

Realization of “trapped rainbow” in 1D slab waveguide with surface dispersion engineering

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Abstract: We present a design of a one dimensional dielectric waveguide that can trap a broadband light pulse with different frequency component stored at different positions, effectively forming a “trapped rainbow” [Nature **450**, 397 (2007)]. The spectrum of the rainbow covers the whole visible range. To do this, we first show that the dispersion of a SiO₂ waveguide with a Si grating placed on top can be engineered by the design parameter of the grating. Specifically, guided modes with zero group velocity(frozen modes) can be realized. Negative Goos-Hänchen shift along the surface of the grating is responsible for such a dispersion control. The frequency of the frozen mode is tuned by changing the lateral feature parameters (period and duty cycle) of the grating. By tuning the grating feature point by point along the waveguide, a light pulse can be trapped with different frequency components frozen at different positions, so that a “rainbow” is formed. The device is expected to have extremely low ohmic loss because only dielectric materials are used. A planar geometry also promises much reduced fabrication difficulty.

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1. Introduction

To use photons as the information carrier is currently under intense study because of the almost unlimited bandwidth and the high energy efficiency. Many devices in an optical network depend on the capability to control the dispersion property of a waveguide. One example is the slow light for which the group velocity of light is much smaller than that in the free space. Applications of slow light devices include optical buffers, nonlinear optics, and optical signal processing. Whereas the slow light is usually realized on Bose-Einstein condensate [1], to build solid-state slow light devices is of great practical importance. The former is based on electromagnetically induced transparency (EIT) and usually demands on bulky, ultra-low temperature apparatus, while the latter is of much lighter weight and is applicable for on-chip integration. Photonic crystal is used almost exclusively for on-chip slow light devices. Usually, the part of the dispersion curve around the edge of the reduced Brillouin zone is used. This part of the dispersion curve are flattened because of the coupling between the forward and backward propagating modes, and exhibits a very small group velocity [2–5]. In 2007, a new idea to realize slow and stopped light was proposed in theory by Tsakmakidis, Boardman and Hess, making use of the anomalous property of metamaterials [6]. In their proposal, the negative Goos-Hänchen shift on a interface between a metamaterial and a regular dielectric is used. It was shown that, for a waveguide made of a metamaterial, when the Goos-Hänchen shift on the side walls of the waveguide compensates completely the forward-leap of the ray in a round trip, the guided mode would become, intuitively, "frozen" on the waveguide and no forward power propagation can be observed. This is actually a description of slow light using the ray picture. Further, a scheme of trapping optical signal of a broad frequency band is proposed: since the operating frequency to freeze the light is related to the waveguide thickness, a waveguide segment of tapered thickness should be able to trap light of a continuous spectrum at different positions along the waveguide, forming a "trapped rainbow".

The idea has since attracted many researchers and different designs have been tried. However, the experimental realization of the original idea of "trapped rainbow" faces great challenge. In the original design, the metamaterial was treated as a homogeneous medium similar to a regular dielectric, while in reality such an artificial material is always composed of discrete inclusions with strong temporal and spatial dispersion. Up to now, the best optical metamaterial uses inclusions of $\sim \lambda_0/3$ in size, where λ_0 is the free space wavelength at the operating frequency. When used to build a waveguide which itself might only be a few wavelengths in width, thus modeling the waveguide as a homogeneous one is problematic. The optical metamaterial usually operates at a frequency up to the near infrared. Little progress has been made for metamaterials working in the visible band with reasonably good property. Also, metamaterials are inevitably dispersive, and there have been no report on metamaterials with negative ϵ and μ that cover the whole visible domain. Further, the ohmic loss related with metamaterial is a formidable factor. In the optical frequency domain, plasmonic materials (gold or silver) are

used almost exclusively to build metamaterials. Their ohmic loss is far from tolerable for the application of “trapped rainbow”, and might erase any feature related to the broadband rainbow trapping without external gain [7]. There have been a few reports on the experimental demonstration of the “trapped rainbow” after the theoretical proposal [8–10]. However, none has actually used the approach proposed in the original paper that was based on the negative Goos-Hänchen shift. Rather, they are realized on the edge of the Brillouin zone of a plasmonic periodic structure. Inevitably, the strong ohmic loss makes the trapping effect very weak.

In this paper, we numerically demonstrate an approach to realize frozen mode based on the negative Goos-Hänchen shift. The proposed approach uses only dielectric materials, thus could have extremely low ohmic loss. The “trapped rainbow” is then realized by a waveguide with chirped or adiabatically tuned design of the frozen mode waveguide. Our approach is the first, complete demonstration for frozen mode and trapped rainbow that uses negative Goos-Hänchen shift, the original idea in [6]. In the following, we first review the idea of using negative Goos-Hänchen shift to construct a frozen mode, and our recent discovery that negative Goos-Hänchen shift, sometimes of giant magnitude, can be realized on the surface of a dielectric decorated by a grating. We then demonstrate rigorously that a “frozen mode” where the Goos-Hänchen shift fully compensates the forward leap of the ray in a round trip inside a slab waveguide indeed corresponds to zero group velocity of the guided mode, and show numerical results of such a frozen mode. Based on this waveguide that supports frozen mode, we then show our designs of “trapped rainbow”, where a broadband pulse is “frozen” with different frequency components trapped on different positions along the device.

2. Goos-Hänchen shift and frozen mode

When a Gaussian beam is totally reflected from a surface, the axis of the reflected beam experiences a lateral shift with respect to the position predicted by geometric optics [11]. Such a phenomenon is named after the discoverers Goos and Hänchen, and has been shown as an example of discrepancy between geometric optics and the wave nature of the light. The shift is related to the change of the reflection phase for different plane wave components of the incident beam. Mathematically, the Goos-Hänchen shift can be evaluated as [12]

$$z_s = -\frac{\partial \phi}{\partial k_x} \quad (1)$$

where ϕ is the phase of the reflection coefficient of the plane wave component with a lateral wavenumber of k_x . Here an $e^{-i\omega t}$ time variation is assumed for the electromagnetic field. The Goos-Hänchen shift is usually positive for the reflection from an interface between two regular dielectrics, while negative on interfaces between regular dielectrics and plasmonic material or metamaterial.

The negative Goos-Hänchen shift has attracted a lot of research interest, one of which is to control the direction of energy flow in a dielectric slab waveguide with respect to the wave vector of the guided mode, as discussed in the same paper that proposed the trapped rainbow [6]. Whereas the original discussion was from a rather intuitive approach, here we would like to give a rigorous mathematical description. Considering a slab waveguide made of a dielectric of refractive index n and thickness h . A guided mode can be described as a plane wave total-internally reflected back and forth on the two interfaces that satisfies the following relationship:

$$2nhk_0 \cos \theta + \phi_1(\theta, k_0) + \phi_2(\theta, k_0) = 2m\pi \quad (2)$$

where k_0 is the free space wavenumber, θ is the angle of incidence, $\phi_j(\theta, k_0)$, $j = 1, 2$ is the phase loss (the phase of the reflection coefficient) on the two side walls, respectively, while m

an integer. When written in terms of k_x , the wavenumber parallel to the waveguide wall, we have

$$2h\sqrt{n^2k_0^2 - k_x^2} + \phi_1(k_x, k_0) + \phi_2(k_x, k_0) = 2m\pi \quad (3)$$

Take the total differential of both sides with respect to k_0 and k_x , we get

$$2nh \frac{\Delta k_0}{\sqrt{1 - k_x^2/(nk_0)^2}} - 2h \frac{k_x}{\sqrt{n^2k_0^2 - k_x^2}} \Delta k_x + \sum_{j=1,2} \left(\frac{\partial \phi_j}{\partial k_0} \Delta k_0 + \frac{\partial \phi_j}{\partial k_x} \Delta k_x \right) = 0 \quad (4)$$

which is a relationship a guided mode must satisfy in addition to Eq. (2). When deriving the former equation, we assume that n does not change with frequency, which is a reasonable assumption for dielectric waveguides. Divide both sides by Δk_x and take the limit of $\Delta k_x \rightarrow 0$, we get

$$\left(\frac{2nh}{\sqrt{1 - k_x^2/(nk_0)^2}} + \sum_{j=1,2} \frac{\partial \phi_j}{\partial k_0} \right) \frac{\partial k_0}{\partial k_x} = 2h \frac{k_x}{\sqrt{n^2k_0^2 - k_x^2}} + \sum_{j=1,2} -\frac{\partial \phi_j}{\partial k_x} \quad (5)$$

Notice that $\partial \phi_i / \partial k_0$ is always positive. This is because $\partial \phi_i / \partial k_0$ is the delay of the center of the Gaussian pulse at the reflection of the interface. For lossless reflection (which is the case here), this delay must be positive for a causal system. This means the sign of the right hand side completely determines the sign of $\partial k_0 / \partial k_x$, which is proportional to the group velocity. For the right hand side, the second term is the Goos-Hänchen shift on the two side walls. Also notice that $k_x / \sqrt{n^2k_0^2 - k_x^2} = k_x / k_y$. Thus, if we let $z_w = hk_x / \sqrt{n^2k_0^2 - k_x^2}$, z_w is actually the forward displacement of the ray when propagating from one side wall to the other (see Fig. 1(a)). Combining the contribution of z_s and z_w together, the right hand side of Eq. (5) gives the total x direction displacement of a ray in a round trip, as we see in Fig. 1. When the waveguide and the surrounding medium are both made of regular dielectrics, the Goos-Hänchen shift is positive, thus the right hand side is always positive. This means we always have $\partial k_0 / \partial k_x > 0$. Things become interesting when we have negative Goos-Hänchen shift on one or both of the side walls, especially when it is of large magnitude so that the total

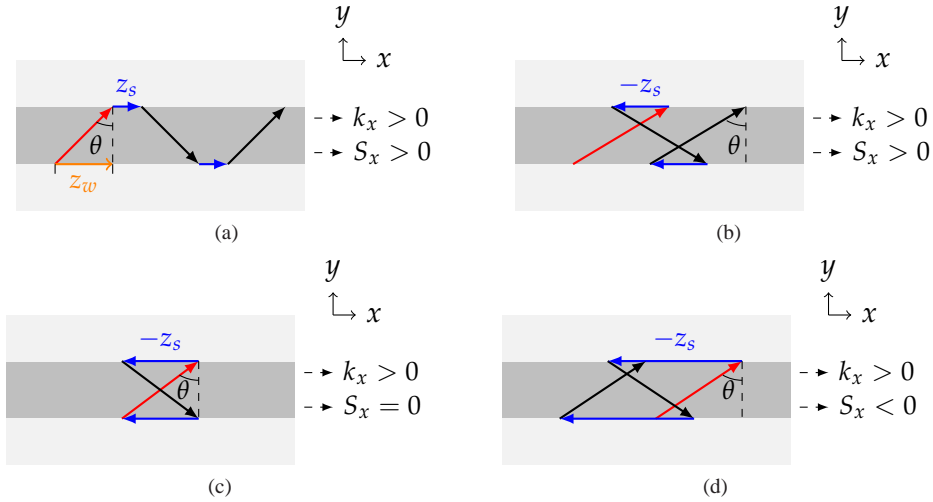


Fig. 1. Energy flow and Goos-Hänchen shift in a planar waveguide. The incidence is colored in red, Goos-Hänchen shift z_s in blue, while the forward displacement z_w in orange. The rays travelling inside the waveguide are in black.

displacement is negative (Fig. 1(d)). In this case, the group velocity would be negative, and the energy propagates anti-parallel to k_x . When the Goos-Hänchen shift is just enough to make the right hand side goes to zero (the situation described by Fig. 1(c)), we have $\partial k_0 / \partial k_x = 0$, and a “frozen mode” of the waveguide is formed. This is a guided mode with finite propagating constant, but zero net power propagation. All these conclusions are consistent with those in [6] but with rigorous mathematical analysis. We would like to point out that the conclusions only hold when the waveguide material has no temporal or spatial dispersion, i.e. $\partial n / \partial k_0 = 0$ and $\partial n / \partial k_x = 0$, as assumed when deriving Eq. (4). This is a reasonable assumption for dielectric waveguide, but not for metamaterial waveguides.

3. Negative Goos-Hänchen shift and frozen mode on a dielectric grating

A negative Goos-Hänchen shift is crucial in building a frozen mode. This can be achieved on the surface of plasmonic materials or metamaterials, but is usually accompanied with large ohmic losses. However, it is possible to make negative Goos-Hänchen shift using completely dielectric devices, as we demonstrated in a recent publication [13]. The system under consideration is shown in Fig. 2(a), where a thin grating made of Si is placed on a substrate of SiO₂. For certain grating design, the phase of the reflection coefficient for incidence from the SiO₂ side is of very different nature compared to that on the SiO₂/Air interface, as we see in Fig. 2(b) in which S polarized incidence is studied, i.e. E_z is the only electric field component. Whereas the phase decreases with the incident angle for a SiO₂/Air interface indicating a positive Goos-Hänchen shift (see --- in Fig. 2(b)), on the SiO₂/Grating interface, the phase increases, exhibiting a negative Goos-Hänchen shift (see — and — in Fig. 2(b)). This is similar to that of a SiO₂/Metamaterial case (see - - - in Fig. 2(b)). The negative Goos-Hänchen shift is related to the guided mode of the grating. For the second band of the guided mode of the grating, the energy propagates to the opposite direction of the wave vector. The part of the dispersion curve for this band that is between the light lines of the free space and the substrate is leaky on the substrate side, and can couple to the incident beam efficiently. According to a commonly accepted explanation, the negative energy propagation with respect to the lateral

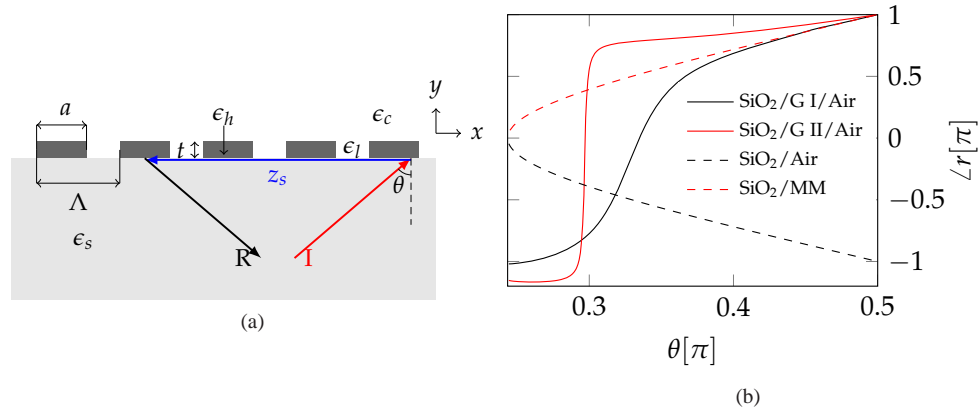


Fig. 2. (a): Schematic of an infinite thick substrate decorated by a dielectric grating. I and R stand for incidence and reflected beam respectively. z_s is the Goos-Hänchen shift. Duty cycle is defined as $\Gamma = a/\Lambda$. (b): Reflection phase vs Incidence angle θ of different interfaces when incidence coming from the SiO₂ substrate. Operating free space wavelength is $1.5\mu m$. Grating_I parameters: $\Lambda_I = 0.53\mu m$, $t_I = 0.097\mu m$, $\Gamma_I = 0.65$. Grating_{II} parameters: $\Lambda_{II} = 0.43\mu m$, $t_{II} = 0.11\mu m$, $\Gamma_{II} = 0.93$.

wave propagating direction is responsible for the negative Goos-Hänchen shift [13, 14]. The amount of Goos-Hänchen shift can be controlled by the grating design: depending on the parameters of the grating, we may have a very large (a steep $\phi-k_x$ curve) or a mediocre (a slow-varying $\phi-k_x$ curve) Goos-Hänchen shift. In fact, the amount of Goos-Hänchen shift ranges from tens of nanometers up to several millimeters. In our study, a Goos-Hänchen shift of more than 5000 times of the free space wavelength [13] has been observed.

With the help of the negative Goos-Hänchen shift on the grating, we can readily realize the frozen mode discussed in the former section, by placing the grating on the sides of a dielectric waveguide. It turns out that grating on one side is enough to realize our goal. One of the designs makes use of a SiO_2 waveguide of 200nm thick and a grating of 40nm in thickness. To calculate the dispersion curve of this grating-decorated waveguide, we first find out the reflection phase $\phi(k_0, k_x)$ on the SiO_2 /Grating interface and the SiO_2 /Air interface, respectively. These values are then used in Eq. (2) to get the dispersion relation. The results are shown in Fig. 3(a) as red cross. Notice how the dispersion curve bends to form a local extreme where $\partial k_0 / \partial k_x$ goes to zero at which a frozen mode is formed. Calculation confirms that the right hand side of Eq. (5) indeed goes to zero at the top of the dispersion curve, which demonstrates the application of Eq. (5) in finding a frozen mode. The numerical evaluation also shows that the right hand side of Eq. (5) is positive for the part of the dispersion curve with k_x smaller than that at the top point, and negative when k_x is larger than that at the top. This is consistent with the positive or negative group velocity the dispersion curve shows (see Fig. 1). The nature of the negative Goos-Hänchen shift can be used to understand the frozen mode. Recall that the negative Goos-Hänchen shift is usually explained [14] by an energy flow beyond the reflection interface that is opposite to k_x . If this power flow compensates completely the forward power flow inside the waveguide, no net power flow is carried by the guided mode, and a zero group velocity is expected.

The evaluation of the dispersion relation using Eq. (2) ignores the high order spatial harmonics of the field around the grating, of course. To see if this poses any important influence, we also evaluated the dispersion relation of the waveguide using full wave analysis. To do

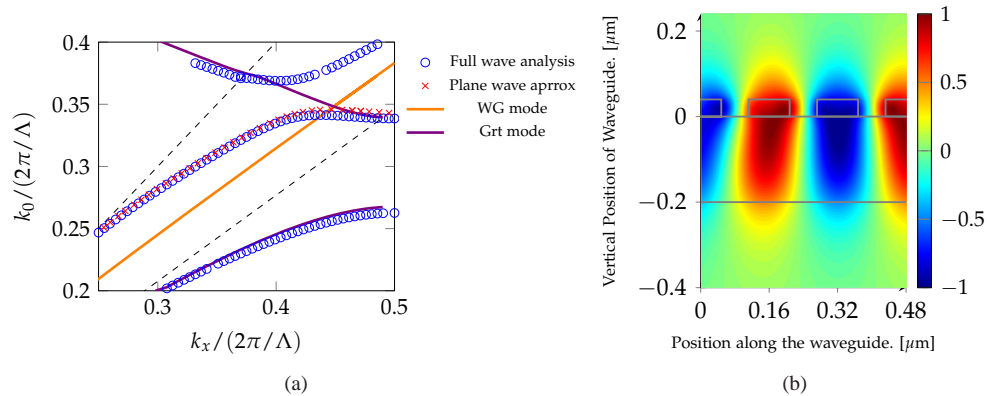


Fig. 3. (a): The dispersion property of waveguide with grating parameters: $t = 0.04\mu\text{m}$, $\Lambda = 0.16\mu\text{m}$, $\Gamma = 0.6$, $\epsilon_h = 10.24$, $\epsilon_l = \epsilon_c = 1$; waveguide parameters: $h = 0.2\mu\text{m}$, $\epsilon_s = 2.09$. The upper and lower dashed lines are light lines of the free space and the waveguide. The blue circles stands for the FDTD result while red cross stands for the plane wave approximation analysis. Solid violet line shows the dispersion of single grating laying on a SiO_2 substrate, while solid orange line shows the dispersion of bare waveguide. (b): The instantaneous field distribution of the frozen mode in three periods of the waveguide.

this, we use MEEP, an open source numerical electromagnetic package based on the finite-difference, time-domain (FDTD) method. The result is shown as circles in the same plot of Fig. 3(a), together with the guided modes of the same Si grating sitting on a SiO₂ substrate of infinite thickness. We can identify the nature of each part of the dispersion curve by examining the field distribution of the guided modes. For the lowest band below the light line of SiO₂, the electromagnetic field is well confined inside the grating, and the dispersion curve overlaps well with the guided mode of the grating on SiO₂ substrate. These modes are below the light line of SiO₂ and does not couple well with the propagating plane waves in the SiO₂ waveguide, thus can not be predicted by Eq. (2). Rather, this is the guided mode of the grating itself. The second band is above the light line of SiO₂, thus the plane waves inside the SiO₂ slab waveguide take part in the formation of this band. Notice that the dispersion curve calculated from Eq. (2) (plotted as \times) indeed overlaps well with that calculated from full-wave analysis. This means the high order spatial harmonics of the field do not contribute obviously in forming the mode, and to use Eq. (2) for mode calculation is safe. The shape of the lower part of the second band is similar to the dispersion curve of a bare SiO₂ slab waveguide of the same thickness, but shifted in k_x because of the changed reflection phase on the SiO₂/grating wall (see Eq. (2)). As the frequency increases, the waveguide mode gets close to the second band of the grating's guided mode where the two anti-cross each other, causing the opening of a bandgap. Notice that, the second band of the grating mode is also where negative Goos-Hänchen shift is observed [13]. The instantaneous field distribution of the frozen mode, i.e. the mode at the top of the lower band, is shown in Fig. 3(b). The Poynting vector evaluated from these simulation results indeed confirm the zero net power flow of this mode.

One important feature needed for a trapped rainbow is the capability to tune the frequency of the frozen mode. In the original paper of trapped rainbow [6], this is realized by varying the thickness of the waveguide. A tapered waveguide requires gray-scale etching, which is difficult in the conventional micro- and nano- fabrication developed for planar geometry. The grating used in our device can tune the operating frequency without thickness variation: we can change the lateral design parameters (the period Λ and the duty cycle Γ) to modulate the frequency of the frozen mode while leaving the waveguide thickness untouched. It appears that the frozen mode frequency can be varied most effectively by changing the period. The effect is shown in Fig. 4, in which the dispersion curves for two waveguides of the same SiO₂ slab and grating thickness (200nm and 40nm, respectively) but different grating periods (\circ : *Period* = 160nm; \triangle : *Period* = 170nm.) The result in this plot is again from MEEP simulation. As we can see, the frequency of the frozen mode (the top of the lower band) is obviously changed. For a period

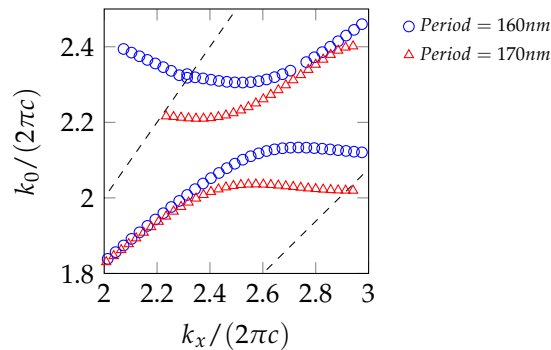


Fig. 4. The dispersion of two types of waveguide in the difference of *Period* = 0.16 μ m (Blue circle) and *Period* = 0.17 μ m (Red triangle).

variation of $\sim 6\%$, the frequency is changed by $\sim 4.5\%$.

4. Trapped rainbow

To construct a trapped rainbow requires building a waveguide on which the frozen mode is of different frequency at different position along the device. In our design, this is realized by placing multiple segments of the waveguide of different grating period one after another. A schematic is shown in the top of Fig. 5(a). As a demonstration, our first device is composed of 14 waveguide segments, and the grating of each segment consists of 10 identical periods. These gratings have the same thickness of $t = 40\text{nm}$ and duty cycle $\Gamma = 0.6$, but the period varies from 130nm to 260nm. The free-space wavelength of the frozen mode that would be supported by waveguides of these different designs range from 395nm to 704nm.

The device is fed from the left by a slab waveguide made of the same material and of the same thickness. A dipole source is placed in the left feeding waveguide and used as the excitation, which gives a broadband pulse with the spectrum covering the whole frequency range interested to the trapping device. We arrange the waveguide segments so that the frozen mode frequency decreases from left to the right, with the segments of higher frequency sitting at the upper stream of the optical power flow. This is because, according to Fig. 3(a) and Fig. 4, each segment actually supports two modes of zero group velocity, one at the top of the lower band while the other at the bottom of the top band. In our design, we use the lower band of every segment, and the arrangement described above promises that the bottom of the upper band of each segment falls inside the bandgap of its neighboring upper stream segment, thus would not be excited. The structure is again simulated in MEEP. In the simulation, we record the field at different positions along the center of the waveguide after the transient field fades out. A Fourier transform then reveals the spectrum at each position. The observed spectrum intensity at different positions along the whole device is shown in the bottom of Fig. 5(a). Here the horizontal axis is the lateral position along the device with the origin at the beginning of the first waveguide, while the vertical axis is the signal frequency. The color shows the spectrum intensity. In the simulation, a pulse signal with approximately flat spectrum in the band of interests is used, so that no frequency component has an advantage in the power intensity. As we walk from left to the right along the device, we can indeed observe 14 discrete steps in the spectrum at positions corresponding to the 14 waveguide segments (the last one is less obvious due to the reflection by the segments ahead of it), going from 760THz (violet color) to 400THz (red color). To give an intuitive understanding to the result, we show the color that would be observed at different positions along the device rendered from the spectrum measured at the very position. The algorithm to render a color from a distribution of spectrum intensity is discussed in [15], and the result is shown in the middle of Fig. 5(a). The result clearly gives a “rainbow” trapped along the device.

The former demonstration uses a piecewise continuous design. To have a rainbow with adiabatic color change, we turn to a device with tapered design. Rather than physically tapering the thickness of the slab waveguide, we use continuously changed grating period along the whole device in the same range as the former example, as we see in the schematic shown on the top of Fig. 5(b). A similar idea was used to make flat focusing lens in one of the authors' former works [16]. We expect the result to be a smooth-out version of the trapped rainbow shown in Fig. 5(a). The result indeed proves our expectation (refer to the middle of Fig. 5(b)). As we see in the bottom of Fig. 5(b), the peak frequency of the spectrum changes as the observation position changes, and a rainbow of continuously varying color can be observed.

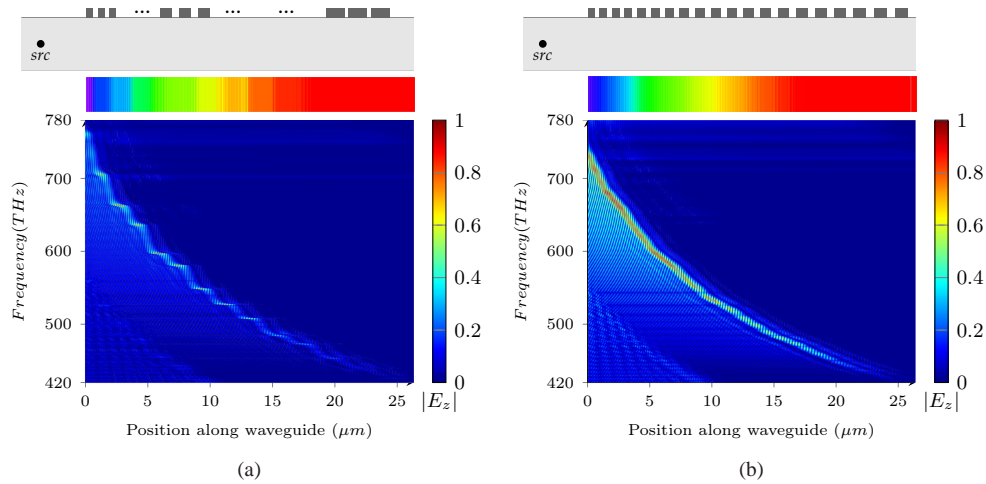


Fig. 5. (a): Top: The schematic of the rainbow-trapping device composed of multiple waveguide segments. Bottom: Full wave analysis of the trapped rainbow obtain by FDTD simulation and Fast Fourier Transform. For the waveguide, $t = 0.2\mu\text{m}$. For the grating, $\Gamma = 0.6$, $t = 0.04\mu\text{m}$, period range is designed from $0.13 \rightarrow 0.26\mu\text{m}$. The Gaussian pulse enters from the left of the structure in the slab waveguide(not shown in the plot). Both figures demonstrate the field distribution in the frequency domain(vertical axis) and the spatial position(horizontal axis). (b): Rainbow trapping by waveguide of continuously varying parameters. The period varies from $0.130\mu\text{m}$ to $0.267\mu\text{m}$ gradually. The other design parameters and the excitation is the same as (a).

5. Conclusion

In this paper we make use of the negative Goos-Hänchen shift on the surface of a dielectric grating to realize a frozen mode, i.e. a guided mode on a waveguide with no net power propagation. Further, by tuning the design parameters of the grating on a waveguide, we can achieve frozen modes with different frequencies sitting at different positions along the waveguide, so that a broad band pulse covering the whole visible spectrum can be caught by the waveguide, with different frequency components stored at different positions. The current design is, to the best of our knowledge, the first demonstration of the “trapped rainbow” proposed in [6] that make explicit use of the negative Goos-Hänchen shift, the mechanism originally proposed in that paper. At the same time, the use of only dielectric materials promises a much lower ohmic loss. The negative Goos-Hänchen shift is realized on the surface of a grating, which has a geometry much easier to be fabricated compared to the usually three dimensional structure of a metamaterial. Tuning the lateral design parameters rather than the thickness further reduces the fabrication difficulty. All these features make the device suitable for practical use in areas such as slow light.

We should point out that the “trapped rainbow” serves as a manifesto of the capability of the grating in controlling the dispersion property of a slab waveguide. According to Eq. (5), the Goos-Hänchen shift, or more generally, the reflection phase ϕ on the surface of the grating, directly determines the behavior of the group velocity. Since the reflection phase is controlled by the design parameters of the grating, Eq. (5) gives us a straightforward method to synthesize the dispersion property of the waveguide as needed. We believe this dispersion engineering approach can have promising applications in optical networks.

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