

# CAPACITY ALLOCATION IN VERTICALLY INTEGRATED RAIL SYSTEMS: A BARGAINING APPROACH

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## Abstract

This paper presents a game-theoretic bargaining approach to allocating rail line capacity in vertically integrated systems. A passenger rail agency negotiates with the host freight railroad to determine train schedules and the associated payment. The objective on the passenger side is to maximize utility, i.e., revenue minus costs of passenger train operations, passenger schedule delay and en-route delay; the freight side minimizes the costs of train departure delay, en-route delay, loss of demand, and track maintenance. Bargaining in both complete and incomplete information settings are considered; the latter arises because the freight railroad may withhold its private cost information. With complete information, we find that the equilibrium payments proposed by the passenger rail agency and the host freight railroad will each be invariant to who initiates the payment bargaining, although the actual payment does depend on who is the initiator. The equilibrium schedule maximizes system welfare. With incomplete information, the passenger rail agency may choose between pooling and separating equilibrium strategies while proposing a payment, depending on its prior belief about the cost type of the freight railroad; whereas the host freight railroad will adopt strategies that do not reveal its cost type. To identify equilibrium schedules, a pooling equilibrium is constructed along with conditions for the existence of equilibrium schedules. We further conduct numerical experiments to obtain additional policy-relevant insights.

## Keywords

Rail line capacity allocation; vertically integrated systems; bargaining game; train schedule; payment; complete and incomplete information

## 1. Introduction

Rail is an essential component in the US passenger and freight transportation system. On the freight side, rail transports about 27% of total freight revenue ton-miles (US DOT 2012), with Class I railroads' revenue ton-miles increased from 1,495 billion in 2001 to 1,729 billion in 2011 (AAR 2012). On the passenger side, Amtrak, the national intercity transportation service provider, has set ten ridership records over the last 11 years, carrying more than 31.5 million passengers in 2013 (Amtrak 2013). Amtrak's operation is characterized by the shared use of infrastructure with freight railroads, as 70% of Amtrak's train miles are produced on tracks which are owned and maintained by host freight railroads (Amtrak 2012). The flourishing of passenger rail has spurred the development of Higher Speed Rail (HrSR) services (FRA 2015), which still run on shared-use tracks but at a higher speed, up to 110 mph (Peterman et al. 2009). Compared to conventional Amtrak trains, introducing HrSR means

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greater speed heterogeneity between passenger and freight operations, which has negative impact on the rail line capacity (Talebian and Zou 2015). Given the HrSR development and the expected continuous growth of freight and passenger rail traffic, having appropriate mechanisms in place has become increasingly critical to efficiently and fairly allocate capacity between passenger and freight operations on shared-use rail lines.

One premise for allocating rail line capacity is to understand capacity. Two aspects are of particular importance. First, the capacity of a rail line depends on how conflict-free train schedules are created; thus capacity is endogenously rather than exogenously determined (Pena-Alcaraz et al. 2014). Second, the consequence of increasing rail service by one operator may not only delay trains on the line, but can also lead to the inability of other operator(s) to schedule as many trains as they originally demand (Talebian and Zou 2015). The latter is termed capacity scarcity (Nash and Matthews 2003; Nash and Sansom 1999). Capacity scarcity is especially relevant when one type of trains is scheduled before another type. In the US, Amtrak services are given higher access priority over freight operations by Public Law (110 Congress 2008).

In addition to the above two aspects, appropriate mechanisms for rail line capacity allocation hinges on how a rail system is structured – more specifically, whether infrastructure ownership and train operations are vertically integrated or separated. The US rail system has long been vertically integrated, with majority of the rail tracks owned by freight railroads, on the ground that such system takes advantage of economies of scale which minimize train operating expenses; in contrast, separation of infrastructure ownership from operations would require effective communication between different entities which may elevate costs. While there are concerns about vertically integrated systems for the lack of competition, the counterargument is that sufficient competition already exists – from alternative modes, different transportation sources, and between adjacent lines (Drew 2009). Among the empirical investigations, Bitzan (2003) finds that the cost of resource use would increase if the US rail system were to be vertically separated; a different view is provided by Ivaldi and Mc Cullough (2001), who argue that there may be no inherent advantages of vertical integration. Although debates on whether vertical separation is needed in the US rail system will continue, for the foreseeable future the existing vertical integration structure is likely to remain unchanged.

The state-of-the-practice allocation of rail line capacity between a host freight railroad and Amtrak is primarily conducted through negotiation based on Public Law 110-432 (110 Congress 2008). In general, the negotiation process, which is due to the legislated power being not absolute, consists of determining passenger train schedules and payment between Amtrak and freight host railroads for track usage and penalty for train delays (US DOT, 2010). The negotiation process is extensively covered in the industry press (e.g., in magazines like *Railway Age and Trains*) and well known to those who follow North American railroads, although published formal disclosure is limited. The academic literature on negotiation-based capacity allocation in vertically integrated systems is also quite scarce – if it exists – which is in sharp contrast with the abundant knowledge on how to allocate capacity in vertically separated systems (see Section 2 below).

The present paper makes the first attempt to fill this gap. We propose a game-theoretic bargaining approach to modeling capacity allocation on a vertically integrated rail line. A mathematical structure generally consistent with the practice is established to characterize how passenger and freight sides bargain to determine train schedules and payment from the passenger rail agency to the host freight railroad. We incorporate a range of cost components that are also in line with the practice. We further recognize that the passenger rail agency may not possess full cost information about the host freight railroad, and extend the bargaining model from a complete to an incomplete information setting. Through analytical solutions and numerical analysis, we obtain a number of policy-relevant insights, which advance the knowledge and help inform future practices on capacity allocation on vertically integrated, shared-use rail lines in the US.

The modeling framework developed in this paper could be of potential use in other contexts as well. One example is the Chinese High-Speed Rail (CHSR) system where the passenger rail operator, which owns and operates CHSR, negotiates with the dedicated freight HSR operator for line capacity allocation. The issue of capacity allocation will likely become more important given the growing use of high speed rail for intercity logistics as driven by the rapid development of e-commerce in China.

Another example is vessel chartering in maritime transportation, where a charterer negotiates with a ship owner on shipping price, volume, and time. More generally, the game-theoretic bargaining framework developed in this paper could be applied to a range of transportation settings where negotiation and/or contracting is involved, such as between shippers and carriers in the trucking industry and in aircraft leasing. The framework may be used to investigate other domains as well including pipelines and electric power grids – basically any time a government seeks to break up a monopoly or provide a product or service which the commercial market has refused to provide.

The remainder of the paper is organized as follows. Section 2 presents a review of existing mechanisms for rail line capacity allocation, based on which the contributions of the present study are highlighted. Section 3 puts forward the modeling framework, ensued by a description of the pre-bargaining input preparation in Section 4. The bargaining model is presented in Section 5. Section 6 offers numerical analysis and results discussions. Summary of major findings and directions for further research are given in Section 7.

## **2. Literature review and contributions of the present study**

Mechanisms for rail line capacity allocation can be broadly categorized into three groups: administrative, value-based, and market-based. Administrative mechanisms employ a set of rules, such as “intercity trains go first” (Gibson 2003), to allocate train paths to operators. Such mechanisms do not rely on values for rail infrastructure capacity, and are mostly employed in rail networks fully owned and directly controlled by governments or government-backed infrastructure managers. In the rest of the section, we focus on reviewing studies in the value-based and market-based groups.

The value-based mechanisms require the existence of a rail infrastructure manager, who places on each train path a value. On receiving the values, train operators decide whether to take the offered paths. Train path values are often determined based on either average or marginal cost. Under average cost valuation, total cost is distributed among trains proportional to track infrastructure usage. As examples, Kennedy (1997) suggests allocating total cost, including costs due to track maintenance, renewal, and traction (electricity) current, to trains according to mileage or passenger-mile shares. Kozan and Burdett (2005) propose four access charging schemes which are corridor-based, section-based, time-based, and weight-based. Under marginal cost valuation, access charges are computed based on the effects of the added train on line congestion as well as the train’s knock-on delay effect throughout the network (Gibson 2003), or based on the incremental infrastructure wear-and-tear cost (Bugarinovic and Boskovic 2015). For estimating the marginal infrastructure wear-and-tear cost, various econometric models have been developed (Gaudry and Quinet 2003; Johansson and Nilsson 2004; Wheat and Smith 2008; Anderson 2011; Anderson et al. 2012).

The market-based mechanisms apply mostly to vertically separated systems, seeking to elicit prices that train operators are willing to pay for their desired train paths, at the same time taking into account congestion impact. One prominent form of market-based mechanisms is auction, with pioneering efforts made by Brewer and Plott (1996) and Nilsson (1999) who consider respectively first-price and second-price auctions. In particular, the second-price auction incentivizes train operators, who are bidders, to truthfully report their valuation of schedules. Borndörfer et al. (2009) present a general Vickrey-Clarke-Groves (VCG) mechanism which provides incentive compatible and efficient allocation of rail line capacity with combinatorial constraints to maintain train path feasibility. (A mechanism is called “incentive compatible” if participants in the mechanism find it their best interests to truthfully reveal their private information, as is asked by the mechanism.) Borndorfer et al. (2006) design a multi-round combinatorial auctioning mechanism that allows flexibility for train departure and arrival times, speed, and routes taken. The research is later extended by Harrod (2013) who employs a hypergraph-based train scheduling model, arguing that commonly used discrete-time dynamic graphs do not fully capture train interactions during block transitions. Most recently, Kuo and Miller-Hooks (2015) propose a combinatorial auctioning scheme with two bid set construction techniques that enable expression of complementary and substitutable relationship among train slots.

Besides auction, non-cooperative game-theoretic models have also been employed to determine allocation of rail line capacity. The earliest model of this type is from Harker and Hong (1994), who propose an internal market structure within a railroad company, which consists of a railroad authority and several divisions (e.g., passenger, intermodal, etc.) each with a different per train value for transit

time and schedule adherence. The divisions compete for train paths; the railroad authority minimizes a weighted sum of train path deviations from ideal timetables subject to train travel time constraints. Rail track charges are determined based on the dual prices of the train travel time constraints, which represent the value of using the track to each train. Bassanini et al. (2002) consider a similar structure and develop a three-stage model, in which train operators request their preferred schedules at the first stage. At the second stage, the infrastructure manager determines access charges and the effective schedules by solving an analytical, probabilistic timetabling model. At the third stage, each train operator sets fare given the access charges and effective schedules. Lang et al. (2013) propose a two-level model to capture interactions among an infrastructure manager, a regulatory agency, and train operators. At the upper level, the regulatory agency sets access charge that maximizes social welfare subject to the break-even constraint for the infrastructure manager. Given the access charge, at the lower level train operators compete to maximize profit.

None of the three approaches, however, is amenable to the US rail system, which is the interest of the present study. The administrative approach is flawed as it does not provide incentives for train operators to efficiently use existing capacity. In addition, the requirement of a government or government-backed entity for capacity allocation is obviously not the case in the US. Within the value-based approach, average cost-based valuation is inadequate, because it neglects rail line capacity constraints and congestion (Gibson 2003; Harrod 2013); marginal-cost based valuation is also difficult to implement as marginal cost depends on the order of trains put on the line. The market-based approach is not suitable either as it applies only to open access markets with separation of rail infrastructure ownership and train operations.

In view of these, this research contributes to the literature of rail line capacity allocation in three major ways. First, a bargaining approach is developed which allocates capacity between a passenger rail agency (PRA) and a host freight railroad (FRR) on a shared-use line which is owned and maintained by FRR. PRA and FRR make alternating proposals, until PRA and FRR reach an agreement on the schedule of passenger trains and the associated payment from PRA to FRR. Second, we investigate bargaining under complete and incomplete information. In the latter case, PRA is assumed to only have a probabilistic belief about FRR's cost type, which is plausible as FRR may not want to disclose its full private information. Third, in addition to deriving close-form solutions, we conduct numerical analysis, which generate policy-relevant insights into the current practice and future improvement of capacity allocation on vertically integrated, shared-use rail lines.

### 3. Modeling framework

Building upon the previous discussions, our modeling framework reflects the following six aspects:

1. *The allocation of rail line capacity is endogenous.* Rail line capacity is determined through train scheduling (Pena-Alcaraz et al. 2014), which specifies train paths. A train path consists of a set of consecutive rail track blocks and the time interval for the train to use each track block. Allocation of rail line capacity refers to assigning a set of train paths between PRA and FRR, provided that these paths do not violate operational feasibility of trains on the line.
2. *PRA is public but FRR is private.* The public nature of PRA is in line with the fact that Amtrak consistently receives subsidies from the US federal government. The public nature means that PRA cares about passenger cost as well as its own profit while calculating utility associated with each schedule. In contrast, the objective of the host FRR is purely to maximize profit.
3. *Passenger trains are given access priority over freight trains.* The US Public Law 110-432 stipulates that Amtrak trains have access priority over freight trains while operating on a shared-use line (110 Congress 2008). In our modeling framework it means that, in occasion of train meets or overpasses, passenger trains should receive priority over freight trains.
4. *PRA makes a payment to FRR for track usage.* The payment from PRA is to compensate the host FRR for the incremental costs caused by accommodating passenger services. The incremental costs include costs due to: i) additional freight train delays, both at departure and en route; ii)

inability of FRR to run as many freight trains as demanded (Nash and Sansom 1999), which leads to a loss of demand and thus profit; and iii) additional track maintenance requirement.

5. *Passenger trains are allowed to stop en route.* Passenger trains stopping en route increases line operational flexibility, which may permit FRR to dispatch more freight trains with less delays. As a result, the incremental cost to FRR will be reduced, leading to lower payment from PRA. However, this should be balanced against increased travel time of passenger trains and associated cost to passengers, which PRA cares about.
6. *Rail passenger demand is sensitive to schedule delay.* For a rail traveler, schedule delay is defined as the time difference between one's preferred departure and the closest train departure (Hendrickson and Kocur 1981) (for further discussion, see subsection 4.2.4). Long schedule delay adds to the travel inconvenience and discourages rail travel. In response, rail travelers may switch to an alternative mode (e.g., auto, air), or decide not to travel at all. Reduced rail ridership means less PRA revenue. Thus PRA should account for the sensitivity of passenger demand to schedule delay when scheduling trains.

With the above aspects, Figure 1 presents the overall approach to allocating rail line capacity between PRA and the host FRR. Before the bargaining begins, necessary inputs need to be prepared. These include the set of feasible passenger train schedules, *FPTS*, which are generated prior to freight train schedules. Given each generated passenger train schedule, the best freight train schedule is then generated (see section 4.3). This order of train schedule generation reflects the access priority given to passenger trains over freight trains in occasion of train meets and overpasses. For each feasible passenger train schedule  $s_i \in FPTS$  and the associated freight train schedule, we compute PRA's utility and FRR's cost.

Our specifications of PRA's utility and FRR's cost draw from existing qualitative descriptions of the PRA-FRR negotiation practice in the US. For example, in the negotiation, a key element in determining the amount of payment is the service quality PRA receives (Bing et al., 2010). PRA compensates FRR more for providing a higher quality of service (US DOT, 2010). PRA's quality of service entails en-route delay cost of passenger trains and passengers, and schedule delay cost of passengers, all of which are considered in our PRA utility function (see Section 4.2). On the other hand, a high PRA quality of service is often at the price of FRR cost, which should be reflected in the payment such that FRR recovers not only the additional operating and maintenance cost, but also the opportunity cost due to capacity granted to PRA (NCHRP, 2007). Our modeling of the FRR-side cost elements is consistent with the above cost considerations: the FRR cost function includes costs of freight train departure delay, en-route delay, loss of demand, and track maintenance (see Section 4.3). Having PRA's utility and FRR's cost functions in place, the two players then proceed to bargaining to achieve an agreed schedule and corresponding payment. In the next two sections, we detail in sequence pre-bargaining input preparation and bargaining.

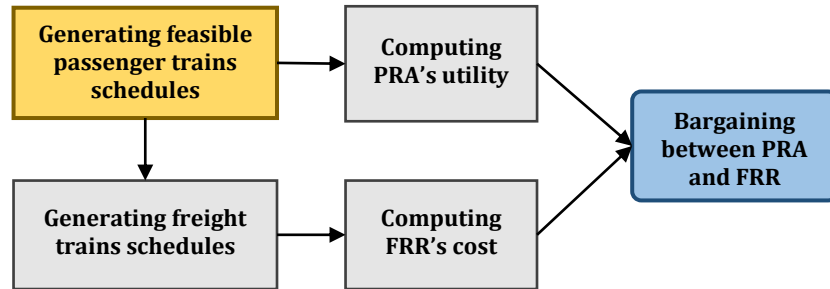


Figure 1: Overall approach to rail line capacity allocation

## 4. Pre-bargaining input preparation

### 4.1 Generating feasible passenger train schedules

We assume a given number of passenger trains running each day on the line of interest. *FPTS* is generated based on our previous work (Talebian and Zou 2015). Among all the schedules in *FPTS*, one schedule is chosen as the baseline schedule  $s_b$ . Details about *FPTS* generation is provided in Appendix A. For each  $s_i \in FPTS$  generated, we compute PRA's utility and FRR's cost as follows.

### 4.2 Computing PRA's utility

#### 4.2.1 PRA's utility structure

Given an  $s_i \in FPTS$ , we consider PRA's utility  $u_{s_i}^P$  to be the difference between PRA's operating revenue ( $OR_{s_i}^P$ ) and the sum of operating cost ( $OC_{s_i}^P$ ), passenger schedule delay cost ( $SDC_{s_i}^P$ ), and passenger en-route delay cost ( $EDC_{s_i}^P$ ). As utility is a relative term, we measure PRA's operating cost, passenger schedule delay cost, and passenger en-route delay cost against their respective values under baseline schedule  $s_b$ . For operating revenue, we measure it against zero, or the operating revenue when PRA is unable to run on the line. Note that choosing a baseline value is equivalent to adding/subtracting a constant term in the utility function. They do not change the relative attractiveness of each feasible passenger train schedule. Measuring  $OR_{s_i}^P$  against zero is an artificial construct. It, together with the measurement of  $OC_{s_i}^P$ ,  $SDC_{s_i}^P$ , and  $EDC_{s_i}^P$ , ensures that the value of  $u_{s_i}^P$  is positive, which is desired in the bargaining model in Section 5. In sum,  $u_{s_i}^P$  is expressed as

$$u_{s_i}^P = OR_{s_i}^P - (OC_{s_i}^P - OC_{s_b}^P) - (SDC_{s_i}^P - SDC_{s_b}^P) - (EDC_{s_i}^P - EDC_{s_b}^P) \quad (1)$$

#### 4.2.2 Operating revenue

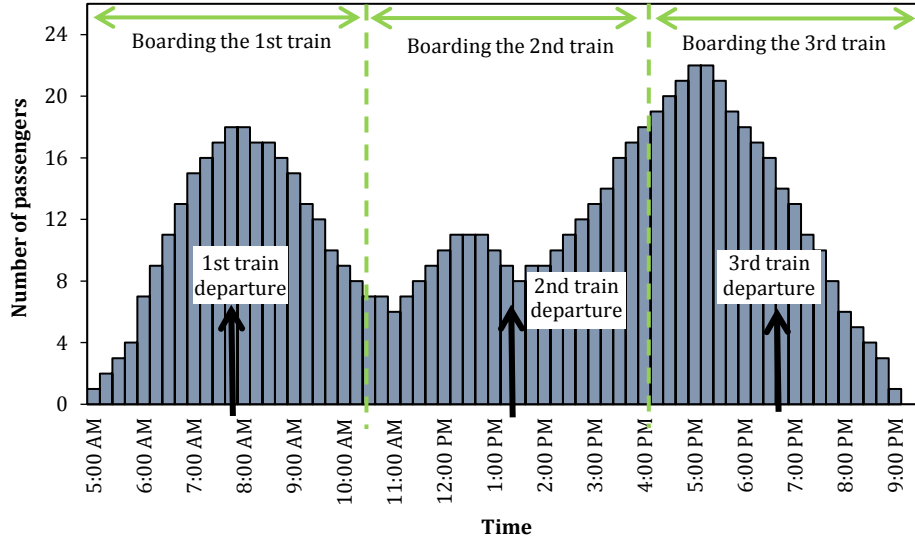
We consider constant rail fare for each origin-destination (OD) pair, and assume that an estimate of OD demand under baseline schedule  $s_b$  is known beforehand. However, the number of rail passengers for each OD is elastic with respect to passenger schedule delay. Once OD demand for a given schedule  $s_i$  is known (see subsections 4.2.4), PRA's operating revenue is calculated by multiplying travel demand on each OD by the corresponding rail fare, and then summing over all ODs.

#### 4.2.3 Operating cost

For a given  $s_i \in FPTS$ , PRA's operating cost is calculated by summing up the train movement cost and train stopping cost. Each cost is obtained by multiplying an appropriate cost factor (in \$/hr) by the total time trains spend in moving/stopping during the trips.

#### 4.2.4 Passenger schedule delay cost

Computing passenger schedule delay cost is based on passenger Preferred Departure Time (PDT) profiles. Each profile corresponds to a combination of a train station and a direction of travel. It describes the number of rail passengers preferring to depart from the station in the direction, in discrete time intervals over the course of a day given an  $s_i \in FPTS$ . Figure 2 illustrates a passenger PDT profile for a station-direction combination, with respect to an  $s_i$  which consists of three passenger trains. A traveler in a time interval chooses the nearest train to board, with the objective of minimizing the time difference between his/her preferred departure and the train departure, i.e., the traveler's schedule delay. In short-haul intercity travel, traveler schedule delay cost is important and even comparable to ticket fare (Kanafani 1983; Talebian and Zou 2015). Total passenger schedule delay costs,  $SDC_{s_i}^P$ , are obtained by summing over all travelers and applying appropriate cost values of schedule delay. Details about computing passenger schedule delay cost can be found in subsection 3.2.1 in Talebian and Zou (2015).



**Figure 2: Illustration of passenger PDT distribution and choice of trains to board (source: Talebian and Zou (2015))**

In the present paper, we further incorporate the fact that the number of travelers preferring to depart in each time interval  $m$  for each station-direction combination  $w$  is elastic with respect to schedule delay cost experienced by the travelers. The elasticity is  $e_{d/w}$ . Specifically, we assume that a baseline passenger train schedule,  $s_b$  and the associated passenger PDT profiles are known. In the baseline PDT profiles, the total number of travelers for  $w$  is the sum of estimated OD demand involved in  $w$ . For a time interval  $m \in T$ , the number of travelers,  $q_{s_b}^{w,m}$ , is calculated by distributing the total number of travelers, based on the ratio of the height of the time interval  $m$  over the sum of heights over all time intervals. With  $s_b$ , we compute schedule delay cost for each traveler,  $s_{s_b}^{w,m}, \forall m \in T, w \in W$ . Now given  $s_i \in FPTS$ , we re-calculate the new schedule delay cost for each traveler:  $s_{s_i}^{w,m}, \forall m \in T, w \in W$ . The number of travelers whose PDT is in  $m$  and for  $w$  is then obtained by

$$q_{s_i}^{w,m} = q_{s_b}^{w,m} \left( 1 - e_{d/w} \left( 1 - \frac{s_{s_i}^{w,m}}{s_{s_b}^{w,m}} \right) \right) \quad \forall w \in W, m \in T, s_i \in FPTS \quad (2)$$

Using  $s_{s_i}^{w,m}, q_{s_i}^{w,m}$ , and appropriate cost values for schedule delay, we compute  $SDC_{s_i}^P$  as discussed before.

#### 4.2.5 Passenger en-route delay cost

Passenger en-route delay occurs when: 1) a train stops at a siding; and 2) a train has longer than the minimum layover at an intermediate station. For a given  $s_i \in FPTS$ , the key to computing passenger en-route delay cost is to know the number of passengers onboard when an en-route delay occurs. Assuming that the PDT profile for each OD follows the PDT profile of the corresponding departing station-direction combination, we can easily compute the exact number of passengers on each train at any location on the line. Recall that travelers choose the train that minimizes their schedule delay cost. Whenever an en-route delay occurs, we multiply the number of passengers onboard by the length of the en-route delay and by the value of passenger travel time to obtain the total passenger delay cost associated with this en-route delay.

#### 4.3 Computing FRR's cost

FRR's cost  $C_{s_i}^F$ , which is conditional on passenger train schedule  $s_i$ , consists of four components: train departure delay cost ( $DDC_{s_i}^F$ ), en-route delay cost ( $EDC_{s_i}^F$ ), loss-of-demand cost ( $LDC_{s_i}^F$ ), and track maintenance cost ( $TMC_{s_i}^F$ ). Inclusion of  $DDC_{s_i}^F$  and  $EDC_{s_i}^F$  is self-explanatory.  $LDC_{s_i}^F$  occurs when FRR is unable to run as many freight trains as demanded, and the loss of demand results in a profit loss.

Because each train imposes some incremental cost for track maintenance on the line,  $TMC_{s_i}^F$  captures the track maintenance cost associated with running passenger trains under  $s_i$  as well as running freight trains given  $s_i$ . Similar to the computation of  $u_{s_i}^P$ , we measure each cost component against a baseline. The baseline here is defined as when there is pure freight traffic on the line. In sum, FRR's cost can be expressed as (subscript 0 indicates the baseline):

$$C_{s_i}^F = (DDC_{s_i}^F - DDC_0^F) + (EDC_{s_i}^F - EDC_0^F) + (LDC_{s_i}^F - LDC_0^F) + (TMC_{s_i}^F - TMC_0^F) \quad (3)$$

To compute each cost component, we need to know the freight train schedule given  $s_i$ . As already mentioned, to account for the access priority of passenger trains over freight trains in meets and overpasses, we insert freight trains between existing passenger trains in a way that minimizes the total cost to freight trains. More specifically, we consider that there is a predetermined number of desired freight trains running on the line, each having an earliest allowed departure and a latest allowed arrival times. FRR seeks a freight train schedule that minimizes the sum of  $DDC_{s_i}^F$ ,  $EDC_{s_i}^F$ , and  $LDC_{s_i}^F$ , constrained by the presence of the passenger train schedule  $s_i$ . Finding such a freight train schedule is based on our previous work (specifically, subsection 3.2.2 of Talebian and Zou (2015)). Note that the cost minimization does not consider  $TMC_{s_i}^F$ : the rationale is that  $DDC_{s_i}^F$ ,  $EDC_{s_i}^F$ , and  $LDC_{s_i}^F$  are tactical cost that FRR deals with on a daily basis; whereas  $TMC_{s_i}^F$ , like the action of track maintenance itself, is considered at a more strategic level.

Once the cost-minimum freight train schedule is obtained, we further compute  $TMC_{s_i}^F$ . We follow Lang et al. (2013) and Kennedy (1997) by considering  $TMC_{s_i}^F$  to consist of fixed and variable costs. The fixed cost does not vary with train schedules, and thus does not appear in  $C_{s_i}^F$ . The variable cost is a linear combination of passenger and freight train miles scheduled on the line.

## 5. Bargaining

In this section, we formally introduce the bargaining model. The bargaining consists of two steps. In the first step, PRA and FRR bargain over the payment from PRA to FRR for each possible schedule. With each possible schedule given a price tag (i.e., a mutually agreed-upon payment from PRA to FRR), bargaining in the second step seeks a mutually agreed-upon schedule. Based on our consultation with former Amtrak employees, the two-step process is consistent with today's Amtrak-freight railroad negotiation practice in which PRA and FRR make alternating proposals and determine train schedule and payment simultaneously. On the other hand, the two-step process makes a simplification that we associate each possible train schedule with a payment prior to schedule bargaining. The benefit of making such a simplification is a significant reduction of the train schedule-payment bundle space, which makes the simultaneous determination of payment and train schedule mathematically tractable. The payment bargaining, which associates each possible train schedule with a payment from PRA to FRR, is a fictitious game taking place in PRA and FRR's minds. It means that PRA and FRR strategize the steps in advance without actually going through the process of making alternating offers, and their bargaining power will determine the payment for each schedule.

We consider bargaining with both complete and incomplete information. In the complete information setting, PRA and FRR know precisely the utility/cost information of the other side. In the incomplete information setting, we conjecture that PRA does not possess full cost information about FRR, but only has a probabilistic belief. This is plausible in the US, where private Class I railroads hold their cost information as confidential (Lai et al. 2013; RSG 2012).

### 5.1 Complete information setting

#### 5.1.1 Payment bargaining

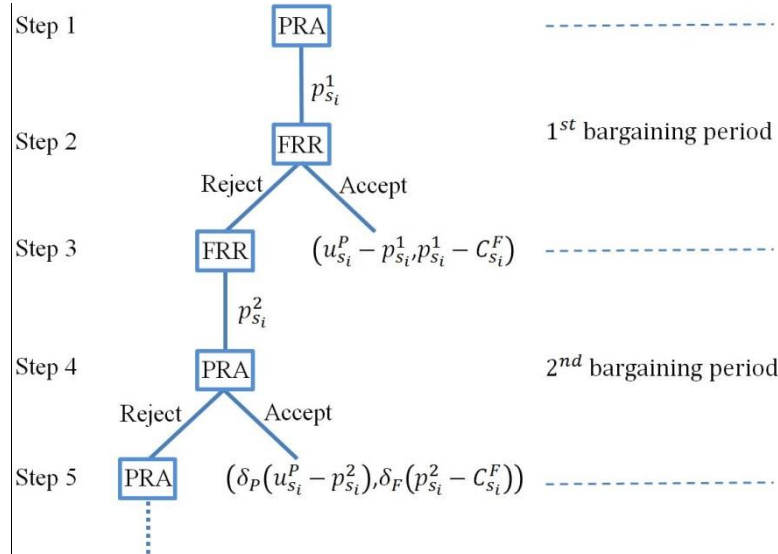
To model payment bargaining between PRA and FRR under complete information, we employ a Rubinstein-style infinite-horizon bargaining game (Rubinstein 1982). Without loss of generality, here we consider a payment bargaining initiated by PRA for a given schedule  $s_i$  (Figure 3). The infinite horizon means that the game has an infinite number of bargaining periods. Figure 3 shows the first



and second bargaining periods. Let  $p_{s_i}^t$  denote the proposed payment from PRA to FRR in the beginning of the  $t^{th}$  period. In odd periods, the proposal is made by PRA; in even periods, the proposal is made by FRR.

The bargaining starts with PRA proposing payment  $p_{s_i}^1$  (Step 1). FRR then decides whether to accept or reject the proposal (Step 2). If PRA accepts  $p_{s_i}^1$ , then the bargaining ends and the payoffs to PRA and FRR will be  $U_{P,s_i}^1 = u_{s_i}^P - p_{s_i}^1$  and  $U_{F,s_i}^1 = p_{s_i}^1 - C_{s_i}^F$ , where  $u_{s_i}^P$  and  $C_{s_i}^F$  are from pre-bargaining input preparation, and known to PRA and FRR. If FRR rejects  $p_{s_i}^1$ , then the bargaining proceeds to the second period and FRR proposes payment  $p_{s_i}^2$  (Step 3). We consider discounting of one's payoff as bargaining goes by, with  $\delta_P$  and  $\delta_F$  ( $0 < \delta_P, \delta_F < 1$ ) being the discount factors for PRA and FRR between two bargaining periods.  $\delta_P$  and  $\delta_F$  values are common knowledge. Introducing  $\delta_P$  and  $\delta_F$  captures the time-value of money, or opportunity costs to PRA and FRR due to disagreement (Gibbons 1992). The opportunity costs include loss of production, lost interest income, and expenses or fees paid to brokers, attorneys, or other agents (Kennan and Wilson 1993). Therefore, if PRA accepts FRR's proposal in Step 4, then the present-value payoffs to PRA and FRR will be  $U_{P,s_i}^2 = \delta_P(u_{s_i}^P - p_{s_i}^2)$  and  $U_{F,s_i}^2 = \delta_F(p_{s_i}^2 - C_{s_i}^F)$ . If PRA rejects FRR's proposal, then PRA makes another payment proposal in the third period. The process of making alternating payment proposals continues until one accepts the other's proposal.

Before delving into the solution of the bargaining, it is worth highlighting the bounds for  $p_{s_i}^t$ .  $p_{s_i}^t$  should be one such that both PRA and FRR receive non-negative payoffs, i.e.,  $u_{s_i}^P - p_{s_i}^t \geq 0$  and  $p_{s_i}^t - C_{s_i}^F \geq 0, \forall t, s_i$ . Otherwise, PRA (or FRR or both) will not have the incentive to be in the bargaining; or if already in the bargaining, it is for sure that the payment proposal will not be accepted. The non-negativity requirement yields  $C_{s_i}^F \leq p_{s_i}^t \leq u_{s_i}^P$ , which further implies that  $C_{s_i}^F \leq u_{s_i}^P$  must hold. If  $u_{s_i}^P < C_{s_i}^F$ , the bargaining will never achieve a mutual agreement.



**Figure 3: Structure of the payment bargaining under complete information: PRA initiates the payment bargaining**

With the above bounds conditions met, it is known that a bargaining game with infinite horizon and alternating proposals is *stationary* (Fudenberg and Tirole 1991). The stationarity property means that PRA always proposes the same payment when it comes to its turn to propose. Likewise for FRR. Consequently all subgames starting with PRA's proposal, including the whole game, will have the same subgame perfect Nash equilibrium (SPNE). Similarly, all subgames initiating with FRR's proposal will also have the same SPNE. It can be further shown that the infinite-horizon bargaining game has a unique SPNE (Fudenberg and Tirole 1991). We use  $p_{s_i}^{1*}$  and  $p_{s_i}^{2*}$  to denote the equilibrium payment

proposals by PRA and FRR. Intuitively, each side accepts a payment proposal from the other side only when the proposal would result in sufficiently high payoff. The criteria used by FRR and PRA to accept the other's proposal are:

- FRR accepts payment  $p_{s_i}^1$  proposed by PRA if and only if  $p_{s_i}^1 - C_{s_i}^F \geq \delta_F(p_{s_i}^{2*} - C_{s_i}^F)$ ,  $\forall s_i \in FTPS$ ;
- PRA accepts payment  $p_{s_i}^2$  proposed by FRR if and only if  $u_{s_i}^P - p_{s_i}^2 \geq \delta_P(u_{s_i}^P - p_{s_i}^{1*})$ ,  $\forall s_i \in FTPS$ .

The first criterion states that after PRA proposes  $p_{s_i}^1$ , FRR will anticipate that if she rejects, then PRA must accept her subsequent proposal  $p_{s_i}^{2*}$ , because otherwise the bargaining would repeat and discounting would reduce both sides' payoffs. Acceptance of the subsequent proposal  $p_{s_i}^{2*}$  would give FRR a SPNE payoff of  $\delta_F(p_{s_i}^{2*} - C_{s_i}^F)$ . Thus, as long as  $p_{s_i}^1 - C_{s_i}^F$  is no less than  $\delta_F(p_{s_i}^{2*} - C_{s_i}^F)$ , FRR will accept  $p_{s_i}^1$ . Similar reasoning can be made for the second criterion.

At the equilibrium, PRA's proposal will make FRR indifferent between accepting and rejecting; likewise for FRR's proposal. Consequently,  $p_{s_i}^{1*} - C_{s_i}^F = \delta_F(p_{s_i}^{2*} - C_{s_i}^F)$  and  $u_{s_i}^P - p_{s_i}^{2*} = \delta_P(u_{s_i}^P - p_{s_i}^{1*})$ . Solving these two equations lead to the equilibrium payments as:

$$p_{s_i}^{1*} = \frac{1}{1 - \delta_F \delta_P} (\delta_F(1 - \delta_P)u_{s_i}^P + (1 - \delta_F)C_{s_i}^F) \quad (4.1)$$

$$p_{s_i}^{2*} = \frac{1}{1 - \delta_F \delta_P} ((1 - \delta_P)u_{s_i}^P + \delta_P(1 - \delta_F)C_{s_i}^F) \quad (4.2)$$

Therefore, the bargaining will proceed as follows: at the outset PRA proposes  $p_{s_i}^{1*}$  to FRR with anticipation of FRR's payment proposal  $p_{s_i}^{2*}$  if FRR rejects  $p_{s_i}^{1*}$ . Facing  $p_{s_i}^{1*}$ , FRR decides to accept the payment proposal, and the bargaining ends.

As expected, the payment from PRA to FRR positively correlates with PRA's utility and FRR's cost. FRR will require a higher payment if a passenger train schedule imposes greater cost to FRR. On the other hand, PRA will be willing to pay more if the schedule gives PRA higher utility. Comparing (4.1) with (4.2) and recalling that  $C_{s_i}^F \leq u_{s_i}^P$  must hold if the bargaining proceeds, we can conveniently show that  $p_{s_i}^{1*} \leq p_{s_i}^{2*}$ . Because of the stationarity nature of the bargaining,  $p_{s_i}^{2*}$  would be the payment proposed by FRR (and accepted by PRA) if the bargaining is initiated by FRR. Then  $p_{s_i}^{1*} \leq p_{s_i}^{2*}$  suggests that the first-mover in the bargaining has an advantage: the equilibrium payment will be lower if PRA starts (thus greater payoff to PRA) and be higher if FRR starts (thus greater payoff to FRR). To make this explicit, the payoffs to PRA and FRR are:

$$U_{P,s_i}^{1*} = u_{s_i}^P - p_{s_i}^{1*} = \frac{1 - \delta_F}{1 - \delta_F \delta_P} (u_{s_i}^P - C_{s_i}^F) \quad (5.1)$$

$$U_{F,s_i}^{1*} = p_{s_i}^{1*} - C_{s_i}^F = \frac{\delta_F(1 - \delta_P)}{1 - \delta_F \delta_P} (u_{s_i}^P - C_{s_i}^F) \quad (5.2)$$

The SPNE payoffs to PRA and FRR in the subgame starting with FRR's proposal, which would also be the payoffs to PRA and FRR if the bargaining starts with FRR proposing a payment, are:

$$U_{P,s_i}^{2*} = u_{s_i}^P - p_{s_i}^{2*} = \frac{\delta_P(1 - \delta_F)}{1 - \delta_F \delta_P} (u_{s_i}^P - C_{s_i}^F) \quad (6.1)$$

$$U_{F,s_i}^{2*} = p_{s_i}^{2*} - C_{s_i}^F = \frac{1 - \delta_P}{1 - \delta_F \delta_P} (u_{s_i}^P - C_{s_i}^F) \quad (6.2)$$

Clearly,  $U_{P,s_i}^{1*} > U_{P,s_i}^{2*}$  and  $U_{F,s_i}^{2*} > U_{F,s_i}^{1*}$ .

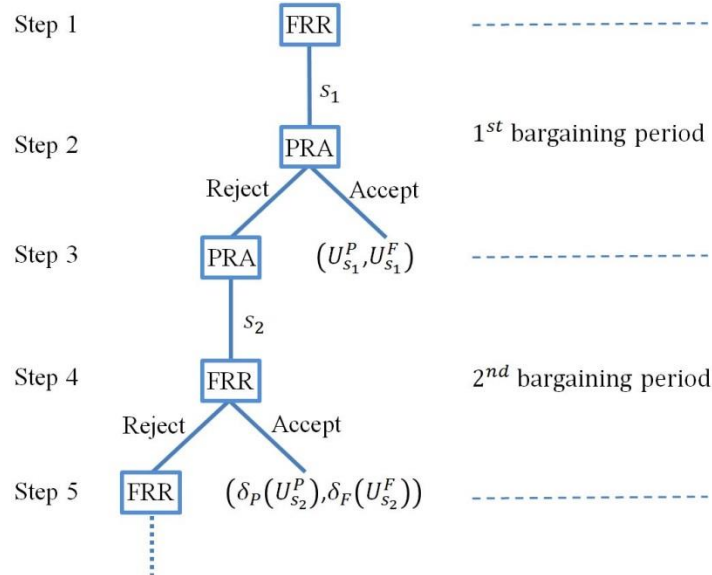
### 5.1.2 Schedule bargaining

Given a price tag (i.e., payment from PRA to FRR) to each possible train schedule, PRA and FRR enter the bargaining to determine a mutually agreed-upon schedule. Again, we consider a Rubinstein-style infinite-horizon bargaining game. Without loss of generality, we assume that the schedule bargaining is initiated by FRR, and that the payment associated with each schedule comes from the payment bargaining initiated by PRA.

Figure 4 shows the structure of the schedule bargaining game. FRR starts the bargaining by proposing a schedule  $s_1 \in \text{FPTS}$  in Step 1. Should PRA accept  $s_1$ , the bargaining ends, resulting in payoffs  $U_{s_1}^P = u_{s_1}^P - p_{s_1}^{1*}$  to PRA and  $U_{s_1}^F = p_{s_1}^{1*} - C_{s_1}^F$  to FRR, where  $p_{s_1}^{1*}$  is from (4.1). If PRA rejects  $s_1$ , then PRA makes the counter-proposal  $s_2$ . The process of making alternating schedule proposals continues until one accepts the other's proposal.

Similar to payment bargaining, the infinite-horizon schedule bargaining game is stationary, i.e., FRR always proposes the same schedule at the beginning of all odd bargaining periods and PRA always proposes the same schedule at the beginning of all even bargaining periods. Using similar reasoning as in subsection 5.1.1, the criteria for PRA and FRR to accept the other's proposal is:

- PRA accepts  $s_1$  proposed by FRR if and only if  $U_{s_1}^P \geq \delta_P U_{s_2}^P$ ;
- FRR accepts  $s_2$  proposed by PRA if and only if  $U_{s_2}^F \geq \delta_F U_{s_1}^F$ ;



**Figure 4: Structure of the schedule bargaining game under complete information: FRR initiates the schedule bargaining**

Note that schedules are discrete. Unlike the payment bargaining, it may not be possible to have a schedule pair  $(s_1^*, s_2^*)$  that exactly makes PRA and FRR indifferent between accepting and rejecting the other's proposal. On the other hand, for whatever schedule  $s_1$  FRR proposes to PRA at the beginning of the schedule bargaining, the payoffs to FRR and PRA can be immediately obtained from (5.1) and (5.2). Since  $\delta_F$  and  $\delta_P$  are constant, the strategy for FRR is to propose  $s_1^* = \text{argmax}_{s_1} (u_{s_1}^P - C_{s_1}^F)$  according to (5.2). Because such  $s_1^*$  achieves the maximum possible payoffs not only for FRR but also for PRA, PRA cannot reject it. By the same token, if it is PRA's turn to propose a schedule  $s_2$ , then PRA will also propose  $s_2^* = \text{argmax}_{s_2} (u_{s_2}^P - C_{s_2}^F)$  according to (5.1). FRR cannot reject  $s_2^*$  because  $s_2^*$  maximizes FRR's payoff as well. Therefore, there exists an equilibrium schedule pair  $(s_1^*, s_2^*)$  in the schedule bargaining game, where  $s_1^* = \text{argmax}_{s_1} (u_{s_1}^P - C_{s_1}^F)$  and  $s_2^* = \text{argmax}_{s_2} (u_{s_2}^P - C_{s_2}^F)$ .  $s_1^*$  and  $s_2^*$  can be identical or different schedules, as long as  $s_1^*$  and  $s_2^*$  yield the same maximum value for  $u_{s_i}^P - C_{s_i}^F$ , which can be viewed as the welfare of the bargaining system. The equilibrium is subgame perfect. We

formalize the above discussion as Proposition 1.

**Proposition 1:** *The schedule bargaining has at least one equilibrium schedule pair  $(s_1^*, s_2^*)$ .  $s_1^*$  and  $s_2^*$  can be identical or different, but they both maximize welfare of the bargaining system, i.e., the difference between PRA's utility and FRR's cost  $(u_{s_i}^P - C_{s_i}^F)$ . The equilibrium schedule is invariant to FRR's and PRA's discount factors.*

Two points are worth mentioning. First, the general bargaining game structure and finding are invariant to the objectives of PRA and FRR. This is because, as shown in the derivations above, the equilibrium schedule pair  $(s_1^*, s_2^*)$  should always be such that  $s_1^* = \arg\max_{s_1} (u_{s_1}^P - C_{s_1}^F)$  and  $s_2^* = \arg\max_{s_2} (u_{s_2}^P - C_{s_2}^F)$ . Having alternative objectives may affect the functional forms of  $u_{s_i}^P$  and  $C_{s_i}^F$  (for example,  $u_{s_i}^P$  would be replaced by an expression of social welfare if we consider PRA as a social welfare maximizer;  $-C_{s_i}^F$  would be substituted by FRA's profit if FRR is viewed as a profit maximizer). Consequently,  $\arg\max_{s_i} (u_{s_i}^P - C_{s_i}^F)$  may yield a different equilibrium schedule. But such a schedule still maximizes welfare of the bargaining system, now just under a new functional form for  $u_{s_i}^P - C_{s_i}^F$ . This is formalized as Corollary 1 below.

**Corollary 1:** *The general bargaining game structure and the finding that the equilibrium schedule pair  $(s_1^*, s_2^*)$  should always be such that  $s_1^* = \arg\max_{s_1} (u_{s_1}^P - C_{s_1}^F)$  and  $s_2^* = \arg\max_{s_2} (u_{s_2}^P - C_{s_2}^F)$  are invariant to the specific objectives for PRA and FRR.*

Second, note that the above discussion uses the fact that  $U_{P,s_i}^{1*}$  and  $U_{F,s_i}^{1*}$  are both proportional to  $u_{s_i}^P - C_{s_i}^F$ . Therefore, no matter who initiates the schedule bargaining, the set of equilibrium schedules and payoffs to PRA and FRR remain the same. This invariance is also true if it is FRR who initiates the payment bargaining. Proposition 2 below summarizes this observation.

**Proposition 2:** *Whether PRA or FRR initiates the schedule bargaining does not change the set of equilibrium schedules, nor the payoffs to PRA and FRR.*

## 5.2 Incomplete information setting

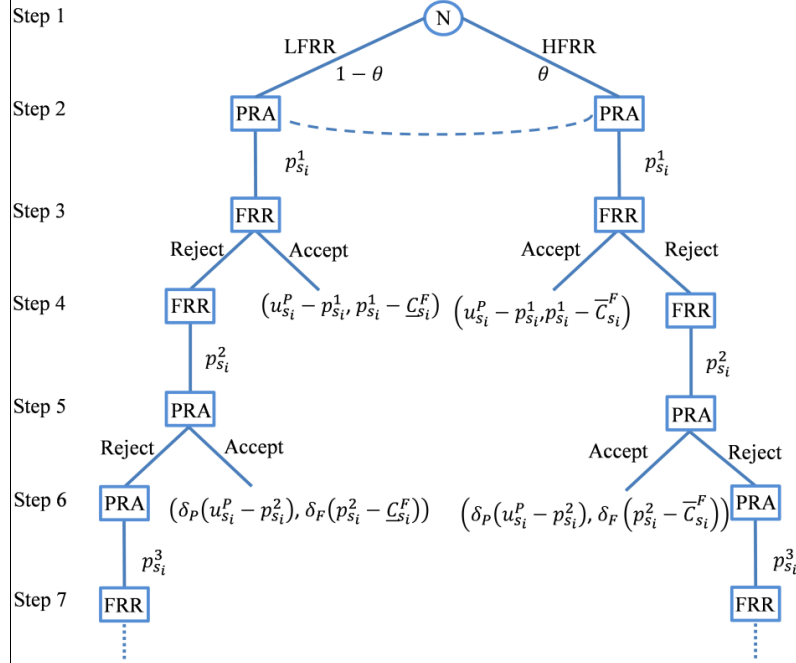
Different from the previous subsection, now we assume that PRA does not have complete information about FRR's cost for each schedule, i.e.,  $C_{s_i}^F, \forall s_i \in FPTS$ , but only knows FRR's cost probabilistically. More specifically, PRA believes that FRR is one of the two types: low-cost FRR (LFRR) and high-cost FRR (HFRR). PRA believes that there is  $\theta$  ( $0 < \theta < 1$ ) probability for FRR to be HFRR and  $1 - \theta$  probability for FRR to be LFRR. PRA's such belief is common knowledge. The costs associated with  $s_i$  to LFRR and HFRR are  $\underline{C}_{s_i}^F$  and  $\bar{C}_{s_i}^F$  ( $0 \leq \underline{C}_{s_i}^F < \bar{C}_{s_i}^F \leq u_{s_i}^P$ ), which is also known to PRA and FRR. As in the complete information case, we first consider an infinite-horizon payment bargaining for each  $s_i \in FPTS$ , which is again a fictitious play. Given a price tag (i.e., mutually agreed-upon payment) to each schedule, PRA and FRR then bargain over schedule.

### 5.2.1 Payment bargaining

Now that PRA has incomplete information about FRR's type, it is likely that who initiates the payment bargaining will result in even more different equilibrium payments than in the complete information setting. Thus we investigate in sequence payment bargaining initiated by PRA and FRR.

#### Case 1: PRA proposes the payment first

Figure 5 illustrates the structure of the payment bargaining initiated by PRA. At Step 1, Nature moves and FRR realizes its type. The dashed line at Step 2 indicates that the two decision nodes for PRA (on what payment to propose) belong to the same information set, i.e., PRA at Step 2 does not know which one of the two nodes is reached. PRA proposes  $p_{s_i}^1$  to FRR. If FRR accepts the proposal, then the payment bargaining ends. Otherwise, FRR makes a counter-proposal. PRA and FRR keep making alternating proposals until one accepts the other's proposal.



**Figure 5: Structure of the payment bargaining under incomplete information: PRA initiates the payment bargaining**

Before delving into the solution of the payment bargaining game, we introduce the following lemma which shows that, when it comes to FRR to propose a payment, FRR's strategy is not to reveal its type.

**Lemma 1:** *When it comes to FRR to propose a payment, the payment will be invariant to FRR's type. No matter what type FRR is, FRR will pretend to be HFRR. In other words, the proposed payment will be one that would be proposed by HFRR in a complete information setting.*

**Proof:** We prove by contradiction. Suppose that FRR would propose different payments according to its type. On seeing the proposed payment, PRA would discover FRR's type. Henceforth, the bargaining would become a Rubinstein-style bargaining (i.e., with complete information). Knowing FRR's type, the equilibrium payment proposed by PRA would be  $\frac{1}{1-\delta_F\delta_P}(\delta_F(1-\delta_P)u_{s_i}^P + (1-\delta_F)\bar{C}_{s_i}^F)$  and  $\frac{1}{1-\delta_F\delta_P}(\delta_F(1-\delta_P)u_{s_i}^P + (1-\delta_F)\underline{C}_{s_i}^F)$  for HFRR and LFRR respectively, following (4.1). Because  $\bar{C}_{s_i}^F > \underline{C}_{s_i}^F$ , FRR would obtain a lower payoff if it were LFRR than were HFRR. However, LFRR can obtain a higher payoff by pretending to be HFRR, and will do so. ■

With Lemma 1, we conjecture two Bayesian equilibria for the payment bargaining. In the first, pooling equilibrium, PRA proposes a payment high enough such that FRR accepts it regardless of type. In the second, separating equilibrium, PRA proposes a payment that only LFRR accepts it. In what follows, we first derive the payment for each equilibrium and then develop the conditions for PRA to propose a payment that corresponds to one of the equilibria.

**Equilibrium 1 (pooling equilibrium):** this occurs when PRA is highly confident that FRR is HFRR. Consequently PRA proposes a payment  $p_{s_i}^1$  (see Step 2 in Figure 5) that is high enough for HFRR to accept. If FRR is indeed HFRR (i.e., the right branch in Figure 5), by accepting  $p_{s_i}^1$  HFRR will immediately achieve payoff  $p_{s_i}^1 - \bar{C}_{s_i}^F$ . If HFRR rejected  $p_{s_i}^1$ , PRA would renew the belief and know for sure that FRR is HFRR. The subsequent bargaining would become a Rubinstein bargaining, for which we already know the equilibrium payment proposed by HFRR would be

$$p_H = \frac{1}{1 - \delta_F \delta_P} \left( (1 - \delta_P) u_{s_i}^P + \delta_P (1 - \delta_F) \bar{C}_{s_i}^F \right) \quad (7)$$

which follows (4.2). Therefore, in Step 2 HFRR will accept  $p_{s_i}^1$  if and only if  $p_{s_i}^1 - \bar{C}_{s_i}^F \geq \delta_F (p_H - \bar{C}_{s_i}^F)$ . At equilibrium, PRA proposes  $p_{s_i}^{*,H}$  such that HFRR is indifferent between accepting and rejecting:  $p_{s_i}^{*,H} - \bar{C}_{s_i}^F = \delta_F (p_H - \bar{C}_{s_i}^F)$ . Substituting the expression for  $p_H$  into this indifference equation yields the equilibrium payment  $p_{s_i}^{*,H}$ :

$$p_{s_i}^{*,H} = \frac{1}{1 - \delta_F \delta_P} \left( \delta_F (1 - \delta_P) u_{s_i}^P + (1 - \delta_F) \bar{C}_{s_i}^F \right) \quad (8)$$

Given  $p_{s_i}^{*,H}$ , PRA's payoff will be  $u_{s_i}^P - p_{s_i}^{*,H}$ .

If FRR is LFRR, FRR will also accept the payment proposal  $p_{s_i}^{*,H}$ . To see this, we simply compare LFRR's payoff between accepting and rejecting the proposal. By accepting  $p_{s_i}^{*,H}$ , LFRR achieves payoff  $p_{s_i}^{*,H} - \underline{C}_{s_i}^F$ ; if LFRR rejected  $p_{s_i}^{*,H}$ , then by Lemma 1 LFRR would propose the same payment  $p_H$  in Step 4. In the subgame starting from Step 4, LFRR's payoff would be  $\delta_F (p_H - \underline{C}_{s_i}^F)$ . But  $p_{s_i}^{*,H} - \underline{C}_{s_i}^F > \delta_F (p_H - \underline{C}_{s_i}^F)$ , which can be shown by substituting (7) and (8) for  $p_H$  and  $p_{s_i}^{*,H}$ :  $p_{s_i}^{*,H} - \underline{C}_{s_i}^F - \delta_F (p_H - \underline{C}_{s_i}^F) = \frac{1}{1 - \delta_F \delta_P} \left( (1 - \delta_P) u_{s_i}^P + \delta_P (1 - \delta_F) \bar{C}_{s_i}^F \right) - \underline{C}_{s_i}^F - \delta_F \left( \frac{1}{1 - \delta_F \delta_P} \left( (1 - \delta_P) u_{s_i}^P + \delta_P (1 - \delta_F) \bar{C}_{s_i}^F \right) - \underline{C}_{s_i}^F \right) = (1 - \delta_F) \left( \frac{1 - \delta_P}{1 - \delta_F \delta_P} (u_{s_i}^P - \underline{C}_{s_i}^F) + \frac{\delta_P (1 - \delta_F)}{1 - \delta_F \delta_P} (\bar{C}_{s_i}^F - \underline{C}_{s_i}^F) \right) > 0$  because  $0 \leq \underline{C}_{s_i}^F < \bar{C}_{s_i}^F \leq u_{s_i}^P$ . Therefore, LFRR will accept  $p_{s_i}^{*,H}$ . PRA's payoff remains  $u_{s_i}^P - p_{s_i}^{*,H}$ .

In sum, the expected payoff for PRA under the pooling equilibrium is  $\theta (u_{s_i}^P - p_{s_i}^{*,H}) + (1 - \theta) (u_{s_i}^P - p_{s_i}^{*,H}) = u_{s_i}^P - p_{s_i}^{*,H}$ , which is invariant of PRA's belief in  $\theta$ .

**Equilibrium 2 (separating equilibrium):** this occurs when PRA highly believes that FRR is LFRR, and proposes a lower payment  $p_{s_i}^1$  than  $p_{s_i}^{*,H}$  in Step 2 such that only LFRR would accept it. If FRR is indeed LFRR (the left branch in Figure 5) and accepts  $p_{s_i}^1$ , then LFRR will immediately achieve payoff  $p_{s_i}^1 - \underline{C}_{s_i}^F$ . If LFRR rejected  $p_{s_i}^1$ , LFRR would propose a payment in Step 4. By Lemma 1 LFRR would pretend to be HFRR. Consequently the equilibrium payment proposed by LFRR in Step 4 would be the same as in the pooling equilibrium:  $p_H = \frac{1}{1 - \delta_F \delta_P} \left( (1 - \delta_P) u_{s_i}^P + \delta_P (1 - \delta_F) \bar{C}_{s_i}^F \right)$ . At equilibrium, PRA proposes  $p_{s_i}^{*,L}$  such that LFRR is indifferent between accepting and rejecting in Step 2:  $p_{s_i}^{*,L} - \underline{C}_{s_i}^F = \delta_F (p_H - \underline{C}_{s_i}^F)$ , which yields

$$p_{s_i}^{*,L} = \frac{1}{1 - \delta_F \delta_P} \left( \delta_F (1 - \delta_P) u_{s_i}^P + \delta_F \delta_P (1 - \delta_F) \bar{C}_{s_i}^F \right) + (1 - \delta_F) \underline{C}_{s_i}^F \quad (9)$$

Because  $\bar{C}_{s_i}^F > \underline{C}_{s_i}^F$ , it can be easily seen that  $p_{s_i}^{*,L} < p_{s_i}^{*,H}$ .

Given  $p_{s_i}^{*,L}$ , PRA's payoff will be  $u_{s_i}^P - p_{s_i}^{*,L}$ .

If FRR is HFRR, then HFRR will not accept  $p_{s_i}^{*,L}$ . This is because  $\delta_F (p_H - \bar{C}_{s_i}^F) > p_{s_i}^{*,L} - \bar{C}_{s_i}^F$ , which can be verified by incorporating (7) and (9) into the inequality. PRA's payoff, which will be realized in Step 6, will be  $\delta_P (u_{s_i}^P - p_H)$ .

In sum, the expected payoff for PRA under the separating equilibrium is  $(1 - \theta) (u_{s_i}^P - p_{s_i}^{*,L}) + \theta \delta_P (u_{s_i}^P - p_H)$ .

Which of the two equilibria will be realized (i.e., whether PRA proposes  $p_{s_i}^{*,H}$  or  $p_{s_i}^{*,L}$ ) depends on the extent to which PRA believes FRR is HFRR, i.e., the value of  $\theta$ . There exists a threshold  $\hat{\theta}_{s_i}$  such that if

PRA's belief for FRR to be HFRR is greater than  $\hat{\theta}_{s_i}$ , then PRA will propose  $p_{s_i}^{*,H}$ ; otherwise, PRA will propose  $p_{s_i}^{*,L}$ . The threshold is obtained by equating PRA's expected payoff in the two equilibria. The resulting equilibrium payment proposed by PRA in Step 2 is

$$p_{s_i|PRA \text{ initiates}}^* = \begin{cases} p_{s_i}^{*,H} & \text{if } \theta \geq \hat{\theta}_{s_i} \\ p_{s_i}^{*,L} & \text{if } \theta < \hat{\theta}_{s_i} \end{cases} \quad (10)$$

where  $\hat{\theta}_{s_i} = \frac{(\bar{C}_{s_i}^F - \underline{C}_{s_i}^F)(1 - \delta_F \delta_P)}{(1 - \delta_P^2)u_{s_i}^P + (\delta_P^2 - \delta_F \delta_P)\bar{C}_{s_i}^F - (1 - \delta_F \delta_P)\underline{C}_{s_i}^F}$ , which is positive on both the numerator and denominator. For the numerator, positivity comes from  $\bar{C}_{s_i}^F > \underline{C}_{s_i}^F$ . For the denominator, it can be rewritten as  $(1 - \delta_P^2)(u_{s_i}^P - \bar{C}_{s_i}^F) + (1 - \delta_F \delta_P)(\bar{C}_{s_i}^F - \underline{C}_{s_i}^F)$ , which is also positive as  $0 \leq \underline{C}_{s_i}^F < \bar{C}_{s_i}^F \leq u_{s_i}^P$ .

### Case 2: FRR proposes the payment first

This payment bargaining also starts with the Nature's move which realizes FRR's type. FRR then proposes payment  $p_{s_i}^1$ . Should PRA accept the proposal, PRA will immediately realize payoff  $u_{s_i}^P - p_{s_i}^1$ . Otherwise, the bargaining proceeds to PRA proposing a payment. Because FRR adopts an unrevealing strategy (Lemma 1), PRA will not know FRR's type when proposing. The subsequent bargaining will be the same as Case 1.

If  $\theta \geq \hat{\theta}_{s_i}$  and PRA rejects FRR's proposal, then PRA will realize an expected payoff of  $\delta_P(u_{s_i}^P - p_{s_i}^{*,H})$ . The equilibrium payment proposed by FRR in the beginning should make PRA indifferent between accepting and rejecting:  $u_{s_i}^P - p_{s_i|FRR \text{ initiates}}^* = \delta_P(u_{s_i}^P - p_{s_i}^{*,H})$ , which yields  $p_{s_i|FRR \text{ initiates}}^* = (1 - \delta_P)u_{s_i}^P + \delta_P p_{s_i}^{*,H}$ .

If  $\theta < \hat{\theta}_{s_i}$  and PRA rejects FRR's proposal, then PRA will realize an expected payoff of  $(1 - \theta)\delta_P(u_{s_i}^P - p_{s_i}^{*,L}) + \theta\delta_P^2(u_{s_i}^P - p_H)$ . The equilibrium payment proposed by FRR in the beginning should make PRA indifferent between accepting and rejecting:  $u_{s_i}^P - p_{s_i|FRR \text{ initiates}}^* = (1 - \theta)\delta_P(u_{s_i}^P - p_{s_i}^{*,L}) + \theta\delta_P^2(u_{s_i}^P - p_H)$ , which yields  $p_{s_i|FRR \text{ initiates}}^* = (1 - \delta_P)(1 + \theta\delta_P)u_{s_i}^P + (1 - \theta)\delta_P p_{s_i}^{*,L} + \theta\delta_P^2 p_H$ .

To summarize,

$$p_{s_i|FRR \text{ initiates}}^* = \begin{cases} (1 - \delta_P)u_{s_i}^P + \delta_P p_{s_i}^{*,H} & \text{if } \theta \geq \hat{\theta}_{s_i} \\ (1 - \delta_P)(1 + \theta\delta_P)u_{s_i}^P + (1 - \theta)\delta_P p_{s_i}^{*,L} + \theta\delta_P^2 p_H & \text{if } \theta < \hat{\theta}_{s_i} \end{cases} \quad (11)$$

Substituting  $p_{s_i}^{*,H}$  and  $p_{s_i}^{*,L}$  by (8) and (9), we obtain

$$p_{s_i|FRR \text{ initiates}}^* = \begin{cases} \frac{1}{1 - \delta_F \delta_P} \left( (1 - \delta_P)u_{s_i}^P + \delta_P(1 - \delta_F)\bar{C}_{s_i}^F \right) & \text{if } \theta \geq \hat{\theta}_{s_i} \\ Au_{s_i}^P + B\bar{C}_{s_i}^F + \delta_P(1 - \delta_F)(1 - \theta)\underline{C}_{s_i}^F & \text{if } \theta < \hat{\theta}_{s_i} \end{cases} \quad (12)$$

where  $A = \frac{1 - \delta_P}{1 - \delta_F \delta_P} (1 + \theta\delta_P(1 + \delta_P)(1 - \delta_F))$ ;  $B = \frac{\delta_P^2(1 - \delta_F)}{1 - \delta_F \delta_P} ((1 - \theta)\delta_F + \theta\delta_P)$ .

### 5.2.2 Schedule bargaining

In this subsection we present schedule bargaining initiated by PRA (Figure 6). Schedule bargaining initiated by FRR can be derived in a similar fashion. Without loss of generality, we assume that payment bargaining begins with FRR. Thus the equilibrium payment follows (12). In the schedule bargaining, PRA realizes an expected payoff  $U_{s_i}^P$  for each schedule  $s_i \in FPTS$ , as shown in (13). FRR's payoff depends on its type. Let  $\underline{U}_{s_i}^F$  and  $\bar{U}_{s_i}^F$  be FRR's payoff when FRR is LFRR and HFRR. Their expressions are given by (14)-(15).

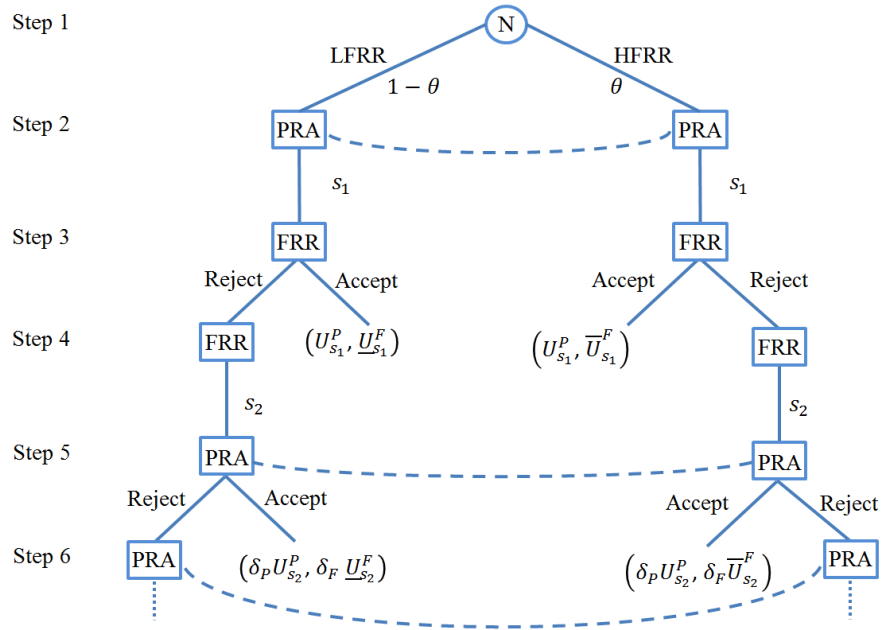
$$U_{s_i}^P = u_{s_i}^P - p_{s_i|FRR \text{ initiates}}^* = \begin{cases} \frac{\delta_P(1-\delta_F)}{1-\delta_F\delta_P} (u_{s_i}^P - \bar{C}_{s_i}^F) & \text{if } \theta \geq \hat{\theta}_{s_i} \\ (1-A)u_{s_i}^P + B\bar{C}_{s_i}^F + \delta_P(1-\delta_F)(1-\theta)\underline{C}_{s_i}^F & \text{if } \theta < \hat{\theta}_{s_i} \end{cases} \quad (13)$$

$$\underline{U}_{s_i}^F = p_{s_i|FRR \text{ initiates}}^* - \underline{C}_{s_i}^F = \begin{cases} \frac{1}{1-\delta_F\delta_P} ((1-\delta_P)u_{s_i}^P + \delta_P(1-\delta_F)\bar{C}_{s_i}^F) - \underline{C}_{s_i}^F & \text{if } \theta \geq \hat{\theta}_{s_i} \\ Au_{s_i}^P + B\bar{C}_{s_i}^F + (\delta_P(1-\delta_F)(1-\theta) - 1)\underline{C}_{s_i}^F & \text{if } \theta < \hat{\theta}_{s_i} \end{cases} \quad (14)$$

$$\bar{U}_{s_i}^F = p_{s_i|FRR \text{ initiates}}^* - \bar{C}_{s_i}^F = \begin{cases} \frac{1-\delta_P}{1-\delta_F\delta_P} (u_{s_i}^P - \bar{C}_{s_i}^F) & \text{if } \theta \geq \hat{\theta}_{s_i} \\ Au_{s_i}^P + (B-1)\bar{C}_{s_i}^F + \delta_P(1-\delta_F)(1-\theta)\underline{C}_{s_i}^F & \text{if } \theta < \hat{\theta}_{s_i} \end{cases} \quad (15)$$

In this paper we construct a pooling equilibrium for schedule bargaining with incomplete information: regardless of FRR's type, FRR proposes the same schedule to PRA and PRA also proposes the same schedule to FRR. Under this construct, no information is revealed while FRR and PRA make alternating proposals. Thus in Steps 2, 5, and 6 in Figure 6, PRA's belief on FRR's type remains the same. The bargaining is stationary. Suppose that there exists an equilibrium schedule pair  $(s_1^*, s_2^*)$ . The criteria for PRA and FRR to accept the other's proposal is as follows:

- PRA accepts any schedule  $s_2$  proposed by FRR if and only if  $U_{s_2}^P \geq \delta_P U_{s_1}^P$ ;
- LFRR (HFRR) accepts any schedule  $s_1$  proposed by PRA if and only if  $\underline{U}_{s_1}^F \geq \delta_F \underline{U}_{s_2}^F$  ( $\bar{U}_{s_1}^F \geq \delta_F \bar{U}_{s_2}^F$ ).



**Figure 6: Structure of the schedule bargaining under incomplete information: PRA initiates the schedule bargaining**

To obtain the equilibrium schedule pair(s)  $(s_1^*, s_2^*)$ , we examine two related types of schedule bargaining under complete information. The first type is between PRA and LFRR; the second type between PRA and HFRR. Let  $\underline{ES}$  and  $\bar{ES}$  denote the sets of equilibrium schedule pairs in these two types of bargaining. Obviously, any schedule pair in  $\underline{ES} \cap \bar{ES}$  will be an equilibrium schedule of the original bargaining in Figure 6.

Recall that in the schedule bargaining under complete information, the equilibrium schedule is the one that maximizes both PRA's and FRR's payoffs, and also the welfare of the bargaining system. For schedule bargaining under incomplete information, this finding will only apply to one case that FRR is



HFRR and  $\theta \geq \hat{\theta}_{s_i}$ . This can be seen in the payoff expressions (13) and (15) for  $U_{s_i}^P$  and  $\bar{U}_{s_i}^F$  when  $\theta \geq \hat{\theta}_{s_i}$ . Both expressions are some constant multiplied by system welfare  $u_{s_i}^P - \bar{C}_{s_i}^F$ , as in (5.1)-(5.2) and (6.1)-(6.2). Other than this case, a system welfare-maximizing schedule is generally not an equilibrium schedule. In fact, in the remaining cases payoffs for PRA and FRR involve both  $\bar{C}_{s_i}^F$  and  $\underline{C}_{s_i}^F$ . Below we propose a sorting/elimination-based method to identify  $\bar{ES}$  and  $\underline{ES}$  separately. Then we put forward two conditions for the existence of the pooling equilibrium.

We focus our description on the sorting/elimination-based method to obtain  $\underline{ES}$ . Exactly the same method applies to finding  $\bar{ES}$ . We first sort schedules  $s_i \in FPTS$  such that  $\underline{U}_{s_i}^F$  values are in descending order. Let  $\underline{U}_{s_o}^F$  denote the vector containing the sorted payoffs;  $\underline{U}_{s_o}^F(n)$  denotes the  $n^{th}$  element in  $\underline{U}_{s_o}^F$ ;  $U_{s_o}^P$  the vector of payoffs to PRA such that the  $n^{th}$  element  $U_{s_o}^P(n)$  corresponds to the  $n^{th}$  sorted schedule. Then we identify elements  $j = 2, 3, \dots, \dim(\underline{U}_{s_o}^F)$  for which  $U_{s_o}^P(j) < U_{s_o}^P(j-1)$ . Note that by sorting,  $\underline{U}_{s_o}^F(j) \leq \underline{U}_{s_o}^F(j-1)$ . If  $U_{s_o}^P(j) < U_{s_o}^P(j-1)$ , the  $j$ th sorted schedule is Pareto dominated by the  $(j-1)$ th sorted schedule. We therefore remove the  $j$ th schedule and the corresponding payoffs from  $\underline{U}_{s_o}^F$  and  $U_{s_o}^P$ . We repeat this until no more schedule can be removed. The resulting payoff vectors are  $\underline{SU}_{s_o}^F$  and  $SU_{s_o}^P$ . Due to eliminations, elements of  $SU_{s_o}^P$  are now in (non-strict) ascending order.

Let us use a simple example in Figure 7 to illustrate the sorting/elimination process. Consider a set of six schedules, each associated with payoffs to LFRR and PRA. First we sort schedules in descending order, based on LFRR's payoff. We then remove schedule  $s_1$  (Pareto dominated by  $s_2$ ), and  $s_3$  and  $s_5$  (both Pareto dominated by  $s_4$ ). None of the remaining schedules  $s_2$ ,  $s_4$ , and  $s_6$  Pareto dominates the others.

Original			After sorting			After elimination		
Schedule	$\underline{U}_{s_i}^F$	$U_{s_i}^P$	Schedule	$\underline{U}_{s_o}^F$	$U_{s_o}^P$	Schedule	$\underline{SU}_{s_o}^F$	$SU_{s_o}^P$
$s_1$	11	7.5	$s_6$	13	7	$s_6$	13	7
$s_2$	12.5	8	$s_2$	12.5	8	$s_2$	12.5	8
$s_3$	8.5	6	$s_1$	11	7.5	$s_4$	10	13
$s_4$	10	13	$s_4$	10	13			
$s_5$	4.5	9	$s_3$	8.5	6			
$s_6$	13	7	$s_5$	4.5	9			

Figure 7: Illustration of the schedule sorting/elimination process (those in green are removed in the process)

Based on the accepting criteria mentioned above, any two elements  $(j, k)$  that satisfy

$$\underline{SU}_{s_o}^F(j) \geq \delta_F \underline{SU}_{s_o}^F(k) \quad (16.1)$$

$$\underline{SU}_{s_o}^P(k) \geq \delta_P \underline{SU}_{s_o}^P(j) \quad (16.2)$$

will give a pair of equilibrium schedules, thus an element in  $\underline{ES}$ . The schedule corresponding to  $j$  will be one proposed by PRA; the schedule corresponding to  $k$  will be one proposed by LFRR. It can be easily seen that as long as  $\underline{SU}_{s_o}^F$  is not an empty vector, such a pair always exists. Indeed, any pair of schedules for which the corresponding payoffs to FRR are in  $\underline{SU}_{s_o}^F$  will form an equilibrium schedule pair. This is because  $\forall j, k = 1, \dots, \dim(\underline{SU}_{s_o}^F)$  (assuming without loss of generality that  $j < k$ ),  $\underline{SU}_{s_o}^F(j) \geq \underline{SU}_{s_o}^F(k) > \delta_F \underline{SU}_{s_o}^F(k)$  and  $\underline{SU}_{s_o}^P(k) \geq \underline{SU}_{s_o}^P(j) > \delta_P \underline{SU}_{s_o}^P(j)$ . In particular, any schedule involved in  $\underline{SU}_{s_o}^F$  will form an equilibrium schedule pair with itself. Proposition 3 below shows that  $\underline{SU}_{s_o}^F$  cannot be empty. Then it follows from the above discussions that  $\underline{ES}$  is non-empty.

**Proposition 3:** The schedule bargaining between PRA and LFRR (or HFRR) always has at least one equilibrium pair.

**Proof:** During the elimination process, the first element in  $\underline{U}_{s_o}^F$  remains intact. Therefore,  $\underline{U}_{s_o}^F(1)$  will always be in  $\underline{SU}_{s_o}^F$ .  $\underline{SU}_{s_o}^F$  is non-empty. Following the above discussion,  $\underline{ES}$  is non-empty. Thus an equilibrium schedule pair exists. ■

Although  $\overline{ES}$  and  $\underline{ES}$  are never empty,  $\overline{ES} \cap \underline{ES}$  may be empty. Proposition 4 provides two conditions under which  $\overline{ES} \cap \underline{ES} \neq \emptyset$ , i.e., a pooling equilibrium exists.

**Proposition 4:** *If either of the following two conditions is met, then schedule bargaining under incomplete information has a pooling equilibrium:*

- a) *There exists a schedule  $s_j \in FPTS$  which Pareto dominates all other schedules in the bargaining between PRA and LFRR, and the same schedule  $s_j$  Pareto dominates all other schedules in the bargaining between PRA and HFRR;*
- b) *For any two schedules  $s_m, s_k \in FPTS | s_m \neq s_k$ , if  $\underline{U}_{s_m}^F > \underline{U}_{s_k}^F$  then  $\overline{U}_{s_m}^F > \overline{U}_{s_k}^F$  also holds.*

**Proof:** Proofs are intuitive:

- a) The Pareto dominance suggests that  $\underline{U}_{s_j}^F > \underline{U}_{s_i}^F, \overline{U}_{s_j}^F > \overline{U}_{s_i}^F, U_{s_j}^P > U_{s_i}^P, \forall s_i \in FPTS | s_i \neq s_j$ . As a result, the sorting/elimination process will have  $s_j$  as the only element in  $\overline{ES}$  and  $\underline{ES}$ . The bargaining between PRA and LFRR has one and only one equilibrium schedule pair  $(s_j, s_j)$ . The same for bargaining between PRA and HFRR. Thus  $\overline{ES} \cap \underline{ES} = (s_j, s_j)$ .
- b) Under this condition, all  $s_i \in FPTS$  will be ordered in the same way in  $\underline{U}_{s_o}^F$  and  $\overline{U}_{s_o}^F$ , which means  $\underline{U}_{s_o}^F(1)$  and  $\overline{U}_{s_o}^F(1)$  are associated with the same schedule. Recall that  $\underline{U}_{s_o}^F(1)$  and  $\overline{U}_{s_o}^F(1)$  will remain intact after the elimination process.  $\underline{U}_{s_o}^F(1)$  will be  $\underline{SU}_{s_o}^F(1)$  and  $\overline{U}_{s_o}^F(1)$  will be  $\overline{SU}_{s_o}^F(1)$ . The schedule corresponding to  $\underline{SU}_{s_o}^F(1)$  will form an equilibrium schedule pair with itself for bargaining between PRA and LFRR. Because the same schedule is associated with  $\overline{SU}_{s_o}^F(1)$ , this schedule and itself also form an equilibrium schedule pair for bargaining between PRA and HFRR. Thus this schedule and itself give an equilibrium schedule pair under the pooling equilibrium. ■

When  $\overline{ES} \cap \underline{ES}$  is empty, a separating equilibrium may exist: LFRR and HFRR propose different schedules to PRA. Investigation of the separating equilibrium, however, is left for future research.

## 6. Numerical analysis

This section presents numerical experiments of the bargaining model. We first describe the problem to be investigated and model parameters. Then, results in both complete and incomplete information settings are presented and discussed. The bargaining model is coded in MATLAB 2013b, while *FPTS* and freight train schedules are generated using IBM ILOG CPLEX Optimizer V12.6. The numerical experiments are executed on a desktop computer with Windows 8.1 OS, Intel Core i7 3.4 GHz processor and 12 GB memory.

### 6.1 Setup

We consider a single-track, shared-use rail line presented in Talebian and Zou (2015). The line consists of 11 blocks, five of which are equally distanced, double-track blocks. Each double-track block is two miles long. The remaining six single-track blocks are each 18 miles long, except for the first and last blocks which are 19 miles. This line has two OD pairs, one for each direction (i.e., from one end to the other end). We consider 1-5 passenger trains running in each direction in a day. A maximum demand of 15 freight trains is desired in each direction on the line. Passenger and freight train speeds are 120 and 60 mph respectively. The planning horizon in a day is 5 am – 9:30 pm. Time is discretized into 5-minute intervals. For a given number of passenger trains, the baseline passenger train schedule  $s_b$  is the same as the optimal schedule solved in Talebian and Zou (2015).

There exist many feasible schedules for passenger trains. We restrict our attention to passenger train schedules that are not too far from the optimal schedule. Specifically, for any schedule  $s_i \in FPTS$  of a given passenger train, its departure is at most 30 minutes earlier than the train's departure time in  $s_b$ ; its arrival is at most 30 minutes later than the train's arrival time in  $s_b$ .

Recall that we allow passenger trains to stop en route. In generating *FPTS*, we introduce a maximum en-route delay (MED) time that each train can have during a trip. A large MED can affect PRA to FRR in at least three possible ways. First, a large MED grants passenger trains greater operational flexibility,

which may reduce passenger schedule delay, leading to higher passenger demand and revenue. Second, a large MED may result in increased passenger en-route delay cost and passenger train operating cost, both lowering PRA's utility. Third, by allowing passenger trains to stop more en route, a large MED may permit more freight trains to run on the line, thus reducing the loss-of-demand cost to FRR. To numerically investigate the net effect, many of the results in the numerical analysis are plotted against MED.

The *Populate* feature of CPLEX is employed to enumerate feasible passenger train schedules generated from the hypergraph-based scheduling model in Appendix A. For a given MED value, we randomly collect up to 10,000 feasible passenger train schedules (note that when the number of passenger trains and MED are both small, there can be less than 10,000 feasible passenger train schedules). Using the Matlab Parallel Computing Toolbox, the average time required to generate a feasible passenger train schedule and the associated freight train schedule, and to compute the associated PRA's utility and FRR's cost is about 0.15 second.

## 6.2 Model parameters

Model parameter values are mostly drawn from the literature. On the passenger side, we construct PDT profiles following the one that is empirically developed by Cascetta and Coppola (2012). We consider passenger values of schedule delay and en-route delay to be \$52/hr and \$67/hr, as used by Corman and D'Ariano (2011) and Vansteenwegen and Van Oudheusden (2007). Rail fare is assumed \$30 per trip. Following Levinson et al., (1997), we estimate that the train unit operating cost while en-route stopping is \$255.5/hr.

On the freight side, the unit costs while a freight train is moving, delayed at departure, delayed en-route, and foregone (i.e., loss of demand) are \$1814.6/hr, \$3424.1/hr, \$5057.2/hr, and \$14856 respectively, based on Talebian and Zou (2015) and AAR (2012). Recall that track maintenance cost is a linear combination of passenger and freight train miles. Grimes and Barkan (2006) report an average track maintenance cost of \$2507 per million gross ton miles on Class I Railroads in 2001. Multiplying this number by the average tonnage hauled by a freight train (3585 tons, according to (AAR 2012)) and dividing it by  $10^6$  lead to an estimated track maintenance cost of \$8.99/train-mile, or \$11.73/train-mile after updating it 2010 value (using a 3% inflation rate). For passenger trains, we use the estimate from Zambreski (2004) and convert it to 2010 value, which amounts to \$3.51/train-mile.

In the bargaining model, we consider the discount factors to be  $\delta_p = 0.9$  and  $\delta_f = 0.8$ . Later we also investigate the sensitivity of the bargaining outcome to different discount factors. In the incomplete information setting, we assume that the FRR is HFRR, and PRA believes that FRR has equal probability for being HFRR and LFRR (i.e.,  $\theta = 0.5$ ). The value of  $\bar{C}_{s_i}^F$  equals the corresponding  $C_{s_i}^F$  in the complete information setting; whereas the value of  $\underline{C}_{s_i}^F$  is set to be the product of  $\bar{C}_{s_i}^F$  and a random number drawn from the uniform distribution in  $[0.8, 0.9]$  for each schedule  $s_i$ .

## 6.3 Results

Let us first look at results with complete information. We consider the case that PRA initiates schedule bargaining and FRR initiates payment bargaining. Figure 8 shows the equilibrium payment from PRA to FRR, in terms of both the total amount (left) and the amount per train (right). Each line represents the payment for a given passenger train frequency, plotted against MED values from 0 to 6 time periods (intervals). String lines representing train schedules at two extreme values of MED are presented in Appendix B. Generally, increasing passenger train frequency leaves less capacity for FRR. FRR's cost will increase, suggesting that greater payment from PRA to FRR be required. This results in a higher amount of total payment. However, the payment does not exhibit a clear changing pattern with the increase in MED. Explanations for this are presented in Appendix C. On the other hand, total payment increases at a lower rate than the passenger train frequency, as indicated by the per train payment. Comparing the calculated payment values with the reported track usage payment made by Amtrak (US DOT 2010), which is \$4.44/train-mile, or \$532 for the 120-mile rail line, Amtrak seems to undercompensate the host FRR. One possible reason is that the current payment focuses on incremental cost to track maintenance (US DOT 2010), whereas our bargaining model encompasses also the delay and loss-of-demand costs to the host FRR.

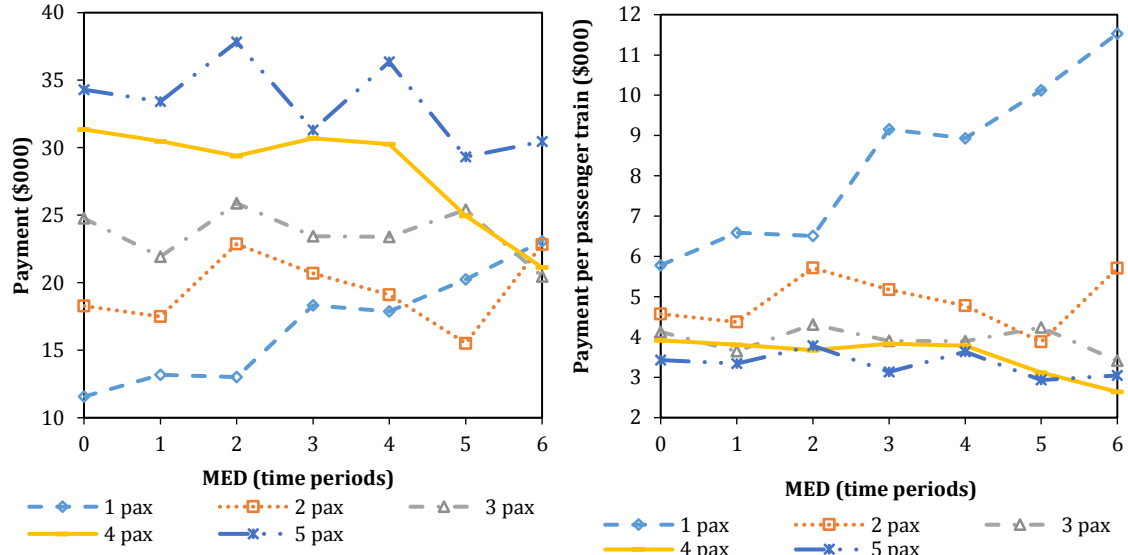
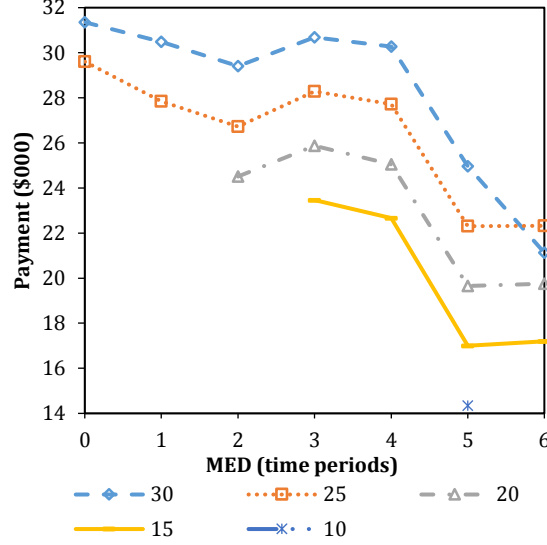


Figure8: Total and per train payment from PRA to FRR with complete information

We further compare the average payment from PRA to FRR on a per passenger basis, with rail fare (\$30). Table 1 reports the payment per passenger with different passenger train frequencies and MEDs. The values range from \$13.5 to \$28.5, or 45.1% to 94.9% of rail fare. Thus a significant portion of PRA's revenue would be used to pay to the host FRR, which puts pressure on PRA to cover its own operating expenses. To further investigate this, we lower rail fare from \$30 to \$10 with an increment of \$5, and plot the payment from PRA to FRR with four passenger trains in Figure 9. We find that, if rail fare is \$20, no equilibrium will be reached for MED being zero or one time period, because PRA's utility would become lower than FRR's cost. The state of no equilibrium expands as we decrease rail fare. With a \$10 rail fare, only one equilibrium exists when MED equals five periods. Therefore, to achieve a bargaining agreement under low fare, external revenue sources (e.g., government subsidies) would be necessary.

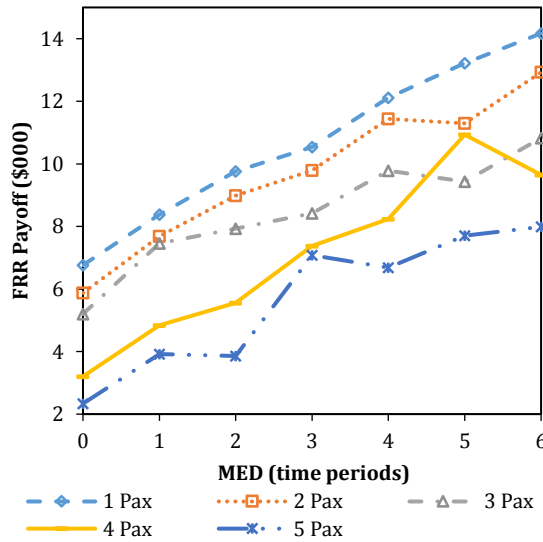
Table 1: Payment from PRA to FRR on a per passenger basis (in \$) with complete information

Number of passenger trains in each direction	Maximum en-route delay (MED)						
	0	1	2	3	4	5	6
1	14.6	16.6	16.3	23.2	22.1	25.3	28.5
2	16.6	15.4	19.8	19.1	18.2	15.2	22.4
3	20.5	15.8	21.2	20.1	19.6	20.3	16.0
4	22.2	20.6	19.6	22.8	21.2	16.8	13.5
5	19.5	20.7	22.9	19.5	24.8	18.2	18.5



**Figure 9: Payment from PRA to FRR under different rail fare with complete information (four passenger trains scheduled)**

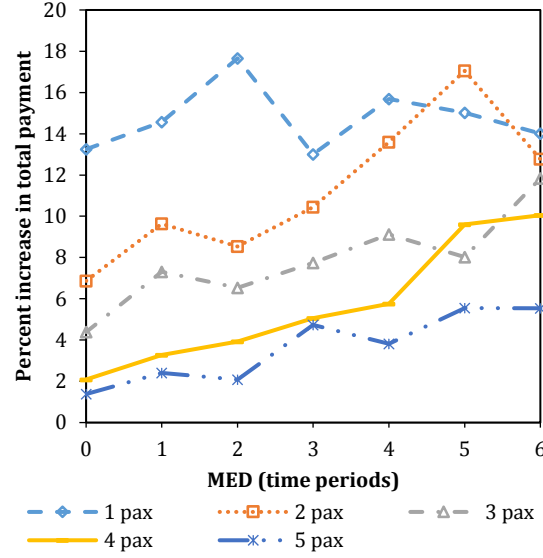
On the FRR side, Figure 10 presents FRR's payoff as a function of MED with different passenger train frequencies. Given a passenger train frequency, FRR's payoff generally increases with MED. Recall our discussions on MED in subsection 6.1 and Equation (6.2). This suggests that the MED effects of reducing passenger schedule delay and allowing more freight trains to run on the line, which contribute to increasing  $u_{s_i}^P$  and reducing  $C_{s_i}^F$ , respectively, dominate the potential effect of greater passenger en-route delay and passenger train operating cost (which decrease  $u_{s_i}^P$ ). Despite the payment from PRA, FRR's payoff decreases with more passenger trains. Given that FRR's payoff is linearly correlated with system welfare, the finding implies that more shared-use operations reduce system welfare.



**Figure 10: FRR's payoff with complete information**

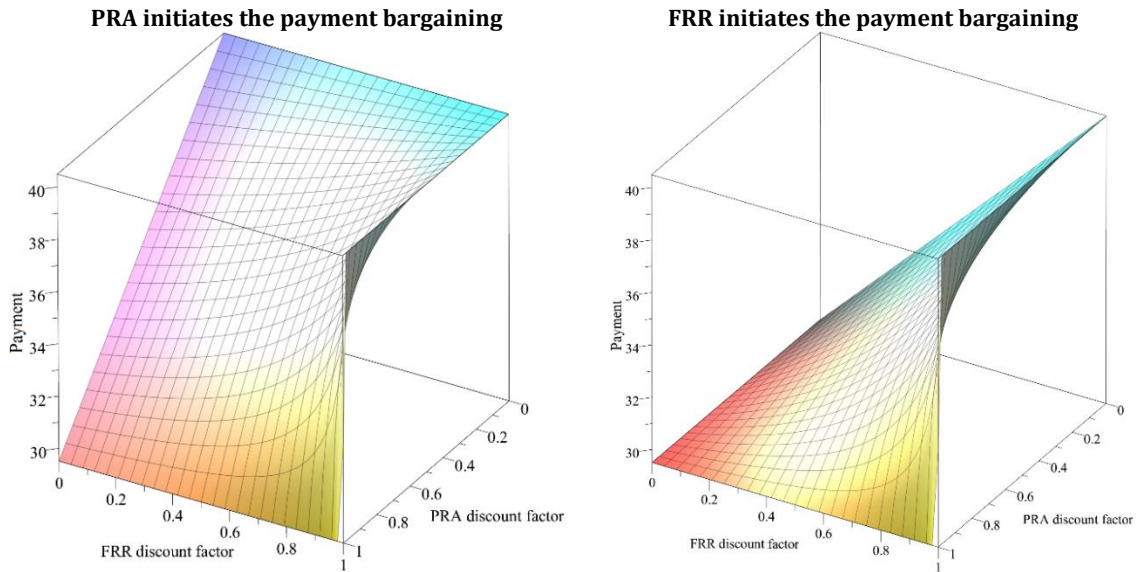
In subsection 5.1.1 we find that although who initiates the payment bargaining does not change the equilibrium schedule, it does affect the amount of payment from PRA to FRR. Figure 11 plots the

percentage increase in total payment when the payment bargaining is initiated by FRR vs. by PRA. The percent increase can be up to 17.7%, which occurs when only one passenger train is scheduled and MED is two time periods. This percent increase in payment diminishes as more passenger trains are scheduled.



**Figure 11: Percent increase in total payment from PRA to FRR, if the payment bargaining is initiated by FRR as opposed to by PRA**

Recall that the discount factors  $\delta_p$  and  $\delta_f$  capture the time-value of money for PRA and FRR, which can be alternatively viewed as the patience of PRA and FRR. Figure 12 plots the payment as a function of  $\delta_p$  and  $\delta_f$ , for both cases of PRA and FRR initiating the payment bargaining. Five passenger trains are scheduled with MED=1. (For other combinations of passenger train frequencies and MED values, similar trends are observed.) Two points are worth highlighting here.



**Figure 12: Payment from PRA to FRR (in \$000) as a function of PRA and FRR's discount factors**

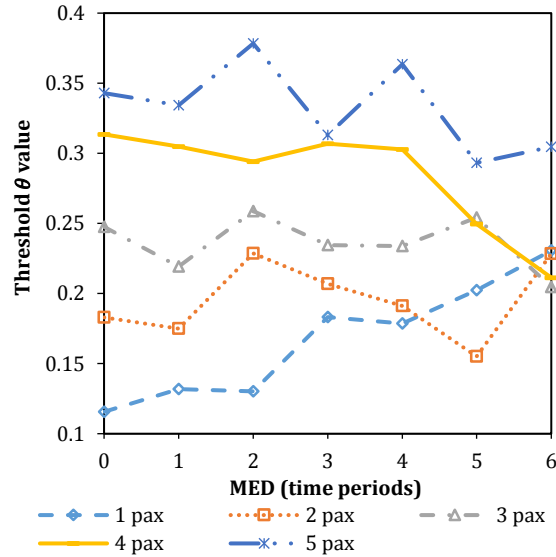
First, if the initiator of the payment bargaining is extremely patient, then depending on whether the payment bargaining is initiated by PRA (in which case  $\delta_p \cong 1$ ) or FRR (in which case  $\delta_f \cong 1$ ), the

payment will be the lowest or the highest. For example, when an extremely patient PRA initiates the payment bargaining, the PRA will demand the lowest possible payment regardless of FRR's discount factor. Referring back to Equation (4.1), this payment would collapse to  $C_{s^*}^F$  (where  $s^*$  denotes the equilibrium schedule), which means that the payment just covers FRR's cost so that FRR still wants to participate in the bargaining. Similarly, if it is an extremely patient FRR who initiates the payment bargaining, then the FRR will demand the highest possible payment, which is  $u_{s^*}^P$  according to Equation (4.2), regardless of PRA's type. In this way, FRR will extract the entire PRA's utility, while keeping PRA in the bargaining.

Second, if the receiving end of the initial payment proposal is extremely impatient, the initial proposer can take advantage of this impatience. When PRA initiates the payment bargaining and FRR is extremely impatient ( $\delta_F \cong 0$ ), then the payment will be the lowest. The amount, again, collapses to  $C_{s^*}^F$  according to Equation (4.1). When it is FRR who initiates the payment bargaining and PRA is extremely impatient ( $\delta_P \cong 0$ ), the payment will be the highest, equal to  $u_{s^*}^P$  according to Equation (4.2). On the contrary, if the receiving end of the initial payment proposal is extremely patient, the payment will be the highest when PRA initiates (or the lowest when FRR initiates).

Turning now to bargaining with incomplete information, Figure 13 illustrates  $\hat{\theta}_{s^*}$ , the threshold value for  $\theta$  with different passenger train frequencies and MED values (here we consider that PRA initiates the schedule bargaining, and FRR initiates the payment bargaining). In general,  $\hat{\theta}_{s^*}$  increases with passenger train frequency. The intuition is as follows. As shown in Talebian and Zou (2015), increasing passenger train frequency leads to higher passenger side benefit but also higher FRR cost, with FRR cost increase at a faster rate. This suggests that the numerator in the  $\hat{\theta}_{s_i}$  expression (under Equation (10)) may increase more than the denominator as we add more passenger trains. In contrast to this,  $\hat{\theta}_{s^*}$  does not exhibit a clear changing trend with respect to MED.

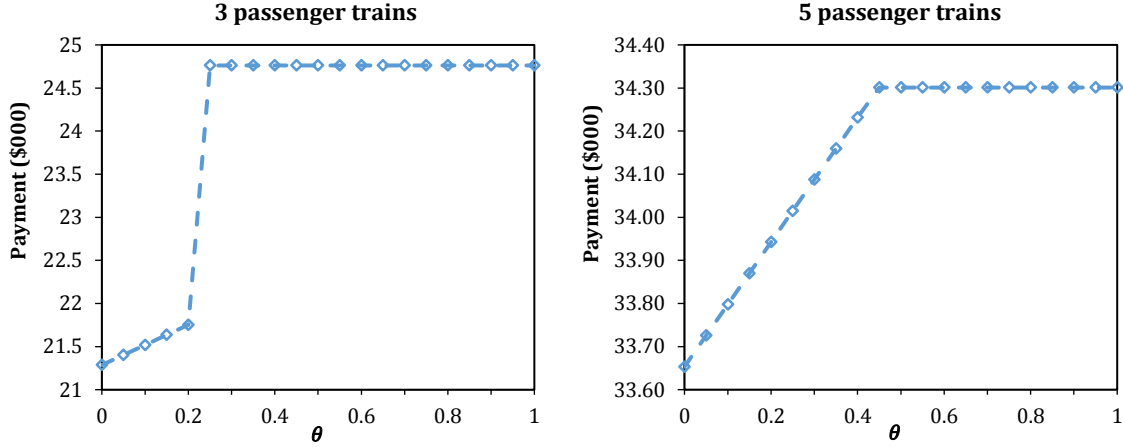
Since we set PRA's prior belief  $\theta=0.5$ , which is always greater than  $\hat{\theta}_{s^*}$  in Figure 13, the payment from PRA to FRR will correspond to the pooling equilibrium, and equal the first line of Equation (12), which is the same as in the complete information setting, except that  $C_{s_i}^F$  is replaced by  $\bar{C}_{s_i}^F$  (see Equation (4.2)).



**Figure 13: Threshold  $\theta$  values as a function of passenger train frequency and MED values (PRA initiates schedule bargaining and FRR initiates payment bargaining)**

Finally, to investigate how PRA's prior belief influences the payment from PRA to FRR, we vary  $\theta$  from 0 to 1 with an increment of 0.05. The amounts of payment for three and five passenger trains with MED=0 are plotted in Figure 14. When  $\theta$  is less than its threshold value, PRA will propose payment according to the separating equilibrium. Having a low belief that FRR is HFRR, PRA would propose a

low payment to FRR when it is PRA's turn to propose. Recognizing this, FRR will also propose a low equilibrium payment. As PRA's belief of FRR being HFRR becomes stronger, FRR increases the equilibrium payment proposed. After PRA's belief exceeds the threshold value, the payment proposed by FRR, as expressed in the first line of Equation (12), will be the same as if PRA knows with certainty that FRR is HFRR (Equation (4.2)). The payment gap between low and high PRA beliefs accounts for up to 14% of the payment under high belief, when three passenger trains are scheduled. The payment gap becomes insignificant (less than 2%) when five passenger trains are scheduled.



**Figure 14: Payment from PRA to FRR as a function of PRA's prior belief about FRR's type (PRA initiates schedule bargaining and FRR initiates payment bargaining)**

Finally, we investigate the payment from PRA to FRR when FRR initiates schedule bargaining and PRA initiates payment bargaining. To illustrate, we examine the payment for the scenario with 3 passenger trains in each direction and  $MED=0$ . The value of threshold  $\theta$  (i.e.,  $\hat{\theta}_{s^*}$ ) is 0.248. For any value of  $\theta < 0.248$ , PRA proposes a payment to FRR according to (9). However, FRR rejects the proposed payment and instead proposes  $p_H = \$24761.7$  in the second bargaining period. The amount of payment under the complete information setting will be \$23721.5 which is 4.2% less than with incomplete information. For any value of  $\theta \geq 0.248$ , PRA proposes a payment to FRR according to (8). In sum, the payment under incomplete information is always greater than the payment under complete information when 1) PRA initiates payment bargaining; and 2) PRA's prior belief is incorrect.

## 7. Conclusion

The continuous growth of rail traffic, both passenger and freight, on shared-use rail lines calls for better understanding of the ways line capacity is allocated between passenger and freight operations. The focus of the present paper is on vertically integrated systems where the host freight railroad owns the track infrastructure and the passenger rail agency pays for using the infrastructure. We propose a bargaining approach to determine passenger and freight train schedules and the associated payment from the passenger rail agency to the host freight railroad. In the bargaining, the passenger rail agency aims to maximize its revenue minus operating cost, passenger schedule delay cost, and passenger en-route delay cost; the freight railroad considers its cost due to departure delay, en-route delay, loss of demand, and track maintenance. The bargaining proceeds in two steps: first, the two sides bargain over payment for each possible schedule; with a price tag given to each possible schedule, bargaining in the second step seeks a mutually agreed-upon schedule. The first-step bargaining would be a fictitious play that is performed in each side's mind. We consider two information settings in the bargaining: 1) both sides know precisely the utility/cost information of the other side; 2) the passenger rail agency does not possess full cost information about the freight railroad, but instead has a probabilistic belief. The latter situation is plausible when private freight railroads hold their cost information confidential.



Analytically solving the bargaining leads to several major findings. In the complete information setting, although the equilibrium payment proposed by the passenger rail agency and the freight railroad will each be invariant to who initiates the bargaining, the actual payment does depend on who is the initiator. The initiator will take the first-mover advantage by raising/lowering the payment. The equilibrium schedule is the one that maximizes system welfare, which is invariant to who initiates the bargaining or discount factors. In the incomplete information setting, we consider pooling and separating equilibria to characterize two possible ways for the passenger rail agency to propose payment. Which way to choose depends on the strength of the prior belief of the passenger rail agency about the freight railroad's cost type. For schedule bargaining, we construct a pooling equilibrium, in which a sorting/elimination-based method is first used to identify equilibrium schedule pairs given the type of the freight railroad. We then provide two conditions for the existence of a pooling equilibrium schedule pair.

Further policy-relevant insights are obtained through numerical analysis. Under our experiment setting, we find that the payment from the passenger rail agency to the host freight railroad holds a significant portion of the rail fare in the complete information setting. If rail fare is not high enough, the bargaining cannot lead to an agreement and the passenger rail agency would need to require external revenue sources. Although the computed payment is much higher than Amtrak actually pays to host freight railroads, accommodating passenger trains still decreases the host freight railroad's payoff as well as system welfare. For the latter, while it is based on a specific rail line configuration, it should provoke further thinking on shared-use vs. dedicated-line operations. In the incomplete information setting, the payment from the passenger rail agency will be lower than with complete information, if the passenger rail agency's belief that the freight railroad has high cost is not strong enough. The difference, however, would be insignificant with more passenger trains scheduled on the line.

In terms of extending the current work, further efforts are suggested in a few directions. First, alternative objectives of PRA and FRR could be considered. Since PRA is assumed public, we may look at maximizing social welfare as PRA's objective. Doing so would require further specification of a passenger demand function and computation of consumer surplus (e.g., by performing integral along the passenger demand curve). On the FRR side, we may refine the characterization of FRR's pricing behavior, so that profit maximization of FRR can be explicitly modeled. Second, it would be interesting to explore the existence of a separating equilibrium for schedule bargaining in the incomplete information setting. Third, it is possible that the freight railroad does not have full information about the passenger rail agency. In this case, a bargaining with two-sided incomplete information would be more appropriate. Lastly, how to use the payment and system welfare as a signal to direct future rail infrastructure investment would be an area of both research interests and practical importance.

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## Appendix A: Hypergraph-based train scheduling model to generate *FPTS*

This appendix provides additional information on using a hypergraph-based train scheduling model, initially introduced by Harrod (2011), to generate feasible passenger train schedules. Specifically, we modify the upper-level model in Talebian and Zou (2015) to allow passenger trains to stop en route. Parameters, sets, and decision variables in the model are documented in Table A.1.

Each feasible passenger train schedule is a set of conflict-free train paths specified on a hypergraph. The hypergraph-based modeling of train paths is capable of explicitly capturing train transitions between blocks. This is not the case with discrete time-block dynamic graphs, which is commonly used in modeling train timetables. Following Talebian and Zou (2015), we assume that each physical train can be composed of multiple subtrains, each moving between two consecutive train stations. A subtrain is indexed by  $r_w^n$  ( $n = 1, \dots, N; w \in W$ ), which denotes the  $n^{th}$  subtrain traveling from the origin block of station pair  $w$  to the destination block of station pair  $w$ .

**Table A.1: notations used in generating feasible passenger train schedules**

Type	Component	Description
Decision variables	$x_{i,j,u,v}^r$	Occupancy arc denoting if (sub)train $r$ enters into block $i$ at $u$ , occupies block $i$ in time interval $[u,v)$ , and the exits into block $j$ at time $v$
	$y_{t,\hat{t}}^{r,\hat{r}}$	Artificial linking arc denoting if the arrival of subtrain $r$ at its destination at time $t$ is linked to the departure of its continuation subtrain $\hat{r}$ departing at time $\hat{t}$
Parameters	$o^w$	Origin block of station pair $w$
	$d^w$	Destination block of station pair $w$
	$EADT^r$	Earliest allowed departure time (EADT) from origin of (sub)train $r$
	$LAAT^r$	Latest allowed arrival time (LAAT) at destination of (sub)train $r$
	$l_{max}^r$	Maximum allowable layover time at a station for passenger subtrain $r$
	$l_{min}^r$	Minimum allowable layover time at a station for passenger subtrain $r$
	$b_t^i$	Capacity (number of trains) of block $i$ at time $t$
	$v_t^i$	Capacity (number of trains) of cell $i$ at time $t$
	$\varepsilon$	Leading transition time margin
	$\gamma$	Lagging transition time margin
	$f^n$	Maximum en-route delay time for $n^{th}$ physical train
	$h^r$	Minimum gap between train (or subtrain) $r$ and following trains
Sets	$T$	The discrete-time horizon, ordered with starting value 1
	$R^p$	The subset of passenger subtrains
	$R^{p,N}$	The set of passenger subtrains traveling in the direction with increasing track block index
	$R^{p,S}$	The set of passenger subtrains traveling in the opposite direction with decreasing track block index, $R^{p,N} \cup R^{p,S} = R^p$
	$B$	The set of all track blocks, ordered by a common reference of travel such as "North" or "South"
	$Z^p$	The set of linked passenger subtrains ( $\{(r, \hat{r})\}$ where $r$ is a terminating subtrain and $\hat{r}$ is an originating subtrain at the same location. Both subtrains refer to the same physical train)
	$\Psi^{p,r}$	The set of all feasible arcs $(i, j, u, v)$ for passenger subtrain $r$
	$\Psi^p$	The union of all sets $\Psi^{p,r}$ , i.e., $\Psi^p = \bigcup_{r \in R^p} \Psi^{p,r}$

Type	Component	Description
	$\mathcal{T}$	The set of network cells
	$L_{r_n^w, r_n^{\hat{w}}}^p$	The set of valid pairs of arrival times for passenger subtrain $r$ and departure of its continuation subtrain $r_n^{\hat{w}} : \{t, \hat{t} \in T   (i, d^w, u, t) \in \tau^{p, r_n^w}, (o^{\hat{w}}, j, \hat{t}, v) \in \tau^{p, r_n^{\hat{w}}}, t + l_{min}^r \leq \hat{t} \leq t + l_{max}^r\}$
	$W$	Set of all station pairs ( $W = W^N \cup W^S$ )

A hypergraph is composed of two types of nodes: blocks and cells. Cells are introduced to capture train transitions between neighboring blocks. To describe train paths on a hypergraph, we use 0-1 binary decision variables  $x_{i,j,u,v}^r$  and  $y_{t,\hat{t}}^{r_n^w, r_n^{\hat{w}}}$ .  $x_{i,j,u,v}^r$  denotes whether subtrain  $r$  enters into block  $i$  at period  $u$ , occupies block  $i$  in time interval  $[u, v)$ , and exits into block  $j$  at period  $v$ .  $y_{t,\hat{t}}^{r, \hat{r}}$  is an artificial arc which links subtrain  $r$  arriving at its station at time  $t$  to its continuation subtrain  $\hat{r}$  departing from the same station at  $\hat{t}$ .

Recall that for a given passenger train, its departure is at most 30 minutes earlier than the train's departure time in  $s_b$ . Let  $EADT^r$  denote this earliest allowed departure time for train  $r$ . We further allow train  $r$  to arrive at its destination at most 30 minutes later than the train's arrival time in  $s_b$ . Let  $LAAT^r$  denote this latest arrival time for train  $r$ . Using  $EADT^r$  and  $LAAT^r$ , we can conveniently generate the set of feasible hyperarcs for each subtrain, i.e.,  $\Psi^{p,r}$ .

We use the set of constraints (A.1)-(A.13) to enumerate feasible passenger train schedules which are built on the set of all feasible hyperarcs  $\Psi^p$ . For each subtrain, constraints (A.1) and (A.2) ensure unique departure of each subtrain from its origin and unique arrival at its destination. Constraint (A.3) guarantees the continuity of each train path. Constraints (A.4) and (A.5) capture train flow conservation at intermediate stations. Constraint (A.6) indicates that decision variables are binary. Constraint (A.7) regulates capacity limit for each block. Constraint (A.8) controls the transitions conducted within the transition window  $[t + 1 - \varepsilon, t + 1 + \gamma]$ . Following Harrod (2013), we assume  $\varepsilon = \gamma = 0$ . Constraints (A.9) and (A.10) manage train headway ( $h^r$ ) in both running directions, which is defined as the minimum separation distance between a pair of leading and following trains measured in blocks. Constraints (A.11) and (A.12) maintain the order of passenger trains while they are running. Constraint (A.13) guarantees that total en-route delay for each physical train is less than a pre-specified maximum en-route delay (MED).

#### Linear network constraints

$$\sum_{(o^w, j, u, v) \in \Psi^{p, r_n^w}} x_{o^w, j, u, v}^{r_n^w} = 1 \quad \forall n = 1:N, \forall w \in W \quad (A.1)$$

$$\sum_{(i, d^w, u, v) \in \Psi^{p, r_n^w}} x_{i, d^w, u, v}^{r_n^w} = 1 \quad \forall n = 1:N, \forall w \in W \quad (A.2)$$

$$\sum_{(a, i, u, t) \in \Psi^{p, r}} x_{a, i, u, t}^r = \sum_{(i, j, t, v) \in \Psi^{p, r}} x_{i, j, t, v}^r \quad \forall r \in R^p, i \in \{B | i \neq o^w, d^w\}, t \in T \quad (A.3)$$

$$\sum_{(i, d^w, u, t) \in \Psi^{p, r}} x_{i, d^w, u, t}^r = \sum_{(t, \hat{t}) \in L_{r, \hat{r}}^p} y_{t, \hat{t}}^{r, \hat{r}} \quad \forall r, \hat{r} \in R^p, (r, \hat{r}) \in Z^p, t \in T \quad (A.4)$$

$$\sum_{(o^{\hat{w}}, j, \hat{t}, v) \in \Psi^{p, r}} x_{o^{\hat{w}}, j, \hat{t}, v}^{\hat{r}} = \sum_{(t, \hat{t}) \in L_{r, \hat{r}}^p} y_{t, \hat{t}}^{r, \hat{r}} \quad \forall r, \hat{r} \in R^p, (r, \hat{r}) \in Z^p, t \in T \quad (A.5)$$

$$x_{i, j, u, v}^r, y_{t, \hat{t}}^{r, \hat{r}} \in \{0, 1\} \quad (A.6)$$

#### Side constraints

$$\sum_{\substack{r \in R^p \\ (i, j, u, v) \in \Psi^{p, r} | u \leq t < v}} x_{i, j, u, v}^r \leq b_t^i \quad \forall i \in B, t \in T \quad (A.7)$$

$$\sum_{\substack{r \in R^{p,N} \\ v \in \{t+1-\varepsilon, \dots, t+1+\gamma\} \\ (i,j,u,v) \in \Psi^{p,r} | j=a+1, j \neq i}} x_{i,j,u,v}^r + \sum_{\substack{r \in R^{p,S} \\ v \in \{t+1-\varepsilon, \dots, t+1+\gamma\} \\ (i,j,u,v) \in \Psi^{p,r} | j=a, j \neq i}} x_{i,j,u,v}^r \leq v_t^a \quad \forall (a, t) \in \mathcal{T} \quad (\text{A.8})$$

$$\sum_{\substack{r \in R^{p,N} | h^r \geq 1 \\ a \in \{i-h^r, \dots, i-1\} \\ (a,j,u,v) \in \Psi^{p,r} | u \leq t < v, a \neq j}} x_{a,j,u,v}^r + \sum_{\substack{r \in R^{p,N} \\ (i,j,u,v) \in \Psi^{p,r} | u \leq t < v}} x_{i,j,u,v}^r \leq b_t^i \quad \forall i \in B, t \in T \quad (\text{A.9})$$

$$\sum_{\substack{r \in R^{p,S} | h^r \geq 1 \\ a \in \{i+1, \dots, i+h^r\} \\ (a,j,u,v) \in \Psi^{p,r} | u \leq t < v, a \neq j}} x_{a,j,u,v}^r + \sum_{\substack{r \in R^{p,S} \\ (i,j,u,v) \in \Psi^{p,r} | u \leq t < v}} x_{i,j,u,v}^r \leq b_t^i \quad \forall i \in B, t \in T \quad (\text{A.10})$$

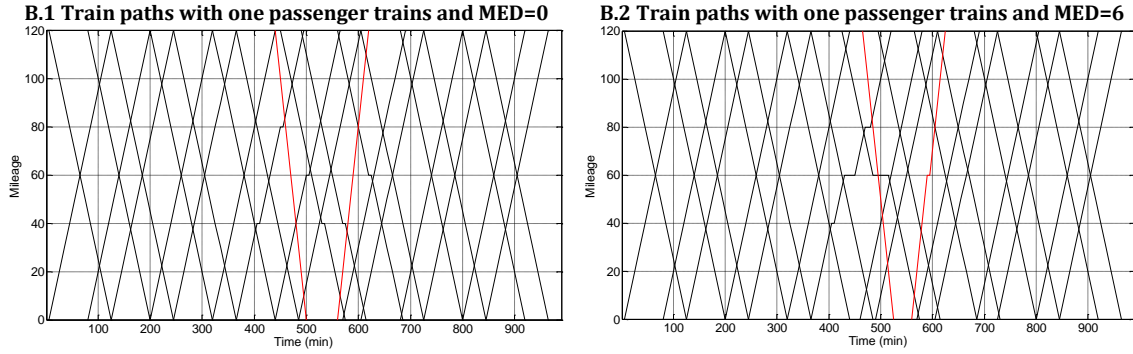
$$\sum_{(o^w, j, u, v) \in \Psi^{p,r_{n-1}^w}} u * x_{o^w, j, u, v}^{r_{n-1}^w} \leq \sum_{(o^w, j, u, v) \in \Psi^{p,r_n^w}} u * x_{o^w, j, u, v}^{r_n^w} \quad \forall w \in W, \{\forall n = 2:N\} \quad (\text{A.11})$$

$$\sum_{(i, d^w, u, v) \in \Psi^{p,r_{n-1}^w}} v * x_{i, d^w, u, v}^{r_{n-1}^w} \leq \sum_{(i, d^w, u, v) \in \Psi^{p,r_n^w}} v * x_{i, d^w, u, v}^{r_n^w} \quad \forall w \in W, \{\forall n = 2:N\} \quad (\text{A.12})$$

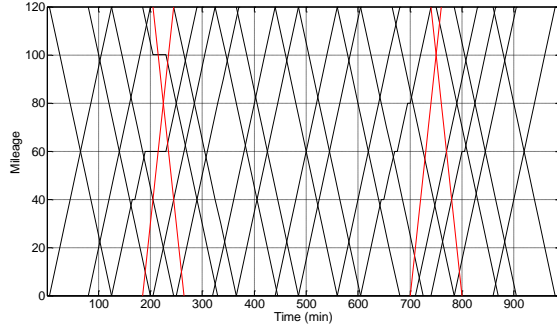
$$\sum_{\substack{w \in W \\ (i,j,u,v) \in \Psi^{p,r_n^w} | i=j}} x_{i,j,u,v}^{r_n^w} \leq f^n \quad \forall n = 1:N \quad (\text{A.13})$$

## Appendix B: String lines representing train schedules under the two extreme values of MED

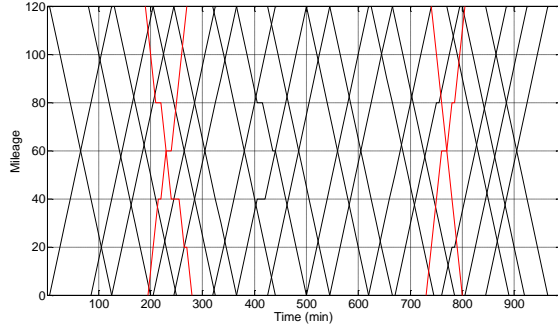
This appendix presents train schedules for different numbers of passenger trains (from one to five, in each direction) with MED equal to 0 and 6 time periods. Red and black lines are for passenger and freight train schedules respectively.



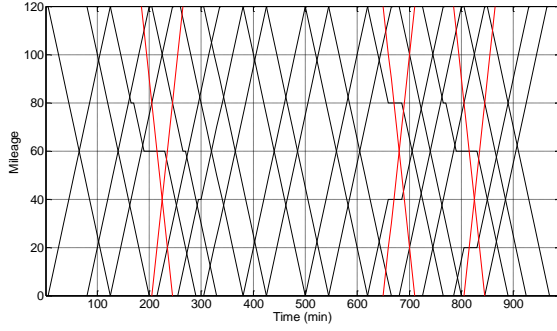
**B.3 Train paths with two passenger trains and MED=0**



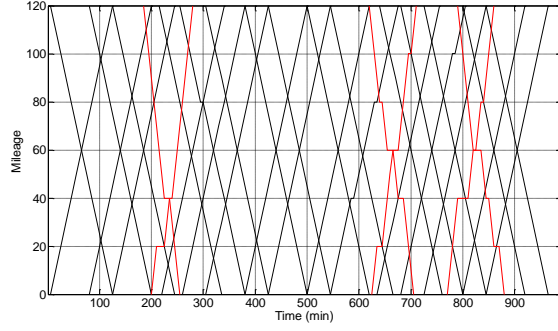
**B.4 Train paths with two passenger trains and MED=6**



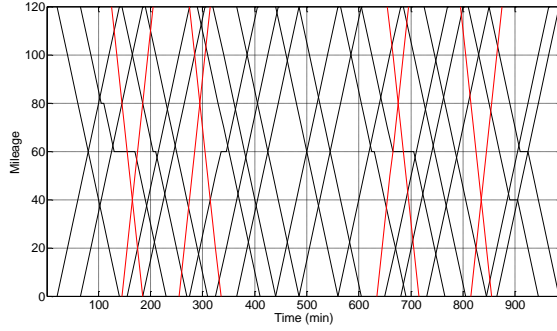
**B.5 Train paths with three passenger trains and MED=0**



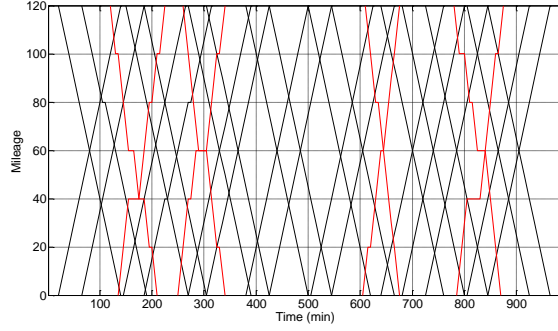
**B.6 Train paths with three passenger trains and MED=6**



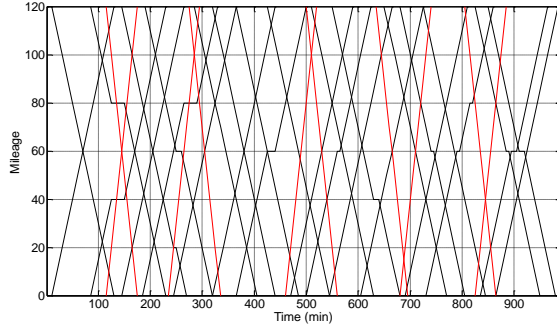
**B.7 Train paths with four passenger trains and MED=0**



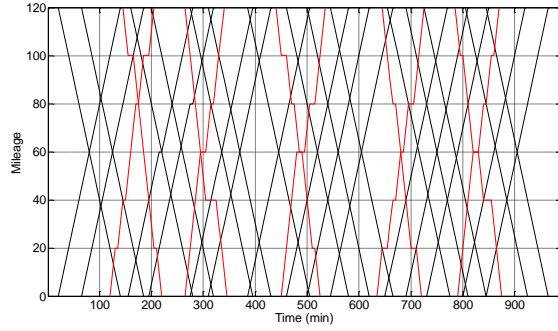
**B.8 Train paths with four passenger trains and MED=6**



**B.9 Train paths with five passenger trains and MED=0**



**B.10 Train paths with five passenger trains and MED=6**



## Appendix C: Explanations for payment not exhibiting a clear changing pattern with increase in MED

Intuitively, a larger MED could drive down the cost to both PRA and FRR, which means a larger PRA's utility  $u_{s_i}^P$  and a small FRR's cost  $C_{s_i}^F$ . However, according to payment expression (Eq. (4.1) and (4.2) depending on who initiates the payment bargaining), the changes in  $u_{s_i}^P$  and  $C_{s_i}^F$  will have opposing effects on the payment. So it is unclear whether a larger MED will reduce payment or not.

More systematically, by allowing for a larger MED, the set of possible train schedules will be enlarged (at least not reduced). As a result,  $\max_{s_i}(u_{s_i}^P - C_{s_i}^F)$ , where  $s_i$  is a feasible passenger train schedule, will be non-decreasing. Recall from Proposition 1 that a bargaining equilibrium schedule  $s_i^*$  maximizes  $u_{s_i}^P - C_{s_i}^F$ , i.e.,  $s_i^* = \operatorname{argmax}_{s_i}(u_{s_i}^P - C_{s_i}^F)$ . Three cases could occur after increasing MED:

1.  $\max_{s_i}(u_{s_i}^P - C_{s_i}^F)$  remains the same with the same equilibrium schedule  $s_i^*$ . Then according to Eq. (4.1) and (4.2), the payment will not change.
2.  $\max_{s_i}(u_{s_i}^P - C_{s_i}^F)$  remains the same but associated with a different equilibrium schedule  $s_i^{*'}$ . As long as  $u_{s_i^{*'}}^P - C_{s_i^{*'}}^F = u_{s_i^*}^P - C_{s_i^*}^F$ , it can be  $u_{s_i^{*'}}^P > u_{s_i^*}^P$  and  $C_{s_i^{*'}}^F > C_{s_i^*}^F$  which results in a larger payment according to Eq. (4.1) and (4.2); or  $u_{s_i^{*'}}^P < u_{s_i^*}^P$  and  $C_{s_i^{*'}}^F < C_{s_i^*}^F$  which results in a smaller payment.
3.  $\max_{s_i}(u_{s_i}^P - C_{s_i}^F)$  is increased with a different equilibrium schedule  $s_i^{*''}$ . Similar to case 2, there are more than one possibility for the changing directions of  $u_{s_i^{*''}}^P$  and  $C_{s_i^{*''}}^F$ , such that  $u_{s_i^{*''}}^P - C_{s_i^{*''}}^F > u_{s_i^*}^P - C_{s_i^*}^F$ . Again according to Eq. (4.1) and (4.2), the changing direction for payment is indeterminate.

Furthermore, please note that up to 10,000 feasible train schedules are randomly collected for each MED value (as mentioned in section 6.1, the actual number of feasible train schedules is less than 10,000 when MED is small, but exceeds 10,000 as MED increases). The randomness in collecting feasible train schedules could be another factor that contributes to the non-monotonic changing pattern of payment with respect to MED.

Based on the above arguments, it is not necessary that higher MED is always associated with reduced payment in Figure 8.