a) Supplier's prices w_p^* (CP), w^* (CR) and w_r^* (CPR)



(b) Supplier's revenue share $\breve{r}^*$ (CPR) and r^* (CR)

Figure 22. Supplier's profit at equilibrium as a function of γ under four contracts

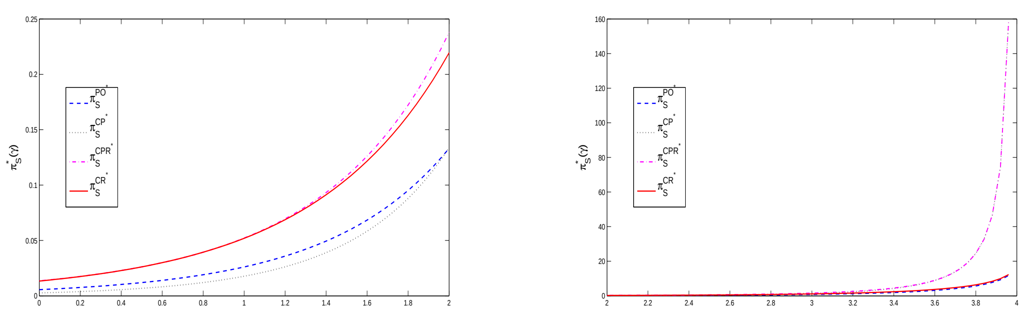
earns a higher profit in the CPR contract than the other types of consignment contracts when retailer differentiation is strong. The CP contract is more beneficial to the supplier as retailers become less differentiated. The comparison of these three types of consignment contracts is summarized in Table Table V.



(a) Retailer i's profit $\pi^*_{R_i}$ for high retailer differentiation

(b) Retailer i's profit $\pi^*_{R_i}$ for low retailer differentiation

Figure 23. Retailer i's profit at equilibrium as a function of γ under four contracts



(a) Supplier's profit π^*_S for high retailer differentiation

(b) Supplier's profit π^*_S for low retailer differentiation

Figure 24. Supplier's profit at equilibrium as a function of γ under four contracts

Decisions and profits	PO contract	CP contract	CR contract	CPR contract	Remark
w_p^* or w^* or w_r^*	increasing	increasing	-	increasing	$w^* > w_p^* > w_r^*$ for small γ $w^* > w_r^* > w_p^*$ for large γ
r^* or $\breve{r}^*$	-	-	increasing	non-increasing	$\breve{r}^* > r^*$ for small γ $r^* > \breve{r}^*$ for large γ
p_i^*	increasing	increasing	increasing	increasing	$p_i^{CP^*} > p_i^{PO^*} > p_i^{CR^*} > p_i^{CPR^*}$ for small γ $p_i^{CP^*} > p_i^{CPR^*} > p_i^{PO^*} > p_i^{CR^*}$ for large γ
Q_i^*	not monotonic	increasing	not monotonic	increasing	$Q_i^{CPR^*} > Q_i^{CR^*} > Q_i^{PO^*} > Q_i^{CP^*}$ for small γ $Q_i^{CP^*} > Q_i^{CPR^*} > Q_i^{CR^*} > Q_i^{PO^*}$ for large γ
z_i^*	decreasing	independent	decreasing	non-decreasing	$z_i^{CP^*} > z_i^{CR^*} > z_i^{PO^*} > z_i^{CPR^*}$ $z_i^{CP^*} \geq z_i^{CPR^*} > z_i^{CR^*} > z_i^{PO^*}$ for large γ
$\pi_{R_i}^*$	not monotonic	increasing	not monotonic	increasing	$\pi_{R_i}^{PO^*} > \pi_{R_i}^{CP^*} > \pi_{R_i}^{CR^*} > \pi_{R_i}^{CPR^*}$ for small γ $\pi_{R_i}^{CP^*} \geq \pi_{R_i}^{CPR^*} > \pi_{R_i}^{PO^*} > \pi_{R_i}^{CR^*}$ for large γ
π_S^*	increasing	increasing	increasing	increasing	$\pi_S^{CPR^*} > \pi_{R_i}^{CR^*} > \pi_S^{PO^*} > \pi_S^{CP^*}$ for small γ $\pi_S^{CP^*} > \pi_{R_i}^{CPR^*} > \pi_S^{CR^*} > \pi_S^{PO^*}$ for large γ
$\frac{\pi_{R_1}^* + \pi_{R_2}^*}{\pi_S^*}$	decreasing	decreasing	decreasing	non-increasing	$PO > CP \geq CR > CPR$ for small γ $PO > CP > CPR > CR$ for large γ

TABLE IV

EFFECT OF PARAMETER γ ON THE EQUILIBRIUM DECISIONS AND PROFITS

Decisions and profits	Retailer differentiation
p_i^*	$p_i^{CP^*} > p_i^{CR^*} > p_i^{CPR^*}$ for more retailer differentiation (small γ) $p_i^{CP^*} > p_i^{CPR^*} > p_i^{CR^*}$ for less retailer differentiation (large γ)
Q_i^*	$Q_i^{CPR^*} > Q_i^{CR^*} > Q_i^{CP^*}$ for more retailer differentiation (small γ) $Q_i^{CP^*} > Q_i^{CPR^*} > Q_i^{CR^*}$ for less retailer differentiation (large γ)
z_i^*	$z_i^{CP^*} > z_i^{CR^*} > z_i^{CPR^*}$ for more retailer differentiation (small γ) $z_i^{CP^*} \geq z_i^{CPR^*} > z_i^{CR^*}$ for less retailer differentiation (large γ)
$\pi_{R_i}^*$	$\pi_{R_i}^{CP^*} > \pi_{R_i}^{CR^*} > \pi_{R_i}^{CPR^*}$ for more retailer differentiation (small γ) $\pi_{R_i}^{CP^*} > \pi_{R_i}^{CPR^*} > \pi_{R_i}^{CR^*}$ for more retailer differentiation (small γ)
π_S^*	$\pi_S^{CPR^*} > \pi_{R_i}^{CR^*} > \pi_S^{CP^*}$ for more retailer differentiation (small γ) $\pi_S^{CP^*} > \pi_{R_i}^{CPR^*} > \pi_S^{CPR^*}$ for less retailer differentiation (large γ)
$\frac{\pi_{R_1}^* + \pi_{R_2}^*}{\pi_S^*}$	$CP \geq CR > CPR$ for more retailer differentiation (small γ) $CP > CPR > CR$ for less retailer differentiation (large γ)

TABLE V

THE COMPARISON OF THREE TYPES OF CONSIGNMENT CONTRACTS

3.7 Extension: Comparison between consignment contracts and revenue-sharing contracts

In sections 3.2 – 3.4, we use the price-only contract as a benchmark for evaluating consignment contracts. In this section, we compare consignment contracts with revenue-sharing contracts.

The key difference between revenue-sharing and consignment contracts is the time of payment and the ownership of inventory. In revenue-sharing contracts, the retailers pay the supplier a wholesale price for each unit ordered in addition to a percentage of the revenue the retailers generate (Cachon and Lariviere, 2005; Yao et al., 2008b; Linh and Hong, 2009; Pan et al., 2010).

Thus, the retailers have full ownership of the inventory and bear all the risk of unsold units remaining . The supplier receive the payment for all the units ordered by retailers, regardless of whether or not the retailer sells them, in addition to a percentage of the retailers' revenue. In a consignment contract, the supplier retains ownership of merchandise even though items are at retail locations. The supplier receives no payment until the items are sold by retailers. Therefore, the risk of underselling is now born by the supplier.

3.7.1 Analytical results

In the revenue sharing (RS) contract, decisions are made in two steps. In the first step, the supplier decides the wholesale price w_p for each unit ordered by retailers and the revenue share r to be received from the retailers for each unit sold to consumers. In the second step, given this wholesale price and revenue share, each retailer simultaneously chooses the retail price p_i and order quantity Q_i. We first derive each retailer's best response and inventory quantity to the supplier's wholesale price and revenue share decisions.

3.7.1.1 Retailer i's selling price and stocking factor decision

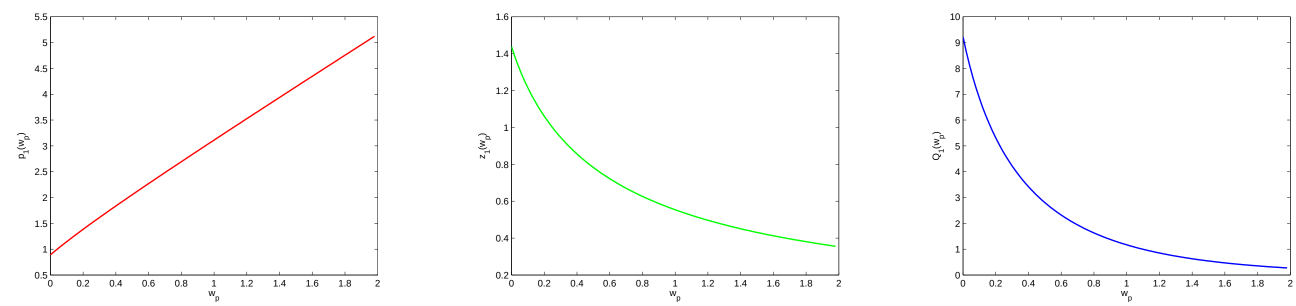
For a given stocking factor z_i, consignment price $w > 0$, revenue share r and price p_{-i} of retailer $-i$, retailer i's unique best response price $\tilde{p}_i(z_i|w, \breve{r})$ is

$$\tilde{p}_i(z_i|w, r) = \frac{1}{\beta} + \frac{(c\alpha_i + w)z_i}{(1-r)(z_i - \Lambda(z_i))}.$$

The retailer i's best response stocking factor $\bar{z}_i$ that maximizes the retailer i's profit for a given w and a given r is uniquely determined as the solution of:

$$\frac{1-r}{(c\alpha_i + w)\beta} + \frac{\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} = \frac{1}{1-F(\bar{z}_i)}.$$

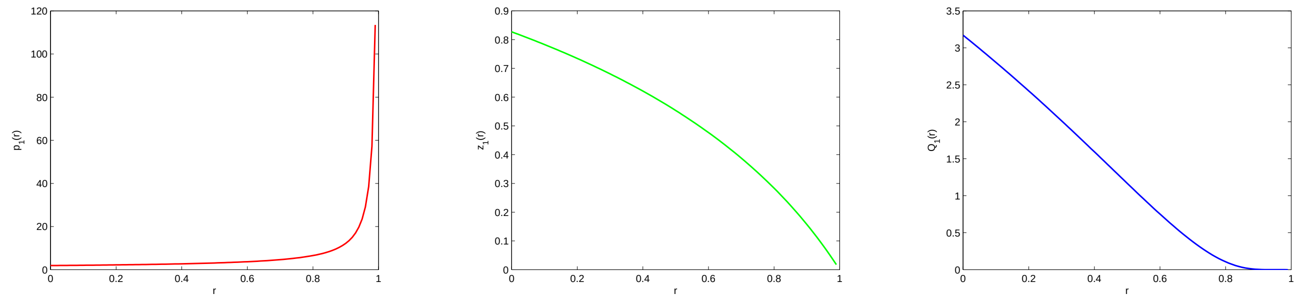
In particular, the RS best response stocking factor depends on both the supplier's wholesale price w and the supplier's revenue share r. This result is consistent with best response obtained in both the PO and the CR contracts. The retailer's best response quantity is $y_i(\mathbf{p})\bar{z}_i$, where $y_i(\mathbf{p}) = ae^{-\beta p_i + \gamma p_{-i}}$.



(a) Best response price $\bar{p_1}$ (b) Best response stocking factor $\bar{z_1}$ (c) Best response quantity $\bar{Q_1}$

Figure 25. Retailer 1's best response price, stocking factor and quantity as a function of the wholesale price w when $\beta = 2,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ and r is fixed at 0.5 in the RS contract

Figures Figure 25 and Figure 26 illustrate that the retailer's best response retail price increases with both the supplier's wholesale price w and revenue share r: when the supplier keeps a higher wholesale price and/or a higher share of the retailer's revenue, the retailers transfer



(a) Best response price $\bar{p_1}$ (b) Best response stocking factor $\bar{z_1}$ (c) Best response quantity $\bar{Q_1}$

Figure 26. Retailer 1's best response price, stocking factor and quantity as a function of the revenue share r when $\beta = 2,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ and w is fixed at 1 in the RS contract

the increasing cost to consumers. The higher price causes the demand to decrease, which leads to a lower quantity at each retailer. While both the expected demand and order quantity decrease with the supplier's wholesale price and revenue share, the order quantity decreases faster than the expected demand. Therefore, the stocking factor decreases with the supplier's revenue.

3.7.1.2 Supplier's revenue sharing fraction decision

At the first step, anticipating the retailers' reaction to her decision, the supplier sets the wholesale price w and revenue share r to maximize her own expected profit $\pi_S(w,r)$, given by

$$\begin{aligned}\pi_S(w,r) &= r\bar{p}_1 E\{\min(D_1,\bar{Q}_1)\} + r\bar{p}_2 E\{\min(D_2,\bar{Q}_2)\} + (w - c(1-\alpha_1-\alpha_2))(\bar{Q}_1+\bar{Q}_2)\\ &= y_1(\bar{\mathbf{p}}))[r\bar{p}_1(\bar{z}_1 - \Lambda(\bar{z}_1)) + (w - c(1-\alpha_1-\alpha_2))\bar{z}_1]\\ &\quad + y_2(\bar{\mathbf{p}}))[r\bar{p}_2(\bar{z}_2 - \Lambda(\bar{z}_2)) + (w - c(1-\alpha_1-\alpha_2))\bar{z}_2]\\ &= ae^{-\beta\bar{p}_1+\gamma\bar{p}_2}[r\bar{p}_1(\bar{z}_1 - \Lambda(\bar{z}_1)) + (w - c(1-\alpha_1-\alpha_2))\bar{z}_1]\\ &\quad + ae^{\beta\bar{p}_2+\gamma\bar{p}_1}[r\bar{p}_2(\bar{z}_2 - \Lambda(\bar{z}_2)) + (w - c(1-\alpha_1-\alpha_2))\bar{z}_2].\end{aligned}$$

To find the equilibrium solutions, denoted by r^* and $w^* = \tilde{w}(r^*)$, we have to maximize $\pi_S(\tilde{w}(r),r)$ over r. Obtaining an analytical solution for this maximization problem is intractable, therefore, we use numerical methods.

3.7.2 Numerical results

Following our previous numerical study, we set $a = 10$, $c = 1$, and $\alpha_1 = \alpha_2 = 0.125$ and the random perturbation on the demand ϵ is assumed to follow a uniform distribution on $[0,2]$. For the analysis of the effect of the price sensitivity parameter, we fix $\gamma = 2$; for the analysis of the effect of retailer differentiation, we fix $\beta = 4$.

Our numerical study shows that the equilibrium supplier's wholesale price w^* is very close to zero for any given β and γ. This means that at equilibrium, the supplier's income (or profit) would almost entirely come from the fraction of the retailers' revenue. The retailers pay almost nothing for each unit purchased from the supplier but remit a share of their revenue to the supplier (for each unit sold to consumers). Indeed, the supplier is better off not charging the retailers for each unit ordered to incentivize them to order more and price adequately. Essentially, the revenue-sharing contract is equivalent to our consignment with revenue share (CR) contract. The results and conclusions of the effect of the price sensitivity parameter and the effect of retailer differentiation on the equilibrium decisions and profits of the supply chain members for the CR contract remain valid for the RS contract.

CHAPTER 4

CHANNEL COORDINATION IN A CONSIGNMENT CONTRACT WITH SUPPLIER COMPETITION AND DUAL CHANNEL SUPPLY CHAIN

In this chapter, we study consignment contracts between two competing suppliers and one retailer, acting as Stackelberg leader, under two different payment terms: (i) a fixed consignment price per unit sold and (ii) a percentage of the revenue earned by the retailer as the suppliers revenue share. We also consider the situation where one of the suppliers sells its products directly to consumers via a direct channel, in addition to selling through the retailer via consignment (dual channel).

4.1 Model assumptions and centralized channel

4.1.1 Model assumptions

Consider a supply chain with one retailer (R) and two suppliers (S_1 and S_2). We assume each supplier produces one product which is substitutable to (but differentiated from) the product of the other supplier, and sells it through a common retailer. The supplier i produces at a constant unit cost of $\$c_{S_i},\ i = 1, 2$, and the retailer incurs a unit cost of $\$c_{R_i},\ i = 1, 2$ for handling and selling product i to consumers.

We consider a demand for the product from each supplier during a single selling season that is price-dependent and uncertain. A multiplicative demand model is widely used in the literature to capture the randomness of the demand (see for example (Petruzzi and Dada, 1999; Wang et al., 2004; Ru and Wang, 2010)). We model the demand for product i at the retailer, denoted by $D_i(\mathbf{p})$ as:

$$D_i(\mathbf{p}) = y_i(\mathbf{p}) \cdot \epsilon, \quad i = 1, 2, \tag{4.1}$$

where $\mathbf{p} = (p_1, p_2)$ and p_i, $i = 1, 2$ is the retail price of supplier i's product, $y_i(\mathbf{p})$ is the expected demand for product i, and ϵ is a random scaling factor, representing randomness of the demand, with a mean value of 1, cumulative distribution function $F(\cdot)$ and probability density function $f(\cdot)$ that have support $[A, B] \subset \Re^+$ with $B > A$. Let $h(x) = xf(x)/[1 - F(x)]$ denote the generalized failure rate function.

We model the expected demand as a linear function of each supplier's own price and its competitor's price. A linear demand has been widely adopted in many studies in the supply chain management literature (see for example (Coughlan, 1992; Raju et al., 1995) in the absence of competition and (Choi, 1996; Niu et al., 2012; Liao and Tseng, 2007; Pan et al., 2010; Bernstein and Federgruen, 2005) in the presence of competition). This type of model captures product differentiation. The expected demand for product i is given by

$$y_i(\mathbf{p}) = a - p_i + \gamma(p_{-i} - p_i), \qquad a,\ \gamma > 0;\ \gamma < 1. \tag{4.2}$$

where p_i is the supplier's own price and p_{-i} is the competitor's price.

Note that in this model, the expected demand for each product is a decreasing function of its own price p_i, and an increasing function of its competitor's price p_{-i}, where $-i = 2$ if $i = 1$ and $-i = 1$ if $i = 2$. Parameter a is the primary demand of each product (i.e., demand if both prices were zero). Parameter γ is related to the degree of product differentiation. As γ increases (approaches 1), the products are more substitutable (less differentiated), therefore the price difference between two products at the retailer has more impact on the demand (see (Choi, 1996; Pan et al., 2010; Liao and Tseng, 2007)). When γ is zero, the two products are completely differentiated and therefore the price difference has no impact.

Assumption 4.1.1. *The demand distribution satisfies the increasing generalized failure rate (IGFR) property:* $h(x) = \frac{xf(x)}{[1-F(x)]}$ *is increasing in* x*.*

This assumption is less restrictive than the increasing failure rate (IFR) assumption in (Vives, 1984; Vives, 1985) and (Petruzzi and Dada, 1999). Many distributions, such as the uniform, exponential, and normal distributions, satisfy both IFR and IGFR conditions.

Assumption 4.1.2. *The price* p_i *for product* i *takes values on* $S_i = [0, p_i^{max}]$*, where* p_i^{max} *is the maximum admissible value of* p_i *and* $y_i(\mathbf{p})|_{\mathbf{p}=\mathbf{p}^{max}}$ *is positive for* $i = 1, 2$*.*

This assumption imposes an upper bound on prices and assures that the expected demand is positive (Vives, 1984).

4.1.2 Centralized channel

In the centralized channel, a decision maker simultaneously chooses the selling price and the quantity for each of the products in order to maximize the expected total channel profit.

Following the standard newsvendor model (Cachon, 2003), the following sequence of events occurs: (1) a central decision maker simultaneously chooses the selling price and the quantity for each of the products; (2) before the start of selling season, the supplier i produces and delivers q_i units of product i to the retailer; (3) demand realizes; (4) transfer payment are made between supplier and retailer based upon the centralized price decisions and demand realization.

The objective is to maximize the expected channel profit which can be written as:

$$\pi_C(\mathbf{p}, \mathbf{q}) = p_1 E\{\min(D_1, q_1)\} - (c_{S_1} + c_{R_1})q_1 + p_2 E\{\min(D_2, q_2)\} - (c_{S_2} + c_{R_2})q_2.$$

Denoting $z_i = q_i/y_i(\mathbf{p})$, the profit can be rewritten as

$$\begin{aligned} \pi_C(\mathbf{p}, \mathbf{z}) &= y_1(\mathbf{p})\{(p_1(z_1 - \Lambda(z_1)) - (c_{S_1} + c_{R_1})z_1\} + y_2(\mathbf{p})\{(p_2(z_2 - \Lambda(z_2)) - (c_{S_2} + c_{R_2})z_2\} \\ &= (a - p_1 + \gamma(p_2 - p_1))\{p_1(z_1 - \Lambda(z_1)) - (c_{S_1} + c_{R_1})z_1\} \\ &\quad +(a - p_2 + \gamma(p_1 - p_2))\{p_2(z_2 - \Lambda(z_2)) - (c_{S_2} + c_{R_2})z_2\}, \end{aligned} \tag{4.3}$$

where $\Lambda(z_i) = \int_A^{z_i} (z_i - x) f(x)\, dx$.

To find the optimal solution, denoted as $(\mathbf{p}^*, \mathbf{z}^*)$, which maximizes (Equation 4.3), we first find the optimal stocking factor $\mathbf{z}^{C^*}(\mathbf{p})$ for a fixed price vector $\mathbf{p}$ and then maximize $\pi_C(\mathbf{p}, \mathbf{z}^{C^*}(\mathbf{p}))$ with respect to $\mathbf{p}$ to find the optimal price $p_i^{C^*},\ i = 1, 2$ (Vives, 1984; Vives, 1985; Petruzzi and Dada, 1999).

In theory, it is possible for the suppliers to produce zero quantities of the products. However, we consider a more interesting case in which the suppliers produce positive quantities of the products, i.e., $q_i > 0,\ i = 1, 2$. Thus, we assume that $c_{S_i} + c_R < p_i$ i.e., the sale of a product covers the costs incurred to ensure that the optimal stocking factor exists and is unique (Porteus, 1990).

Proposition 4.1.3. *For any given price $p_i > c_{S_i} + c_R$, supplier i's unique optimal stocking factor $z_i^{C^*}(p_i)$, is given by*

$$z_i^{C^*}(p_i) = F^{-1}(\theta_i), \tag{4.4}$$

where $\theta_i = 1 - \frac{c_{S_i} + c_{R_i}}{p_i}$.

According to Proposition 4.1.3, the first best stocking factor (ratio of the quantity supplied and the expected demand) for supplier i's product is nondecreasing in its relative total margin, i.e., the ratio of supplier i's unit margin (difference between unit selling price and unit production cost and retailer's cost) to the unit selling price. Indeed, when the relative total margin from selling supplier i's product increases, both supplier i and the retailer would want to increase the quantity ordered.

The optimal quantity $q_i^{C^*}$ can be easily derived from $q_i^{C^*} = y_i C^*(\mathbf{p}) z_i^{C^*}$.

Proposition 4.1.4. *The optimal prices* $p_1^{C^*}$ *and* $p_2^{C^*}$ *that maximize the centralized profit* $\pi_C(\mathbf{p}, \mathbf{z}^{C^*}(\mathbf{p}))$ *must satisfy the first-order necessary conditions, given by*

$$1 + \frac{c_{S_1} + c_{R_1}}{p_1} \frac{F^{-1}(\theta_1)}{\int_A^{F^{-1}(\theta_1)} x f(x)\, dx} + \frac{\gamma p_2 \int_A^{F^{-1}(\theta_2)} x f(x)\, dx}{(a - p_1 + \gamma(p_2 - p_1)) \int_A^{F^{-1}(\theta_1)} x f(x)\, dx} = \frac{(1+\gamma)p_1}{a - p_1 + \gamma(p_2 - p_1)}$$
$$1 + \frac{c_{S_2} + c_{R_2}}{p_2} \frac{F^{-1}(\theta_2)}{\int_A^{F^{-1}(\theta_2)} x f(x)\, dx} + \frac{\gamma p_1 \int_A^{F^{-1}(\theta_1)} x f(x)\, dx}{(a - p_2 + \gamma(p_1 - p_2)) \int_A^{F^{-1}(\theta_2)} x f(x)\, dx} = \frac{(1+\gamma)p_2}{a - p_2 + \gamma(p_1 - p_2)} \quad (4.5)$$

Proposition 4.1.4 gives, implicitly, the unique first best price for each of the suppliers' products as the solution of a system of two equations. This allows us to find, numerically, the centralized solution.

As noted by (Lariviere and Porteus, 2001; Petruzzi and Dada, 1999; Yao et al., 2008b), obtaining an analytical solution for a maximization problem of $\pi_C(\mathbf{p}, z_i^{C^*}(\mathbf{p}))$ is intractable. Therefore, we solve it using numerical methods as described in Section 4.4

4.2 Decentralized channel with consignment contracts

In the decentralized channel, each of the competing suppliers produces and sells a product through a common retailer under consignment. The suppliers retain full ownership of the inventory that is placed at retailer's. As a result, the suppliers bear the risk associated with demand uncertainty while the retailer incur only a holding cost for over-ordered merchandise. We consider two types of consignment contracts, namely consignment price (CP) contracts and consignment with revenue share (CR) contracts.

We model the decision making of this two-tier supply chain as a Retailer-Stackelberg game. Following the standard newsvendor model (Cachon, 2003), the following sequence of events takes place: (1) the retailer, acting as a leader, offers a consignment contract specifying the terms of payment to her from each supplier for every unit of product sold to customers; (2) each supplier, acting as a follower, chooses the quantity q_i and the retail price p_i; (3) before the start of selling season, supplier i produces and delivers q_i units of product i to the retailer; (4) demand realizes; (5) transfer payments are made between the retailer and suppliers according to the agreed upon contract.

In this study, the retailer and two suppliers play, vertically, a Stackelberg game with the retailer as a leader and the two suppliers as followers. Horizontally, the two suppliers play a Nash game, i.e., they simultaneously decide their prices and quantities. We solve this equilibrium problem to find the Stackelberg/Nash equilibrium. The next sections present equilibrium solutions for two types of consignment contracts and derive their implications.

4.2.1 Consignment price (CP) contracts

Under CP contracts, decisions are made in two sequential steps. In the first step, the retailer decides the consignment price w corresponding to the amount of payment to be received from the suppliers for each unit sold to consumers. In the second step, given this consignment price, each supplier simultaneously selects the retail price p_i and quantity q_i. We find the equilibrium solution by using backward induction. We first derive each suppliers best response price and inventory quantity to the retailers' consignment price decision.

Prime examples of CP contracts in practice include the Home Depot and Bonanza.com. The Home Depot implements the Pay-By-Scan (PBS) program with its seasonal plant growers. Once products are sold, Home Depot makes a payment to the suppliers and retains the difference between this payment and the retail price. Similarly, Bonanza.com, online shopping website, only collects a fixed commission (not a percentage share) once products are sold to consumers.

4.2.1.1 Supplier i's selling price and stocking factor decision

At the second step of the decision sequence, for a given consignment price w selected by the retailer, supplier i selects the retail price p_i and quantity q_i to maximize his own expected profit:

$$\pi_{S_i}(\mathbf{p}, q_i|w) \quad = \quad (p_i - w)E\{\min(D_i(\mathbf{p}), q_i)\} - c_{S_i}q_i.$$

Since $z_i = q_i/y_i(\mathbf{p})$, the profit can also be rewritten as

$$\begin{aligned} \pi_{S_i}(\mathbf{p}, z_i|w) &= y_i(\mathbf{p})\{(p_i - w)(z_i - \Lambda(z_i)) - c_{S_i}z_i\} \\ &= (a - p_i + \gamma(p_{-i} - p_i))\{(p_i - w)(z_i - \Lambda(z_i)) - c_{S_i}z_i\}. \end{aligned}$$

To find the best response, denoted by $(\bar{p}_i, \bar{z}_i)$, that maximizes $\pi_{S_i}(\mathbf{p}, z_i|w)$ for a given w and p_{-i}, we first derive the retailer's best response stocking factor $\tilde{z}_i(p_i|w)$ for a given $p_i,\ p_{-i}$ and w; we then find the best response retail price $\bar{p}_i$ that maximizes $\pi_{S_i}(\mathbf{p}|w, \tilde{z}_i(p_i|w))$. Supplier i's best response (as a follower) to the retailer's consignment price w is denoted by $(\bar{p}_i^*, \bar{z}_i^*)$. Note that $\bar{p}_i$ and $\bar{z}_i$ are functions of w and p_{-i} and $\bar{p}_i^*$ and $\bar{z}_i^*$ are functions of w, but we omit to explicitly show the dependency to keep the notation simpler. The results are summarized in the propositions below.

In theory, it is possible for the suppliers to produce zero quantities of the products at equilibrium. However, we consider a more interesting case in which the suppliers produce positive quantities of the products, i.e., $q_i > 0,\ i = 1, 2$. Thus, we assume that $0 < w + c_{S_i} < p_i$ i.e., the selling price covers the total costs incurred to ensure that the optimal stocking factor exists and is unique.

Proposition 4.2.1.

- *For any given price p_i and consignment price $w > 0$ where $w + c_{S_i} < p_i$, supplier i's unique best stocking factor $\tilde{z}_i(p_i|w)$ is given by*

$$\tilde{z}_i(p_i|w) = F^{-1}(\phi_i), \tag{4.6}$$

- *The supplier i's best response price $\bar{p}_i$ that maximizes $\pi_{S_i}(\mathbf{p}|w, \tilde{z}_i(p_i|w))$ for a given consignment price w and price p_{-i} of supplier $-i$ is uniquely determined as the solution of:*

$$1 + \frac{c_{S_i}}{p_i - w} \frac{F^{-1}(\phi_i)}{\int_A^{F^{-1}(\phi_i)} x f(x)\, dx} = \frac{(1+\gamma)(p_i - w)}{a - p_i + \gamma(p_{-i} - p_i)}, \tag{4.7}$$

where $\phi_i = 1 - \frac{c_{S_i}}{p_i - w}$.

Proposition 4.2.1 implies that supplier i's best response stocking is nondecreasing with its relative margin, i.e., the ratio of supplier i's unit margin (the difference between unit selling price and unit consignment price and unit production cost) to the unit payment received. This reflects the fact that the supplier would want to increase the quantity supplied to the retailer when its relative margin for each unit sold increases.

Using (Equation 4.6) and (Equation 4.7), we obtain the best response stocking factor to a consignment price w and price p_{-i} of supplier $-i$ as

$$\bar{z}_i = \tilde{z}_i(\bar{p}_i|w) = F^{-1}(\bar{\phi}_i) = F^{-1}\left(1 - \frac{c_{S_i}}{\bar{p}_i - w}\right). \tag{4.8}$$

Therefore, the unique joint best response prices for suppliers 1 and 2 (as the followers) to the retailer's consignment price w, denoted by $(\bar{p}_1^*,\ \bar{p}_2^*)$, respectively, is the solution to the system of equations

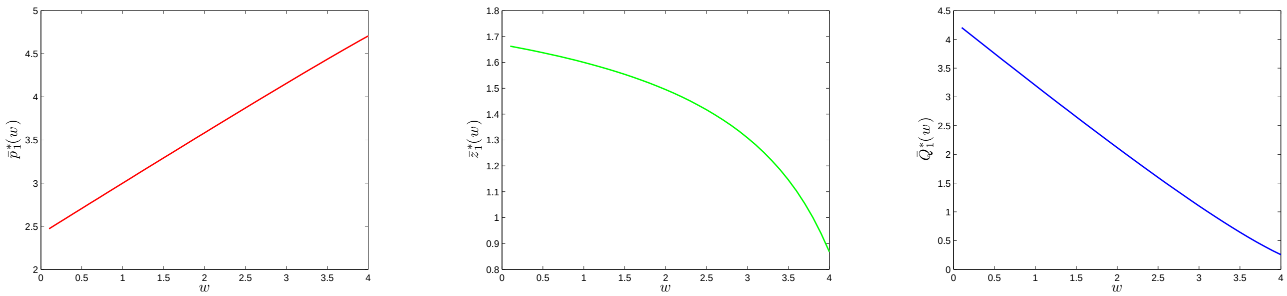
$$\begin{aligned} 1 + \frac{c_{S_1}}{p_1 - w} \frac{F^{-1}(\phi_1)}{\int_A^{F^{-1}(\phi_1)} xf(x)\,dx} &= \frac{(1+\gamma)(p_1 - w)}{a - p_1 + \gamma(p_2 - p_1)}, \\ 1 + \frac{c_{S_2}}{p_2 - w} \frac{F^{-1}(\phi_2)}{\int_A^{F^{-1}(\phi_2)} xf(x)\,dx} &= \frac{(1+\gamma)(p_2 - w)}{a - p_2 + \gamma(p_1 - p_2)}. \end{aligned} \tag{4.9}$$

This allow us to find, numerically, the best response price.

The corresponding best response stocking factor $\bar{z}_i^*$ to a consignment price w, where $i = 1, 2$ is

$$\bar{z}_i^* = \tilde{z}_i(\bar{p}_i{}^*|w) = F^{-1}(\bar{\phi}_i^*) = F^{-1}\left(1 - \frac{c_{S_i}}{\bar{p}_i^* - w}\right). \tag{4.10}$$

Figure Figure 27 illustrates the supplier's best response price, stocking factor, and quantity as a function of the retailer's consignment price w. We observe that the supplier's best



(a) Retailer 1's best response price $\bar{p}_1^*$ to a given consignment price w

(b) Retailer 1's best response stocking factor $\bar{z}_1^*$ to a given consignment price w

(c) Retailer 1's best response quantity $\bar{q}_1^*$ to a given consignment price w

Figure 27. The retailer 1's best response price, stocking factor and stocking quantity to a given consignment price w where $\gamma = 0.5, a = 5, c_{S_1} = c_{S_2} = 0.4$ in the CP contract

response price increases with w. Clearly, as the retailer's consignment price increases, the supplier must transfer this cost increase to consumers by increasing the retail price. The higher retail price causes the demand to go down which leads to a lower quantity supplied by each supplier. Consequently, both expected demand and quantity decrease with w. However, the quantity decreases faster than the expected demand. Thus, the stocking factor decreases with the retailer's consignment price.

Retailer's consignment price decision At the first step, anticipating the suppliers' reaction to its decision, the retailer sets the consignment price w to maximize its own expected profit $\pi_R(w)$, given by

$$\begin{aligned}
\pi_R(w) &= w[E\{min(D_1(\bar{\mathbf{p}}^*), \bar{q}_1^*)\} + E\{min(D_2(\bar{\mathbf{p}}^*), \bar{q}_2^*)\}] - c_{R_1}\bar{q}_1^* - c_{R_2}\bar{q}_2^* \\
&= w[y_1(\bar{\mathbf{p}}^*)(\bar{z}_1^* - \Lambda(\bar{z}_1^*)) + y_2(\bar{\mathbf{p}}^*)(\bar{z}_2^* - \Lambda(\bar{z}_2^*))] - c_{R_1}y_1(\bar{\mathbf{p}}^*)\bar{z}_1^* + c_{R_2}y_2(\bar{\mathbf{p}}^*)\bar{z}_2^* \\
&= (a - \bar{p}_1^* + \gamma(\bar{p}_2^* - \bar{p}_1^*))[w(\bar{z}_1^* - \Lambda(\bar{z}_1^*)) - c_{R_1}\bar{z}_1^*] \\
&\quad + (a - \bar{p}_2^* + \gamma(\bar{p}_1^* - \bar{p}_2^*))[w(\bar{z}_2^* - \Lambda(\bar{z}_2^*)) - c_{R_2}\bar{z}_2^*].
\end{aligned}$$

To find the equilibrium solution, denoted by w^*, we seek to maximize $\pi_R(w)$ over w. Since $\bar{p}_i^*$ and $\bar{z}_i^*$ are only known as implicit functions of w given by (Equation 4.9) and (Equation 4.10), we find w^* numerically.

Once w^* is computed, equations (Equation 4.9) and (Equation 4.10) allow us to determine the equilibrium price and the equilibrium stocking factor for each of the supplier's products.

4.2.2 Consignment with revenue share (CR) contract

In this section, we consider another type of consignment contract, known as consignment with revenue share (CR) contract. Under CR contracts, decisions are made in two sequential steps. In the first step, the retailer decides the revenue share r of the suppliers' revenue that he will receive for each unit sold to consumers. In the second step, given this revenue share, each supplier simultaneously selects the retail price p_i and quantity q_i. We find the equilibrium solution by using backward induction. We first derive each supplier's best response price and inventory quantity to the retailers revenue share decision.

One of the most predominant examples of CR contracts is Amazon Marketplace. The merchants (suppliers) decide the quantity of products and the selling price. Amazon.com collects a certain percentage from the final sale price only when products are sold to consumers.

4.2.2.1 Supplier i's selling price and stocking factor decision

At the second step, for a given revenue share r selected by the retailer, supplier i selects the retail price p_i and quantity q_i to maximize his own expected profit which is given by

$$\pi_{S_i}(\mathbf{p}, q_i|r) \quad = \quad (1-r)p_i E\{min(D_i(\mathbf{p}), q_i)\} - c_{S_i} q_i.$$

Since $z_i = q_i/y_i(\mathbf{p})$, the profit can be rewritten as

$$\begin{aligned} \pi_{S_i}(\mathbf{p}, z_i|r) &= y_i(\mathbf{p})\{(1-r)p_i(z_i - \Lambda(z_i)) - c_{S_i} z_i\} \\ &= (a - p_i + \gamma(p_{-i} - p_i))\{(1-r)p_i(z_i - \Lambda(z_i)) - c_{S_i} z_i\}. \qquad (4.11) \end{aligned}$$

To find the best response, denoted by $(\bar{p}_i, \bar{z}_i)$, that maximizes $\pi_{S_i}(\mathbf{p}, z_i|r)$ for a given r and p_{-i}, we first derive the retailer's best response stocking factor $\tilde{z}_i(p_i|r)$ for a given p_i, p_{-i} and r; we then find the best response retail price $\bar{p}_i$ that maximizes $\pi_{S_i}(\mathbf{p}|r, \tilde{z}_i(p_i|r))$. The supplier's best response (as a follower) to the retailer's revenue sharing fraction r is denoted by $(\bar{p}_i^*, \bar{z}_i^*)$. Note that $\bar{p}_i$ and $\bar{z}_i$ are functions of r and p_{-i} and $\bar{p}_i^*$ and $\bar{z}_i^*$ are functions of r, but we omit to explicitly show the dependency to keep the notation simpler. The results are summarized in the propositions below.

In theory, it is possible for the suppliers to produce zero quantities of the products. However, we consider a more interesting case in which the suppliers produce positive quantities of the products, i.e., $q_i > 0,\ i = 1, 2$. Thus, we assume that $0 < \frac{c_{S_i}}{1-r} < p_i$ i.e., the sale covers the cost incurred to ensure that the best response stocking factor exists and is unique.

Proposition 4.2.2.

- *For any given price* $p_i > \frac{c_{S_i}}{1-r}$ *and revenue sharing proportion* $0 < r < 1$, *retailer* i*'s unique best response stocking factor* $\tilde{z}_i(p_i|r)$ *is given by*

$$\tilde{z}_i(p_i|r) = F^{-1}(\rho_i), \tag{4.12}$$

- *The supplier* i*'s best response price* $\bar{p}_i$ *that maximizes the supplier* i*'s profit* $\pi_{S_i}(\tilde{z}_i(p_i|r), \mathbf{p}|r)$ *for a given revenue share* r *and price* p_{-i} *of supplier* $-i$ *is uniquely determined as the solution of:*

$$1 + \frac{c_{S_i}}{(1-r)p_i} \frac{F^{-1}(\rho_i)}{\int_A^{F^{-1}(\rho_i)} x f(x)\, dx} = \frac{(1+\gamma)p_i}{a - p_i + \gamma(p_{-i} - p_i)}, \tag{4.13}$$

where $\rho_i = 1 - \frac{c_{S_i}}{(1-r)p_i}$.

Proposition 4.2.2 suggests that supplier i's best response stocking stocking is nondecreasing with its relative margin, i.e., the ratio of supplier i's unit margin (the difference between unit selling price and unit revenue share fraction and unit production cost) to the unit payment received. This reflects the fact that the supplier would want to increase the quantity supplied to the retailer when the relative margin for each unit sold increases.

Using (Equation 4.12) and (Equation 4.13), we obtain that the best response stocking factor to a revenue sharing proportion r is

$$\bar{z}_i = \tilde{z}_i(\bar{p}_i|r) = F^{-1}(\bar{\rho}_i) = F^{-1}\left(1 - \frac{c_{S_i}}{(1-r)\bar{p}_i}\right). \tag{4.14}$$

Therefore, the unique joint best response prices for suppliers 1 and 2 (as the followers) to the retailer's revenue share fraction r, denoted by $(\bar{p}_1^*,\ \bar{p}_2^*)$, respectively, is the solution to the system of equations

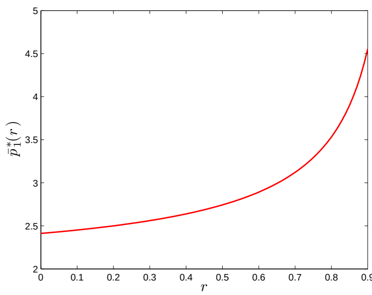
$$\begin{aligned} 1 + \frac{c_{S_1}}{(1-r)p_1}\frac{F^{-1}(\rho_1)}{\int_A^{F^{-1}(\rho_1)} xf(x)\,dx} &= \frac{(1+\gamma)p_1}{a - p_1 + \gamma(p_2 - p_1)}, \\ 1 + \frac{c_{S_2}}{(1-r)p_2}\frac{F^{-1}(\rho_2)}{\int_A^{F^{-1}(\rho_2)} xf(x)\,dx} &= \frac{(1+\gamma)p_2}{a - p_2 + \gamma(p_1 - p_2)}. \end{aligned} \tag{4.15}$$

This allows us to find, numerically, the best response price.

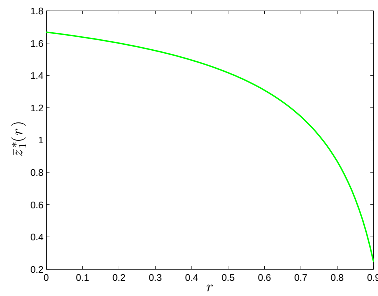
The corresponding best response stocking factor $\bar{z}_i^*$ to a revenue sharing proportion r, where $i = 1, 2$ is

$$\bar{z}_i^* = \tilde{z}_i(\bar{p}_i^*|r) = F^{-1}(\bar{\rho}_i^*) = F^{-1}\left(1 - \frac{c_{S_i}}{(1-r)\bar{p}_i^*}\right) \tag{4.16}$$

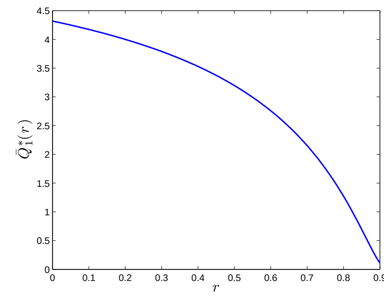
Figure Figure 28 illustrates the supplier's best response price, stocking factor, and quantity as a function of the retailer's revenue share r. When the retailer keeps a higher share of the suppliers revenue, the suppliers transfer this increasing revenue loss to consumers. The higher



(a) Retailer 1's best response price $\bar{p}_1^*$ to a given revenue share r



(b) Retailer 1's best response stocking factor $\bar{z}_1^*$ to a given revenue share r



(c) Retailer 1's best response quantity $\bar{q}_1^*$ to a given revenue share r

Figure 28. The retailer 1's best response price, stocking factor and stocking quantity to a given revenue share r where $\gamma = 0.5, a = 5, c_{S_1} = c_{S_2} = 0.4$ in the CR contract

retail price causes the demand to go down, which leads to a lower quantity ordered from each supplier. While both the expected demand and quantity decrease with the suppliers revenue share r, the quantity decreases faster than the expected demand. Therefore, the stocking factor decreases with the retailer's revenue share.

4.2.2.2 Retailer's revenue sharing fraction decision

At the first step, anticipating the suppliers' reaction to its decision, the retailer sets the revenue sharing fraction r to maximize its own expected profit $\pi_R(r)$, given by

$$\begin{aligned}
\pi_R(r) &= r\bar{p}_1^* E\{\min(D_1(\bar{\mathbf{p}}^*), \bar{q}_1^*)\} - c_{R_1}\bar{q}_1^* + r\bar{p}_2^* E\{\min(D_2(\bar{\mathbf{p}}^*), \bar{q}_2^*)\} - c_{R_2}\bar{q}_2^* \\
&= y_1(\bar{\mathbf{p}}^*))[r\bar{p}_1^*(\bar{z}_1^* - \Lambda(\bar{z}_1^*)) - c_{R_1}\bar{z}_1^*] + y_2(\bar{\mathbf{p}}^*))[r\bar{p}_2^*(\bar{z}_2^* - \Lambda(\bar{z}_2^*)) - c_{R_2}\bar{z}_2^*] \\
&= (a - \bar{p}_1^* + \gamma(\bar{p}_2^* - \bar{p}_1^*))[r\bar{p}_1^*(\bar{z}_1 - \Lambda(\bar{z}_1^*)) - c_{R_1}\bar{z}_1^*] \\
&\quad + (a - \bar{p}_2^* + \gamma(\bar{p}_1^* - \bar{p}_2^*))[r\bar{p}_2^*(\bar{z}_2^* - \Lambda(\bar{z}_2^*)) - c_{R_2}\bar{z}_2^*].
\end{aligned}$$

To find the equilibrium solution, denoted by r^*, we would have to maximize $\pi_S(r)$ over r. Since $\bar{p}_i^*$ and $\bar{z}_i^*$ are only known as implicit functions of r given by (Equation 4.15) and (Equation 4.16), obtaining an analytical solution for this maximization problem is intractable. Thus, we use numerical methods.

Once r^* is computed, using equations (Equation 4.15) and (Equation 4.16) allow us to solve, numerically, the equilibrium price and the equilibrium stocking factor for each of supplier's products.

4.2.2.3 Special case: CR contracts for the case of deterministic demand and symmetric cost structure

In order to shed some light on the retailer's optimal revenue share r^*, we now consider a special case where the two suppliers have a symmetric cost structure ($c_{S_1} = c_{S_2} = c_S$) and the retailer incurs the same costs of handling and selling products from both suppliers ($c_{R_1} = c_{R_2} = c_R$). In this section, we assume that the demand function is deterministic, i.e., $\epsilon = 1$ and $D_i(\mathbf{p}) = y_i(\mathbf{p})$. Since there is no stochastic, the quantity ordered by the retailer matches the demand and the only decision left to suppliers is the retail price. The best response retail prices satisfy $\bar{p}_1^* = \bar{p}_2^* = \bar{p}^*$ by symmetry.

Using (Equation 4.15), the best response retail price for each supplier, $\bar{p}^*$ to the retailer's revenue share r is given by

$$\bar{p}_i^* = \frac{1}{2+\gamma}\left[a + \frac{(1+\gamma)c_S}{1-r}\right]. \tag{4.17}$$

It follows from (Equation 4.17) that the retailer's profit can be expressed as

$$\pi_R(w) \quad = \quad 2(a - \bar{p}^*)[r\bar{p}^* - c_R]. \tag{4.18}$$

Using (Equation 4.17) in (Equation 4.18), we find

$$\pi_R(r) \quad = \quad 2\left\{a - \frac{1}{2+\gamma}\left[a + \frac{(1+\gamma)c_S}{1-r}\right]\right\}\left\{r\left[\frac{1}{2+\gamma}\left[a + \frac{(1+\gamma)c_S}{1-r}\right]\right] - c_R\right\}.$$

Proposition 4.2.3. *The retailer's equilibrium revenue share r^* is the solution of:*

$$\frac{1+\gamma}{2+\gamma}(a^2 + c_S) = \frac{(1+\gamma)c_S}{(2+\gamma)(1-r)^2}\left\{\left[\frac{(3+\gamma)a}{2+\gamma} + \frac{(1+\gamma)(1+r)c_S}{(2+\gamma)(1-r)}\right] - c_R\right\}. \tag{4.19}$$

Proposition 4.2.3 gives, implicitly, the retailer's equilibrium revenue share. This allows us to find, numerically, the equilibrium revenue share in the case of deterministic demand and symmetric cost structure

4.3 Dual channel supply chain (DC)

Although suppliers who enter consignment contracts can increase sales volume and profits by gaining access to consumers and transferring the responsibility and the cost of storage to the retailer, it could be beneficial for the supplier to additionally sell its products directly to consumers and thereby eliminate the margin (charged by the retailer). Recall that we consider a supply chain with one common retailer (R) and two suppliers (S_1 and S_2). Assume that one supplier, S_1, introduces an additional direct channel to sell products directly to consumers while continuing to sell through the retailer via consignment. The cost of producing and selling a unit of product 1 through its direct channel (direct sale unit cost) is $\$c_{S_1^d}$. We assume that $\$c_{S_1^d} > \$c_{S_1}$. This indicates that the cost incurred at supplier 1's direct channel is higher than that in its retail consignment channel due to additional expenses.

In order to measure the channel performance, we consider two settings – centralized and decentralized. In the centralized case, a decision-maker simultaneously chooses the selling price and the quantity for each of the products in order to maximize the expected total channel profit for both suppliers and the retailer. On the other hand, in the decentralized case, each of the competing suppliers chooses the selling price and the quantity for his product in order to maximize his expected profit. We are interested in: (i) The channel efficiency i.e., does the supplier additional direct channel help improve channel efficiency? and (ii) the effect of the supplier's direct channel on the channel decisions and profits.

Let $y_1(\mathbf{p})$, $y_2(\mathbf{p})$ and $y_1^d(\mathbf{p})$, respectively, denote the expected demand for product 1, product 2 at the retailer and the expected demand for product 1 in the direct channel. Using an extended linear demand functions that consider both product differentiation and store differentiation (Choi, 1996; Liao and Tseng, 2007), we model the expected demand for product i at the retailer, $y_i(\mathbf{p})$ for $i = 1, 2$, and the expected demand for product 1 at its direct channel, $y_1^d(\mathbf{p})$, as

$$\begin{aligned} y_1(\mathbf{p}) &= a - p_1 + \gamma(p_2 - p_1) + \alpha(p_1^d - p_1) \\ y_2(\mathbf{p}) &= a - p_2 + \gamma(p_1 - p_2) + \alpha\gamma(p_1^d - p_2) \\ y_1^d(\mathbf{p}) &= a - p_1^d + \alpha(p_1 - p_1^d) + \alpha\gamma(p_2 - p_1^d), \qquad a, \gamma, \alpha > 0;\ \gamma, \alpha < 1 \end{aligned} \tag{4.20}$$

where γ is related to product differentiation and α is related to store differentiation. Parameter a is the primary demand of each product. As α increases, the stores (direct channel and the retailer) are less differentiated (more substitutable), therefore the price competition is more intense. $\alpha\gamma$ is related to the cross effect of the degree of store and product differentiation.

The demand for product i at the retailer and the demand for product 1 at its direct channel, respectively, are defined as:

$$\begin{aligned} D_1^{DC}(\mathbf{p}) &= y_1(\mathbf{p}) \cdot \epsilon \\ D_2^{DC}(\mathbf{p}) &= y_2(\mathbf{p}) \cdot \epsilon \\ D_1^{d\ DC}(\mathbf{p}) &= y_1^d(\mathbf{p}) \cdot \epsilon, \quad \mathbf{p} = (p_1, p_2, p_1^d). \end{aligned} \tag{4.21}$$

Centralized channel

In the centralized direct channel, a decision maker simultaneously chooses the selling price and the quantity for each of the products in order to maximize the expected total channel profit for both suppliers and the retailer. The following sequence of events occurs: (1) a central decision maker simultaneously chooses the selling price and the quantity for each of the products; (2) before the start of selling season, supplier i produces q_i units of product i for selling through the retailer via consignment and supplier 1 produces q_1^d units of product 1 for selling in its direct channel; (3) demand realizes.

The central planner's objective is to maximize the total expected channel profit, which is the sum of both the suppliers' and the retailer's profits. The central planner simultaneously chooses the selling price p_i, the quantity q_i for product i, where $i = 1, 2$ as well as the selling price p_1^d and the quantity q_1^d. The objective is to maximize the expected total channel profit which can be written as:

$$\begin{aligned}\pi_C^{DC}(\mathbf{p}, \mathbf{q}) &= p_1 E\{\min(D_1^{DC}, q_1)\} - (c_{S_1} + c_{R_1})q_1 + p_2 E\{\min(D_2^{DC}, q_2)\} - (c_{S_2} + c_{R_2})q_2 \\ &\quad + p_1^d E\{\min(D_1^{d\ DC}, q_1^d)\} - c_{S_1^d} q_1^d.\end{aligned}$$

It follows from Proposition 3.3.3 that for any given price $p_i > c_{S_i} + c_{R_i}$, supplier i's unique optimal stocking factor $z_i^{C/DC^*}(p_i)$ is given by

$$z_i^{C/DC^*}(p_i) = F^{-1}(\nu_i), \quad i = 1, 2; \tag{4.22}$$

for any given price $p_1^d > c_{S_1}^d$, supplier i's unique optimal stocking factor $z_i^{C/DC^*}(p_i)$ is given by

$$z_1^{d\ C/DC^*}(p_1^d) = F^{-1}(\nu_1^d), \tag{4.23}$$

where $\nu_i = 1 - \frac{c_{S_i} + c_{R_i}}{p_i}$, for $i = 1, 2$ and $\nu_1^d = 1 - \frac{c_{S_1^d}}{p_1^d}$.

Following Proposition 3.3.4, the unique optimal prices p_1^{C/DC^*}, p_2^{C/DC^*} and $p_1^{d\ C/DC^*}$ that maximize the centralized direct channel profit $\pi_{C/DC}(\mathbf{p}, \mathbf{z}^{C^*}(\mathbf{p}))$ must satisfy the first-order necessary conditions

We solve this system of three equations, numerically, to find the optimal prices and thus the quantities in Section 4.4.

Decentralized channel

In the decentralized channel, each of the competing suppliers chooses the selling price and the quantity for its product in order to maximize its expected profit. The following sequence of events occurs: (1) each of the suppliers simultaneously chooses the selling price and the quantity for his product (for direct and indirect channel in the case of supplier 1); (2) before the start of the selling season, supplier i produces q_i units of product i for selling through the retailer via consignment ($i = 1, 2$) and supplier 1 produces q_1^d units of product 1 for selling through its direct channel; (3) demand realizes.

Supplier 1's objective:

Supplier 1's objective is to maximize the total profit which is the sum of the profits from the direct channel (d) and indirect channel (IC) profits. Thus, supplier 1 chooses the selling prices p_1^d and p_1 as well as the quantities q_1^d and q_1, for both direct and indirect consignment channels respectively. The objective is to maximize the expected total profit which is given by:

$$\pi_{S_1}^T(\mathbf{p}, \mathbf{q}_1) \quad = \quad \pi_{S_1}^d(\mathbf{p}, q_1^d) + \pi_{S_1}^{IC}(\mathbf{p}, q_1),$$

where $\mathbf{p} = 9p_1^d, p_1, p_2)$ and $\mathbf{q}_1 = (q_1^d, q_1)$.

Consignment price (CP) contracts: supplier 1's expected profit can be written as:

$$\pi_{S_1}^T(\mathbf{p}, \mathbf{q}_1|w) \quad = \quad p_1^d E\{min(D_1^{d\ DC}, q_1^d)\} - c_{S_1^d} q_1^d + (p_1 - w)E\{\min(D_1^{IC}(\mathbf{p}), q_1)\} - c_{S_1} q_1.$$

Consignment with revenue share (CR) contracts: supplier 1's expected profit can be written as:

$$\pi_{S_1}^T(\mathbf{p}, \mathbf{q}_1|r) \quad = \quad p_1^d E\{min(D_1^{d\ DC}, q_1^d)\} - c_{S_1^d} q_1^d + (1 - r)p_1 E\{min(D_1^{DC}(\mathbf{p}), q_1)\} - c_{S_1} q_1.$$

We first derive the best response stocking factor $\bar{z}_1^d$ for the direct channel. It follows from Proposition 3.3.6 for $w = 0$ and $r = 0$ that, for any given price $p_1^d > c_{S_1^d}$ supplier i's unique best response stocking factor $\bar{z}_1^d(p_1)$ is given by

$$\bar{z}_1^{d\ DC}(p_1^d) = F^{-1}(\nu_1^d), \tag{4.24}$$

where $\nu_1^d = 1 - \frac{c_{S_1^d}}{p_1^d}$

The best response stocking factor $\bar{z}_1$ for the indirect channel is as given by Proposition 3.3.6. Next, we derive the best response price for the direct channel and the best response price for the indirect channel under CP and CR contracts.

- **Consignment price (CP) contracts**: the unique joint best response prices for supplier 1 that maximizes the supplier 1's profit $\pi_{S_1}^T(\bar{\mathbf{z}}_1(\mathbf{p}), \mathbf{p}|w)$, for a given price p_2 of supplier 2 and a consignment price w, must satisfy the first-order necessary conditions, given by:

$$\begin{aligned}
1 + \frac{c_{S_1^d}}{p_1^d} \frac{F^{-1}(\nu_1^d)}{\int_A^{F^{-1}(\nu_1^d)} x f(x)\,dx} &+ \frac{\alpha(p_1 - w)\int_A^{F^{-1}(\phi_1)} x f(x)\,dx}{(a - p_1^d + \alpha(p_1 - p_1^d) + \alpha\gamma(p_2 - p_1^d))\int_A^{F^{-1}(\nu_1^d)} x f(x)\,dx} \\
&= \frac{(1+\alpha+\alpha\gamma)p_1^d}{a - p_1^d + \alpha(p_1 - p_1^d) + \alpha\gamma(p_2 - p_1^d)}, \\
1 + \frac{c_{S_1}}{p_1 - w} \frac{F^{-1}(\phi_1)}{\int_A^{F^{-1}(\phi_1)} x f(x)\,dx} &+ \frac{\alpha p_1^d \int_A^{F^{-1}(\nu_1^d)} x f(x)\,dx}{(a - p_1^d + \alpha(p_1 - p_1^d) + \alpha\gamma(p_2 - p_1^d))\int_A^{F^{-1}(\phi_1)} x f(x)\,dx} \\
&= \frac{(1+\gamma+\alpha)(p_1 - w)}{a - p_1 + \gamma(p_2 - p_1) + \alpha(p_1^d - p_1)}
\end{aligned}$$

where $\nu_1^d = 1 - \frac{c_{S_1^d}}{p_1^d}$ and $\phi_i = 1 - \frac{c_{S_i}}{p_i - w}$.

- **Consignment with revenue share (CR) contracts**: the unique joint best response prices for supplier 1 that maximizes the supplier 1's profit $\pi_{S_1}^T(\bar{\mathbf{z}}_1(\mathbf{p}), \mathbf{p}|r)$, for a given price p_2 of supplier 2 and a revenue share fraction r, must satisfy the first-order necessary conditions, given by:

$$\begin{aligned}
1 + \frac{c_{S_1^d}}{p_1^d} \frac{F^{-1}(\nu_1^d)}{\int_A^{F^{-1}(\nu_1^d)} xf(x)\,dx} &+ \frac{\alpha(1-r)p_1 \int_A^{F^{-1}(\phi_1)} xf(x)\,dx}{(a - p_1^d + \alpha(p_1 - p_1^d) + \alpha\gamma(p_2 - p_1^d)) \int_A^{F^{-1}(\nu_1^d)} xf(x)\,dx} \\
&= \frac{(1+\alpha+\alpha\gamma)p_1^d}{a - p_1^d + \alpha(p_1 - p_1^d) + \alpha\gamma(p_2 - p_1^d)}, \\
1 + \frac{c_{S_1}}{p_1 - w} \frac{F^{-1}(\phi_1)}{\int_A^{F^{-1}(\phi_1)} xf(x)\,dx} &+ \frac{\alpha p_1^d \int_A^{F^{-1}(\nu_1^d)} xf(x)\,dx}{(a - p_1^d + \alpha(p_1 - p_1^d) + \alpha\gamma(p_2 - p_1^d)) \int_A^{F^{-1}(\phi_1)} xf(x)\,dx} \\
&= \frac{(1+\gamma+\alpha)(p_1 - w)}{a - p_1 + \gamma(p_2 - p_1) + \alpha(p_1^d - p_1)}
\end{aligned}$$

where $\nu_1^d = 1 - \frac{c_{S_1^d}}{p_1^d}$ and $\rho_i = 1 - \frac{c_{S_i}}{(1-r)p_i}$.

Supplier 2's objective function is the same as in sections 4.2.1 and 4.2.2. Using backward induction, the retailer, anticipating the supplier's reaction to its decision, sets the consignment price w (CP contract) or the revenue share fraction r (CR contract) to maximize her own expected profit π_R. We use numerical methods to solve for the equilibrium solution, denoted by w^* or r^* and we then solve, numerically, the equilibrium price and the equilibrium stocking factor for each of the supplier's products.

4.4 Numerical results

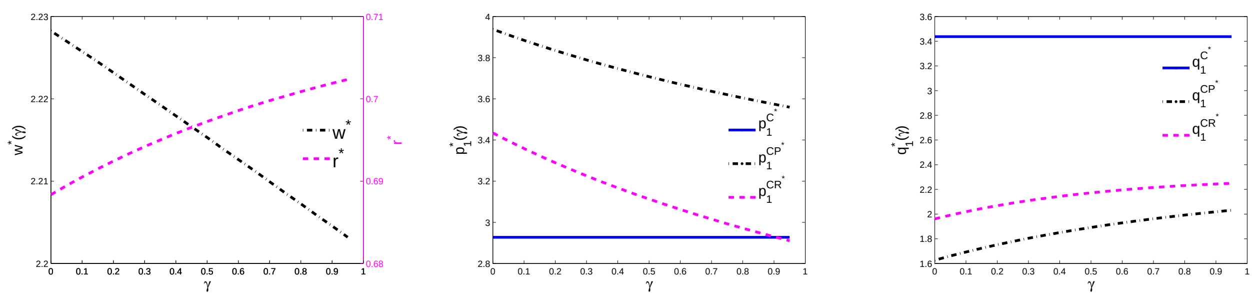
In this section, we obtain numerically the equilibrium quantities and we interpret the findings. The efficiency of the decentralized system is defined as $Eff = \pi_T^{D^*}/\pi_T^{C^*}$.

Our numerical study is intended to gain an understanding of the impact of product differentiation γ on the equilibrium decisions and profits. We then consider the effect of this factor on the efficiency of the decentralized channel. Moreover, we are interested in how demand variability affects the equilibrium decisions, profits and channel efficiency. Subsequently, we examine how the presence of a direct channel affects equilibrium decisions, profits and channel efficiency (the effect of store differentiation α). We are also interested whether the presence of a direct channel always benefits some supplier chain members.

The random perturbation on the demand, ϵ, is assumed to follow a uniform distribution on $[1-C, 1+C]$ for $0 \leq C \leq 1$. Following previous numerical studies (Choi, 1991; Li et al., 2009; Zhang et al., 2010; Adida and Ratisoontorn, 2011), we set $a = 5$, $c_{S_1} = c_{S_2} = 0.4$ and $c_{R_1} = c_{R_2} = 0.1$. We choose $A = 0$ and $B = 2$ in order to ensure that the perturbation on the demand has a mean value of 1.

4.4.1 The effect of product differentiation

The major difference between our work and previous studies in consignment contracts (Wang et al., 2004; Ru and Wang, 2010; Adida and Ratisoontorn, 2011) that we incorporate supplier competition into our model. Although (Wang, 2006) considers supplier competition in his consignment study, he assumes that products from competing suppliers are perfect complements. Our model, however, relaxes this assumption by considering substitutable, but differentiated, products from two suppliers. The parameter γ is related product differentiation. That is, if products are less differentiated (price competition is more intense), then γ is larger (closer to 1). The value of γ varies within $[0,\ 1)$ to ensure $\gamma < 1$.



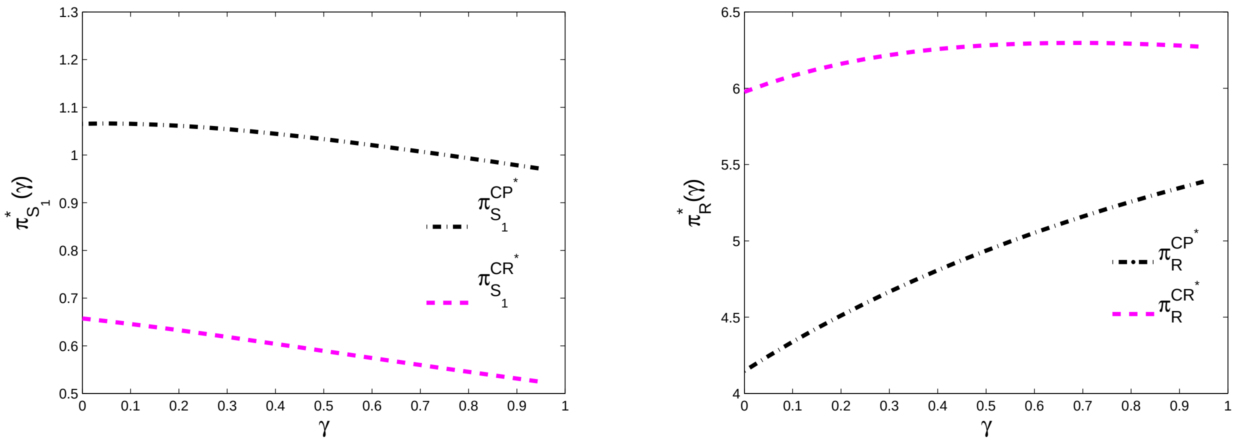
(a) Retailer's consignment price w^* or revenue share r^*

(b) Retail price p_1^*

(c) Quantity q_1^*

Figure 29. Retailer's consignment price (or revenue share), retail price and quantity as a function of γ under CP and CR contracts

We now examine the effect of product differentiation on equilibrium prices, quantities and profits. Figure Figure 29(a) suggests that the retailer's consignment price (CP contract) decreases in γ while the revenue share (CR contract) increases in γ. This opposite patterns can be explained by: (1) The retailer could take advantage of a more intense level of competition between suppliers (products are less differentiated at a higher value of γ) by increasing its revenue share. (2) On the other hand, as the level of competition increases the suppliers' natural response is to decrease their retail price. The retailer, anticipating the suppliers' reaction, would



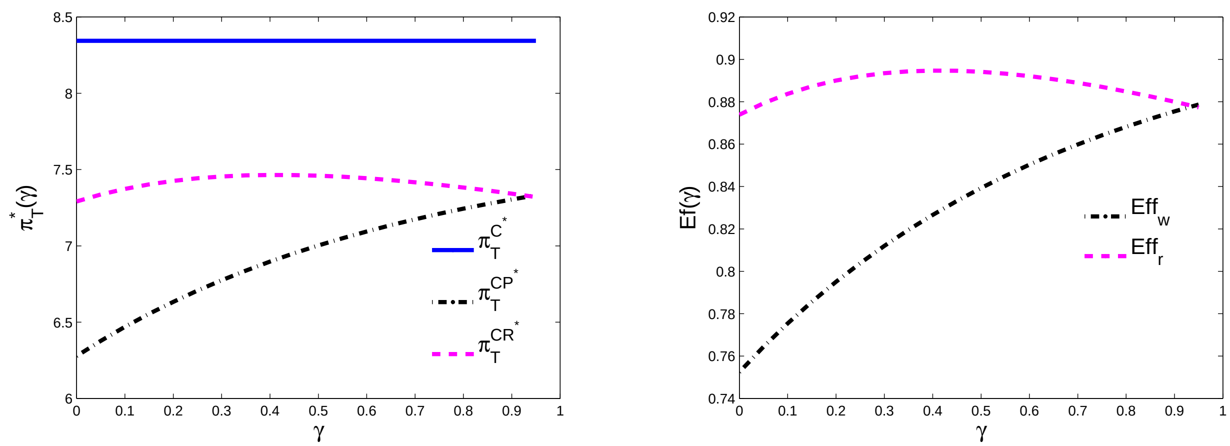
(a) Supplier 1's profit $\pi^*_{S_i}$

(b) Retailer's profit π^*_R

Figure 30. Supplier i's profit (for $i = 1$, by symmetry, suppliers 1 and 2 have the same profits) and retailer's profit as a function of γ under CP and CR contracts

lower its consignment price. Figure Figure 29(c) depicts the effect of competition (product differentiation) on the quantity q_i^*. That is, as products are less differentiated (higher value of γ), the equilibrium quantity increases. This result is intuitive since a more intense level of supplier competition generally increases the quantity (Choi, 1996; Liao and Tseng, 2007).

Figure Figure 30(a) illustrates that the suppliers' profits under both types of consignment contracts decreases in γ. On the other hand, the retailer's profit under the CP contract increases in γ and is not monotonic in γ under the CR contract (Figure Figure 30(b)). The fact that the retailer's profit in the CR contract is non-monotonic in γ is due to the combination of two opposite effects of γ on profits. On the one hand, an increase of γ may lead to an increase of the



(a) Total channel profit π_T^*

(b) Channel efficiency *Eff*

Figure 31. Total channel profit as a function of γ and channel efficiency under CP and CR contracts

(expected) demand which may increase the retailer's profit. On the other hand, as γ increases, the retail price decreases which, in turn, could cause the retailer's profit to go down.

We now compare the decisions and the total profits in the centralized and the decentralized channels. Unsurprisingly, the retail price in the centralized channel is lower than the retail price in the decentralized channel due to double marginalization (Figure Figure 29(b)). Also as expected, the quantity in the decentralized channel (in CP and CR contracts) is smaller than that in the centralized channel (Figure Figure 29(c)). It is interesting to note that the retail price in the centralized system is independent in γ (Figure Figure 29(b)) because the two products have symmetric costs and as a result, have the same retail price, which eliminates the effect of product differentiation on the price, quantity and total profit in the centralized channel (Figures Figure 29(b), (c) and Figure 31(a)).

The total profit in the decentralized CP contracts increases in γ because the retailer's profit sharply increases in γ and this increase dominates the decrease in the suppliers' profits in γ. Similarly, the total profit under the CR contract is not monotonic in γ because of the non-monotonicity of the retailer's profit. In the decentralized supply chain, the total profit in the CR contract is always higher than that in the CP contract, regardless of the degree of product differentiation (Figure Figure 31(a)).

Figure Figure 31(b) shows the impact of product differentiation on supply chain efficiency. The channel efficiency for CP contract increases in γ. This means that increased competition between the two suppliers helps improve the channel efficiency. This conclusion is consistent with the study by (Van Ryzin and Mahajan, 1999; Yao et al., 2008b) that the channel efficiency increase as products become less differentiated (higher level of supplier competition). The channel efficiency for CR contract, however, is not monotonic in γ due to non-monotinicity of the total channel profit. The efficiency of the decentralized CR contract is always higher than the efficiency of the CP contract. However, the channel efficiency of the CP contract is higher (closer to that of the CR contract) as products become less differentiated.

The effects of product differentiation (supplier competition) on the decisions, profits and supply chain efficiency are summarized in Table Table VI.

Decisions/profits	Centralized Channel	CP contract	CR contract	Remark
w^* or r^*	–	decreasing	increasing	
p_i^*	independent	decreasing	decreasing	$p_i^{CP^*} > p_i^{CR^*} > p_i^{C^*}$
q_i^*	independent	increasing	increasing	$q_i^{C^*} > q_i^{CR^*} > q_i^{CP^*}$
$\pi_{S_i}^*$	–	decreasing	decreasing	$\pi_{S_i}^{CP^*} > \pi_{S_i}^{CR^*}$
π_R^*	–	increasing	not monotonic	$\pi_R^{CR^*} > \pi_R^{CP^*}$
π_T^*	independent	increasing	not monotonic	$\pi_T^{CR^*} > \pi_T^{CP^*}$
Eff	–	increasing	not monotonic	$Eff_{CR} > Eff_{CP}$

TABLE VI

EFFECT OF PARAMETER γ ON THE EQUILIBRIUM DECISIONS,PROFITS AND CHANNEL EFFICIENCY

4.4.2 The effect of the presence of a direct channel

One of our contributions to the literature is to introduce a dual channel supply chain in the study of consignment contracts. Here, we consider a supply chain in which one supplier can sell products directly to consumers through its direct channel, in addition to selling through the retailer via consignment. Thus, we introduce another level of competition (store differentiation) between the supplier's direct channel and the retailer, in addition to supplier competition (product differentiation between the two suppliers).

In this section, we study the impact of store differentiation between the supplier's direct channel and the retailer, with the existence of product differentiation between the two suppliers. We then examine whether the presence of an additional direct channel always benefits the supplier and the other supply chain members? Additionally, we also benchmark the channel profit and efficiency against those of the corresponding supply chain without a direct channel.

The parameter α represents the level of store differentiation, i.e., it captures store competition between the supplier's direct channel and the retailer. That is, if these two channels are less differentiated (competition is more intense), then α is larger (closer to 1). The value of α varies within $[0,\ 1)$ to ensure $\alpha < 1$. The value of γ is fixed at 0.5 in order to avoid introducing intense product differentiation (and competition) that could bias the effect of store differentiation. In this study, we set $c_{S_1^d} = 0.45$.

4.4.2.1 The effect of store differentiation

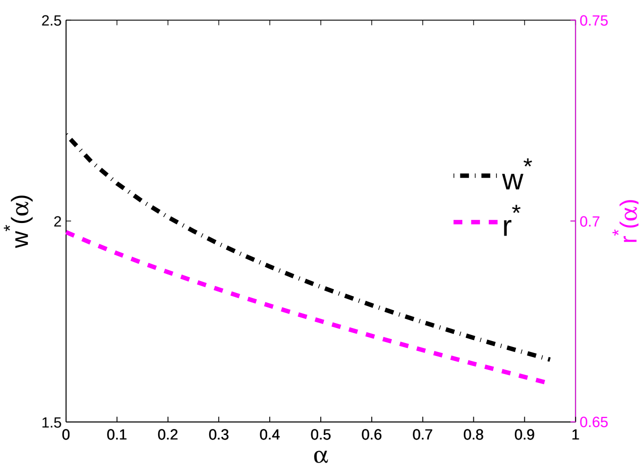
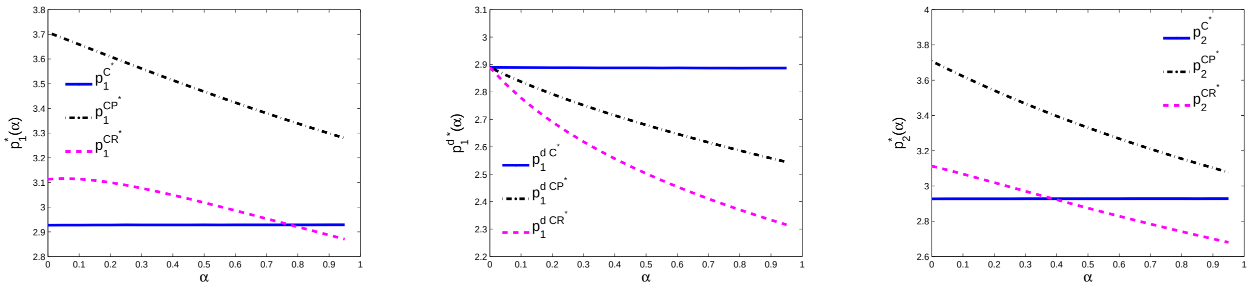


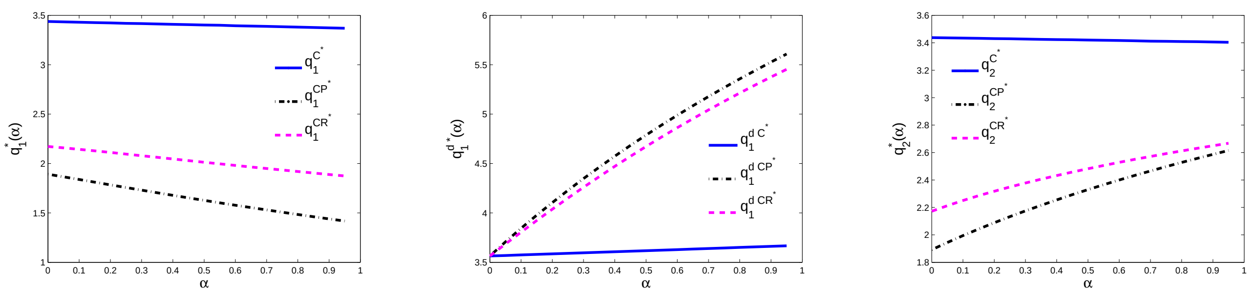
Figure 32. Retailer's consignment price w^* (or revenue share r^*)

We study the effect of store differentiation (parameter α) on equilibrium prices, quantities and profits. Figure Figure 32 suggests that the retailer's consignment price (CP contract) and



(a) Retail price p_1^* (b) Retail price p_1^{d*} (c) Retail price p_2^*

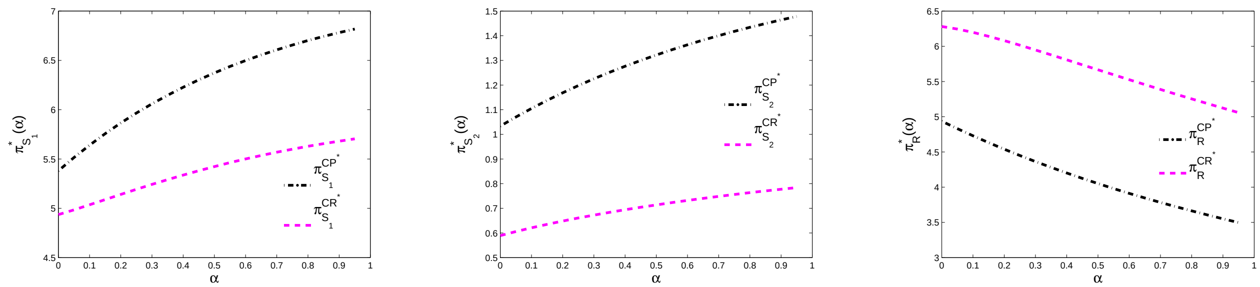
Figure 33. Retail price for supplier i as a function of α under CP and CR contracts



(a) Quantity q_1^* (b) Quantity q_1^{d*} (c) Quantity q_2^*

Figure 34. Quantity for supplier i's product as a function of α under CP and CR contracts

revenue share (CR contract) decrease in α. It is clear that when the supplier's direct channel and the retailer become less differentiated (higher channel competition), the retailer needs to lower the consignment price/revenue share charged to the suppliers. As a result, the retail price decreases ((Figures Figure 33(a) and (c)). This finding is consistent with the result of



(a) Supplier 1's profit $\pi^*_{S_1}$ (b) Supplier 2's profit $\pi^*_{S_2}$ (c) Retailer's profit π^*_R

Figure 35. Supplier i's profit and retailer's profit as a function of α under CP and CR contracts

(Choi, 1996) and (Liao and Tseng, 2007) who study the impact of store differentiation on the equilibrium under a wholesale price contract.

Figure Figure 34 depicts the effect of channel competition (store differentiation) on the quantity. As the supplier's direct channel and the retailer become less differentiated (higher competition: a higher value of α), supplier 1 direct channel's quantity and supplier's 2 quantity increase. On the other hand, the supplier 1 indirect channel's quantity decreases in α. This could be because supplier 1 has a higher incentive to sell products through its own direct channel when its direct channel and the retailer are less differentiated.

The profits for both suppliers increase as the level of channel competition increases in both types of consignment contracts (Figures Figure 35(a) and (b)). The retailer's profit, however, decreases as channel competition is more intense (Figure Figure 35(c)). As expected, the degree of store differentiation has a positive impact on the suppliers' profits but a negative impact on

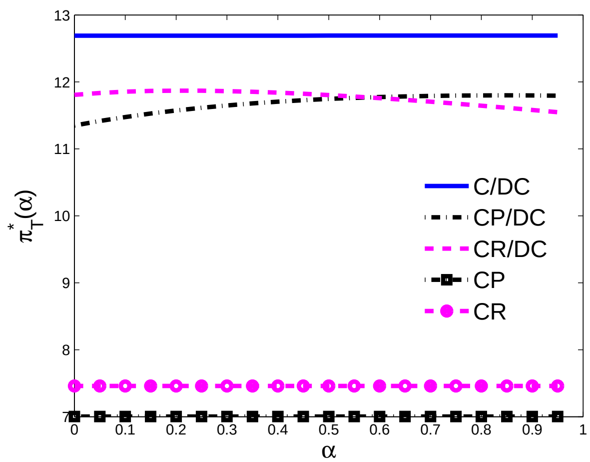


Figure 36. Total channel profit as a function of α

the retailer's profit. The finding is in line with previous research (Choi, 1996; Pan et al., 2010; Liao and Tseng, 2007) which states that channel competition leads to an increase in the supplier's profit and a decrease in the retailer's profit.

We now compare the decisions and the total profits in the centralized and the decentralized channels. It is worthwhile to note that the retail price in the decentralized channel could potentially be lower than the retail price in the centralized channel, due to the effect of intensified channel competition (Figures Figure 33(a) and (c)).

The total channel profit under CP and CR contracts is not monotonic in α (Figure Figure 36). In other words, channel competition between the direct and retail channels does not necessarily help improve the total supply chain's profit. When the direct and retail channels are more differentiated, a larger retailer's profit drives the total supply chain profits to increase with the level of channel competition. When the direct and retail channels are less differentiated, however, the retailer's profit contributes less to the total profits and the trend thus follows the suppliers' profits which decreases with the level of channel competition.

The effects of store differentiation (channel competition) on the decisions, profits and supply chain efficiency are summarized in Table Table VII.

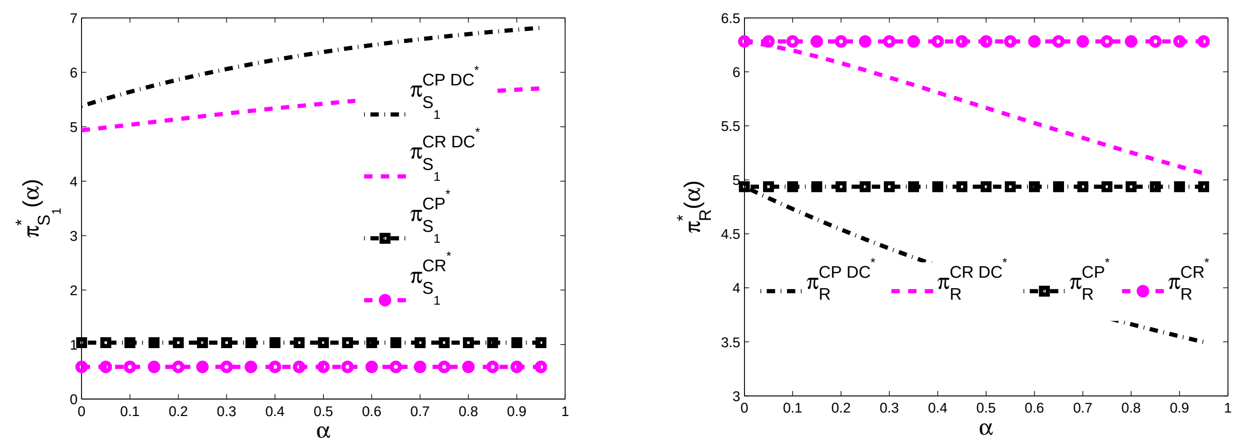
Decisions/profits	Centralized Channel	CP contract	CR contract	Remark
w^* or r^*	–	decreasing	decreasing	
p_1^*	independent	decreasing	decreasing	$p_1^{CP^*} > p_1^{CR^*} > p_1^{C^*}$, small α $p_1^{CP^*} > p_i^{C^*} > p_i^{CR^*}$, large α
p_1^{d*}	independent	decreasing	decreasing	$p_1^{d\ C^*} > p_1^{d\ CP^*} > p_1^{d\ CR^*}$
p_2^*	independent	decreasing	decreasing	$p_2^{CP^*} > p_2^{CR^*} > p_2^{C^*}$, small α $p_2^{CP^*} > p_2^{C^*} > p_2^{CR^*}$, large α
q_1^*	decreasing	decreasing	decreasing	$q_1^{C^*} > q_1^{CR^*} > q_1^{CP^*}$
q_1^{d*}	increasing	increasing	increasing	$q_1^{d\ CP^*} > q_1^{d\ CR^*} > q_1^{d\ C^*}$
q_2^*	decreasing	increasing	increasing	$q_2^{C^*} > q_2^{CR^*} > q_2^{CP^*}$
$\pi_{S_1}^*$	–	increasing	increasing	$\pi_{S_2}^{CP^*} > \pi_{S_1}^{CR^*}$
$\pi_{S_2}^*$	–	increasing	increasing	$\pi_{S_2}^{CP^*} > \pi_{S_1}^{CR^*}$
π_R^*	–	decreasing	decreasing	$\pi_R^{CR^*} > \pi_R^{CP^*}$
π_T^*	independent	not monotonic	not monotonic	$\pi_T^{CR^*} > \pi_T^{CP^*}$, small α $\pi_T^{CP^*} > \pi_T^{CR^*}$, large α
Eff	–	not monotonic	not monotonic	$Eff_{CR} > Eff_{CP}$, small α $Eff_{CP} > Eff_{CR}$, large α

TABLE VII

EFFECT OF PARAMETER α ON THE EQUILIBRIUM DECISIONS,PROFITS AND CHANNEL EFFICIENCY

4.4.2.2 Performance comparison of the two channel structures

Adding a direct channel could lead to larger demand due to access to new consumers, and thus a higher profit to the supplier. On the other hand, higher competition on the retail market could lower price prices and hence profits. We examine whether adding a direct channel to the supply chain always benefits both suppliers and/or the retailer. We then compare the channel performance with a supply chain without direct channel.



(a) Supplier 1's profit $\pi^*_{S_1}$

(b) Retailer's profit π^*_R

Figure 37. Supplier 1's profit and retailer's profit as a function of α under the indirect and dual channels

It is expected that supplier 1's profit increases since adding a direct channel helps the supplier gain access to a larger consumer base (as shown in Figure Figure 37(a)). The re-

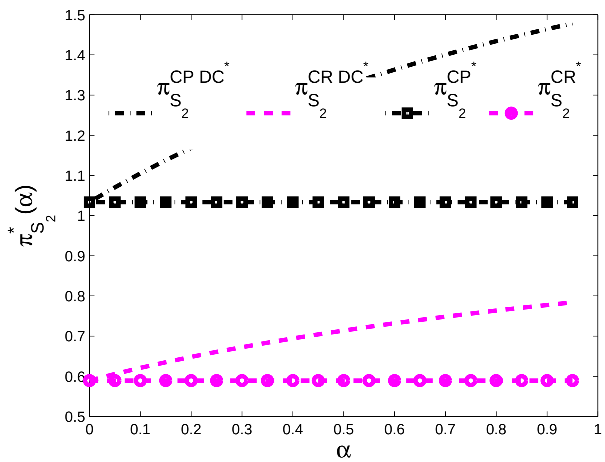


Figure 38. Supplier 2's profit as a function of α under the indirect and dual channels

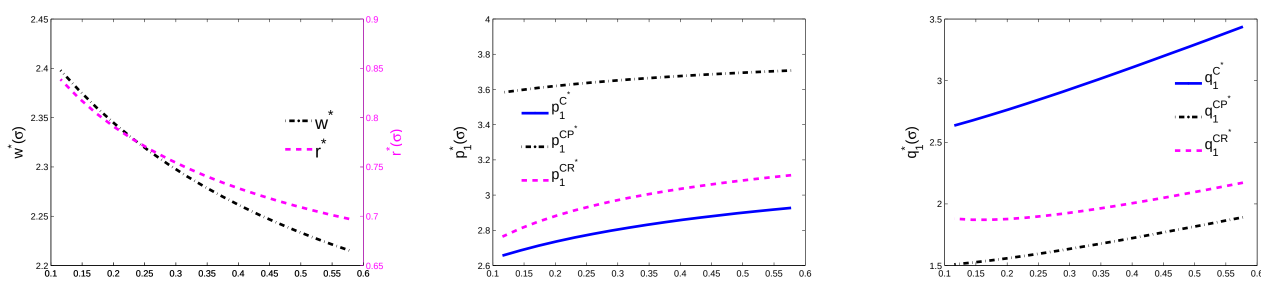
tailer's profit decreases as a new channel increases competition in the retail market (Figure Figure 37(b)). It is more interesting, however, that supplier 2 also benefits from this direct channel (Figure Figure 38). This implies that the direct effect of α (which causes the demand to increase) dominates the indirect effect of α (high channel competition which drives the demand to decrease). Consequently, the positive impact of a direct channel on both suppliers' profits helps improve the total channel profit (Figure Figure 36).

4.4.3 The effect of demand uncertainty

We first consider the impact of demand uncertainty via the standard deviation of the perturbation parameter σ on the retailer's equilibrium consignment price (CP contract), revenue share (CR contract), retail price, quantity, profits and supply chain efficiency, in the consignment channel. The mean value of the perturbation is fixed to 1 and the value of demand variability σ varies within $[0.12 - 0.58]$. The value of γ is set at 0.5, in order to avoid introducing intense product differentiation and competition that could bias the effect of demand uncertainty.

We then consider the effect of demand uncertainty on the equilibrium decisions, profits and channel efficiency in the dual supply chain channel. The range of values of demand variability is the same as the previous study. The values of γ and α are set at 0.5, in order to avoid introducing intense product and store competition that could bias the effect of demand uncertainty.

Finally, we compare the effect of demand uncertainty on the equilibrium decisions, profits and channel efficiency, under each type of the consignment contract. We are interested in how the benefit of dual channel supply chain varies with different degrees of demand uncertainty.

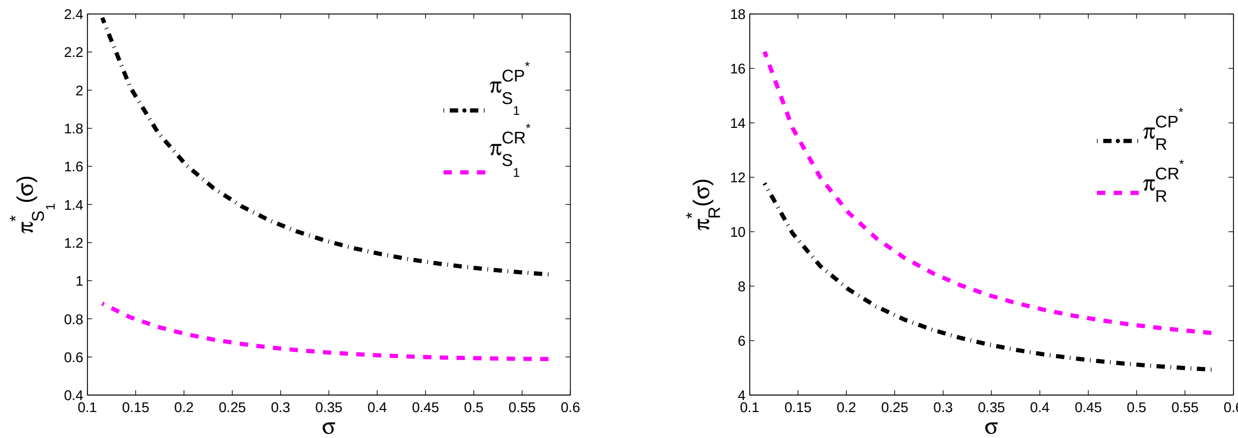


(a) Retailer's consignment price w^* or revenue share r^*

(b) Retail price p_1^*

(c) Quantity q_1^*

Figure 39. Retailer's consignment price (or revenue share) and retail price as a function of σ under CP and CR contracts

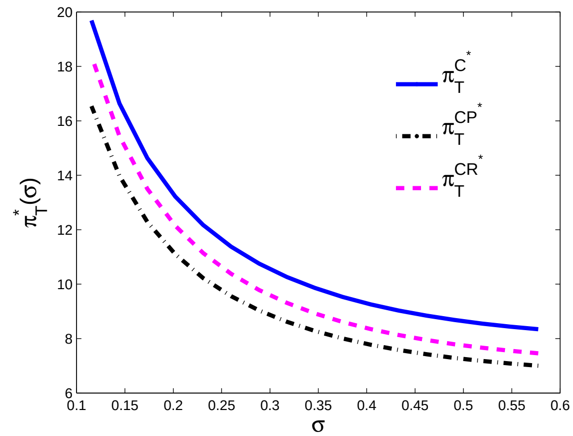


(a) Supplier i's profit $\pi^*_{S_i}$

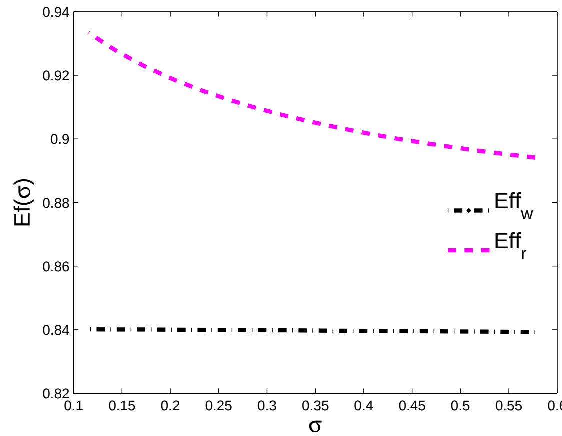
(b) Retailer's profit π^*_R

Figure 40. Supplier i's profit (for $i = 1$, by symmetry, suppliers 1 and 2 have the same profits) and retailer's profit as a function of σ under CP and CR contracts

Figure Figure 39(a) shows that the retailer's consignment price (CP contract) and revenue share (CR contract) decrease as demand variability increases. One explanation is that under consignment contracts, the retailer incurs no risk associated with demand uncertainty, whereas the suppliers bears all risk associated with demand uncertainty, hence suppliers' profits go down. To give an incentive to suppliers to continue selling, the retailer decreases its consignment price (and its revenue share) to help share risk with the suppliers from the increased variability in demand. The retail price, however, increases with the increase in demand uncertainty. Indeed, the suppliers must increase their retail price to compensate for increased risk associated with demand uncertainty.



(a) Total channel profit π_T^*



(b) Channel efficiency *Eff*

Figure 41. Total channel profit in centralized and decentralized channels and channel efficiency as a function of σ

Figure Figure 39(b) also demonstrates that the retail price in the centralized system increases in σ. This suggests that the central decision maker increases the retail price as demand uncertainty increases, in order to compensate for all risk associated with this uncertainty.

An increase in demand uncertainty increases the quantity q_i^* that suppliers deliver to the retailer, under the CP contract in order to prevent stockouts (Figure Figure 39(c)). The findings showing that retail price and quantity increase with the increased demand uncertainty are also consistent with findings by (Yao et al., 2008b).

Figures Figure 40(a) and (b) illustrate the effects of σ on the suppliers' profits and the retailer's profit in a decentralized system, respectively. It is expected that the profits of the suppliers and the retailer decrease as demand uncertainty increases. Figure Figure 41(a) shows that the channel profits (both centralized and decentralized systems) decrease with the increased demand uncertainty. The total profit under the CR contract is higher than that under the CP contract, regardless of the level of demand variability.

Figure Figure 41(b) depicts the impact of demand uncertainty on supply chain efficiency. The channel efficiency under both types of consignment contracts decreases in σ. It is known that demand uncertainty can reduce the channel efficiency for the decentralized channel. This result further suggests the interesting finding that the channel efficiency under the CP contract is less sensitive to the demand variability than the CR contract.

The effects of demand uncertainty on the equilibrium decisions, profits and supply chain efficiency are summarized in Table Table VIII.

Decisions/profits	Centralized Channel	CP contract	CR contract	Remark
w^* or r^*	–	decreasing	decreasing	
p_i^*	increasing	increasing	increasing	$p_i^{CP*} > p_i^{CR*} > p_i^{C*}$
q_i^*	increasing	increasing	non monotonic	$q_i^{C*} > q_i^{CR*} > q_i^{CP*}$
$\pi_{S_i}^*$	–	decreasing	decreasing	$\pi_{S_i}^{CP*} > \pi_{S_i}^{CR*}$
π_R^*	–	decreasing	decreasing	$\pi_R^{CR*} > \pi_R^{CP*}$
π_T^*	decreasing	decreasing	decreasing	$\pi_T^{CR*} > \pi_T^{CP*}$
Eff	–	decreasing	decreasing	$Eff_{CR} > Eff_{CP}$

TABLE VIII

EFFECT OF DEMAND UNCERTAINTY σ ON THE EQUILIBRIUM DECISIONS AND PROFITS

CHAPTER 5

CONCLUSIONS

Consignment contracts have received increasing attention in the Supply Chain Management literature, with most of this attention being focused on a channel structure with a single supplier and a single retailer. While upstream competition has been recently discussed, downstream retailer competition has not. The first part of the thesis contributes to research in consignment contracts and retail competition by providing insights on how the presence of retail competition and retailer differentiation affect the decisions and performance of the supply chain. We build a game-theoretic model in order to analyze the channel decisions and performance in three different contracts: price-only, consignment price, and consignment with revenue share contracts.

Although upstream competition among suppliers has been discussed to some extent, existing work relies upon some restrictive assumptions, such as perfect complementarity of products and deterministic demand. Despite the popularity of direct sales channel, no existing consignment study has considered a supply chain in which a supplier can sell directly to consumers through a direct channel, in addition to selling through the retailer via consignment contracts. The second part of the thesis contributes to the literature on consignment contracts and competition by providing insights on how the presence of supplier competition affects the decisions and performance of the supply chain. In addition, we are interested in how the presence of a direct channel for one supplier affects the channel decisions and profits. We build a game-theoretic model in order to analyze the channel decisions and performance in two different consignment contracts: consignment price, and consignment with revenue share contracts. We summarize our findings below.

(1) There is no particular type of contract that dominates the others from all players' perspective. The benefit of each consignment contract critically depends upon the level of retailer differentiation. The CP contract is preferable to all supply chain members when retailer differentiation is weak. When retailer differentiation is strong, the PO contract yields higher profits to the retailers than the two types of consignment contracts. The supplier, however, earns the highest profit in the CR contract when retailer differentiation is strong.

(2) In order to understand how the different payment terms in consignment contracts affect the decisions of the channel members and supply chain performance, we consider two different payment schemes: fixed (CP contract) and proportional (CR contract). The comparison of these two types of contracts is summarized in Table Table IX. Our numerical study shows that the CP contract yields higher profits to the retailers than the CR contract, regardless of the level of retailer differentiation. The benefit of each type of consignment contract to the supplier, however, depends upon the level of retailer differentiation. When retailer differentiation is strong, the CR contract yields a higher profit to the supplier; when it is weak, the CP contract is more beneficial.

(3) The effect of retailer differentiation on the decisions of the supplier and the retailers are as follows: with less retailer differentiation, the supplier increases the price (wholesale or consignment) or revenue share charged to the retailers. An increase of the supplier's price or

revenue share leads the retailers to increase the retail price. The order quantity in the CP contract increases when the level of retailer differentiation decreases, but it is non monotonic in the PO and CR contracts.

(4) The supplier earns a higher profit when the level of retailer differentiation decreases, for all three contracts. The retailers also earn a higher profit when the level of retailer differentiation decreases in the CP contract; however, the retailers' profits in the PO and CR contracts is not monotonic with retailer differentiation. Furthermore, our numerical study suggests that the benefits of the a lower retailer differentiation are not equally distributed across all supply chain members: the retailers collect a smaller share of the total profits as the differentiation decreases.

Decisions and profits	Retailer differentiation
p_i^*	$p_i^{CP^*} > p_i^{CR^*}$
Q_i^*	$Q_i^{CR^*} > Q_i^{CP^*}$ for more retailer differentiation (small γ) $Q_i^{CR^*} < Q_i^{CP^*}$ for less retailer differentiation (large γ)
z_i^*	$z_i^{CP^*} > z_i^{CR^*}$
$\pi_{R_i}^*$	$\pi_{R_i}^{CP^*} > \pi_{R_i}^{CR^*}$
π_S^*	$\pi_S^{CR^*} > \pi_S^{CP^*}$ for more retailer differentiation (small γ) $\pi_S^{CR^*} < \pi_S^{CP^*}$ for less retailer differentiation (large γ)
$\frac{\pi_{R_1}^*+\pi_{R_2}^*}{\pi_S^*}$	$CP > CR$

TABLE IX

THE COMPARISON OF TWO TYPES OF CONSIGNMENT CONTRACTS

(5) The suppliers' profits decrease as products are less differentiated. The impact of supplier competition on the retailer's profit, however, critically depends upon the type of consignment contract. Specifically, supplier competition helps improve the retailer's profit in the CP contract, but not necessarily in the CR contract. This is due to the combination of two opposite effects of supplier competition on the profits. On the one hand, a higher supplier competition may lead to an increase of the (expected) demand which may increase the retailer's profit. On the other hand, as competition between two suppliers increases, the retail price decreases which, in turn, could cause the retailer's profit to decrease.

(6) As expected, channel competition has an opposite impact on the suppliers' and the retailer's profits. That is, high channel competition always improves the suppliers' profits and decreases the retailer's profit. More interestingly, we find that high channel competition between the direct and retail channels does not necessarily help improve the total supply chain's profits. When the direct and retail channels are more differentiated, a larger retailer's profit drives the total supply chain profits to increase with the level of channel competition. When the direct and retail channels are less differentiated, however, the retailer's profit contributes less to the total profits and the trend thus follows the suppliers' profits which decreases with the level of channel competition.

(7) Adding a direct channel to the supply chain, in addition to selling through the retailer via consignment introduces another level of competition (channel competition) to the existence of product differentiation between two suppliers. It is intuitive that the suppliers' profits increase, since adding a direct channel helps one supplier gain access to a larger consumer base. The

retailer's profit decreases as a new channel increases competition in the retail market. It is more interesting, however, that the supplier (without a direct channel) also benefits from its competitor's direct channel. This implies that the direct effect of α (which causes the demand to increase) dominates the indirect effect of α (high channel competition which drives the demand to decrease). Consequently, the positive impact of a direct channel on both suppliers' profits helps improve the total channel profit.

Clearly, the results and insights obtained in the thesis are based on a specific demand function, i.e., non-linear and linear demand with multiplicative demand uncertainty (Ru and Wang, 2010; Chen, 2011). As suggested in (Wang et al., 2004) and (Wang, 2006), obtaining closed form solutions for some demand functional forms, such as linear and additive demand model, is intractable even for the simplest setting with one retailer and one supplier. However, (Wang et al., 2004) conduct numerical experiments and show that the properties and insights generated from their iso-price-elastic and multiplicative demand model still hold strongly for other demand models such as the linear with multiplicative demand model. It would be interesting to examine whether their conclusion still remains valid for the case of two suppliers.

In future research, one could consider different power structures (e.g., Nash game) (Choi, 1991; Wang, 2006; Pan et al., 2010). We also assume that the common retailer faces symmetric demands for products from different suppliers. It may be of interest to study the effect of demand asymmetry among retailers on decisions and profits. Finally, this model could be extended to a situation in which the retailer faces multiple competing suppliers or the supplier faces multiple competing retailers and possibly to an even more general setting with multiple agents at both levels of the supply chain.

APPENDICES

Appendix A

PROOFS OF CHAPTER THREE

Proof of Lemma 3.3.1. . We consider three possible cases.

1. For $z_i < A$, $z_i - \Lambda(z_i) = z_i - \int_A^{z_i}(z_i - x)f(x)\,dx = z_i - 0 = z_i > 0$

2. For $A \le z_i \le B$, $z_i - \Lambda(z_i) = z_i - \int_A^{z_i}(z_i - x)f(x)\,dx = z_i - z_iF(z_i) + \int_A^{z_i} xf(x)\,dx$.

   Thus, $z_i - \Lambda(z_i) > 0$ as $F(z_i) \le 1$ and $\int_A^{z_i} xf(x)\,dx \ge 0$

3. For $z_i > B$, $z_i - \Lambda(z_i) = z_i - \int_A^{B}(z_i - x)f(x)\,dx = z_i - z_i + \int_A^{B} xf(x)\,dx = \int_A^{B} xf(x)\,dx$

   Thus, $z_i - \Lambda(z_i) > 0$ since $\int_A^{B} xf(x)\,dx = E(\epsilon) = 1$ □

Proof of Proposition 3.3.3. . For any given stocking factor z_i, we take the partial derivative of $\pi_{R_i}(p_i, z_i|w)$ with respect to p_i as

$$\frac{\partial \pi_{R_i}(p_i, z_i|w_p)}{\partial p_i} = ae^{-\beta p_i + \gamma p_j}\left[z_i - \Lambda(z_i) - \beta\{p_i(z_i - \Lambda(z_i)) - (c\alpha_i + w_p)z_i\}\right].$$

Since $ae^{-\beta p_i + \gamma p_j} > 0$, $\frac{\partial \pi_{R_i}(p_i, z_i|w)}{\partial p_i} = 0$ when $p_i = \frac{1}{\beta} + \frac{(c\alpha_i + w_p)z_i}{z_i - \Lambda(z_i)} \equiv \tilde{p}_i(z_i|w_p)$. Moreover, $\frac{\partial \pi_{R_i}}{\partial p_i} > 0$ for all $p_i < \tilde{p}_i(z_i|w_p)$ and $\frac{\partial \pi_{R_i}}{\partial p_i} < 0$ for all $p_i > \tilde{p}_i(z_i|w_p)$, so $\tilde{p}_i(z_i|w_p)$ is the unique maximizer of $\pi_{R_i}(p_i, z_i|w)$ for fixed z_i, w_p and p_{-i}. □

Appendix A (Continued)

Proof of Proposition 3.3.4. . We want to derive $\bar{z}_i$ that maximizes $\pi_{R_i}(\tilde{p}_i(z_i|w_p), z_i|w_p)$. By the chain rule, we have

$$\frac{d\pi_{R_i}(\tilde{p}_i(z_i|w_p), z_i|w_p)}{dz_i} = \frac{\partial\pi_{R_i}(\tilde{p}_i(z_i|w_p), z_i|w_p)}{\partial p_i} \cdot \frac{d\tilde{p}_i(z_i|w_p)}{dz_i} + \frac{\partial\pi_{R_i}(\tilde{p}_i(z_i|w_p), z_i|w_p)}{\partial z_i}.$$

The first term is zero since $\frac{\partial\pi_{R_i}(\tilde{p}_i(z_i|w_p), z_i|w_p)}{\partial p_i} = 0$, due to optimality of $\tilde{p}_i(z_i|w_p)$.

Thus, we have

$$\begin{aligned}
\frac{d\pi_{R_i}(\tilde{p}_i(z_i|w_p), z_i|w_p)}{dz_i} &= \frac{\partial\pi_{R_i}(\tilde{p}_i(z_i|w_p), z_i|w_p)}{\partial z_i} \\
&= ae^{-\beta\tilde{p}_i(z_i|w_p))+\gamma p_{-i}(z_{-i})} \left\{ \left[\frac{1}{\beta} + \frac{(c\alpha_i + w_p)z_i}{z_i - \Lambda(z_i)}\right] [1 - F(z_i)] - (c\alpha_i + w_p) \right\} \\
&= \frac{ae^{-\beta\tilde{p}_i(z_i|w_p)+\gamma p_{-i}(z_{-i})}}{\beta(z_i - \Lambda(z_i))} \cdot g(z_i),
\end{aligned}$$

where $g(z_i) = [z_i - \Lambda(z_i) + \beta(c\alpha_i + w_p)z_i]\,[1 - F(z_i)] - \beta(c\alpha_i + w_p)(z_i - \Lambda(z_i))$. Since the ratio $\frac{ae^{-\beta\tilde{p}_i(z_i|w_p)+\gamma p_{-i}(z_i)}}{\beta(z_i-\Lambda(z_i))}$ in the above expression is always positive since we know from Lemma 1. that $z_i - \Lambda(z_i)$ is positive, first-order condition requires that the optimal $\bar{z}_i$ satisfy $g(\bar{z}_i) = 0$, which gives (Equation 3.8). Such a $\bar{z}_i$ always exists in the support interval (A, B) of $F(\cdot)$, because $g(z_i)$ is continuous, and $g(A) = A > 0$ and $g(B) = -\beta c\alpha_i < 0$, since the mean value of ϵ is equal to 1. To verify the uniqueness of $\bar{z}_i$, we have

$$\begin{aligned} g'(z_i) &= [1-F(z_i)]\Big\{[1-F(z_i)] - h(z_i)\big[\beta(c\alpha_i + w_p)z_i + z_i - \Lambda(z_i)\big]\Big\} \\ g''(z_i) &= -h(z_i)g'(z_i) \\ &\quad +[1-F(z_i)]\Big\{-f(z_i) - h'(z_i)\big[\beta(c\alpha_i + w_p)z_i + z_i - \Lambda(z_i)\big] - h(z_i)\big[\beta(c\alpha_i + w_p) + 1 - F(z_i)\big]\Big\}, \end{aligned}$$

where $h(z_i) = f(z_i)/[1-F(z_i)]$ is the failure rate of the demand distribution. From Assumption 3.1.1, $h'(z_i) > 0$, then $g''(z_i) < 0$ whenever $g'(z_i) = 0$, implying that $g(z_i)$ is a unimodal function. We have proved that $\bar{z}_i$ is a unique maximizer of $\pi_{R_i}(\tilde{p}_i(z_i|w_p), z_i|w_p)$. □

Proof of Corollary 3.3.5. . (Equation 3.5) can be rearranged as

$$\frac{1}{1-F(\bar{z}_i)} - \frac{\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} = \frac{1}{(c\alpha_i + w_p)\beta}.$$

The right-hand side is a decreasing function of w_p, thus $G(\bar{z}_i)$ decreases in w_p, where G was defined in Lemma 2. The result then follows from Lemma 3.3.2. □

Proof of Proposition 3.3.6. . The proof is similar to the proof of Proposition 1 and is therefore omitted. □

Appendix A (Continued)

Proof of Proposition 3.3.7. . We want to derive $\bar{z}_i$ that maximizes $\pi_{R_i}(\tilde{p_i}(z_i|w), z_i|w)$. By the chain rule, we have

$$\frac{d\pi_{R_i}(\tilde{p_i}(z_i|w), z_i|w)}{dz_i} = \frac{\partial\pi_{R_i}(\tilde{p_i}(z_i|w), z_i|w)}{\partial p_i} \cdot \frac{d\tilde{p_i}(z_i|w)}{dz_i} + \frac{\partial\pi_{R_i}(\tilde{p_i}(z_i|w), z_i|w)}{\partial z_i}.$$

The first term is zero since $\frac{\partial\pi_{R_i}(\tilde{p_i}(z_i|w), z_i|w)}{\partial p_i} = 0$, due to optimality of $\tilde{p_i}(z_i|w)$. Thus, we have

$$\begin{aligned} \frac{d\pi_{R_i}(\tilde{p_i}(z_i|w), z_i|w)}{dz_i} &= \frac{\partial\pi_{R_i}(\tilde{p_i}(z_i|w), z_i|w)}{\partial z_i} \\ &= ae^{-\beta\tilde{p_i}(z_i|w)+\gamma p_{-i}(z_{-i})}\left\{\left[\frac{1}{\beta} + \frac{c\alpha_i z_i}{z_i - \Lambda(z_i)}\right][1 - F(z_i)] - c\alpha_i\right\} \\ &= \frac{ae^{-\beta\tilde{p_i}(z_i|w)+\gamma p_{-i}(z_i)}}{\beta(z_i - \Lambda(z_i))} \cdot L(z_i), \end{aligned}$$

where $L(z_i) = [z_i - \Lambda(z_i) + \beta c\alpha_i z_i][1-F(z_i)] - \beta c\alpha_i(z_i - \Lambda(z_i))$. Since the ratio $\frac{ae^{-\beta\tilde{p_i}(z_i|w)+\gamma p_{-i}(z_i)}}{\beta(z_i-\Lambda(z_i))}$ in the above expression is always positive, first-order condition requires that the optimal $\bar{z}_i*$ satisfy $L(\bar{z}_i) = 0$, which gives (Equation 3.8). Such a $\bar{z}_i$ always exists in the support interval (A, B) of $F(\cdot)$, because $L(z_i)$ is continuous, and $L(A) = A > 0$ and $g(B) = -\beta c\alpha_i\mu < 0$, where μ is the mean value of ϵ. To verify the uniqueness of $\bar{z}_i$, we have

$$\begin{aligned} L'(z_i) &= [1 - F(z_i)]\Big\{[1 - F(z_i)] - h(z_i)\big[\beta c\alpha_i z_i + z_i - \Lambda(z_i)\big]\Big\} \\ L''(z_i) &= -h(z_i)g'(z_i) + [1 - F(z_i)]\Big\{ - f(z_i) - h'(z_i)\big[\beta c\alpha_i z_i + z_i - \Lambda(z_i)\big] - h(z_i)\big[\beta c\alpha_i + 1 - F(z_i)\big]\Big\}, \end{aligned}$$

Appendix A (Continued)

where $h(z_i) = f(z_i)/[1 - F(z_i)]$ is defined as the failure rate of the demand distribution. From Assumption 3.1.1, $h'(z_i) > 0$, then $L''(z_i) < 0$ whenever $L'(z_i) = 0$, implying that $L(z_i)$ is a unimodal function. We have proved that $\bar{z}_i$ is a unique maximizer of $\pi_{R_i}(\tilde{p}_i(z_i|w), z_i|w)$. $\square$

Proof of Proposition 3.3.8. . At the second step, $\bar{z}_i$ chosen by the retailer i does not depend on the consignment price w set by the supplier at the first step. Since $\bar{p}_i = \tilde{p}_i(\bar{z}_i|w) = \frac{1}{\beta} + \frac{c\alpha_i \bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} + w$, the first derivative of $\pi_S(w)$ with respect to w can be written as

$$\begin{aligned}
\frac{d\pi_S(w)}{dw} &= ae^{-\beta\left[\frac{1}{\beta}+\frac{c\alpha_1\bar{z}_1}{\bar{z}_1-\Lambda(\bar{z}_1)}+w\right]+\gamma\left[\frac{1}{\beta}+\frac{c\alpha_2\bar{z}_2}{\bar{z}_2-\Lambda(\bar{z}_2)}+w\right]}\Big[(\bar{z}_1 - \Lambda(\bar{z}_1)) \\
&\qquad\qquad -(\beta-\gamma)\big[w(\bar{z}_1 - \Lambda(\bar{z}_1)) - c(1-\alpha_1-\alpha_2)\bar{z}_1\big]\Big] \\
&\quad + ae^{\beta\left[\frac{1}{\beta}+\frac{c\alpha_2\bar{z}_2}{\bar{z}_2-\Lambda(\bar{z}_2)}+w\right]+\gamma\left[\frac{1}{\beta}+\frac{c\alpha_1\bar{z}_1}{\bar{z}_1-\Lambda(\bar{z}_1)}+w\right]}\Big[(\bar{z}_2 - \Lambda(\bar{z}_2)) \\
&\qquad\qquad -(\beta-\gamma)\big[w(\bar{z}_2 - \Lambda(\bar{z}_2)) - c(1-\alpha_1-\alpha_2)\bar{z}_2\big]\Big] \\
&= ae^{-\beta[1/\beta+w]+\gamma[1/\beta+w]}\Big\{e^{-\left(\frac{\beta\alpha_1\bar{z}_1}{(\bar{z}_1-\Lambda(\bar{z}_1))}-\frac{\gamma\alpha_2\bar{z}_2}{(\bar{z}_2-\Lambda(\bar{z}_2))}\right)c}\Big[(\bar{z}_1 - \Lambda(\bar{z}_1)) \\
&\qquad -(\beta-\gamma)\big[w(\bar{z}_1 - \Lambda(\bar{z}_1)) - c(1-\alpha_1-\alpha_2)\bar{z}_1\big]\Big] \\
&\qquad + e^{-\left(\frac{\beta\alpha_2\bar{z}_2}{(\bar{z}_2-\Lambda(\bar{z}_2))}-\frac{\gamma\alpha_1\bar{z}_1}{(\bar{z}_1-\Lambda(\bar{z}_1))}\right)c}\Big[(\bar{z}_2 - \Lambda(\bar{z}_2)) \\
&\qquad -(\beta-\gamma)\big[w(\bar{z}_2 - \Lambda(\bar{z}_2)) - c(1-\alpha_1-\alpha_2)\bar{z}_2\big]\Big]\Big\}.
\end{aligned}$$

Since $ae^{-\beta[1/\beta+w]+\gamma[1/\beta+w]} > 0$, $\frac{d\pi_S(w)}{dw} = 0$ implies that

$$w^* = \frac{k_1[(\bar{z}_1 - \Lambda(\bar{z}_1)) + (\beta-\gamma)c(1-\alpha_1-\alpha_2)\bar{z}_1] + k_2[(\bar{z}_2 - \Lambda(\bar{z}_2)) + (\beta-\gamma)c(1-\alpha_1-\alpha_2)\bar{z}_2]}{(\beta-\gamma)[k_1(\bar{z}_1 - \Lambda(\bar{z}_1)) + k_2(\bar{z}_2 - \Lambda(\bar{z}_2))]},$$

Appendix A (Continued)

where $k_i = e^{-\left(\frac{\beta\alpha_i\bar{z}_i}{(\bar{z}_i-\Lambda(\bar{z}_i))} - \frac{\gamma\alpha_{-i}\bar{z}_{-i}}{(\bar{z}_{-i}-\Lambda(\bar{z}_{-i}))}\right)c}$, $i = 1, 2$ and $\bar{z}_i$ as in (Equation 3.8). Moreover, $\frac{d\pi_S}{dw} > 0$ for all $w < w^*$ and $\frac{d\pi_S}{dw} < 0$ for all $w > w^*$, so w^* is the unique maximizer of π_S. □

Proof of Proposition 3.3.9. .

- From (Equation 3.8), $\frac{1}{c\alpha\beta}$ is a decreasing function in β. Thus $G(z^*)$ decreases in β. The result then follows from Lemma 3.3.2.
- To show that $w^*$is decreasing in β, we show that $\frac{\partial w^*}{\partial\beta} \le 0$

  $\frac{\partial w^*}{\partial\beta} = -\frac{1}{(\beta-\gamma)^2} + c(1-\alpha)\left[\frac{zF(z)-\Lambda(z)}{(z-\Lambda(z))^2}\right]\frac{\partial z^*}{\partial\beta} \le 0$ since $\frac{zF(z)-\Lambda(z)}{(z-\Lambda(z))^2} = \frac{\int_A^z xf(x)\,dx}{(z-\Lambda(z))^2} \ge 0$ and $\frac{\partial z^*}{\partial\beta} \le 0$.
- To show that $p^*$is decreasing in β, we show that is equivalent to $\frac{\partial p^*}{\partial\beta} \le 0$

  $\frac{\partial p^*}{\partial\beta} = -\frac{1}{\beta^2} + c\alpha\left[\frac{zF(z)-\Lambda(z)}{(z-\Lambda(z))^2}\right]\frac{\partial z^*}{\partial\beta} + \frac{\partial w^*}{\partial\beta} \le 0$ since $\frac{\partial z^*}{\partial\beta} \le 0$ and $\frac{\partial w^*}{\partial\beta} \le 0$.

□

Proof of Proposition 3.3.10. .

- To show that $w^*$is increasing in γ, we show that $\frac{\partial w^*}{\partial\gamma} \ge 0$ (see (Equation 3.11)).
- To show that $p^*$is increasing in γ, we show that $\frac{\partial p^*}{\partial\gamma} \ge 0$ (see (Equation 3.12)).
- To show that $p^*/w^*$is decreasing in γ, we show that $\frac{\partial[p^*/w^*]}{\partial\gamma} \le 0$.

  $\frac{\partial[p^*/w^*]}{\partial\gamma} = -\frac{1}{(\beta-\gamma)^2}\left[\frac{1}{\beta} + \frac{c\alpha z^*}{z^*-\Lambda(z^*)}\right] \le 0$.
- To show that $Q^*$is increasing in γ, we show that $\frac{\partial Q^*}{\partial\gamma} \ge 0$.

  $\frac{\partial Q^*}{\partial\gamma} = ae^{-(\beta-\gamma)[\frac{1}{\beta}+\frac{1}{\beta-\gamma}+\frac{cz^*}{z^*-\Lambda(z^*)}]}z^*\left\{-(\beta-\gamma)\frac{\partial p^*}{\partial\gamma} + [\frac{1}{\beta} + \frac{1}{\beta-\gamma} + \frac{cz^*}{z^*-\Lambda(z^*)}]\right\}$.

  $= ae^{-(\beta-\gamma)[\frac{1}{\beta}+\frac{1}{\beta-\gamma}+\frac{cz^*}{z^*-\Lambda(z^*)}]}z^*\{\frac{1}{\beta} + \frac{cz^*}{z^*-\Lambda(z^*)}\}$.

- To show that $\pi_R^{d^*}$ ∗is increasing in γ, we show that $\frac{\partial \pi_{R_i}^{d^*}}{\partial \gamma} \geq 0$.
  $\frac{\partial \pi_{R_i}^{d^*}}{\partial \gamma} = ae^{-(\beta-\gamma)[\frac{1}{\beta}+\frac{1}{\beta-\gamma}+\frac{cz^*}{z^*-\Lambda(z^*)}]}\frac{1}{\beta}\left(\frac{1}{\beta}+\frac{cz^*}{z^*-\Lambda(z^*)}\right)[z^*-\Lambda(z^*)] \geq 0$, where $i = 1, 2$.

- To show that $\pi_S^{d^*}$ is increasing in γ, we show that $\frac{\partial \pi_S^{d^*}}{\partial \gamma} \geq 0$.
  $\frac{\partial \pi_S^{d^*}}{\partial \gamma} = 2ae^{-(\beta-\gamma)[\frac{1}{\beta}+\frac{1}{\beta-\gamma}+\frac{cz^*}{z^*-\Lambda(z^*)}]}\left\{\frac{1}{(\beta-\gamma)^2}+\frac{1}{\beta-\gamma}\left(\frac{1}{\beta}+\frac{cz^*}{z^*-\Lambda(z^*)}\right)\right\}[z^*-\Lambda(z^*)] \geq 0.$

□

Proof of Proposition 3.3.11. . We take the partial derivative of (Equation 3.13) with respect to p_i and get

$$\frac{\partial \pi_{R_i}(p_i, z_i|r)}{\partial p_i} = ae^{-\beta p_i+\gamma p_j}\left[(1-r)(z_i-\Lambda(z_i)) - \beta\{(1-r)p_i(z_i-\Lambda(z_i)) - c\alpha_i z_i\}\right].$$

Since $ae^{-\beta p_i+\gamma p_j} > 0$, $\frac{\partial \pi_{R_i}(p_i, z_i/r)}{\partial p_i} = 0$ implies that $\tilde{p}_i(z_i|r) = \frac{1}{\beta} + \frac{c\alpha_i z_i}{(1-r)(z_i-\Lambda(z_i))}$, which gives us (Equation 3.14). Moreover, $\frac{\partial \pi_{R_i}}{\partial p_i} > 0$ for all $p_i < \tilde{p}_i(z_i|r)$ and $\frac{\partial \pi_{R_i}}{\partial p_i} < 0$ for all $p_i > \tilde{p}_i(z_i|r)$, so $\tilde{p}_i(z_i|r)$ is the unique maximizer of $\pi_{R_i}(p_i, z_i|r)$ for fixed z_i, r and p_{-i}. □

Proof of Proposition 3.3.12. . We want to derive $\bar{z}_i$ that maximizes $\pi_{R_i}(\tilde{p}_i(z_i|r), z_i|r)$. By the chain rule, we have

$$\frac{d\pi_{R_i}(\tilde{p}_i(z_i|r), z_i|r)}{dz_i} = \frac{\partial \pi_{R_i}(\tilde{p}_i(z_i|r), z_i|r)}{\partial p_i} \cdot \frac{d\tilde{p}_i(z_i|r)}{dz_i} + \frac{\partial \pi_{R_i}(\tilde{p}_i(z_i|r), z_i|r)}{\partial z_i}.$$

Appendix A (Continued)

The first term is zero, $\frac{\partial \pi_{R_i}(\tilde{p}_i(z_i|r), z_i|r)}{\partial p_i} = 0$, due to optimality of $\tilde{p}_i(z_i|r)$. Thus, we have

$$\begin{aligned}\frac{d\pi_{R_i}(\tilde{p}_i(z_i|r), z_i|r)}{dz_i} &= \frac{\partial \pi_{R_i}(\tilde{p}_i(z_i|r), z_i|r)}{\partial z_i} \\ &= ae^{-\beta\tilde{p}_i(z_i|r)+\gamma p_{-i}}\left\{(1-r)\left[\frac{1}{\beta} + \frac{c\alpha_i z_i}{(1-r)(z_i-\Lambda(z_i))}\right][1-F(z_i)] - c\alpha_i\right\} \\ &= \frac{ae^{-\beta\tilde{p}_i(z_i|r)+\gamma p_{-i}}}{\beta(z_i-\Lambda(z_i))}\cdot H(z_i),\end{aligned}$$

where $H(z_i) = [(1-r)(z_i-\Lambda(z_i)) + \beta c\alpha_i z_i]\,[1-F(z_i)] - \beta c\alpha_i(z_i-\Lambda(z_i))$.Since the ratio $\frac{ae^{-\beta\tilde{p}_i(z_i)+\gamma p_{-i}}}{\beta(1-r)(z_i-\Lambda(z_i))}$ in the above expression is always positive, first-order condition requires that the optimal $\bar{z}_i$ satisfy $H(z_i) = 0$, which gives us (Equation 3.15). Such a $\bar{z}_i$ always exists in the support interval (A, B) of $F(\cdot)$, because $H(z_i)$ is continuous for any given r where $0 < r < 1$, and $H(A) = A(1 - r + \alpha_i c\beta r) > 0$ and $H(B) = -\beta c\alpha_i(1-r) < 0$, since the mean value of ϵ is 1. To verify the uniqueness of $\bar{z}_i$, we have

$$\begin{aligned}H'(z_i) &= [1-F(z_i)]\Big\{(1-r)(1-F(z_i)) - h(z_i)\big[\beta c\alpha_i z_i + (1-r)(z_i-\Lambda(z_i))\big]\Big\} \\ H''(z_i) &= -h(z_i)H'(z_i) + [1-F(z_i)]\Big\{-f(z_i)(1-r) - h'(z_i)\big[\beta c\alpha_i z_i + (1-r)(z_i-\Lambda(z_i))\big] \\ &\quad -h(z_i)\big[\beta c\alpha_i + (1-r)(1-F(z_i))\big]\Big\},\end{aligned}$$

where $h(z_i) = f(z_i)/[1-F(z_i)]$ is defined as the failure rate of the demand distribution. From Assumption 3.1.1, $h'(z_i) > 0$, then $H''(z_i) < 0$ whenever $H'(z_i) = 0$, implying that $H(z_i)$ is a unimodal function. We have proved that $\bar{z}_i$ is a unique maximizer of $\pi_{R_i}(\tilde{p}_i(z_i|r), z_i|r)$. $\square$

Appendix A (Continued)

Proof of Proposition 3.3.14. . Consider the supplier's profit function

$$\pi_S(r) \;=\; ae^{-(\beta-\gamma)1/\beta}\left\{\sum_{i=1}^{2} e^{\frac{-(\beta\alpha_i-\gamma\alpha_{-i})c}{(1-r)}}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_i-\alpha_{-i})c\right]\right\}.$$

If $\frac{\alpha_1}{\alpha_2} \notin \left(\frac{\gamma}{\beta}, \frac{\beta}{\gamma}\right)$, meaning that $-\beta c\left(\alpha_i - \frac{\gamma}{\beta}\alpha_{-i}\right) > 0$, then

$$\begin{aligned}\lim_{r\to 1}\pi_S(r) &= ae^{-(\beta-\gamma)1/\beta}\left\{\sum_{i=1}^{2}\lim_{r\to 1} e^{\frac{-(\beta\alpha_i-\gamma\alpha_{-i})c}{(1-r)}}\left[\lim_{r\to 1} r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-\lim_{r\to 1}(1-\alpha_i-\alpha_{-i})c\right]\right\}\\ &= +\infty.\end{aligned}$$

Thus, if $\frac{\alpha_1}{\alpha_2} \notin \left(\frac{\gamma}{\beta}, \frac{\beta}{\gamma}\right)$, then the supplier's profit function is unbounded.

Appendix A (Continued)

Assume that $\frac{\gamma}{\beta} < \frac{\alpha_1}{\alpha_2} < \frac{\beta}{\gamma}$, the first order condition requires the optimal revenue share r^* satisfy the following equation:

$$
\begin{aligned}
\frac{d\pi_S}{dr} &= ae^{-\beta[\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}]+\gamma[\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}]}\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2} \\
&\qquad +\left[\frac{-\beta\alpha_1 c}{(1-r)^2}+\frac{\gamma\alpha_2 c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\} \\
&\quad +ae^{-\beta[\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}]+\gamma[\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}]}\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2} \\
&\qquad +\left[\frac{-\beta\alpha_2 c}{(1-r)^2}+\frac{\gamma\alpha_1 c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\} \\
&= ae^{-(\beta-\gamma)\frac{1}{\beta}}\left\{e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}\right.\right. \\
&\qquad \left.-\left[\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\} \\
&\quad +e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}\right. \\
&\qquad \left.\left.-\left[\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\}\right\} \\
&= 0.
\end{aligned}
$$

There exists $r^* \in [0,1]$ such that $\frac{d\pi_S}{dr}|_{r=r^*} = 0$ because $\frac{d\pi_S}{dr}$ is continuous in r, $\frac{d\pi_S}{dr}|_{r=0} > 0$ and $\frac{d\pi_S}{dr}|_{r=1} < 0$. To verify that $\pi_S(r)$ is strictly increasing for $0 \leq r < r^*$ and strictly decreasing for $r^* < r \leq 1$, we need to show that $\pi_S(r)$ is a unimodal function.

Appendix A (Continued)

Consider

$$
\begin{aligned}
\frac{d^2\pi_S}{dr^2} &= e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\Bigg\{\frac{2\alpha_1 c}{(1-r)^2}+\frac{2\alpha_1 cr}{(1-r)^3}-\frac{2(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right] \\
&\qquad -\frac{2(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\left[\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}\right] \\
&\qquad +\frac{(\beta\alpha_1-\gamma\alpha_2)^2c^2}{(1-r)^4}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\Bigg\} \\
&\quad +e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\Bigg\{\frac{2\alpha_2 c}{(1-r)^2}+\frac{2\alpha_2 cr}{(1-r)^3}-\frac{2(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right] \\
&\qquad -\frac{2(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\left[\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}\right] \\
&\qquad +\frac{(\beta\alpha_2-\gamma\alpha_1)^2c^2}{(1-r)^4}\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\Bigg\} \\
\frac{d^2\pi_S}{dr^2} &= e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left\{\frac{2\alpha_1 c}{(1-r)^2}+\frac{2\alpha_1 cr}{(1-r)^3}-\frac{2(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\} \\
&\quad -e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left(\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right)\left\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}\right\} \\
&\quad -e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left(\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right)\Bigg\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2} \\
&\qquad -\left[\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\Bigg\} \\
&\quad +e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left\{\frac{2\alpha_2 c}{(1-r)^2}+\frac{2\alpha_2 cr}{(1-r)^3}-\frac{2(\beta\alpha_2-\gamma\alpha_1 c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\} \\
&\quad -e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left(\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\right)\left\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}\right\} \\
&\quad -e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left(\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\right)\Bigg\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2} \\
&\qquad -\left[\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\Bigg\}.
\end{aligned}
$$

At $\bar{r}^*$ such that $\frac{d\pi_S}{dr}=0$, the third and sixth terms become zero.

Appendix A (Continued)

$$
\begin{aligned}
\frac{d^2\pi_S}{dr^2}|_{r=\bar{r}^*} \quad = \quad & e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left\{\frac{2\alpha_1 c}{(1-r)^2}+\frac{2\alpha_1 cr}{(1-r)^3}-\frac{2(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\} \\
& -e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left(\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right)\left\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}\right\} \\
& +e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left\{\frac{2\alpha_2 c}{(1-r)^2}+\frac{2\alpha_2 cr}{(1-r)^3}-\frac{2(\beta\alpha_2-\gamma\alpha_1 c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\} \\
& -e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left(\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\right)\left\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}\right\}.
\end{aligned}
$$

At $\bar{r}^*$,

$$
\begin{aligned}
& e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}-\left[\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\} \\
& +e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}-\left[\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\}=0.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
& e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left\{\frac{2\alpha_1 c}{(1-r)^2}+\frac{2\alpha_1 cr}{(1-r)^3}-\frac{2(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\} \\
& +e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left\{\frac{2\alpha_2 c}{(1-r)^2}+\frac{2\alpha_2 cr}{(1-r)^3}-\frac{2(\beta\alpha_2-\gamma\alpha_1 c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\} \\
& =-\frac{2}{\beta(1-r)}\left(e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}+e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\right).
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\frac{d^2\pi_S}{dr^2}|_{\bar{r}^*} \quad = \quad & -\frac{2}{\beta(1-r)}\left(e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}+e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\right) \\
& -e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left(\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right)\left\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}\right\} \\
& -e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left(\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\right)\left\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}\right\}.
\end{aligned}
$$

Since $e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}$, $e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}$, $\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}$ and $\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}$ are positive, $\frac{d^2\pi_S}{dr^2}|_{\bar{r}^*}$ is negative. $\frac{d^2\pi_S}{dr^2} < 0$ whenever $\frac{d\pi_S}{dr} = 0$. Therefore, $\pi_S(r)$ itself is a unimodal function. The proof is complete. □

Proof of Corollary 3.3.13. . (Equation 3.15) can be rearranged as

$$
\frac{1}{1-F(\bar{z}_i)} - \frac{\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} = \frac{1-r}{c\alpha_i\beta}.
$$

$\frac{1-r}{c\alpha_i\beta}$ is a monotonic decreasing function in r. In conjunction with Lemma 3.3.2, there is a one-to-one correspondence between $\bar{z}_i$ and r, given that any other parameters remain unchanged. Therefore, we can further say that decreases $\bar{z}_i$ in r. The proof is complete. □

Appendix B

PROOFS OF CHAPTER FOUR

Proof of Proposition 4.1.3. For any given price p_i, we take the partial derivative of π_C with respect to z_i as

$$\frac{\partial \pi_C(\mathbf{p},\mathbf{z})}{\partial z_i} \quad = \quad (a - p_i + \gamma(p_{-i} - p_i))[p_i(1 - F(z_i)) - (c_{S_i} + c_{R_i})].$$

Since $c_{S_i} + c_R < p_i,\ \theta_i \in [0,1]$ and since $a - p_i + \gamma(p_{-i} - p_i) > 0$, $\frac{\partial \pi_C}{\partial z_i} = 0$, when $z_i = F^{-1}(\theta_i) \equiv z_i^{C^*}(p_i)$. Moreover, $\frac{\partial \pi_C}{\partial z_i} > 0$ for all $z_i < z_i^{C^*}(p_i)$ and $\frac{\partial \pi_C}{\partial z_i} < 0$ for all $z_i > z_i^{C^*}(p_i)$, so $z_i^{C^*}(p_i)$ is a unique maximizer of π_C for a fixed p_i. □

Proof of Proposition 4.1.4. Consider the centralized profit function when $\mathbf{z} = \mathbf{z}^{C^*}(\mathbf{p})$.

$$\begin{aligned}
\bar{\pi}_C(\mathbf{p}) \quad &\equiv \quad \pi_C(\mathbf{p}, \mathbf{z}^{C^*}(\mathbf{p})) \\
&= \quad (a - p_1 + \gamma(p_2 - p_1)) \left\{ p_1 \left(F^{-1}(\theta_1) - \theta_1 F^{-1}(\theta_1) + \int_A^{F^{-1}(\theta_1)} x f(x)\, dx \right) - (c_{S_1} + c_{R_1}) F^{-1}(\theta_1) \right\} \\
&\quad\ + (a - p_2 + \gamma(p_1 - p_2)) \left\{ p_2 \left(F^{-1}(\theta_2) - \theta_2 F^{-1}(\theta_2) + \int_A^{F^{-1}(\theta_2)} x f(x)\, dx \right) - (c_{S_2} + c_{R_2}) F^{-1}(\theta_2) \right\} \\
&= \quad (a - p_1 + \gamma(p_2 - p_1)) \left\{ p_1 \int_A^{F^{-1}(\theta_1)} x f(x)\, dx \right\} + (a - p_2 + \gamma(p_1 - p_2)) \left\{ p_2 \int_A^{F^{-1}(\theta_2)} x f(x)\, dx \right\}.
\end{aligned}$$

Appendix B (Continued)

The first-order necessary condition requires supplier i's optimal price p_i^{C*} to satisfy the following equation:

$$\begin{aligned}
\frac{\partial \bar{\pi}_C(\mathbf{p})}{\partial p_i} &= (a - p_i + \gamma(p_{-i} - p_i))\left\{\int_A^{F^{-1}(\theta_i)} xf(x)\,dx + \frac{c_{S_i} + c_{R_i}}{p_i}F^{-1}(\theta_i)\right\} \\
&\quad -(1+\gamma)p_i \int_A^{F^{-1}(\theta_i)} xf(x)\,dx + \gamma p_{-i}\int_A^{F^{-1}(\theta_{-i})} xf(x)\,dx \\
&= 0.
\end{aligned}$$

Since $a - p_i + \gamma(p_{-i} - p_i) > 0$ and $\int_A^{F^{-1}(\theta_i)} xf(x)\,dx > 0$, the first-order conditions for the optimal prices p_i^{C*} can be rewritten as:

$$1 + \frac{c_{S_i} + c_{R_i}}{p_i}\frac{F^{-1}(\theta_i)}{\int_A^{F^{-1}(\theta_i)} xf(x)\,dx} + \frac{\gamma p_{-i}\int_A^{F^{-1}(\theta_{-i})} xf(x)\,dx}{(a - p_i + \gamma(p_{-i} - p_i))\int_A^{F^{-1}(\theta_i)} xf(x)\,dx} = \frac{(1+\gamma)p_i}{a - p_i + \gamma(p_{-i} - p_i)}.$$

The proof is complete. $\square$

Proof of Proposition 4.2.1.

- For any given price p_i, we take the partial derivative of π_{S_i} with respect to z_i as

$$\frac{\partial \pi_{S_i}(\mathbf{p}, z_i|w)}{\partial z_i} = (a - p_i + \gamma(p_{-i} - p_i))[(p_i - w)(1 - F(z_i)) - c_{S_i}].$$

  Since $0 < w + c_{S_i} < p_i$, $\phi_i \in [0,1]$ and since $a - p_i + \gamma(p_{-i} - p_i) > 0$, $\frac{\partial \pi_{S_i}(\mathbf{p},z_i|w)}{\partial z_i} = 0$, when $z_i = F^{-1}(\phi_i) \equiv \tilde{z}_i(p_i|w)$. Moreover, $\frac{\partial \pi_{S_i}(\mathbf{p},z_i|w)}{\partial z_i} > 0$ for all $z_i < \tilde{z}_i(p_i|w)$ and

Appendix B (Continued)

$\frac{\partial \pi_{S_i}(\mathbf{p}, z_i|w)}{\partial z_i} < 0$ for all $z_i > \tilde{z}_i(p_i|w)$, so $\tilde{z}_i(p_i|w)$ is a unique maximizer of π_S for a fixed p_i and w.

- We want to derive $\bar{p}_i$ that maximizes $\pi_{S_i}(\mathbf{p}|w, \tilde{z}_i(p_i|w))$ which is given by

$$\begin{aligned}
\bar{\pi}_{S_i}(\mathbf{p}|w) &\equiv \pi_{S_i}(\tilde{z}_i(p_i|w), \mathbf{p}|w) \\
&= (a - p_i + \gamma(p_{-i} - p_i))\Big\{(p_i - w)\Big[F^{-1}(\phi_i) - \phi_i F^{-1}(\phi_i) + \int_A^{F^{-1}(\phi_i)} x f(x)\, dx\Big] \\
&\qquad - c_{S_i} F^{-1}(\phi_i)\Big\}. \\
&= (a - p_i + \gamma(p_{-i} - p_i)) \left\{ (p_i - w) \int_A^{F^{-1}(\phi_i)} x f(x)\, dx \right\}.
\end{aligned}$$

Applying a monotonic logarithmic transformation to $\bar{\pi}_{S_i}(\mathbf{p}|w)$ we obtain

$$ln(\pi_{S_i}(\mathbf{p}|w)) = ln\,(a - p_i + \gamma(p_{-i} - p_i)) + ln \left((p_i - w) \int_A^{F^{-1}(\phi_i)} x f(x)\, dx \right).$$

For any given consignment price w and p_{-i}, we take the partial derivative of $ln(\bar{\pi}_{S_i}(\mathbf{p}|w))$ with respect to p_i as

$$\begin{aligned}
\frac{\partial ln(\bar{\pi}_{S_i}(\mathbf{p}|w))}{\partial p_i} &= \frac{-(1+\gamma)}{a - p_i + \gamma(p_{-i} - p_i)} + \frac{\int_A^{F^{-1}(\phi_i)} x f(x)\, dx + \frac{c_{S_i}}{p_i - w} F^{-1}(\phi_i)}{(p_i - w) \int_A^{F^{-1}(\phi_i)} x f(x)\, dx} \\
&= \frac{1}{p_i - w} \left\{ \frac{-(1+\gamma)(p_i - w)}{a - p_i + \gamma(p_{-i} - p_i)} + \frac{\int_A^{F^{-1}(\phi_i)} x f(x)\, dx + \frac{c_{S_i}}{p_i - w} F^{-1}(\phi_i)}{\int_A^{F^{-1}(\phi_i)} x f(x)\, dx} \right\} \\
&= \frac{1}{p_i - w} \left\{ \frac{-(1+\gamma)(p_i - w)}{a - p_i + \gamma(p_{-i} - p_i)} + 1 + \frac{c_{S_i}}{p_i - w} \frac{F^{-1}(\phi_i)}{\int_A^{F^{-1}(\phi_i)} x f(x)\, dx} \right\} \\
&= \frac{1}{p_i - w} \Psi_i(\mathbf{p}|w),
\end{aligned}$$

Appendix B (Continued)

where $\Psi_i(\mathbf{p}|w) = \frac{-(1+\gamma)(p_i-w)}{a-p_i+\gamma(p_{-i}-p_i)} + 1 + \frac{c_{S_i}}{p_i-w}\frac{F^{-1}(\phi_i)}{\int_A^{F^{-1}(\phi_i)} xf(x)\,dx}$ and $\phi_i = 1 - \frac{c_{S_i}}{p_i-w}$.

Since $\frac{1}{p_i-w} > 0$, the first order condition requires that the optimal $\bar{p}_i$ satisfies $\Psi_i(\mathbf{p}|w) = 0$ which gives (Equation 4.7). To verify that there always exists $p_i \in [w + c_{S_i}, p_i^{max}]$ such that $\Psi_i(\mathbf{p}|w) = 0$, we consider

$$\lim_{p_i \to w+c_{S_i}} \Psi_i(\mathbf{p}|w) = \lim_{p_i \to w+c_{S_i}} \frac{-(1+\gamma)(p_i-w)}{a-p_i+\gamma(p_{-i}-p_i)} + 1 + \lim_{p_i \to w+c_{S_i}} \frac{c_{S_i}}{p_i-w}\frac{F^{-1}(\phi_i)}{\int_A^{F^{-1}(\phi_i)} xf(x)\,dx}.$$

The first term is a finite negative number since $a - p_i + \gamma(p_{-i} - p_i) > 0$. The last term becomes very large $(+\infty)$ as $p_i \to w + c_{S_i}$ because $\lim_{p_i \to w+c_{S_i}} \frac{c_{S_i}}{p_i-w} = 1$ and $\lim_{p_i \to w+c_{S_i}} F^{-1}(\phi_i) = A$. Therefore, $\lim_{p_i \to w+c_{S_i}} \Psi_i(\mathbf{p}|w) = +\infty > 0$.

Similarly, consider

$$\lim_{p_i \to p_i^{max}} \Psi(\mathbf{p}|w) = \lim_{p_i \to p_i^{max}} \frac{-(1+\gamma)(p_i-w)}{a-p_i+\gamma(p_{-i}-p_i)} + 1 + \lim_{p_i \to p_i^{max}} \frac{c_{S_i}}{p_i-w}\frac{F^{-1}(\phi_i)}{\int_A^{F^{-1}(\phi_i)} xf(x)\,dx}.$$

The first term is a very large negative number $(-\infty)$ since $\lim_{p_i \to p_i^{max}} a-p_i+\gamma(p_{-i}-p_i) = 0^+$ and the last term is finite. Therefore, $\lim_{p_i \to p_i^{max}} \Psi_i(\mathbf{p}|w) = -\infty < 0$.

We have proved that $\bar{p}_i$ satisfying $\Psi_i(\mathbf{p}|w) = 0$ always exists in the interval $[w+c_{S_i}, p_i^{max}]$ because $\Psi(\mathbf{p}|w)$ is continuous in p_i, and $\lim_{p_i \to w+c_{S_i}} \Psi_i(\mathbf{p}|w) > 0$ and $\lim_{p_i \to p_i^{max}} \Psi_i(\mathbf{p}|w) < 0$.

Appendix B (Continued)

To verify the uniqueness of $\bar{p}_i$, we intend to show that $\Psi_i(\mathbf{p}|w)$ is decreasing in p_i (i.e., $\frac{\Psi_i(\mathbf{p}|w)}{\partial p_i} < 0$). We have

$$\begin{aligned}\Psi_i(\mathbf{p}|w) &= \frac{-(1+\gamma)(p_i - w)}{a - p_i + \gamma(p_{-i} - p_i)} + 1 + \frac{c_{S_i}}{p_i - w} \frac{F^{-1}(\phi_i)}{\int_A^{F^{-1}(\phi_i)} xf(x)\,dx} \\ &= m(\mathbf{p}|w) + 1 + g_i(p_i|w),\end{aligned}$$

where $m(\mathbf{p}|w) = \frac{-(1+\gamma)(p_i-w)}{a-p_i+\gamma(p_{-i}-p_i)}$ and $g_i(p_i|w) = \frac{c_{S_i}}{p_i-w}\frac{F^{-1}(\phi_i)}{\int_A^{F^{-1}(\phi_i)} xf(x)\,dx}$.

We can easily show that $m(\mathbf{p}|w)$ is decreasing in p_i since $\frac{\partial m(\mathbf{p}|w)}{\partial p_i} = \frac{-(1+\gamma)(a-(1+\gamma)w+\gamma p_{-i}}{(a-p_i+\gamma(p_{-i}-p_i))^2} < 0$. We then want to show that $g(p_i|w)$ is decreasing in p_i. Consider

$$\begin{aligned}\frac{dg(p_i|w)}{dp_i} &= \frac{\frac{c_{S_i}}{p_i-w}\left[\frac{c_{S_i}}{(p_i-w)^2}\frac{1}{f(F^{-1}(\phi_i))}\int_A^{F^{-1}(\phi_i)} xf(x)\,dx - \frac{c_{S_i}}{(p_i-w)^2}(F^{-1}(\phi_i))^2\right]}{\left(\int_A^{F^{-1}(\phi_i)} xf(x)\,dx\right)^2} \\ &\quad - \frac{c_{S_i}}{(p_i-w)^2}\frac{F^{-1}(\phi_i)}{\int_A^{F^{-1}(\phi_i)} xf(x)\,dx}.\end{aligned}$$

Let $u = F^{-1}(\phi_i) = F^{-1}(1 - \frac{c_{S_i}}{p_i-w})$, then $\frac{c_{S_i}}{p_i-w} = 1 - F(u)$ and $\frac{du}{dp_i} = \frac{c_{S_i}}{(p_i-w)^2 f(F^{-1}(\phi_i))} > 0$. Thus, $\frac{dg_i(p_i|w)}{dp_i}$ can be rewritten as

$$\frac{dg(p_i|w)}{dp_i} = \left[(1 - h(u))\int_A^u xf(x)\,dx - u^2 f(u)\right]\frac{1 - F(u)}{\left(\int_A^u xf(x)\,dx\right)^2}\frac{du}{dp_i}$$

where $h(u) = \frac{uf(u)}{1-F(u)}$.

$\frac{dg_i(p_i|w)}{dp_i}$ is now similar to $\frac{dg(p)}{dp}$ in the proof of Lemma 1 in (Chen et al., 2004). Thus, $\frac{dg(p_i|w)}{dp_i}$ is decreasing in p_i (see (Chen et al., 2004) for a detailed proof). We have thus proved that $\Psi_i(\mathbf{p})$ is decreasing in p_i. Therefore, the proof that $\bar{p}_i$ is a unique maximizer of $\pi_{S_i}(\mathbf{p}|w, \tilde{z}_i(p_i|w))$ for fixed p_{-i} and w is complete.

□

Proof of Proposition 4.2.2. The proof is similar to Proposition 3.3.6 and therefore omitted. □

CITED LITERATURE

Adida, E. and Ratisoontorn, N.: Consignment contracts with retail competition. European Journal of Operational Research, 215(1):136–148, 2011.

Bendavid, Y., Boeck, H., and Philippe, R.: Rfid-enabled traceability system for consignment and high value products: A case study in the healthcare sector. Journal of Medical Systems, 0(0):1–17, 2011.

Bernstein, F. and Federgruen, A.: Decentralized supply chains with competing retailers under demand uncertainty. Management Science, 51(1):18–29, 2005.

Besbes, O. and Maglaras, C.: Revenue optimization for a make-to-order queue in an uncertain market environment. Operations Research, 57(6):1438–1450, 2009.

eds. J. H. J. Blackstone and J. F. Cox APICS Dictionary. Virginia, APICS, eleventh edition, 2005.

Boorstin, J.: An auto-parts store your mother could love. Fortune, 148(10):163, 2003.

Cachon, G. P.: Supply chain coordination with contracts. In Handbook in Operations Research and Management Science: Supply Chain Management, eds. S. C. Graves and A. G. Kok, pages 229–339. Amsterdam, Elsevier, 2003.

Cachon, G. P.: The allocation of inventory risk in a supply chain: push, pull, and advance-purchase discount contracts. Management Science, 50(2):222–238, 2004.

Cachon, G. P. and Kok, A. G.: Competing manufacturers in a retail supply chain: On contractual form and coordination. Management Science, 56(3):571–589, 2010.

Cachon, G. P. and Lariviere, M. A.: Supply chain coordination with revenue-sharing contracts: Strengths and limitations. Management Science, 51(1):30–44, 2005.

Cai, G.: Channel selection and coordination in dual-channel supply chains. Journal of Retailing, 86(1):22–36, 2010.

CITED LITERATURE (Continued)

Chen, F. Y., Yan, H., and Yao, L.: A newsvendor pricing game. IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans, 34(4):450–456, 2004.

Chen, J.: Return with wholesale-price-discount contract in a newsvendor problem. International Journal of Production Economics, 130(1):104–111, 2011.

Chiang, W., Chhajed, D., and Hess, J.: Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. Marketing Science, 49(1):1–20, 2003.

Choi, S. C.: Price competition in a channel structure with a common retailer. Marketing Science, 10(4):271–296, 1991.

Choi, S. C.: Price competition in a duopoly common retailer channel. Journal of Retailing, 72(2):117–134, 1996.

Coughlan, A. T.: Retail pricing: Does channel length matter. Managerial and Decision Economic, 13(3):201–214, 1992.

Cowan, S.: When does third-degree price discrimination reduce social welfare, and when does it raise it? Economics series working paper 410, Washington University of Oxford, Department of Economics, 2008.

Dana Jr., J. D. and Spier, K. E.: Revenue sharing and vertical control in the video rental market. Journal of Industrial Economics, 49(3):223–245, 2001.

Dumrongsiri, A., Fan, M., Jain, A., and Moinzadeh, K.: Asupplychainmodel with direct and retailchannels. European Journal of Operational Research, 187(3):691–718, 2008.

Fahey, J.: Stock now, pay later. Forbes, 172(9):60, 2003.

Gallego, G. and Van Ryzin, G.: Optimal dynamic pricing of inventories with stochastic demand over finite horizons. Management Science, 40(8):999–1020, 1994.

Gerchak, Y. and Wang, Y.: Revenue-sharing vs. wholesale-price contracts in assembly systems with random demand. Production and Operations Management, 13(1):23–33, 2004.

Gosman, M. L. and Kelly, T.: Big customers and their suppliers: A case examining business relationships and their financial effects. Issues in Accounting Education, 17(1):41–56, 2002.

CITED LITERATURE (Continued)

Greenhut, M. L., Norman, G., and Hung, C.: The economics of imperfect competition: a spatial approach. Journal of Economic Behavior & Organization, 9(3):321–323, 1988.

Gümüs, M., Jewkes, E. M., and Bookbinder, J. H.: Impact of consignment inventory and vendor-managed inventory for a two-party supply chain. International Journal of Production Economics, 113(2):502–517, 2008.

Hastings, J.: Wholesale price discrimination and regulation: Implications for retail gasoline prices. Yale University Working Paper, 2009.

Huang, C., Yu, G., Wang, S., and Wang, X.: Disruption management for supply chain coordination with exponential demand function. Acta Mathematica Scientia, 26(4):655–669, 2006.

Jeuland, A. and Shugan, S.: Competitive pricing behavior in distribution systems. In Issues in Pricing: Theory and Research, ed. T. M. Devinney, pages 219–238. Lexington Books, 1988.

Karlin, S. and Carr, C. R.: Prices and optimal inventory policies. In Studies in Applied Probability and Management Science, eds. K. J. Arrow, S. Karlin, and H. Scarf, chapter 10, pages 159–172. Stanford University Press, 1962.

Katok, E. and Wu, D. Y.: Contracting in supply chains: a laboratory investigation. Management Science, 55(12):1953–1968, 2009.

Kurata, H., Yao, D., and Liu, J. J.: Pricing policies under direct vs. indirect channel competition and national vs. store brand competition. European Journal of Operational Research, 180(1):262–281, 2007.

Lariviere, M. A. and Porteus, E. L.: Selling to the newsvendor: An analysis of price-only contracts. Manufacturing and Service Operations Management, 3(4):293–305, 2001.

Lau, H. S. and Lau, A. H. L.: The effect of reducing demand uncertainty in a manufacturer-retailer channel for single-period products. Computers and Operations Research, 29(11):1583–1620, 2002.

Li, S., Zhu, Z., and Huang, L.: Supply chain coordination and decision making under consignment contract with revenue sharing. International Journal of Production Economics, 120(1):88–99, 2009.

CITED LITERATURE (Continued)

Liao, C. and Tseng, B.: Strategic selection of direct selling and private brand in retail market under retailer stackelberg. Department of Economics and Graduate Institute of International Economics Working Paper, National Chung Cheng University, 2007.

Linh, C. T. and Hong, Y.: Channel coordination through a revenue sharing contract in a two-period newsboy problem. European Journal of Operational Research, 198(3):822–829, 2009.

McGuire, T. W. and Staelin, R.: Industry equilibrium analysis of downstream vertical integration. Marketing Science, 2(2):161–191, 1983.

Meyer, D. and Fischer, J.: Strategic decentralization in channels. Federal Trade Commission Bureau of Economics Working Paper #271, 2004.

Moorthy, K. S.: Strategic decentralization in channels. Marketing Science, 7(4):335–355, 1988.

Narayanan, V. G., Raman, A., and Singh, J.: Agent costs in a supply chain with demand uncertainty and price competition. Management Science, 51(1):120–132, 2005.

Niu, R. H., Zhao, X., Castillo, I., and Joro, T.: Pricing and inventory strategies for a two-stage dual-channel supply chain. Asia-Pacific Journal of Operational Research, 29(1):xxx–xxx, 2012.

Özer, O. and Wei, W.: Strategic commitments for an optimal capacity decision under asymmetric forecast information. Management Science, 52(8):1238–1257, 2006.

Pan, K., Lai, K. K., Leung, S. C. H., and Xiao, D.: Revenue-sharing versus wholesale price mechanisms under different channel power structures. European Journal of Operational Research, 203(2):532–538, 2010.

Pasternack, B. A.: Using revenue sharing to achieve channel coordination for a newsboy type inventory model. In Supply Chain Management: Models, Applications, and Research Directions, eds. J. Geunes, P. M. Pardalos, and H. E. Romeijn, pages 117–136. Kluwer Academic Publishers, 2002.

Perakis, G. and Roels, G.: The price of anarchy in supply chains: Quantifying the efficiency of price-only contracts. Management Science, 53(8):1249–1268, 2007.

Petruzzi, N. and Dada, M.: Pricing and the newsvendor problem: A review with extensions. Operations Research, 47(2):184–194, 1999.

CITED LITERATURE (Continued)

Porteus, E. L.: Stochastic inventory theory. In Handbooks in Operations Research and Management Science, eds. D. P. Heyman and M. J. Sobel, pages 605–652. North-Holland, Elsevier, 1990.

Raju, J. S., Raj, S., and Sanjay, D. K.: The introduction and performance of store brands. Marketing Science, 41(6):957–978, 1995.

Ru, J. and Wang, Y.: Consignment contracting: Who should control inventory in the supply chain? European Journal of Operational Research, 201(3):760–769, 2010.

Simon, C.: Dynamic pricing with demand learning under competition. Doctoral dissertation, Massachusetts Institute of Technology, 2007.

Su, X. and Zhang, F.: Strategic customer behavior, commitment, and supply chain performance. Management Science, 54(10):1759–1773, 2008.

Talluri, K. T. and Van Ryzin, G. J.: The theory and practice of revenue management. Boston, Springer, 2005.

Tsay, A. A.: The quantity flexibility contract and supplier-customer incentives. Management Science, 45(10):1339–1358, 1999.

Tsay, A. A. and Agrawal, N.: Channel dynamics under price and service competition. Manufacturing and Service Operations Management, 2(4):372–391, 2000.

Tsay, A. A. and Agrawal, N.: Channel conflict and coordination in the e-commerce age. Production and Operations Management, 13(1):93–110, 2004.

Tsay, A. A. and Agrawal, N.: Modeling conflict and coordination in multi-channel distribution systems: A review. In Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era, eds. D. Simchi-Levi, S. D. Wu, and Z.-J. Shen, pages 13–66. Kluwer Academic Publishers, 2004.

University of California, S. F. M. C.: Materiel service or resources and education guide. `http://www.ucsfmedicalcenter.org/matserv/misc/or_guides/or_resource_and_information_guide.doc`, 2006. [Online; accessed April 12, 2011].

Van Ryzin, G. and Mahajan, S.: On the relationship between inventory costs and variety benefits in retail assortments. Management Science, 45(11):1496–1509, 1999.

CITED LITERATURE (Continued)

Villas-Boas, S. B.: An empirical investigation of the welfare effects of banning wholesale price discrimination. Department of Agricultural and Resource Economics, UCB, UC Berkeley, 2008.

Vives, X.: Duopoly information equilibrium: Cournot and bertrand. Journal of Economic Theory, 34(1):71–94, 1984.

Vives, X.: On the efficiency of bertrand and cournot equilibria with product differentiation. Journal of Economic Theory, 36(1):166–175, 1985.

Wang, Y.: Joint pricing-production decisions in supply chains of complementary products with uncertain demand. Operations Research, 54(6):1110–1127, 2006.

Wang, Y., Jiang, L., and Shen, Z.: Channel performance under consignment contract with revenue sharing. Management Science, 50(1):34–47, 2004.

Wikinvest: Amazon.com (amzn). `http://www.wikinvest.com/stock/Amazon.com_(AMZN)`, 2012. [Online; accessed January 29, 2012].

Xiao, T., Yu, G., Sheng, Z., and Xia, Y.: Coordination of a supply chain with one-manufacturer and two retailers under demand promotion and disruption management decisions. Annals of Operations Research, 135(1):87–109, 2005.

Yao, Z., Leung, S. C. H., and Lai, K. K.: The effectiveness of revenue-sharing contract to coordinate the price-setting newsvendor products' supply chain. Supply Chain Management: An International Journal, 134(4):263–271, 2008.

Yao, Z., Leung, S. C. H., and Lai, K. K.: Manufacturer's revenue sharing contract and retail competition. European Journal of Operational Research, 186(2):637–651, 2008.

Zhang, D.: Essays On Supply Contracts And Dynamic Pricing. Doctoral dissertation, University of Iowa, 2008.

Zhang, D., de Matta, R., and Lowe, T. J.: Channel coordination in a consignment contract. European Journal of Operational Research, 207(2):897–905, 2010.

VITA

NAME:	Nantaporn Ratisoontorn
EDUCATION:	Ph.D., Industrial Engineering and Operations Research University of Illinois at Chicago, Chicago, Illinois, 2012
	M.S., Computer Science University of Chicago, Chicago, Illinois, 2007
	M.S., Information Systems Management (Honors) National Institute of Development Administration, Bangkok, Thailand, 2004
	B.S., Mathematics (Honors) Mahidol University, Bangkok, Thailand, 2002
TEACHING:	Department of Mechanical and Industrial Engineering, University of Illinois at Chicago, 2010–2012
	Department of Information and Decision Sciences, Liautaud Graduate School of Business, University of Illinois at Chicago 2008–2011
MEMBERSHIP:	International Conference on Information System (ICIS)
	Institute of Operations Research and the Management Sciences (INFORMS)
PUBLICATIONS:	"Consignment Contracts with Retail Competition" (with E. Adida), European Journal of Operational Research, Vol. 215(1): 136-148, 2011
	"Supply Chain Coordination under Consignment Contracts with Supplier Competition" (with E. Adida). Working paper.
PRESENTATIONS:	"Consignment Contracts with Retail Competition" POMS Annual Conference April 2012, Chicago IL.
	"Consignment Contracts with Retail Competition" INFORMS Annual Meeting November 2011, Charlotte NC.