# Electromagnetic Scattering by Various Cylindrical Posts Located Inside a Parallel-Plate Waveguide


BY

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THESIS

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*to all those who dedicate their lives to bring peace in this world*

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AA

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# LIST OF ABBREVIATIONS

| | |
|---|---|
| Z | intrinsic impedance of free space |
| $Z_{in}$ | intrinsic impedance of the post |
| j | imaginary unit (= $\sqrt{-1}$) |
| $e^{+j\omega t}$ | time-dependence factor (omitted throughout) |
| $\lambda$ | wavelength |
| k | wavenumber (=$\frac{2\pi}{\lambda}$) |
| $\varepsilon_n$ | Neumann symbol ($\varepsilon_0 = 1$; $\varepsilon_n = 2$ $for\, n = 1, 2, 3...$) |
| x,y,z | rectangular coordinates |
| $\rho$,$\phi$,z | circular-cylinder coordinates |
| u,v,z | elliptic-cylinder coordinates |
| $\nabla$ | gradient operator |
| $\nabla\times$ | curl operator |
| $\nabla\cdot$ | divergence operator |
| $\underline{E}^i$ | incident electric field vector |
| $\underline{H}^i$ | incident magnetic field vector |
| $\underline{E}^s$ | scattered electric field vector |
| $\underline{H}^s$ | scattered magnetic field vector |

## LIST OF ABBREVIATIONS (Continued)

| | |
|---|---|
| $\underline{E} = \underline{E}^i + \underline{E}^s$ | total electric field vector |
| $\underline{H} = \underline{H}^i + \underline{H}^s$ | total magnetic field vector |
| $J_n$ | Bessel function of the first kind |
| $H_n^{(2)}$ | Hankel function of the second kind |
| $Re_m^{(1)}$ | even radial Mathieu function of the first kind |
| $Ro_m^{(1)}$ | odd radial Mathieu function of the first kind |
| $Re_m^{(4)}$ | even radial Mathieu function of the fourth kind |
| $Ro_m^{(4)}$ | odd radial Mathieu function of the fourth kind |
| $Se_m$ | even angular Mathieu function |
| $So_m$ | odd angular Mathieu function |
| $Ne_m$ | even normalization coefficient |
| $No_m$ | odd normalization coefficient |

## SUMMARY

In this thesis, electromagnetic scattering of a mode propagating inside a parallel plate waveguide by posts of different cylindrical cross-sections is considered, in the phasor domain. The analysis is conducted for both TM and TE modes and exact solutions are obtained for various cylindrical posts. In particular, exact expressions for surface current densities on PEC surfaces and numerical results of the same are provided.

For the case of both circular-cylinder and elliptic-cylinder post, solutions are first obtained when the linear, homogeneous and isotropic material of the post is isorefractive to the surrounding medium. The case of PEC posts is a special case of the previous solution and is obtained by considering intrinsic impedance of the post to be zero, $Z_{in} = 0$. It should be noted that the case of metal strip is the limiting case of a post of elliptical cross section and is obtained when the elliptic-cylinder post collapses onto a strip.

# CHAPTER 1

# INTRODUCTION

Electromagnetic scattering theory enables one to investigate and explain electromagnetic field behavior in the presence of material objects. The nature of electromagnetic fields, and especially of surface currents and far fields is of interest in a myriad of applications. In general, scattering means re-radiation of the incident field from an object, which is also known as scatterer. The re-radiated field may change the properties of the incident field like amplitude, polarization and direction of propagation. The scattered field carries the information about the properties of the object which interacts with the EM field and this forms the basis of classical applications of electromagnetic scattering like remote sensing, radar etcetera.

Historically, the boundary-value problem has been solved for diffraction from simple shapes such as circular cylinders and spheres. The problem of diffraction of a normally incident plane wave by a homogenous dielectric circular cylinder was solved by Rayleigh (1). A more general case of scattering of plane wave obliquely incident on homogenous and isotropic circular-cylinder was analyzed by Wait (2), in which he proved that whether the incident field is E-polarized or H-polarized, the scattered field contains a cross-polarized component which always vanishes in the particular case of normal incidence. Uslenghi (3), extended Wait′s result to penetrable cylinders of arbitrary cross section, in which he proved that polarization decoupling can be achieved if the cylinder is made of homogenous material, which is either isorefractive or anti-isorefractive to the surrounding medium.

In this work, scattering of a mode propagating inside a parallel plate waveguide by a cylindrical post located inside the waveguide and oriented perpendicularly to the waveguide plates is analyzed , in the phasor domain and with a time-dependence factor $e^{+j\omega t}$ omitted throughout. The analysis is conducted for both TE and TM modes. The case of TEM mode is a trivial case and is similar to the case of scattering of a normally incident plane wave from the cylindrical post.

Three different shapes of the cross-sectional area of the post are considered: circular, elliptical, and segment (corresponding to a flat strip as a limiting case of a post of elliptical cross section). The analysis consists of three steps. First, a propagating mode is decomposed into the sum of two plane waves that are obliquely incident on the cylindrical post. Second, the scattering of each plane wave by a post of infinite length is determined. Finally, it is verified that the superposition of the two scattered fields yields an overall scattered field which satisfies the boundary conditions on the two plates of the waveguide.

Exact solutions are presented for metallic posts of all three cross-sectional shapes. For circular cylinder and elliptic cylinder posts, a solution is also given when the linear, homogeneous and isotropic material of the post is isorefractive to the surrounding medium.

The geometry of the problem can also be viewed as infinitely long cylindrical posts of different shapes being perpendicularly truncated by two metal planes. In a similar work, Uslenghi (4) investigated scattering by metallic cylinders perpendicularly truncated by a single metal plane.

# CHAPTER 2

# GENERAL THEORY

In this section, the general theory of the problem is developed. Considering a combination of two plane waves traveling inside a parallel plate waveguide and obliquely incident on a cylindrical post, general solutions are derived in curvilinear coordinates for both E- and H-polarizations. Curviliniear coordinates are chosen over rectangular coordinates because solutions in case of cylindrical cross-sectional posts are easier to express in curvilinear coordinate systems.

## 2.1 E-polarization

Consider the incident plane wave

$$\underline{E}^{(e)i} = (-\hat{x}\cos\theta_0\cos\phi_0 - \hat{y}\cos\theta_0\sin\phi_0 - \hat{z}\sin\theta_0)e^{-jk\hat{k}^i\cdot\underline{r}} \tag{2.1a}$$

$$\underline{H}^{(e)i} = Y(-\hat{x}\sin\phi_0 + \hat{y}\cos\phi_0)e^{-jk\hat{k}^i\cdot\underline{r}} \tag{2.1b}$$

where,

$$\hat{k}^i = -\hat{x}\cos\theta_0\cos\phi_0 - \hat{y}\cos\theta_0\sin\phi_0 - \hat{z}\sin\theta_0 \tag{2.2a}$$

$$\underline{r} = x\hat{x} + y\hat{y} + z\hat{z} \tag{2.2b}$$

$$\hat{k}^i\cdot\underline{r} = -x\cos\theta_0\cos\phi_0 - y\cos\theta_0\sin\phi_0 - z\sin\theta_0 \tag{2.2c}$$

**Solution in curvilinear coordinates**

Consider a cylinder with generators parallel to the z-axis and truncated by a metal plane z = 0. In

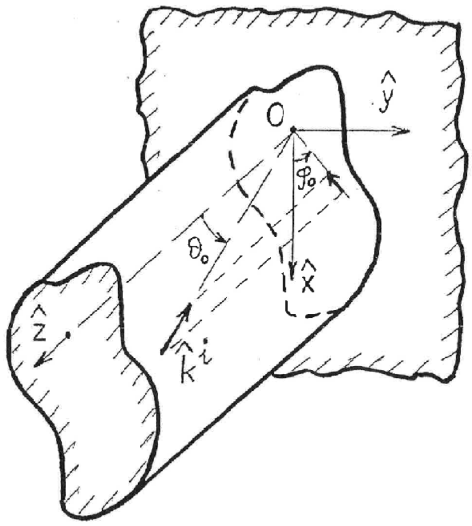


Figure 1: Plane wave incident on a cylinder truncated by a flat surface

curvilinear orthogonal coordinates $(u_1, u_2, z)$ with $\hat{u_1} \times \hat{u_2} = \hat{z}$, assume that the total field and the incident field at normal incidence ($\theta_0 = \frac{\pi}{2}$) on the untruncated (infinite) cylinder are given by :

$$E_z^{(e)}|_{2D} = u^{(e)}\,(u_1, u_2; k) \tag{2.3a}$$

$$E_z^{(e)\,i}|_{2D} = e^{j\,k\,(x\,\cos\,\phi_0 + y\,\sin\,\phi_0)} \tag{2.3b}$$

then the total field for E-polarization in the geometry of Fig. 1 is (4) :

$$E^{(e)} = \frac{2\cos\theta_0}{k\sin\theta_0}\sin(kz\cos\theta_0)\,\nabla_t\,u^{(e)}(u_1,u_2;k\sin\theta_0) + 2\sin\theta_0\cos(kz\cos\theta_0)\,u^e(u_1,u_2;k\sin\theta_0)\hat{z} \tag{2.4a}$$

$$H^{(e)} = \frac{-2jY}{k\sin\theta_0}\cos(kz\cos\theta_0)\,(\hat{z}\times\nabla_t)\,u^{(e)}(u_1,u_2;k\sin\theta_0) \tag{2.4b}$$

where $\nabla_t$ is the transverse gradient operator.

Parallel Plate Waveguide

Consider propagation inside the parallel plate waveguide of Fig. 2.

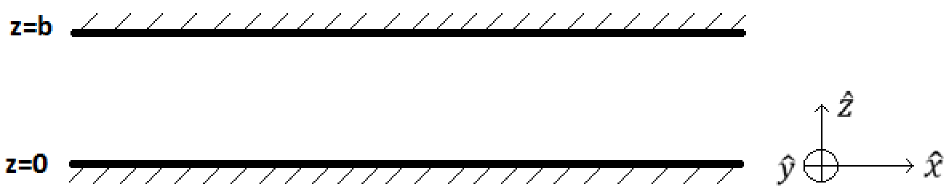


Figure 2: Side view of parallel plate waveguide

In case of E-polarization and $\phi_0 = \pi$ :

$$\underline{E}^{(e)} = E_x^{(e)}\,\hat{x} + E_z^{(e)}\,\hat{z} \tag{2.5a}$$

$$\underline{H}^{(e)} = H_y^{(e)}\,\hat{y} \tag{2.5b}$$

where the exprssions of $H_y^{(e)}, E_x^{(e)}$ and $E_z^{(e)}$ are as follows :

$$H_y^{(e)} = -Y\,\cos(\beta\,z)\,e^{-j\,k_t\,x} \tag{2.6a}$$

$$E_x^{(e)} = j\,\frac{\beta}{k}\,\sin(\beta\,z)\,e^{-j\,k_t\,x} \tag{2.6b}$$

$$E_z^{(e)} = \frac{k_t}{k}\,\cos(\beta\,z)\,e^{-j\,k_t\,x} \tag{2.6c}$$

with,

$$\beta^2 + k_t^2 = k^2 \tag{2.7a}$$

$$\beta = \frac{m\,\pi}{b}, m = 0,1,2,3,...(m = 0\ is\ the\ TEM\ mode) \tag{2.7b}$$

Using Euler's formulas

$$\sin(\beta\,z) = \frac{1}{2\,j}\,(e^{j\,\beta\,z} - e^{-j\,\beta\,z}) \tag{2.8a}$$

$$\cos(\beta\,z) = \frac{1}{2}\,(e^{j\,\beta\,z} + e^{-j\,\beta\,z}) \tag{2.8b}$$

in (2.6) we obtain,

$$H_y^{(e)} = \frac{Y}{2}\, e^{-j(k_t x - \beta z)} + \frac{Y}{2}\, e^{-j(k_t x + \beta z)} \tag{2.9a}$$

$$E_x^{(e)} = \frac{-\beta}{2k}\, e^{-j(k_t x - \beta z)} + \frac{\beta}{2k}\, e^{-j(k_t x + \beta z)} \tag{2.9b}$$

$$E_z^{(e)} = \frac{k_t}{2k}\, e^{-j(k_t x - \beta z)} + \frac{k_t}{2k}\, e^{-j(k_t x + \beta z)} \tag{2.9c}$$

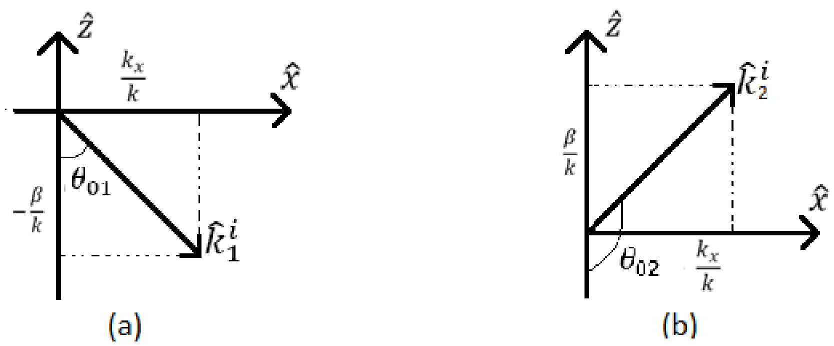


Figure 3: Two incident plane waves, (a) Plane wave 1 is propagating in the direction $\hat{k}_1^i$ and makes an angle $\theta_{01}$ with the negative z-axis. (b) Plane wave 2 is propagating in the direction $\hat{k}_2^i$ and makes an angle $\theta_{02}$ with the negative z-axis.

Equations (2.9) show that the electromagnetic field is a superposition of two plane waves, as shown in Fig. 3, where :

$$\hat{k}_1^i = \frac{k_t}{k}\,\hat{x} - \frac{\beta}{k}\,\hat{z} \tag{2.10a}$$

$$\hat{k}_2^i = \frac{k_t}{k}\,\hat{x} + \frac{\beta}{k}\,\hat{z} \tag{2.10b}$$

$$\theta_{02} = \pi - \theta_{01} \tag{2.10c}$$

$$\cos\theta_2 = -\cos\theta_1 = -\frac{\beta}{k} \tag{2.10d}$$

$$\sin\theta_2 = \sin\theta_1 = \frac{k_t}{k} \tag{2.10e}$$

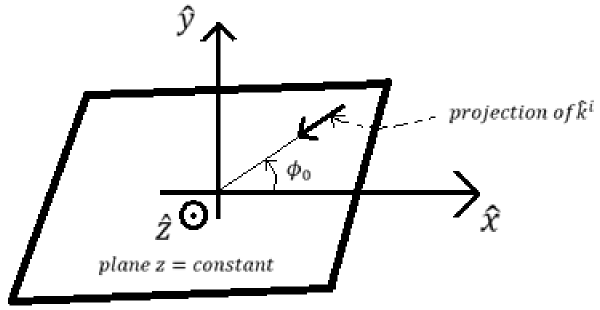


Figure 4: Top view of parallel plate waveguide

It follows that for an arbitrary $\phi_0$ the incident fields can be rewritten as,

$$
\begin{aligned}
\underline{E}^{(e)i} &= \frac{1}{2}\sum_{l=1}^{2}(-\hat{x}\cos\theta_{0l}\,\cos\phi_0 - \hat{y}\cos\theta_{0l}\,\sin\phi_0 - \hat{z}\,\sin\theta_{0l})e^{-jk\hat{k}_l^i\cdot\underline{r}} \\
&= \left[-\frac{j\beta}{k}\,(\hat{x}\cos\phi_0 + \hat{y}\sin\phi_0)\,\sin(\beta z) + \frac{k_t}{k}\,\hat{z}\cos(\beta z)\right]e^{jk_t\,(x\cos\phi_0 + y\sin\phi_0)}
\end{aligned} \tag{2.11a}
$$

$$
\begin{aligned}
\underline{H}^{(e)i} &= \frac{Y}{2}\,(-\hat{x}\sin\phi_0 + \hat{y}\cos\phi_0)\sum_{l=1}^{2} e^{-jk\hat{k}_l^i\cdot\underline{r}} \\
&= Y\,\cos(\beta z)\,(-\hat{x}\sin\phi_0 + \hat{y}\cos\phi_0)\,e^{jk_t\,(x\cos\phi_0 + y\sin\phi_0)}
\end{aligned} \tag{2.11b}
$$

At z = 0,b, the electric field becomes parallel to $\hat{z}$ and the boundary conditions are satisfied.

At n = 0 ($\beta = 0$, TEM mode),

$$\underline{E}^{(e)i}|_{n=0} = \hat{z}\,e^{jk_t\,(x\cos\phi_0 + y\sin\phi_0)} \tag{2.12a}$$

$$\underline{H}^{(e)i}|_{n=0} = Y\,(-\hat{x}\sin\phi_0 + \hat{y}\cos\phi_0)\,e^{jk_t\,(x\cos\phi_0 + y\sin\phi_0)} \tag{2.12b}$$

If $\phi_0 = \pi$,

$$\underline{E}^{(e)i}|_{\phi_0=\pi} = \left[\frac{j\beta}{k}\,\hat{x}\sin(\beta z) + \frac{k_t}{k}\,\hat{z}\cos(\beta z)\right]e^{-jk_t x} \tag{2.13a}$$

$$\underline{H}^{(e)i}|_{\phi_0=\pi} = -\hat{z}Y\,\cos(\beta z)\,e^{-jk_t x} \tag{2.13b}$$

which coincide with (2.6).

In this case of E-polarization, the expressions for total incident fields (2.11), which are a combination of two plane waves are found by dividing the expressions of each plane wave by 2 and adding them.

In curvilinear coordinates the solution will be :

$$\underline{E}^{(e)} = -\frac{\beta}{k\,k_t}\sin(\beta z)\,\nabla_t\,u^{(e)}(u_1,\,u_2;\,k_t) + \hat{z}\,\frac{k_t}{k}\,2\cos(\beta z)\,u^{(e)}(u_1,\,u_2;\,k_t) \tag{2.14a}$$

$$\underline{H}^{(e)} = -\frac{2\,jY}{k_t}\cos(\beta z)(\hat{z}\times\nabla_t)\,u^{(e)}(u_1,\,u_2;\,k_t) \tag{2.14b}$$

## 2.2 H-polarization

The incident plane wave for H-polarization is :

$$\underline{H}^{(h)i} = Y\,(-\hat{x}\cos\theta_0\cos\phi_0 - \hat{y}\cos\theta_0\sin\phi_0 - \hat{z}\sin\theta_0)e^{-jk\hat{k}^i\cdot\underline{r}} \tag{2.15a}$$

$$\underline{E}^{(h)i} = (\hat{x}\sin\phi_0 - \hat{y}\cos\phi_0)e^{-jk\hat{k}^i\cdot\underline{r}} \tag{2.15b}$$

**Solution in curvilinear coordinates**

We assume that the total field and the incident field at normal incidence on the untruncate (infinite) cylinder are :

$$H_z^{(h)}|_{2D} = Y\,u^{(h)}\,(u_1,u_2;k) \tag{2.16a}$$

$$H_z^{(h)\,i}|_{2D} = Y\,e^{jk\,(x\cos\phi_0 + y\sin\phi_0)} \tag{2.16b}$$

then the total field for H-polarization in the geometry of Fig. 1 is (4) :

$$E^{(h)} = \frac{-2}{k\sin\theta_0}\sin(kz\cos\theta_0)\,(\hat{z}\times\nabla_t)\,u^{(h)}\,(u_1,u_2;k\sin\theta_0) \tag{2.17a}$$

$$\begin{aligned} H^{(h)} = {} & \frac{2\,jY\cos\theta_0}{k\sin\theta_0}\cos(kz\cos\theta_0)\,\nabla_t\,u^{(h)}\,(u_1,u_2;k\sin\theta_0) \\ & + 2\,jY\sin\theta_0\sin(kz\cos\theta_0)\,u^{(h)}\,(u_1,u_2;k\sin\theta_0)\hat{z} \end{aligned} \tag{2.17b}$$

Parallel Plate Waveguide

In case of H-polarization, for propagation inside the parallel plate waveguide of Fig. 2 :

$$\underline{H}^{(h)} = H_x^{(h)}\,\hat{x} + H_z^{(h)}\,\hat{z} \tag{2.18a}$$

$$\underline{E}^{(h)} = E_y^{(h)}\,\hat{y} \tag{2.18b}$$

where the exprssions of $E_y^{(h)}, H_x^{(h)}$ and $H_z^{(h)}$ are as follows :

$$E_y^{(h)} = \sin(\beta\, z)\, e^{-j\,k_t\,x} \tag{2.19a}$$

$$\begin{aligned} H^{(h)} &= \frac{j\,Y}{k}\,\nabla \times \left[E_y^{(h)}(x,y)\hat{y}\right] \\ &= \frac{j\,Y}{k}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} \\ &= \frac{j\,Y}{k}\left[\hat{x}\left(-\frac{\partial\,E_y^{(h)}}{\partial\,z}\right) + \hat{z}\left(\frac{\partial\,E_y^{(h)}}{\partial\,x}\right)\right] \end{aligned} \tag{2.19b}$$

$$H_x^{(h)} = -j\,Y\,\frac{\beta}{k}\,\cos(\beta\,z)\,e^{-j\,k_t\,x} \tag{2.19c}$$

$$H_z^{(h)} = Y\,\frac{k_t}{k}\,\sin(\beta\,z)\,e^{-j\,k_t\,x} \tag{2.19d}$$

$\beta$ and $k_t$ are given by (2.7). Note that m = 0 yields zero fields (there is no TEM mode for H-polarization). Hence :

$$\beta = \frac{m\pi}{b} \tag{2.20a}$$

$$\beta b = m\pi, \;\; (m = 1, 2, 3, ...) \tag{2.20b}$$

It can be easily verified that

$$E_y^{(h)}|_{z=0,b} = 0 \tag{2.21}$$

Using Euler's formulas in equation (2.19a), (2.19c) and (2.19d) we obtain,

$$E_y^{(h)} = \frac{1}{2j} e^{-j(k_t x - \beta z)} - \frac{1}{2j} e^{-j(k_t x + \beta z)} \tag{2.22a}$$

$$H_x^{(h)} = \frac{1}{2j} Y \frac{\beta}{k} e^{-j(k_t x - \beta z)} + \frac{1}{2j} Y \frac{\beta}{k} e^{-j(k_t x + \beta z)} \tag{2.22b}$$

$$H_z^{(h)} = \frac{1}{2j} Y \frac{k_t}{k} e^{-j(k_t x - \beta z)} - \frac{1}{2j} Y \frac{k_t}{k} e^{-j(k_t x + \beta z)} \tag{2.22c}$$

Just like the E-polarization case, plane wave 1 propagates in the direction $\hat{k}_1^i$, makes an angle $\theta_{01}$ with the negative z-axis and plane wave 2 propagates in the direction $\hat{k}_2^i$, makes an angle $\theta_{02}$ with the negative z-axis. Equations (2.10) remain valid for H-polarization.

So, the incident fields can be rewritten as,

$$
\begin{aligned}
\underline{E}^{(h)i} &= \frac{1}{2j}\,(\hat{x}\sin\phi_0 - \hat{y}\cos\phi_0)\left[e^{-jk\hat{k}_1^i\cdot\underline{r}} - e^{-jk\hat{k}_2^i\cdot\underline{r}}\right] \\
&= \frac{1}{2j}\,(\hat{x}\sin\phi_0 - \hat{y}\cos\phi_0)\left[e^{+jk_t\,(x\cos\phi_0 + y\sin\phi_0)}\left(e^{j\beta z} - e^{-j\beta z}\right)\right] \\
&= \frac{1}{2j}\,(\hat{x}\sin\phi_0 - \hat{y}\cos\phi_0)\,2j\,\sin(\beta z)e^{+jk_t\,(x\cos\phi_0 + y\sin\phi_0)}
\end{aligned}
\tag{2.23a}
$$

$$
\underline{E}^{(h)i} = (\hat{x}\sin\phi_0 - \hat{y}\cos\phi_0)\,\sin(\beta z)e^{+jk_t\,(x\cos\phi_0 + y\sin\phi_0)}
$$

$$
\begin{aligned}
\underline{H}^{(h)i} = \frac{Y}{2j}&\left\{\left[-\frac{\beta}{k}(\hat{x}\cos\phi_0 + \hat{y}\,sin\phi_0) + \frac{k_t}{k}\,\hat{z}\right]e^{-jk\hat{k}_1^i\cdot\underline{r}}\right. \\
&\left.-\left[\frac{\beta}{k}(\hat{x}\cos\phi_0 + \hat{y}\,sin\phi_0) + \frac{k_t}{k}\,\hat{z}\right]e^{-jk\hat{k}_2^i\cdot\underline{r}}\right\} \\
= \frac{Y}{2j}&e^{+jk_t\,(x\cos\phi_0 + y\sin\phi_0)}\left\{-\frac{\beta}{k}\,(\hat{x}\cos\phi_0 + \hat{y}\,sin\phi_0)\left[e^{-jk\hat{k}_1^i\cdot\underline{r}} + e^{-jk\hat{k}_2^i\cdot\underline{r}}\right]\right. \\
&\left.+\frac{k_t}{k}\,\hat{z}\left[e^{-jk\hat{k}_1^i\cdot\underline{r}} - e^{-jk\hat{k}_2^i\cdot\underline{r}}\right]\right\}
\end{aligned}
\tag{2.23b}
$$

$$
\underline{H}^{(h)i} = Y\,e^{+jk_t\,(x\cos\phi_0 + y\sin\phi_0)}\left[\frac{j\beta}{k}\,\cos(\beta z)(\hat{x}\cos\phi_0 + \hat{y}\,sin\phi_0) + \frac{k_t}{k}\,\sin(\beta z)\,\hat{z}\right]
$$

If $\phi_0 = \pi$,

$$
\underline{E}^{(h)i}|_{\phi_0=\pi} = \hat{y}\,\sin(\beta z)\,e^{-jk_t x} \tag{2.24a}
$$

$$
\underline{H}^{(h)i}|_{\phi_0=\pi} = Y\left[\frac{-j\beta}{k}\,\hat{x}\cos(\beta z) + \frac{k_t}{k}\,\hat{z}\sin(\beta z)\right]e^{-jk_t x} \tag{2.24b}
$$

In this case of H-polarization, the expressions for total incident fields (2.23), which are a combination of two plane waves are found by dividing the expressions of each plane wave by 2j and subtracting them. In curvilinear coordinates the solutions will be :

$$\begin{aligned}\underline{E}^{(h)} = \frac{1}{2j}\left[\frac{-2}{k_t}\sin(kz\cos\theta_{01})(\hat{z}\times\nabla_t)\,u^{(h)}(u_1,u_2;k_t)\right]\\ -\frac{1}{2j}\left[\frac{-2}{k_t}\sin(kz\cos\theta_{02})(\hat{z}\times\nabla_t)\,u^{(h)}(u_1,u_2;k_t)\right]\end{aligned} \tag{2.25a}$$

$$\begin{aligned}\underline{H}^{(h)} &= \frac{1}{2j}\left[\frac{2jY}{k_t}\cos\theta_{01}\cos(kz\cos\theta_{01})\,\nabla_t\,u^{(h)}(u_1,u_2;k_t)\right.\\ &\qquad\left.+\,2jY\sin\theta_{01}\sin(kz\cos\theta_{01})\,u^{(h)}(u_1,u_2;k_t)\,\hat{z}\right]\\ &= -\frac{1}{2j}\left[\frac{2jY}{k_t}\cos\theta_{02}\cos(kz\cos\theta_{02})\,\nabla_t\,u^{(h)}(u_1,u_2;k_t)\right.\\ &\qquad\left.+\,2jY\sin\theta_{02}\sin(kz\cos\theta_{02})\,u^{(h)}(u_1,u_2;k_t)\,\hat{z}\right]\end{aligned} \tag{2.25b}$$

Using equations (2.10) in (2.25) we obtain,

$$\underline{E}^{(h)} = \frac{2j}{k_t}\sin(\beta z)(\hat{z}\times\nabla_t)\,u^{(h)}(u_1,u_2;k_t) \tag{2.26a}$$

$$\begin{aligned}\underline{H}^{(h)} = Y\left[\frac{2\beta}{k\,k_t}\cos(\beta z)\,\nabla_t\,u^{(h)}(u_1,u_2;k_t)\right.\\ \left.+\frac{2k_t}{k}\sin(\beta z)\,u^{(h)}(u_1,u_2;k_t)\,\hat{z}\right]\end{aligned} \tag{2.26b}$$

# CHAPTER 3

# SCATTERING BY A CIRCULAR-CYLINDER POST INSIDE A PARALLEL PLATE WAVEGUIDE

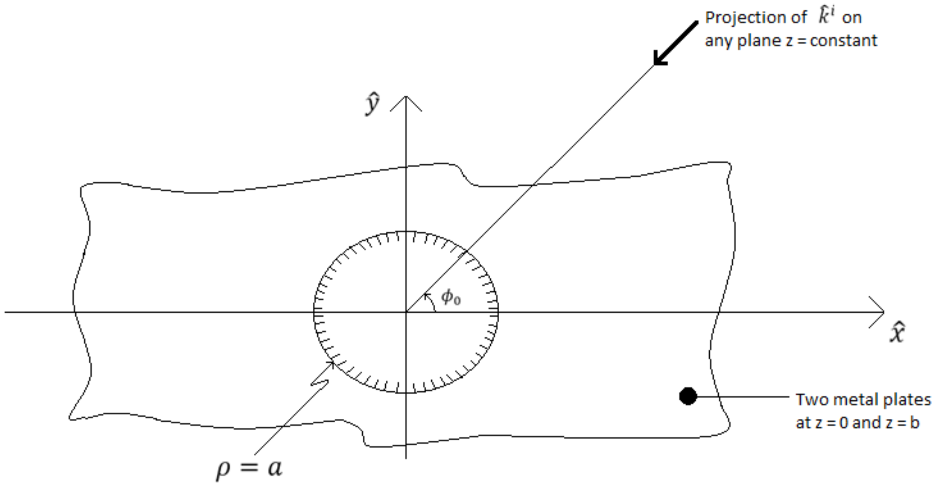


Figure 5: Top view of a circular-cylinder post inside a parallel plate waveguide

In the section, exact solutions are obtained for the case of a circular-cylinder post inside a parallel plate waveguide. The geometry of the problem is shown in Fig. 5, here $a$ is the radius of cylinder and $\phi_0$ is the incidence angle. The problem is solved for both isorefractive and metallic circular-cylinder posts.

The procedure is detailed below, separately for E- and H-polarizations.

In case of a circular-cylinder post , we consider circular-cylinder coordinates $(\rho, \phi, z)$ :

$$x = \rho \cos\phi \tag{3.1a}$$

$$y = \rho \sin\phi \tag{3.1b}$$

$$z = z \tag{3.1c}$$

where $0 \le \rho < \infty,\ 0 \le \phi \le 2\pi,\ 0 \le z \le b$

$$\nabla_t = \hat{\rho}\,\frac{\partial}{\partial\rho} + \frac{\hat{\phi}}{\rho}\,\frac{\partial}{\partial\phi} \tag{3.2a}$$

$$\hat{z} \times \nabla_t = -\frac{\hat{\rho}}{\rho}\,\frac{\partial}{\partial\phi} + \hat{\phi}\,\frac{\partial}{\partial\rho} \tag{3.2b}$$

## 3.1 E-polarization

The general solution in case of E-polarization is :

$$u^{(e)}(\rho, \phi; k) = \sum_{n=0}^{\infty} \varepsilon_n\, j^n\, \zeta_n^{(e)}(k\rho\,; ka) \cos n(\phi - \phi_0) \tag{3.3}$$

where $\varepsilon_0 = 1$, $\varepsilon_{n \geqslant 1} = 2$.

In general,

$$\zeta_n^{(e)}(k\rho\,;ka) = J_n(k\rho) + a_n^{(e)} H_n^{(2)}(k\rho) \quad \text{for } \rho \geq a \tag{3.4a}$$

$$\zeta_n^{(e)}(k\rho\,;ka) = b_n^{(e)} J_n(k\rho) \quad \text{for } \rho \leq a \tag{3.4b}$$

where $J_n$ is the Bessel function of first kind, it represents oscillatory behavior of the fields and is analogous to $\sin(k\rho)$. $J_n$ is chosen over the second kind of Bessel function $Y_n$, because it is non-singular at $\rho = 0$, which is required in expressing finite fields inside the cylindrical post. $H_n^{(2)}$ is the Hankel function of the second kind, this represents an outward-travelling wave and is analogous to $e^{-jk\rho}$ (5). $a_n^{(e)}$ and $b_n^{(e)}$ are the modal expansion coefficients and the superscript (e) stands for E-polarization.

### 3.1.1 Isorefractive Post

We assume that the post is made of a material that is isorefractive to the surrounding space, i:e its wavenumber is k but its intrinsic impedance $Z_{in} = Y_{in}^{-1}$ is different from Z. The incident field is given by (2.11) as follows :

$$\underline{E}^{(e)i} = \left[ -\frac{j\beta}{k}(\hat{x}\cos\phi_0 + \hat{y}\sin\phi_0)\sin(\beta z) + \frac{k_t}{k}\hat{z}\cos(\beta z) \right] e^{jk_t(x\cos\phi_0 + y\sin\phi_0)}$$

$$\underline{H}^{(e)i} = Y\cos(\beta z)(-\hat{x}\sin\phi_0 + \hat{y}\cos\phi_0)\, e^{jk_t(x\cos\phi_0 + y\sin\phi_0)}$$

The z component of the incident and scattered electric fields and of the total electric field inside the post take the following forms after transformation :

$$E_z^{(e)\,i} = \frac{2\,k_t}{k}\cos(\beta\,z)\sum_{n=0}^{\infty}\varepsilon_n\,j^n\,J_n(\,k_t\,\rho\,)\cos n(\phi-\phi_0) \tag{3.5a}$$

$$E_z^{(e)\,s} = \frac{2\,k_t}{k}\cos(\beta\,z)\sum_{n=0}^{\infty}\varepsilon_n\,j^n\,a_n^{(e)}\,H_n^{(2)}(\,k_t\,\rho\,)\cos n(\phi-\phi_0) \tag{3.5b}$$

$$E_z^{(e)\,in} = \frac{2\,k_t}{k}\cos(\beta\,z)\sum_{n=0}^{\infty}\varepsilon_n\,j^n\,b_n^{(e)}\,J_n(\,k_t\,\rho\,)\cos n(\phi-\phi_0) \tag{3.5c}$$

Since there is an impedance shift at the boundary of isorefractive circular-cylinder post, we use Maxwell's equations

$$\nabla\times\underline{H} = j\,k\,Y\,\underline{E} \tag{3.6a}$$

$$\nabla\times\underline{E} = -\,j\,k\,Z\,\underline{H} \tag{3.6b}$$

to obtain:

$$H_\phi^{(e)\,i,s} = j\,\frac{Y}{k}\left(\frac{\partial\,E_\rho^{(e)\,i,s}}{\partial\,z}-\frac{\partial\,E_z^{(e)\,i,s}}{\partial\,\rho}\right) \tag{3.7}$$

$$E_\rho^{(e)\,i,s} = \frac{1}{j\,k\,Y}\left(\frac{1}{\rho}\frac{\partial\,H_z^{(e)\,i,s}}{\partial\,\phi}-\frac{\partial\,H_\phi^{(e)\,i,s}}{\partial\,z}\right) \tag{3.8}$$

Since $H_z = 0$ in the case of E-polarization, (3.8) reduces to the following form :

$$E_\rho^{(e)\,i,s} = \frac{j}{k\,Y}\left(\frac{\partial\,H_\phi^{(e)\,i,s}}{\partial\,z}\right) \tag{3.9}$$

Substituting (3.9) in (3.7) we get,

$$H_{\phi}^{(e)\,i,s} = j\,\frac{Y}{k}\left(\frac{\partial}{\partial z}\left(\frac{j}{kY}\,\frac{\partial H_{\phi}^{(e)\,i,s}}{\partial z}\right) - \frac{\partial E_{z}^{(e)\,i,s}}{\partial \rho}\right) \tag{3.10}$$

Considering that the z variation of all field components is such that $\frac{\partial^2}{\partial z^2} = -\beta^2$, we obtain :

$$H_{\phi}^{(e)\,i,s} = \left(\frac{\beta}{k}\right)^2 H_{\phi}^{(e)\,i,s} - j\,\frac{Y}{k}\,\frac{\partial E_{z}^{(e)\,i,s}}{\partial \rho}$$

hence :

$$H_{\phi}^{(e)\,i} = -\,j\,\frac{Y\,k}{k_t^2}\,\frac{\partial E_{z}^{(e)\,i}}{\partial \rho}, \qquad \rho \geq a \tag{3.11a}$$

$$H_{\phi}^{(e)\,s} = -\,j\,\frac{Y\,k}{k_t^2}\,\frac{\partial E_{z}^{(e)\,s}}{\partial \rho}, \qquad \rho \geq a \tag{3.11b}$$

and similarly :

$$H_{\phi}^{(e)\,in} = -\,j\,\frac{Y_{in}\,k}{k_t^2}\,\frac{\partial E_{z}^{(e)\,in}}{\partial \rho}, \qquad \rho \leq a \tag{3.11c}$$

The modal expansion coefficients can be determined by imposing the continuity of tangential components of electric and magnetic field across the interface $\rho = a$, such that :

$$E_{z}^{(e)\,in} = E_{z}^{(e)\,i} + E_{z}^{(e)\,s} \tag{3.12a}$$

$$H_{\phi}^{(e)\,in} = H_{\phi}^{(e)\,i} + H_{\phi}^{(e)\,s} \tag{3.12b}$$

the modal expansion coefficients are found to be :

$$a_n^{(e)} = \frac{(\chi - 1) J_n(k_t a) J_n'(k_t a)}{J_n(k_t a) H_n^{(2)\prime}(k_t a) - \chi J_n'(k_t a) H_n^{(2)}(k_t a)} \tag{3.13a}$$

$$b_n^{(e)} = \frac{-2j}{\pi k_t a \big(J_n(k_t a) H_n^{(2)\prime}(k_t a) - \chi J_n'(k_t a) H_n^{(2)}(k_t a)\big)} \tag{3.13b}$$

where,

$$\prime = \frac{\partial}{\partial \rho}$$

$$\chi = \frac{Z}{Z_{in}}$$

If there is no post ($Z = Z_{in}$) then,

$$(a_n^{(e)})_{\chi = 1} = 0 \tag{3.14a}$$

$$(b_n^{(e)})_{\chi = 1} = 1 \tag{3.14b}$$

Also, it should be noted that the case of a PEC circular cylinder ($Z_{in} = 0$) is a particular case of an isorefractive circular-cylinder :

$$(a_n^{(e)})_{\chi \to \infty} = -\frac{J_n(k_t a)}{H_n^{(2)}(k_t a)} \tag{3.15a}$$

$$(b_n^{(e)})_{\chi \to \infty} = 0 \tag{3.15b}$$

### 3.1.2 Metallic Post

For a PEC cylinder,

$$\zeta_n^{(e)}(k\rho\,;ka) = J_n(k\rho) - \frac{J_n(ka)}{H_n^{(2)}(ka)} H_n^{(2)}(k\rho) \tag{3.16}$$

Utilizing the general theory for E-polarization by substituting (3.2) in (2.14) we obtain,

$$\begin{aligned}\underline{E}^{(e)} = &-\frac{2\beta}{k}\sin(\beta z)\left\{\hat{\rho}\sum_{n=0}^{\infty}\varepsilon_n\, j^n\,\zeta_n^{(e)\,\prime}(k_t\rho\,;k_t a)\cos n(\phi-\phi_0)\right.\\ &\left.\frac{-2\hat{\phi}}{k_t\rho}\sum_{n=1}^{\infty} n\, j^n\,\zeta_n^{(e)}(k_t\rho\,;k_t a)\sin n(\phi-\phi_0)\right\}\\ &+\frac{2k_t}{k}\cos(\beta z)\,\hat{z}\sum_{n=0}^{\infty}\varepsilon_n\, j^n\,\zeta_n^{(e)}(k_t\rho\,;k_t a)\cos n(\phi-\phi_0)\end{aligned} \tag{3.17a}$$

$$\begin{aligned}\underline{H}^{(e)} = &-2\,jY\cos(\beta z)\left\{\hat{\rho}\,\frac{2}{k_t\rho}\sum_{n=1}^{\infty} n\, j^n\,\zeta_n^{(e)}(k_t\rho\,;k_t a)\sin n(\phi-\phi_0)\right.\\ &\left.+\hat{\phi}\sum_{n=0}^{\infty}\varepsilon_n\, j^n\,\zeta_n^{(e)\,\prime}(k_t\rho\,;k_t a)\cos n(\phi-\phi_0)\right\}\end{aligned} \tag{3.17b}$$

where $'$ represents $\frac{\partial}{\partial\,(k_t\,\rho)}$

Surface current densities

On the cylinder ($\rho = a$),

$$\begin{aligned}\underline{J}^{(e)}|_{\rho=a} &= \hat{\rho}\times\underline{H}^{(e)}|_{\rho=a}\\ &= \frac{4Y}{\pi k_t a}\,cos(\beta z)\,\hat{z}\sum_{n=0}^{\infty}\frac{\varepsilon_n\, j^n}{H_n^{(2)}(k_t a)}\,cos\, n(\phi-\phi_0)\end{aligned} \tag{3.18}$$

On the plates ( z = 0, b ),

$$\underline{J}^{(e)}|_{z=0,b} = \pm\hat{z}\times\underline{H}^{(e)}|_{z=0,b}$$

$$= \mp 2\,jY \begin{bmatrix} 1 \\ (-1)^m \end{bmatrix} \Bigg\{ -\hat{\rho}\sum_{n=0}^{\infty} \varepsilon_n\, j^n\, \zeta_n^{(e)\,\prime}(k_t\,\rho\,;k_t\,a)\cos n(\phi-\phi_0) \tag{3.19}$$

$$+\,\hat{\phi}\,\frac{2}{k_t\,\rho}\sum_{n=1}^{\infty} n\, j^n\, \zeta_n^{(e)}(k_t\,\rho\,;k_t\,a)\sin n(\phi-\phi_0)\Bigg\}$$

where,

$$\zeta_n^{(e)\,\prime}(k_t\,a\,;k_t\,a) = \frac{J_n'(k_t\,a)H_n^{(2)}(k_t\,a) - J_n(k_t\,a)H_n^{(2)\prime}(k_t\,a)}{H_n^{(2)}(k_t\,a)}$$

$$= \frac{2\,j}{\pi\,k_t\,a\,H_n^{(2)}(k_t\,a)}$$

At the junction between post and plates ( $\rho$ = a ; z = 0, b ),

$$\underline{J}^{(e)}|_{\rho=a,\ z\to 0,b} = \frac{4Y}{\pi\,k_t\,a}\begin{bmatrix} 1 \\ (-1)^m \end{bmatrix}\hat{z}\sum_{n=0}^{\infty}\frac{\varepsilon_n\, j^n}{H_n^{(2)}(k_t\,a)}\,cos\,n(\phi-\phi_0) \tag{3.20a}$$

$$\underline{J}^{(e)}|_{z=0,b,\rho\to a} = \pm 2\,jY\begin{bmatrix} 1 \\ (-1)^m \end{bmatrix}\hat{\rho}\sum_{n=0}^{\infty}\varepsilon_n\, j^n\,\frac{2\,j}{\pi\,k_t\,a\,H_n^{(2)}(k_t\,a)}\cos n(\phi-\phi_0)$$

$$= \mp\frac{4Y}{\pi\,k_t\,a}\begin{bmatrix} 1 \\ (-1)^m \end{bmatrix}\hat{\rho}\sum_{n=0}^{\infty}\frac{\varepsilon_n\, j^n}{H_n^{(2)}(k_t\,a)}\cos n(\phi-\phi_0) \tag{3.20b}$$

therefore, $\underline{J}^{(e)}$ is continuous across the junction. This continuity of surface currents is a verification of the exact solution.

## 3.2 H-polarization

The general solution in case of H-polarization,

$$u^{(h)}\,(\rho,\phi;k) = \sum_{n=0}^{\infty} \varepsilon_n\, j^n\, \zeta_n^{(h)}(k\,\rho\,;\,k\,a)\,\cos n(\phi\,-\,\phi_0) \tag{3.21}$$

where, $\varepsilon_0 = 1$, $\varepsilon_{n\geqslant 1} = 2$

In general,

$$\zeta_n^{(h)}(k\,\rho\,;\,k\,a) = J_n(\,k\,\rho\,) + a_n^{(h)}(k)\,H_n^{(2)}(\,k\,\rho\,) \quad \text{for } \rho \geq a \tag{3.22a}$$

$$\zeta_n^{(h)}(k\,\rho\,;\,k\,a) = b_n^{(h)}(k)\,J_n(\,k\,\rho\,) \quad \text{for } \rho \leq a \tag{3.22b}$$

### 3.2.1 Isorefractive Post

The incident field is given by (2.23) as follows :

$$\underline{E}^{(h)i} = (\hat{x}\,\sin\phi_0 - \hat{y}\,\cos\phi_0)\,\sin(\beta\,z)e^{+j\,k_t\,(\,x\,\cos\phi_0\,+\,y\,\sin\phi_0)}$$

$$\underline{H}^{(h)i} = Y\left[\frac{j\,\beta}{k}\,\cos(\beta\,z)(\hat{x}\,\cos\phi_0\,+\,\hat{y}\,sin\phi_0)\,+\,\frac{k_t}{k}\,\sin(\beta\,z)\,\hat{z}\right]e^{+j\,k_t\,(\,x\,\cos\phi_0\,+\,y\,\sin\phi_0)}$$

The z component of the magnetic field takes the following form after transformation :

$$H_z^{(h)\,i} = \frac{2\,Y\,k_t}{k}\,\sin(\beta\,z)\sum_{n=0}^{\infty}\varepsilon_n\,j^n\,J_n(\,k_t\,\rho\,)\,\cos n(\phi\,-\,\phi_0) \tag{3.23a}$$

$$H_z^{(h)\,s} = \frac{2\,Y\,k_t}{k}\,\sin(\beta\,z)\sum_{n=0}^{\infty}\varepsilon_n\,j^n\,a_n^{(h)}\,H_n^{(2)}(\,k_t\,\rho\,)\,\cos n(\phi\,-\,\phi_0) \tag{3.23b}$$

$$H_z^{(h)\,in} = \frac{2\,Y_{in}\,k_t}{k}\,\sin(\beta\,z)\sum_{n=0}^{\infty}\varepsilon_n\,j^n\,b_n^{(h)}\,J_n(\,k_t\,\rho\,)\,\cos n(\phi\,-\,\phi_0) \tag{3.23c}$$

Using Maxwell's equations we obtain :

$$E_\phi^{(h)\,i,s} = \frac{1}{j\,k\,Y}\left(\frac{\partial\,H_\rho^{(h)\,i,s}}{\partial\,z} - \frac{\partial\,H_z^{(h)\,i,s}}{\partial\,\rho}\right) \tag{3.24}$$

$$H_\rho^{(h)\,i,s} = \frac{j\,Y}{k}\left(\frac{1}{\rho}\frac{\partial\,E_z^{(h)\,i,s}}{\partial\,\phi} - \frac{\partial\,E_\phi^{(h)\,i,s}}{\partial\,z}\right) \tag{3.25}$$

Since $E_z\,=\,0$ in case of H-polarization, (3.25) reduces to the following form :

$$H_\rho^{(h)\,i,s} = -\,\frac{j\,Y}{k}\left(\frac{\partial\,E_\phi^{(h)\,i,s}}{\partial\,z}\right) \tag{3.26}$$

Substituting (3.26) in (3.24) and considering that the z variation of all field components is such that $\frac{\partial^2}{\partial\,z^2}\,=\,-\beta^2$, we obtain :

$$E_\phi^{(h)\,i} = -\,\frac{1}{k^2}\,\frac{\partial^2 E_\phi^{(h)\,i}}{\partial\,z^2} - \frac{1}{j\,k\,Y}\,\frac{\partial\,H_z^{(h)\,i}}{\partial\,\rho}$$

hence :

$$E_{\phi}^{(h)\,i} = \frac{j\,k}{Y\,k_t^2}\,\frac{\partial\,H_z^{(h)\,i}}{\partial\,\rho}, \qquad \rho \,\geq\, a \tag{3.27a}$$

$$E_{\phi}^{(h)\,s} = \frac{j\,k}{Y\,k_t^2}\,\frac{\partial\,H_z^{(h)\,s}}{\partial\,\rho}, \qquad \rho \,\geq\, a \tag{3.27b}$$

and similarly :

$$E_{\phi}^{(h)\,in} = \frac{j\,k}{Y_{in}\,k_t^2}\,\frac{\partial\,H_z^{(h)\,in}}{\partial\,\rho}, \qquad \rho \,\leq\, a \tag{3.27c}$$

The modal expansion coefficients can be determined by imposing the continuity of tangential components of electric and magnetic field across the interface $\rho\,=\,a$, such that :

$$H_z^{(h)\,in} = H_z^{(h)\,i} + H_z^{(h)\,s} \tag{3.28a}$$

$$E_{\phi}^{(h)\,in} = E_{\phi}^{(h)\,i} + E_{\phi}^{(h)\,s} \tag{3.28b}$$

the modal expansion coefficients are found to be :

$$a_n^{(h)} = \frac{(\chi_0-1)\,J_n(\,k_t\,a\,)\,J_n'(\,k_t\,a\,)}{J_n(\,k_t\,a\,)\,H_n^{(2)\,\prime}(\,k_t\,a\,)\,-\,\chi_0\,J_n'(\,k_t\,a\,)\,H_n^{(2)}(\,k_t\,a\,)} \tag{3.29a}$$

$$b_n^{(e)} = \frac{-\,2\,j\,\chi_0}{\pi\,k_t\,a\big(J_n(\,k_t\,a\,)\,H_n^{(2)\,\prime}(\,k_t\,a\,)\,-\,\chi_0\,J_n'(\,k_t\,a\,)\,H_n^{(2)}(\,k_t\,a\,)\big)} \tag{3.29b}$$

where,

$$\prime = \frac{\partial}{\partial \rho}$$
$$\chi_0 = \frac{Z_{in}}{Z}$$

If there is no post ($Z = Z_{in}$) then,

$$(a_n^{(h)})_{\chi_0=1} = 0 \tag{3.30a}$$

$$(b_n^{(h)})_{\chi_0=1} = 1 \tag{3.30b}$$

Also, it should be noted that the case of PEC circular cylinder ($Z_{in} = 0$) is a particular case of the isorefractive circular cylinder :

$$(a_n^{(h)})_{\chi_0=0} = -\frac{J_n'(k_t a)}{H_n^{(2)\prime}(k_t a)} \tag{3.31a}$$

$$(b_n^{(e)})_{\chi_0=0} = 0 \tag{3.31b}$$

### 3.2.2 Metallic Post

For a PEC cylinder,

$$\zeta_n^{(h)}(k\rho\,;ka) = J_n(k\rho) - \frac{J_n'(ka)}{H_n^{(2)\prime}(ka)} H_n^{(2)}(k\rho) \tag{3.32}$$

where $'$ represents $\frac{\partial}{\partial\,(k_t\,\rho)}$

Utilizing the general theory for H-polarization by substituting (3.21) in (2.25) we obtain,

$$\underline{E}^{(h)} = \frac{2\,j}{k_t}\sin(\beta\,z)\left[-\frac{\hat{\rho}}{\rho}\frac{\partial}{\partial\,\phi} + \hat{\phi}\,\frac{\partial}{\partial\,\rho}\right]\sum_{n=0}^{\infty}\varepsilon_n\,j^n\,\zeta_n^{(h)}(k_t\,\rho\,;\,k_t\,a)\,\cos n(\phi\,-\,\phi_0)$$

$$= 2\,j\,\sin(\beta\,z)\left[\frac{2\,\hat{\rho}}{k_t\,\rho}\sum_{n=1}^{\infty} n\,j^n\,\zeta_n^{(h)}(k_t\,\rho\,;\,k_t\,a)\,\sin n(\phi\,-\,\phi_0)\right. \qquad (3.33a)$$

$$\left. + \hat{\phi}\sum_{n=0}^{\infty}\varepsilon_n\,j^n\,\zeta_n^{(h)\,\prime}(k_t\,\rho\,;\,k_t\,a)\,\cos n(\phi\,-\,\phi_0)\right]$$

$$\underline{H}^{(h)} = Y\left[\frac{2\,\beta}{k\,k_t}\cos(\beta\,z)\left(\hat{\rho}\,\frac{\partial}{\partial\,\rho} + \frac{\hat{\phi}}{\rho}\,\frac{\partial}{\partial\,\phi}\right)\sum_{n=0}^{\infty}\varepsilon_n\,j^n\,\zeta_n^{(h)}(k_t\,\rho\,;\,k_t\,a)\,\cos n(\phi\,-\,\phi_0)\right.$$

$$\left. + \frac{2\,k_t}{k}\sin(\beta\,z)\,\hat{z}\sum_{n=0}^{\infty}\varepsilon_n\,j^n\,\zeta_n^{(h)}(k_t\,\rho\,;\,k_t\,a)\,\cos n(\phi\,-\,\phi_0)\right]$$

$$= Y\,\frac{2\,\beta}{k}\,cos(\beta\,z)\left[\hat{\rho}\sum_{n=0}^{\infty}\varepsilon_n\,j^n\,\zeta_n^{(h)\,\prime}(k_t\,\rho\,;\,k_t\,a)\,\cos n(\phi\,-\,\phi_0)\right. \qquad (3.33b)$$

$$\left. - \frac{2\,\hat{\phi}}{k_t\,\rho}\sum_{n=1}^{\infty} n\,j^n\,\zeta_n^{(h)}(k_t\,\rho\,;\,k_t\,a)\,\sin n(\phi\,-\,\phi_0)\right]$$

$$+ Y\,\frac{2\,k_t}{k}\sin(\beta\,z)\,\hat{z}\sum_{n=0}^{\infty}\varepsilon_n\,j^n\,\zeta_n^{(h)}(k_t\,\rho\,;\,k_t\,a)\,\cos n(\phi\,-\,\phi_0)$$

<u>Surface current densities</u>

On the cylinder ( $\rho\,=\,a$ ),

$$\underline{J}^{(h)}|_{\rho\,=\,a} = \hat{\rho}\times\underline{H}^{(h)}|_{\rho\,=\,a}$$

$$= \frac{4\,j\,Y}{\pi\,ka}\left[\frac{2\,\beta}{k_t^2 a}\cos(\beta\,z)\,\hat{z}\sum_{n=1}^{\infty}\frac{n\,j^n}{H_n^{(2)\,\prime}(\,k_t\,a\,)}\sin n(\phi\,-\,\phi_0)\right. \qquad (3.34)$$

$$\left. + \sin(\beta\,z)\,\hat{\phi}\sum_{n=0}^{\infty}\frac{\varepsilon_n\,j^n}{H_n^{(2)\,\prime}(\,k_t\,a\,)}\cos n(\phi\,-\,\phi_0)\right]$$

where,

$$\zeta_n^{(h)\,\prime}(k_t\,a\,;k_t\,a) = \frac{-2\,j}{\pi\,k_t\,a H_n^{(2)\,\prime}(\,k_t\,a\,)}$$

On the plates ( z = 0, b ),

$$\begin{aligned}\underline{J}^{(h)}|_{z=0,b} &= \pm\hat{z}\times\underline{H}^{(h)}|_{z=0,b}\\ &= \pm Y\,\frac{2\beta}{k}\begin{bmatrix}1\\ \\(-1)^m\end{bmatrix}\Bigg[\hat{\phi}\sum_{n=0}^{\infty}\varepsilon_n\,j^n\,\zeta_n^{(h)\,\prime}(k_t\,\rho\,;k_t\,a)\cos n(\phi-\phi_0) \qquad (3.35)\\ &\qquad + \hat{\rho}\,\frac{2}{k_t\,\rho}\sum_{n=1}^{\infty}n\,j^n\,\zeta_n^{(h)}(k_t\,\rho\,;k_t\,a)\sin n(\phi-\phi_0)\Bigg]\end{aligned}$$

At the junction between post and plates ( $\rho$ = a ; z = 0, b ),

$$\underline{J}^{(h)}|_{\rho=a,\;z\to 0,b} = Y\,\frac{8\,j\,\beta}{\pi\,k\,k_t^2\,a^2}\begin{bmatrix}1\\ \\(-1)^m\end{bmatrix}\hat{z}\sum_{n=1}^{\infty}\frac{n\,j^n}{H_n^{(2)\,\prime}(\,k_t\,a\,)}\sin n(\phi-\phi_0) \qquad (3.36a)$$

$$\underline{J}^{(h)}|_{\,z=0,b,\rho\to a} = \mp Y\,\frac{8\,j\,\beta}{\pi\,k\,k_t^2\,a^2}\begin{bmatrix}1\\ \\(-1)^m\end{bmatrix}\hat{\rho}\sum_{n=1}^{\infty}\frac{n\,j^n}{H_n^{(2)\,\prime}(\,k_t\,a\,)}\sin n(\phi-\phi_0) \qquad (3.36b)$$

therefore, $\underline{J}^{(h)}$ is continuous across the junction.

# CHAPTER 4

# SCATTERING BY AN ELLIPTIC-CYLINDER POST INSIDE A PARALLEL PLATE WAVEGUIDE

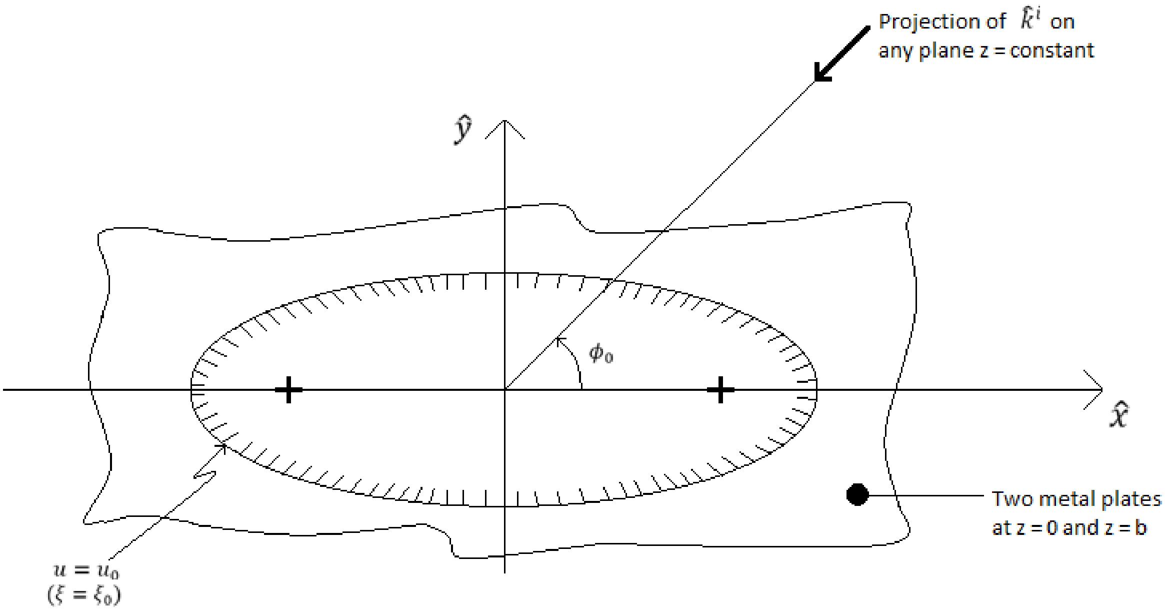


Figure 6: Top view of an elliptic-cylinder post inside a parallel plate waveguide

In this section, exact solutions are obtained for the case of an elliptic-cylinder post inside a parallel plate waveguide. The geometry of the problem is shown in Fig. 6, here $u_0$ is the surface of elliptic-cylinder post and $\phi_0$ is the incidence angle. The problem is solved for both isorefractive and metallic elliptic-cylinder posts. The procedure is detailed below separately for E- and H-polarizations. For an elliptic-cylinder post, we introduce elliptic-cylinder coordinates (u, v, z) :

$$x = \frac{d}{2} \cosh u \cos v \tag{4.1a}$$

$$y = \frac{d}{2} \sinh u \sin v \tag{4.1b}$$

$$z = z \tag{4.1c}$$

where $0 \le u < \infty$, $0 \le v \le 2\pi$ and $0 \le z \le b$.

Some authors use,

$$\xi = \cosh u \tag{4.2a}$$

$$\eta = \cos v \tag{4.2b}$$

where $1 \leq \xi < \infty$ *and* $-1 \leq \eta \leq +1$.

Note, however, that $\eta$ is not a monotonic function of v, so that :

$$\frac{\partial}{\partial u} = \sqrt{\xi^2 - 1}\,\frac{\partial}{\partial \xi} \tag{4.3a}$$

$$\frac{\partial}{\partial v} = \mp\sqrt{1 - \eta^2}\,\frac{\partial}{\partial \eta}, \quad \begin{cases} - & \text{for } 0 \leq v \leq \pi \\ + & \text{for } \pi \leq v \leq 2\pi \end{cases} \tag{4.3b}$$

## 4.1 E-polarization

An E-polarized mode can be considered as a sum of two plane waves, whose projection of $\hat{k}^i$ on any plane z = constant forms the angle $\phi_0$ $(0 \leq \phi_0 \leq \frac{\pi}{2})$ with the negative x - axis, whereas $\hat{k}^i$ forms the angle $\theta_0 = \theta_{01}$ or $\theta_0 = \theta_{02}$ with the negative z - axis, where :

$$\text{plane wave 1}: \theta_0 = \theta_{01} = \arccos\frac{\beta}{k} \tag{4.4a}$$

$$\text{plane wave 2}: \theta_0 = \theta_{02} = \pi - \theta_{01} \tag{4.4b}$$

$$\beta = \frac{m\pi}{b}\ (m = 0, 1, 2, 3, ...) \tag{4.4c}$$

hence,

$\sin\theta_{01} = \sin\theta_{02} = \frac{k_t}{k}$

$\cos\theta_{01} = -\cos\theta_{02} = \frac{\beta}{k}$

k is the wavenumber, $\beta$ is the longitudinal wave number (in the direction of the z - axis) and $k_t$ is the transverse wave number.

In the absence of a post, the total field is the incident field, which is obtained from (2.11) :

$$\underline{E}^{(e)i} = \left[ -\frac{j\beta}{k} (\hat{x}\cos\phi_0 + \hat{y}\sin\phi_0)\sin(\beta z) + \frac{k_t}{k}\hat{z}\cos(\beta z) \right] e^{j k_t (x\cos\phi_0 + y\sin\phi_0)}$$

$$\underline{H}^{(e)i} = Y \cos(\beta z)\,(-\hat{x}\sin\phi_0 + \hat{y}\cos\phi_0)\, e^{j k_t (x\cos\phi_0 + y\sin\phi_0)}$$

where $Z = Y^{-1}$ is the intrinsic impedance of medium inside the waveguide.

If the post is a circular cylinder, we have a rotational symmetry. So the expressions can be made simpler by considering $\phi_0 = 0$, in which case :

$$\underline{E}^{(e)i}|_{\phi_0=0} = \left[ -\frac{j\beta}{k}\hat{x}\sin(\beta z) + \frac{k_t}{k}\hat{z}\cos(\beta z) \right] e^{j k_t x} \tag{4.5a}$$

$$\underline{H}^{(e)i}|_{\phi_0=0} = \hat{y} Y \cos(\beta z)\, e^{j k_t x} \tag{4.5b}$$

The incident field (2.11) can be written in terms of Mathieu functions :

$$\underline{E}^{(e)i} = 2\sqrt{2\pi}\,\frac{k_t}{k}\left[ \cos(\beta z)\,\hat{z} - \frac{\beta}{k_t\,\gamma\sqrt{\xi^2 - \eta^2}} \sin(\beta z)\left( \hat{u}\frac{\partial}{\partial u} + \hat{v}\frac{\partial}{\partial v} \right) \right] \sum^{i} \tag{4.6a}$$

$$\underline{H}^{(e)i} = \frac{2j}{\gamma}\sqrt{\frac{2\pi}{\xi^2 - \eta^2}}\, Y \cos(\beta z)\left( \hat{u}\frac{\partial}{\partial u} - \hat{v}\frac{\partial}{\partial v} \right) \sum^{i} \tag{4.6b}$$

where,

$$\sum^{i} = \sum_{m=0}^{\infty} \left[ \frac{j^m}{N_m^{(e)}} Re_m^{(1)} (\gamma, \xi) Se_m (\gamma, \eta) Se_m (\gamma, \cos\phi_0) \right. \\ \left. + \frac{j^m}{N_m^{(o)}} Ro_m^{(1)} (\gamma, \xi) So_m (\gamma, \eta) So_m (\gamma, \cos\phi_0) \right] \tag{4.7}$$

$$\gamma = \frac{k_t\, d}{2}, \text{ (d = interfocal distance)} \tag{4.8}$$

$Re_m^{(1)}$ , $Ro_m^{(1)}$ are the radial Mathieu functions of the first kind. $Se_m$ , $So_m$ are the angular Mathieu functions; the subscripts e and o indicate the even and odd functions respectively. $N_m^{(e),(o)}$ are the normalization coefficients. This notation used is from Stratton, 1941(6).

If an elliptic-cylinder post is present (with surface $u = u_0$ or $\xi = \xi_0$), then the total field inside the waveguide and outside the post ($u_0 \leq u < \infty$) is the sum of incident field (4.6) and the scattered field ($\underline{E}^s$ , $\underline{H}^s$) :

$\underline{E} = \underline{E}^i + \underline{E}^s$ and $\underline{H} = \underline{H}^i + \underline{H}^s$.

The scattered field ($\underline{E}^s$ , $\underline{H}^s$) is given by (4.6) upon replacing $\sum^i$ with $\sum^s$, where,

$$\sum^{s} = \sum_{m=0}^{\infty} j^m \left[ \frac{a_m^{(e)}}{N_m^{(e)}} Re_m^{(4)} (\gamma, \xi) Se_m (\gamma, \eta) Se_m (\gamma, \cos\phi_0) \right. \\ \left. + \frac{a_m^{(o)}}{N_m^{(o)}} Ro_m^{(4)} (\gamma, \xi) So_m (\gamma, \eta) So_m (\gamma, \cos\phi_0) \right] \tag{4.9}$$

and $Re_m^{(4)}$ , $Ro_m^{(4)}$ are the radial Mathieu functions of the fourth kind.

**Solution in curvilinear coordinates**

For $\xi \ \geq \ \xi_0$ :

$$u^{(e)}(\xi,\,\eta,\,k_t) \;=\; \sum_{m=0}^{\infty}\left[\frac{j^m}{N_m^{(e)}}\,(Re_m^{(1)}\,(\,\gamma,\xi\,)\,+\,a_m^{(e)}\,Re_m^{(4)}\,(\,\gamma,\xi\,))\,Se_m\,(\,\gamma,\eta\,)\,Se_m\,(\,\gamma,\cos\phi_0\,)\right] + \sum_{m=1}^{\infty}\left[\frac{j^m}{N_m^{(o)}}\,(Ro_m^{(1)}\,(\,\gamma,\xi\,)\,+\,a_m^{(o)}\,Ro_m^{(4)}\,(\,\gamma,\xi\,))\,So_m\,(\,\gamma,\eta\,)\,So_m\,(\,\gamma,\cos\phi_0\,)\right] \tag{4.10}$$

For $\xi \ \leq \ \xi_0$ :

$$u^{(e)}(\xi,\,\eta,\,k_t) \;=\; \sum_{m=0}^{\infty}\left[\frac{j^m}{N_m^{(e)}}\,(\,b_m^{(e)}\,Re_m^{(1)}\,(\,\gamma,\xi\,))\,Se_m\,(\,\gamma,\eta\,)\,Se_m\,(\,\gamma,\cos\phi_0\,)\right] + \sum_{m=1}^{\infty}\left[\frac{j^m}{N_m^{(o)}}\,(\,b_m^{(o)}\,Re_m^{(1)}\,(\,\gamma,\xi\,))\,So_m\,(\,\gamma,\eta\,)\,So_m\,(\,\gamma,\cos\phi_0\,)\right] \tag{4.11}$$

### 4.1.1 Isorefractive Post

We assume that the post is made of a material that is isorefractive to the surrounding space, i.e. its wavenumber is k but its intrinsic impedance $Z_{in} \;=\; Y_{in}^{-1}$ is different from Z. The total field inside the post is given by (4.6) upon replacing $\sum^i$ with $\sum^{in}$ and $Y$ with $Y_{in}$, where,

$$\sum^{in} = \sum_{m=0}^{\infty} j^m \left[\frac{b_m^{(e)}}{N_m^{(e)}}\,Re_m^{(1)}\,(\,\gamma,\xi\,)\,Se_m\,(\,\gamma,\eta\,)\,Se_m\,(\,\gamma,\cos\phi_0\,) + \frac{b_m^{(o)}}{N_m^{(o)}}\,Ro_m^{(1)}\,(\,\gamma,\xi\,)\,So_m\,(\,\gamma,\eta\,)\,So_m\,(\,\gamma,\cos\phi_0\,)\right] \tag{4.12}$$

Note that $N_m^{(e),(o)}$ are functions of $\gamma$, that has the same value inside and outside the isorefractive post. Also, note that the radial functions of fourth kind are excluded in (4.12), because the terms of the series must remain finite in the special case of a circular post.

The modal expansion coefficients can be determined by imposing the continuity of the tangential components of electric and magnetic fields across the interface $u = u_0$, such that :

$$\underline{E}_z^{in} = \underline{E}_z^i + \underline{E}_z^s \tag{4.13a}$$

$$\underline{H}_v^{in} = \underline{H}_v^i + \underline{H}_v^s \tag{4.13b}$$

the modal expansion coefficients are found to be :

$$a_m^{(e)} = \frac{(\chi - 1)\, Re_m^{(1)}(\gamma,\xi_0)\; Re_m^{(1)\prime}(\gamma,\xi_0)}{Re_m^{(1)}(\gamma,\xi_0)\, Re_m^{(4)\prime}(\gamma,\xi_0) - \chi\, Re_m^{(1)\prime}(\gamma,\xi_0)\, Re_m^{(4)}(\gamma,\xi_0)} \tag{4.14a}$$

$$a_m^{(o)} = \frac{(\chi - 1)\, Ro_m^{(1)}(\gamma,\xi_0)\; Ro_m^{(1)\prime}(\gamma,\xi_0)}{Ro_m^{(1)}(\gamma,\xi_0)\, Ro_m^{(4)\prime}(\gamma,\xi_0) - \chi\, Ro_m^{(1)\prime}(\gamma,\xi_0)\, Ro_m^{(4)}(\gamma,\xi_0)} \tag{4.14b}$$

$$b_m^{(e)} = \frac{-j}{Re_m^{(1)}(\gamma,\xi_0)\, Re_m^{(4)\prime}(\gamma,\xi_0) - \chi\, Re_m^{(1)\prime}(\gamma,\xi_0)\, Re_m^{(4)}(\gamma,\xi_0)} \tag{4.14c}$$

$$b_m^{(o)} = \frac{-j}{Ro_m^{(1)}(\gamma,\xi_0)\, Ro_m^{(4)\prime}(\gamma,\xi_0) - \chi\, Ro_m^{(1)\prime}(\gamma,\xi_0)\, Ro_m^{(4)}(\gamma,\xi_0)} \tag{4.14d}$$

where,

$$\prime = \frac{\partial}{\partial u}$$

$$\chi = \frac{Z}{Z_{in}}$$

In the case when there is no post ($Z = Z_{in}$) :

$$(a^{(e)}\,,\,a^{(o)})_{\chi=1} = 0 \tag{4.15a}$$

$$(b^{(e)}\,,\,b^{(o)})_{\chi=1} = 1 \tag{4.15b}$$

<u>Surface current densities on the parallel plates</u>

On the plate $z = 0+$ :

$$\begin{aligned} \underline{J}^{(e)}\big|_{z=0} &= \hat{z} \times \underline{H}^{(e)}\big|_{z=0} \\ &= \frac{2\,j}{\gamma}\sqrt{\frac{2\,\pi}{\xi^2 - \eta^2}} \begin{bmatrix} Y \\ \\ Y_{in} \end{bmatrix} (\hat{u}\,\frac{\partial}{\partial\,u} + \hat{v}\,\frac{\partial}{\partial\,v}) \begin{bmatrix} \Sigma^i + \Sigma^s \\ \\ \Sigma^{in} \end{bmatrix} \begin{matrix} ,u \geq u_0 \\ \\ ,u \leq u_0 \end{matrix} \end{aligned} \tag{4.16}$$

i) Outside the post ($u \geq u_0$) :

$$\begin{aligned} J_u^{(e)}\big|_{z=0} = \frac{2\,jY}{\gamma}\sqrt{\frac{2\,\pi}{\xi^2 - \eta^2}} \sum_{m=0}^{\infty} \Bigg[ & \frac{j^m}{N_m^{(e)}} \big[ Re_m^{(1)\prime}(\gamma,\xi) + a_m^{(e)}\, Re_m^{(4)\prime}(\gamma,\xi) \big]\, Se_m(\gamma,\eta)\, Se_m(\gamma,\cos\phi_0) \\ & + \frac{j^m}{N_m^{(o)}} \big[ Ro_m^{(1)\prime}(\gamma,\xi) + a_m^{(o)}\, Ro_m^{(4)\prime}(\gamma,\xi) \big]\, So_m(\gamma,\eta)\, So_m(\gamma,\cos\phi_0) \Bigg] \end{aligned} \tag{4.17}$$

$$J_v^{(e)}\big|_{z=0} = \frac{2\,jY}{\gamma}\sqrt{\frac{2\,\pi}{\xi^2-\eta^2}}\sum_{m=0}^{\infty}\left[\frac{j^m}{N_m^{(e)}}\left[Re_m^{(1)}\,(\,\gamma,\xi\,)+a_m^{(e)}\,Re_m^{(4)}\,(\,\gamma,\xi\,)\right]Se_m'\,(\,\gamma,\eta\,)\,Se_m\,(\,\gamma,\cos\phi_0\,)\right.$$
$$\left.+\frac{j^m}{N_m^{(o)}}\left[Ro_m^{(1)}\,(\,\gamma,\xi\,)+a_m^{(o)}\,Ro_m^{(4)}\,(\,\gamma,\xi\,)\right]So_m'\,(\,\gamma,\eta\,)\,So_m\,(\,\gamma,\cos\phi_0\,)\right] \tag{4.18}$$

where $'$ on the radial function means $\frac{\partial}{\partial u}$ and $'$ on the angular function means $\frac{\partial}{\partial v}$.

ii) Inside the post ($u \leq u_0$) :

$$J_u^{(e)}\big|_{z=0} = \frac{2\,j\,Y_{in}}{\gamma}\sqrt{\frac{2\,\pi}{\xi^2 - \eta^2}}\sum_{m=0}^{\infty}\left[\frac{j^m}{N_m^{(e)}}\,b_m^{(e)}\,Re_m^{(1)\prime}(\gamma,\xi)\,Se_m(\gamma,\eta)\,Se_m(\gamma,\cos\phi_0) + \frac{j^m}{N_m^{(o)}}\,b_m^{(o)}\,Ro_m^{(1)\prime}(\gamma,\xi)\,So_m(\gamma,\eta)\,So_m(\gamma,\cos\phi_0)\right] \tag{4.19}$$

$$J_v^{(e)}\big|_{z=0} = \frac{2\,j\,Y_{in}}{\gamma}\sqrt{\frac{2\,\pi}{\xi^2 - \eta^2}}\sum_{m=0}^{\infty}\left[\frac{j^m}{N_m^{(e)}}\,b_m^{(e)}\,Re_m^{(1)}(\gamma,\xi)\,Se_m^{\prime}(\gamma,\eta)\,Se_m(\gamma,\cos\phi_0) + \frac{j^m}{N_m^{(o)}}\,b_m^{(o)}\,Ro_m^{(1)}(\gamma,\xi)\,So_m^{\prime}(\gamma,\eta)\,So_m(\gamma,\cos\phi_0)\right] \tag{4.20}$$

For the current densities on the plate z = b-, multiply the RHS of (4.17 - 4.20) by

$$-\cos(\beta\,z) = -\cos(m\,\pi) = -(-1)^m,$$

where m is the integer in (4.4).

### 4.1.2 Metallic Post

The case of a PEC post follows as a particular case of isorefractive post by considering $Z_{in} = 0$. Therefore, by substituting $Z_{in} = 0$ in the equation (4.14), the modal expansion coefficients take the following form :

$$a_m^{(e)}\Big|_{PEC} = -\frac{Re_m^{(1)}(\gamma,\xi_0)}{Re_m^{(4)}(\gamma,\xi_0)} \tag{4.21a}$$

$$a_m^{(o)}\Big|_{PEC} = -\frac{Ro_m^{(1)}(\gamma,\xi_0)}{Ro_m^{(4)}(\gamma,\xi_0)} \tag{4.21b}$$

$$b_m^{(e)}\Big|_{PEC} = 0 \tag{4.21c}$$

$$b_m^{(o)}\Big|_{PEC} = 0 \tag{4.21d}$$

Surface current density on surface $u = u_0$ of the PEC elliptic-cylinder post

$$\begin{aligned}
\underline{J}^{(e)}\big|_{u=u_0} &= \hat{u} \times \underline{H}^{(e)}\big|_{u=u_0} \\
&= -\hat{z}\,\frac{2\,jY}{\gamma}\sqrt{\frac{2\,\pi}{\xi_0^2-\eta^2}}\cos(\beta\,z)\,\frac{\partial}{\partial\,u}\Big(\sum^{i}+\sum^{s}\Big)\big|_{u=u_0} \\
&= -\hat{z}\,\frac{2\,jY}{\gamma}\sqrt{\frac{2\,\pi}{\xi_0^2-\eta^2}}\cos(\beta\,z) \\
&\times \sum_{m=0}^{\infty}\Bigg[\frac{j^m}{N_m^{(e)}}\big[Re_m^{(1)\prime}(\gamma,\xi)+a_m^{(e)}\,Re_m^{(4)\prime}(\gamma,\xi)\big]\,Se_m(\gamma,\eta)\,Se_m(\gamma,\cos\phi_0) \\
&\qquad + \frac{j^m}{N_m^{(o)}}\big[Ro_m^{(1)\prime}(\gamma,\xi)+a_m^{(o)}\,Ro_m^{(4)\prime}(\gamma,\xi)\big]\,So_m(\gamma,\eta)\,So_m(\gamma,\cos\phi_0)\Bigg]
\end{aligned} \tag{4.22}$$

Note that at the junctions between the PEC cylinder and the plates, $J_v = 0$ and the current normal to the junction is continuous in crossing the intersection line.

### 4.2 H-polarization

Similar to the E-polarized mode, an H-polarized mode can also be considered as a sum of two plane waves. So equations (4.4) are valid for this case as well. In absence of the post, the incident field can obtained from (2.23) :

$$\underline{H}^{(h)i} = Y \left[ \frac{j\beta}{k} \cos(\beta z)(\hat{x}\cos\phi_0 + \hat{y}\, sin\phi_0) + \frac{k_t}{k} \sin(\beta z)\,\hat{z} \right] e^{+j k_t\,(x\cos\phi_0 + y\sin\phi_0)}$$

$$\underline{E}^{(h)i} = (\hat{x}\sin\phi_0 - \hat{y}\cos\phi_0)\,\sin(\beta z) e^{+j k_t\,(x\cos\phi_0 + y\sin\phi_0)}$$

where $Z = Y^{-1}$ is the intrinsic impedance of medium inside the waveguide.

If the post is a circular-cylinder, we have a rotational symmetry. So the expressions can be made simpler by considering $\phi_0 = 0$, in which case :

$$\underline{H}^{(e)i}|_{\phi_0=0} = Y \left[ \frac{j\beta}{k}\,\hat{x}\sin(\beta z) + \frac{k_t}{k}\,\hat{z}\cos(\beta z) \right] e^{j k_t x} \tag{4.23a}$$

$$\underline{E}^{(e)i}|_{\phi_0=0} = -\,\hat{y}\cos(\beta z)\, e^{j k_t x} \tag{4.23b}$$

The incident field ( 2.23 ) can be expressed in terms of Mathieu functions :

$$\underline{H}^{(h)\,i} = 2Y\sqrt{2\pi}\,\frac{k_t}{k} \left[ \sin(\beta z)\,\hat{z} + \frac{\beta}{k_t\,\gamma\sqrt{\xi^2 - \eta^2}} \cos(\beta z) \left( \hat{u}\frac{\partial}{\partial u} + \hat{v}\frac{\partial}{\partial v} \right) \right] \sum^{i} \tag{4.24a}$$

$$\underline{E}^{(h)\,i} = \frac{2j}{\gamma}\sqrt{\frac{2\pi}{\xi^2 - \eta^2}}\,\sin(\beta z) \left( -\,\hat{u}\frac{\partial}{\partial v} + \hat{v}\frac{\partial}{\partial u} \right) \sum^{i} \tag{4.24b}$$

If an elliptic-cylinder post is present ( with surface $u = u_0$ or $\xi = \xi_0$ ), then the total field inside the waveguide and outside the post ($u_0 \leq u < \infty$) is the sum of incident field (4.24) and scattered field ($\underline{E}^s$ , $\underline{H}^s$ ) :

$\underline{E} = \underline{E}^i + \underline{E}^s$ and $\underline{H} = \underline{H}^i + \underline{H}^s$

The scattered field ($\underline{E}^s$ , $\underline{H}^s$ ) is given by (4.24) upon replacing $\sum^i$ with $\sum^s$. Where, $\sum^i$ and $\sum^s$ are defined in the equations (4.7) and (4.9) respectively.

**Solution in curvilinear coordinates**

For $\xi \geq \xi_0$ :

$$\begin{aligned} u^{(h)}(\xi, \eta, k_t) = & \sum_{m=0}^{\infty} \left[ \frac{j^m}{N_m^{(e)}} (Re_m^{(1)}(\gamma, \xi) + a_m^{(e)} Re_m^{(4)}(\gamma, \xi)) Se_m(\gamma, \eta) Se_m(\gamma, \cos\phi_0) \right] \\ & + \sum_{m=1}^{\infty} \left[ \frac{j^m}{N_m^{(o)}} (Ro_m^{(1)}(\gamma, \xi) + a_m^{(o)} Ro_m^{(4)}(\gamma, \xi)) So_m(\gamma, \eta) So_m(\gamma, \cos\phi_0) \right] \end{aligned} \tag{4.25}$$

For $\xi \leq \xi_0$ :

$$\begin{aligned} u^{(h)}(\xi, \eta, k_t) = & \sum_{m=0}^{\infty} \left[ \frac{j^m}{N_m^{(e)}} (b_m^{(e)} Re_m^{(1)}(\gamma, \xi)) Se_m(\gamma, \eta) Se_m(\gamma, \cos\phi_0) \right] \\ & + \sum_{m=1}^{\infty} \left[ \frac{j^m}{N_m^{(o)}} (b_m^{(o)} Re_m^{(1)}(\gamma, \xi)) So_m(\gamma, \eta) So_m(\gamma, \cos\phi_0) \right] \end{aligned} \tag{4.26}$$

### 4.2.1 Isorefractive Post

Assuming that the elliptic-cylinder post is made of isorefractive material, we can obtain the total fields inside the post by replacing $\sum^i$ with $\sum^{in}$ in (4.24), where $\sum^{in}$ is defined in (4.12). The modal expansion coefficients can be determined by imposing the continuity of the tangential components of electric and magnetic fields across the interface $u = u_0$, such that :

$$\underline{H}_z^{in} = \underline{H}_z^i + \underline{H}_z^s \tag{4.27a}$$

$$\underline{E}_v^{in} = \underline{E}_v^i + \underline{E}_v^s \tag{4.27b}$$

where each field component is as follows :

$$\underline{H}_z^i = 2Y\sqrt{2\pi}\,\frac{k_t}{k}\,\sin(\beta z)\sum^{i} \tag{4.28a}$$

$$\underline{H}_z^s = 2Y\sqrt{2\pi}\,\frac{k_t}{k}\,\sin(\beta z)\sum^{s} \tag{4.28b}$$

$$\underline{H}_z^{in} = 2Y_{in}\sqrt{2\pi}\,\frac{k_t}{k}\,\sin(\beta z)\sum^{in} \tag{4.28c}$$

$$\underline{E}_v^i = \frac{2j}{\gamma}\sqrt{\frac{2\pi}{\xi^2-\eta^2}}\,\cos(\beta z)\,\frac{\partial}{\partial u}\sum^{i} \tag{4.28d}$$

$$\underline{E}_v^s = \frac{2j}{\gamma}\sqrt{\frac{2\pi}{\xi^2-\eta^2}}\,\sin(\beta z)\,\frac{\partial}{\partial u}\sum^{s} \tag{4.28e}$$

$$\underline{E}_v^{in} = \frac{2j}{\gamma}\sqrt{\frac{2\pi}{\xi^2-\eta^2}}\,\sin(\beta z)\,\frac{\partial}{\partial u}\sum^{in} \tag{4.28f}$$

the modal expansion coefficients are found to be :

$$a_m^{(e)} = \frac{(\chi_0 - 1)\, Re_m^{(1)}(\gamma,\xi_0)\; Re_m^{(1)\prime}(\gamma,\xi_0)}{Re_m^{(1)}(\gamma,\xi_0)\, Re_m^{(4)\prime}(\gamma,\xi_0) - \chi_0\, Re_m^{(1)\prime}(\gamma,\xi_0)\, Re_m^{(4)}(\gamma,\xi_0)} \tag{4.29a}$$

$$a_m^{(o)} = \frac{(\chi_0 - 1)\, Ro_m^{(1)}(\gamma,\xi_0)\; Ro_m^{(1)\prime}(\gamma,\xi_0)}{Ro_m^{(1)}(\gamma,\xi_0)\, Ro_m^{(4)\prime}(\gamma,\xi_0) - \chi_0\, Ro_m^{(1)\prime}(\gamma,\xi_0)\, Ro_m^{(4)}(\gamma,\xi_0)} \tag{4.29b}$$

$$b_m^{(e)} = \frac{-j\,\chi_0}{Re_m^{(1)}(\gamma,\xi_0)\, Re_m^{(4)\prime}(\gamma,\xi_0) - \chi_0\, Re_m^{(1)\prime}(\gamma,\xi_0)\, Re_m^{(4)}(\gamma,\xi_0)} \tag{4.29c}$$

$$b_m^{(o)} = \frac{-j\,\chi_0}{Ro_m^{(1)}(\gamma,\chi_0)\, Ro_m^{(4)\prime}(\gamma,\xi_0) - \chi_0\, Ro_m^{(1)\prime}(\gamma,\xi_0)\, Ro_m^{(4)}(\gamma,\xi_0)} \tag{4.29d}$$

where,

$$\prime = \frac{\partial}{\partial u}$$

$$\chi_0 = \frac{Z_{in}}{Z}$$

In the case when there is no post ($Z = Z_{in}$) :

$$(a^{(e)}\,,\, a^{(o)})_{\chi_0 = 1} = 0 \tag{4.30a}$$

$$(b^{(e)}\,,\, b^{(o)})_{\chi_0 = 1} = 1 \tag{4.30b}$$

Surface current densities on the parallel plates

On the plate $z = 0+$:

$$\underline{J}^{(h)}\big|_{z=0} = \hat{z} \times \underline{H}^{(h)}\big|_{z=0}$$
$$= \frac{2\beta}{k\gamma}\sqrt{\frac{2\pi}{\xi^2 - \eta^2}} \begin{bmatrix} Y \\ \\ Y_{in} \end{bmatrix} \left(\hat{v}\frac{\partial}{\partial u} - \hat{u}\frac{\partial}{\partial v}\right) \begin{bmatrix} \Sigma^i + \Sigma^s \\ \\ \Sigma^{in} \end{bmatrix} \begin{matrix} , u \geq u_0 \\ \\ , u \leq u_0 \end{matrix} \tag{4.31}$$

i) Outside the post ($u \geq u_0$) :

$$J_u^{(h)}\big|_{z=0} = -\frac{2\beta Y}{k\gamma}\sqrt{\frac{2\pi}{\xi^2 - \eta^2}} \sum_{m=0}^{\infty} \Bigg[ \frac{j^m}{N_m^{(e)}} \left[ Re_m^{(1)}(\gamma,\xi) + a_m^{(e)} Re_m^{(4)}(\gamma,\xi) \right] Se_m'(\gamma,\eta)\, Se_m(\gamma,\cos\phi_0)$$
$$+ \frac{j^m}{N_m^{(o)}} \left[ Ro_m^{(1)}(\gamma,\xi) + a_m^{(o)} Ro_m^{(4)}(\gamma,\xi) \right] So_m'(\gamma,\eta)\, So_m(\gamma,\cos\phi_0) \Bigg] \tag{4.32}$$

$$J_v^{(h)}\big|_{z=0} = \frac{2\beta Y}{k\gamma}\sqrt{\frac{2\pi}{\xi^2 - \eta^2}} \sum_{m=0}^{\infty} \Bigg[ \frac{j^m}{N_m^{(e)}} \left[ Re_m^{(1)\prime}(\gamma,\xi) + a_m^{(e)} Re_m^{(4)\prime}(\gamma,\xi) \right] Se_m(\gamma,\eta)\, Se_m(\gamma,\cos\phi_0)$$
$$+ \frac{j^m}{N_m^{(o)}} \left[ Ro_m^{(1)\prime}(\gamma,\xi) + a_m^{(o)} Ro_m^{(4)\prime}(\gamma,\xi) \right] So_m(\gamma,\eta)\, So_m(\gamma,\cos\phi_0) \Bigg] \tag{4.33}$$

where $'$ on the radial function means $\frac{\partial}{\partial u}$ and $'$ on the angular function means $\frac{\partial}{\partial v}$

ii) Inside the post ($u \leq u_0$) :

$$
\begin{aligned}
J_u^{(h)}\big|_{z=0} = -\frac{2\,\beta\,Y_{in}}{k\,\gamma}\sqrt{\frac{2\,\pi}{\xi^2 - \eta^2}}\sum_{m=0}^{\infty}\Bigg[ & \frac{j^m}{N_m^{(e)}}\, b_m^{(e)}\, Re_m^{(1)}\,(\,\gamma,\xi\,)\, Se_m'\,(\,\gamma,\eta\,)\, Se_m\,(\,\gamma,\cos\phi_0\,) \\
& + \frac{j^m}{N_m^{(o)}}\, b_m^{(o)}\, Ro_m^{(1)}\,(\,\gamma,\xi\,)\, So_m'\,(\,\gamma,\eta\,)\, So_m\,(\,\gamma,\cos\phi_0\,)\Bigg]
\end{aligned}
\tag{4.34}
$$

$$
\begin{aligned}
J_v^{(h)}\big|_{z=0} = \frac{2\,\beta\,Y_{in}}{k\,\gamma}\sqrt{\frac{2\,\pi}{\xi^2 - \eta^2}}\sum_{m=0}^{\infty}\Bigg[ & \frac{j^m}{N_m^{(e)}}\, b_m^{(e)}\, Re_m^{(1)\prime}\,(\,\gamma,\xi\,)\, Se_m\,(\,\gamma,\eta\,)\, Se_m\,(\,\gamma,\cos\phi_0\,) \\
& + \frac{j^m}{N_m^{(o)}}\, b_m^{(o)}\, Ro_m^{(1)\prime}\,(\,\gamma,\xi\,)\, So_m\,(\,\gamma,\eta\,)\, So_m\,(\,\gamma,\cos\phi_0\,)\Bigg]
\end{aligned}
\tag{4.35}
$$

For the current densities on the plate z = b-, multiply the RHS of (4.32 - 4.35) by

$$
-\cos(\,\beta\,z\,) = -\cos(\,m\,\pi\,) = -(-1)^m
$$

where m is the integer in (4.4).

### 4.2.2 Metallic Post

The case of a PEC post follows as the particular case of the isorefractive post by considering $Z_{in} = 0$. Therefore, by substituting $Z_{in} = 0$ in (4.24), the modal expansion coefficients take the following form :

$$a_m^{(e)}\Big|_{PEC} = -\frac{Re_m^{(1)\prime}(\gamma,\xi_0)}{Re_m^{(4)\prime}(\gamma,\xi_0)} \tag{4.36a}$$

$$a_m^{(o)}\Big|_{PEC} = -\frac{Ro_m^{(1)\prime}(\gamma,\xi_0)}{Ro_m^{(4)\prime}(\gamma,\xi_0)} \tag{4.36b}$$

$$b_m^{(e)}\Big|_{PEC} = 0 \tag{4.36c}$$

$$b_m^{(o)}\Big|_{PEC} = 0 \tag{4.36d}$$

Surface current density on the surface $u = u_0$ of PEC elliptic-cylinder post

$$\underline{J}^{(h)}\big|_{u=u_0} = \hat{u} \times \underline{H}^{(h)}\big|_{u=u_0}$$
$$= -\hat{v}\,\frac{2\sqrt{2\pi}\,Y\,k_t}{k}\sin(\beta z)\,\Big(\sum^{i} + \sum^{s}\Big)\Big|_{u=u_0} + \hat{z}\,\frac{2\beta Y}{k\gamma}\sqrt{\frac{2\pi}{\xi_0^2 - \eta^2}}\cos(\beta z)\,\frac{\partial}{\partial v}\Big(\sum^{i} + \sum^{s}\Big)\Big|_{u=u_0} \tag{4.37}$$

$$J_z^{(h)}\big|_{u=u_0} = \frac{2\beta Y}{k\gamma}\sqrt{\frac{2\pi}{\xi_0^2 - \eta^2}}\cos(\beta z)\sum_{m=0}^{\infty}\Bigg[\frac{j^m}{N_m^{(e)}}\big[Re_m^{(1)}(\gamma,\xi) + a_m^{(e)}\,Re_m^{(4)}(\gamma,\xi)\big]\,Se'_m(\gamma,\eta)\,Se_m(\gamma,\cos\phi_0)$$
$$+ \frac{j^m}{N_m^{(o)}}\big[Ro_m^{(1)}(\gamma,\xi) + a_m^{(o)}\,Ro_m^{(4)}(\gamma,\xi)\big]\,So'_m(\gamma,\eta)\,So_m(\gamma,\cos\phi_0)\Bigg] \tag{4.38}$$

$$J_v^{(h)}\big|_{u=u_0} = -\frac{2\sqrt{2\pi}\,Y\,k_t}{k}\sin(\beta z)\sum_{m=0}^{\infty}\Bigg[\frac{j^m}{N_m^{(e)}}\big[Re_m^{(1)}(\gamma,\xi) + a_m^{(e)}\,Re_m^{(4)}(\gamma,\xi)\big]\,Se_m(\gamma,\eta)\,Se_m(\gamma,\cos\phi_0)$$
$$+ \frac{j^m}{N_m^{(o)}}\big[Ro_m^{(1)}(\gamma,\xi) + a_m^{(o)}\,Ro_m^{(4)}(\gamma,\xi)\big]\,So_m(\gamma,\eta)\,So_m(\gamma,\cos\phi_0)\Bigg] \tag{4.39}$$

Note that at the junctions between the PEC cylinder and the plates, $J_v = 0$ and the current normal to the junction is continuous in crossing the intersection line.

# CHAPTER 5

# SCATTERING BY A METALLIC STRIP POST INSIDE A PARALLEL PLATE WAVEGUIDE

An additional simplification occurs when the elliptic-cylinder post collapses onto a flat metal strip of width d, i.e. when $u_0 = 0$ ($\xi_0 = 1$). In this section, exact solutions are obtained for the case of strip post inside a parallel plate waveguide. The problem is solved for metallic strip post and the procedure is detailed below separately for E- and H-polarizations.

## 5.1 E-polarization

In case of E-polarization, the modal expansion coefficients are found by substituting $\xi_0 = 1$ in (4.21). Furthermore, in this case :

$$Ro_m^{(1)}(\gamma, 1) = 0 \tag{5.1}$$

which implies that,

$$\left. a_m^{(o)} \right|_{strip} = 0 \tag{5.2}$$

Therefore, the modal expansion coefficients take the following form :

$$a_m^{(e)}\Big|_{strip} = -\frac{Re_m^{(1)}(\gamma, 1)}{Re_m^{(4)}(\gamma, 1)} \tag{5.3a}$$

$$a_m^{(o)}\Big|_{strip} = 0 \tag{5.3b}$$

$$b_m^{(e)}\Big|_{strip} = 0 \tag{5.3c}$$

$$b_m^{(o)}\Big|_{strip} = 0 \tag{5.3d}$$

<u>Surface current density on the surface of the metal strip</u>

The expression of surface current density in the case of a metal strip is obtained by substituting (5.3) in (4.22) and using ($\xi_0 = 1$) :

$$\underline{J}^{(e)}\big|_{u_0=0} = -\hat{z}\,\frac{2\,jY}{\gamma}\sqrt{\frac{2\,\pi}{1-\eta^2}}\cos(\beta\,z)\sum_{m=0}^{\infty}\Bigg[\frac{j^{m+1}}{N_m^{(e)}\,Re_m^{(4)\prime}(\gamma,1)}\,Se_m(\gamma,\eta)\,Se_m(\gamma,\cos\phi_0) + \frac{j^m}{N_m^{(o)}}\left[Ro_m^{(1)\prime}(\gamma,\xi)\,So_m(\gamma,\eta)\,So_m(\gamma,\cos\phi_0)\right]\Bigg] \tag{5.4}$$

Note that

$$Ro_m^{(1)}(\gamma,1) = 0 \tag{5.5a}$$

$$Re_m^{(1)\prime}(\gamma,1) = 0 \tag{5.5b}$$

$$Re_m^{(1)}\,Re_m^{(4)\prime} - Re_m^{(4)}\,Ro_m^{(1)\prime} = -j \tag{5.5c}$$

$$Ro_m^{(1)}\,Ro_m^{(4)\prime} - Ro_m^{(4)}\,Ro_m^{(1)\prime} = -j \tag{5.5d}$$

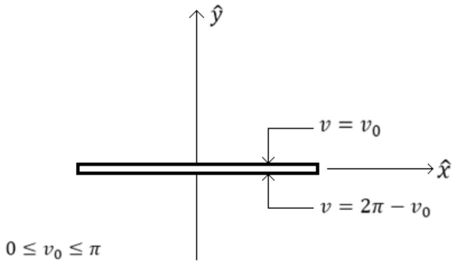


Figure 7: Metal strip of width d

$$Se_m(c,\eta)\big|_{v=2\pi-v_0} = Se_m(c,\eta)\big|_{v=v_0} \tag{5.5e}$$

$$So_m(c,\eta)\big|_{v=2\pi-v_0} = -So_m(c,\eta)\big|_{v=v_0} \tag{5.5f}$$

## 5.2 H-polarization

In case of H-polarization, the modal expansion coefficients are found by substituting $\xi_0 = 1$ in the (4.36). Furthermore, in this case :

$$Re_m^{(1)\prime}(\gamma, 1) = 0 \tag{5.6}$$

which implies that,

$$\left. a_m^{(e)} \right|_{strip} = 0 \tag{5.7}$$

Therefore, the modal expansion coefficients take the following form :

$$\left. a_m^{(e)} \right|_{strip} = 0 \tag{5.8a}$$

$$\left. a_m^{(o)} \right|_{strip} = -\frac{Re_m^{(1)\prime}(\gamma, 1)}{Re_m^{(4)\prime}(\gamma, 1)} \tag{5.8b}$$

$$\left. b_m^{(e)} \right|_{strip} = 0 \tag{5.8c}$$

$$\left. b_m^{(o)} \right|_{strip} = 0 \tag{5.8d}$$

Surface current density on surface of the metal strip

The expression of surface current density in the case of a metal strip ($\xi_0 = 1$) is obtained by substituting (5.8) in (4.38) and (4.39) :

$$J_z^{(h)}|_{u_0=0} = \frac{2\,\beta\,Y}{k\,\gamma}\sqrt{\frac{2\,\pi}{1-\eta^2}}\cos(\beta\,z)\sum_{m=0}^{\infty}\Bigg[\frac{j^m}{N_m^{(e)}}\big[Re_m^{(1)}(\gamma,\xi)\big]\,Se_m'(\gamma,\eta)\,Se_m(\gamma,\cos\phi_0) - \frac{j^{m+1}}{N_m^{(o)}}\Big[\frac{1}{Ro_m^{(4)\prime}(\gamma,1)}\Big]\,So_m'(\gamma,\eta)\,So_m(\gamma,\cos\phi_0)\Bigg] \tag{5.9}$$

$$J_v^{(h)}|_{u_0=0} = -\frac{2\sqrt{2\,\pi}\,Y\,k_t}{k}\sin(\beta\,z)\sum_{m=0}^{\infty}\Bigg[\frac{j^m}{N_m^{(e)}}\big[Re_m^{(1)}(\gamma,\xi)\big]\,Se_m(\gamma,\eta)\,Se_m(\gamma,\cos\phi_0) - \frac{j^{m+1}}{N_m^{(o)}}\Big[\frac{1}{Ro_m^{(4)\prime}(\gamma,1)}\Big]\,So_m(\gamma,\eta)\,So_m(\gamma,\cos\phi_0)\Bigg] \tag{5.10}$$

# CHAPTER 6

## NUMERICAL RESULTS

The surface currents are computed from the exact solutions given in Chapters 3, 4 and 5 for E- and H-polarization, respectively. The computation is carried out in MATLAB R2017a, with the aid of the results for Mathieu functions calculation provided in (7) (8) (9).

For the case of a PEC circular-cylinder, the radius of the cylinder is a = $0.79\,\lambda$, the angles of incidence are $\phi_0 = \pi$, $\theta_{01} = 38.2^\circ$ and the plate separation is b = $1.27\,\lambda$. The angle of incidence $\theta_{01}$ is such that the quantization number is m = 2. Fig. 8 shows $\rho$ and $z$ directed surface currents on the parallel plates and the cylinder respectively and Fig. 9 shows $\phi$ directed surface currents on both parallel plates and the cylinder for the case of E-polarization. It is important to note that there is continuity between $J_\rho$ on the parallel plates and $J_z$ on the cylinder across the junctions. Also, it is noteworthy that in case of E-polarization, $J_\phi$ on the cylinder completely vanishes. Fig. 10 shows $\rho$ and $z$ directed surface currents on the parallel plates and the cylinder respectively and Fig. 11 shows $\phi$ directed surface currents on both parallel plates and the cylinder for the case of H-polarization. Once again, we observe the continuity in the currents $J_\rho$ and $J_z$ on the parallel plates and the cylinder respectively, across the intersection lines. But unlike the case of E-polarization, here $J_\phi \neq 0$ on the cylinder.

For the case of a metal strip, the width of the strip is d = $0.47\,\lambda$, the angles of incidence are $\phi_0 = -\frac{\pi}{2}$, $\theta_{01} = 38.2^\circ$ and the plate separation is b = $0.63\,\lambda$. The angle of incidence $\theta_{01}$ is such that the quantization number is m = 1. Fig. 12 shows $\rho$ and $z$ directed surface currents on the parallel plates and the metal strip respectively and Fig. 13 shows $\phi$ directed surface currents on both parallel plates and the strip for the case of E-polarization. It should be noted that there is a continuity between $J_\rho$ on the parallel plates and $J_z$ on the metal strip across the junctions. Also, $J_\phi$ on the plate vanishes in the case of E- polarization, which is similar to the cylindrical case. Fig. 14 shows $J_\rho$ and $J_z$ on the parallel plates and the strip respectively. Fig. 15 shows $J_\phi$ on both parallel plates and the strip for the case of H-polarization. Once again, we observe a continuity in the currents $J_\rho$ and $J_z$ on the parallel plates and the strip respectively, across the junctions. But unlike the case of E-polarization, here $J_\phi \neq 0$ on the strip.

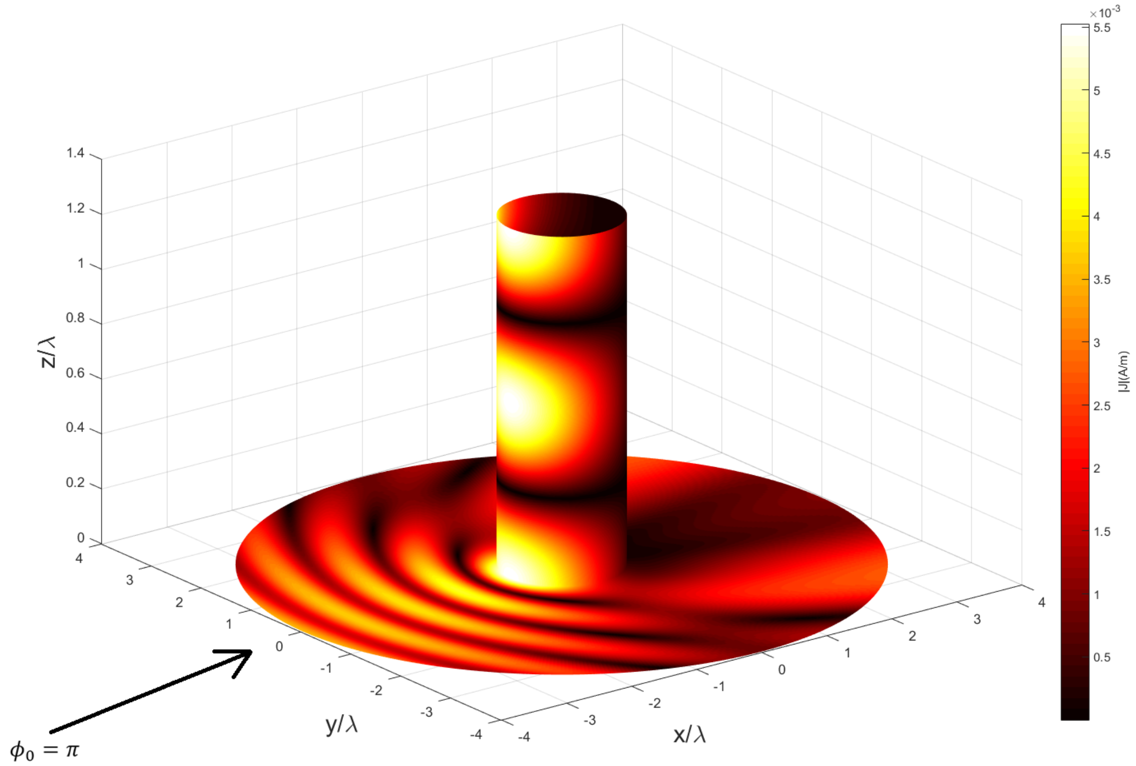


Figure 8: $|J_{\rho/z}|$ on PEC Surfaces in case of cylindrical post (E-polarization)

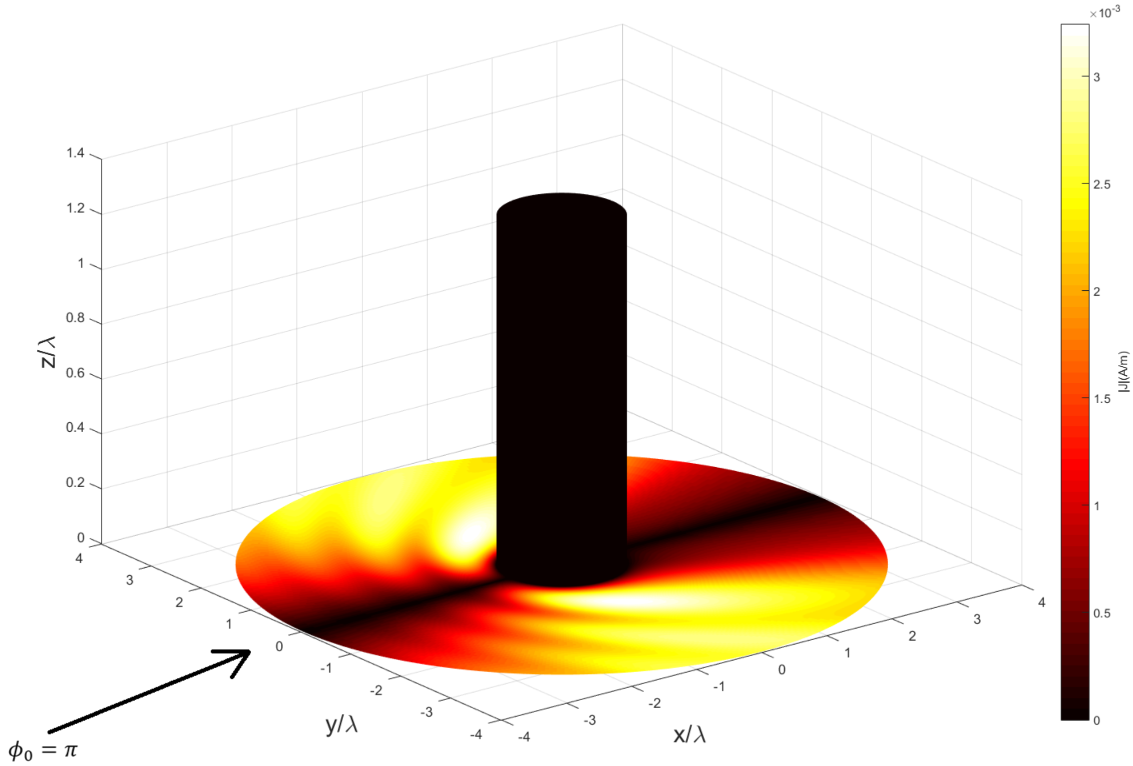


Figure 9: $|J_\phi|$ on PEC Surfaces in case of cylindrical post (E-polarization)

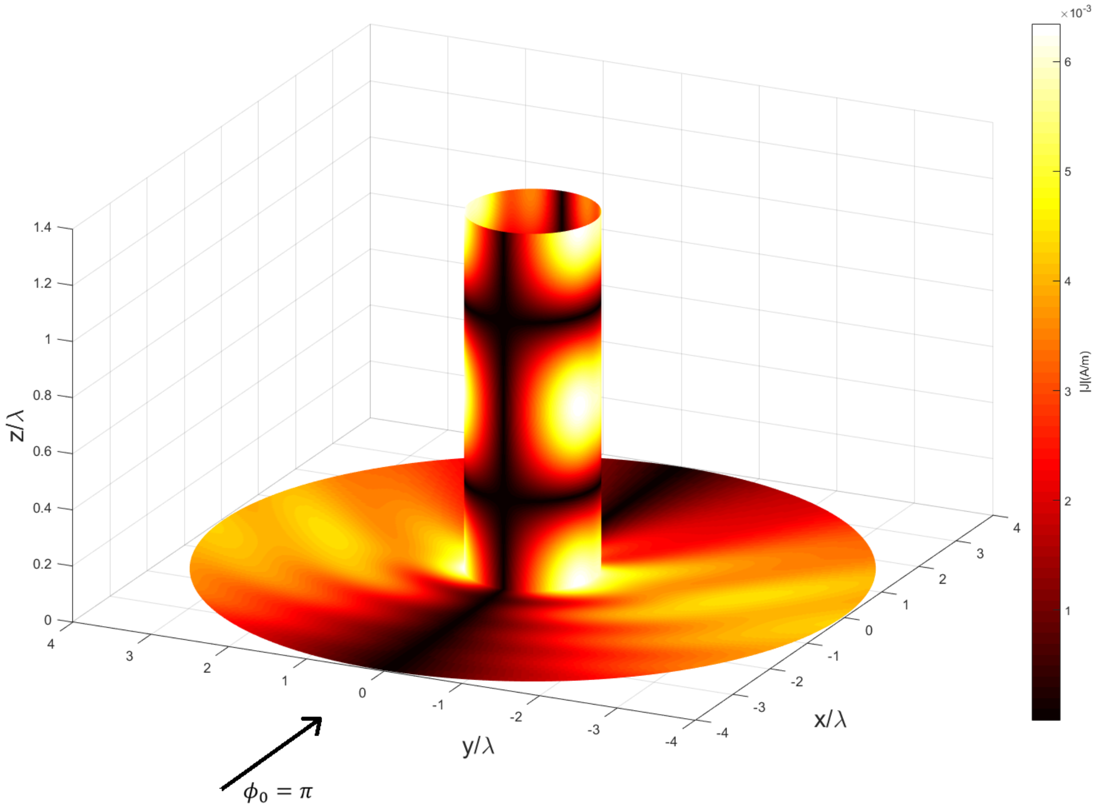


Figure 10: $|J_{\rho/z}|$ on PEC Surfaces in case of cylindrical post (H-polarization)

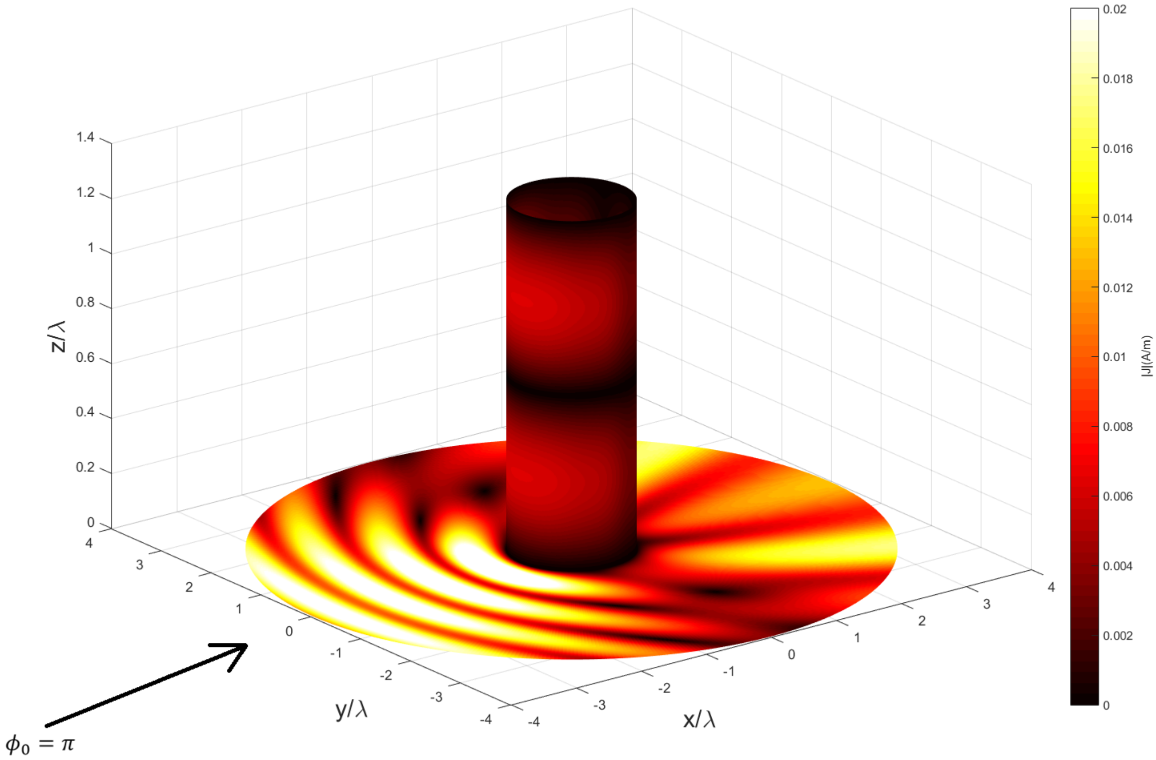


Figure 11: $|J_\phi|$ on PEC Surfaces in case of cylindrical post (H-polarization)

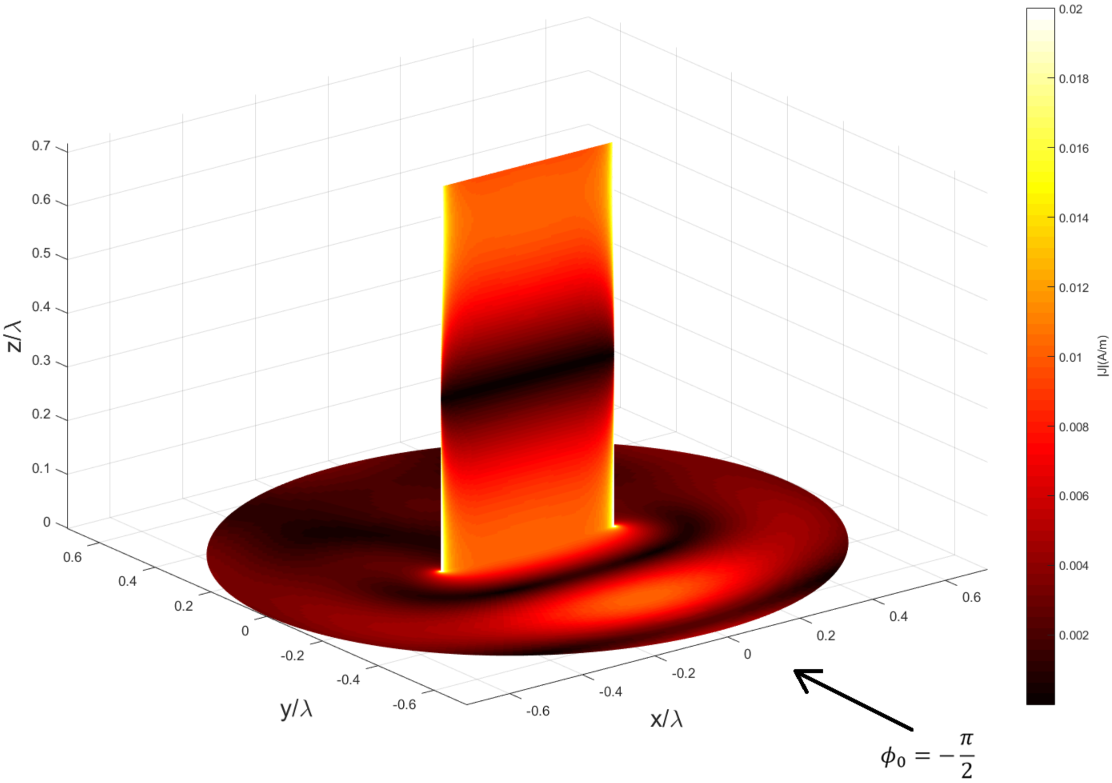


Figure 12: $|J_{u/z}|$ on PEC Surfaces in case of metal strip post (E-polarization)

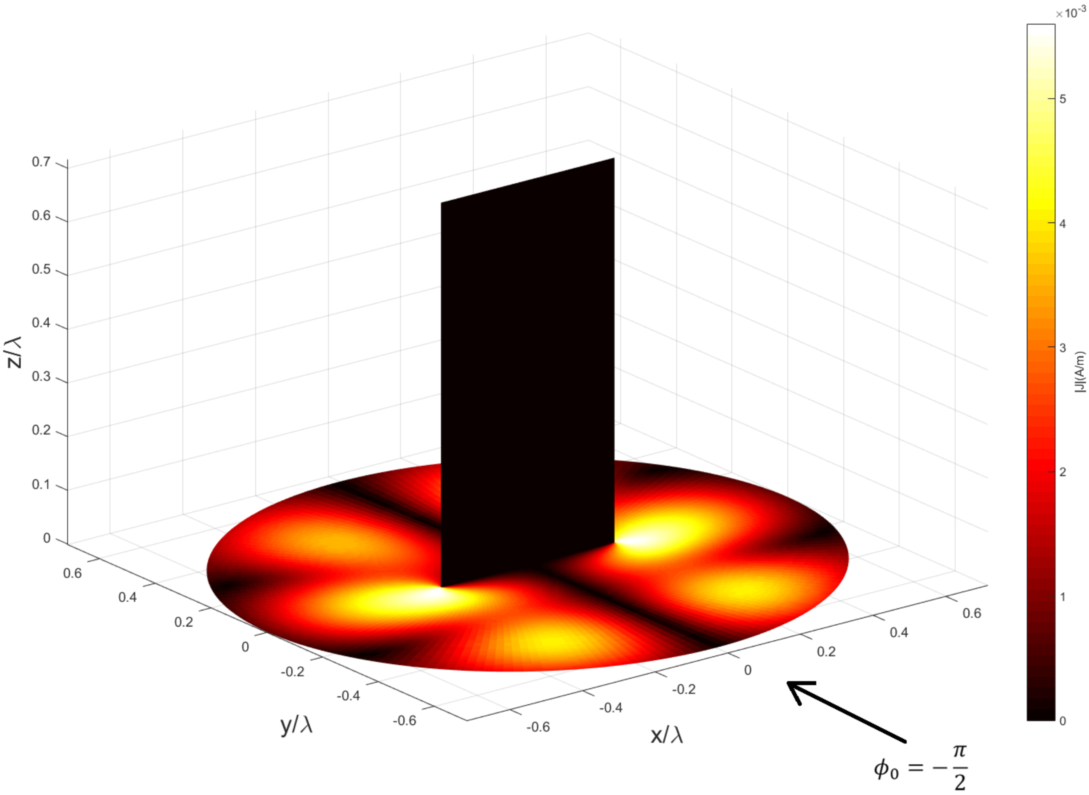


Figure 13: $|J_v|$ on PEC Surfaces in case of metal strip post (E-polarization)

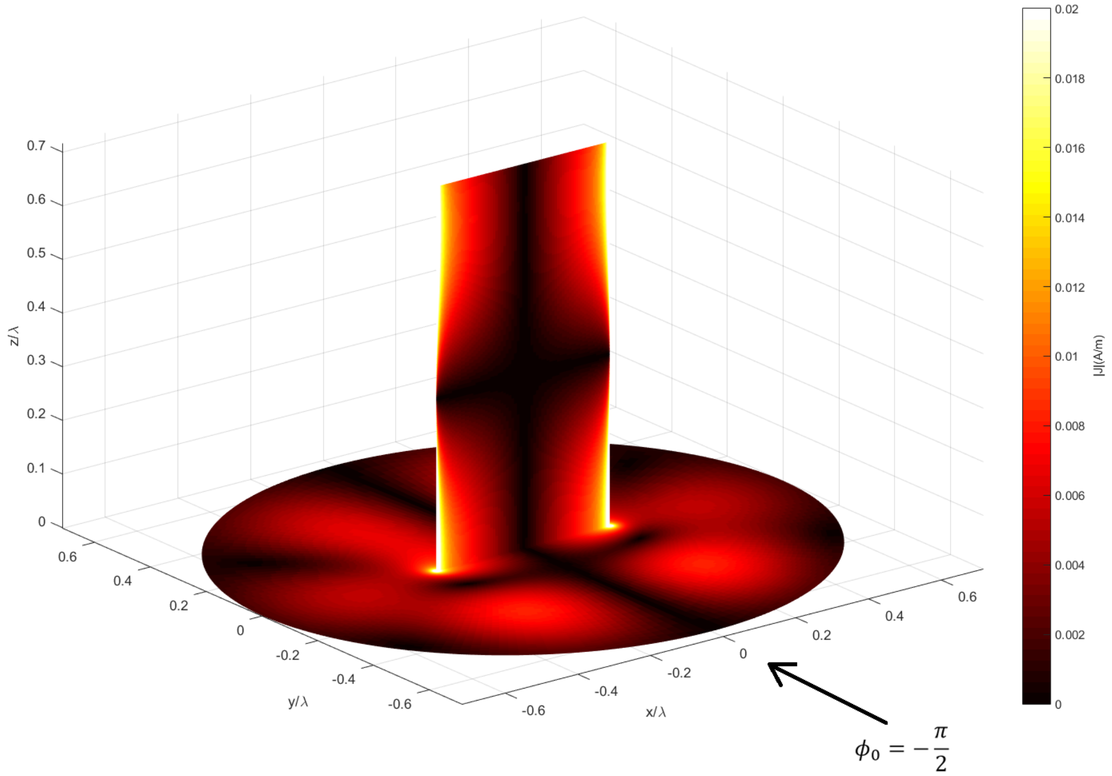


Figure 14: $|J_{u/z}|$ on PEC Surfaces in case of metal strip post (H-polarization)

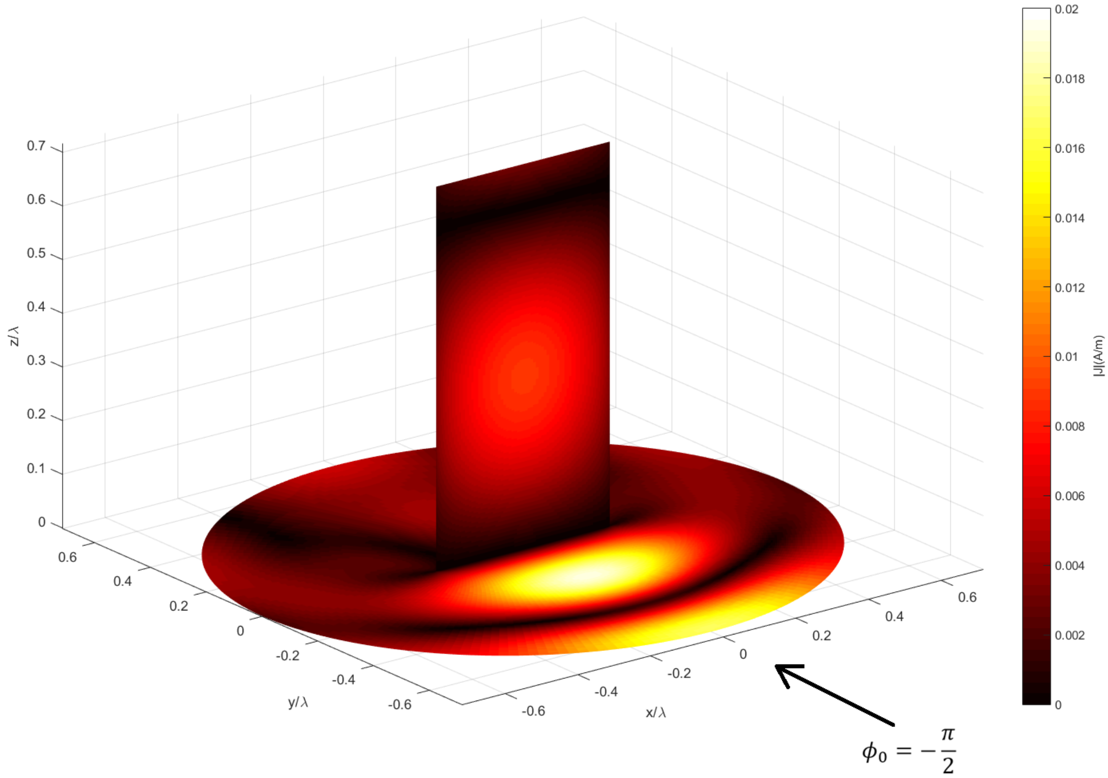


Figure 15: $|J_v|$ on PEC Surfaces in case of metal strip post (H-polarization)

# CHAPTER 7

# CONCLUSION

Exact analytical solutions have been derived for scattering of a mode propagating inside a parallel plate waveguide by a cylindrical post located inside the waveguide. Three different shapes of the cross-sectional area of the post are considered: circular, elliptical and strip. The solution is obtained when the linear, homogeneous and isotropic material of the cylindrical or elliptical post is isorefractive to the surrounding medium; hence, the solution for a PEC post is a particular case of the more general solution. The analysis for both TE and TM modes with numerical results are presented for each one them.
Some of the results reported in this thesis were orally presented at two conferences (10) (11).

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