# Fiscal Volatility Diminishes Fiscal Multipliers 

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For my parents.

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## LIST OF ABBREVIATIONS

| AIC | Akaike Information Criterion |
| :--- | :--- |
| API | Application-Programming Interface |
| BIC | Bayesian Information Criterion |
| EU | European Union |
| GDP | Hannan and Quinn criterion |
| HQC | Impulse Response Function |
| IRF | Likstrumental Variable |
| IV | Modified Akaike Information Criterion |
| LR | Modified Bayesian Information Criterion |
| MAIC | National Accounts |
| MBIC | Norganization for Economic Cooperation and De- |
| NA | Near Vector Auto-Regression Treaty Organization Price Parity |
| NATO | NPCD |

# LIST OF ABBREVIATIONS (Continued) 

PWT

RI

SIPRI

SLR

SVAR

TVAR

VAR

Penn World Tables

Rational Inattention

Stockholm International Peace Research Institute

Sims's Likelihood Ratio Test

Structural Vector Auto-Regressive

Threshold Vector Auto-Regressive

Vector Auto-Regressive

## SUMMARY

The study investigates the observed heterogeneity in fiscal multipliers due to rational inattention. The previously untested prediction that fiscal volatility diminishes the multiplier is tested using a panel spanning 62 countries and 60 years. Endogeneity between output and government purchases is addressed using defense spending, near vector autoregressive models, and instrumental variable techniques. Fiscal volatility, output gap, and trade openness are found to diminish the fiscal multiplier. Conditional on volatility, multipliers range from less than zero to 0.9 at full employment. Multipliers exceed one in severe recessions. (JEL E22, E62, H50)

## CHAPTER 1

## INTRODUCTION

The American Recovery and Reinvestment Act of 2009 highlighted the long running debate over the merits of fiscal stimulus. President Obama's Council of Economic Advisers recommended the $\$ 840$ B package based on an estimated fiscal multiplier of $1.6^{1}$ (Romer and Bernstein, 2009). Robert Barro, whose multiplier estimates lie closer to 0.5, campaigned against the stimulus (Barro and Redlick, 2009, 2011). Others, such as Afonso (2009), even suggested the merits of expansionary fiscal consolidation. In subsequent years, renewed interest in fiscal policy has pivoted research from the classic question What is the the fiscal multiplier? to What are the factors that determine the size of the fiscal multiplier?

The theory that fiscal expansions are less effective at stimulating economic growth when agents are closely tracking fiscal policy is tested. The result, based on the application of Sims's theory of rational inattention (RI) to macroeconomics, is supported by the empirical results (Sims, 2003). The study is the first empirical evidence to support this, hitherto unproven, theoretical result of RI.

RI relaxes the assumption that agents have unlimited ability to acquire and process information. Limited-capacity agents ration their attention across competing signals based on each

[^0]signal's entropy and the expected benefits of increasingly precise tracking. RI macroeconomic models demonstrate that when a signal's volatility is low, agents allocate comparatively less attention to its level; resulting in sub-optimal behavior in the face of shocks (Mackowiak and Wiederholt, 2010). When fiscal volatility is high, agents rationally allocate a larger portion of their limited attention capacity to tracking the level of fiscal shocks and the fiscal multiplier is low, in agreement with classical theory (Dworczak, 2011).

Using fiscal volatility to measure the attention agents devote to the fiscal policy signal, the results indicate that when recent fiscal volatility is high, the multiplier is low, agreeing with the classical fiscal multiplier. When volatility is low however, the estimated multiplier is high, in closer agreement with Keynesian multipliers.

A central problem in estimating fiscal multipliers is the possibly endogenous relationship between government purchases and output. Unbiased regression results require that right hand side (RHS) variables are independent of the error term. Since tax coffers grow alongside economic output, governments facing a growing economy may expand their purchases more so than they would have in the absence of growth. This paper incorporates several methods for combating such endogeneity. A near vector auto-regressive (NVAR) model is developed by predicting fiscal shocks and then estimating a specification based on these prediction errors in lieu of actual fiscal shocks. Barro and Redlick (2011) suggests that endogeneity can be overcome by focusing on a subset of total government purchases that is less likely to respond to output growth than overall government purchases. In particular he examines defense spending for the US. This paper expands on this approach by considering the defense purchase of multiple pan-
els of countries. In addition to using defense spending in lieu of total government purchases, defense spending can also be used as an instrumental variable for government purchases since it is correlated with total government purchases but not the error term. Defense based approaches prove dramatically more effective for estimating multipliers based for the US data than for the panels. The difference is elaborated upon in Chapter 6.

Theoretical and empirical results suggest that fiscal spending is more effective in stimulating the economy during recessions than in expansions (Auerbach and Gorodnichenko, 2010, 2012b; Baum et al., 2012; Batini et al., 2012; Beetsma et al., 2006; Candelon and Lieb, 2013; Fazzari et al., 2012; Kandil, 2001). The research is not unanimous on this point (Owyang et al., 2013). In a similar vein, recent empirical evidence in the literature supports the canonical result that openness to trade reduces the multiplier (Ilzetzki et al., 2013; Karras, 2011, 2014). Chapter 5 integrates both slackness and openness into the identification methodology. The empirical evidence in Chapter 5 support the theory that both factors reduce the multiplier.

Several datasets are incorporated:

- Barro and Redlick data which spans from 1912 to 2006 for the US
- Penn World Tables (US) data for 62 countries spanning 1952 to 2011 (Feenstra, Inklaar, and Timmer, 2013)
- North Atlantic Treaty Organisation (NATO) panel of defense spending data for 14 countries spanning 1956 to 2011
- World Bank's World Development Indicators (WDI) panel of defense spending data for 14 countries spanning 1988 to 2013.

The empirical results indicate that the multiplier ranges from -0.4 in countries facing high fiscal volatility to 1.6 in those facing low volatility. Estimates based solely on the 94 years of annual US data range from 0.3 , under high volatility, to 0.9 when volatility is low. Multipliers additionally contingent upon slackness and openness are estimated to range from - 2.8 to 1.3 based on the PWT panel estimates and -0.4 to 1.6 for the US time series.

Chapter 2 briefly discusses the relevant theoretical and empirical work on the effects of fiscal stimulus. Chapter 3 discusses data sources. Chapter 4 develops identification strategies. Chapter 5 presents the main results. Chapter 6 modifies the core dynamic model (Equation 4.6 ) to account for possible endogeneity by estimating near vector autoreggressions (NVAR), defense spending models, and defense spending instrumental variable models. Chapter 6 next develops and implements tests for outliers and structural breaks. Finally Chapter 7 concludes.

## CHAPTER 2

## FISCAL MULTIPLIER LITERATURE

This Chapter surveys the theoretical and empirical literature on fiscal multipliers. Section 2.1 is limited to benchmark neoclassical, Keynesian, and Rational Inattention (RI) models. Multipliers due to RI are shown to exhibit neoclassical tendencies when fiscal volatility is high and similarities to the Keynesian multiplier when fiscal volatility is low. ${ }^{1}$

Section 2.2 reviews the rapidly growing empirical literature on the size of the fiscal multiplier. Evidence based solely on US data is considered separately from studies employing panel data and each section is further divided among studies estimating a single overall multiplier and those concerned with contingent multipliers. ${ }^{2}$

### 2.1 Theoretical Fiscal Multipliers

This section provides theoretical motivation for the empirical tests in Chapter 4. Agents maximize their welfare by dynamically selecting their work effort, leisure, goods consumption, and savings. In simple terms, the fiscal multiplier simply tracks how fiscal shocks can produce an increase in hours worked and thus output. New Keynesian extensions and, to a lesser extent,

[^1]scenarios under RI are shown to result in suboptimal consumer behaviors that increase the fiscal multiplier.

### 2.1.1 Classical Fiscal Multipliers

The canonical infinite-horizon, permanent income hypothesis model ${ }^{1}$ demonstrates that a permanent increase in government purchases has zero effect on output and is fully absorbed by a fall in consumption. In this model a temporary increase in government purchases can cause a temporary, though less than one to one increase in output as consumption falls to a lower level in all future periods.

Ramey (2011a) discusses the possibility of higher positive short-run multipliers due to the expectation of hump shaped distortionary taxes. In this scenario taxes remain low in the short run before rising to fund government expansion in the medium term. Individuals that know their taxes will be higher in the future will substitute more labor to the present when taxes are relatively low thus increasing the short run effects of fiscal policy. Citing Marianne Baxter and Baxter and King (1999) and Burnside, Eichenbaum, and Fisher (2000), Ramey suggests a short run neoclassical fiscal multiplier ranging from -2.5 to 1.2 with the long-run multiplier strictly less than 1.0. Similarly, Hall (2009) derives simple static fiscal multipliers for the both neoclassical and new Keynesian models and demonstrates upper bounds of 1. In general neoclassical multipliers are strictly less than 1 , oftentimes less than 0.5 , and sometimes less than 0 .

[^2]
### 2.1.2 New Keynesian and Keynesian Fiscal Multipliers

New Keynesian models share similar optimization and rational expectations bases with neoclassical models. They also typically produce similar fiscal multipliers below one. Cogan, Cwik, Taylor, and Wieland (2010) uses the Smets and Wouters (2007) dynamic macroeconomic model $^{1}$ to demonstrate that in the most widely used dynamic models, new Keynesian fiscal multipliers agree with neoclassical estimates of a fiscal multiplier less than one.

Christiano, Eichenbaum, and Rebelo (2009), show that the multiplier in a New Keynesian model becomes large when the economy hits the zero nominal interest rate. In the Christiano et al. (2009) model, a zero nominal interest rate results in a deflationary spiral in which desired saving continuously lower the level of output. Under such a scenario fiscal shocks result in a rise in output and expected inflation. This drives down the real interest rate breaking the deflationary spiral and spurring private spending. Section 2.2 discusses the empirical evidence for this effect.

New Keynesian theorists, such as Mankiw (2000), suggest incorporating rule-of-thumb consumers alongside the standard infinite-horizon Ricardian consumers typical in new Keynesian models. Rule-of-thumb consumers do not borrow or save and instead, consume their full labor income. This suggestion is based on the fact that a significant percentage of U.S. households have near zero net wealth ${ }^{2}$. Gali and Gertler (1999) incorporate this feature in their theoretical

[^3]derivations of the effects of fiscal policy on consumption. They obtain output multipliers as high as 2.0 by assuming that half the consumers are rule-of-thumb variety and that the elasticity of labor supply is infinite. This is to say that they assume work is fully demand determined.

These non-optimizing adjustments bring new Keynesian model closer to approximating the basic Old Keynesian multiplier. This Keynesian fiscal multiplier is $\frac{1}{(1-m p c)}$ in a closed economy with no distortionary taxes ${ }^{1}$. And the multiplier falls in the face of taxes and openness.

Additional explanations for large fiscal multipliers include involuntary unemployment, underutilized resources, and the possible provision of public goods. Most economic models treat unemployment as optimally planned leisure. Involuntary unemployment, say due to strict labor laws, and under-utilization of capital have limited theoretical backgrounds in macroeconomics and are likely fertile grounds for new research.

So far, fiscal spending has been treated as utility free dig and fill spending. Government purchases of public goods in the sense of being non-rival and non-excludable would certainly result in larger fiscal multipliers. And any excludable projects whose impact exceeds 1.0 should be expected to attract private capital. It seems unlikely that the projects aimed at economic stabilization are truly public good projects with positive net present value; otherwise they should be instituted regardless of the state of the economy.

[^4]
### 2.1.3 Rational Inattention Fiscal Multipliers

Rational inattention theory relaxes the assumption that agents have unlimited ability to acquire and process information. In practice, several implementations achieve this goal. A discrete-time approach is to assume agents update their information infrequently while a continuous time approach involves continuous reallocation of limited Shannon capacity (Sims, 2010).

Dworczak (2011) uses the Maćkowiak and Wiederholt (2009) macroeconomic model to analyze fiscal policy under rational inattention and finds that low fiscal volatility results in a large fiscal multiplier. This result makes intuitive sense. Optimal decisions in the current period depend on shocks in previous periods. Agents allocate their limited attention in order to maximize their welfare. At zero volatility agents do not perceive any change in government purchases nor do they anticipate any future tax increases, et cetera. Thus they do not downwardly revise their consumption. Increased wages are then perceived as due to an increase in the value of labor's marginal product and thus effort and output increase. At high levels of volatility, government-spending shocks are frequent. Agents optimally allocate their attention to the level of government purchases and they revise their consumption plans in response to any shocks.

In contrast to the Dworczak model, the empirical results presented Chapter 4 suggest longrun differences in the effects of fiscal policy. Several ad hoc explanations for this difference exist. Many of the standard old Keynesian arguments from Subsection 2.1.2 may apply over the long run. For instance, permanent investments in response to the net demand shock could augment
the capital stock and future output. Nevertheless, the salient lesson can be summarized as: the old Keynesian fiscal multiplier holds during periods of low volatility and the neoclassical fiscal multiplier holds in the face of high volatility. The next section reviews the empirical estimates of the fiscal multiplier in the literature.

### 2.2 Empirical Fiscal Multiplier Estimates

In the past several years, renewed interest into the effects of fiscal policy have been redirected from the goal of estimating an unconditional fiscal multiplier to identifying the factors that determine the array of fiscal reactions observed over time and across countries. The remaining sections will first consider literature that attempts to estimate a single fiscal multiplier before considering literature on contingent multipliers.

### 2.2.1 Cross-Country Fiscal Multiplier Estimates

Cross-country samples offer a much larger variety of fiscal policy episodes for analysis. This section catalogues studies which exploit the advantages of international data.

Only two studies in this section estimate an unconditional fiscal multiplier. In one, Perotti (2007) uses an SVAR approach on data from four OECD countries to estimate an average spending multiplier at impact of 0.7 that grows to 1.2. Perroti also stresses however that his estimates vary greatly over time and across counties with a range from -2.3 to 3.7 .

In the other study suggesting an unconditional multiplier, Beetsma, Giuliodori, and Klaassen (2006) calculate a multiplier of 0.358 based on annual data on EU countries from 1980 to 2002. Beetsma, Giuliodori, and Klaassen (2006) also find that fiscal stimulus in one country has economically and statistically significant effects on output in other countries. Table IV summarizes

## TABLE I

Unconditional Fiscal Multiplier Estimates - Panel Data

| Source | Data | Estimate |
| :--- | :--- | :--- |
| Perotti (2007) | $(1947-2006)^{q}$ | 0.7 on impact and 1.2 in long-run |
|  | 4 countries |  |
| Beetsma et al. (2006) | $(1980-2002)^{a}$ | 0.358 and fiscal expansions lead to in- |
|  | 15 EU coun- | creases in bilateral exports |
|  | tries |  |

the basic features of the contributions that use panel data to estimate an unconditional fiscal. The estimates range from a low of 0.358 to a high of 1.2 .

The remaining papers aim to quantify how specific factors, such as the nominal interest rate or whether the economy is in the midst of a recession, affect the fiscal multiplier according to panel data Afonso (2009) uses annual data on EU15 countries from 1970 to 2005 to estimate the expansionary effects of fiscal consolidations. Alfonso finds that consolidations result in growth. In contrast Guajardo, Leigh, and Pescatori (2010), in their study of 15 OECD countries spanning 1980 to 2009, find that fiscal consolidations reduce output in the long run whether they come from tax hikes or reduced government purchases. Alesina and Ardagna (2010) study OECD countries over the 1970 to 2007 period and find fiscal stimuli based on tax cuts increase growth more than those based upon spending increases.

In the only analysis, apart from the present paper, to use defense data for a panel of countries, Almunia and Benetrix (2010) use several identification techniques in their analysis of
data for 27 countries spanning 1925 to 1939. In addition to using VAR techniques to investigate the effect of total fiscal shocks they perform VAR and instrumental variable (IV) regressions on defense data compiled from the 1924 to 1940 editions of the League of Nations Armaments Yearbook. Almunia and Benetrix find that the multiplier is 1.2 to 2.5 for defense expenditures but only 0.13 to 0.43 for total government purchases. Almunia and Benetrix note that, "Fiscal policy made little difference during the 1930s because it was not deployed on the requisite scale, not because it was ineffective."

Several papers reach the same conclusions that fiscal spending is more effective in recessions than during expansions. Baum, Poplawski-Ribeiro, and Weber (2012) employ a threshold variable based on the state of the economy. Their study examines quarterly data from G7 countries (minus Italy) spanning 1996 to 2011. They find that the multiplier ranges from -2.0 in expansions to 2.2 in recessions. Tagkalakis (2008) finds that the multiplier is stronger during bad times in an analysis of 19 OECD countries from 1970 to 2002. Tagkalakis (2008) also finds that fiscal policy is more effective under credit constraints. Batini, Callegari, and Melina (2012) consider quarterly data for the US, Japan, and the Euro area in their regime switching VAR model. They find that the spending multiplier is approximately ten times higher than during recession, at 1.6-2.6, as compared to during expansions when its 0.16 to 0.35 . In the first of a series of papers, Auerbach and Gorodnichenko (2012b) adapt an SVAR to annual data on 31 OECD countries from 1984 to 2011. There results allow for a maximum multiplier of 3.5 in recessions with a baseline level around 1 during normal times and expansions.

Karras (2014) uses a panel of 179 countries spanning more than 40 years to demonstrate that openness increases the spillover effect as suggested by basic theory. In Auerbach and Gorodnichenko (2012a), fiscal stimulus in one country is found to have economically and statistically significant effects on output in other countries mirroring the result of Beetsma et al. (2006). Furthermore, the strength of the spillover is shown to increase when either the recipient or source countries are in recession.

Favero, Giavazzi, and Perego (2011) study 8 OECD countries over the 1978 to 2009 period and rather than suggesting a particular multiplier, they argue that several factors, including debt dynamics, openness, and fiscal reaction functions, are largely responsible for determining the sign and magnitude of any so-called multiplier.

Ilzetzki, Mendoza, and Végh (2013) use an SVAR specification and data from 20 developed and 24 developing countries spanning 10 years to demonstrate that the fiscal spending multiplier is dependent on the level of development, exchange rate regime, openness to trade, and public indebtedness. Ilzetzki et al. (2013) use an indicator variable to measure openness. They find that the multiplier is large for industrial economies, closed economies, and those with fixed exchange rates but zero for those with flexible rates. They also find that the fiscal multiplier is zero in high-debt countries. They recommend that developing economies adopt a-cyclical fiscal policies and, in downturns, use fiscal stimulus consisting of government investment. In a study of 62 developed economies over the 1951 to 2007 period, Karras (2011) finds that the fiscal multiplier is extremely sensitive to a country's level of trade openness by employing a continuous trade openness variable. Karras's continuous measure of openness is defined as

TABLE II
Conditional Fiscal Multiplier Estimates - Panel Data

| Source | Data | Estimate |
| :---: | :---: | :---: |
| Afonso (2009) | 15 EU countries | Implicitly $<0$. Afonso finds spending cuts to be expansionary |
|  | $(1970-2007){ }^{a} 30$ | $<1$. They show that tax cuts increase |
| Alesina and Ardagna (2010) | OECD countries $(1925-1939)^{a} \quad 27$ | growth more than spending increases 1.2-2.5 for defense spending but only 0.13- |
| Almunia and Benetrix (2010) | countries | 0.43 for total government purchases |
| Auerbach and Gorodnichenko (2012a) | OECD countries | spillovers and economic state in source and recipient countries increase multiplier |
| Auerbach and Gorodnichenko (2012b) | $(1984-2011)^{a} \quad 31$ OECD countries | multiplier is larger in recessions than during normal times or in expansions |
| Batini et al. (2012) | (1975-2010) ${ }^{q}$ US, EU, and Japan $(1966-2011)^{q} \quad$ G7 | multiplier is 10 times larger during recessions than during expansions 2.2 in downturns and -2.0 in expansions |
| Baum et al. (2012) | $(\text { minus Italy })$ | finds that fiscal multiplier is decreasing in |
| Favero et al. (2011) | $\begin{aligned} & \text { OECD countries } \\ & (1980-2009)^{a} \quad 15 \end{aligned}$ | debt and openness <br> $>0$. Finds that fiscal consolidation re- |
| Guajardo et al. (2010) | countries | duces output. |
| Ilzetzki et al. (2013) | $(1966-2006)^{q} \quad 44$ countries | finds fiscal multiplier falls with openness to trade, exchange rate flexibility, public indebtedness, and rises in level of development. |
| Karras (2011) | $(1951-2011)^{a} \quad 62$ countries | finds that a $10 \%$ increase in openness reduces multiplier by more than $5 \%$. Multipliers range from 0.6 for open to 1.5 for a closed economies |
| Karras (2014) | $\begin{array}{ll} (1970-2005)^{a} & 179 \\ \text { countries } \end{array}$ | demonstrates that openness increases the spillover effect as suggested by basic the- |
| Tagkalakis (2008) | $(1970-2002)^{a} \quad 19$ OECD countries | Demonstrates that binding liquidity constraints increase effectiveness of fiscal policy and that fiscal policy boosts private consumption in recessions more than in expansions. |

the sum of imports and exports as a fraction of output. Specifically, Karras finds that a $10 \%$ increase in openness decreases the fiscal multiplier by more the $5 \%$. These results imply a range of multipliers from 0.6 to 1.5 for countries with openness between $10 \%$ and $100 \%$.

Table IV summarizes the basic features of the contributions that use panel data to estimate the fiscal multiplier contingent on additional economic factors. The estimates range from a low of less than 0 to a high of 3.8.

TABLE III

Unconditional Fiscal Multiplier Estimates - US Data

| Source | Data | Estimate | Notes |
| :--- | :--- | :--- | :--- |
| Barro (1981) | $(1889-1978)^{a}$ | $<1$ | uses defense spending |
|  | $(1889-1978)^{q}$ | $0.54 \quad 0.74$ | uses defense spending, defense |
| Barro and Redlick (2011) |  |  | news |
|  | $(1947-1997)^{q}$ | 1.25 after | 0.96 at impact |
| Blanchard and Perotti (2002) |  | 4 years |  |

Burnside, Eichenbaum, and Fisher (2000)

Edelberg, Eichenbaum, and Fisher (1999)

Ellahie and Ricco (2013)

Fatás and Mihov (2001)
Fisher and Peters (2010)
Ramey (2011a)
$(1939-2006)^{q} \quad 1.6 \quad$ uses defense spending, defense news
$(1959-2012)^{q} \quad 0.28 \quad$ state and local fiscal multipliers up to 4
$(1960-1997)^{q}>1$
$(1960-2008)^{q} \quad 0.6 \quad$ uses defense spending
$(1939-2006)^{q} \quad 0.6-1.2 \quad$ uses defense spending, defense news

Romer and Bernstein (2009)
$(1947-1995)^{q} \quad 0.125 \quad 0.5$ imputed from IRFs
estimate is the average of estimates by a leading private forecasting firm and the FRB/US model

### 2.2.2 United States Fiscal Multiplier Estimates

Barro (1981) uses annual defense spending from 1889 to 1978 to estimate that the fiscal multiplier is positive, less than 1.0 , and is lower for permanent spending increases. In the literature, defense spending is often used in lieu of overall government purchases due to the possibly endogenous response of non-defense purchases, to output growth, especially at the state and local levels. Barro and Redlick (2011) increase the precision of Barro's earlier (1981) estimate to between 0.54 and 0.74 using quarterly data from 1913 to 2008. Ramey (2011a) goes a step further in combatting potential endogeneity by employing a narrative approach to capture news about future increases in government purchases. The defense news variable proceeds actual defense purchases and is based on quarterly US defense news from 1939 to 2006. Ramey finds that the narrative shocks variable Granger-causes the shocks in typical VAR analyses. If the VAR approach captures shocks too late, it misses the fall in consumption and other effects that occur as soon as the news is learned. Ramey estimates fiscal multipliers in the range of 0.6 to 1.2 .

Edelberg, Eichenbaum, and Fisher (1999) use Ramey's data and information on residential and non-residential investment in a vector autoregressive (VAR) model with date uncertainty to estimate a fiscal spending multiplier of 1.6. Fisher and Peters (2010) also build on Ramey's defense news variable by identifying fiscal shocks using statistical innovations to the accumulated excess returns of large US defense contractors. Their approach yields a fiscal spending multiplier of 0.6 over a 5 -year horizon.

Typically the empirical literature employs government purchases in the measure of fiscal multipliers, as opposed to defense spending or defense news. An alternative method to address the issue of endogeneity is using a VAR model that corrects for the effects of output on government purchases. Burnside, Eichenbaum, and Fisher (2000) employ such an approach with quarterly data on total government purchases, hours worked, and real wages, for the period beginning in 1947 and ending 1995. They do not explicitly report a fiscal multiplier but a long-run estimate of 0.5 can be imputed from their impulse response functions (IRF).

Contrasting with Burnside et al. (2000), Fatás and Mihov (2001) employ their VAR model on quarterly data from 1947 to 1995 specifically for the purpose of estimating the impact of government purchases on output and private consumption. They find that the fiscal multiplier is firmly larger than one and that private consumption increases. These results contrast with all but Keynesian results in section 2.1, since the neoclassical, New Keynesian, and RI models all predict a fall in private consumption.

Blanchard and Perotti (2002) consider a VAR based on quarterly data from 1947 to 1997 to estimate a contemporaneous multiplier of 0.96 that reaches its peak after 4 years at about 1.25.

Ellahie and Ricco (2013) use quarterly federal, state, and local data from 1959 to 2012 in a Bayesian VAR model to estimate a 0.28 long-run federal fiscal multiplier and state and local multipliers as high as 4.

## TABLE IV

Conditional Fiscal Multipliers Estimates - US DATA

| Source | Data | Estimate \& Notes |
| :--- | :--- | :--- |
| Auerbach and $\quad$ Gorodnichenko <br> $(2010)$ | $(1950-2010)^{q}$ | 0.5 at impact, as high as 2.5 in recessions, <br> and $<0$ in expansions |
| Bilbiie (2008) | $(1957-2004)^{a}$ | 0.4 until 1979 then 0.15 after 1982 |
| Candelon and Lieb (2013) | $(1968-2010)^{q}$ | $>1$ in recessions and $<1$ in expansions |
| Fazzari et al. (2012) | $(1967-2011)^{q}$ | 1.5 facing underutilization and 1.2 facing <br> full employment |
| Hall (2009) | $(1930-2008)^{q}$ | 0.71 .0 but, at the zero lower bound, Hall <br> derives a theoretical multiplier of 1.7 |
| Kandil (2001) | $(1955-1996)^{q}$ | $>0$ in recessions. Notes asymmetry, <br> stressing that fiscal tightening results in <br> pronounced contraction |
| Monacelli and Perotti (2008) | $(1954-2006)^{q}$ | peaks at 2 then falls to 0. Multiplier is in- <br> creasing in share of government purchases <br> spent on non-traded goods |
| Mountford and Uhlig (2009) | $(1955-2000)^{q}$ | 0.5 at impact though $<0$ in long run. Tax <br> cuts have multiplier of 3.8 |

Table III summarizes the basic features of the contributions making unconditional estimates of the fiscal multiplier based on US data. The estimates vary from a low of 0.125 to a high of 1.6.

The remaining US empirical literature in this review does not aim to quantify a single unconditional fiscal multiplier. Instead these papers consider how specific factors, such as the nominal interest rate or whether the economy is in the midst of a recession, affect the fiscal multiplier.

Considered first is whether the sign of the fiscal shock may influence the multiplier. Kandil (2001) considers whether the effects of fiscal contractions are different from fiscal expansions. Using quarterly data from 1955 to 1996, Kandil's VAR results suggest that fiscal expansions raise the interest rate crowding out private investment while contractions have no effect on the interest rate. Consequently, private spending falls during expansions yet there is no corresponding increase in private spending observed during contractions, thus exacerbating the contraction. In sum these results point to a negative fiscal spending multiplier that is Bilbiie (2008) asks whether the multiplier has changed over time. Using a structural VAR (SVAR) on annual data from 1957 to 2004, Bilbiie estimates that the fiscal multiplier is 0.4 for the period before 1979 and -0.15 after 1982. Bilbiie expands on his empirical results by generating a dynamic stochastic general equilibrium (DSGE) model to incorporate price rigidities and limited asset market participation. The DSGE results suggests that increased asset market participation accounts for the change in fiscal transmission observed since the early 1980s.

When output consists of traded and non-traded good, fiscal policy may have unexpected effects on the terms of trade. Monacelli and Perotti (2008) consider quarterly US data from 1954 to 2006. Using service and non-service goods as proxies for non-traded and traded goods respectively, they find that fiscal expansion results in a fall in the price of traded versus nontraded goods and thus an appreciation of the terms of trade. In spite of government increasing output, the bias towards service and away from traded goods results in an appreciation of the price of own traded goods and thus a relative decline in the price of imports. Their overall fiscal multiplier peaks at 2.0 but eventually falls to 0 .

Alesina and Ardagna (2010) show that tax cuts stimulate economic activity more than equivalent increases in government purchases. This is at odds with the baseline Ricardian equivalence ( RE ) approach. RE posits that the method of financing government spending does not affect the decision to consume.

Mountford and Uhlig (2009) use a VAR approach that distinguishes tax changes from spending changes and finds that tax cuts result in a multiplier of 3.8 while spending increases only result in a fiscal multiplier of 0.33 . Their analysis is based on quarterly data from 1955 to 2000 .

Several papers have tested whether the multiplier is different in recessions versus during expansions. Papers by Auerbach and Gorodnichenko (2010) and by Fazzari, Morley, and Panovska (2012) both examine quarterly postwar data. Auerbach and Gorodnichenko find that during recessions the fiscal multiplier is initially 1 to 1.5 and 2.5 after twenty quarters. During expansions, they find it to be 0 to 0.5 on impact and negative soon after. In contrast, Fazzari et al. estimate a recession fiscal multiplier of 1.5 and an expansion multiplier of 1.2. Using a dataset similar to Fazzari et al., Candelon and Lieb (2013) employ a regime-switching error-correction framework to estimate a recession fiscal multiplier greater than 1 and an expansionary fiscal multiplier less than one.

As discussed in Section 2.1, fiscal policy is expected to have strong effects when an economy is facing the zero lower bound. Due to the inherent limitations of data on the US experience, US data cannot be used to measure this effect. Hall (2009) provides a theoretical estimate of as high as 1.7 in his paper that also estimates simple multipliers using US defense data.

Chapter 3 summarizes the basic features of the contributions that use US data to estimate the fiscal multiplier contingent on additional economic factors. The estimates range from a low of less than 0 to a high of 3.8.

## CHAPTER 3

## DATA

The empirical analysis in Chapter 5 and Chapter 6 considers from four datasources. The Penn World Tables (PWT) panel and Barro and Redlick's US data are the primary datasources while the World Development Indicators (WDI) and North Atlantic Treaty Organization (NATO) panels are used to test the robustness of the main results. Table V through Table VII summarize the relevant variables from each panel. While these variables are defined formally Chapter 4, here are brief definitions:

- Output growth $y$ is the percentage change in real GDP
- Fiscal shock $g$ refers to a change in the level of government purchases relative to GDP. Suppose for instance government purchases are $20 \%$ of GDP in year 1 and $22 \%$ of constant GDP in year 2 , then the fiscal shock is $2 \% .{ }^{1}$
- $\sigma_{g}$ is the volatility of fiscal shocks. It is measured as the standard deviation of the previous 6 fiscal shocks. The 6 -year window is selected according to the analysis in Chapter 3 . The appendix provides $R$ code which, when run from any modern installation of the free $R$ software, downloads and assembles the datasets, outputs a .CSV file of the relevant variables, and generates Chapter 4's main empirical analyses.

[^5]
### 3.1 Penn World Tables 1952-2011

Feenstra, Inklaar, and Timmer (2013) highlight the PWTs fundamental ability to measure differences in standards of living across countries and time by careful use of national accounts (NA) data. Johnson and Larson (2009) note that certain estimates vary substantially across different versions of the PWTs. Feenstra et al. (2013) demonstrate that revisions provided by certain NA offices around the world are the dominant reason for the differences noted by Johnson and Larson and that the current version represents the best possible estimates available today.

Feenstra et al. (2013) Table 5, recommends the variable rgdpna, for studies requiring real gross domestic product (GDP) based growth rates over time as well as for comparing growth rates across countries. Other variables based on shares of national income are independent of PPP considerations, thus all other series that will be used alongside the PWT data are in terms of output shares or are already in real terms.

All Other PWT variables in use are in terms of shares of rgdpna. These are real government purchases as a share of output csh_g, real exports as a share of output csh_x, and real imports as a share of output csh_m. The PWT data span from 1952-2011 for 62 countries and as many as 149 countries over shorter timespans. The sample is diverse in terms of average annual growth rate of real GDP, average fiscal shocks, and fiscal volatility. For example China's growth rate averaged $8 \%$ while the Democratic Republic of Congo experienced less than $1 \%$ average growth. Additionally, fiscal volatility varies greatly across the panel. This variation is not limited to countries experiencing dissimilar growth patterns. Average fiscal spending volatility in India,

## PWT Scatterplot Matrix (105 omitted)



Figure 1. PWT - Scatterplot Matrix
for instance, was relatively stable at $0.81 \%$ while fiscal volatility in Israel averaged $3.74 \%$. Yet both countries experienced high average growth rates of $5.19 \%$ and $6.35 \%$ respectively.

Figure 1 displays several two-dimensional scatterplots ${ }^{1}$ of the primary PWT variables. Variable names and associated histograms occupy the diagonal. The lower triangular entries plot the observations with the row variable on the $y$-axis and the column variable on the $x$-axis. Therefore the growth variable, $y$, is the $y$-axis varaible along the bottom row. The lower trangular plots also contain locally smoothed regression (LOESS) lines. The upper triangular entries contain correlation coefficients for the variables along the diagonal.

[^6]
## TABLE V

PWT - List of Countries and Sample Means (1952-2011)

| Country | Growth | Fiscal <br> Shock | $\sigma_{g}$ | Country | Growth | Fiscal <br> Shock | $\sigma_{g}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Argentina | $3.05 \%$ | $0.29 \%$ | $1.08 \%$ | Japan | $4.98 \%$ | $0.70 \%$ | $0.60 \%$ |
| Australia | 3.58 | 0.55 | 0.36 | Kenya | 3.80 | 0.73 | 1.26 |
| Austria | 3.19 | 0.47 | 0.42 | Korea, Republic of | 7.06 | 0.84 | 0.46 |
| Belgium | 2.78 | 0.59 | 0.57 | Luxembourg | 3.72 | 0.49 | 0.65 |
| Bolivia | 2.71 | 0.86 | 1.28 | Mauritius | 4.25 | 0.64 | 1.20 |
| Brazil | 4.98 | 0.79 | 1.32 | Mexico | 4.31 | 0.57 | 0.36 |
| Canada | 3.41 | 0.57 | 0.47 | Morocco | 4.72 | 1.06 | 3.33 |
| Chile | 4.22 | 0.57 | 1.17 | Netherlands | 3.16 | 0.63 | 0.73 |
| China | 8.00 | 1.78 | 1.65 | New Zealand | 2.81 | 0.57 | 0.72 |
| Colombia | 4.14 | 0.46 | 0.54 | Nigeria | 3.95 | 0.63 | 2.76 |
| Congo | 0.96 | 0.35 | 1.06 | Norway | 3.28 | 0.62 | 1.30 |
| Costa Rica | 4.86 | 0.79 | 0.51 | Pakistan | 5.08 | 0.99 | 1.86 |
| Cyprus | 4.82 | 0.74 | 1.94 | Panama | 5.89 | 1.15 | 1.53 |
| Denmark | 2.64 | 0.63 | 0.65 | Paraguay | 4.21 | 0.45 | 1.08 |
| Dominican Republic | 5.60 | 0.74 | 1.52 | Peru | 3.82 | 0.49 | 1.24 |
| Ecuador | 4.53 | 0.86 | 1.30 | Philippines | 4.12 | 0.50 | 0.87 |
| Egypt | 5.91 | 1.60 | 1.91 | Portugal | 3.63 | 0.72 | 0.64 |
| El Salvador | 3.11 | 2.33 | 4.20 | South Africa | 3.29 | 0.35 | 0.32 |
| Ethiopia | 4.04 | 0.55 | 1.64 | Spain | 3.90 | 0.65 | 0.47 |
| Finland | 3.19 | 0.68 | 0.52 | Sri Lanka | 4.83 | 1.03 | 2.29 |
| France | 3.13 | 0.62 | 0.42 | Sweden | 2.72 | 0.70 | 0.55 |
| Germany | 2.93 | 0.49 | 0.37 | Switzerland | 2.41 | 0.21 | 0.27 |
| Greece | 3.66 | 0.55 | 0.69 | Taiwan | 7.29 | 1.22 | 0.71 |
| Guatemala | 4.04 | 0.74 | 0.69 | Thailand | 7.05 | 1.27 | 0.95 |
| Honduras | 4.08 | 0.75 | 1.07 | Trinidad and Tobago | 4.32 | 0.57 | 1.09 |
| Iceland | 3.72 | 0.82 | 0.76 | Turkey | 4.73 | 0.71 | 0.79 |
| India | 5.19 | 1.10 | 0.81 | Uganda | 4.28 | 1.03 | 1.40 |
| Ireland | 4.00 | 0.59 | 0.63 | United Kingdom | 2.31 | 0.44 | 0.58 |
| Israel | 6.35 | 2.08 | 3.74 | United States of America | 3.07 | 0.35 | 0.42 |
| Italy | 3.12 | 0.48 | 0.43 | Uruguay | 2.13 | 0.24 | 0.95 |
| Jamaica | 2.18 | 0.83 | 1.34 | Venezuela | 3.58 | 0.71 | 2.35 |
|  |  |  |  |  |  |  |  |

### 3.2 World Development Indicators 1988-2011

The World Bank offers simple web-based access to its WDI database via an applicationprogramming interface (API). In order to ensure the reliability of the empirical results presented inChapter 5 several WDI indicators are used to augment the model in Chapter 5. These variables are defense expenditure as a percent of GDP MS.MIL.XPND.GD.ZS, nominal interest rate FR.INR.RINR, total tax rate as a percentage of commercial profits IC.TAX.TOTL.CP.ZS, and tax revenue a percent of GDP. The WDI data span from 1960-2013 for more than 200 economies however the defense spending variable begins in 1988.

Figure 2 displays a scatterplot matrix of the primary WDI variables. Variables' names and their histograms occupy the diagonal plots while the lower triangular entries plot the observations against one another. The row variable occupies the $y$-axis and the column variable is on the $x$-axis. The upper triangular entries contain correlation coefficients for the variables along the diagonal.

## TABLE VI

WDI - List of countries and sample means(1988-2011)

| Country | Growth | Defense Shock | $\sigma_{d}$ | Country | Growth | Defense Shock | $\sigma_{d}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Albania | 5.77\% | 1.05\% | 2.23\% | Kuwait | 4.48\% | -0.21\% | 6.25\% |
| Angola | 9.69 | 1.75 | 11.86 | Laos | 6.91 | 0.86 | 1.34 |
| Argentina | 3.82 | 0.32 | 1.83 | Lebanon | 3.99 | 0.10 | 2.73 |
| Australia | 3.36 | 0.51 | 0.30 | Lesotho | 3.72 | 0.88 | 1.43 |
| Austria | 2.15 | 0.31 | 0.54 | Luxembourg | 3.62 | 0.64 | 0.71 |
| Bahrain | 4.94 | -0.14 | 1.71 | Madagascar | 2.95 | 0.16 | 1.51 |
| Bangladesh | 5.64 | -0.07 | 0.76 | Malaysia | 5.06 | 0.63 | 0.50 |
| Belgium | 1.92 | 0.47 | 0.80 | Mali | 4.97 | 0.52 | 2.85 |
| Bolivia | 3.86 | 0.34 | 0.74 | Malta | 2.81 | 0.99 | 1.29 |
| Botswana | 5.62 | 0.29 | 1.84 | Mauritius | 4.53 | 0.41 | 0.48 |
| Brazil | 3.15 | 0.47 | 2.30 | Mexico | 2.56 | 0.48 | 0.56 |
| Brunei | 1.61 | 0.51 | 3.98 | Mongolia | 6.07 | 1.86 | 5.63 |
| Bulgaria | 2.53 | 0.04 | 4.17 | Morocco | 3.93 | 0.12 | 0.50 |
| Burkina Faso | 6.03 | 1.04 | 1.21 | Nepal | 4.21 | 0.54 | 1.13 |
| Cameroon | 3.81 | 0.49 | 0.70 | Netherlands | 2.24 | 0.72 | 0.93 |
| Canada | 2.64 | 0.49 | 0.40 | New Zealand | 2.56 | 0.70 | 0.56 |
| Cape Verde | 6.78 | 1.88 | 2.89 | Nigeria | 6.66 | 1.08 | 2.34 |
| Chile | 4.68 | -0.14 | 0.67 | Norway | 2.31 | 0.14 | 1.34 |
| China | 9.89 | 1.80 | 1.29 | Oman | 4.46 | -0.40 | 2.10 |
| Colombia | 3.34 | 0.59 | 0.48 | Pakistan | 4.11 | 0.56 | 2.15 |
| Cyprus | 3.40 | 0.78 | 1.56 | Paraguay | 2.90 | 0.14 | 0.53 |
| Denmark | 1.42 | 0.51 | 0.92 | Peru | 5.01 | -0.16 | 0.81 |
| Dominican Republic | 5.77 | 0.81 | 0.88 | Philippines | 4.36 | -0.29 | 0.85 |
| Ecuador | 3.58 | -0.45 | 0.99 | Poland | 4.56 | 0.82 | 0.55 |
| Egypt | 4.81 | 0.77 | 0.80 | Portugal | 1.79 | 0.59 | 0.77 |
| El Salvador | 2.48 | 5.13 | 11.29 | Romania | 2.88 | 0.53 | 4.39 |
| Ethiopia | 7.57 | 0.91 | 2.61 | Rwanda | 10.19 | 3.56 | 5.54 |
| Fiji | 2.02 | 0.52 | 1.63 | Saudi Arabia | 3.17 | 0.48 | 1.40 |
| Finland | 2.87 | 0.68 | 0.74 | Singapore | 5.80 | 1.06 | 1.54 |
| France | 1.68 | 0.42 | 0.62 | South Africa | 3.27 | 0.37 | 0.35 |
| Germany | 1.42 | 0.37 | 0.33 | Spain | 2.61 | 0.65 | 0.52 |
| Ghana | 5.77 | 0.35 | 1.38 | Sri Lanka | 5.37 | 0.08 | 1.84 |
| Greece | 2.02 | 0.51 | 1.15 | Swaziland | 2.55 | -0.07 | 4.45 |
| Guatemala | 3.66 | 1.05 | 0.67 | Sweden | 2.79 | 0.61 | 0.91 |
| Hungary | 2.25 | 0.51 | 0.91 | Switzerland | 1.77 | 0.17 | 0.36 |
| India | 7.01 | 1.40 | 0.87 | Tanzania | 5.95 | 1.04 | 2.82 |
| Iran | 4.66 | 0.83 | 1.36 | Thailand | 3.43 | 0.66 | 0.59 |
| Ireland | 5.29 | 0.49 | 0.58 | Tunisia | 4.33 | 0.25 | 0.41 |
| Israel | 4.03 | 0.88 | 0.77 | Turkey | 4.52 | 1.04 | 0.92 |
| Italy | 0.99 | 0.28 | 0.70 | Uganda | 7.04 | 1.82 | 1.15 |
| Jamaica | 0.54 | 1.57 | 2.68 | United Kingdom | 2.28 | 0.59 | 0.55 |
| Japan | 0.78 | 0.45 | 0.25 | United States of America | 2.45 | 0.32 | 0.23 |
| Jordan | 5.18 | -0.24 | 2.24 | Uruguay | 2.72 | -0.38 | 0.91 |
| Kenya | 3.62 | 0.41 | 0.66 | Venezuela | 2.75 | -0.78 | 1.64 |
| Korea, Republic of | 4.77 | 0.74 | 0.38 |  |  |  |  |



Figure 2. WDI - Scatterplot Matrix

TABLE VII

| NATO - list of countries and sample means (1956-2011) |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Country | Growth | Defense <br> Shock | $\sigma_{d}$ | Country | Growth |  |  | | Defense |
| :--- |
| Shock |$\sigma_{d}$.

### 3.3 North Atlantic Treaty Organization 1956-2013

The North Atlantic Treaty Organization compiles data on the defense spending of select countries. The largest possible balanced panel from this data sets is established by selecting the 14 countries whose data spans from 1956 to 2011.

Figure 3 displays a scatterplot matrix of the primary NATO variables. Variables' names and histograms occupy the diagonal plots while the lower triangular entries plot the observations against one another. The row variable occupies the $y$-axis and the column variable is on the $x-a x i s$. The upper triangular entries contain correlation coefficients for the variables along the diagonal.


Figure 3. NATO - Scatterplot Matrix

### 3.4 Barro and Redlick's United States Data 1912-2006

Barro and Redlick (2011) compiled annual US data spanning from 1912 to 2012. The data allow for comparison of the cross-country results in the first part of Chapter 5 with an in-depth US analysis. Barro's average marginal tax rate variable is superior to the crude measures of the tax rate that are available from other data sources such as the WDI and OECD. Unfortunately, no similar data are readily available for the panel of countries considered in this paper Figure 3 displays several two-dimensional (2D) scatterplots of the US data. Variable names and associated histograms occupy the diagonal. The lower triangular entries plot the observations with the row variable on the $y$-axis and the column variable on the $x$-axis. Therefore the growth variable, $y$, is the $y$-axis varaible along the bottom row. The lower trangular plots also contain locally smoothed regression (LOESS) lines. The upper triangular entries contain correlation coefficients for the variables along the diagonal.

## US Scatterplot Matrix



Figure 4. US - Scatterplot Matrix

## CHAPTER 4

## IDENTIFICATION METHODOLOGY

### 4.1 The Baseline Empirical Model

Following the simple empirical specification found in Hall (2009) and Karras (2011) first consider:

$$
\begin{equation*}
\frac{Y_{i, t}-Y_{i, t-1}}{Y_{i, t-1}}=v_{i}+w_{t}+m \frac{G_{i, t}-G_{i, t-1}}{Y_{i, t-1}}+\epsilon_{i, t} \tag{4.1}
\end{equation*}
$$

Where

- $Y_{i, t}$ is per capita real GDP for country $i$ and year $t$,
- $G_{i, t}$ is per capita real government purchases for country i and year t ,
- $m$ is the multiplier to be estimated,
- $v_{i}$ represents country specific effects,
- $w_{t}$ represents time specific effects,
- and $\epsilon$ is an error term.

The shock in government purchases denoted by $\frac{G_{i, t}-G_{i, t-1}}{Y_{i, t-1}}$ in Equation 4.1 uses Y for its denominator. This implies that $m$ measures the percentage change in Y due to a fiscal shock that is $1 \%$ of Y in size. Thus $m$ has the standard interpretation as the fiscal multiplier (Barro and Redlick, 2011; Hall, 2009; Karras, 2011).

In order to simplify the notation of more involved specifications, lower-case letters are used to represent output growth and the fiscal shock in terms of $Y$. Define $y_{i, t}=\frac{Y_{i, t}-Y_{i, t-1}}{Y_{i, t-1}}$ and $g_{i, t}=\frac{G_{i, t}-G_{i, t-1}}{Y_{i, t-1}}$, then the equation becomes:

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+m g_{i, t}+\epsilon_{i, t} \tag{4.2}
\end{equation*}
$$

In order to estimate a multiplier that is contingent on the level of another variable consider:

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+m g_{i, t}+m^{x} g_{i, t} * x_{i, t}+\epsilon_{i, t} \tag{4.3}
\end{equation*}
$$

Now the multiplier is contingent on the level of $x$ for country i at time t and the multiplier is now given by $m_{i, t}=m+m^{x} * x_{i, t}$. For instance consider $x_{i, t}=\sigma_{g_{i, t}}^{k}$, where

$$
\sigma_{g_{i, t}}^{k}=\sqrt{\frac{1}{(k-1)} \sum_{n=1}^{k}\left(g_{i, t-n}-\frac{\sum_{l=1}^{k} g_{i, t-l}}{k}\right)^{2}}
$$

Thus, $\sigma_{g_{i, t}}^{k}$ measures the volatility of fiscal shocks during the preceding $k$ years. Note that $\sigma_{g_{i, t}}^{k}$ does not contain any information from year t . And the central static model becomes

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+m g_{i, t}+m^{\sigma} g_{i, t} * \sigma_{g_{i, t}}^{k}+\epsilon_{i, t} . \tag{4.4}
\end{equation*}
$$

This implies that the fiscal multiplier for country $i$ at time $t$ conditional on the level of volatility in fiscal shocks over the past $k$ years is $m+m^{\sigma} g_{i, t} * \sigma_{i, t}$.

### 4.2 The Dynamic Empirical Model

In order to capture dynamic effects of past fiscal shocks, it is necessary to consider a dynamic specification. Let

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J} m_{j} g_{i, t-j}+\epsilon_{i, t} \tag{4.5}
\end{equation*}
$$

Conditional on the variable $\sigma_{g_{i, t-j}}^{k}$ this equation used for estimating the fiscal multiplier becomes

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} g_{i, t-j}+m_{j}^{\sigma} g_{i, t-j} \sigma_{g_{i, t-j}}^{k}\right)+\epsilon_{i, t} \tag{4.6}
\end{equation*}
$$

where J is the number of lags, $\alpha_{j}, m_{j}$, and $m_{j}^{\sigma}$ are the new coefficients to be estimated. Formally this implies a fiscal multiplier of

$$
\begin{equation*}
\frac{\sum_{j=1}^{J}\left(m_{j}+m_{j}^{\sigma} * \sigma_{g_{i, t}}^{k}\right)}{1-\sum_{j=1}^{J} \alpha_{j}} \tag{4.7}
\end{equation*}
$$

### 4.3 Models for Addressing Endogeneity

As discussed briefly in Chapter 2, ensuring unbiased results requires that the right hand side (RHS) variables are independent of the error term. The problem is rooted in the possible endogeneity of total government purchases to output. Tax coffers grow alongside the economy which might lead governments to expand spending in ways that they would not have in the absence of growth. This paper employs several approaches to combat this problem. The first
involves using a near-vector autoregression (NVAR) system of equations based on Karras (2011) to deal with possible bias from output to government purchases. NVARs, also referred to as the quasi-VAR models, are a special cases of the traditional VAR model. In the next approach defense purchases are used in lieu of government purchases since defense purchases are less likely to respond systematically to output growth and are thus exogenous. The third technique employed to combat endogenity relies on the fact that defense spending is often correlated with overall government purchases. Therefore it may be used as an instrumental variable (IV) for government purchases. IV techniques allow consistent estimation when variables in the regression are correlated with the error. IV techniques will be implemented for all datasets that include defense purchases data.

### 4.3.1 The Near-Vector Autoregressive Model

The near-vector autoregressive model (NVAR) is a tool used to mitigate the problem of endogeneity. It imposes linear restrictions which allow for different numbers of regressors in each equation. For present purposes let

$$
\begin{equation*}
g_{i, t}=a_{i}+b_{t}+\sum_{j=1}^{J} \phi_{j} y_{i, t-j}+\sum_{j=1}^{J} \delta_{j} g_{i, t-j}+u_{i, t}^{g} . \tag{4.8}
\end{equation*}
$$

Equation 4.8 can be used to predict fiscal shocks as a function of past $y$. Next, using the prediction errors from this step, we estimate

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} \widehat{u_{i, t-j}}+m_{j}^{\sigma} \widehat{u_{i, t-j}} \sigma_{\widehat{u_{i, t-j}}}{ }^{k}\right)+\epsilon_{i, t} \tag{4.9}
\end{equation*}
$$

where $a_{i}$ and $b_{i}$ are vectors of time and country level effects and $\phi_{j}$ and $\delta_{j}$ are estimated parameters used to calculate the $\widehat{u}_{i, t}^{g}$. Thus $\widehat{u}_{i, t}^{g}$ can be interpreted as exogenous fiscal shocks, so that the NVAR specification (Equation 4.8, Equation 4.9) can be viewed as free from endogenous fiscal bias.

### 4.3.2 The Defense Spending Model

The next approach to addressing the problem of endogeneity uses defense spending data in place of government purchases. Equation 4.10- Equation 4.13 are identical to those in Section 3.2 except that the fiscal shock is by an adjusted defense shock variable defined $d_{i, t}=\frac{D_{i, t}-D_{i, t-1}}{Y_{i, t-1}}$. Where $D_{i, t}$ is defense spending by country $i$ in year $t$. Using $d_{i, t}$ the four models, this translates into

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+m d_{i, t}+\epsilon_{i, t} \tag{4.10}
\end{equation*}
$$

for the static model and conditional on volatility

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+m d_{i, t}+m^{\sigma} d_{i, t} * \sigma_{d_{i, t}}^{k}+\epsilon_{i, t} . \tag{4.11}
\end{equation*}
$$

In terms of the dynamic model we have

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J} m_{j} d_{i, t-j}+\epsilon_{i, t} \tag{4.12}
\end{equation*}
$$

and conditional on the variable $\sigma_{d_{i, t-j}}^{k}$ this becomes

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} d_{i, t-j}+m_{j}^{\sigma} d_{i, t-j} \sigma_{d_{i, t-j}}^{k}\right)+\epsilon_{i, t} \tag{4.13}
\end{equation*}
$$

The empirical literature often makes the case that estimates based on defense spending do a better job in combatting possible endogeneity bias than do estimates based on total government purchases.

### 4.3.3 The Instrumental Variable Model

The final technique employed to combat endogenity bias relies on the fact that defense spending is often correlated with overall government purchases. Therefore it may be used as an instrumental variable (IV) for government purchases. Instrumental variable techniques allow consistent estimation when variables in the regression are correlated with the error. IV techniques will be implemented for all datasets that include defense purchases data.

The standard two-stage least-squares (2SLS) IV technique is used. In the first stage each potentially endogenous variables, $g$ and $\sigma_{g}$, are regressed on the lagged growth terms and the defense spending terms. The predicted variables are then replaced in the dynamic model with and without interactions.

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} \widehat{g_{i, t-j}}\right)+\epsilon_{i, t} \tag{4.14}
\end{equation*}
$$

and

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} \widehat{g_{i, t-j}}\right)+\sum_{j=0}^{J}\left(m_{j}^{\sigma} g_{i, t-j \sigma_{g_{i, t-j}}}\right)+\epsilon_{i, t} \tag{4.15}
\end{equation*}
$$

### 4.4 Lag and Volatility Window Selection

This section considers an array of information criteria for determining the optimal volatility window $k$ and maximum lag $J$. Although lag order is sometimes chosen as a function of researchers' hypotheses or to ensure significant results for specific factors, the selections of $J$ and $k$ in this paper are based on the variety of results suggested by the information criteria proposed here. ${ }^{1}$

### 4.4.1 Lag and Volatility Window Selection Techniques

The eight selection criteria considered are the Akaike information criterion (AIC), the Bayesian information criterion (BIC), the Hannan and Quinn criterion (HQC), the modified Akaike information criterion (MAIC), the modified Bayesian information criterion (MBIC), likelihood ratio test (LR), the Sims likelihood ratio test (SLR), and the adjusted $R^{2}$ Where $N=C *(\operatorname{range}(t))$ is the number of observations, $J$ is the maximum number of lags, $S S T_{J}$ is

[^7]
## TABLE VIII

## Lag Selection Criteria

| AdjustedR $^{2}=\frac{S S R_{J}}{N-J K-K} / \frac{S S T_{J}}{N-1}$ | $L R=N\left(\ln \left(\frac{S S R_{J}}{N}\right)-\ln \left(\frac{S S R_{J-1}}{N}\right)\right)$ |
| :--- | :--- |
| $A I C(J)=N \ln \left(\frac{S S R_{J}}{N}\right)+2 J$ | $M A I C(J)=\ln \left(\frac{S S R_{J}}{N}\right)+\frac{2\left(\beta_{0}^{2} \sum_{J+1}^{N} \widehat{\vartheta}_{t-1}^{2}\right)}{S S R_{J}}+\frac{2 J}{N-J}$ |
| $B I C(J)=N \ln \left(\frac{S S R_{J}}{N}\right)+J \ln (N)$ | $M B I C(J)=\ln \left(\frac{S S R_{J}}{N}\right)+\frac{\ln (N-J)\left(\beta_{0}^{2} \sum_{J+1}^{N} \hat{y}_{t-1}^{2}\right)}{S S R_{J}}+\frac{2 J}{N-J}$ |
| $H Q C(J)=N \ln \left(\frac{S S R_{J}}{N}\right)+2 J \ln (\ln N)$ | $S L R=(N-J * K)\left(\ln \left(\frac{S S R_{J}}{N}\right)-\ln \left(\frac{S S R_{J-1}}{N}\right)\right)$ |

the total sum of the squares for a model with $J$ lags, $S S R_{J}$ is the sum of the squared residuals, and $K$ is the number of parameters. ${ }^{12}$

The HQC is an often cited criteria similar to the AIC and BIC. Like the BIC, but not the AIC, it is not asymptotically efficient though it often leads to results in between each. Ng and Perron (1995) show that traditional information based rules such as AIC, HQC, and the BIC criterion systematically select values of J that are smaller than those chosen with other methods such as sequential F-testing, Hall (1994)' general to specific (GS) method, or Sims (1980) modified likelihood ratio test. Ng and Perron (2001) recommend a class of modified information criteria (MIC) where the penalty factors are sample dependent. This is achieved

[^8]by acknowledging that the bias in the summed autoregressive coefficients is dependent on $J$ and sensitive to the deterministic variables.

The SLR is a modification of the traditional LR test recommended by Sims (1980). LR tests for lag length use the mean sum of squared errors as a quasi-maximum likelihood estimate to generate a statistic which is asymptotically $\chi^{2}$ distributed. A large general model is compared to an identical model less one lag. If a small difference in fit between the models is detected, then progressively smaller models are tested until either a large difference is detected or the number of lags reaches 0 . Differences are measured by the p -value of the computed $\chi^{2}$ statistic. When the p-value falls below 0.05 , the fit of the larger model is deemed worthy of its added complexity and the search is ended. The SLR statistic is systematically smaller than the LR statistic. Sims acknowledges that this difference biases the SLR in favor of the null hypothesis and thus more parsimonious models. The next section applies both methodologies to the PWT dataset and finds that a lag of 2 is optimal for both the LR and the SLR.

### 4.4.2 Lag and Volatility Window Selection Results

Models including interaction variables require the selection of a volatility window $k .{ }^{1} \mathrm{Be}-$ fore considering $k$ however, the optimal lag structure for the $y$ time-series, Equation 4.13, is estimated.

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J} m_{j} g_{i, t-j}+\epsilon_{i, t} . \tag{4.16}
\end{equation*}
$$

[^9]

Figure 5. Lag Selection Information Criterion

Applications of the AIC, BIC, and their modified versions to the largest possible panel are displayed in Figure 5.

The criteria in Figure 5 do not agree on an optimal lag length. While the traditional AIC and BIC suggest $J=1$, the MAIC suggests $J=6$, and MBIC, $J=4$. These varying results lead to the following methodology. Each criterion is considered for complete models at a variety of lag lengths, $J$, and volatility windows, $k$. 3D surfaces built from the lag length, $k$-volatility
window size, and the criteria statistics are plotted on the $x, y$, and $z$ axes, respectively in

## Figure 6.

Optimums are indicated by minimum values in panels (a) (e), maximum values in panel $(\mathrm{f})$, and values less than 0.05 in (g) and (h). The results in panels (b), (e), (g), and (h) each indicate a maximal lag of 2 while the others indicate somewhat higher lags dependent on the chosen volatility window, K. Models with $\mathrm{K}>6$ demonstrate markedly worse results for all criteria with global optima in every panel at $\mathrm{K}=6$. Taken together, the results strongly suggest the use of 2 lags and a volatility window of 6 years.

In Chapter 5 , the estimating equations, optimal lag structure, and optimal volatility window established in this chapter are applied across the datasets described in Chapter 3.


Figure 6. Lag and Volatility Window Selection

## CHAPTER 5

## EMPIRICAL RESULTS

Chapter 5 applies the techniques established in Chapter 4 to the PWT and US datasets described in Chapter 3. Section 5.1 carefully walks through the results and interpretation of the PWT estimations. Random effects (RE) and fixed effects (FE) models are estimated for each specification. Subsequent sections of Chapter 5 incorporate additional insights into the estimation of the fiscal multiplier discussed in Chapter 2 before Chapter 6 explores tests of robustness.

### 5.1 Penn World Tables 1952-2011

The PWT panel spans 62 countries and 60 years (from 1950 to 2011). ${ }^{1}$. The data are used to make estimates of the traditional fiscal multiplier and fiscal multipliers contingent on fiscal volatility. Both static and dynamic models are considered. The possibility of the endogeneity of government purchases is considered in full in Chapter 6.

### 5.1.1 Penn World Tables - Baseline Empirical Results

Table IX displays the estimates of Equation 4.2 and Equation 4.4 for the PWT data. Recall that the coefficient on the fiscal shock, $m$, can be interpreted as the fiscal spending multiplier in these equations.

$$
y_{i, t}=v_{i}+w_{t}+m g_{i, t}+m^{\sigma} g_{i, t} * \sigma_{i, t}^{k}+\epsilon_{i, t}
$$

[^10]
## TABLE IX

|  | Dependent variable: Output Growth |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Equation 4.2 |  |  | Model (4) |  |  |
|  | OLS | FE | RE | OLS | FE | RE |
| Fiscal Shock | $\begin{aligned} & 0.346^{* * *} \\ & (0.026) \end{aligned}$ | $\begin{aligned} & 0.268^{* * *} \\ & (0.025) \end{aligned}$ | $\begin{aligned} & 0.317^{* * *} \\ & (0.025) \end{aligned}$ | $\begin{aligned} & 0.665^{* * *} \\ & (0.038) \end{aligned}$ | $\begin{aligned} & 0.544^{* * *} \\ & (0.036) \end{aligned}$ | $\begin{aligned} & 0.607^{* * *} \\ & (0.037) \end{aligned}$ |
| Fiscal Shock <br> * Volatility |  |  |  | $\begin{gathered} -2.474^{* * *} \\ (0.216) \end{gathered}$ | $\begin{gathered} -2.090^{* * *} \\ (0.202) \end{gathered}$ | $\begin{gathered} -2.233^{* *} \\ (0.210) \end{gathered}$ |
| Observations | 3,472 | 3,472 | 3,472 | 3,472 | 3,472 | 3,472 |
| Adjusted $\mathrm{R}^{2}$ | 0.048 | 0.033 | 0.043 | 0.083 | 0.062 | 0.073 |
| F Statistic | $174.776^{* * *}$ | 119.059*** | 155.471*** | $156.253^{* * *}$ | $114.703^{* *}$ | 137.132*** |
| Note: |  |  |  |  | ${ }^{*} \mathrm{p}<0.1$; ${ }^{* *} \mathrm{p}$ | .5; ${ }^{* * *} \mathrm{p}<0.01$ |

$m$ measures the response of output to a one unit increase in government purchases.
Given $m^{\text {sigma }}$ is restricted to 0 , the estimated multipliers range from 0.268 to 0.346 depending on the treatment of the time and country specific effects. The estimates are significantly different from both 0 and 1 . This suggests that output rises in response to fiscal stimulus but by less than the amount of the increase in government purchases. Therefore other components of GDP must fall along with the increase in government spending. The full equation is used to test whether the multiplier is a function of each economy's recent volatility in fiscal spending. The fiscal multiplier is now $m_{i, t}=m+m^{\sigma} * \sigma_{i, t}$. This multiplier is a function of $\sigma_{i, t}$ and is thus country and time specific.

The last three columns of Table IX model the fiscal multiplier as dependent on the volatility of fiscal shocks. The coefficients on the volatility interaction terms for the PWT model are negative, large, and significant. Thus, these findings support the hypothesis that volatility in fiscal shocks reduces the fiscal multiplier. The multiplier for a hypothetical country with zero volatility is simply the coefficient on the government purchases shock, 0.544 .

In order to illustrate the impact of volatility on multipliers in the baseline static model, consider the range of multipliers implied for the countries with the minimum and maximum mean volatilities over time. These contemporaneous multipliers range from 0.538 for Switzerland (volatility $=0.00265$ ) to 0.456 for El Salvador (volatility $=0.042$ ).

The top panel of Figure 7 displays the relationship between fiscal multipliers and fiscal volatility. According to all three specifications, fiscal volatility diminishes the fiscal multiplier. The middle and bottom panels of Figure 7 display the data split into halves and quarters respectively. Each plot displays a progression of corresponding regressions lines with confidence bands. The slopes of the estimated lines are equivalent to the fiscal multiplier and diminish with fiscal volatility. ${ }^{1}$

[^11]

Figure 7. PWT - Interaction Effect Visualizations

### 5.1.2 Penn World Tables - Dynamic Empirical Results

While the baseline static model provides a useful benchmark for comparison, the dynamic model with 2 lags and a volatility window of $\mathrm{k}=6$ years offers a more realistic perspective. ${ }^{1}$ Chapter 3 establishes the method for extending the baseline model to a dynamic model as well as the method for selecting the optimal lag and volatility window.

$$
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} g_{i, t-j}+m_{j}^{\sigma} g_{i, t-j} \sigma_{g_{i, t-j}}\right)+\epsilon_{i, t}
$$

Table X reports the parameter estimates for the PWT dynamic model. The left columns show the dynamic effect of government purchases on output with $m_{j}^{\sigma}$ constrained to 0 . The fixed effects model parameters imply a long-run multiplier of 0.49. ${ }^{2}$

The third and fourth columns of Table X allow the fiscal multiplier to depend on the volatility of fiscal shocks. Recall that the long-run multiplier is given by

$$
\frac{\sum_{j=1}^{J}\left(m_{j}+m_{j}^{\sigma} * \sigma_{g_{i, t}}^{k}\right)}{1-\sum_{j=1}^{J} \alpha_{j}}
$$

[^12]
## TABLE X

PWT - Dynamic Empirical Results

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Equation 4.5 |  | Equation 4.6 |  |
|  | FE | RE | FE | RE |
| Output Growth (lag 1) | $\begin{aligned} & 0.207^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.248^{* * *} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.206 * * * \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.242^{* * *} \\ & (0.017) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{aligned} & 0.048^{* * *} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.055^{* * *} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.051^{* * *} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.054^{* * *} \\ & (0.017) \end{aligned}$ |
| Fiscal Shock | $\begin{aligned} & 0.254^{* * *} \\ & (0.025) \end{aligned}$ | $\begin{aligned} & 0.287^{* * *} \\ & (0.025) \end{aligned}$ | $\begin{aligned} & 0.666^{* * *} \\ & (0.042) \end{aligned}$ | $\begin{aligned} & 0.697^{* * *} \\ & (0.043) \end{aligned}$ |
| Fiscal Shock (lag 1) | $\begin{gathered} 0.035 \\ (0.025) \end{gathered}$ | $\begin{gathered} 0.029 \\ (0.026) \end{gathered}$ | $\begin{gathered} -0.008 \\ (0.049) \end{gathered}$ | $\begin{gathered} 0.009 \\ (0.050) \end{gathered}$ |
| Fiscal Shock (lag 2) | $\begin{aligned} & 0.076^{* * *} \\ & (0.025) \end{aligned}$ | $\begin{aligned} & 0.089^{* * *} \\ & (0.026) \end{aligned}$ | $\begin{aligned} & 0.118^{* * *} \\ & (0.045) \end{aligned}$ | $\begin{aligned} & 0.134^{* * *} \\ & (0.046) \end{aligned}$ |
| Fiscal Shock *Volatility |  |  | $-4.197^{* * *}$ | $-4.294^{* *}$ |
|  |  |  | (0.376) | (0.387) |
| Fiscal Shock <br> *Volatility (lag 1) |  |  | $-1.841^{* * *}$ | $-2.136^{* * *}$ |
|  |  |  | (0.495) | (0.507) |
| Fiscal Shock <br> *Volatility (lag 2) |  |  | $-3.748^{* * *}$ | $-3.983^{* * *}$ |
|  |  |  | (0.546) | (0.558) |
| Observations | 3,348 | 3,348 | 3,348 | 3,348 |
| Adjusted $\mathrm{R}^{2}$ | 0.088 | 0.124 | 0.125 | 0.158 |
| F Statistic | $64.783^{* * *}$ | 94.973*** | 60.101*** | $78.840^{* * *}$ |
| Note: |  |  | *p<0.1; ${ }^{* *} \mathrm{p}$ | 5; *** $<0.01$ |

The multiplier varies by country and over time since it is based on the recent fiscal volatility in each country and via past growth rates. Apart from the coefficient on the first lag of the fiscal shock variable, all of the parameters are highly significant and the overall model fit is much better than that of the baseline model. The parameters on the volatility interaction terms are negative, large, and significant. These findings therefore support the hypothesis that recent volatility in government purchases reduces the fiscal multiplier. Based on Equation 4.7, the multiplier for a hypothetical country with zero volatility is 1.05 .

Figure 8 displays the inverse relationship between fiscal volatility and fiscal multipliers according to the dynamic model. Fiscal volatility diminishes the fiscal multiplier in all three specifications.

Figure 9 displays two plots of the PWT data set. The scattered points are color coded so that observations with volatility ranging high to low and red to blue. The left panel contains the plane generated by estimating a modified Equation 4.4, while the right panel illustrates the response surface with interactions estimated by Equation 4.6.

The Hausman test is used to determine whether the fixed effects (FE) specification or the random effects (RE) specification is preferred. The statistic

$$
H=\left(\widehat{\beta}_{F E}-\widehat{\beta_{R E}}\right)^{\prime}\left(V_{F E}-V_{R E}\right)^{-1}\left(\widehat{\beta_{F E}}-\widehat{\beta_{R E}}\right)
$$



Figure 8. PWT - Dynamic Model Interaction Effects Visualizations


Figure 9. PWT - Data and the Fitted Dynamic Model
is aymptotically $\chi^{2}$ distributed. Its value of 70.3 implies p -value of $4.35 \mathrm{e}-12$ and thus the null hypothesis of the RE model is rejected. The remaining tables and figures based on Table X consider only the FE estimates.

Table XI lists implied long-run multipliers for a number of countries in order to highlight the impact of volatility on the multiplier. A five-number summary of the mean volatilities across time for each country is used to calculate typical multipliers for the representative countries. Multipliers range from 1.01 for Switzerland (volatility $=0.00265$ ) to 0.493 for El Salvador $($ volatility $=0.042)$.

## TABLE XI

PWT - Long-run Multipliers across Mean Volatilities

|  | Zero Vol. | 0th \%-ile | 25th \%-ile | 50th $\%$-ile | 75th \%-ile | 100th \%-ile |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Country | NA | Switzerland | Sweden | India | Brazil | El Salvador |
| Mean Volatility | 0 | 0.00265 | 0.00549 | 0.00813 | 0.0132 | 0.042 |
| Multiplier | 1.05 | 1.01 | 0.974 | 0.939 | 0.872 | 0.493 |



Figure 10. PWT - Impulse Responses to a $1 \%$ Fiscal Shock

### 5.1.3 Penn World Tables - Impulse Response Functions and Historical Estimates

Figure 10 tracks the impulse responses of output to $1 \%$ fiscal shocks for the fixed effects PWT dynamic model. Impulse responses demonstrate the evolution of variables in dynamic systems over time. The volatilities used to generate the responses are based on a five-number summary of the mean volatilities for each country across time. Notice that long-run multipliers are significantly larger for countries with lower fiscal volatility. Whether or not the multiplier is $>1$ is unclear. In all of the figures, government purchases volatility clearly dampens the effect of fiscal policy as predicted by the theory.

Panel (a) of Figure 11 illustrates the long-run multipliers implied by the PWT dynamic model contingent on historical data for the largest, median, and smallest economies in the PWT panel. The average multiplier across the countries in panel (a) is 0.954 and the minimum


Figure 11. PWT - Implied Historical Multipliers for Representative Economies
and maximum multipliers are 0.774 and 1.03 respectively. Panel (b) illustrates the long-run multipliers implied by the PWT dynamic model contingent on historical data for the poorest, median, and wealthiest economies in terms of per capita income. The average multiplier across the countries in panel (b) is 0.891 and the minimum and maximum multipliers are 0.55 and 1.04 respectively.

### 5.2 United States 1912-2006

### 5.2.1 United States - Baseline Static Model Empirical Results

The US data spans 94 years (from 1912 to 2006). Table XII reports the parameter estimates for the baseline static model.

$$
y_{i, t}=v_{i}+w_{t}+m g_{i, t}+m^{\sigma} g_{i, t} * \sigma_{g_{i, t}}^{k}+\epsilon_{i, t}
$$

The left column of Table XII shows the contemporaneous effect of government purchases on output, with $m^{\sigma}=0$. The fixed effects estimation suggests a multiplier of 0.56 . The right column of Table XII models the fiscal multiplier as dependent on the volatility of fiscal shocks. The interaction is insignificant in the baseline model.

TABLE XII

US - Baseline Model Empirical Results

|  | Dependent variable: Output Growth |  |
| :--- | :---: | :---: |
|  | Equation 4.2 | Equation 4.4 |
| Fiscal Shock | $0.560^{* * *}$ | $0.567^{* * *}$ |
| Fiscal Shock | $(0.089)$ | $(0.090)$ |
| * Volatility |  | 0.080 |
|  |  | $(0.106)$ |
| Observations | 89 | 89 |
| Adjusted R |  |  |
| Residual Std. Error | 0.302 | 0.299 |
| F Statistic | 0.041 | 0.041 |
| Note: | $39.094^{* * *}$ | $19.736^{* * *}$ |
| ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |  |  |

### 5.2.2 United States - Dynamic Model Empirical Results

While the baseline static model provides a useful benchmark for comparison, the dynamic model with 2 lags and a volatility window of $\mathrm{k}=6$ years offers a more realistic perspective. Table XIII reports the parameter estimates the US dynamic model.

$$
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} g_{i, t-j}+m_{j}^{\sigma} g_{i, t-j} \sigma_{g_{i, t-j}}^{k}\right)+\epsilon_{i, t}
$$

The left column measures the dynamic effect of government purchases on output with the $m_{j}^{\sigma}$ constrained to 0 . The model parameters imply a long-run multiplier of 0.537 .

The right column of Table XIII allows the fiscal multiplier to depend on the volatility of fiscal shocks. The parameters on the volatility interaction terms are negative, large, and significant on net. These findings therefore support the hypothesis that recent volatility in government spending reduces the fiscal multiplier. Based on Equation 4.7, the long-run multiplier for a hypothetical country with zero volatility is 1.06 . Table XIV highlights the impact of volatility on the multiplier. It lists the implied multipliers for a five-number summary of all the volatilities recorded in the US. The implied long-run multipliers range from 1.05 for 1997 (volatility $=$ 0.0015 ) to 0.369 in 1952 (volatility $=0.111$ ).

## TABLE XIII

US - Dynamic Empirical Results

|  | Dependent variable: Output Growth |  |
| :--- | :---: | :---: |
|  | Equation 4.10 | Equation 4.11 |
|  | FE | FE |
| Output Growth (lag 1) | $0.333^{* * *}$ | $0.322^{* * *}$ |
|  | $(0.114)$ | $(0.115)$ |
| Output Growth (lag 2) | -0.112 | -0.143 |
|  | $(0.112)$ | $(0.114)$ |
| Fiscal Shock | $0.458^{* * *}$ | $0.828^{* * *}$ |
|  | $(0.124)$ | $(0.247)$ |
| Fiscal Shock (lag 1) | -0.001 | 0.011 |
|  | $(0.133)$ | $(0.273)$ |
| Fiscal Shock (lag 2) | 0.0003 | 0.051 |
|  | $(0.114)$ | $(0.201)$ |
| Fiscal Shock * Volatility |  | $-4.950^{*}$ |
|  |  | $(2.773)$ |
| Fiscal Shock * Volatility (lag 1) |  | 0.810 |
|  |  | $(2.858)$ |
| Fiscal Shock * Volatility (lag 2) |  | -0.915 |
|  |  | $(2.279)$ |
| Observations | 87 | 87 |
| Adjusted R ${ }^{2}$ | ${ }^{2}$ | 0.346 |
| Residual Std. Error | 0.344 | 0.040 |
| F Statistic | 0.040 | $6.678^{* * *}$ |
| Note: | $10.018^{* * *}$ |  |

TABLE XIV

US - Long-run Multipliers across All Historical Volatilities

|  | Zero Vol. | 0 th $\%$-ile | 25 th $\%$-ile | 50th $\%$-ile | 75 th $\%$-ile | 100th $\%$-ile |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year |  | 1997 | 1994 | 1964 | 1926 | 1952 |
| Volatility | 0 | 0.0015 | 0.00278 | 0.00553 | 0.0257 | 0.111 |
| Multiplier | 1.06 | 1.05 | 1.04 | 1.03 | 0.9 | 0.369 |

### 5.2.3 United States - Impulse Response Functions and Historical Estimates

Impulse responses demonstrate the evolution of variables in dynamic systems over time. Figure 12 tracks the response of output to fiscal shock of $1 \%$ of GDP for the fixed effects estimations of the US dynamic model with 2 lags contingent on a 6 year volatility window.

Figure 13 tracks the long-run multipliers implied by the US dynamic model contingent on a 6 -year volatility window for the US. The average multiplier across time is 0.598 and the minimum and maximum multipliers are 0.319 and 1.05 respectively.

Figure 12. US - Fixed Effects Impulse Responses to a 1\% Fiscal Shock


Figure 13. US - Implied Historical Multipliers


### 5.3 Extensions

### 5.3.1 Ancillary Controls

This section seeks to estimate the model parameters with more precision in order to narrow the range of policy advice based on this research. Chapter 2 catalogs a broad list of fiscal multiplier determinants proposed in the literature. None of these factors apart from fiscal shocks have, so far, been included in this analysis. The existence of such factors, such as a country's level of openness, does not undermine the results since any omitted variables orthogonal to the right-hand side variables do not bias the estimated effects (Romer and Romer, 2010; Barro, 2011). The omission of these ostensibly orthogonal factors does however suggest that estimates for specific countries could be made more precise. And in the case of the PWT estimates, in whlich estimated multipliers for many countries are not significantly different from 1 , additional precision could result in very different policy implications.

This section expands on the volatility-contingent dynamic model by including additional variables such as slack in the economy, openness, tax rates, and tax revenue as a share of GDP. Additional contextual data can help to demonstrate the relative importance of considering recent fiscal volatility in crafting policy. Such expanded, policy-focused models, which estimate more precise multipliers for specific countries over time, can be used to judge whether past fiscal expansions were advisable and whether or not future expansions are warranted.

### 5.3.1.1 Slackness in the Economy

Several papers reach the conclusion that fiscal spending is more effective in recessions than during expansions. In order to test this hypothesis, consider the generic equation developed in Chapter 3:

$$
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} g_{i, t-j}+m_{j}^{x} g_{i, t-j} x_{g_{i, t-j}}^{k}\right)+\epsilon_{i, t}
$$

The variable $x$ is an interaction term that either magnifies or diminishes the multiplier. Next the model is augmented with an additional interaction term.

$$
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} g_{i, t-j}+m_{j}^{x} g_{i, t-j} x_{g_{i, t-j}}+m_{j}^{q} g_{i, t-j} q_{g_{i, t-j}}^{k}\right)+\epsilon_{i, t} .
$$

In this equation, $q$ is a second interaction term and $m^{q}$ has the same interpretation with respect to $q$ as $m^{x}$ does with $x$. In particular, Table XV sets $x$ to its familiar value as $\sigma_{g}$ and uses $q$ to measure the output or unemployment gap. The log-output gap, output growth gap, and unemployment gap are defined as their instantaneous values minus their trend values. For instance, an over heated economy has

- a positive log output gap,
- a positive output growth gap, and
- a negative unemployment gap.


Figure 14. US - Filtered series for the US

TABLE XV

Ancillary Controls: Measures of Slackness

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) |
| Output Growth (lag 1) | $\begin{aligned} & 0.352^{* * *} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.246^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.234^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.339^{* * *} \\ & (0.028) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{gathered} 0.021 \\ (0.028) \end{gathered}$ | $\begin{aligned} & 0.061^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.061^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{gathered} 0.015 \\ (0.028) \end{gathered}$ |
| Fiscal Shock | $\begin{gathered} 0.064^{* *} \\ (0.026) \end{gathered}$ | $\begin{aligned} & 0.191^{* * *} \\ & (0.024) \end{aligned}$ | $\begin{aligned} & 0.464^{* * *} \\ & (0.046) \end{aligned}$ | $\begin{aligned} & 0.331^{* * *} \\ & (0.061) \end{aligned}$ |
| Fiscal Shock (lag 1) | $\begin{gathered} -0.009 \\ (0.033) \end{gathered}$ | $\begin{gathered} -0.032 \\ (0.024) \end{gathered}$ | $\begin{gathered} 0.056 \\ (0.048) \end{gathered}$ | $\begin{gathered} 0.043 \\ (0.075) \end{gathered}$ |
| Fiscal Shock (lag 2) | $\begin{aligned} & 0.092^{* * *} \\ & (0.034) \end{aligned}$ | $\begin{aligned} & 0.067^{* * *} \\ & (0.025) \end{aligned}$ | $\begin{gathered} 0.080^{*} \\ (0.047) \end{gathered}$ | $\begin{gathered} 0.114^{*} \\ (0.068) \end{gathered}$ |
| Unemployment Gap | $\begin{gathered} -0.027 \\ (0.050) \end{gathered}$ |  |  | $\begin{gathered} -0.054 \\ (0.052) \end{gathered}$ |
| Unemployment Gap (lag 1) | $\begin{aligned} & 0.161^{* * *} \\ & (0.055) \end{aligned}$ |  |  | $\begin{aligned} & 0.130^{* *} \\ & (0.057) \end{aligned}$ |
| Unemployment Gap (lag 2) | $\begin{array}{r} -0.092^{*} \\ (0.049) \end{array}$ |  |  | $\begin{gathered} -0.081 \\ (0.058) \end{gathered}$ |
| Hodrick-Prescott Output Gap |  | $\begin{gathered} 6.834^{* * *} \\ (0.698) \end{gathered}$ | $\begin{aligned} & 6.581^{* * *} \\ & (0.714) \end{aligned}$ |  |
| Hodrick-Prescott Output Gap (lag 1) |  | $\begin{gathered} -8.296^{* * *} \\ (0.692) \end{gathered}$ | $\begin{gathered} -7.416^{* * *} \\ (0.717) \end{gathered}$ |  |
| Hodrick-Prescott Output Gap (lag 2) |  | $\begin{array}{r} -1.316^{*} \\ (0.705) \end{array}$ | $\begin{gathered} -0.811 \\ (0.745) \end{gathered}$ |  |
| Fiscal Shock*Volatility |  |  | $\begin{gathered} -2.899^{* * *} \\ (0.385) \end{gathered}$ | $\begin{gathered} -2.186^{* * *} \\ (0.462) \end{gathered}$ |
| Fiscal Shock*Volatility (lag 1) |  |  | $\begin{gathered} -2.293^{* * *} \\ (0.486) \end{gathered}$ | $\begin{gathered} -1.535^{* *} \\ (0.718) \end{gathered}$ |
| Fiscal Shock*Volatility (lag 2) |  |  | $\begin{gathered} -2.846^{* * *} \\ (0.546) \\ \hline \end{gathered}$ | $\begin{gathered} -2.173^{* * *} \\ (0.717) \\ \hline \end{gathered}$ |
| Observations | 1,357 | 3,050 | 3,050 | 1,357 |
| Adjusted R ${ }^{2}$ | 0.135 | 0.160 | 0.175 | 0.149 |
| F Statistic | $26.668^{* * *}$ | $73.184^{* * *}$ | 59.372 ${ }^{* * *}$ | $21.863^{* * *}$ |
| Note: |  |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}$ | $5 ;{ }^{* * *} \mathrm{p}<0.01$ |

According to the literature, these factors dampen the fiscal multiplier thus interactions on each variable are expected to produce

- negative coefficients on log output gap interacted with fiscal shock terms
- negative coefficients on output growth gap interacted with fiscal shock terms
- positive coefficients on unemployment gap interacted with fiscal shock terms

Actual and trend estimates for US log output, US output growth and US unemployment are plotted in Figure 14. Table XV employs the Hodrick-Prescott filter ${ }^{1}$ for calculating the trend values of output and unemployment. Summing the coefficients for each interaction variable and its associated lags determines the sign of the effect each factor has on the multiplier. Four models are considered. Slackness interactions are modeled using the output gap and, for comparison, unemployment. Next each of these equations is modeled along side the familiar fiscal volatility interaction.

$$
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} g_{i, t-j}+m_{j}^{\sigma} g_{i, t-j} \sigma_{g_{i, t-j}}+m_{j}^{g a p} g_{i, t-j} g a p_{g_{i, t-j}}^{k}\right)+\epsilon_{i, t}
$$

In all four unemployment gap equations, the results support the hypotheses that unemployment increases the multiplier, an output gap decreases the multiplier, and fiscal volatility diminishes the multiplier. The results, found in Table XV also seem to indicate that a Hodrick-Prescott (HP) filtered output gap results in a better fit than the HP unemployment gap.

[^13]
### 5.3.1.2 Openness

Chapter 2 discusses the influence of openness on the multiplier in the Keyensian theoretical model as well in the literature. Specifically, openness is defined as the sum of imports and exports in an economy. If the mechanism underlying the multiplier is the repeated use of money spent on government purchases, then as a larger share of income is spent on imports, the residual that is re-spent in domestically falls. Karras (2011, 2014) and Ilzetzki et al. (2013) find that that the fiscal multiplier is very sensitive to countries' openness. The results in this section consider the impact of openness by itself as well as modeled alongside volatility.

$$
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} g_{i, t-j}+m_{j}^{\sigma} g_{i, t-j} \sigma_{g_{i, t-j}}+m_{j}^{o p e n} g_{i, t-j} \text { open } g_{g_{i, t-j}}^{k}\right)+\epsilon_{i, t} .
$$

Indeed the empirical results in Section 5.3 support the openness hypothesis. The multipliers in this estimation are decidedly less than 1 and the sums of the respective interaction terms are each less than zero thus implying that openness reduces the multiplier.

### 5.3.1.3 Taxation

Several papers in the literature target taxes as an important determinant of an economy's path (Barro and Redlick, 2011; Ramey, 2011a). Extended analysis of tax revenue as a share of GDP and total taxes on commercial profits reveals very little about either growth or the impact of the fiscal multiplier. The influence of tax rates on growth and on the multiplier are investigated by modeling the levels of these rates, changes in their levels, and interactions of
both. Several models including taxation are attempted in order to consider the overall impact of each tax variable. In particular, variations of

$$
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} g_{i, t-j}+m_{j}^{\sigma} g_{i, t-j} \sigma_{g_{i, t-j}}+\beta_{j}^{t a x} t a x_{i, t-j}\right)+\epsilon_{i, t}
$$

are employed. Table XVI estimates the naive model of the taxes on growth in the presence of government purchases. Table XVII examines the impact of tax rates and revenue on the fiscal multiplier using the interactions technique.

$$
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} g_{i, t-j}+m_{j}^{\sigma} g_{i, t-j} \sigma_{g_{i, t-j}}+m_{j}^{t a x} g_{i, t-j} t a x_{g_{i, t-j}}^{k}\right)+\epsilon_{i, t} .
$$

Nevertheless, there are virtually no instances in which any tax variable is statistically different from zero. In the rare instance in which a statistically significant tax result exists, the coefficients on tax revenue as a share of GDP and change in tax revenue as a share of GDP are marginally greater than zero. It is important to note that limited availability of tax data ${ }^{1}$ may explain the insignificant results.

[^14]
## TABLE XVI

PWT and WDI - Ancillary Controls: Taxes Impact on Output (1950-2011)

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) |
| Output Growth (lag 1) | $\begin{gathered} -0.085 \\ (0.068) \end{gathered}$ | $\begin{gathered} -0.137^{* *} \\ (0.069) \end{gathered}$ | $\begin{gathered} -0.288^{* * *} \\ (0.085) \end{gathered}$ | $\begin{gathered} -0.323^{* * *} \\ (0.085) \end{gathered}$ |
| Output Growth (lag 2) | $\begin{gathered} -0.088 \\ (0.083) \end{gathered}$ | $\begin{gathered} -0.106 \\ (0.081) \end{gathered}$ | $\begin{gathered} -0.244^{* *} \\ (0.094) \end{gathered}$ | $\begin{gathered} -0.216^{* *} \\ (0.093) \end{gathered}$ |
| Fiscal Shock | $\begin{gathered} 0.008 \\ (0.018) \end{gathered}$ | $\begin{gathered} 0.010 \\ (0.018) \end{gathered}$ | $\begin{gathered} 0.007 \\ (0.021) \end{gathered}$ | $\begin{gathered} 0.007 \\ (0.021) \end{gathered}$ |
| Fiscal Shock (lag 1) | $\begin{gathered} 0.014 \\ (0.017) \end{gathered}$ | $\begin{gathered} 0.014 \\ (0.017) \end{gathered}$ | $\begin{gathered} 0.015 \\ (0.019) \end{gathered}$ | $\begin{gathered} 0.012 \\ (0.018) \end{gathered}$ |
| Fiscal Shock (lag 2) | $\begin{gathered} -0.001 \\ (0.019) \end{gathered}$ | $\begin{gathered} 0.002 \\ (0.018) \end{gathered}$ | $\begin{gathered} 0.002 \\ (0.021) \end{gathered}$ | $\begin{gathered} 0.001 \\ (0.020) \end{gathered}$ |
| Tax Rate | $\begin{gathered} -0.0001 \\ (0.0003) \end{gathered}$ |  |  |  |
| Tax Rate (lag 1) | $\begin{gathered} 0.0002 \\ (0.0005) \end{gathered}$ |  |  |  |
| Tax Rate (lag 2) | $\begin{aligned} & -0.00003 \\ & (0.001) \end{aligned}$ |  |  |  |
| Tax Rev |  | $\begin{gathered} 0.001 \\ (0.001) \end{gathered}$ |  |  |
| Tax Rev (lag 1) |  | $\begin{aligned} & 0.004^{* * *} \\ & (0.001) \end{aligned}$ |  |  |
| Tax Rev (lag 2) |  | $\begin{gathered} -0.0004 \\ (0.001) \end{gathered}$ |  |  |
| Tax Rate Change |  |  | $\begin{gathered} -0.0002 \\ (0.0004) \end{gathered}$ |  |
| Tax Rate Change (lag 1) |  |  | $\begin{gathered} 0.00001 \\ (0.0004) \end{gathered}$ |  |
| Tax Rate Change (lag 2) |  |  | $\begin{gathered} -0.0005 \\ (0.001) \end{gathered}$ |  |
| Tax Rev Change |  |  |  | $\begin{aligned} & -0.001 \\ & (0.002) \end{aligned}$ |
| Tax Rev Change (lag 1) |  |  |  | $\begin{gathered} 0.003^{*} \\ (0.002) \end{gathered}$ |
| Tax Rev Change (lag 2) |  |  |  | $\begin{gathered} 0.003^{*} \\ (0.002) \\ \hline \end{gathered}$ |
| Observations | 257 | 257 | 197 | 197 |
| Adjusted R ${ }^{2}$ | 0.015 | 0.051 | 0.084 | 0.108 |
| F Statistic | 0.486 | 1.768* | $2.369^{* *}$ | $3.168^{* * *}$ |
| Note: |  |  | ${ }^{*} \mathrm{p}<0.1$; ${ }^{* *} \mathrm{p}$ | ; ${ }^{* * *} \mathrm{p}<0.01$ |

## TABLE XVII

PWT and WDI - Ancillary Controls: Tax Impact on Multiplier (1950-2011)

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) |
| Output Growth (lag 1) | $\begin{gathered} -0.088 \\ (0.068) \end{gathered}$ | $\begin{gathered} -0.105 \\ (0.068) \end{gathered}$ | $\begin{gathered} -0.295^{* * *} \\ (0.084) \end{gathered}$ | $\begin{aligned} & -0.297^{* * *} \\ & (0.085) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{gathered} -0.084 \\ (0.082) \end{gathered}$ | $\begin{gathered} -0.113 \\ (0.081) \end{gathered}$ | $\begin{gathered} -0.218^{* *} \\ (0.093) \end{gathered}$ | $\begin{gathered} -0.274^{* * *} \\ (0.092) \end{gathered}$ |
| Fiscal Shock | $\begin{gathered} -0.078 \\ (0.183) \end{gathered}$ | $\begin{gathered} -0.201 \\ (0.157) \end{gathered}$ | $\begin{gathered} 0.011 \\ (0.021) \end{gathered}$ | $\begin{gathered} -0.123 \\ (0.098) \end{gathered}$ |
| Fiscal Shock (lag 1) | $\begin{gathered} -0.009 \\ (0.234) \end{gathered}$ | $\begin{gathered} -0.206 \\ (0.154) \end{gathered}$ | $\begin{gathered} 0.020 \\ (0.019) \end{gathered}$ | $\begin{gathered} 0.118 \\ (0.072) \end{gathered}$ |
| Fiscal Shock (lag 2) | $\begin{gathered} 0.071 \\ (0.128) \end{gathered}$ | $\begin{gathered} 0.099 \\ (0.131) \end{gathered}$ | $\begin{gathered} 0.004 \\ (0.021) \end{gathered}$ | $\begin{gathered} 0.033 \\ (0.098) \end{gathered}$ |
| Tax Rate*Fiscal Shock | $\begin{gathered} 0.002 \\ (0.005) \end{gathered}$ |  |  |  |
| Tax Rate*Fiscal Shock (lag 1) | $\begin{gathered} 0.001 \\ (0.007) \end{gathered}$ |  |  |  |
| Tax Rate*Fiscal Shock (lag 2) | $\begin{gathered} -0.002 \\ (0.004) \end{gathered}$ |  |  |  |
| Tax Rev*Fiscal Shock |  | $\begin{gathered} 0.016 \\ (0.012) \end{gathered}$ |  |  |
| Tax Rev*Fiscal Shock (lag 1) |  | $\begin{gathered} 0.016 \\ (0.012) \end{gathered}$ |  |  |
| Tax Rev*Fiscal Shock (lag 2) |  | $\begin{gathered} -0.008 \\ (0.010) \end{gathered}$ |  |  |
| Tax Rate Change *Fiscal Shock |  |  | $\begin{gathered} -0.037 \\ (0.039) \end{gathered}$ |  |
| Tax Rate Change |  |  | -0.071* |  |
| *Fiscal Shock (lag 1) |  |  | (0.040) |  |
| Tax Rate Change |  |  | -0.024 |  |
| *Fiscal Shock (lag 2) |  |  | (0.041) |  |
| Tax Rev Change |  |  |  | -0.121 |
| *Fiscal Shock |  |  |  | (0.089) |
| Tax Rev Change |  |  |  | 0.084 |
| *Fiscal Shock (lag 1) |  |  |  | (0.080) |
| Tax Rev Change |  |  |  | 0.037 |
| *Fiscal Shock (lag 2) |  |  |  | (0.090) |
| Observations | 257 | 257 | 197 | 197 |
| Adjusted $\mathrm{R}^{2}$ | 0.018 | 0.033 | 0.095 | 0.098 |
| F Statistic | 0.581 | 1.122 | $2.718^{* * *}$ | $2.839^{* * *}$ |
| Note: |  |  | *p<0.1; **p | 5; ***p<0.01 |

### 5.3.2 The Extended Model

Subsection 5.3.1 offers evidence that fiscal volatility, output gap, and openness diminish the fiscal multiplier. Thus the initial extended model is defined as,

$$
\begin{align*}
y_{i, t}=v_{i}+w_{t} & +\sum_{j=1}^{J} \alpha_{j} y_{i, t-j} \\
& +\sum_{j=0}^{J}\left(m_{j} g_{i, t-j}+m_{j}^{\sigma} g_{i, t-j} \sigma_{g_{i, t-j}}+m_{j}^{\text {open }} g_{i, t-j} \text { open } g_{g_{i, t-j}}^{k}+m_{j}^{\text {gap }} g_{i, t-j} \text { gap } g_{g_{i, t-j}}^{k}\right) \\
& +\epsilon_{i, t} . \tag{5.1}
\end{align*}
$$

Applying extended model to the PWT panel and US data yields the empirical results found in Table XVIII column 6 and Table XIX column 3 respectively. Only two of the three variables hold in the context of the PWT data, thus the extended model is redefined as

$$
\begin{equation*}
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} g_{i, t-j}+m_{j}^{\sigma} g_{i, t-j} \sigma_{g_{i, t-j}}+m_{j}^{g a p} g_{i, t-j} g a p_{g_{i, t-j}}^{k}\right)+\epsilon_{i, t} . \tag{5.2}
\end{equation*}
$$

The coefficients on the output gap interactions closely match the results from the literature discussed in Chapter 2. The dynamic effects are highly significant and possess the expected signs. In the long run, both fiscal volatility and output gap diminish the multiplier.

## TABLE XVIII

## PWT - Extended Models

|  | Dependent variable: Output Growth |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (5) | (6) |
| Output Growth (lag 1) | $\begin{aligned} & 0.206^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.201^{* * *} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.202^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.222^{* * *} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.212^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & \hline 0.207^{* * *} \\ & (0.017) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{aligned} & 0.051^{* * *} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.053^{* * *} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.054^{* * *} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.071^{* * *} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.072^{* * *} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.083^{* * *} \\ & (0.017) \end{aligned}$ |
| Fiscal Shock | $\begin{aligned} & 0.666^{* * *} \\ & (0.042) \end{aligned}$ | $\begin{aligned} & 0.700^{* * *} \\ & (0.041) \end{aligned}$ | $\begin{aligned} & 0.730^{* * *} \\ & (0.043) \end{aligned}$ | $\begin{aligned} & 0.229^{* * *} \\ & (0.024) \end{aligned}$ | $\begin{aligned} & 0.545^{* * *} \\ & (0.043) \end{aligned}$ | $\begin{aligned} & 0.597^{* * *} \\ & (0.043) \end{aligned}$ |
| Fiscal Shock (lag 1) | $\begin{gathered} -0.008 \\ (0.049) \end{gathered}$ | $\begin{gathered} -0.090^{* *} \\ (0.042) \end{gathered}$ | $\begin{gathered} -0.052 \\ (0.051) \end{gathered}$ | $\begin{gathered} -0.029 \\ (0.025) \end{gathered}$ | $\begin{gathered} 0.044 \\ (0.048) \end{gathered}$ | $\begin{gathered} -0.054 \\ (0.051) \end{gathered}$ |
| Fiscal Shock (lag 2) | $\begin{aligned} & 0.118^{* * *} \\ & (0.045) \end{aligned}$ | $\begin{array}{r} -0.015 \\ (0.034) \end{array}$ | $\begin{gathered} 0.027 \\ (0.051) \end{gathered}$ | $\begin{aligned} & 0.074^{* * *} \\ & (0.025) \end{aligned}$ | $\begin{gathered} 0.075 \\ (0.046) \end{gathered}$ | $\begin{array}{r} -0.073 \\ (0.050) \end{array}$ |
| Fiscal Shock | $-4.197^{* * *}$ |  | $-1.423^{* *}$ |  | $-3.414^{* * *}$ | 0.940 |
| *Volatility | (0.376) |  | (0.667) |  | (0.373) | (0.676) |
| Fiscal Shock | $-1.841^{* * *}$ |  | -1.631 |  | $-2.407^{* * *}$ | 0.605 |
| *Volatility (lag 1) | (0.495) |  | (1.198) |  | (0.485) | (1.208) |
| Fiscal Shock | $-3.748^{* * *}$ |  | $-1.408^{*}$ |  | $-3.142^{* * *}$ | 0.440 |
| *Volatility (lag 2) | (0.546) |  | (0.742) |  | (0.543) | (0.734) |
| Fiscal Shock |  | $-0.060^{* * *}$ | $-0.049^{* * *}$ |  |  | $-0.060^{* * *}$ |
| *Openness |  | (0.005) | (0.008) |  |  | (0.008) |
| Fiscal Shock |  | $0.043^{* * *}$ | $0.046^{* * *}$ |  |  | $0.033^{* *}$ |
| *Openness (lag 1) |  | (0.005) | (0.013) |  |  | (0.013) |
| Fiscal Shock |  | 0.010* | -0.002 |  |  | 0.034** |
| *Openness (lag 2) |  | (0.005) | (0.013) |  |  | (0.014) |
| Fiscal Shock |  |  |  | $6.042^{* * *}$ | $5.908^{* * *}$ | $5.058^{* * *}$ |
| *Output Gap |  |  |  | (0.692) | (0.701) | (0.738) |
| Fiscal Shock |  |  |  | $-8.493{ }^{* * *}$ | $-7.416^{* * *}$ | $-8.841^{* * *}$ |
| *Output Gap (lag 1) |  |  |  | (0.682) | (0.699) | (0.726) |
| Fiscal Shock |  |  |  | -1.280* | -0.668 | $-1.729^{* *}$ |
| *Output Gap (lag 2) |  |  |  | (0.694) | (0.724) | (0.754) |
| Observations | 3,348 | 3,348 | 3,348 | 3,348 | 3,348 | 3,348 |
| Adjusted R ${ }^{2}$ | 0.125 | 0.134 | 0.135 | 0.147 | 0.168 | 0.186 |
| F Statistic | $60.101^{* * *}$ | $65.194^{* * *}$ | $47.855^{* * *}$ | $72.548^{* * *}$ | $61.805^{* * *}$ | $55.033^{* * *}$ |

Note:
${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$

TABLE XIX

US - Extended Model

|  | Dependent variable: Output Growth |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (5) | (6) |
| Output Growth (lag 1) | $\begin{aligned} & 0.322^{* * *} \\ & (0.116) \end{aligned}$ | $\begin{aligned} & 0.322^{* * *} \\ & (0.117) \end{aligned}$ | $\begin{aligned} & 0.322^{* * *} \\ & (0.120) \end{aligned}$ | $\begin{aligned} & 0.323^{* * *} \\ & (0.117) \end{aligned}$ | $\begin{aligned} & 0.331^{* * *} \\ & (0.118) \end{aligned}$ | $\begin{aligned} & 0.341^{* * *} \\ & (0.117) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{gathered} -0.142 \\ (0.115) \end{gathered}$ | $\begin{gathered} -0.121 \\ (0.118) \end{gathered}$ | $\begin{gathered} -0.108 \\ (0.120) \end{gathered}$ | $\begin{gathered} -0.111 \\ (0.117) \end{gathered}$ | $\begin{gathered} -0.130 \\ (0.117) \end{gathered}$ | $\begin{gathered} -0.101 \\ (0.114) \end{gathered}$ |
| Fiscal Shock | $\begin{aligned} & 0.826^{* * *} \\ & (0.250) \end{aligned}$ | $\begin{aligned} & 0.832^{* * *} \\ & (0.298) \end{aligned}$ | $\begin{gathered} 1.694^{*} \\ (0.926) \end{gathered}$ | $\begin{aligned} & 0.477^{* * *} \\ & (0.145) \end{aligned}$ | $\begin{aligned} & 1.515^{*} \\ & (0.887) \end{aligned}$ | $\begin{gathered} 1.338 \\ (0.839) \end{gathered}$ |
| Fiscal Shock (lag 1) | $\begin{gathered} 0.019 \\ (0.305) \end{gathered}$ | $\begin{gathered} 0.144 \\ (0.542) \end{gathered}$ | $\begin{gathered} -0.291 \\ (0.951) \end{gathered}$ | $\begin{gathered} -0.052 \\ (0.168) \end{gathered}$ | $\begin{array}{r} -0.483 \\ (0.795) \end{array}$ | $\begin{array}{r} -0.549 \\ (0.751) \end{array}$ |
| Fiscal Shock (lag 2) | $\begin{gathered} 0.038 \\ (0.278) \end{gathered}$ | $\begin{gathered} 0.285 \\ (0.502) \end{gathered}$ | $\begin{gathered} -0.186 \\ (0.881) \end{gathered}$ | $\begin{gathered} 0.055 \\ (0.141) \end{gathered}$ | $\begin{gathered} -0.048 \\ (0.460) \end{gathered}$ | $\begin{gathered} -0.023 \\ (0.399) \end{gathered}$ |
| Fiscal Shock <br> *Volatility | $\begin{gathered} -4.957^{*} \\ (2.793) \end{gathered}$ | $\begin{gathered} -5.956 \\ (4.498) \end{gathered}$ | $\begin{gathered} -5.706 \\ (4.892) \end{gathered}$ |  | $\begin{array}{r} -4.124 \\ (2.938) \end{array}$ |  |
| Fiscal Shock | 0.775 | -0.156 | $(4.892)$ 0.692 |  | -0.614 |  |
| *Volatility (lag 1) | (2.927) | (4.959) | (5.798) |  | (3.363) |  |
| Fiscal Shock | -0.809 | -3.399 | -0.742 |  | 0.649 |  |
| *Volatility (lag 2) | (2.806) | (4.503) | (5.142) |  | (3.191) |  |
| Fiscal Shock |  | -1.398 | $-0.548$ | 2.016 |  |  |
| *Output Gap |  | (4.218) | (4.314) | (2.163) |  |  |
| Fiscal Shock |  | -0.859 | 1.132 | -1.965 |  |  |
| *Output Gap (lag 1) |  | (4.150) | (5.124) | (2.216) |  |  |
| Fiscal Shock |  | -2.666 | -3.408 | -0.301 |  |  |
| *Output Gap (lag 2) |  | (2.595) | (2.907) | (2.143) |  |  |
| Fiscal Shock |  |  | -13.633 |  | -11.562 | -13.201 |
| *Openness |  |  | (14.384) |  | (13.998) | (12.528) |
| Fiscal Shock |  |  | 7.224 |  | 10.112 | 8.974 |
| *Openness (lag 1) |  |  | (12.882) |  | (12.373) | (10.840) |
| Fiscal Shock |  |  | 1.605 |  | -1.459 | -0.682 |
| *Openness (lag 2) |  |  | (7.253) |  | (5.861) | (5.495) |
| Observations | 86 | 86 | 86 | 86 | 86 | 86 |
| Adjusted R ${ }^{2}$ | 0.336 | 0.322 | 0.308 | 0.324 | 0.321 | 0.325 |
| Residual Std. Error | 0.040 | 0.041 | 0.041 | 0.041 | 0.041 | 0.040 |
| F Statistic | $6.380^{* * *}$ | $4.663^{* * *}$ | $3.707^{* * *}$ | $6.082^{* * *}$ | $4.652^{* * *}$ | $6.119^{* * *}$ |
| Note: |  |  |  |  | ${ }^{*} \mathrm{p}<0.1$; ${ }^{* *} \mathrm{p}<0$ | ; ${ }^{* * *} \mathrm{p}<0.01$ |



Figure 15. Implied Long-run Multipliers

### 5.3.2.1 Multipliers Conditional on Output Gap and Fiscal Volatility

Table XX and Table XXI display arrays of multipliers based on fiscal volatilities and output gaps for the PWT and US data respectively. Rows vary by fiscal volatility and columns by output gap. Estimated multipliers are greater than one when economies are experiencing recessions and low fiscal volatility. The PWT estimates suggest that multipliers are less than one in normal times and during expansions. Estimates based on the US suggest higher multipliers for all combinations of volatility and slackness.

TABLE XX

PWT - Extended Model Multipliers

|  |  | Output Gap |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Volatility | $\mathbf{- 4 \%}$ | $\mathbf{- 3 \%}$ | $\mathbf{- 2 \%}$ | $\mathbf{- 1 \%}$ | $\mathbf{0}$ | $\mathbf{1 \%}$ |
| 0.00024 | 1.050 | 1.020 | 0.985 | 0.954 | 0.924 | 0.894 |
| 0.00271 | 1.010 | 0.984 | 0.954 | 0.923 | 0.893 | 0.863 |
| 0.00374 | 1 | 0.971 | 0.941 | 0.911 | 0.880 | 0.850 |
| 0.00570 | 0.977 | 0.947 | 0.916 | 0.886 | 0.856 | 0.825 |
| 0.00882 | 0.938 | 0.908 | 0.877 | 0.847 | 0.816 | 0.786 |
| 0.01490 | 0.862 | 0.832 | 0.801 | 0.771 | 0.741 | 0.710 |

TABLE XXI

US - Extended Model Multipliers

|  |  | Output Gap |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Volatility | Year | $\mathbf{- 4 \%}$ | $\mathbf{- 2 \%}$ | $\mathbf{0}$ | $\mathbf{2 \%}$ | $\mathbf{4 \%}$ |  |
| 0.0015 | 1997 | 1.690 | 1.570 | 1.450 | 1.330 | 1.210 |  |
| 0.0030 | 1928 | 1.680 | 1.560 | 1.430 | 1.310 | 1.190 |  |
| 0.0079 | 1935 | 1.620 | 1.500 | 1.380 | 1.260 | 1.140 |  |
| 0.0352 | 1942 | 1.330 | 1.200 | 1.080 | 0.962 | 0.841 |  |
| 0.0985 | 1945 | 0.637 | 0.516 | 0.395 | 0.273 | 0.152 |  |
| 0.1790 | 1947 | -0.233 | -0.355 | -0.476 | -0.597 | -0.718 |  |

### 5.3.2.2 Multipliers Across Time

Figure 15 plots historical US multipliers for the full model (Equation 4.9 ) based on the PWT data and the Barro and Redlick US data. Multipliers based on US data are usually a bit more than 0.6 higher than those based on the PWT data. However the Barro and Redlick based multipliers are far more variable. The adjusted $R^{2}$ for the US model is larger than that for the PWT specification however most of its coefficients are insignificant from zero, while the coefficients of interest in the final PWT model are highly significant. Interestingly, the conjectured multipliers closely correspond with one another during the 1956-2006 period over which they overlap. In particular, dips in the late 50 s and early 70s multiplier estimates correspond and feature prominently in both panels.

## CHAPTER 6

## TESTS OF ROBUSTNESS

This Chapter tests the robustness of Chapter 5's empirical results using controls for endogeneity, checks for structural breaks, and techniques for outlier removal.

### 6.1 Endogeneity Controls

Endogeneity is a perennial issue in the measurement of fiscal multipliers. As economic growth leads to increases in tax coffers, governments may expand their spending in ways that they would not have in the absence of growth. In order to ensure unbiased statistical results special techniques are required to ensure that the right hand side (RHS) variables are independent of the error term.

The first method used to address endogeneity employs a near-vector autoregressive (NVAR) system of equations based on Karras (2011). NVARs, also referred to as the quasi-VAR models, are a special case of the traditional VAR model. Next defense purchases are used in lieu of government purchases since defense purchases are less likely to respond systematically to output growth. The last technique employs instrumental variables (IV). Defense spending is often correlated with overall government purchases, thus it may prove a viable IV. IVs allow consistent estimation when variables in the regression are correlated with the error.

### 6.1.1 Penn World Tables Near-Vector Autoregressive Empirical Results

The NVAR model is developed in Section 4.3. Table XXII reports the parameter estimates for the PWT NVAR baseline static model.

$$
y_{i, t}=v_{i}+w_{t}+m \widehat{u_{i, t}}+m^{\sigma} \widehat{u_{i, t}} \sigma_{\widehat{u_{i, t}}}=\epsilon_{i, t}
$$

The left three columns of Table XXII show the contemporaneous effects of government purchases on output, when $m^{\sigma}$ is constrained to 0 . Under this scenario, the fixed effects estimation suggests a multiplier of 0.254 .

## TABLE XXII

PWT NVAR - Baseline Empirical Results

|  | Dependent variable: Output Growth |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Model (1) |  |  | Model (2) |  |  |
|  | OLS | FE | RE | OLS | FE | RE |
| Fiscal Shock | $\begin{aligned} & 0.338^{* * *} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.254^{* * *} \\ & (0.026) \end{aligned}$ | $\begin{aligned} & 0.308^{* * *} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.628^{* * *} \\ & (0.041) \end{aligned}$ | $\begin{aligned} & 0.529^{* * *} \\ & (0.039) \end{aligned}$ | $\begin{aligned} & 0.586^{* * *} \\ & (0.041) \end{aligned}$ |
| Fiscal Shock <br> * Volatility |  |  |  | $\begin{gathered} -1.229^{* * *} \\ (0.131) \\ \hline \end{gathered}$ | $\begin{gathered} -1.117^{* * *} \\ (0.123) \\ \hline \end{gathered}$ | $\begin{gathered} -1.150^{* * *} \\ (0.128) \end{gathered}$ |
| Observations | 3,224 | 3,224 | 3,224 | 3,224 | 3,224 | 3,224 |
| Adjusted R ${ }^{2}$ | 0.045 | 0.029 | 0.040 | 0.070 | 0.054 | 0.063 |
| F Statistic | $150.927^{* * *}$ | 97.972*** | 133.598*** | $121.741^{* * *}$ | 91.336*** | 108.874*** |
| Note: |  |  |  |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}$ | 05; *** $<0.01$ |

The right three columns of Table XXII model the fiscal multiplier as dependent on the volatility of fiscal shocks. The volatility interaction coefficients for the static PWT NVAR model are negative, large, and significant. Thus, the findings in Table XXII support the hypothesis that fiscal volatility reduces the fiscal multiplier. The multiplier for a hypothetical country with zero volatility is simply the coefficient on the government spending shock, 0.529 .

In order to illustrate the impact of volatility on multipliers in the baseline static model, consider the range of multipliers implied for the countries with the minimum and maximum mean volatilities over time. These contemporaneous multipliers range from 0.526 for Switzerland (volatility $=0.00265$ ) to 0.482 for El Salvador (volatility $=0.042$ ).

While the baseline static model provides a useful benchmark for comparison, the dynamic model with $\mathrm{J}=2$ lags and a volatility window of $\mathrm{k}=6$ years offers a more realistic perspective.

$$
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} \widehat{u_{i, t-j}}+m_{j}^{\sigma} \widehat{u_{i, t-j}} \sigma_{\widehat{u_{i, t-j}}}{ }^{k}\right)+\epsilon_{i, t} .
$$

Table XXIII reports the parameter estimates for the PWT NVAR dynamic model. The left two columns show the dynamic effects of defense spending on output when $m_{j}^{\sigma}$ is constrained to 0 . The fixed effects model parameters imply a long-run multiplier of 0.37 .

The third and fourth columns of Table XXIII allow the fiscal multiplier to depend on the volatility of fiscal shocks. The volatility interaction coefficients are negative, large, and significant. Thus these findings further support the hypothesis that fiscal volatility reduces the

TABLE XXIII

PWT NVAR - Dynamic Empirical Results

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Equation 4.9 |  | Equation 4.9 |  |
|  | FE | RE | FE | RE |
| Output Growth (lag 1) | $\begin{aligned} & 0.232^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.274^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.256^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.291^{* * *} \\ & (0.018) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{aligned} & 0.056^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.063^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.071^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.076^{* * *} \\ & (0.018) \end{aligned}$ |
| Fiscal Shock | $\begin{aligned} & 0.243^{* * *} \\ & (0.025) \end{aligned}$ | $\begin{aligned} & 0.280^{* * *} \\ & (0.026) \end{aligned}$ | $\begin{aligned} & 0.673^{* * *} \\ & (0.045) \end{aligned}$ | $\begin{aligned} & 0.698^{* * *} \\ & (0.046) \end{aligned}$ |
| Fiscal Shock (lag 1) | $\begin{gathered} 0.006 \\ (0.025) \end{gathered}$ | $\begin{gathered} -0.001 \\ (0.026) \end{gathered}$ | $\begin{gathered} -0.170^{* * *} \\ (0.046) \end{gathered}$ | $\begin{gathered} -0.151^{* * *} \\ (0.047) \end{gathered}$ |
| Fiscal Shock (lag 2) | $\begin{gathered} 0.015 \\ (0.032) \end{gathered}$ | $\begin{gathered} 0.023 \\ (0.033) \end{gathered}$ | $\begin{gathered} -0.115^{* *} \\ (0.057) \end{gathered}$ | $\begin{gathered} -0.075 \\ (0.058) \end{gathered}$ |
| Fiscal Shock <br> * Volatility |  |  | $-1.767^{* * *}$ | $-1.821^{* * *}$ |
|  |  |  | (0.187) | (0.192) |
| Fiscal Shock <br> * Volatility (lag 1) |  |  | 0.001 | -0.182 |
|  |  |  | (0.201) | (0.206) |
| Fiscal Shock <br> * Volatility (lag 2) |  |  | -0.589 | $-1.222^{*}$ |
|  |  |  | (0.695) | (0.712) |
| Observations | 3,100 | 3,100 | 3,100 | 3,100 |
| Adjusted R ${ }^{2}$ | 0.092 | 0.132 | 0.128 | 0.164 |
| F Statistic | $62.715^{* * *}$ | $94.173^{* * *}$ | $57.305^{* * *}$ | $76.008^{* * *}$ |
| Note: |  |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}$ | ; ${ }^{* * *} \mathrm{p}<0.01$ |

multiplier. Based on Equation 4.7, the multiplier for a hypothetical country with zero volatility is 0.577 .

The Hausman test is used to determine whether the fixed effects (FE) specification or the random effects (RE) specification is preferred. The Hausman statistic of 204 implies p-value of $8.32 \mathrm{e}-40$ and thus the null hypothesis of the RE model is rejected. The remaining tables and figures based on Table XXIII consider the FE model.

Table XXIV lists implied long-run multipliers for a number of countries in order to highlight the impact of volatility on the multiplier. A five-number summary of the mean volatilities across time for each country is used to calculate typical multipliers for the representative countries. Multipliers range from 0.568 for Switzerland (volatility $=0.00265$ ) to 0.43 for El Salvador $($ volatility $=0.042)$.

TABLE XXIV

PWT NVAR - Long-run Multipliers across Mean Volatilities

|  | Zero Vol. | 0th \%-ile | 25th \%-ile | 50th \%-ile | 75th \%-ile | 100th \%-ile |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Country | NA | Switzerland | Sweden | India | Brazil | El Salvador |
| Mean Volatility | 0 | 0.00265 | 0.00549 | 0.00813 | 0.0132 | 0.042 |
| Multiplier | 0.577 | 0.568 | 0.558 | 0.548 | 0.531 | 0.43 |



Figure 16. PWT NVAR - Impulse Responses to a 1\% Fiscal Shock

Figure 16 tracks the impulse responses of output to $1 \%$ fiscal shocks for the fixed effects PWT NVAR dynamic model. Impulse responses demonstrate the evolution of variables in dynamic systems over time. The volatilities used to generate the responses are based on a fivenumber summary of the mean volatilities for each country across time. Notice that long-run multipliers are significantly larger for countries with lower fiscal volatility.

Panel (a) of Figure 17 illustrates the long-run multipliers implied by the PWT NVAR dynamic model contingent on historical data for the smallest, median, and largest economies in the PWT NVAR panel. The average multiplier across the countries in panel (a) is 0.561 and the minimum and maximum multipliers are 0.53 and 0.574 respectively. Panel (b) illustrates the long-run multipliers implied by the PWT NVAR dynamic model contingent on historical data for the poorest, median, and wealthiest economies in terms of per capita income. The


Figure 17. PWT NVAR - Implied Historical Multipliers for Representative Economies
average multiplier across the countries in panel (b) is 0.55 and the minimum and maximum multipliers are 0.491 and 0.576 respectively.

### 6.1.2 United States Near-Vector Autoregressive Baseline Static Model Empirical Results

The US defense spending data spans 94 years (from 1912 to 2006). Table XXV reports the parameter estimates for the baseline static model.

$$
y_{i, t}=v_{i}+w_{t}+m g_{i, t}+m^{\sigma} g_{i, t} * \sigma_{g_{i, t}}^{k}+\epsilon_{i, t}
$$

The left column shows the contemporaneous effect of government purchases on output, with $m^{\sigma}=0$. The fixed effects estimation suggests a multiplier of 0.499.

TABLE XXV

US NVAR Baseline Model Empirical Results

|  | Dependent variable: Output Growth |  |
| :--- | :---: | :---: |
|  | Model (1) | Model (2) |
| Fiscal Shock | $0.499^{* * *}$ | $0.505^{* * *}$ |
| Fiscal Shock | $(0.148)$ | $(0.148)$ |
| * Volatility |  | 0.192 |
|  |  | $(0.178)$ |
| Observations | 85 | 85 |
| Adjusted R |  |  |
| Residual Std. Error | 0.110 | 0.112 |
| F Statistic | 0.047 | 0.047 |
| Note: | $11.400^{* * *}$ | $6.290^{* * *}$ |
| ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |  |  |

The right column of Table XXV models the fiscal multiplier as dependent on the volatility of fiscal shocks. The interaction coefficient is not significantly different from zero. And the multiplier for a hypothetical country with zero volatility is simply the coefficient on the government spending shock, 0.505 .

While the baseline static model provides a useful benchmark for comparison,the dynamic model with 2 lags and a volatility window of $\mathrm{k}=6$ years offers a more realistic perspective. Table XXVI reports the parameter estimates for the US NVAR dynamic model.

$$
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} \widehat{u_{i, t-j}}+m_{j}^{\sigma} \widehat{u_{i, t-j}} \sigma_{\widehat{u_{i, t-j}}}{ }^{k}\right)+\epsilon_{i, t}
$$

The left column shows the dynamic effect of government purchases on output with the $m_{j}^{\sigma}$ constrained to 0 . The fixed effects model parameters imply a long-run multiplier of 1.27.

The right column of Table XXVI allows the fiscal multiplier to depend on the volatility of fiscal shocks. The parameters on the volatility interaction terms are negative, large, and significant. These findings therefore support the hypothesis that recent volatility in government spending reduces the fiscal multiplier. The multiplier for a hypothetical country with zero volatility is 2.98. Table XXVII highlights the impact of volatility on the multiplier. It lists the implied multipliers for a five-number summary of all the volatilities across time in the US. The implied multipliers range from 2.95 in 1997 (volatility $=0.0015$ ) to 0.846 in 1952 (volatility $=$ 0.111 ).

## TABLE XXVI

US NVAR - Dynamic Empirical Results

|  | Dependent variable: Output Growth |  |
| :--- | :---: | :---: |
|  | Equation 4.9 | Equation 4.9 |
| Output Growth (lag 1) | $0.485^{* * *}$ | $0.629^{* * *}$ |
|  | $(0.111)$ | $(0.127)$ |
| Output Growth (lag 2) | -0.091 | -0.165 |
|  | $(0.102)$ | $(0.113)$ |
| Fiscal Shock | $0.509^{* * *}$ | $1.281^{* * *}$ |
|  | $(0.126)$ | $(0.395)$ |
| Fiscal Shock (lag 1) | 0.205 | -0.069 |
|  | $(0.137)$ | $(0.405)$ |
| Fiscal Shock (lag 2) | -0.112 | 0.305 |
|  | $(0.137)$ | $(0.366)$ |
| Fiscal Shock * Volatility |  | $-8.206^{*}$ |
|  |  | $(4.202)$ |
| Fiscal Shock * Volatility (lag 1) |  | 1.146 |
|  |  | $(4.306)$ |
| Fiscal Shock * Volatility (lag 2) |  | -5.046 |
|  |  | $(3.844)$ |
| Observations | 83 | 83 |
| Adjusted R |  | 0.385 |
| Residual Std. Error | 0.356 | 0.039 |
| F Statistic | 0.040 | $7.427^{* * *}$ |
| Note: | $10.071^{* * *}$ | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |

## TABLE XXVII

US NVAR - Long-run Multipliers across All Historical Volatilities

|  | Zero Vol. | 0th $\%$-ile | 25 th $\%$-ile | 50 th $\%$-ile | 75 th $\%$-ile | 100th $\%$-ile |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year |  | 1997 | 1994 | 1964 | 1926 | 1952 |
| Volatility | 0 | 0.0015 | 0.00278 | 0.00553 | 0.0257 | 0.111 |
| Multiplier | 2.98 | 2.95 | 2.93 | 2.87 | 2.49 | 0.846 |

Figure 18. US NVAR - Fixed Effects Impulse Responses to a 1\% Fiscal Shock


Impulse responses demonstrate the evolution of variables in dynamic systems over time. 6.1.2 tracks the response of output to fiscal shock of $1 \%$ of GDP for the fixed effects estimations of the US NVAR dynamic model with 2 lags contingent on a 6 year volatility window. The reported volatilities reflect a five-number summary of the mean volatilities of the countries in this dataset. Recall that shocks, g, are defined as an increase in government spending equal to $1 \%$ of GDP and not simply a $1 \%$ increase in the level of government spending.

### 6.1.3 United States Defense Spending Empirical Results

The US defense spending data spans 94 years (from 1912 to 2006). Table XXVIII reports the parameter estimates for the US Defense Spending baseline static model.

$$
y_{i, t}=v_{i}+w_{t}+m d_{i, t}+m^{\sigma} d_{i, t} * \sigma_{d_{i, t}}^{k}+\epsilon_{i, t}
$$

The left column of Table XXVIII shows the contemporaneous effect of government purchases on output, with $m^{\sigma}$ constrained to 0 . The fixed effects estimation suggests a multiplier of 0.582. The right column estimates the defense spending multiplier dependent on the volatility of defense spending shocks. The interaction term is insignificant. The multiplier for a hypothetical year with zero volatility is simply the coefficient on the government spending shock, 0.589.

## TABLE XXVIII

US Defense Spending Baseline Model Empirical Results

|  | Dependent variable: Output Growth |  |
| :--- | :---: | :---: |
|  | Equation 4.10 | Equation 4.11 |
| Defense Shock | $0.582^{* * *}$ | $0.589^{* * *}$ |
|  | $(0.090)$ | $(0.091)$ |
| Defense Shock |  | 0.073 |
| * Volatility |  | $(0.101)$ |
| Observations | 88 | 88 |
| Adjusted R | 0.318 | 0.314 |
| Residual Std. Error | 0.041 | 0.041 |
| F Statistic | $41.570^{* * *}$ | $20.924^{* * *}$ |
| Note: | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |  |

While the baseline static model provides a useful benchmark for comparison, the dynamic model with 2 lags and a volatility window of $\mathrm{k}=6$ years offers a more realistic perspective. Table XXIX reports the parameter estimates for the dynamic US defense spending model.

$$
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} g_{i, t-j}+m_{j}^{\sigma} g_{i, t-j} \sigma_{g_{i, t-j}}^{k}\right)+\epsilon_{i, t}
$$

The left column shows the dynamic effect of government purchases on output with the $m_{j}^{\sigma}$ constrained to 0 . The parameter estimates imply a long-run multiplier of 0.549 .

## TABLE XXIX

US Defense Spending - Dynamic Empirical Results

|  | Dependent variable: Output Growth |  |
| :--- | :---: | :---: |
|  | Equation 4.12 | Equation 4.13 |
| Output Growth (lag 1) | $0.326^{* * *}$ | $0.310^{* * *}$ |
|  | $(0.116)$ | $(0.117)$ |
| Output Growth (lag 2) | -0.091 | -0.112 |
|  | $(0.114)$ | $(0.116)$ |
| Defense Shock | $0.421^{* * *}$ | $0.711^{* * *}$ |
|  | $(0.126)$ | $(0.263)$ |
| Defense Shock (lag 1) | 0.027 | 0.037 |
|  | $(0.149)$ | $(0.343)$ |
| Defense Shock (lag 2) | -0.062 | -0.021 |
|  | $(0.128)$ | $(0.297)$ |
| Defense Shock * Volatility |  | -3.754 |
|  |  | $(2.664)$ |
| Defense Shock * Volatility (lag 1) |  | 0.617 |
|  |  | $(3.112)$ |
| Defense Shock * Volatility (lag 2) |  | -0.675 |
|  |  | $(2.888)$ |
| Observations | 86 | 86 |
| Adjusted R ${ }^{2}$ | 0.334 | 0.327 |
| Residual Std. Error | 0.040 | 0.040 |
| F Statistic | $9.523^{* * *}$ | $6.152^{* * *}$ |
| Note: | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |  |

The right column of Table XXIX allows the defense spending multiplier to depend on the volatility of defense spending shocks. The parameters on the volatility interaction terms are negative and large yet insignificant. These findings therefore thinly support the hypothesis that recent volatility in government spending reduces the defense spending multiplier. Based on Equation 4.7, the multiplier for a hypothetical country with zero volatility is 0.958 . Table XXX highlights the impact of volatility on the multiplier. It lists the implied multipliers for a five-number summary of all the volatilities recorded in the US. The implied long-run multipliers range from 0.956 in 1934 (volatility $=0.000474$ ) to 0.421 in 1952 (volatility $=0.113$ ).

## TABLE XXX

US Defense Spending - Implied Historical Long-run Multipliers

|  | Zero Vol. | 0 th $\%$-ile | 25 th $\%$-ile | 50th $\%$-ile | 75th $\%$-ile | 100th $\%$-ile |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year |  | 1934 | 1979 | 1990 | 1955 | 1952 |
| Volatility | 0 | 0.000474 | 0.00146 | 0.00308 | 0.0276 | 0.113 |
| Multiplier | 0.958 | 0.956 | 0.951 | 0.943 | 0.827 | 0.421 |

Impulse responses demonstrate the evolution of variables in dynamic systems over time. 6.1.3 tracks the response of output to defense spending shock of $1 \%$ of GDP for the fixed effects


Figure 19. US Defense Spending - Impulse Responses to a $1 \%$ Shock
estimations of the US Defense Spending dynamic model. The reported volatilities reflect a five-number summary of the volatilities recorded for the US.

Table XXXI considers whether the multiplier due to defense spending is different than that due to non-defense spending. While the defense spending multiplier is significantly greater than zero, the hypothesis that the multipliers across spending types are identical cannot be rejected.

Table XXXII considers whether the effect of defense spending volatility on the multiplier is different than that due to non-defense spending volatility. The hypothesis that both effects are zero cannot be rejected.

Table XXXIII considers whether the multiplier due to defense spending in the dynamic model is different than that due to non-defense spending. While the defense spending multiplier
is significantly greater than zero, the hypothesis that the multipliers across spending types are identical cannot be rejected.

Table XXXIV considers whether the effect of defense spending volatility on the multiplier is different than that due to non-defense spending volatility for the dynamic model. The hypothesis that neither defense volatility nor non-defense volatility is significantly different from zero cannot be rejected.

TABLE XXXI

US Defense Spending - Static Empirical Results

|  | Dependent variable: Output Growth |  |
| :--- | :---: | :---: |
|  | Non-Defense | Non-Def. and Def. |
|  | FE | FE |
| Non-Defense Shock | -0.902 | 0.092 |
|  | $(0.664)$ | $(0.577)$ |
| Defense Shock |  | $0.587^{* * *}$ |
|  |  | $(0.095)$ |
| Observations | 88 | 88 |
| Adjusted R2 | 0.010 | 0.310 |
| Residual Std. Error | 0.049 | 0.041 |
| F Statistic | 1.848 | $20.562^{* * *}$ |
| Note: | $\mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |  |

## TABLE XXXII

US Defense Spending - Defense and Non-Defense Static Empirical Results with Interactions

|  | Dependent variable: Output Growth |  |
| :--- | :---: | :---: |
|  | Non-Defense |  |
|  | FE | FE |
| Non-Def. Shock | 0.558 | 0.279 |
|  | $(0.604)$ | $(0.644)$ |
| Non-Def. Shock*Non-Def. Vol. | $52.258^{* * *}$ | 13.350 |
|  | $(8.480)$ | $(32.192)$ |
| Def. Shock |  | 0.609 |
|  |  | $(0.606)$ |
| Def. Shock*Def. Vol. |  | -1.907 |
|  |  | $(3.734)$ |
| Observations | 0.307 | 88 |
| Adjusted R |  |  |
| Residual Std. Error | 0.041 | 0.310 |
| F Statistic | $20.308^{* * *}$ | 0.041 |
| Note: | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |  |

## TABLE XXXIII

US Defense Spending - Defense Static Empirical Results with Interactions

|  | Dependent variable: Output Growth |  |
| :--- | :---: | :---: |
|  | FE | FE |
| Output Growth (lag 1) | $0.505^{* * *}$ | $0.324^{* * *}$ |
|  | $(0.115)$ | $(0.118)$ |
| Output Growth (lag 2) | -0.156 | -0.096 |
|  | $(0.115)$ | $(0.117)$ |
| Non-Def. Shock | -0.443 | 0.440 |
|  | $(0.817)$ | $(0.812)$ |
| Non-Def. Shock (lag 1) | -0.604 | 0.076 |
|  | $(0.780)$ | $(0.779)$ |
| Non-Def. Shock (lag 2) | 0.649 | 0.836 |
|  | $(0.600)$ | $(0.581)$ |
| Def. Shock |  | $0.467^{* * *}$ |
|  |  | $(0.136)$ |
| Def. Shock (lag 1) |  | 0.011 |
|  |  | $(0.151)$ |
| Def. Shock (lag 2) |  | 0.008 |
|  |  | $(0.141)$ |
| Observations | 86 | 86 |
| Adjusted R ${ }^{2}$ | 0.328 |  |
| Residual Std. Error | 0.209 | 0.040 |
| F Statistic | 0.044 | $6.189^{* * *}$ |
| Note: | $5.499^{* * *}$ | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |

TABLE XXXIV

US Defense Spending - Dynamic Empirical Results

|  | Dependent variable: Output Growth |  |
| :---: | :---: | :---: |
|  | Non-Defense |  |
|  | FE | FE |
| Output Growth (lag 1) | $\begin{gathered} 0.294^{* *} \\ (0.117) \end{gathered}$ | $\begin{aligned} & 0.322^{* * *} \\ & (0.120) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{gathered} -0.114 \\ (0.113) \end{gathered}$ | $\begin{gathered} -0.155 \\ (0.120) \end{gathered}$ |
| Non-Def. Shock | $\begin{gathered} 0.788 \\ (0.806) \end{gathered}$ | $\begin{gathered} 1.289 \\ (0.866) \end{gathered}$ |
| Non-Def. Shock (lag 1) | $\begin{gathered} 0.761 \\ (0.830) \end{gathered}$ | $\begin{gathered} 0.897 \\ (0.873) \end{gathered}$ |
| Non-Def. Shock (lag 2) | $\begin{gathered} 1.237^{*} \\ (0.645) \end{gathered}$ | $\begin{gathered} 0.621 \\ (0.762) \end{gathered}$ |
| Non-Def. Shock*Non-Def. Vol. | $\begin{aligned} & 44.370^{* * *} \\ & (13.621) \end{aligned}$ | $\begin{gathered} -48.566 \\ (57.326) \end{gathered}$ |
| Non-Def. Shock*Non-Def. Vol. (lag 1) | $\begin{gathered} 7.715 \\ (14.512) \end{gathered}$ | $\begin{aligned} & 111.965^{*} \\ & (64.150) \end{aligned}$ |
| Non-Def. Shock*Non-Def. Vol. (lag 2) | $\begin{gathered} 2.331 \\ (13.946) \end{gathered}$ | $\begin{gathered} -25.694 \\ (37.091) \end{gathered}$ |
| Def. Shock |  | $\begin{gathered} 1.973^{*} \\ (1.042) \end{gathered}$ |
| Def. Shock (lag 1) |  | $\begin{gathered} -1.492 \\ (1.147) \end{gathered}$ |
| Def. Shock (lag 2) |  | $\begin{gathered} -0.025 \\ (0.814) \end{gathered}$ |
| Def. Shock*Def. Vol |  | $\begin{array}{r} -13.765^{*} \\ (7.010) \end{array}$ |
| Def. Shock*Def. Vol (lag 1) |  | $\begin{aligned} & 10.088 \\ & (7.698) \end{aligned}$ |
| Def. Shock*Def. Vol (lag 2) |  | $\begin{gathered} 0.424 \\ (5.360) \\ \hline \end{gathered}$ |
| Observations | 86 | 86 |
| Adjusted $\mathrm{R}^{2}$ | 0.354 | 0.346 |
| Residual Std. Error | 0.040 | 0.040 |
| F Statistic | $6.814^{* * *}$ | $4.218^{* * *}$ |
| Note: | * $\mathrm{p}<0$. | 0.05; ${ }^{* * *} \mathrm{p}<0.01$ |

### 6.1.4 United States Instrumental Variable Empirical Results

The US data spans 94 years (from 1912 to 2006). Table XXXV reports the parameter estimates for the baseline static model described in Section 4.3. The left column shows the contemporaneous effect of government purchases on output, with $m^{\sigma}=0$. The simple contemporaneous estimation suggests a multiplier of 0.572 .

## TABLE XXXV

US Defense Spending IV Baseline Model Empirical Results

|  | Dependent variable: Output Growth |  |
| :--- | :---: | :---: |
|  | Equation 4.14 | Equation 4.15 |
| Fiscal Shock | $0.572^{* * *}$ | $0.903^{* * *}$ |
|  | $(0.091)$ | $(0.228)$ |
| Fiscal Shock |  | -3.726 |
| * Volatility |  | $(2.613)$ |
|  |  | 88 |
| Observations | 89 | 0.321 |
| Adjusted R | 0.041 |  |
| Residual Std. Error | 0.041 | $21.747^{* * *}$ |
| F Statistic | $39.838^{* * *}$ | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |

The right column of Table XXXV models the fiscal multiplier as dependent on the volatility of fiscal shocks ${ }^{1}$. The coefficients on the volatility interaction terms for the US Defense Spending IV model are negative, large, and significant. These findings support the hypothesis that volatility in government spending reduces the fiscal multiplier. The multiplier for a hypothetical country with zero volatility is simply the coefficient on the government spending shock 0.903.

While the baseline static model provides a useful benchmark for comparison, the dynamic model with 2 lags and a volatility window of $\mathrm{k}=6$ years offers a more realistic perspective. Table XXXVI reports the parameter estimates for the US IV dynamic model. The left column shows the dynamic effect of government purchases on output when the $m_{j}^{\sigma}$ are constrained to 0 . The fixed effects model parameters imply a long-run multiplier of 0.567 .

The right column of Table XXXVI allows the fiscal multiplier to depend on the volatility of fiscal shocks. The parameters on the volatility interaction terms are large and negative on net but not significant. These findings therefore thinly support the hypothesis that recent volatility in government spending reduces the fiscal multiplier. The multiplier for a hypothetical country with zero volatility is 1.07 . Table XXXVII highlights the impact of volatility on the multiplier. It lists the implied multipliers for a five-number summary of all the volatilities recorded in the panel. Next, the multipliers implied by a five-number summary of mean volatilities over time in each country are reported. Across all volatilities in the data, the implied multipliers range from 1.06 in 1997 (volatility $=0.0015)$ to 0.389 in $1952($ volatility $=0.111)$.

[^15]
## TABLE XXXVI

US Defense Spending IV - Dynamic Empirical Results

|  | Dependent variable: Output Growth |  |
| :--- | :---: | :---: |
|  | Equation 4.14 | Equation 4.15 |
| Output Growth (lag 1) | $0.333^{* * *}$ | $0.327^{* * *}$ |
|  | $(0.115)$ | $(0.116)$ |
| Output Growth (lag 2) | -0.098 | -0.125 |
|  | $(0.113)$ | $(0.116)$ |
| Fiscal Shock | $0.443^{* * *}$ | $0.785^{* * *}$ |
|  | $(0.128)$ | $(0.266)$ |
| Fiscal Shock (lag 1) | 0.019 | 0.039 |
|  | $(0.137)$ | $(0.330)$ |
| Fiscal Shock (lag 2) | -0.037 | 0.029 |
|  | $(0.117)$ | $(0.305)$ |
| Fiscal Shock * Volatility |  | -4.593 |
|  |  | $2.907)$ |
| Fiscal Shock * Volatility (lag 1) |  | 0.547 |
|  |  | $(3.088)$ |
| Fiscal Shock * Volatility (lag 2) |  | -1.145 |
|  |  | $(3.013)$ |
| Observations | 87 | 86 |
| Adjusted R |  | 0.335 |
| Residual Std. Error | 0.343 | 0.040 |
| F Statistic | 0.040 | $6.228^{* * *}$ |
| Note: | $9.960^{* * *}$ | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |

## TABLE XXXVII

US Defense Spending IV - Implied Historical Long-run Multipliers

|  | Zero Vol. | 0th $\%$-ile | 25 th $\%$-ile | 50th $\%$-ile | 75 th $\%$-ile | 100th $\%$-ile |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year |  | 1997 | 1994 | 1964 | 1926 | 1952 |
| Volatility | 0 | 0.0015 | 0.00278 | 0.00553 | 0.0257 | 0.111 |
| Multiplier | 1.07 | 1.06 | 1.06 | 1.04 | 0.915 | 0.389 |



Figure 20. US Defense Spending IV - Fixed Effects Impulse Responses to a 1\% Fiscal Shock


Figure 21. US Defense Spending IV - Implied Historical Multipliers

Impulse responses demonstrate the evolution of variables in dynamic systems over time. Figure 20 tracks the response of output to fiscal shock of $1 \%$ of GDP for the fixed effects estimations of the US Defense Spending IV dynamic model with 2 lags contingent on a 6 year volatility window. The reported volatilities reflect a five-number summary of the mean volatilities of the countries in this dataset. Recall that shocks, g , are defined as an increase in government spending equal to $1 \%$ of GDP and not simply a $1 \%$ increase in the level of government spending.

### 6.1.5 World Development Indicators Empirical Results

The WDI panel spans 89 countries and 24 years (from 1988 to 2011). The defense spending model is developed in Section 4.3. Table XXXVIII reports the parameter estimates for the WDI baseline static model.

$$
y_{i, t}=v_{i}+w_{t}+m d_{i, t}+m^{\sigma} d_{i, t} * \sigma_{d_{i, t}}^{k}+\epsilon_{i, t}
$$

The left three columns of Table XXXVIII show the contemporaneous effects of government purchases on output, with $m^{\sigma}=0$. The fixed effects estimation suggests a multiplier of 0.208 .

## TABLE XXXVIII

| WDI - Baseline Empirical Results |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Dependent variable: Output Growth |  |  |  |  |  |
|  | OLS | FE | RE | OLS | Fquation 4.11 | RE |
| Defense Shock | $0.260^{*}$ | $0.208^{*}$ | 0.131 | 0.185 | 0.060 | -0.030 |
|  | $(0.134)$ | $(0.111)$ | $(0.119)$ | $(0.167)$ | $(0.138)$ | $(0.148)$ |
| Defense Shock |  |  |  | 1.178 | $2.315^{*}$ | $2.506^{*}$ |
| * Volatility |  |  |  | $(1.547)$ | $(1.283)$ | $(1.375)$ |
| Observations | 1,513 | 1,513 | 1,513 | 1,513 | 1,513 | 1,513 |
| Adjusted R ${ }^{2}$ | 0.002 | 0.002 | 0.001 | 0.003 | 0.004 | 0.003 |
| F Statistic | $3.746^{*}$ | $3.532^{*}$ | 1.213 | 2.162 | $3.398^{* *}$ | 2.267 |
| Note: |  |  |  |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |  |

The right three columns of Table XXXVIII model the defense spending multiplier as dependent on the volatility of defense spending shocks. The multiplier for a hypothetical country with zero volatility is simply the coefficient on the government spending shock, 0.0601.

In order to illustrate the impact of volatility on multipliers in the baseline static model, consider the range of multipliers implied for the countries with the minimum and maximum
mean volatilities over time. These contemporaneous multipliers range from 0.0604 for Japan (volatility $=0.000116$ ) to 0.272 for Kuwait (volatility $=0.0914$ ).

While the baseline static model provides a useful benchmark for comparison, the dynamic model with $\mathrm{J}=2$ lags and a volatility window of $\mathrm{k}=6$ years offers a more realistic perspective.

$$
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} d_{i, t-j}+m_{j}^{\sigma} d_{i, t-j} \sigma_{d_{i, t-j}}^{k}\right)+\epsilon_{i, t}
$$

Table XXXIX reports the parameter estimates for the WDI dynamic model. The left two columns show the dynamic effects of defense spending on output when $m_{j}^{\sigma}$ is constrained to 0 . The fixed effects model parameters imply a long-run multiplier of -0.0943 .

The third and fourth columns of Table XXXIX allow the defense spending multiplier to depend on the volatility of defense spending shocks. The volatility interaction coefficients are negative and large but insignificant. Based on Equation 4.7, the multiplier for a hypothetical country with zero volatility is 0.036 .

The Hausman test is used to determine whether the fixed effects (FE) specification or the random effects (RE) specification is preferred. The Hausman statistic of 5.42 implies p-value of 0.712 and thus the null hypothesis of the RE model is not rejected.

Table XL lists implied long-run multipliers for a number of countries in order to highlight the impact of volatility on the multiplier. A five-number summary of the mean volatilities across time for each country is used to calculate typical multipliers for the representative countries.

## TABLE XXXIX

WDI - Dynamic Empirical Results

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Equation 4.12 |  | Equation 4.13 |  |
|  | FE | RE | FE | RE |
| Output Growth (lag 1) | $\begin{aligned} & 0.220^{* * *} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.255^{* * *} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.220^{* * *} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.254^{* * *} \\ & (0.027) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{gathered} 0.005 \\ (0.028) \end{gathered}$ | $\begin{gathered} 0.019 \\ (0.027) \end{gathered}$ | $\begin{gathered} 0.005 \\ (0.028) \end{gathered}$ | $\begin{gathered} 0.018 \\ (0.027) \end{gathered}$ |
| Defense Shock | $\begin{gathered} 0.114 \\ (0.122) \end{gathered}$ | $\begin{gathered} 0.034 \\ (0.132) \end{gathered}$ | $\begin{gathered} 0.274 \\ (0.217) \end{gathered}$ | $\begin{gathered} 0.009 \\ (0.232) \end{gathered}$ |
| Defense Shock (lag 1) | $\begin{gathered} -0.104 \\ (0.124) \end{gathered}$ | $\begin{gathered} -0.069 \\ (0.134) \end{gathered}$ | $\begin{gathered} -0.184 \\ (0.157) \end{gathered}$ | $\begin{gathered} -0.078 \\ (0.169) \end{gathered}$ |
| Defense Shock (lag 2) | $\begin{gathered} -0.083 \\ (0.115) \end{gathered}$ | $\begin{gathered} -0.079 \\ (0.123) \end{gathered}$ | $\begin{gathered} -0.062 \\ (0.143) \end{gathered}$ | $\begin{gathered} -0.103 \\ (0.154) \end{gathered}$ |
| Defense Shock * Volatility |  |  | -3.612 | 0.512 |
|  |  |  | (3.989) | (4.283) |
| Defense Shock <br> * Volatility (lag 1) |  |  | 0.664 | 0.200 |
|  |  |  | (1.264) | (1.361) |
| Defense Shock <br> * Volatility (lag 2) |  |  | -0.327 | 0.383 |
|  |  |  | (1.328) | (1.429) |
| Observations | 1,335 | 1,335 | 1,335 | 1,335 |
| Adjusted R ${ }^{2}$ | 0.047 | 0.069 | 0.047 | 0.068 |
| F Statistic | 13.168*** | 19.711*** | 8.344*** | $12.240^{* * *}$ |
| Note: |  |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}$ | 5; ${ }^{* * *} \mathrm{p}<0.01$ |

Multipliers range from 0.0355 for Japan (volatility $=0.000116$ ) to -0.35 for Kuwait (volatility $=0.0914)$.

TABLE XL

WDI - Long-run Multipliers across Mean Volatilities

|  | Zero Vol. | 0th \%-ile | 25th \%-ile | 50th \%-ile | 75th \%-ile | 100th \%-ile |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Country | NA | Japan | Tunisia | Greece | Laos | Kuwait |
| Man Volatility | 0 | 0.000116 | 0.000935 | 0.00191 | 0.00399 | 0.0914 |
| Multiplier | 0.036 | 0.0355 | 0.032 | 0.0279 | 0.0191 | -0.35 |

Figure 22 tracks the impulse responses of output to $1 \%$ defense spending shocks for the fixed effects WDI dynamic model. Impulse responses demonstrate the evolution of variables in dynamic systems over time. The volatilities used to generate the responses are based on a fivenumber summary of the mean volatilities for each country across time. Notice that long-run multipliers are significantly larger for countries with lower defense spending volatility.

Table XLI considers whether the multiplier due to defense spending is different than that due to non-defense spending. While the defense spending multiplier is significantly greater than zero, the hypothesis that the multipliers across spending types are identical cannot be rejected.


Figure 22. WDI - Impulse Responses to a 1\% Defense Shock

Table XLII considers whether the effect of defense spending volatility on the multiplier is different than that due to non-defense spending volatility. Non-defense spending volatility significantly reduces the multiplier, while defense spending volatility significantly increases the multiplier.

Table XLIII considers whether the multiplier due to defense spending in the dynamic model is different than that due to non-defense spending. While the defense spending multiplier is significantly greater than zero, the hypothesis that the multipliers across spending types are identical cannot be rejected.

Table XLIV considers whether the effect of defense spending volatility on the multiplier is different than that due to non-defense spending volatility for the dynamic model. Nondefense spending volatility significantly reduces the multiplier, while defense spending volatility significantly increases the multiplier.

## TABLE XLI

WDI - Defense and Non-Defense Static Empirical Results

|  | Dependent variable: |  |  | Output Growth |
| :--- | :---: | :---: | :---: | :---: |
|  | Non-Defense |  | Non-Def. and Def. |  |
|  | FE | RE.nonm | FE | RE.nonm.m |
| Non-Defense Shock | 0.030 | $0.057^{* * *}$ | 0.028 | $0.056^{* * *}$ |
|  | $(0.020)$ | $(0.022)$ | $(0.020)$ | $(0.022)$ |
| Defense Shock |  |  | $0.203^{*}$ | 0.121 |
|  |  |  | $(0.111)$ | $(0.118)$ |
| Observations | 1,513 | 1,513 | 1,513 | 1,513 |
| Adjusted R ${ }^{2}$ | 0.001 | 0.005 | 0.004 | 0.005 |
| F Statistic | 2.145 | $6.956^{* * *}$ | $2.752^{*}$ | $4.000^{* *}$ |
| Note: |  |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |  |

TABLE XLII

WDI - Defense and Non-Defense Static Empirical Results with Interactions

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Non-Defense |  | Non-Def. and Def. |  |
|  | FE | RE | FE | RE |
| Non-Def. Shock | $\begin{gathered} 0.028 \\ (0.020) \end{gathered}$ | $\begin{aligned} & 0.056^{* *} \\ & (0.022) \end{aligned}$ | $\begin{gathered} 0.033 \\ (0.020) \end{gathered}$ | $\begin{aligned} & 0.060^{* * *} \\ & (0.022) \end{aligned}$ |
| Non-Def. Shock*Non-Def. Volatility | $\begin{gathered} 0.838 \\ (0.693) \end{gathered}$ | $\begin{gathered} 0.602 \\ (0.742) \end{gathered}$ | $\begin{gathered} -3.359^{* *} \\ (1.549) \end{gathered}$ | $\begin{gathered} -2.627 \\ (1.655) \end{gathered}$ |
| Def. Shock |  |  | $\begin{gathered} 0.331^{*} \\ (0.187) \end{gathered}$ | $\begin{gathered} 0.177 \\ (0.199) \end{gathered}$ |
| Def. Shock*Def. Volatility |  |  | $\begin{aligned} & 4.752^{* * *} \\ & (1.712) \end{aligned}$ | $\begin{gathered} 4.365^{* *} \\ (1.832) \end{gathered}$ |
| Observations | 1,513 | 1,513 | 1,513 | 1,513 |
| Adjusted R ${ }^{2}$ | 0.002 | 0.005 | 0.009 | 0.009 |
| F Statistic | 1.804 | $3.806^{* *}$ | $3.356^{* * *}$ | $3.422^{* * *}$ |
| Note: |  |  | ${ }^{*} \mathrm{p}<0.1$; ${ }^{* *} \mathrm{p}$ | ; ${ }^{* * *} \mathrm{p}<0.01$ |

## TABLE XLIII

WDI - Defense and Non-Defense Empirical Results with Interactions

|  | Dependent variable: Output Growth |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Non-Def. |  | Non-Def. and Def. |  |
|  | FE.nonm | RE.nonm | FE.nonm.m | RE.nonm.m |
| Output Growth (lag 1) | $0.218^{* * *}$ | $0.253^{* * *}$ | $0.219^{* * *}$ | $0.253^{* * *}$ |
|  | $(0.028)$ | $(0.027)$ | $(0.028)$ | $(0.027)$ |
| Output Growth (lag 2) | 0.002 | 0.018 | 0.004 | 0.018 |
|  | $(0.028)$ | $(0.027)$ | $(0.028)$ | $(0.027)$ |
| Non-Def. Shock | 0.021 | $0.044^{*}$ | 0.018 | $0.043^{*}$ |
|  | $(0.022)$ | $(0.024)$ | $(0.022)$ | $(0.024)$ |
| Non-Def. Shock (lag 1) | 0.018 | -0.0003 | 0.020 | 0.0004 |
|  | $(0.022)$ | $(0.023)$ | $(0.022)$ | $(0.023)$ |
| Non-Def. Shock (lag 2) | -0.004 | -0.003 | -0.010 | -0.006 |
|  | $(0.022)$ | $(0.023)$ | $(0.022)$ | $(0.024)$ |
| Def. Shock |  |  | 0.136 | 0.031 |
|  |  |  | $(0.124)$ | $(0.134)$ |
| Def. Shock (lag 1) |  |  | -0.109 | -0.074 |
|  |  |  | $(0.125)$ | $(0.134)$ |
| Def. Shock (lag 2) |  |  | -0.083 | -0.077 |
|  |  |  | $(0.115)$ | $(0.123)$ |
| Observations | 1,335 | 0.071 | 1,335 | 1,335 |
| Adjusted R ${ }^{2}$ |  |  | 0.048 | 0.072 |
| F Statistic | 0.046 |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;$ | ${ }^{* * *} \mathrm{p}<0.01$ |

TABLE XLIV

WDI - Defense and Non-Defense Dynamic Empirical Results

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Non-Defense |  | Non-Def. and Def. |  |
|  | FE | RE | FE | RE |
| Output Growth (lag 1) | $\begin{aligned} & 0.218^{* * *} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.252^{* * *} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.209^{* * *} \\ & (0.029) \end{aligned}$ | $\begin{aligned} & 0.244^{* * *} \\ & (0.027) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{gathered} 0.005 \\ (0.028) \end{gathered}$ | $\begin{gathered} 0.019 \\ (0.027) \end{gathered}$ | $\begin{gathered} -0.004 \\ (0.028) \end{gathered}$ | $\begin{gathered} 0.010 \\ (0.027) \end{gathered}$ |
| Non-Def. Shock | $\begin{gathered} 0.020 \\ (0.022) \end{gathered}$ | $\begin{gathered} 0.044^{*} \\ (0.024) \end{gathered}$ | $\begin{gathered} 0.028 \\ (0.022) \end{gathered}$ | $\begin{gathered} 0.051^{* *} \\ (0.024) \end{gathered}$ |
| Non-Def. Shock (lag 1) | $\begin{gathered} 0.019 \\ (0.022) \end{gathered}$ | $\begin{gathered} 0.0001 \\ (0.023) \end{gathered}$ | $\begin{gathered} 0.023 \\ (0.022) \end{gathered}$ | $\begin{gathered} 0.003 \\ (0.023) \end{gathered}$ |
| Non-Def. Shock (lag 2) | $\begin{gathered} -0.006 \\ (0.022) \end{gathered}$ | $\begin{gathered} -0.003 \\ (0.024) \end{gathered}$ | $\begin{gathered} 0.002 \\ (0.022) \end{gathered}$ | $\begin{gathered} 0.004 \\ (0.024) \end{gathered}$ |
| Non-Def. Shock <br> $*$ Non-Def. Volatility | -0.343 | -0.533 | -4.637 | $-5.064$ |
|  | (0.876) | (0.946) | (3.133) | (3.366) |
| Non-Def. Shock <br> *Non-Def. Volatility (lag 1) | $-1.537^{*}$ | -1.236 | $-3.997^{*}$ | $-2.619$ |
| Non-Def. Shock <br> $*$ Non-Def. Volatility (lag 2) | (0.867) | (0.932) | (2.361) | (2.547) |
|  | -1.472* | -1.310 | $-5.660^{* * *}$ | $-5.056^{* * *}$ |
|  | (0.755) | (0.813) | (1.705) | (1.839) |
| Def. Shock |  |  | $\begin{gathered} 0.451^{* *} \\ (0.230) \end{gathered}$ | $\begin{gathered} 0.153 \\ (0.246) \end{gathered}$ |
| Def. Shock (lag 1) |  |  | $\begin{aligned} & 0.00004 \\ & (0.192) \end{aligned}$ | $\begin{gathered} 0.032 \\ (0.206) \end{gathered}$ |
| Def. Shock (lag 2) |  |  | $\begin{gathered} 0.317^{*} \\ (0.190) \end{gathered}$ | $\begin{gathered} 0.243 \\ (0.204) \end{gathered}$ |
| Def. Shock <br> *Def. Volatility |  |  | 3.344 | 9.684 |
|  |  |  | (10.713) | (11.516) |
| Def. Shock <br> *Def. Volatility (lag 1) |  |  | 3.148 | 1.718 |
| Def. Shock <br> *Def. Volatility (lag 2) |  |  | (2.222) | (2.397) |
|  |  |  | 4.554** | $5.047^{* *}$ |
|  |  |  | (2.027) | (2.175) |
| Observations | 1,335 | 1,335 | 1,335 | 1,335 |
| Adjusted $\mathrm{R}^{2}$ | 0.049 | 0.073 | 0.059 | 0.077 |
| F Statistic | 8.719*** | 13.170*** | $6.015^{* * *}$ | 7.942*** |

### 6.1.6 World Development Indicators Instrumental Variable Empirical Results

The WDI IV panel spans 89 countries and 23 years (from 1989 to 2011). The IV model is developed in Section 4.3. Table XLV reports the parameter estimates for the WDI IV baseline static model. The left three columns of Table XLV show the contemporaneous effects of government purchases on output, with $m^{\sigma}=0$. The fixed effects estimation suggests a multiplier of 0.176.

TABLE XLV

WDI IV - Baseline Empirical Results

|  | Dependent variable: Output Growth |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | OLS | FE | RE | OLS | FE | RE |
| Defense Shock | $0.221^{*}$ | $0.176^{*}$ | 0.111 | -0.284 | -1.095 | -1.118 |
|  | $(0.115)$ | $(0.095)$ | $(0.101)$ | $(0.974)$ | $(1.201)$ | $(1.154)$ |
| Defense Shock |  |  |  | 3.107 | 7.550 | 7.525 |
| * Volatility |  |  |  | $(5.876)$ | $(7.022)$ | $(6.873)$ |
| Observations | 1,513 | 1,513 | 1,513 | 1,513 | 1,513 | 1,513 |
| Adjusted R ${ }^{2}$ | 0.009 | 0.002 | 0.005 | 0.0003 | 0.002 | 0.002 |
| F Statistic | -19.005 | -45.799 | $-1,509.512$ | -126.867 | -410.628 | -754.092 |
| Note: |  |  |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |  |  |

The right three columns of Table XLV model the defense spending multiplier as dependent on the volatility of defense spending shocks. ${ }^{1}$ The multiplier for a hypothetical country with zero volatility is simply the coefficient on the government spending shock, -1.1 .

While the baseline static model provides a useful benchmark for comparison, the dynamic model with $\mathrm{J}=2$ lags and a volatility window of $\mathrm{k}=6$ years offers a more realistic perspective.

Table XLVI reports the parameter estimates for the WDI IV dynamic model. The left two columns show the dynamic effects of defense spending on output when $m_{j}^{\sigma}$ is constrained to 0 . The fixed effects model parameters imply a long-run multiplier of -0.276 .

The third and fourth columns of Table XLVI allow the defense spending multiplier to depend on the volatility of defense spending shocks. The FE volatility interaction coefficients are negative and large but insignificant. Based on Equation 4.7, the multiplier for a hypothetical country with zero volatility is 0.392 .

The Hausman test is used to determine whether the fixed effects (FE) specification or the random effects (RE) specification is preferred. The Hausman statistic of 10.2 implies p-value of 0.254 and thus the null hypothesis of the RE model is not rejected.

Table XLVII lists implied long-run multipliers for a number of countries in order to highlight the impact of volatility on the multiplier. A five-number summary of the mean volatilities across time for each country is used to calculate typical multipliers for the representative countries. Multipliers range from 0.385 (volatility $=0.00232$ ) to 0.0113 (volatility $=0.119)$.

[^16]
## TABLE XLVI

WDI IV - Dynamic Empirical Results

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Equation 4.14 |  | Equation 4.15 |  |
|  | FE | RE | FE | RE |
| Output Growth (lag 1) | $\begin{aligned} & 0.227^{* * *} \\ & (0.034) \end{aligned}$ | $\begin{aligned} & 0.264^{* * *} \\ & (0.035) \end{aligned}$ | $\begin{aligned} & 0.208^{* * *} \\ & (0.066) \end{aligned}$ | $\begin{aligned} & 0.274^{* * *} \\ & (0.051) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{gathered} 0.015 \\ (0.033) \end{gathered}$ | $\begin{gathered} 0.031 \\ (0.034) \end{gathered}$ | $\begin{gathered} -0.023 \\ (0.079) \end{gathered}$ | $\begin{gathered} 0.049 \\ (0.098) \end{gathered}$ |
| Defense Shock | $\begin{gathered} 0.028 \\ (0.156) \end{gathered}$ | $\begin{gathered} -0.008 \\ (0.171) \end{gathered}$ | $\begin{gathered} 0.611 \\ (0.850) \end{gathered}$ | $\begin{gathered} -0.070 \\ (0.762) \end{gathered}$ |
| Defense Shock (lag 1) | $\begin{gathered} -0.160 \\ (0.169) \end{gathered}$ | $\begin{gathered} -0.119 \\ (0.180) \end{gathered}$ | $\begin{gathered} -0.734 \\ (0.675) \end{gathered}$ | $\begin{gathered} -0.229 \\ (0.620) \end{gathered}$ |
| Defense Shock (lag 2) | $\begin{gathered} -0.077 \\ (0.117) \end{gathered}$ | $\begin{gathered} -0.077 \\ (0.124) \end{gathered}$ | $\begin{gathered} 0.443 \\ (1.150) \end{gathered}$ | $\begin{gathered} -0.169 \\ (1.086) \end{gathered}$ |
| Defense Shock <br> * Volatility |  |  | $\begin{gathered} -3.762 \\ (5.724) \end{gathered}$ | $\begin{gathered} 0.554 \\ (5.216) \end{gathered}$ |
| Defense Shock <br> * Volatility (lag 1) |  |  | $\begin{gathered} 3.564 \\ (4.220) \end{gathered}$ | $\begin{gathered} 0.883 \\ (3.914) \end{gathered}$ |
| Defense Shock <br> * Volatility (lag 2) |  |  | $\begin{array}{r} -2.421 \\ (6.546) \\ \hline \end{array}$ | $\begin{gathered} 0.669 \\ (6.364) \\ \hline \end{gathered}$ |
| Observations | 1,335 | 1,335 | 1,335 | 1,335 |
| Adjusted $\mathrm{R}^{2}$ | 0.020 | 0.056 | 0.003 | 0.041 |
| F Statistic | -2.414 | -265.535 | -47.642 | -165.524 |
| Note: |  |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}$ | ; ${ }^{* * *} \mathrm{p}<0.01$ |

## TABLE XLVII

WDI IV - Long-run Multipliers across Mean Volatilities

|  | Zero Vol. | 0th $\%$-ile | 25th $\%$-ile | 50th \%-ile | 75 th $\%$-ile | 100 th $\%$-ile |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Country | NA | NA | France | Denmark | Botswana | Angola |
| Mean Volatility | 0 | 0.00232 | 0.00623 | 0.00924 | 0.0184 | 0.119 |
| Multiplier | 0.392 | 0.385 | 0.372 | 0.363 | 0.333 | 0.0113 |



Figure 23. WDI IV - Impulse Responses to a 1\% Defense Shock


Figure 24. WDI IV - Implied Historical Multipliers for Representative Economies

Figure 23 tracks the impulse responses of output to $1 \%$ defense spending shocks for the fixed effects WDI IV dynamic model. Impulse responses demonstrate the evolution of variables in dynamic systems over time. The volatilities used to generate the responses are based on a five-number summary of the mean volatilities for each country across time. Notice that long-run multipliers are significantly larger for countries with lower defense spending volatility.

Panel (a) of Figure 24 illustrates the long-run multipliers implied by the WDI IV dynamic model contingent on historical data for the smallest, median, and largest economies in the WDI IV panel. The average multiplier across the countries in panel (a) is 0.38 and the minimum and maximum multipliers are 0.362 and 0.391 respectively. Panel (b) illustrates the long-run multipliers implied by the WDI IV dynamic model contingent on historical data for the poorest, median, and wealthiest economies in terms of per capita income. The average multiplier across the countries in panel (b) is 0.117 and the minimum and maximum multipliers are -2.85 and 0.392 respectively.

### 6.1.7 North Atlantic Treaty Organization Empirical Results

The NATO panel spans 14 countries and 55 years (from 1957 to 2011). The defense spending model is developed in Section 4.3. Table XLVIII reports the parameter estimates for the NATO baseline static model.

$$
y_{i, t}=v_{i}+w_{t}+m d_{i, t}+m^{\sigma} d_{i, t} * \sigma_{d_{i, t}}^{k}+\epsilon_{i, t}
$$

The left three columns of Table XLVIII show the contemporaneous effects of government purchases on output, with $m^{\sigma}=0$. The fixed effects estimation suggests a multiplier of 1.72.

TABLE XLVIII

|  | Dependent variable: Output Growth |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Equation 4.10 |  |  | Equation 4.11 |  |  |
|  | OLS | FE | RE | OLS | FE | RE |
| Defense Shock | $\begin{aligned} & 1.604 * * * \\ & (0.397) \end{aligned}$ | $\begin{aligned} & 1.717^{* * *} \\ & (0.321) \end{aligned}$ | $\begin{aligned} & 1.478^{* * *} \\ & (0.394) \end{aligned}$ | $\begin{gathered} 1.628^{* *} \\ (0.727) \end{gathered}$ | $\begin{aligned} & 2.252^{* * *} \\ & (0.598) \end{aligned}$ | $\begin{gathered} 1.406^{*} \\ (0.722) \end{gathered}$ |
| Defense Shock <br> * Volatility |  |  |  | $\begin{gathered} -4.971 \\ (128.565) \\ \hline \end{gathered}$ | $\begin{array}{r} -109.311 \\ (102.998) \\ \hline \end{array}$ | $\begin{array}{r} 15.165 \\ (127.276) \\ \hline \end{array}$ |
| Observations | 686 | 686 | 686 | 686 | 686 | 686 |
| Adjusted R ${ }^{2}$ | 0.023 | 0.040 | 0.020 | 0.023 | 0.041 | 0.020 |
| F Statistic | $16.328^{* * *}$ | $28.541^{* * *}$ | $14.089 * * *$ | 8.153*** | $14.836^{* * *}$ | 7.040*** |

The right three columns of Table XLVIII model the defense spending multiplier as dependent on the volatility of defense spending shocks. The volatility interaction coefficients for the static NATO model are negative and large but insignificant. The multiplier for a hypothetical country with zero volatility is simply the coefficient on the government spending shock, 2.25.

In order to illustrate the impact of volatility on multipliers in the baseline static model, consider the range of multipliers implied for the countries with the minimum and maximum mean volatilities over time. These contemporaneous multipliers range from 2.14 for Belgium (volatility $=0.00101$ ) to 1.76 for Turkey (volatility $=0.00453$ ).

While the baseline static model provides a useful benchmark for comparison, the dynamic model with $\mathrm{J}=2$ lags and a volatility window of $\mathrm{k}=6$ years offers a more realistic perspective.

$$
y_{i, t}=v_{i}+w_{t}+\sum_{j=1}^{J} \alpha_{j} y_{i, t-j}+\sum_{j=0}^{J}\left(m_{j} d_{i, t-j}+m_{j}^{\sigma} d_{i, t-j} \sigma_{d_{i, t-j}}^{k}\right)+\epsilon_{i, t}
$$

Table XLIX reports the parameter estimates for the NATO dynamic model. The left two columns show the dynamic effects of defense spending on output when $m_{j}^{\sigma}$ is constrained to 0 . The fixed effects model parameters imply a long-run multiplier of 2.26.

The third and fourth columns of Table XLIX allow the defense spending multiplier to depend on the volatility of defense spending shocks. The volatility interaction coefficients are negative and large but insignificant. Based on Equation 4.7, the multiplier for a hypothetical country with zero volatility is 3.38 .

## TABLE XLIX

NATO - Dynamic Empirical Results


The Hausman test is used to determine whether the fixed effects (FE) specification or the random effects (RE) specification is preferred. The Hausman statistic of 72.6 implies p-value of $1.5 \mathrm{e}-12$ and thus the null hypothesis of the RE model is rejected. The remaining tables and figures based on Table XLIX consider the FE model.

Table L lists implied long-run multipliers for a number of countries in order to highlight the impact of volatility on the multiplier. A five-number summary of the mean volatilities across time for each country is used to calculate typical multipliers for the representative countries. Multipliers range from 3.17 for Belgium (volatility $=0.00101$ ) to 2.41 for Turkey (volatility $=$ $0.00453)$.

## TABLE L

NATO - Long-run Multipliers across Mean Volatilities

|  | Zero Vol. | 0th $\%$-ile | 25 th $\%$-ile | 50th $\%$-ile | 75th $\%$-ile | 100 th $\%$-ile |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Country | NA | Belgium | Denmark | Netherlands | Greece | Turkey |
| Mean Volatility | 0 | 0.00101 | 0.00113 | 0.0016 | 0.00333 | 0.00453 |
| Multiplier | 3.38 | 3.17 | 3.14 | 3.04 | 2.67 | 2.41 |

Figure 25 tracks the impulse responses of output to $1 \%$ defense spending shocks for the fixed effects NATO dynamic model. Impulse responses demonstrate the evolution of variables


Figure 25. NATO - Impulse Responses to a $1 \%$ Defense Shock
in dynamic systems over time. The volatilities used to generate the responses are based on a five-number summary of the mean volatilities for each country across time. Notice that long-run multipliers are significantly larger for countries with lower defense spending volatility.

Panel (a) of Figure 26 illustrates the long-run multipliers implied by the NATO dynamic model contingent on historical data for the smallest, median, and largest economies in the NATO panel. The average multiplier across the countries in panel (a) is 3.38 and the minimum and maximum multipliers are 3.35 and 3.38 respectively. Panel (b) illustrates the long-run multipliers implied by the NATO dynamic model contingent on historical data for the poorest, median, and wealthiest economies in terms of per capita income. The average multiplier across the countries in panel (b) is 3.37 and the minimum and maximum multipliers are 3.34 and 3.38 respectively.


Figure 26. NATO - Implied Historical Multipliers for Representative Economies

Table LI considers whether the multiplier due to defense spending is different than that due to non-defense spending. Both multipliers are significantly greater than zero. And the defense spending multiplier is significantly greater than the non-defense spending multiplier.

Table LII considers whether the effect of defense spending volatility on the multiplier is different than that due to non-defense spending volatility. The hypothesis that both effects are zero cannot be rejected.

Table LIII considers whether the multiplier due to defense spending in the dynamic model is different than that due to non-defense spending. Both long-run multipliers are significantly greater than zero. The defense spending multiplier is significantly greater than the non-defense spending multiplier.

Table LIV considers whether the effect of defense spending volatility on the multiplier is different than that due to non-defense spending volatility for the dynamic model. The
hypothesis that neither defense volatility nor non-defense volatility is significantly different from zero cannot be rejected.

TABLE LI

NATO - Defense and Non-Defense Static Empirical Results

|  | Dependent variable: Output Growth |  |  |  |
| :--- | :---: | :---: | :--- | :---: | :---: |
|  | Non-Defense |  | Non-Def. and Def. |  |
|  | FE | RE | FE | RE |
| Non-Defense Shock | $0.209^{*}$ | $0.284^{* *}$ | $0.373^{* * *}$ | $0.357^{* * *}$ |
|  | $(0.122)$ | $(0.132)$ | $(0.122)$ | $(0.132)$ |
| Defense Shock |  |  | $1.945^{* * *}$ | $1.624^{* * *}$ |
|  |  |  | $(0.328)$ | $(0.396)$ |
| Observations | 686 | 686 | 686 | 686 |
| Adjusted $\mathrm{R}^{2}$ | 0.004 | 0.007 | 0.053 | 0.031 |
| F Statistic | $2.937^{*}$ | $4.600^{* *}$ | $19.141^{* * *}$ | $10.793^{* * *}$ |
| Note: |  |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |  |

TABLE LII

NATO - Defense and Non-Defense Static Empirical Results with Interactions

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Non-Defense |  | Non-Def. and Def. |  |
|  | FE | RE | FE | RE |
| Non-Def. Shock | $\begin{aligned} & 0.344^{* * *} \\ & (0.122) \end{aligned}$ | $\begin{gathered} 0.339^{* *} \\ (0.132) \end{gathered}$ | $\begin{aligned} & 0.369^{* * *} \\ & (0.122) \end{aligned}$ | $\begin{aligned} & 0.362^{* * *} \\ & (0.133) \end{aligned}$ |
| Non-Def. Shock*Non-Def. Volatility | $\begin{aligned} & 273.076^{* * *} \\ & (51.766) \end{aligned}$ | $\begin{aligned} & 210.377^{* * *} \\ & (63.170) \end{aligned}$ | $\begin{gathered} 84.415 \\ (89.325) \end{gathered}$ | $\begin{gathered} -14.674 \\ (112.754) \end{gathered}$ |
| Def. Shock |  |  | $\begin{aligned} & 1.990^{* * *} \\ & (0.715) \end{aligned}$ | $\begin{gathered} 1.464^{*} \\ (0.888) \end{gathered}$ |
| Def. Shock*Def. Volatility |  |  | $\begin{aligned} & -99.710 \\ & (103.550) \end{aligned}$ | $\begin{gathered} 50.204 \\ (128.163) \end{gathered}$ |
| Observations | 686 | 686 | 686 | 686 |
| Adjusted R ${ }^{2}$ | 0.043 | 0.022 | 0.055 | 0.031 |
| F Statistic | $15.446^{* * *}$ | $7.889^{* * *}$ | $9.966^{* * *}$ | $5.420^{* * *}$ |
| Note: |  |  | ${ }^{*} \mathrm{p}<0.1$; ${ }^{* *} \mathrm{p}$ | ; ${ }^{* * *} \mathrm{p}<0.01$ |

## TABLE LIII

NATO - Defense and Non-Defense Dynamic Empirical Results with Interactions

|  | Dependent variable: Output Growth |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Non-Def. |  | Non-Def. and Def. |  |
|  | FE.nonm | RE.nonm | FE.nonm.m | RE.nonm.m |
| Output Growth (lag 1) | $0.196^{* * *}$ | $0.314^{* * *}$ | $0.168^{* * *}$ | $0.295^{* * *}$ |
|  | $(0.041)$ | $(0.039)$ | $(0.041)$ | $(0.039)$ |
| Output Growth (lag 2) | 0.018 | 0.063 | 0.023 | 0.043 |
|  | $(0.041)$ | $(0.039)$ | $(0.041)$ | $(0.039)$ |
| Non-Def. Shock | $0.222^{*}$ | 0.123 | $0.346^{* * *}$ | 0.160 |
|  | $(0.121)$ | $(0.129)$ | $(0.121)$ | $(0.131)$ |
| Non-Def. Shock (lag 1) | -0.113 | -0.140 | -0.003 | -0.091 |
|  | $(0.121)$ | $(0.127)$ | $(0.122)$ | $(0.129)$ |
| Non-Def. Shock (lag 2) | -0.019 | 0.017 | -0.037 | 0.007 |
|  | $(0.121)$ | $(0.128)$ | $(0.122)$ | $(0.128)$ |
| Def. Shock |  |  | $1.781^{* * *}$ | $0.976^{* *}$ |
|  |  |  | $(0.328)$ | $(0.386)$ |
| Def. Shock (lag 1) |  |  | $0.678^{* *}$ | $0.912^{* *}$ |
|  |  |  | $(0.337)$ | $(0.388)$ |
| Def. Shock (lag 2) |  |  | -0.502 | 0.062 |
|  |  |  | $(0.335)$ | $(0.385)$ |
| Observations | 658 | 0.120 | 658 | 658 |
| Adjusted R ${ }^{2}$ |  |  | 0.093 | 0.138 |
| F Statistic |  |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;$ | ${ }^{* * *} \mathrm{p}<0.01$ |

## TABLE LIV

NATO - Defense and Non-Defense Dynamic Empirical Results with Interactions

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Non-Defense |  | Non-Def. and Def. |  |
|  | FE | RE | FE | RE |
| Output Growth (lag 1) | $\begin{aligned} & 0.172^{* * *} \\ & (0.041) \end{aligned}$ | $\begin{aligned} & 0.301^{* * *} \\ & (0.039) \end{aligned}$ | $\begin{aligned} & 0.167^{* * *} \\ & (0.041) \end{aligned}$ | $\begin{aligned} & 0.295^{* * *} \\ & (0.039) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{gathered} 0.020 \\ (0.041) \end{gathered}$ | $\begin{gathered} 0.047 \\ (0.040) \end{gathered}$ | $\begin{gathered} 0.024 \\ (0.041) \end{gathered}$ | $\begin{gathered} 0.045 \\ (0.040) \end{gathered}$ |
| Non-Def. Shock | $\begin{aligned} & 0.335^{* * *} \\ & (0.121) \end{aligned}$ | $\begin{gathered} 0.158 \\ (0.131) \end{gathered}$ | $\begin{aligned} & 0.342^{* * *} \\ & (0.123) \end{aligned}$ | $\begin{gathered} 0.152 \\ (0.132) \end{gathered}$ |
| Non-Def. Shock (lag 1) | $\begin{gathered} -0.065 \\ (0.122) \end{gathered}$ | $\begin{gathered} -0.110 \\ (0.129) \end{gathered}$ | $\begin{gathered} -0.011 \\ (0.125) \end{gathered}$ | $\begin{gathered} -0.102 \\ (0.131) \end{gathered}$ |
| Non-Def. Shock (lag 2) | $\begin{gathered} -0.031 \\ (0.123) \end{gathered}$ | $\begin{gathered} 0.012 \\ (0.129) \end{gathered}$ | $\begin{gathered} -0.042 \\ (0.124) \end{gathered}$ | $\begin{gathered} -0.008 \\ (0.130) \end{gathered}$ |
| Non-Def. Shock <br> *Non-Def. Volatility | $\begin{aligned} & 248.075^{* * *} \\ & (52.238) \end{aligned}$ | $\begin{gathered} 120.719^{*} \\ (61.692) \end{gathered}$ | $\begin{gathered} 69.647 \\ (92.166) \end{gathered}$ | $\begin{aligned} & -13.128 \\ & (111.031) \end{aligned}$ |
| Non-Def. Shock <br> *Non-Def. Volatility (lag 1) | $\begin{gathered} 91.366^{*} \\ (53.043) \end{gathered}$ | $\begin{gathered} 123.476^{* *} \\ (61.428) \end{gathered}$ | $\begin{gathered} 68.835 \\ (91.842) \end{gathered}$ | $\begin{gathered} 33.494 \\ (110.969) \end{gathered}$ |
| Non-Def. Shock <br> *Non-Def. Volatility (lag 2) | $\begin{array}{r} -43.547 \\ (54.077) \end{array}$ | $\begin{gathered} 23.158 \\ (62.467) \end{gathered}$ | $\begin{gathered} 32.690 \\ (93.074) \end{gathered}$ | $\begin{gathered} 24.114 \\ (111.815) \end{gathered}$ |
| Def. Shock |  |  | $\begin{aligned} & 2.097^{* * *} \\ & (0.743) \end{aligned}$ | $\begin{gathered} 1.005 \\ (0.885) \end{gathered}$ |
| Def. Shock (lag 1) |  |  | $\begin{gathered} 0.578 \\ (0.751) \end{gathered}$ | $\begin{aligned} & 1.708^{*} \\ & (0.895) \end{aligned}$ |
| Def. Shock (lag 2) |  |  | $\begin{gathered} -0.592 \\ (0.724) \end{gathered}$ | $\begin{gathered} -0.213 \\ (0.864) \end{gathered}$ |
| Def. Shock <br> *Def. Volatility |  |  |  | 21.057 |
|  |  |  | (109.265) | (128.769) |
| Def. Shock <br> *Def. Volatility (lag 1) |  |  |  | -211.279 |
|  |  |  | (110.891) | (131.226) |
| Def. Shock <br> *Def. Volatility (lag 2) |  |  | $\begin{aligned} & -15.571 \\ & (106.701) \\ & \hline \end{aligned}$ | $\begin{array}{r} 50.966 \\ (126.925) \\ \hline \end{array}$ |
| Observations | 658 | 658 | 658 | 658 |
| Adjusted R ${ }^{2}$ | 0.082 | 0.132 | 0.096 | 0.141 |
| F Statistic | $7.375^{* * *}$ | $12.498^{* * *}$ | $5.071^{* * *}$ | $7.755^{* * *}$ |
| Note: |  |  | *p<0.1; ${ }^{* *} \mathrm{p}<0$ | 5; ${ }^{* * *} \mathrm{p}<0.01$ |

### 6.1.8 North Atlantic Treaty Organization Instrumental Variable Empirical Results

The NATO IV panel spans 14 countries and 55 years (from 1957 to 2011). The IV model is developed in Section 4.3. Table LV reports the parameter estimates for the NATO IV baseline static model. The left three columns of Table LV show the contemporaneous effects of government purchases on output, with $m^{\sigma}=0$. The fixed effects estimation suggests a multiplier of 4.39 .

TABLE LV

NATO IV - Baseline Empirical Results

|  | Dependent variable: Output Growth |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Equation 4.14 |  |  | Equation 4.15 |  |  |
|  | OLS | FE | RE | OLS | FE | RE |
| Defense Shock | $\begin{aligned} & 2.672^{* * *} \\ & (0.787) \end{aligned}$ | $\begin{aligned} & 4.393^{* * *} \\ & (1.352) \end{aligned}$ | $\begin{aligned} & 2.514^{* * *} \\ & (0.775) \end{aligned}$ | $\begin{gathered} -12.218 \\ (77.174) \end{gathered}$ | $\begin{gathered} -6.122 \\ (103.600) \end{gathered}$ | $\begin{array}{r} -11.443 \\ (73.001) \end{array}$ |
| Defense Shock * Volatility |  |  |  | $\begin{gathered} 3,634.361 \\ (19,379.990) \\ \hline \end{gathered}$ | $\begin{gathered} 4,201.086 \\ (43,280.010) \\ \hline \end{gathered}$ | $\begin{array}{r} 3,806.548 \\ (20,640.870) \\ \hline \end{array}$ |
| Observations | 686 | 686 | 686 | 686 | 686 | 686 |
| Adjusted R ${ }^{2}$ | 0.019 | 0.020 | 0.017 | 0.002 | 0.014 | 0.001 |
| F Statistic | -189.871 | -382.051 | -683.476 | -336.942 | -309.456 | -341.210 |

The right three columns of Table LV model the defense spending multiplier as dependent on the volatility of defense spending shocks. ${ }^{1}$ The multiplier for a hypothetical country with zero volatility is simply the coefficient on the government spending shock, -6.12.

In order to illustrate the impact of volatility on multipliers in the baseline static model, consider the range of multipliers implied for the countries with the minimum and maximum mean volatilities over time. These contemporaneous multipliers range from 7.31 for NA (volatility $=$ $0.0032)$ to 52.5 for Norway (volatility $=0.014)$.

While the baseline static model provides a useful benchmark for comparison, the dynamic model with $\mathrm{J}=2$ lags and a volatility window of $\mathrm{k}=6$ years offers a more realistic perspective.

Table LVI reports the parameter estimates for the NATO IV dynamic model. The left two columns show the dynamic effects of defense spending on output when $m_{j}^{\sigma}$ is constrained to 0 . The fixed effects model parameters imply a long-run multiplier of 4.09.

The third and fourth columns of Table LVI allow the defense spending multiplier to depend on the volatility of defense spending shocks. The volatility interaction coefficients are negative and large but insignificant. Based on Equation 4.7, the multiplier for a hypothetical country with zero volatility is -14.7 .

The Hausman test is used to determine whether the fixed effects (FE) specification or the random effects (RE) specification is preferred. The Hausman statistic of 0.397 implies p-value of 1 and thus the null hypothesis of the RE model is not rejected.

[^17]
## TABLE LVI

NATO IV - Dynamic Empirical Results

|  | Dependent variable: Output Growth |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Equation 4.14 |  | Equation 4.15 |  |
|  | FE | RE | FE | RE |
| Output Growth (lag 1) | 0.150 | $0.239^{* * *}$ | 0.664 | 0.240 |
|  | $(0.092)$ | $(0.053)$ | $(2.119)$ | $(0.342)$ |
| Output Growth (lag 2) | -0.022 | -0.102 | 0.124 | 0.442 |
|  | $(0.113)$ | $(0.081)$ | $(1.728)$ | $(2.107)$ |
| Defense Shock | $4.130^{* *}$ | 1.811 | -0.104 | 2.733 |
|  | $(1.605)$ | $(1.150)$ | $(20.490)$ | $(33.694)$ |
| Defense Shock (lag 1) | -0.469 | 1.167 | 1.571 | 7.126 |
|  | $(1.590)$ | $(1.450)$ | $(15.532)$ | $(28.761)$ |
| Defense Shock (lag 2) | -0.091 | 0.124 | -4.577 | -5.311 |
|  | $(2.132)$ | $(1.429)$ | $(17.793)$ | $(38.442)$ |
| Defense Shock |  |  | -433.543 | $-1,142.817$ |
| * Volatility |  |  | $(2,396.219)$ | $(4,375.347)$ |
| Defense Shock |  |  | $-1,063.347$ | $-1,082.871$ |
| * Volatility (lag 1) |  |  | $(4,652.755)$ | $(6,558.121)$ |
|  |  |  | -167.034 | -844.880 |
| Defense Shock |  |  | $(4,377.376)$ | $(13,947.020)$ |
| * Volatility (lag 2) |  |  | 658 | 658 |
| Observations |  |  | 0.041 | -70.744 |

Table LVII lists implied long-run multipliers for a number of countries in order to highlight the impact of volatility on the multiplier. A five-number summary of the mean volatilities across time for each country is used to calculate typical multipliers for the representative countries. Multipliers range from - 39.9 for NA (volatility $=0.0032$ ) to -124 for Norway ( volatility $=0.014$ ).

TABLE LVII

NATO IV - Long-run Multipliers across Mean Volatilities

|  | Zero Vol. | 0 th $\%$-ile | 25th $\%$-ile | 50th $\%$-ile | 75th $\%$-ile | 100th $\%$-ile |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Country | NA | NA | Italy | Belgium | Denmark | Norway |
| Mean Volatility | 0 | 0.0032 | 0.0042 | 0.00572 | 0.0069 | 0.014 |
| Multiplier | -14.7 | -39.9 | -47.7 | -59.7 | -69 | -124 |

Figure 27 tracks the impulse responses of output to $1 \%$ defense spending shocks for the fixed effects NATO IV dynamic model. Impulse responses demonstrate the evolution of variables in dynamic systems over time. The volatilities used to generate the responses are based on a fivenumber summary of the mean volatilities for each country across time. Notice that long-run multipliers are significantly larger for countries with lower defense spending volatility.

Panel (a) of Figure 28 illustrates the long-run multipliers implied by the NATO IV dynamic model contingent on historical data for the smallest, median, and largest economies in the


Figure 27. NATO IV - Impulse Responses to a 1\% Defense Shock

NATO IV panel. The average multiplier across the countries in panel (a) is -19.5 and the minimum and maximum multipliers are -35 and -15.7 respectively. Panel (b) illustrates the long-run multipliers implied by the NATO IV dynamic model contingent on historical data for the poorest, median, and wealthiest economies in terms of per capita income. The average multiplier across the countries in panel (b) is -20.5 and the minimum and maximum multipliers are -41 and -15.7 respectively.


Figure 28. NATO IV - Implied Historical Multipliers for Representative Economies

### 6.1.9 Review of Defense Spending Results

The empirical results based on defense spending panels are not as statistically persuasive as those estimated for either the standard dynamic ( Equation 4.6 ), the NVAR (Equation 4.8 ), or the US defense spending models. Nevertheless, the net effect of volatility in defense spending for all of the dynamic models continues to be negative, albeit insignificantly so.

The poor fit realized by panel defense spending is most striking when contrasted with the US defense spending estimates whose fit is virtually the same as for the estimates based on government purchases. While the WDI and NATO defense spending data have fewer observations than the PWT data, data quantity is insufficient for explaining for their poor explanatory power. Smaller than the PWT, the NATO dataset contains 8 times the number of observations as the US data.

The poor explanatory power of the defense data seems to be rooted in the low variability of defense spending in countries. Figure 29 contains scatter-plot matrices for the WDI data,


Figure 29. Scatterplots - WDI and US Data
on the left, and for the US, on the right. ${ }^{1}$ The diagonal contains histograms for each variable. The below diagonal cells plot the variables $y, g$, and $d$ against one another with a LOESS $^{2}$ smoother. The upper triangular cells display the correlations between each variable. Defense shocks are almost completely uncorrelated with either output growth or fiscal shocks in most countries. The WDI defense spending shocks' correlation with output growth is 0.07 and its correlation with fiscal shocks is 0.12 . Whereas for the US, these correlations are 0.57 and 0.99 respectively. This difference may explain why defense based estimates tend to be insignificant for the panels, yet sometimes significant for the US.

The final technique employed to combat endogeneity bias relies on the fact that defense spending is sometimes correlated with overall government purchases and may thus be used

[^18]${ }^{2}$ Local regression
as an instrumental variable (IV) for government purchases. IV techniques allow consistent estimation when variables in the regression are correlated with the error.

The standard two-stage least-squares (2SLS) IV technique is used. In the first stage, the potentially endogenous variable, $g$ is regressed on the lagged growth terms and the defense spending terms. The predicted variables are then replaced in the dynamic model. While the dynamic IV results appear to support the RI hypothesis that fiscal volatility reduces the fiscal multiplier, they are not statistically significant.

### 6.2 Tests for Structural Breaks and Outliers

Chapter 5's dynamic FE estimations unanimously support the RI hypothesis that fiscal volatility dampens the fiscal multiplier. This section seeks to determine whether the empirical results are driven by structural breaks or outliers. The models assume that the relationship between growth and the parameters on the continuous RHS variables is identical for all observations in the panel. ${ }^{1}$ This section investigates the validity of this assumption. Specifically, it tests whether the parameters are homogenous across the panel and whether particular observations act as influential outliers. Several techniques are implemented. The null hypothesis is the absence of a structural break. The alternative hypotheses translate to the presence of a single structural break. This is as opposed to an alternative effort to identify multiple structural breaks simultaneously.

[^19]The first approach is based on the Chow (1960) test for structural change and the others are extension of this test. The theoretical distribution of the Chow test requires knowledge of the structural change point a priori. Andrews (1993) and Andrews and Ploberger (1994) examine the asymptotic distributions of related test statistics that seek potential structural change points at all points in some interval. The test statistics,

$$
\begin{gathered}
\sup F=\sup _{t_{1}<i<t_{2}} F_{i} \\
\operatorname{ave} F=\frac{1}{t_{1}-t_{2}+1} \sum_{i=t_{1}}^{t_{2}} F_{i} \\
\exp F=\log \left(\frac{1}{t_{1}-t_{2}+1} \sum_{i=t_{1}}^{t_{2}} \exp \left(0.5 * F_{i}\right)\right)
\end{gathered}
$$

are rejected when their values exceed critical values provided by Andrews (1993), Hansen (2013), and Andrews (2003). Andrews and Ploberger (1994) showed that the supF, aveF, and expF have the null distribution (F). The supF statistic is the most powerful among these tests. ${ }^{1}$ Hansen suggests testing supF using $t_{1}=\pi_{0} n$ and $t_{2}=\left(1-\pi_{0}\right) n$ where $n$ is the number of observations and $\pi_{0} \in(0,1)$. Hansen further recommends $\pi_{0}=0.15$, since breaks too near the beginning or end of a sample would be misleading. Andrews (2003) updates the asymptotic critical values originally reported in Andrews (1993) by increasing the underlying simulations from 10,000 to 100,000 . Under the two-way fixed effects (FE) model $p$, the dimension of $\beta$, far

[^20]exceeds the maximum $p$ of 30 provided in either Andrews (1993) or Andrews (2003). Instead, the dynamic OLS model with $9 \beta$ parameters is used for these structural break tests. Without the benefit of control variables, the fit of the OLS model will naturally be more sensitive than an FE model to underlying time and country specific effects. Additionally, the critical values tend to increase with $p$. Thus the test statistics will be larger and the critical values will be smaller under an OLS model rather than FE model. This implies that the OLS test will be more powerful while the possibility of rejecting a true hypothesis of no breaks is increased. The $10 \%, 5 \%$, and $1 \%$ critical values Andrews (2003) reports for $\pi_{0}=0.15, p=9$ are 23.2, 25.54, and 30.42 respectively.

In order to apply the test to panel data, the data are first sorted by country along a given dimension and then by time within the observations for each country. The dimensions considered are GDP per capita $\left(\frac{Y}{\text { population }}\right)$, total GDP $(Y)$, average ${ }^{1} \operatorname{GDP}$ growth ( $y$ ), average fiscal shocks $(g)$, average fiscal volatility $\left(\sigma_{g}\right)$, average government purchases as a share of GDP $\left(\frac{G}{Y}\right)$, and average defense spending as a share of $\operatorname{GDP}\left(\frac{D}{G}\right)$. After the data are sorted, Chow tests for structural breaks are then calculated at each observation from $0.015 * n$ to $0.085 * n$. The Andrews and Ploberger statistics are calculated and compared to the critical values noted above. Additionally, a single break at each year is explored. This approach samples the data at a lower frequency than the Andrews and Ploberger (1994) methodology and is therefore less sensitive and less likely to reject the null.

[^21]The next approach to identifying structural breaks involves plotting and subjectively assessing the evolution of the estimated model parameters as additional observations are added. While this methodology does not correspond to a specific test statistic, it provides insight into the overall stability of the model. It is worth noting that the weight of each observation in determining the parameter estimates falls monotonically as observations are added. Thus, deviations later in the series should draw more attention than among initial observations.

The final, similarly subjective, technique simply splices the sorted data into quartiles. This method imposes breaks at the $25^{t h}, 50^{t h}$, and $75^{\text {th }}$ percentiles and then reports the four sets of FE coefficients and goodness-of-fit statistics for subjective assessment.

### 6.2.1 Gross Domestic Product per Capita based Search for Structural Breaks

Figure 30 plots the Chow F-statistics for structural change across the PWT observations sorted by GDP per Capita . The calculated supF, aveF, and expF are 13.64, 1.85, and 2.834 respectively. Comparing the observed statistics to the Andrews (2003) critical values of 23.2 $(10 \%), 25.54(5 \%)$, and $30.42(1 \%)$, the null hypothesis of no structural breaks is rejected.

Figure 31 plots the evolution of parameter estimates across GDP per Capita. The plots in Figure 31 are relatively unremarkable. Table LVIII estimates the FE model for four subsets of the data. The results are largely consistent across subsets of the data. In particular, the significant coefficients on the interaction terms are largely negative. Thus Table LVIII supports the hypothesis that fiscal volatility dampens the fiscal multiplier.

## PWT - F-Statistics across GDP per Capita



Figure 30. PWT - F-Statistics across GDP per Capita


Figure 31. PWT - Evolution of Parameter Estimates across GDP per Capita

## TABLE LVIII

PWT - Dynamic Empirical Results by Quartiles across GDP per Capita


### 6.2.2 Total Gross Domestic Product based Search for Structural Breaks

Figure 32 plots the Chow F-statistics for structural change across the PWT observations sorted by Total GDP . The calculated supF, aveF, and expF are $13.27,1.77$, and 2.67 respectively. Comparing the observed statistics to the Andrews (2003) critical values of 23.2 $(10 \%), 25.54(5 \%)$, and $30.42(1 \%)$, the null hypothesis of no structural breaks is rejected.

PWT - F-Statistics across Total GDP


Figure 32. PWT - F-Statistics across Total GDP

Figure 33 plots the evolution of parameter estimates across Total GDP. The plots in Figure 33 are relatively unremarkable. Table LIX estimates the FE model for four subsets of the data. The results are largely consistent across subsets of the data. In particular, the significant coefficients on the interaction terms are largely negative. Thus Table LIX supports the hypothesis that fiscal volatility dampens the fiscal multiplier.


Figure 33. PWT - Evolution of Parameter Estimates across Total GDP

## TABLE LIX

PWT - Dynamic Empirical Results by Quartiles across Total GDP

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | First Quartile <br> Q1 | Second Quartile <br> Q2 | Third Quartile Q3 | Fourth Quartile |
| Output Growth (lag 1) | $\begin{aligned} & 0.239^{* * *} \\ & (0.036) \end{aligned}$ | $\begin{aligned} & 0.125^{* * *} \\ & (0.037) \end{aligned}$ | $\begin{aligned} & 0.202^{* * *} \\ & (0.036) \end{aligned}$ | $\begin{aligned} & 0.303^{* * *} \\ & (0.035) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{aligned} & 0.103^{* * *} \\ & (0.036) \end{aligned}$ | $\begin{gathered} -0.010 \\ (0.036) \end{gathered}$ | $\begin{gathered} -0.022 \\ (0.036) \end{gathered}$ | $\begin{gathered} 0.060^{*} \\ (0.035) \end{gathered}$ |
| Fiscal Shock | $\begin{aligned} & 0.434^{* * *} \\ & (0.089) \end{aligned}$ | $\begin{aligned} & 0.891^{* * *} \\ & (0.098) \end{aligned}$ | $\begin{aligned} & 0.420^{* * *} \\ & (0.147) \end{aligned}$ | $\begin{aligned} & 2.262^{* * *} \\ & (0.222) \end{aligned}$ |
| Fiscal Shock (lag 1) | $\begin{gathered} 0.030 \\ (0.114) \end{gathered}$ | $\begin{gathered} -0.073 \\ (0.104) \end{gathered}$ | $\begin{array}{r} -0.265^{*} \\ (0.151) \end{array}$ | $\begin{aligned} & -0.913^{* * *} \\ & (0.243) \end{aligned}$ |
| Fiscal Shock (lag 2) | $\begin{gathered} 0.030 \\ (0.097) \end{gathered}$ | $\begin{aligned} & 0.207^{* *} \\ & (0.104) \end{aligned}$ | $\begin{gathered} -0.058 \\ (0.148) \end{gathered}$ | $\begin{gathered} 0.117 \\ (0.230) \end{gathered}$ |
| Fiscal Shock <br> * Volatility | $\begin{gathered} -3.021^{* * *} \\ (0.705) \end{gathered}$ | $\begin{gathered} -5.918^{* * *} \\ (2.278) \end{gathered}$ | $\begin{array}{r} -2.364 \\ (6.498) \end{array}$ | $\begin{gathered} -75.173^{* * *} \\ (12.829) \end{gathered}$ |
| Fiscal Shock <br> * Volatility (lag 1) | $\begin{gathered} -1.704^{*} \\ (1.006) \end{gathered}$ | $\begin{gathered} 1.642 \\ (2.290) \end{gathered}$ | $\begin{aligned} & 14.588^{* *} \\ & (6.604) \end{aligned}$ | $\begin{aligned} & 53.143^{* *} \\ & (13.920) \end{aligned}$ |
| Fiscal Shock <br> * Volatility (lag 2) | $\begin{gathered} -2.632^{* * *} \\ (0.971) \end{gathered}$ | $\begin{aligned} & -2.016 \\ & (2.227) \end{aligned}$ | $\begin{gathered} -2.521 \\ (6.703) \end{gathered}$ | $\begin{array}{r} -22.840^{*} \\ (12.838) \end{array}$ |
| Observations | 836 | 837 | 836 | 837 |
| Adjusted $\mathrm{R}^{2}$ | 0.112 | 0.154 | 0.074 | 0.234 |
| F Statistic | $13.297^{* * *}$ | $19.436^{* * *}$ | 8.475*** | $32.972^{* * *}$ |
| Note: |  |  | ${ }^{*} \mathrm{p}<0.1$; ** | <0.05; ${ }^{* * *} \mathrm{p}<0.01$ |

### 6.2.3 $\frac{G}{\Gamma}$ based Search for Structural Breaks

Figure 34 plots the Chow F-statistics for structural change across the PWT observations sorted by G share of Y . The calculated supF, ave F, and expF are 9.60, 0.48, and 1.04 respectively. Comparing the observed statistics to the Andrews (2003) critical values of 23.2 $(10 \%), 25.54(5 \%)$, and $30.42(1 \%)$, the null hypothesis of no structural breaks is rejected.

PWT - F-Statistics across G share of Y


Figure 34. PWT - F-Statistics across G share of Y

Figure 35 plots the evolution of parameter estimates across G share of Y. The plots in Figure 35 are relatively unremarkable. Table LX estimates the FE model for four subsets of the data. The results are largely consistent across subsets of the data. In particular, the significant coefficients on the interaction terms are largely negative. Thus Table LX supports the hypothesis that fiscal volatility dampens the fiscal multiplier.


Figure 35. PWT - Evolution of Parameter Estimates across G share of Y

## TABLE LX



### 6.2.4 Average Growth Based Search for Structural Breaks

Figure 36 plots the Chow F-statistics for structural change across the PWT observations sorted by Average Growth . The calculated supF, aveF, and expF are 22.22, 5.98, and 7.49 respectively. Comparing the observed statistics to the Andrews (2003) critical values of 23.2 $(10 \%), 25.54(5 \%)$, and $30.42(1 \%)$, the null hypothesis of no structural breaks is rejected.

PWT - F-Statistics across Average Growth


Figure 36. PWT - F-Statistics across Average Growth

Figure 37 plots the evolution of parameter estimates across Average Growth . The plots in Figure 37 are relatively unremarkable. Table LXI estimates the FE model for four subsets of the data. The results are largely consistent across subsets of the data. In particular, the significant coefficients on the interaction terms are largely negative. Thus Table LXI supports the hypothesis that fiscal volatility dampens the fiscal multiplier.


Figure 37. PWT - Evolution of Parameter Estimates across Average Growth

## TABLE LXI

PWT - Dynamic Empirical Results by Quartiles across Average Growth


### 6.2.5 Average Fiscal Shock Based Search for Structural Breaks

Figure 38 plots the Chow F-statistics for structural change across the PWT observations sorted by Average Fiscal Shocks . The calculated supF, aveF, and expF are $10.85,1.85$, and 2.61 respectively. Comparing the observed statistics to the Andrews (2003) critical values of $23.2(10 \%), 25.54(5 \%)$, and $30.42(1 \%)$, the null hypothesis of no structural breaks is rejected.

Figure 39 plots the evolution of parameter estimates across Average Fiscal Shocks . The plots in Figure 39 are relatively unremarkable. Table LXII estimates the FE model for four subsets of the data. The results are largely consistent across subsets of the data. In particular, the significant coefficients on the interaction terms are largely negative. Thus Table LXII supports the hypothesis that fiscal volatility dampens the fiscal multiplier.

## PWT - F-Statistics across Average Fiscal Shocks



Figure 38. PWT - F-Statistics across Average Fiscal Shocks


Figure 39. PWT - Evolution of Parameter Estimates across Average Fiscal Shocks

## TABLE LXII

PWT - Dynamic Empirical Results by Quartiles across Average Fiscal Shocks

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | First Quartile | Second Quartile | Third Quartile | Fourth Quartile |
|  | Q1 | Q2 | Q3 | Q4 |
| Output Growth (lag 1) | $\begin{aligned} & 0.274^{* * *} \\ & (0.037) \end{aligned}$ | $\begin{aligned} & 0.187^{* * *} \\ & (0.036) \end{aligned}$ | $\begin{aligned} & 0.159^{* * *} \\ & (0.036) \end{aligned}$ | $\begin{aligned} & 0.193^{* * *} \\ & (0.036) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{gathered} 0.019 \\ (0.037) \end{gathered}$ | $\begin{aligned} & 0.136^{* * *} \\ & (0.036) \end{aligned}$ | $\begin{gathered} -0.008 \\ (0.036) \end{gathered}$ | $\begin{gathered} 0.048 \\ (0.035) \end{gathered}$ |
| Fiscal Shock | $\begin{gathered} 0.528^{* *} \\ (0.222) \end{gathered}$ | $\begin{aligned} & 0.848^{* * *} \\ & (0.182) \end{aligned}$ | $\begin{aligned} & 0.595^{* * *} \\ & (0.178) \end{aligned}$ | $\begin{aligned} & 0.634^{* * *} \\ & (0.058) \end{aligned}$ |
| Fiscal Shock (lag 1) | $\begin{gathered} -0.130 \\ (0.222) \end{gathered}$ | $\begin{gathered} 0.180 \\ (0.187) \end{gathered}$ | $\begin{gathered} -0.049 \\ (0.179) \end{gathered}$ | $\begin{gathered} 0.082 \\ (0.073) \end{gathered}$ |
| Fiscal Shock (lag 2) | $\begin{gathered} -0.071 \\ (0.220) \end{gathered}$ | $\begin{gathered} -0.548^{* * *} \\ (0.184) \end{gathered}$ | $\begin{gathered} -0.070 \\ (0.170) \end{gathered}$ | $\begin{aligned} & 0.198^{* * *} \\ & (0.066) \end{aligned}$ |
| Fiscal Shock <br> * Volatility | $\begin{gathered} 15.108 \\ (11.935) \end{gathered}$ | $\begin{array}{r} -5.061 \\ (7.967) \end{array}$ | $\begin{gathered} -0.448 \\ (7.685) \end{gathered}$ | $\begin{gathered} -4.112^{* * *} \\ (0.483) \end{gathered}$ |
| Fiscal Shock <br> * Volatility (lag 1) | $\begin{gathered} 9.516 \\ (12.036) \end{gathered}$ | $\begin{array}{r} -14.414^{*} \\ (8.110) \end{array}$ | $\begin{gathered} -7.182 \\ (7.428) \end{gathered}$ | $\begin{gathered} -2.549^{* * *} \\ (0.671) \end{gathered}$ |
| Fiscal Shock <br> * Volatility (lag 2) | $\begin{gathered} 6.408 \\ (11.937) \end{gathered}$ | $\begin{gathered} 31.582^{* * *} \\ (7.937) \end{gathered}$ | $\begin{gathered} -4.928 \\ (7.037) \end{gathered}$ | $\begin{gathered} -4.454^{* * *} \\ (0.704) \end{gathered}$ |
| Observations | 836 | 837 | 836 | 837 |
| Adjusted $\mathrm{R}^{2}$ | 0.140 | 0.138 | 0.068 | 0.185 |
| F Statistic | $17.226^{* * *}$ | $17.015^{* * *}$ | 7.721*** | $24.345^{* * *}$ |
| Note: |  |  | ${ }^{*} \mathrm{p}<0.1$; ${ }^{*}$ | <0.05; *** $\mathrm{p}<0.01$ |

### 6.2.6 Average Fiscal Volatility Based Search for Structural Breaks

Figure 40 plots the Chow F-statistics for structural change across the PWT observations sorted by Average Fiscal Volatility. The calculated supF, aveF, and expF are 13.74, 1.57, and 2.22 respectively. Comparing the observed statistics to the Andrews (2003) critical values of $23.2(10 \%), 25.54(5 \%)$, and $30.42(1 \%)$, the null hypothesis of no structural breaks is rejected.

Figure 41 plots the evolution of parameter estimates across Average Fiscal Vol. . The plots in Figure 41 are relatively unremarkable. Table LXIII estimates the FE model for four subsets of the data. The results are largely consistent across subsets of the data. In particular, the significant coefficients on the interaction terms are largely negative. Thus Table LXIII supports the hypothesis that fiscal volatility dampens the fiscal multiplier.

## PWT - F-Statistics across Average Fiscal Vol.



Figure 40. PWT - F-Statistics across Average Fiscal Vol.


Figure 41. PWT - Evolution of Parameter Estimates across Average Fiscal Vol.

## TABLE LXIII

PWT - Dynamic Empirical Results by Quartiles across Average Fiscal Vol.

|  | Dependent variable: Output Growth |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | First Quartile | Second Quartile | Third Quartile | Fourth Quartile |
|  | Q1 | Q2 | Q3 | Q4 |
| Output Growth (lag 1) | $0.247^{* * *}$ | $0.213^{* * *}$ | $0.206^{* * *}$ | $0.136^{* * *}$ |
|  | $(0.036)$ | $(0.036)$ | $(0.036)$ | $(0.036)$ |
| Output Growth (lag 2) | -0.023 | $0.076^{* *}$ | $0.089^{* *}$ | 0.016 |
|  | $(0.036)$ | $(0.036)$ | $(0.036)$ | $(0.036)$ |
| Fiscal Shock | $0.832^{* *}$ | $1.567^{* * *}$ | $0.836^{* * *}$ | $0.601^{* * *}$ |
|  | $(0.354)$ | $(0.277)$ | $(0.217)$ | $(0.066)$ |
| Fiscal Shock (lag 1) | 0.069 | $-0.684^{* *}$ | $-0.442^{* *}$ | 0.014 |
|  | $(0.362)$ | $(0.288)$ | $(0.218)$ | $(0.079)$ |
| Fiscal Shock (lag 2) | 0.113 | -0.216 | 0.100 | $0.149^{* *}$ |
|  | $(0.331)$ | $(0.279)$ | $(0.215)$ | $(0.072)$ |
| Fiscal Shock | 1.114 | $-51.634^{*}$ | -4.176 | $-3.705^{* * *}$ |
| * Volatility | $(65.004)$ | $(30.482)$ | $(12.469)$ | $(0.566)$ |
|  | 24.356 | $76.994^{* *}$ | $22.657^{*}$ | $-1.824^{* *}$ |
| Fiscal Shock | $(65.378)$ | $(30.566)$ | $(12.589)$ | $(0.765)$ |
| * Volatility (lag 1) | $-101.434^{*}$ | 5.407 | -0.029 | $-3.592^{* * *}$ |
| Fiscal Shock | $(57.428)$ | $(29.376)$ | $(12.332)$ | $(0.826)$ |
| * Volatility (lag 2) | 836 | 837 | 836 | 837 |
|  | 0.101 | 0.146 | 0.122 | 0.115 |
| Observations | $11.825^{* * *}$ | $18.259^{* * *}$ | $14.792^{* * *}$ | $13.758^{* * *}$ |
| Adjusted R ${ }^{2}$ |  |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |  |
| F Statistic |  |  |  |  |

## TABLE LXIV

PWT - Dynamic OLS Results across GDP per Capita Quartiles

|  |  | Dependent variable: Output Growth |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | First Quartile | Second Quartile | Third Quartile | Fourth Quartile |
|  | Q1 | Q2 | Q3 | Q4 |
| Output Growth (lag 1) | $0.159^{* * *}$ | $0.174^{* * *}$ | $0.173^{* * *}$ | $0.206^{* * *}$ |
|  | $(0.035)$ | $(0.040)$ | $(0.028)$ | $(0.023)$ |
| Output Growth (lag 2) | $0.091^{* * *}$ | 0.039 | $0.060^{* *}$ | 0.017 |
|  | $(0.035)$ | $(0.038)$ | $(0.027)$ | $(0.022)$ |
| Fiscal Shock | $0.558^{* * *}$ | $0.664^{* * *}$ | $0.746^{* * *}$ | $0.850^{* * *}$ |
|  | $(0.064)$ | $(0.091)$ | $(0.079)$ | $(0.070)$ |
| Fiscal Shock (lag 1) | $0.149^{*}$ | $0.182^{*}$ | -0.049 | $-0.146^{* *}$ |
|  | $(0.077)$ | $(0.094)$ | $(0.082)$ | $(0.073)$ |
| Fiscal Shock (lag 2) | 0.085 | 0.083 | 0.051 | 0.113 |
|  | $(0.070)$ | $(0.101)$ | $(0.085)$ | $(0.073)$ |
| Fiscal Shock | $-3.731^{* * *}$ | -3.833 | $-4.955^{*}$ | $-6.054^{* * *}$ |
| * Volatility | $(0.534)$ | $(2.866)$ | $(2.630)$ | $(1.870)$ |
|  | $-2.757^{* * *}$ | -2.302 | 2.162 | 1.968 |
| Fiscal Shock | $(0.724)$ | $(2.779)$ | $(2.599)$ | $(1.890)$ |
| * Volatility (lag 1) | $-3.621^{* * *}$ | -1.877 | $-5.191^{* *}$ | $-3.980^{* *}$ |
|  | $(0.753)$ | $(2.795)$ | $(2.603)$ | $(1.842)$ |
| Fiscal Shock | 864 | 702 | 1,350 | 2,052 |
| * Volatility (lag 2) | 0.130 | 0.174 | 0.135 | 0.141 |
| Observations | $16.342^{* * *}$ | $18.903^{* * *}$ | $26.590^{* * *}$ | $42.327^{* * *}$ |
| Adjusted R ${ }^{2}$ |  |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}<0.05 ;{ }^{* * *} \mathrm{p}<0.01$ |  |
| F Statistic |  |  |  |  |

### 6.2.7 Time Based Search for Structural Breaks

Figure 42 displays the supF test and Figure 42 displays the evolution of the model parameters across time. Next, Figure 43 estimates the model for each quartile of the data.

Figure 43 plots the evolution of parameter estimates across Time. The results are inconsistent across subsets of the data. In particular, the significant coefficients on the interaction terms are largely negative. The only structural Y break indicated by the OLS based supF test is due to the time variable.

Examination of Table LXV reveals that the structural break detected by the OLS supF test is due to a minority of the data from the 1952-1965 period. Thus Table LXV supports the hypothesis that fiscal volatility dampens the fiscal multiplier.

PWT Chow F-Statistics across Year


Figure 42. PWT - Evolution of Parameter Estimates over Time
Parameter Evolution across Data Sorted by Year


$\alpha_{2} \times y_{t-2}$



$$
\mathrm{m}_{1}^{\sigma_{8}} \mathrm{~g}_{\mathrm{t}-1} \times \sigma_{\mathrm{g}_{\mathrm{t}-1}}
$$



$\mathrm{m}_{1} \times \mathrm{g}_{\mathrm{t}-1}$ $\mathrm{m}_{2}^{\sigma_{\mathrm{g}}} \mathrm{g}_{\mathrm{t}-2} \times \sigma_{\mathrm{g}_{\mathrm{t}-2}}$



Figure 43. Structural Break Detection across Time

## TABLE LXV

PWT - Dynamic Model OLS Results Quartiles across Time

|  |  |  | Dependent variable: Output Growth |  |
| :--- | :---: | :---: | :---: | :---: |
|  | First Quartile | Second Quartile | Third Quartile | Fourth Quartile |
|  | Q1 | Q2 | Q3 | Q4 |
| Output Growth (lag 1) | $-0.096^{* *}$ | $0.243^{* * *}$ | $0.071^{*}$ | $0.280^{* * *}$ |
|  | $(0.039)$ | $(0.040)$ | $(0.039)$ | $(0.037)$ |
| Output Growth (lag 2) | $-0.083^{* *}$ | $-0.094^{* *}$ | -0.017 | 0.021 |
|  | $(0.039)$ | $(0.037)$ | $(0.039)$ | $(0.037)$ |
| Fiscal Shock | $0.819^{* * *}$ | $0.766^{* * *}$ | $0.629^{* * *}$ | $0.167^{* * *}$ |
| Fiscal Shock (lag 1) | $(0.122)$ | $(0.134)$ | $(0.197)$ | $(0.063)$ |
|  | $0.345^{* * *}$ | $-0.351^{* *}$ | 0.157 | $0.222^{* *}$ |
| Fiscal Shock (lag 2) | $(0.120)$ | $(0.139)$ | $(0.204)$ | $(0.089)$ |
|  | -0.191 | 0.108 | 0.268 | -0.010 |
| Fiscal Shock | $(0.160)$ | $(0.126)$ | $(0.198)$ | $(0.078)$ |
| * Volatility | $14.169^{* * *}$ | $-7.462^{* * *}$ | 2.632 | $-1.430^{* * *}$ |
| Fiscal Shock | $(4.661)$ | $(2.692)$ | $(9.016)$ | $(0.500)$ |
| * Volatility (lag 1) | -4.834 | $6.253^{* *}$ | $-16.208^{*}$ | $-2.123^{* * *}$ |
| Fiscal Shock | $(3.458)$ | $(2.729)$ | $(9.127)$ | $(0.784)$ |
| * Volatility (lag 2) | 6.105 | -0.569 | -11.799 | $-1.557^{* *}$ |
| Observations | $(4.346)$ | $(2.549)$ | $(9.319)$ | $(0.774)$ |
| Adjusted R 2 | 744 | 745 | 745 | 745 |
| F Statistic | 0.204 | 0.109 | 0.080 | 0.110 |
| Note: | $24.588^{* * *}$ | $11.516^{* * *}$ | $8.199^{* * *}$ | $11.630^{* * *}$ |

### 6.2.8 Test for Outliers

Several standard statistical tests for detecting outliers exist. A common test for detecting outliers involves flagging any observation whose t-score surpasses $3 .{ }^{1}$ After detecting an outlier, the entire country is removed from the subsequent analysis in order to preserve a balanced panel. Applying the test to the PWT data results in the removal of Cyprus, Israel, Morocco, and El Salvador (translating to $6.5 \%$ of the data).


Figure 44. With (left) and Without (right) Outliers

Probability plots for the PWT fiscal shocks (left) and volatilities (right) are presented in Figure 44 with and without outliers. Due to their derivation, the volatility data are clearly nonnormal. Their probability plots resemble those of the squared normal data shown in the top

[^22]right panel. Based on the t-score greater than or equal to 3 criteria, $6.5 \%$ of the data are marked as outliers. These observations have been removed in the third and fourth panels of the bottom row. After removing the offending data, the fiscal shock data remain non-normal. The sum of the volatility interaction coefficients in Table LXVI is decisively negative and significant. The results fail to invalidate the RI hypothesis that fiscal volatility diminishes the fiscal multiplier.

## TABLE LXVI

PWT - Dynamic Empirical Results (Without Outliers)

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Equation 4.5 |  | Equation 4.6 |  |
|  | FE | RE | FE | RE |
| Output Growth (lag 1) | $\begin{aligned} & 0.224^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.254^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.225^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.253^{* * *} \\ & (0.018) \end{aligned}$ |
| Output Growth (lag 2) | $\begin{aligned} & 0.069^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.065^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.068^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.063^{* * *} \\ & (0.018) \end{aligned}$ |
| Fiscal Shock | $\begin{aligned} & 0.686^{* * *} \\ & (0.047) \end{aligned}$ | $\begin{aligned} & 0.707^{* * *} \\ & (0.049) \end{aligned}$ | $\begin{aligned} & 0.973^{* * *} \\ & (0.090) \end{aligned}$ | $\begin{aligned} & 0.981^{* * *} \\ & (0.091) \end{aligned}$ |
| Fiscal Shock (lag 1) | $\begin{gathered} -0.076 \\ (0.049) \end{gathered}$ | $\begin{gathered} -0.053 \\ (0.050) \end{gathered}$ | $\begin{gathered} -0.092 \\ (0.093) \end{gathered}$ | $\begin{gathered} -0.038 \\ (0.094) \end{gathered}$ |
| Fiscal Shock (lag 2) | $\begin{gathered} -0.063 \\ (0.049) \end{gathered}$ | $\begin{gathered} -0.029 \\ (0.050) \end{gathered}$ | $\begin{gathered} -0.184^{* *} \\ (0.091) \end{gathered}$ | $\begin{gathered} -0.124 \\ (0.092) \end{gathered}$ |
| Fiscal Shock <br> * Volatility |  |  | -15.810*** | $-15.147^{* * *}$ |
| Fiscal Shock <br> * Volatility (lag 1) |  |  | 1.085 | -0.649 |
|  |  |  | (4.290) | (4.398) |
| Fiscal Shock <br> * Volatility (lag 2) |  |  | 6.828 | 5.413 |
|  |  |  | (4.226) | (4.332) |
| Observations | 3,132 | 3,132 | 3,132 | 3,132 |
| Adjusted R ${ }^{2}$ | 0.130 | 0.161 | 0.134 | 0.164 |
| F Statistic | $94.256^{* * *}$ | $119.846^{* * *}$ | $61.200^{* * *}$ | $76.965^{* * *}$ |

## CHAPTER 7

## CONCLUSION

The present study has considered evidence for the hypothesis that fiscal expansions are less effective when agents are closely tracking fiscal policy. Specifically, based on the RI framework, the result that fiscal volatility diminishes the fiscal multiplier was tested empirically.

Table LXVII lists the study's primary estimates of the long-run multiplier. The multipliers were calculated using Equation 4.7 to combine representative observed levels of fiscal volatility, $\sigma_{g}$, with the dynamic FE parameter estimates.

The extended model multipliers in Table LXVII were calculated by fixing slackness to zero. Long-run multipliers fall significantly in the presence of high fiscal volatility in all models apart from those based on panel defense spending. Multipliers based on panel defense spending do fall in the presence of high fiscal volatility, however the effect is insignificant.

Table LXVIII and Table LXIX display arrays of multipliers based on fiscal volatilities and output gaps for the PWT and US data respectively. Rows vary by fiscal volatility and columns by output gap. Estimated multipliers are greater than one when economies are experiencing recessions and low fiscal volatility. The PWT estimates suggest that multipliers are less than one in normal times and during expansions. Estimates based on the US suggest higher multipliers for all combinations of volatility and slackness.

Several techniques aimed at addressing possible endogeneity between government purchases and output are implemented in Chapter 6. In particular, the near vector autoregressive (NVAR)

## TABLE LXVII

Estimates of Long-Run Multipliers Conditional on Fiscal Volatility

| Data <br> Source | Type of <br> Specification | Base <br> Observations | $0^{t h}$ <br> Percentile <br> Volatility | $25^{t h}$ <br> Percentile <br> Volatility | Median <br> Volatility <br> Volatility | $75^{t h}$ <br> Percentile <br> Volatility | $100^{t h}$ <br> Percentile <br> Volatility |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PWT | Extended Model | 3844 | 0.92 | 0.87 | 0.84 | 0.76 | -5.92 |
| PWT | Fiscal Shocks | 3844 | 1.04 | 0.99 | 0.953 | 0.878 | -6.16 |
| PWT | NVAR | 3844 | 0.58 | 0.56 | 0.55 | 0.53 | -1.34 |
| US | Fiscal Shocks | 94 | 1.05 | 1.04 | 1.03 | 0.9 | -0.052 |
| US | NVAR | 94 | 2.95 | 2.93 | 2.87 | 2.49 | 0.846 |
| US | Extended Model | 94 | 1.54 | 1.52 | 1.49 | 1.25 | -0.56 |
| US | Defense Shocks | 94 | 0.96 | 0.95 | 0.94 | 0.83 | 0.42 |
| US | IV | 94 | 1.07 | 1.06 | 1.04 | 0.93 | 0.39 |
| WDI | Defense Shocks | 2088 | 0.036 | 0.33 | 0.03 | 0.02 | -1.68 |
| WDI | IV | 2088 | 0.39 | 0.37 | 0.36 | 0.34 | -1.37 |
| NATO | Defense Shocks | 784 | 3.32 | 3.21 | 3.1 | 2.87 | 0.429 |
| NATO | IV | 784 | -21.9 | -40.9 | -54.3 | -75.5 | -275 |

TABLE LXVIII

PWT - Extended Model Multipliers

|  |  | Output Gap |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Volatility | $\mathbf{- 4 \%}$ | $\mathbf{- 3 \%}$ | $\mathbf{- 2 \%}$ | $\mathbf{- 1 \%}$ | $\mathbf{0}$ | $\mathbf{1 \%}$ |
| 0.00024 | 1.050 | 1.020 | 0.985 | 0.954 | 0.924 | 0.894 |
| 0.00271 | 1.010 | 0.984 | 0.954 | 0.923 | 0.893 | 0.863 |
| 0.00374 | 1 | 0.971 | 0.941 | 0.911 | 0.880 | 0.850 |
| 0.00570 | 0.977 | 0.947 | 0.916 | 0.886 | 0.856 | 0.825 |
| 0.00882 | 0.938 | 0.908 | 0.877 | 0.847 | 0.816 | 0.786 |
| 0.01490 | 0.862 | 0.832 | 0.801 | 0.771 | 0.741 | 0.710 |

TABLE LXIX

US - Extended Model Multipliers

|  |  | Output Gap |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Volatility | Year | $\mathbf{- 4 \%}$ | $\mathbf{- 2 \%}$ | $\mathbf{0}$ | $\mathbf{2 \%}$ | $\mathbf{4 \%}$ |  |
| 0.0015 | 1997 | 1.690 | 1.570 | 1.450 | 1.330 | 1.210 |  |
| 0.0030 | 1928 | 1.680 | 1.560 | 1.430 | 1.310 | 1.190 |  |
| 0.0079 | 1935 | 1.620 | 1.500 | 1.380 | 1.260 | 1.140 |  |
| 0.0352 | 1942 | 1.330 | 1.200 | 1.080 | 0.962 | 0.841 |  |
| 0.0985 | 1945 | 0.637 | 0.516 | 0.395 | 0.273 | 0.152 |  |
| 0.1790 | 1947 | -0.233 | -0.355 | -0.476 | -0.597 | -0.718 |  |

model was used to control for endogeneity by filtering predictable ${ }^{1}$ shocks to government purchases before estimating the dynamic model specification, Equation 4.8. In an alternative approach, data on defense spending shocks provided by the World Bank, NATO, and Barro and Redlick (2011) were used to proxy fiscal shocks under the assumption that defense spending is less likely to respond to output than government purchases. ${ }^{2}$ Finally, defense spending was used as an IV for fiscal shocks and volatility. The results indicate that multipliers are slightly lower when controlling for endogeneity; however the RI result persists.

[^23]Defense shocks are almost completely uncorrelated with either output growth or fiscal shocks in most countries. The WDI defense spending shocks' correlation with output growth is 0.07 and its correlation with fiscal shocks is 0.12 . Whereas for the US, these correlations are 0.57 and 0.99 respectively. Chapter 6 expands on this point to help explain why defense based estimates are insignificant for the panels, yet significant for the US.

Based on the estimates in Chapter 5, the results firmly support the hypothesis that fiscal volatility diminishes the multiplier. In order to test the robustness of these findings and to ensure that the results are neither driven by outliers nor applicable only to some systematic subset of countries, several techniques were applied to the Penn World Tables (PWT) panel, the largest dataset in terms of observations. A traditional search for outliers was performed that suggested the removal of Cyrpus, Israel, Morraco, and El Salvador (amounting to 6.5 percent of the data $)^{1}$. The volatility interaction effects remained highly significant and negative in the absence of these flagged outliers.

Both traditional and modern techniques were employed to detect possible structural breaks (Andrews, 1993, 2003; Hansen, 2013). The search involved sorting the data across time, GDP per capita $\left(\frac{Y}{\text { population }}\right)$, total GDP $(Y)$, GDP growth $(y)$, fiscal shocks $(g)$, fiscal volatility $\left(\sigma_{g}\right)$, and government purchases as a share of GDP $\left(\frac{G}{Y}\right)$. The 7 searches, each consisting of more than 5 thousand estimations, were performed for the pooled constant dynamic model (OLS). Structural breaks were detected for the OLS model sorted by time. Using a more

[^24]traditional search for breaks that inspects the data by performing the OLS estimation on each quartile indicated that the interaction terms on the first 15 years ${ }^{1}$ were non-negative. Historical explanations seem responsible as detailed in Chapter 6. Less than 25 percent of the data disagree with the overall parameter estimates, thus there is insufficient evidence to claim that the main results are driven by outliers or a minority of the data.

Despite the array of estimates ranging from $<0$ to $>1$ in Table LXVII, clear policy implications can be gleaned. The typical fiscal multiplier is estimated to be $0.84^{2}$ However, the overarching message of the study is that the potency of fiscal policy is contingent on specific conditions within each country and fiscal volatility in particular. Thus no simple rule of thumb, such as the multiplier is less than 1, suffices. Policy decisions should be made on a case by case basis.

[^25]
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## APPENDICES

## Appendix A

## R CODE

```
######
# This code is designed to run in any modern installation of R. It will
    replicate the core analysis. For the 4000+ lines of code that generate
    the complete analysis including automatic and fully formatted LATEX,
    PDF, and HTML output, please contact the author at richardschwinn@gmail
    .com.
######
# 0.1 Models & Parameters
model.1 = y ~ g
model.2 = y ~ g + sg.g
model.3 = y ~ lag(y,1:lags) + lag(g,0:lags)
model.4=y ~ lag(y, 1:lags ) + lag(g,0:lags) + lag(sg.g,0:lags)
K = 6 # Volatility Window
lags = 2 # Number of Lags
balanced = 1
######
# 0.2 Auto Package Instalation
# First-time users of R require special installation of required packages.
NEW.to.R<- function(x){ if (!require(x, character.only = TRUE)) {install.
    packages(x, dep = TRUE); if(!require(x, character.only = TRUE)) stop('
    Package„not\_found')}}
NEW. to .R('pwt8')
NEW. to.R('caTools')
NEW. to.R(' plm ')
#####
# 1.1 Data Download
# The Penn World Tables (PWT) data are distributed via the 'pwt8' library.
data('pwt8.0')
df = as.data.frame(pwt8.0)
df$Y = df$rgdpna # real GDP level variable
df$G=df$csh_g*df$Y # real government spending variable
df$iso3code = df$isocode
#####
# 1.2 Variable Preparation
# Growth
```


## Appendix A (Continued)

```
df$y = NA
for(country in unique(df$iso3code)) # 'unique, a vector of countries
{ rows = df[df$iso3code = country,] # creates a matrix for each country
rows = rows[order(rows$year),] # ensures that the rows are sorted
n}=\mathrm{ nrow(rows) # counts the number of years spanned
rows$y[2:n] = (rows$Y[2:n] - rows$Y[1:n-1])/rows$Y[1:n-1] # Missing - >NA
df[df$iso3code == country,] = rows} # replaces info for each country
# Fiscal Shocks
df$g = NA
for(country in unique(df$iso3code))
{ rows = df[df$iso3code = country,]
rows = rows[order(rows$year),]
n = nrow(rows)
rows$g[2:n] = (rows$G[2:n] - rows$G[1:n-1])/rows$Y[1:n-1]
df[df$iso3code == country,] = rows }
# Volatility
df$sg=NA; int = NA
df = df[complete.cases(df[,'G']),]
for(country in unique(df$iso3code)) # 'unique' a vector of countries
{rows = df[df$iso3code = country,] # creates a matrix for each country
rows = rows[order(rows$year),] # ensures that the rows are sorted
int = runsd(rows$g,K, align='right') # takes the rolling standard deviation
for(i in 2:(length(int)))
{rows$sg[i]=int[i-1] } # volatility is based on K year estimate
#rows$sg[1:K]=NA # turning this off makes early k's smaller
df[df$iso3code == country,] = rows} # replaces info for each country in df
df$sg.g <- df$sg*df$g
#####
# 1.3 Clean-up
startyear = min(df$year)+lags
df = df[df$year > startyear,]
firstyear = min(df$year)
lastyear = max(df$year)
if (balanced =1){ # removes countries that do not span time
new = NA
for(country in unique(df$iso3code)) {
rows = df[df$iso3code = country,]
rows = rows[order(rows$year),]
if(\boldsymbol{min}(\mathrm{ rows$year)=firstyear) {}
if( (lastyear-firstyear +1)= length(rows[,1]))
{new = rbind(new,rows)} } }
df = new [-1,]}
df = df[df$year > 1955,]
df = df[\mathbf{complete.cases(df[,c('iso3code',''y','g','sg','year')]),]}]
df = subset (df, select = c('y','g','sg.g','iso3code',' year''))
remove(new, pwt8.0,rows)
```


## Appendix A (Continued)

81
82
83

```
######
# 2.0 Analysis
p.df = pdata.frame(df, index = c('iso3code', 'year'), drop.index = FALSE,
    row.names = TRUE) # Changes data into panel format
# Static Models
OLS.s = plm(model.1, data = p.df, model = 'pooling')
FE.s = plm(model.1, data = p.df, model = 'within', effect = 'twoways')
RE.s = plm(model.1,data=p.df,model='random',random.method ='nerlove')
OLS.s.int = plm(model.2, data = p.df, model = 'pooling')
FE.s.int = plm(model.2, data = p.df, model = 'within', effect = 'twoways')
RE.s.int = plm(model.2,data=p.df,model='random',random.method ='nerlove')
# Dynamic Models
OLS = plm(model.3, data = p.df, model = ', pooling')
FE = plm(model.3, data = p.df, model = 'within', effect = 'twoways')
RE = plm(model.3, data = p.df, model ='random',random.method ='nerlove')
OLS.int = plm(model.4, data = p.df, model = ', pooling')
FE.int = plm(model.4, data = p.df, model = 'within', effect = 'twoways')
RE.int = plm(model.4,data=p.df,model='random',random.method='nerlove')
summary(FE.s.int)
summary(FE.int)
```


## Appendix B

## ADDITIONAL ANALYSIS

## B. 1 Additional Measures of Fiscal Volatility

The primary measure of volatility used throughout the analysis is sensitive to growth that is equal across output and government purchases but uneven over time. The effect is small, yet it is worthwhile to consider two similar measures, Equation B. 1 and Equation B.1, which do not suffer from the uneven growth problem.

$$
\begin{equation*}
\sigma_{p p_{1}}=\sqrt{\frac{\sum_{n=1}^{k}\left(\widehat{g_{i, t-n}}-\frac{\sum_{l=1}^{k} \widehat{g_{i, t-l}}}{k}\right)^{2}}{(k-1)}} . \tag{B.1}
\end{equation*}
$$

where

$$
\widehat{g_{i, t}}=\frac{G_{i, t}}{Y_{, i t}}-\frac{G_{i, t-1}}{Y_{i, t-1}}
$$

and

$$
\begin{equation*}
\sigma_{p p_{2}}=\sqrt{\frac{1}{k-1} \sum_{n=1}^{k}\left(\frac{G_{t-n}}{Y_{t-n}}-\frac{\sum_{l=1}^{k} \frac{G_{t-l}}{Y_{t-l}}}{k}\right)^{2}} \tag{B.2}
\end{equation*}
$$

In each case, Equation 4.6 is estimated as usual except for one modification: Either $\sigma_{p p}$ or $\sigma_{g_{i_{i}, t}}^{k}$ is substituted for $\sigma_{g_{i, t}}^{k}$. Table LXX contains the estimates based on $\sigma_{p p_{1}}$. The empirical results employing these alternative volatility measures support the hypothesis that the multiplier falls in response to fiscal volatility.

## Appendix B (Continued)

## TABLE LXX

PWT - Dynamic Empirical Results based on $\sigma_{p p_{1}}$

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\text { FE Equation } 4.5 \mathrm{RE}$ |  | Equation 4.6 |  |
| Output Growth (lag 1) | 0.207 | $0.248{ }^{\text {* }}$ | $0.205^{*}$ | 0.241 |
|  | (0.018) | (0.017) | (0.018) | (0.017) |
| Output Growth (lag 2) | $0.048^{* * *}$ | $0.055^{* * *}$ | $0.051 * * *$ | $0.054{ }^{* * *}$ |
|  | (0.017) | (0.017) | (0.017) | (0.017) |
| Fiscal Shock | $0.254 * * *$ | $0.287^{* * *}$ | 0.665*** | 0.695*** |
|  | (0.025) | (0.025) | (0.043) | (0.044) |
| Fiscal Shock (lag 1) | 0.035 | 0.029 | -0.001 | 0.014 |
|  | (0.025) | (0.026) | (0.049) | (0.050) |
| Fiscal Shock (lag 2) | 0.076*** | 0.089*** | 0.118*** | 0.132*** |
|  | (0.025) | (0.026) | (0.045) | (0.046) |
| Fiscal Shock * Vol. |  |  | $-4.394^{* * *}$ | $-4.484^{* * *}$ |
|  |  |  | (0.411) | (0.423) |
| Fiscal Shock * Vol. (lag 1) |  |  | $-2.022^{* * *}$ | $-2.313^{* * *}$ |
|  |  |  | (0.535) | (0.547) |
| Fiscal Shock <br> * Vol. (lag 2) |  |  | $-4.042^{* * *}$ | $-4.266^{* * *}$ |
|  |  |  | (0.593) | (0.605) |
| Observations | 3,348 | 3,348 | 3,348 | 3,348 |
| Adjusted $\mathrm{R}^{2}$ | 0.088 | 0.124 | 0.122 | 0.156 |
| F Statistic | $64.783^{* * *}$ | $94.973^{* * *}$ | 58.644*** | 77.373*** |
| Note: |  |  | ${ }^{*} \mathrm{p}<0.1 ;{ }^{* *} \mathrm{p}$ | 5; ${ }^{* * *} \mathrm{p}<0.01$ |

## Appendix B (Continued)

## TABLE LXXI

PWT - Dynamic Empirical Results based on $\sigma_{p p_{2}}$

|  | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | FE ${ }^{\text {Equation } 4.5} \mathrm{RE}$ |  | FE Equation 4.6 |  |
| Output Growth (lag 1) | 0.207, | (0.248 | 0.210 ** | 0.248** |
|  | (0.018) | (0.017) | (0.017) | (0.017) |
| Output Growth (lag 2) | $0.048^{* * *}$ | $0.055 * * *$ | 0.058** | 0.061*** |
|  | (0.017) | (0.017) | (0.017) | (0.017) |
| Fiscal Shock | $0.254^{* * *}$ | $0^{0.287 * * *}$ | 0.625*** | 0.639*** |
|  | (0.025) | (0.025) | (0.041) | (0.042) |
| Fiscal Shock (lag 1) | 0.035 | 0.029 | -0.104** | $-0.100^{* *}$ |
|  | (0.025) | (0.026) | (0.042) | (0.043) |
| Fiscal Shock (lag 2) | $0.076 * * *$ | 0.089*** | $-0.033$ | $-0.017$ |
|  | (0.025) | (0.026) | (0.043) | (0.044) |
| Fiscal Shock * Vol. |  |  | $-1.647^{* * *}$ | $-1.581^{* * *}$ |
|  |  |  | (0.151) | (0.156) |
| Fiscal Shock <br> * Vol. (lag 1) |  |  | 0.172 | 0.141 |
|  |  |  | (0.151) | (0.155) |
| Fiscal Shock <br> * Vol. (lag 2) |  |  | 0.009 | -0.0004 |
|  |  |  | (0.139) | (0.143) |
| Observations | 3,348 | 3,348 | 3,348 | 3,348 |
| Adjusted $\mathrm{R}^{2}$ | 0.088 | 0.124 | 0.122 | 0.152 |
| F Statistic | $64.783^{* * *}$ | 94.973*** | 58.225*** | $75.021^{* * *}$ |
| Note: |  |  | *p<0.1; ${ }^{* *} \mathrm{p}$ | $5 ; * * *$ p<0.01 |

## Appendix B (Continued)

## TABLE LXXII

Ancillary Controls: Slackness and Openness

| Output Growth (lag 1) | Dependent variable: Output Growth |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | (2) | (3) | (4) ${ }^{\text {0, }}$ |
|  | $\begin{aligned} & 0.227^{* * *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.222^{* *} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.237^{* * *} \\ & (0.020) \end{aligned}$ | $\begin{aligned} & 0.228^{* *} \\ & (0.018) \end{aligned}$ |
| Output Growth (lag 2) | $0.040^{* *}$ | $0.044^{* *}$ | $0.051^{* *}$ | $0.072^{* * *}$ |
|  | (0.018) | (0.018) | (0.020) | (0.018) |
| Fiscal Shock | $0.622^{* * *}$ | $0.691^{* * *}$ | $0.690^{* * *}$ | $0.521^{* * *}$ |
|  | (0.045) | (0.046) | (0.064) | (0.047) |
| Fiscal Shock (lag 1) | -0.007 | -0.051 | -0.258*** | -0.037 |
|  | (0.050) | (0.052) | (0.070) | (0.051) |
| Fiscal Shock (lag 2) | $0.139^{* *}$ | 0.049 | 0.020 | -0.065 |
|  | (0.046) | (0.052) | (0.070) | (0.051) |
| Fiscal Shock*Volatility | -3.891 *** | -1.281* | $-5.417^{* * *}$ | 1.224* |
|  | (0.387) | (0.666) | (1.779) | (0.677) |
| Fiscal Shock*Volatility (lag 1) | $-1.708^{* * *}$ | -1.286 | 7.604*** | 1.061 |
|  | (0.499) | (1.197) | (1.942) | (1.210) |
| Fiscal Shock*Volatility (lag 2) | $-3.629^{* * *}$ | -1.384* | -0.307 | 0.621 |
|  | (0.550) | (0.743) | (1.748) | (0.735) |
| Openness |  | $-0.046^{* * *}$ | $-0.053^{* * *}$ | $-0.056^{* * *}$ |
|  |  | (0.008) | (0.009) | (0.008) |
| Openness (lag 1) |  | $0.040 * * *$ | 0.077*** | 0.023* |
|  |  | (0.013) | (0.019) | (0.013) |
| Openness (lag 2) |  | 0.001 | 0.009 | $0.037^{* * *}$ |
|  |  | (0.013) | (0.041) | (0.014) |
| Baxter-King Output Gap |  |  | 5.263*** |  |
|  |  |  | $(0.755)$ |  |
| Baxter-King Output Gap (lag 1) |  |  | $-7.426^{* * *}$ |  |
| Baxter-King Output Gap (lag 2) |  |  | - $0.40 .746^{* * *}$ |  |
|  |  |  | (0.807) |  |
| Hodrick-Prescott Output Gap |  |  |  | $5.720^{* * *}$ |
| Hodrick-Prescott Output Gap (lag 1) |  |  |  | $(0.759)$ $-8.754^{* * *}$ |
|  |  |  |  | (0.747) |
| Hodrick-Prescott Output Gap (lag 2) |  |  |  | -1.841** |
|  |  |  |  | (0.779) |
| Observations | 3,050 | 3,050 | 2,684 | 3,050 |
| Adjusted $\mathrm{R}^{2}$ | 0.126 | 0.136 | 0.190 | 0.192 |
| F Statistic | $55.529^{* * *}$ | 44.032*** | $45.393^{* * *}$ | $52.245^{* * *}$ |
| Note: |  |  | *p<0.1; ** p | ; *** $\mathrm{p}<0.01$ |

## Appendix B (Continued)

## B.1.1 Insignificance of Main Effects

## TABLE LXXIII

PWT - Main Effects are Insignificant

|  | Dependent variable: Output Growth |
| :---: | :---: |
| Output Growth (lag 1) | $\frac{\text { resuts }}{0.207^{* * *}}$ |
|  | (0.018) |
| Output Growth (lag 2) | 0.053*** |
| Fiscal Shock | ${ }_{0}^{(0.017)} 0$ |
|  | (0.044) |
| Fiscal Shock (lag 1) | $-0.026$ |
|  | (0.051) |
| Fiscal Shock (lag 2) | 0.065 |
| Volatility | 0.105 |
|  | (0.126) |
| Volatility ( $\operatorname{lag} 1$ ) | 0.151 |
|  | (0.210) |
| Volatility ( $\operatorname{lag} 2$ ) | -0.198 |
| Fiscal Shock*Volatility | (0.151) |
|  | (0.388) |
| Fiscal Shock*Volatility (lag 1) | $-1.416{ }^{* *}$ |
|  | ${ }^{(0.557)}{ }^{2} 896 * *$ |
| Fiscal Shock*Volatility (lag 2) | $-2.896^{* * *}$ |
| Observations | 3,348 |
| Adjusted $\mathrm{R}^{2}$ | 0.127 |
| F Statistic | $\frac{44.375^{* * *}}{}$ |

VITA

## Richard Schwinn

## Education

| University of Illinois | Chicago, Illinois |
| :--- | ---: |
| Ph.D. Economics,Dissertation: Fiscal Volatility Diminishes Fiscal Multipliers | 2015 |
| Committee: Georgios Karras (Chair), Jin Man Lee, Lawrence Officer, Paul Pieper, \& Houston Stokes |  |
| Advisor: Georgios Karras | Chicago, Illinois |
| University of Illinois | 2012 |
| M.S. Mathematics,Specialty: Economics |  |
| Advisor: Jie Yang | Chicago, Illinois |
| University of Illinois | 2003 |
| M.A. Economics,Thesis: Economic Conditions and Voting Behavior in Gubernatorial Races | Oxford, Ohio |
| Advisor: Ali Akarca | 2002 |
| Miami University | B.S. Business,Specialties: Economics, French, and Japanese |

## Academic Experience

Northeastern Illinois Univ.
FACULTY INSTRUCTOR

| Courses Taught: ○ Principles of Macroeconomics <br> - Principles of Microeconomics <br> - Business Statistics I <br> - Business Statistics II <br> - Statistical Consulting | - Price Theory <br> - Cost-Benefit Analysis <br> - Econometrics <br> - Public Finance <br> - Managerial Economics |  |
| :---: | :---: | :---: |
| Loyola University <br> VISITING LECTURER |  | Chicago, Illinois 2015 - Present |
| Courses Taught: ○ Intermediate Macroeconomics University of Illinois VISITING LECTURER | - Business Fluctuations | Chicago, Illinois $2011-2013$ |
| Courses Taught: ○ Intermediate Microeconomics <br> - Introduction to Statistics | - Econometrics |  |
| Lincoln Institute of Land Policy RESEARCHER |  | Chicago, Illinois 2005 |
| Institute of Public Safety Partnerships RESEARCHER |  | Chicago, Illinois 2004-2005 |
| Académie de Versailles ASSISTANT ANGLAIS |  | Paris, France 2003-2004 |

## Professional Employment

| Avant Credit | Chicago, Illinois |
| :--- | ---: |
| STATISTICAL CONSULTANT | $2013-2015$ |
| ASP Magazines |  |
| REPORTER and PHOTOGRAPHER | $2000-2008$ |
| Chicago Mercantile Exchange | Chicago, Illinois |
| TRADE CHECKER | $1996-2000$ |

## Professional Affiliations

American Statistical Association (ASA)


[^0]:    ${ }^{1}$ The term fiscal multiplier is used interchangeably with government purchases multiplier. The later implies no change in taxes. When relevant, the distinction is noted.

[^1]:    ${ }^{1}$ Chapter 5 tests this hypothesis
    ${ }^{2}$ Focus is restricted to national-level fiscal policy although Ramey (2011b) describes several papers considering cross-state evidence.

[^2]:    ${ }^{1}$ See Barro-Ramsey model (Barro, 1974)

[^3]:    ${ }^{1}$ Woodford (2010) cites the Smets and Wouters (2007) model as being the DSGE model of consensus in the economics profession as well as the most representative of New Keynesian views.
    ${ }^{2}$ Estimates range from $10 \%$ to $50 \%$.

[^4]:    ${ }^{1} m p c$ is the marginal propensity to consume

[^5]:    ${ }^{1}$ As opposed to $10 \%$, which is the raw growth rate of G.

[^6]:    ${ }^{1}$ In order to better visualize the relationships, outliers are omitted

[^7]:    ${ }^{1}$ See Ivanov and Kilian (2005) for a careful discussion of lag selection criteria.

[^8]:    ${ }^{1} \frac{S S R}{N}$ is the quasi-maximum likelihood estimate of the innovation covariance matrix. Also note LR and SLR are $\chi_{(J K+K)}^{2}$
    ${ }^{2}(J K+K)$ is the number of coefficients to be estimated per equation

[^9]:    ${ }^{1}$ The largest panel in the study is based on the Penn World Tables as described in Chapter 4. Only volatility windows of three or more years are considered since windows of two years yield no information beyond a single lagged difference.

[^10]:    ${ }^{1}$ The initial six years are used to measure $\sigma_{g}$

[^11]:    ${ }^{1}$ The plots in Figure 7 and related figures were generated using tools developed by Breheny and Burchett (2013).

[^12]:    ${ }^{1}$ Nickell (1981) shows that the demeaning process in RE and FE models results in downward bias of the autoregressive coefficients. In a first order model where $T=$ data timespan, the inconsistency of $\alpha$ as the number of countries $\rightarrow \infty$ is approximately $\left(\frac{1+\alpha}{T-1}\right)$. Supposing the timespans noted in Chapter 4 and an $\alpha=0.3$, which is higher than any calculated in the study, the maximum potential bias ranges from -0.054 to -0.021 . These differences are quite small though it should be noted that such differences would downwardly bias estimates of the long-run multiplier. Results attempting to address this bias directly using GMM estimation do not differ significantly from the RE and FE estimates.
    ${ }^{2}$ Based on the multiplier defined: $\left(\sum_{j=1}^{J} m_{j}\right) \div\left(1-\sum_{j=1}^{J} \alpha_{j}\right)$

[^13]:    ${ }^{1}$ The recommended $\lambda=6.25$ for annual data is used.

[^14]:    ${ }^{1}$ All tax data are taken from the WDI.

[^15]:    ${ }^{1}$ Note that $\mathrm{J}=0$ and the autoregressive terms are dropped in the referenced equations.

[^16]:    ${ }^{1}$ Note that $\mathrm{J}=0$ and the autoregressive terms are dropped in the referenced equations.

[^17]:    ${ }^{1}$ Note that $\mathrm{J}=0$ and the autoregressive terms are dropped in the referenced equations.

[^18]:    ${ }^{1}$ Outliers have been removed from the WDI plots to aid visualization.

[^19]:    ${ }^{1}$ After accounting for time and country specific effects.

[^20]:    ${ }^{1}$ Powerful in the statistical sense that the probability of a Type II error has been reduced.

[^21]:    ${ }^{1}$ The average for each country across the time spanned by the panel.

[^22]:    ${ }^{1}$ at $n>3000$ this is equivalent to a z -score. Using another popular measure: the mean absolute deviation and median test, I must remove 35 countries. When I do so, the signs on the coefficients remain however the overall model loses its significant.

[^23]:    ${ }^{1}$ Predictable in the sense that lagged fiscal shocks and growth were used to make predictions about government purchases.
    ${ }^{2}$ See Equation 4.12.

[^24]:    ${ }^{1}$ Parametric algorithms for identifying outliers tend to assume normality, yet volatility is plainly non-normal, due to its derivation, thus making traditional tests for outliers even more sensitive.

[^25]:    ${ }^{1}$ The data range from 1956 to 1969 , with volatility measures based on the 1952 to 1968 era.
    ${ }^{2}$ This estimate is based on median levels of volatility and slackness for the PWT data. The typical estimate for the US estimates is 1.49 .

