Outage Analysis of Block-Fading Gaussian Interference Channels

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***Abstract*—This paper considers the asymptotic behavior of the outage probability of a two-source block-fading single-antenna Gaussian interference channel in the high-SNR regime by means of the diversity-multiplexing tradeoff. A general setting where the user rates and the average channel gains are not restricted to be symmetric is investigated. This asymmetric scenario allows to analyze networks with "mixed" interference, i.e., when different sources are at different distance from their intended destination, that are not possible under the commonly used symmetric assumption. Inner and outer bounds for the diversity are derived. The outer bound is based on the recent "to within one bit" capacity result of Etkin *et al.* for the unfaded Gaussian channel and is a re-derivation of a known bound for which an error is pointed out. The inner bound is based on the Han and Kobayashi achievable region both without rate splitting and with a rate spitting inspired by the "to within one bit" capacity result. An analytical comparison of the diversity upper and lower bounds for a general channel seems difficult; by numerical evaluations, the two bounds are shown to coincide for a fairly large set of channel parameters.**

***Index Terms*—Interference channel; Diversity Multiplexing Tradeoff;**

I. Introduction

Wireless networks deal with two fundamental limitations that make the communication problem challenging and interesting. On the one hand, simultaneous communications from uncoordinated users create undesired interference. In today's cellular and ad-hoc networks orthogonalization techniques, such as F/T/C/SDMA (frequency, time, code, space division multiple access), are employed to avoid interference. However, although leading to simple network architectures, interference avoidance techniques are suboptimal in terms of achievable rates. On the other hand, the relative strength of the intended signal and the interference signals changes over time due to fading. This makes fixed channel access strategies suboptimal, especially when the interferers are very weak or very strong. Thus, understanding how to deal simultaneously with interference and with fading holds the key to the deployment of future broadband wireless networks. The simplest model for analyzing these problems jointly is the two-source Block-Fading Gaussian InterFerence Channel (BF-GIFC).

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A. Past Work

The Han-Kobayashi (HK) [3] scheme with superposition coding, rate splitting, joint decoding, and time sharing, gives the largest known achievable rate region for a general IFC. For Gaussian channels, with and without fading, achievable regions are usually obtained by considering the HK scheme with jointly Gaussian inputs and without time sharing.

Unfaded GIFC: Several outer bounds are known in the literature for the GIFC without fading [4], [5], [6], [7], [8], [9]. In particular, Etkin *et al.* [6] recently showed that a simple rate splitting strategy in the HK scheme is "to within one bit" of a novel outer bound region for any channel parameter. The key idea of [6] is that the messages that are treated as noise at some receiver are assigned a transmit power such that they are received below the noise floor of the non-intended destination. In doing so, roughly speaking, the effective noise floor at a receiver is at most doubled, thus reducing the SNR (Signal to Noise Ratio) by a factor of at most two, which gives a rate penalty of at most one bit/sec/Hz.

Fading GIFC: GIFCs with fading were recently considered in [10], [11], [12], [13], [14], [15], [1], [16], [17], [2], [18].

For fast-fading channels, the ergodic (Shannon) capacity is the measure of the ultimate system performance. In [10], it was shown that the ergodic sum-rate capacity of a K-source fading GIFC scales linearly with the number of sources. In [11] the capacity of parallel (multi-carrier) GIFCs with three sources was considered and it was shown with an example that separate encoding for each subchannel (carrier) is suboptimal. In [12], the sum-rate capacity of a two-source strong ergodic fading GIFCs was shown to be equal to that of the corresponding compound MAC. In [13], power allocation policies based on capacity inner and outer bounds for ergodic fading GIFCs with perfect transmitter CSI (Channel State Information) were derived.

For slow-fading channels, the proper measure of performance is the outage capacity. In particular, the DMT (Diversity Multiplex Tradeoff) proposed by Zheng and Tse [19] quantifies the tradeoff between rate and outage probability in the high-SNR regime. In [14] the DMT of symmetric two-source BF-GIFCs was studied based on the "to within one bit" outer bound of [6]. The authors of [14] claimed that the derived DMT is actually achievable because the one bit penalty for using a simple HK strategy vanishes at high SNR. However, the achievability of the "to within one bit" outer bound of [6] requires a very specific rate splitting in the HK scheme that depends on the instantaneous fading values. Hence, as pointed out in [15], [1], [16], [17], [2] the DMT derived in [14] is achievable only if the transmitters know the instantaneous fading values perfectly. In the case of no CSIT[1] the DMT of [14] is an upper-bound on the actual DMT.

The DMT of BF-GIFCs without CSIT was investigated in [15], [1], [16], [17], [2], [18]. All these parallel and independent works report similar results, albeit with different proof methods, and provide different insights into the problem. In [15], it was proved that joint decoding of all messages at all destinations achieves the DMT outer-bound of [14] in very strong interference. In [1] a diversity lower bound based on the HK region without rate splitting was shown to be optimal in symmetric networks with sufficiently strong interference. In [16], it was shown that multilevel superposition coding achieves the DMT of any two-source BF-GIFC; however, the diversity is explicitly evaluated only for symmetric channels and for two-level superposition (which corresponds to the HK scheme). In [17], it was shown the optimum DMT for channels where the average SNR's and the average INR's (Interference to Noise Ratio) are the same at all receivers can be achieved if the transmitters are provided with a one-bit quantization for each channel gain. In [2] we derived a diversity lower bound based on the HK region with a rate splitting inspired by [6] and showed it to be optimal for a fairly large class of asymmetric networks. In [18], it was showed that the DMT of BF-GIFCs reduces to that of a Multiple Access Channel (MAC) if the transmitters are not aware of the channel gains and that rate splitting can be ignored for asymptotic analysis if all SNRs and INRs are the same.

The works [14], [15], [17], [18] focused on two-source *symmetric* networks, that is to say, networks for which there is a complete symmetry among the users in terms of average SNR, average INR, transmit rate and diversity. In this work, we consider fully *asymmetric* GIFCs.[2] It should be pointed out that our results are not just a simple generalization of the symmetric network results. Our setting covers all possible classes of channels and includes channels not possible under the symmetric assumptions, such as the case of "mixed" interference. Mixed interference occurs in practice when sources are at different distance from their intended destination and it is the most practical scenario for wireless networks.

B. Contributions

In this work we assume a GIFC with independent Rayleigh fading. Extensions so as to include correlated Rayleigh fading can be carried out similarly to [20] (and references therein).[3] Moreover, the analysis for the Rayleigh fading channel can be easily extended to channels where the fading power gains have a distribution with an exponential tail [21, eq.(47)].

We assume that the channel is block fading with perfect CSIR and no CSIT. In this case, if the instantaneous fading realization is such that the transmission rates cannot be reliably decoded, the system is said to experience an *outage* [22]. In an outage setting without CSIT, it is not clear whether a fixed rate splitting strategy can actually achieve the DMT upper bound of [14]–whose achievability "to within one bit" requires a rate splitting that is a function of the instantaneous channel gains.

Since the capacity region of the interference channel is not known in general, we bound the diversity by considering inner and outer bounds on the capacity region. Our contributions can be summarized as follows:

1) We derive a diversity upper bound by extending the result of [14] to asymmetric networks, i.e., where the user rates and the average SNRs and INRs are not restricted to be the same. We also point out an error in one of the expressions presented in [14], i.e., the outer bound in [14] can be tightened.
2) We derive a diversity lower bound based on the HK achievable region. We consider both the case without rate splitting and the case with rate-spitting. Our rate splitting is inspired by the recent "to within one bit" capacity result of [6].
3) An analytical comparison of our diversity upper and lower bounds seems difficult because of the many parameters involved and because of the complexity of the diversity expressions. By numerical evaluations, we show that for a very wide range of channel parameters, our inner and outer bound meet. In particular, we show that the proposed rate splitting improves the achievable DMT for channels with weak and mixed interference over the no-rate splitting case. The Z-IFC (where one of the receivers does not experience interference) is analyzed in detail.

The rest of the paper is organized as follows: Section II presents the system model and the problem formulation; Section III and IV present the DMT upper and lower bound, respectively; Section V presents numerical results for symmetric and asymmetric channels, as well as for the Z-IFC; Section VI concludes the paper.

II. Channel Model

A two-source single-antenna Rayleigh fading GIFC in standard form has outputs:

$$Y_u = H_{u1}X_1 + H_{u2}X_2 + Z_u \in \mathbb{C},\ u \in \{1,2\}, \quad (1)$$

where, without loss of generality, the white Gaussian noise Z_u has zero mean and unit variance, and the input X_u is subject to the average power constraint $\mathbb{E}[|X_u|^2] \le 1,\ u \in \{1,2\}$. Fig.1 shows the channel in (1). We assume that the channel is block-fading and that each codeword spans one fading block only, i.e., no coding across multiple blocks is allowed. The receivers are assumed to perfectly know the fading realization affecting their received signal for the current block, while the transmitters only know the joint fading statistics. Moreover, blocks are assumed to be long enough so that decoding is possible if the rates are inside the capacity region of the

[1]CSIT stands for CSI at the Transmitter and CSIR for CSI at the Receiver.

[2]Asymmetric scenarios were also considered in [16], even though analytical closed form results were only provided for symmetric networks.

[3]In [20] it was shown that correlation does not affect the exponent of the outage probability but only causes a "power offset" that depends on the determinant of the fading covariance matrix. The same techniques can be used in the context of this work to show that fading correlation does not affect the DMT. Thus, without loss of generality, the DMT can be studied by considering independent Rayleigh fading only.

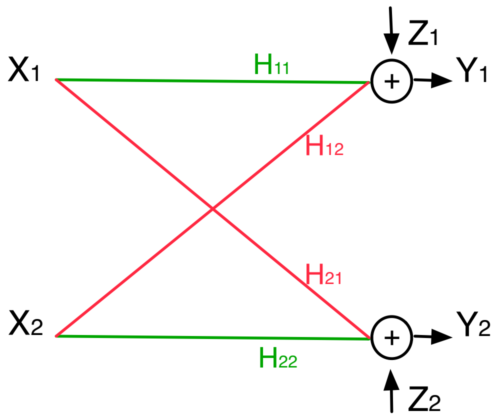

Fig. 1. The two-user Gaussian Interference Channel model.

corresponding unfaded channel with gains given by the fading realization on the block [22], which we shall refer to as the *instantaneous capacity region*. An *outage* occurs if the transmit rates are outside the instantaneous capacity region. The evaluation of the outage probability in analytical form is a difficult problem in a general. However, the outage probability can be tightly approximated in the high-SNR regime by using the DMT framework proposed in [19].

A. High-SNR parameterization

Before giving the precise definition of DMT we specify what we mean by high-SNR in an asymmetric network. In our analysis, we use the following parameterization. Let x be a real-valued constant larger than one. The average power at which the signal from source u is received at destination c is:

$$\mathbb{E}[|H_{cu}X_u|^2] = \mathbb{E}[|H_{cu}|^2]\mathbb{E}[|X_u|^2] = \mathbb{E}[|H_{cu}|^2] \triangleq x^{\beta_{cu}}, \quad (2)$$

with $\beta_{cu} \in \mathbb{R}^+$, $(c,u) \in \{1,2\}\times\{1,2\}$. With (2), owing to the normalization of the noise power and the unit transmit power, the parameters $\{\beta_{cu}\}_{(c,u)\in\{1,2\}\times\{1,2\}}$ relate to the average SNRs and INRs as follows: at receiver c, $c \in \{1,2\}$, the desired signal from source c is received with an average power of $\mathbb{E}[|H_{cc}X_c|^2] = x^{\beta_{cc}} \triangleq \mathrm{SNR}_c$, while the interfering signal from source $u = 3-c$ is received with an average power of $\mathbb{E}[|H_{cu}X_u|^2] = x^{\beta_{cu}} \triangleq \mathrm{INR}_c$.

A symmetric scenario corresponds to $\mathrm{SNR}_1 = \mathrm{SNR}_2 = \mathrm{SNR}$ and $\mathrm{INR}_1 = \mathrm{INR}_2 = \mathrm{INR}$; in this case, one can set without loss of generality $\beta_{11} = \beta_{22} = 1$ and $\beta_{21} = \beta_{12} = \alpha \geq 0$ [14]. In the symmetric case thus, x coincides with SNR and $\mathrm{INR} = \mathrm{SNR}^{\alpha}$; the high-SNR regime is obtained by letting $x = \mathrm{SNR} \to +\infty$.

Here we consider a general *asymmetric* scenario with $\beta_{11} \neq \beta_{22}$ and $\beta_{21} \neq \beta_{21}$; again the high-SNR regime is obtained by letting $x \to +\infty$.

We also parameterize the fading gains as:

$$H_{cu} = \sqrt{x^{\beta_{cu}-\gamma_{cu}}}\, e^{j\theta_{cu}}. \quad (3)$$

Since the fading gains are assumed to be independent, also the pair of random variables $(\gamma_{cu}, \theta_{cu})$, $(c,u) \in \{1,2\}\times\{1,2\}$, are independent. Moreover, for a Rayleigh fading model, the normalized fading power $|H_{cu}|^2/\mathbb{E}[|H_{cu}|^2] = x^{-\gamma_{cu}}$ and the phase $\angle H_{cu} = \theta_{cu}$ are independent, with θ_{cu} uniformly distributed on $[0, 2\pi]$ and $x^{-\gamma_{cu}}$ a negative exponential random variable with unit mean.

In high-SNR, the rates scale logarithmically with SNR [19], thus we parameterize them as:

$$R_u \triangleq \log(1 + x^{r_u}), \quad r_u \in \mathbb{R}^+, \quad u \in \{1,2\}. \quad (4)$$

Although we impose that the "channel gains" β's in (2) and the "rates" r's in (4) should be non-negative, the results derived in the following can be extended to any β's and r's by replacing each β with $[\beta]^+$ and each r with $[r]^+$, where we defined $[t]^+ = \max\{t, 0\}$ for any $t \in \mathbb{R}$.

B. Diversity

With the high-SNR parameterization in (2), (3), and (4), the diversity, or the exponent of the outage probability at high-SNR, is as follows. Since the transmitters do not have CSIT, the transmission rates r_1 and r_2 are chosen only based on the statistical properties of the channel, and not on the instantaneous fading realization. This implies that, for a fixed r_1 and r_2, there is a non-vanishing probability that a receiver is not able to decode its intended message. The probability of outage $\mathbb{P}_{\mathrm{out}}(r_1, r_2)$ is the probability of decoding failure, which occurs when the rates are outside the instantaneous capacity region [22]. The diversity is thus given by:

$$d(r_1, r_2) = \lim_{x\to+\infty} \frac{-\log(\mathbb{P}_{\mathrm{out}}(r_1, r_2))}{\log(x)}. \quad (5)$$

Because the capacity region of the GIFC is not known in general, we will bound it with known inner and outer bounds, thus yielding upper and lower bounds on the diversity. The capacity of the unfaded GIFC is outer bounded by the "to within one bit" region of [6], indicated as $\mathcal{R}_{\mathrm{ETW}}$ and defined in (14). The best known achievable region for the GIFC is due to Han and Kobayashi [3], referred to as $\mathcal{R}_{\mathrm{HK}}$ and defined in (9). With this, we bound the outage probability as:

$$\begin{aligned} \mathbb{P}[(\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}) &\notin \mathcal{R}_{\mathrm{ETW}}] \\ &\leq \mathbb{P}_{\mathrm{out}}(r_1, r_2) \leq \\ \mathbb{P}[(\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}) &\notin \mathcal{R}_{\mathrm{HK}}], \end{aligned}$$

and thus the diversity is bounded by:

$$d_{\mathrm{HK}}(r_1, r_2) \leq d(r_1, r_2) \leq d_{\mathrm{ETW}}(r_1, r_2), \quad (6)$$

where $d_{\mathrm{ETW}}(r_1, r_2)$ and $d_{\mathrm{HK}}(r_1, r_2)$ are defined similarly to $d(r_1, r_2)$ in (5) but with $\mathbb{P}[(\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}) \notin \mathcal{R}_{\mathrm{ETW}}]$ and $\mathbb{P}[(\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}) \notin \mathcal{R}_{\mathrm{HK}}]$, respectively, instead of $\mathbb{P}_{\mathrm{out}}(r_1, r_2)$.

The rest of the paper is devoted to the evaluation of $d_{\mathrm{ETW}}(r_1, r_2)$ (Theorem 1 in Section III) and $d_{\mathrm{HK}}(r_1, r_2)$ (Theorem 2 and Theorem 3 in Section IV). In Section V we investigate by numerical evaluations for which subset of $(\beta_{11}, \beta_{12}\beta_{21}, \beta_{22})$ one has $d_{\mathrm{ETW}}(r_1, r_2) = d_{\mathrm{HK}}(r_1, r_2) = d(r_1, r_2)$.

III. Diversity Upper Bound

In this section we extend the diversity upper bound of [14] to asymmetric networks and point out an error in [14, expression for d_d in Th.1].

theorem 1. *The diversity of a BF-GIFC is upper bounded by:*

$$d_{\rm ETW}(r_1, r_2) = \min_{\ell \in \{(16a),\ldots,(16g)\}} \{d_\ell\},$$

where

$$d_{(16a)} = [\beta_{11} - r_1]^+, \quad (7a)$$
$$d_{(16b)} = [\beta_{22} - r_2]^+, \quad (7b)$$
$$d_{(16c)} = f_s(\beta_{21}), \quad (7c)$$
$$d_{(16d)} = f_s(\beta_{12}), \quad (7d)$$
$$d_{(16e)} = f_s(\beta_{21} + \beta_{21}), \quad (7e)$$
$$d_{(16f)} = \text{eq.(18)}, \quad (7f)$$
$$d_{(16g)} = \text{eq.(18) with the role of the users swapped}, \quad (7g)$$

where

$$f_s(x) \triangleq \max\Big\{[\beta_{11} - r_s]^+ + [\beta_{22} - r_s]^+ + [x - r_s]^+, \beta_{11} + \beta_{22} - x - r_s\Big\}\Big|_{r_s = r_1 + r_2}. \quad (8)$$

Proof: The proof can be found in Appendix A. ■

Remark 1: For symmetric networks, i.e., $\beta_{11} = \beta_{22} = 1$, $\beta_{12} = \beta_{21} = \alpha$ and $r_1 = r_2 = r$ (i.e., $r_s = 2r$ and $r_f = r_g = 3r$), every term in (7) reduces to the corresponding expression in [14, Th.1], except for $d_{(16f)}$ in (7f). Indeed, it is straightforward to see that

$$d_{(16a),\rm sym} = d_{(16b),\rm sym} = [1 - r]^+$$

is like d_a in [14, Th.1], and that

$$d_{(16c),\rm sym} = d_{(16d),\rm sym} = 2\ \max\left\{[A]^+ + [B]^+, A + |B|\right\}\Big|_{A = 1-2r, B = \frac{\alpha}{2} - r},$$

is like d_b in [14, Th.1] given by

$$d_b = 2[1 - r - \min\{r, \alpha/2\}]^+ + [\alpha - 2r]^+.$$

Similarly, one can show that $d_{(16e),\rm sym}$ coincides with d_c in [14], i.e., $d_{(16e),\rm sym}$ is like $d_{(16c),\rm sym}$ but with α replaced by 2α. However, $d_{(16f),\rm sym} = d_{(16g),\rm sym}$ is not equivalent to d_d in [14, Th.1]. In fact, it turns out that d_d in [14] is not correct. Consider the following numerical example: let $r = 0.4$ and $\alpha = 0.5$. The optimization problem for d_d in [14] is:

$$d_d = \min\{\gamma_{11} + \gamma_{12} + \gamma_{21} + \gamma_{22}\}$$

subject to

$$[[1 - \gamma_{11}]^+ - [\alpha - \gamma_{12}]^+]^+ + \max\{[1 - \gamma_{11}]^+, [\alpha - \gamma_{21}]^+\} + \max\{[\alpha - \gamma_{12}]^+, [1 - \gamma_{22}]^+ - [\alpha - \gamma_{21}]^+\} \leq 3r.$$

It can be easily verified that $\gamma_{11} = 0.4$, $\gamma_{12} = \gamma_{21} = \gamma_{22} = 0$ is a feasible solution that gives $d_d \leq (\gamma_{11} + \gamma_{12} + \gamma_{21}, \gamma_{22}) = 0.4$. However for $r = 0.4$ and $\alpha = 0.5$, one has

$$d_d = \max\Big\{[1 - \frac{3r}{2}]^+ + [1 - 3r]^+ + [2\alpha - 3r]^+, \min\left\{[3(1-r) - \min\{3r, 2\alpha\}]^+, \max\{1, 2 - 3r - \min\{3r, 2\alpha\}\}\right\}\Big\} = 0.8,$$

which cannot be correct because $d_d \leq 0.4$. Although d_d in [14] is not correct, the diversity upper bound in [14, Th.1] still holds since the diversity is upper bounded by $d_{(16f),\rm sym}$ which in turn is upper bounded by d_d.

IV. Diversity Lower Bound

As an inner bound to the capacity region of a GIFC, we consider the HK region:

$$\mathcal{R}_{\rm HK} = \bigcup_{P = P_Q P_{W_1, X_1|Q} P_{W_2, X_2|Q}} \mathcal{R}_{\rm HK}(P), \quad (9)$$

where [23, Lemma 4]:

$$\mathcal{R}_{\rm HK}(P) = \big\{(R_1, R_2) \in \mathbb{R}_+^2 :$$
$$R_1 \leq D_1 \quad (10a)$$
$$R_2 \leq D_2 \quad (10b)$$
$$R_1 + R_2 \leq A_1 + G_2 \quad (10c)$$
$$R_1 + R_2 \leq G_1 + A_2 \quad (10d)$$
$$R_1 + R_2 \leq E_1 + E_2 \quad (10e)$$
$$2R_1 + R_2 \leq A_1 + G_1 + E_2 \quad (10f)$$
$$R_1 + 2R_2 \leq E_1 + A_2 + G_2 \quad (10g)$$
$$R_1 \leq A_1 + E_2 \quad (10h)$$
$$R_2 \leq E_1 + A_2\big\}, \quad (10i)$$

for:

$$A_1 = I(X_1; Y_1|W_1, W_2, Q), \quad A_2 = I(X_2; Y_2|W_1, W_2, Q),$$
$$D_1 = I(X_1; Y_1|W_2, Q), \quad D_2 = I(X_2; Y_2|W_1, Q),$$
$$E_1 = I(X_1, W_2; Y_1|W_1, Q), \quad E_2 = I(X_2, W_1; Y_2|W_2, Q),$$
$$G_1 = I(X_1, W_2; Y_1|Q), \quad G_2 = I(X_2, W_1; Y_2|Q).$$

The region $\mathcal{R}_{\rm HK}$ is difficult to evaluate because it requires an optimization with respect to the joint distribution $P = P_Q P_{W_1,X_1|Q} P_{W_2,X_2|Q}$, where Q is a time-sharing random variable and (W_1, W_2) has the meaning of common information decoded at both receivers.[4] In order to have a region that can be easily evaluated, it is customary to assume jointly Gaussian inputs and no time sharing. We follow this approach and set W_u and T_u to be independent random variables, with W_u a zero-mean Gaussian with variance $P_{u,\rm c}$ and T_u a zero-mean Gaussian with variance $P_{u,\rm p}$; we let $X_u = W_u + T_u$ such that the total power constraint is met with equality, i.e., $P_{u,\rm p} + P_{u,\rm c} = 1$, for $u \in \{1, 2\}$. We further parameterize the ratio of the average private power to the total average power for a given user $u \in \{1, 2\}$ as

$$\alpha_u = \frac{P_{u,\rm p}}{P_{u,\rm p} + P_{u,\rm c}} = \frac{1}{1 + x^{b_u}} \in [0, 1], \quad b_u \in \mathbb{R}. \quad (11)$$

[4] The last two inequalities in $\mathcal{R}_{\rm HK}(P)$ can be removed without enlarging the achievable region $\mathcal{R}_{\rm HK}$, i.e., this is possible only if one takes the union of $\mathcal{R}_{\rm HK}(P)$ over all possible input distributions P [23, Remark 3]. However, for a fixed distribution P they cannot be removed.

With (11), the average received power of user u's private and common messages, $u \in \{1,2\}$, at receiver c, $c \in \{1,2\}$, are

$$\mathbb{E}[|H_{cu}|^2]P_{u,\mathrm{p}} = \alpha_u\,\mathbb{E}[|H_{cu}|^2] = \frac{x^{\beta_{cu}}}{1+x^{+b_u}}$$
$$\mathbb{E}[|H_{cu}|^2]P_{u,\mathrm{c}} = (1-\alpha_u)\,\mathbb{E}[|H_{cu}|^2] = \frac{x^{\beta_{cu}}}{1+x^{-b_u}}.$$

We consider the following cases:

1) (Subsection IV-B) the case without rate splitting, which corresponds to setting either $W_u = \emptyset$ (i.e., private message only, that is, interference is treated as noise) or $W_u = X_u$ (i.e., common message only, that is, interference is decoded jointly with the intended message);
2) (Subsection IV-C) inspired by [6], we consider the power split $b_1 = \beta_{21}$ and $b_2 = \beta_{12}$, so that the *average* interfering private power at the non-intended receiver is below the noise floor, that is, $\mathbb{E}[|H_{cu}|^2]P_{u,\mathrm{p}} = \frac{x^{b_u}}{1+x^{b_u}} \leq 1$, for $c \neq u \in \{1,2\}$.

A. Simple time sharing

For future comparison, a TDMA strategy that let user 1 transmit alone for a fixed fraction $\tau \in [0,1]$ of the time, and let user 2 transmit for the remaining time achieves diversity (see Appendix B):

$$d_{\mathrm{tdma}} = \max_{\tau\in[0,1]} \min\{[\beta_{11} - r_1/\tau]^+, [\beta_{22} - r_2/(1-\tau)]^+\}.$$

It follows easily that $d_{\mathrm{tdma}} > 0$ only for $r_1/\beta_{11} + r_2/\beta_{22} < 1$ in which case the optimal $\tau \in [0,1]$ solves $\beta_{11} - r_1/\tau = \beta_{22} - r_2/(1-\tau)$. In the symmetric case $d_{\mathrm{tdma,sym}} = [1-2r]^+$.

B. HK region without rate splitting

theorem 2. *By considering the HK region with Gaussian inputs, without time sharing, and without rate splitting, we can lower bound the diversity of a BF-GIFC by:*

$$d_{\mathrm{HK-wors}} = \min\big\{\max\{d_{\mathrm{NI1}}, d_{\mathrm{MAC1}}\},\, \max\{d_{\mathrm{NI2}}, d_{\mathrm{MAC2}}\}\big\},$$

where

$$\begin{aligned} d_{\mathrm{NI}u} &= [\beta_{uu} - r_u - \beta_{u,3-u}]^+,\\ d_{\mathrm{MAC}u} &= \min\{[\beta_{uu} - r_u]^+,\\ &\qquad [\beta_{u,3-u} - (r_1+r_2)]^+ + [\beta_{uu} - (r_1+r_2)]^+\}, \end{aligned}$$

for $u \in \{1,2\}$ (i.e., "wors" stands for "without rate splitting", "NI" for "noisy interference" and "MAC" for "multiple access channel").

Proof: The proof can be found in Appendix C. ∎

Remark 2: The diversity $d_{\mathrm{HK-wors}}$ has the following intuitive interpretation: each user $u \in \{1,2\}$ chooses the best strategy between treating the interference as noise ($d_{\mathrm{NI}u}$) and joint decoding ($d_{\mathrm{MAC}u}$); the overall diversity without rate splitting $d_{\mathrm{HK-wors}}$ is dominated by the the worst user.

C. HK region with ETW-inspired rate splitting

theorem 3. *By considering the HK region with Gaussian inputs, without time sharing, and with a rate split inspired by the "to within one bit" capacity result of [6], we can lower bound the diversity of a BF-GIFC by:*

$$d_{\mathrm{HK-etw}} = \min_{\ell\in\{(20\mathrm{a}),\ldots,(20\mathrm{g})\}}\{d_\ell\},$$

where

$$d_{(20\mathrm{a})} = [\beta_{11} - r_1]^+, \tag{12a}$$
$$d_{(20\mathrm{b})} = [\beta_{22} - r_2]^+, \tag{12b}$$
$$\begin{aligned} d_{(20\mathrm{c})} = \max\{&[\beta_{22} - r_s]^+ + [\beta_{21} - r_s]^+,\\ &\beta_{11} - r_s + [\beta_{22} - r_s]^+,\\ &\beta_{11} - r_s + (\beta_{22} - \beta_{21}) + [\beta_{21} - r_s]^+\}\} \end{aligned} \tag{12c}$$
$$\begin{aligned} d_{(20\mathrm{d})} = \max\{&[\beta_{11} - r_s]^+ + [\beta_{12} - r_s]^+,\\ &\beta_{22} - r_s + [\beta_{11} - r_s]^+,\\ &\beta_{22} - r_s + (\beta_{11} - \beta_{12}) + [\beta_{12} - r_s]^+\}\} \end{aligned} \tag{12d}$$
$$\begin{aligned} d_{(20\mathrm{e})} = \min_{\alpha\in[0,1]} \Big\{&[\beta_{22} - \beta_{12} - \alpha r_s]^+ + [\beta_{21} - \alpha r_s]^+\\ &+ [\beta_{11} - \beta_{21} - (1-\alpha)r_s]^+ + [\beta_{12} - (1-\alpha)r_s]^+\Big\}, \end{aligned} \tag{12e}$$
$$\begin{aligned} d_{(20\mathrm{f})} \min_{\alpha\in[0,1]} \Big\{&[\beta_{22} - \beta_{12} - \alpha r_f]^+ + [\beta_{21} - \alpha r_f]^+\\ &+ \min\Big\{[\beta_{11} + \beta_{12} - \beta_{21} - (1-\alpha)r_f]^+,\\ &\max\{[\beta_{11} - (1-\alpha)r_f]^+, [\beta_{11} - \frac{\beta_{21} + (1-\alpha)r_f}{2}\}\\ &+ [\beta_{12} - (1-\alpha)r_f]^+\Big\}\Big\}, \end{aligned} \tag{12f}$$
$$\begin{aligned} d_{(20\mathrm{g})} = \min_{\alpha\in[0,1]} \Big\{&[\beta_{11} - \beta_{21} - \alpha r_g]^+ + [\beta_{12} - \alpha r_g]^+\\ &+ \min\Big\{[\beta_{22} + \beta_{12} - \beta_{21} - (1-\alpha)r_g]^+,\\ &\max\{[\beta_{22} - (1-\alpha)r_g]^+, [\beta_{22} - \frac{\beta_{12} + (1-\alpha)r_g}{2}\}\\ &+ [\beta_{21} - (1-\alpha)r_g]^+\Big\}\Big\}. \end{aligned} \tag{12g}$$

Proof: The proof can be found in Appendix D. ∎

Remark 3: The actual achievable diversity with the HK scheme in (6) satisfies

$$d_{\mathrm{HK}} \geq \max\{d_{\mathrm{HK-wors}}, d_{\mathrm{HK-etw}}\}.$$

In the online draft of this paper [24], we considered a general rate split in the HK region. The resulting expression for the diversity is quite complex and not particularly insightful. For this reason we omitted it from this paper. In the next section we show by numerical evaluations that the diversity lower bound $d_{\mathrm{HK-etw}}$ coincides with the diversity upper bound d_{ETW} for a very large set of channel parameters $(\beta_{11}, \beta_{12}, \beta_{21}, \beta_{22})$.

V. Numerical results

In this section we present numerical evaluations of the diversity upper bound d_{ETW} in (15) from Theorem 1, the

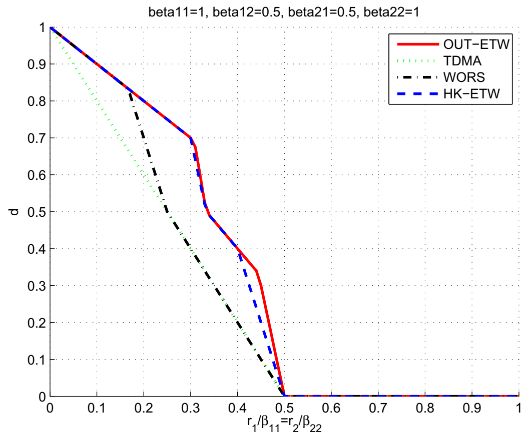

Fig. 2. Symmetric channel in weak interference: $\beta_{11} = \beta_{22} = 1, \beta_{12} = \beta_{21} = 1/2$.

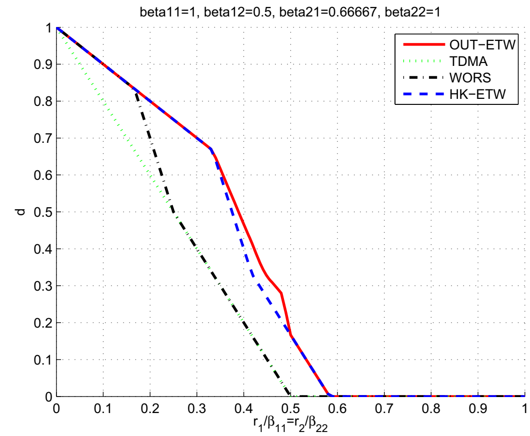

Fig. 3. Asymmetric channel in weak interference: $\beta_{11} = \beta_{22} = 1, \beta_{12} = 1/2, \beta_{21} = 2/3$.

diversity lower bound without rate splitting $d_{\text{HK}-\text{wors}}$ in (19) from Theorem 2, and the diversity lower bound with the rate split inspired by the "to within one bit" result $d_{\text{HK}-\text{etw}}$ in (21) from Theorem 3, for different values of the channel parameters $(\beta_{11}, \beta_{12}, \beta_{21}, \beta_{22})$. For easy of exposition, we consider the symmetric-rate case $r_1 = r_2 = r$ and equal direct-link case $\beta_{11} = \beta_{22} = 1$.

A. Symmetric channels

We first consider the fully symmetric channel with $\beta_{12} = \beta_{21} = \beta \geq 0$. Fig. 2 shows the diversity vs. the common multiplexing gain $r_1 = r_2 = r$ for $\beta = 1/2$. We notice that the upper bound d_{ETW} and the lower bound $d_{\text{HK}-\text{etw}}$ coincide at low rate, i.e., $r \leq 0.3$ and at medium rate, $r \in [0.33, 0.4]$. In general, we noticed that in weak interference, i.e., $\beta < 1$, at low rate the most stringent constraint is the "single user diversity" $d_{(16\text{a})} = d_{(16\text{b})}$, at medium rate is the "sum-rate diversity" $d_{(16\text{f})} = d_{(16\text{g})}$. while at high rate is $d_{(16\text{e})}$, In strong interference, i.e., $\beta \geq 1$, at low rate the most stringent constraint is the "single user diversity" $d_{(16\text{a})} = d_{(16\text{b})}$, while at medium and high rate is the "sum-rate diversity" $d_{(16\text{c})} = d_{(16\text{d})}$, while at high rate is $d_{(16\text{e})}$; in strong interference rate splitting is not needed.

The difference between the diversity upper bound in [14] and our d_{ETW}, with d_{ETW} being the tightest, can be seen for example at $\beta = 2/3$.

In [15] it was shown that the HK scheme without rate splitting is optimal for $\beta \geq 2$. By simulation, we found that d_{ETW} is achievable for symmetric channels by HK scheme without rate splitting for $\beta \geq 3/2 = 1.5$ (see also Fig. 5).

B. Asymmetric channels

In Figs. 3 (weak interference) and 4 (mixed interference) we plot the diversity vs. the common multiplexing gain $r_1 = r_2 = r$ for asymmetric channels with $\beta_{11} = \beta_{22} = 1$ and $\beta_{12} \neq \beta_{21}$. We notice that the upper bound d_{ETW} and the lower bound $d_{\text{HK}-\text{etw}}$ coincide at low rate and at high rate; for sufficiently strong interference upper and lower bound coincide for all

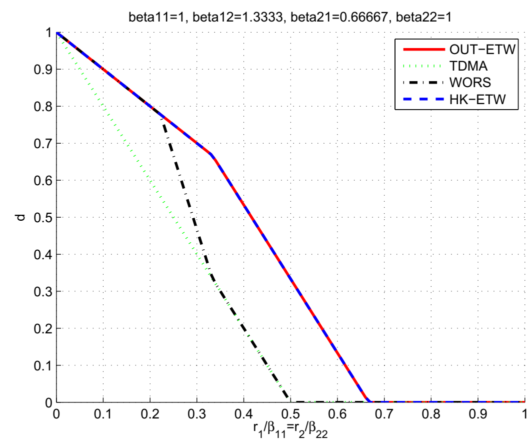

Fig. 4. Asymmetric channel in mixed interference: $\beta_{11} = \beta_{22} = 1, \beta_{12} = 4/3, \beta_{21} = 2/3$.

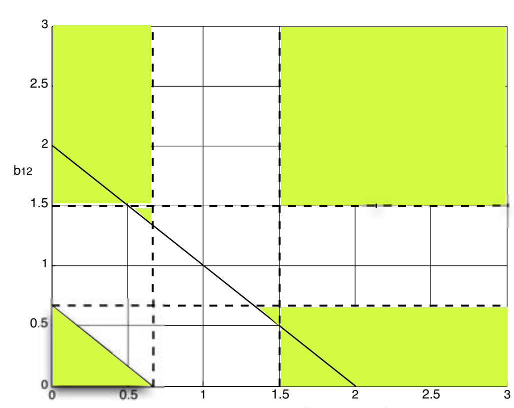

Fig. 5. Shaded/solid-filled area: range of (β_{12}, β_{21}) where inner and outer bound coincides for $\beta_{11} = \beta_{22} = 1$ at all rates.

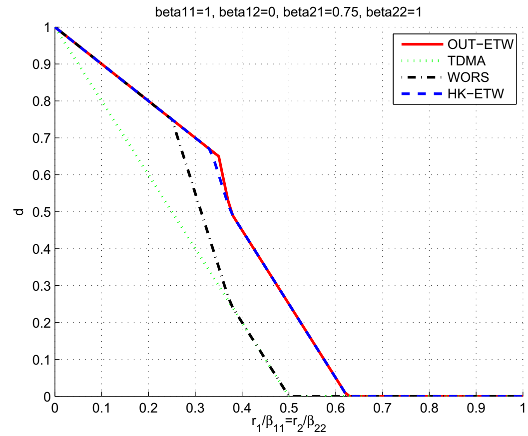

Fig. 6. Z-channel in weak interference: $\beta_{11} = \beta_{22} = 1, \beta_{12} = 3/4, \beta_{21} = 0$.

rates. In Fig. 5 the shaded/solid filled area corresponds to the parameter range for (β_{12}, β_{21}) where inner and outer bound coincides for $\beta_{11} = \beta_{22} = 1$ at all rates. We see that the simple rate split in the HK region inspired by the "to within one bit" capacity result is optimal for a fairly large set of parameters.

In very strong interference, i.e., $\min\{\beta_{12}, \beta_{21}\} \geq \beta_{11}+\beta_{22}$, the interference is so strong that each user can completely remove the unintended signal before decoding its own signal. In this case the capacity region, and hence the diversity, is the Cartesian product of the single user capacities without interference.

C. The Z-IFC

Consider the channel with $\beta_{21} = 0$. In this case the outer bound in (16) has only three active constraints: (16a), (16b) and (16d) (it is easy to see that the remaining constraints are redundant because $\beta_{21} = 0$ implies $X_{21} = 0$ in (16)). The diversity is thus upper bounded by:

$$d_{\text{ETW}}|_{\beta_{21}=0} = \min\left\{[\beta_{11} - r_1]^+, [\beta_{22} - r_2]^+, f_s(\beta_{12})\right\}$$

where $f_s(\beta_{12})$ is defined in (8). Similarly, the inner bound in (20) has only three active constraints: (20a), (20b) and (20d) and the diversity is thus lower bounded by:

$$d_{\text{HK-etw}}|_{\beta_{21}=0} = \min\left\{[\beta_{11} - r_1]^+, [\beta_{22} - r_2]^+, g_s(\beta_{12})\right\},$$

where

$$\begin{aligned} g_s(\beta_{12}) = \max\{&[\beta_{11} - r_s]^+ + [\beta_{12} - r_s]^+, \\ &[\beta_{11} - r_s]^+ + \beta_{22} - r_s, \\ &\beta_{11} - r_s + \beta_{22} - \beta_{12} + [\beta_{12} - r_s]^+\}. \end{aligned}$$

In Fig. 6 we plot the diversity vs. the common multiplexing gain $r_1 = r_2 = r$ for a Z-channel with parameters $\beta_{11} = \beta_{22} = 1, \beta_{12} = 3/4, \beta_{21} = 0$. We notice that in general, lower and upper bound do not coincide at medium rate. At low rate they coincide because both bounds are dominated by the "single user diversity" $d_{(16\text{a})} = d_{(16\text{b})}$, while at high rate they coincide because both bounds are dominated by the "sum-rate diversity" $d_{(16\text{d})}$. As proved in Remark 6 in the Appendix, $d_{(20\text{d})} = d_{(16\text{d})}$ for $\min\{\beta_{12}, \beta_{22}\} \leq r_s$ (in the case of Fig. 6 this corresponds to $r \geq 3/8 = 0.3750$.)

VI. Conclusion

In this paper, we analyzed the diversity-multiplexing trade-off of two-source block-fading Gaussian interference channels without channel state information at the transmitters. As opposed to previous works, we considered generic asymmetric networks. We found that, a simple inner bound based on the HK scheme with fixed power split achieves the outer bound based on perfect channel state information at the transmitter for wide range of channel parameters.

Acknowledgments

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Appendix A
Proof of Theorem 1

The "to within one bit" outer bounds of [6] for the weak, the mixed and the strong interference regimes can be further bounded by using the following inequality:

$$\left[\log\frac{1+\text{SNR}_i}{1+\text{INR}_j}\right]^+ \leq \log\left(1+\frac{\text{SNR}_i}{1+\text{INR}_j}\right), \qquad (13)$$

as originally proposed in [14, region $\mathcal{R}_o(G)$ in eq(1)] so as to have a single outer bound region for all possible parameter regimes (weak, mixed and strong). By using the high-SNR parameterization in (2), (3), and (4), for each fading realization $\gamma = (\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22})$, the outer bound of [14, $\mathcal{R}_o(G)$ in eq(1)] can be rewritten as:

$$\mathcal{R}_{\rm ETW} = \Big\{(\gamma_{11},\gamma_{12},\gamma_{21},\gamma_{22}) \in \mathbb{R}^4 :$$
$$\log(1+x^{r_1}) \le \log(1+x^{\beta_{11}-\gamma_{11}}) \tag{14a}$$
$$\log(1+x^{r_2}) \le \log(1+x^{\beta_{22}-\gamma_{22}}) \tag{14b}$$
$$\log(1+x^{r_1})+\log(1+x^{r_2}) \le \log(1+\frac{x^{\beta_{11}-\gamma_{11}}}{1+x^{\beta_{21}-\gamma_{21}}}) + \log(1+x^{\beta_{22}-\gamma_{22}}+x^{\beta_{21}-\gamma_{21}}) \tag{14c}$$
$$\log(1+x^{r_1})+\log(1+x^{r_2}) \le \log(1+\frac{x^{\beta_{22}-\gamma_{22}}}{1+x^{\beta_{12}-\gamma_{12}}}) + \log(1+x^{\beta_{11}-\gamma_{11}}+x^{\beta_{12}-\gamma_{12}}) \tag{14d}$$
$$\log(1+x^{r_1})+\log(1+x^{r_2}) \le \log(1+x^{\beta_{12}-\gamma_{12}}+\frac{x^{\beta_{11}-\gamma_{11}}}{1+x^{\beta_{21}-\gamma_{21}}}) + \log(1+x^{\beta_{21}-\gamma_{21}}+\frac{x^{\beta_{22}-\gamma_{22}}}{1+x^{\beta_{12}-\gamma_{12}}}) \tag{14e}$$
$$2\log(1+x^{r_1})+\log(1+x^{r_2}) \le \log(1+\frac{x^{\beta_{11}-\gamma_{11}}}{1+x^{\beta_{21}-\gamma_{21}}}) + \log(1+x^{\beta_{11}-\gamma_{11}}+x^{\beta_{12}-\gamma_{12}}) + \log(1+x^{\beta_{21}-\gamma_{21}}+\frac{x^{\beta_{22}-\gamma_{22}}}{1+x^{\beta_{12}-\gamma_{12}}}) \tag{14f}$$
$$\log(1+x^{r_1})+2\log(1+x^{r_2}) \le \log(1+\frac{x^{\beta_{22}-\gamma_{22}}}{1+x^{\beta_{12}-\gamma_{12}}}) + \log(1+x^{\beta_{22}-\gamma_{22}}+x^{\beta_{21}-\gamma_{21}}) + \log(1+x^{\beta_{12}-\gamma_{12}}+\frac{x^{\beta_{11}-\gamma_{11}}}{1+x^{\beta_{21}-\gamma_{21}}})\Big\}. \tag{14g}$$

With the region in (14), we next evaluate $d_{\rm ETW}(r_1,r_2)$ in (6). By using the Laplace's integration method as in [19] we obtain:

$$d_{\rm ETW}(r_1,r_2) = \min_{\gamma\in(\widetilde{\mathcal{R}}_{\rm ETW})^c} \{\gamma_{11}+\gamma_{12}+\gamma_{21}+\gamma_{22}\} \tag{15}$$

where $\widetilde{\mathcal{R}}_{\rm ETW}$ is the large-x approximation of $\mathcal{R}_{\rm ETW}$ in (14) given by

$$\widetilde{\mathcal{R}}_{\rm ETW} = \Big\{(\gamma_{11},\gamma_{12},\gamma_{21},\gamma_{22}) \in \mathbb{R}_+^4 : \ X_{ij} \triangleq [\beta_{ij}-\gamma_{ij}]^+,$$
$$r_1 \le X_{11} \tag{16a}$$
$$r_2 \le X_{22} \tag{16b}$$
$$r_s \triangleq r_1+r_2 \le [X_{11}-X_{21}]^+ + \max\{X_{21},X_{22}\} \tag{16c}$$
$$r_s \triangleq r_1+r_2 \le [X_{22}-X_{12}]^+ + \max\{X_{12},X_{11}\} \tag{16d}$$
$$r_s \triangleq r_1+r_2 \le \max\{X_{12},X_{11}-X_{21}\} + \max\{X_{21},X_{22}-X_{12}\} \tag{16e}$$
$$r_f \triangleq 2r_1+r_2 \le [X_{11}-X_{21}]^+ + \max\{X_{11},X_{12}\} + \max\{X_{21},X_{22}-X_{12}\} \tag{16f}$$
$$r_g \triangleq r_1+2r_2 \le [X_{22}-X_{12}]^+ + \max\{X_{22},X_{21}\} + \max\{X_{12},X_{11}-X_{21}\}\Big\}. \tag{16g}$$

The optimization problem in (15) can be solved as follows: since the complement of $\widetilde{\mathcal{R}}_{\rm ETW}$ in (16) is the union of the complement of the conditions (16a) through (16g), by applying the method of [25] we can show that the diversity in (15) evaluates to:

$$d_{\rm ETW}(r_1,r_2) = \min_{\ell\in\{(16a),\ldots,(16g)\}}\{d_\ell\},$$
$$d_\ell \triangleq \beta_{11}+\beta_{12}+\beta_{21}+\beta_{22}+ - \max\{X_{11}+X_{12}+X_{21}+X_{22}\},$$

where the maximization for the problem d_ℓ is over $\{X_{cu}\}$, $\{c,u\} \in \{1,2\}\times\{1,2\}$, subject to "the ℓ-th subequation of (16) is NOT satisfied". We now proceed to evaluate each d_ℓ, for $\ell \in \{(16a),\ldots,(16g)\}$. Notice that here we index the different diversity terms with the equation number of the corresponding rate constraint.

A. Solution for $d_{(16a)}$ and $d_{(16b)}$

The diversity $d_{(16a)}$ (corresponding to the constraint in (16a)) is:

$$\begin{aligned} d_{(16a)} =& \beta_{11} - \max\{X_{11}\} \\ & \text{subj. to } 0 \le X_{11} \le \beta_{11}, \quad X_{11} \le r_1, \\ =& \beta_{11} - \min\{\beta_{11}, r_1\} = \max\{0, \beta_{11}-r_1\} \\ =& [\beta_{11}-r_1]^+, \end{aligned}$$

as in (7a).

The optimal value is attained by $\gamma_{11}^{(\text{opt.(16a)})} = [\beta_{11}-r_1]^+$.

Similarly to $d_{(16a)}$ but with the role of the users swaped, the diversity $d_{(16b)}$ (corresponding to the constraint in (16b)) is given by:

$$d_{(16b)} = [\beta_{22}-r_2]^+,$$

as in (7b) and the optimal value is attained by $\gamma_{22}^{(\text{opt.(16b)})} = [\beta_{22}-r_2]^+$.

B. Solution for $d_{(16c)}$, $d_{(16d)}$, and $d_{(16e)}$

The diversity $d_{(16c)}$ (corresponding to the constraint in (16c)) involves the following linear program:

$$\begin{aligned} J =& \max\{X_{11}+X_{21}+X_{22}\} \\ & \text{subj. to } 0 \le X_{11} \le \beta_{11},\ 0 \le X_{21} \le \beta_{21},\ 0 \le X_{22} \le \beta_{22}, \\ & \text{and to } \max\{X_{11},X_{21}\} + \max\{X_{22},X_{21}\} - X_{21} \le r_s, \end{aligned}$$

where $r_s \triangleq r_1+r_2$. We start by re-writing the last constraint as follows:

$$\begin{aligned} & \max\{X_{11},X_{21}\} + \max\{X_{22},X_{21}\} \le r_s + X_{21} \\ \iff & \begin{cases} X_{11}+X_{22} \le r_s + X_{21} \\ X_{21}+X_{22} \le r_s + X_{21} \\ X_{11}+X_{21} \le r_s + X_{21} \\ X_{21}+X_{21} \le r_s + X_{21} \end{cases} \\ \iff & \begin{cases} X_{11}+X_{22} \le r_s + X_{21} \\ \max\{X_{11},X_{21},X_{22}\} \le r_s \end{cases}, \end{aligned}$$

which, together with $X_{11} \le \beta_{11}$ and $X_{22} \le \beta_{22}$, implies that $X_{11}+X_{22}$ is upper bounded by:

$$X_{11}+X_{22} \le \min\{r_s+X_{21}, \min\{r_s,\beta_{11}\}+\min\{r_s,\beta_{22}\}\}.$$

With this, the optimization problem now becomes:

$$J=\max\Big\{\min\{X_{21}+\min\{r_s,\beta_{11}\}+\min\{r_s,\beta_{22}\},\ r_s+2X_{21}\}\Big\}$$
$$\text{subj. to } 0\le X_{21}\le\min\{r_s,\beta_{21}\}.$$

Because both functions in the min-espression for J achieve their maximum value at $X_{21}=\min\{r_s,\beta_{21}\}$, we have that $\max\min=\min\max$ and thus

$$J=\min\Big\{\min\{r_s,\beta_{11}\}+\min\{r_s,\beta_{22}\}+\min\{r_s,\beta_{21}\},\ r_s+2\min\{r_s,\beta_{21}\}\Big\}.$$

Finally, by subtructing the above expression for J from $\beta_{11}+\beta_{21}+\beta_{22}$, we obtain:

$$d_{(16c)}=f_s(\beta_{21}),$$

where:

$$\begin{aligned}f_s(t)&\triangleq\max\Big\{[\beta_{11}-r_s]^++[\beta_{22}-r_s]^++[t-r_s]^+,\\&\qquad\beta_{11}-r_s+\beta_{22}-r_s+|t-r_s|\Big\}\\&=\max\Big\{[\beta_{11}-r_s]^++[\beta_{22}-r_s]^++[t-r_s]^+,\\&\qquad\beta_{11}+\beta_{22}-t-r_s\Big\}\\&=\max\Big\{0,\beta_{11}+\beta_{22}+t-3r_s,\\&\qquad\max\{\beta_{22}+t,\beta_{11}+t,\beta_{11}+\beta_{22}\}-2r_s,\\&\qquad\max\{\beta_{11},\beta_{22},t,\beta_{11}+\beta_{22}-t\}-r_s,\Big\},\end{aligned}$$

as in (7c).

The optimal value is attained by:

$$\begin{aligned}&\gamma_{11}^{(\text{opt.}(16c))}+\gamma_{22}^{(\text{opt.}(16c))}\\&=\max\big\{[\beta_{11}-r_s]^++[\beta_{22}-r_s]^+,\\&\qquad(\beta_{11}-r_s)+(\beta_{22}-r_s)+[r_s-\beta_{21}]^+\big\},\\&\gamma_{21}^{(\text{opt.}(16c))}=[\beta_{21}-r_s]^+.\end{aligned}$$

The diversity $d_{(16d)}$ (corresponding to the constraint in (16d)) is as $d_{(16c)}$ but with β_{21} replaced by β_{12}, i.e., with the role of the users swapped, that is:

$$d_{(16d)}=f_s(\beta_{12}),$$

as in (7d).

The diversity $d_{(16e)}$ (corresponding to the constraint in (16e)) is as $d_{(16c)}$ but with $\beta_{21}+\beta_{12}$ instead of β_{21}, that is:

$$d_{(16e)}=f_s(\beta_{12}+\beta_{21}),$$

as in (7e).

Remark 4: In Theorem 2, we show that when receiver 2 jointly decodes both messages, one of the terms in the achievable diversity is

$$d_{\text{sumrate MAC2}}=[\beta_{21}-r_s]^++[\beta_{22}-r_s]^+.$$

It can be easily seen that $d_{\text{sumrate MAC2}}=f_s(\beta_{21})$ if

$$\beta_{11}\le\min\{\beta_{21},r_s\},$$

that is, in strong interference or for high rate. Similarly, $d_{\text{sumrate MAC1}}=f_s(\beta_{12})$ if

$$\beta_{22}\le\min\{\beta_{12},r_s\}.$$

Remark 5: It can be easily seen that the function $f_s(x)$, $x\in\mathbb{R}$, is non-decreasing for

$$x\ge x_{\text{lw}}\triangleq r_s-([\beta_{11}-r_s]^-+[\beta_{22}-r_s]^-),$$

(with $[x]^+=\max\{0,x\}\ge0$ and $[x]^-=\max\{0,-x\}\ge0$, $x\in\mathbb{R}$) and non-increasing for

$$x\le x_{\text{up}}\triangleq r_s,$$

i.e., it is actually flat for $x_{\text{lw}}<x<x_{\text{up}}$. This implies that:

$$\begin{aligned}&\min\{f_s(\beta_{12}),f_s(\beta_{21}),f_s(\beta_{12}+\beta_{21})\}\\&=f_s(\beta_{12}+\beta_{21})\\&\qquad\text{for }\beta_{12}+\beta_{21}\le x_{\text{up}},\\&=\min\{f_s(\max\{\beta_{12},\beta_{21}\}),f_s(\beta_{12}+\beta_{21})\}\\&\qquad\text{for }\beta_{12}+\beta_{21}>x_{\text{up}}\text{ and }\max\{\beta_{12},\beta_{21}\}<x_{\text{lw}},\\&=f_s(\max\{\beta_{12},\beta_{21}\})\\&\qquad\text{for }\beta_{12}+\beta_{21}>x_{\text{up}}\text{ and }x_{\text{lw}}\le\max\{\beta_{12},\beta_{21}\}\le x_{\text{up}},\\&=\min\{f_s(\beta_{12}),f_s(\beta_{21})\}\\&\qquad\text{for }\max\{\beta_{12},\beta_{21}\}>x_{\text{up}},\text{ and }\min\{\beta_{12},\beta_{21}\}<x_{\text{lw}},\\&=f_s(\min\{\beta_{12},\beta_{21}\})\\&\qquad\text{for }\max\{\beta_{12},\beta_{21}\}>x_{\text{up}},\text{ and }\min\{\beta_{12},\beta_{21}\}\ge x_{\text{lw}}.\end{aligned}$$

This allows analytical comparison of inner and outer bounds in some parameter regimes.

C. Solution for $d_{(16f)}$ and $d_{(16g)}$

The diversity $d_{(16f)}$ (corresponding to the constraint in (16f)) involves the following linear program:

$$\begin{aligned}&J=\max\{X+Y+Z+W\}\\&\quad\text{subj. to }0\le X\le\beta_{11},\ 0\le W\le\beta_{12},\\&\quad\text{and to }0\le Y\le\beta_{21},\ 0\le Z\le\beta_{22},\\&\quad\text{and to }\max\{X,Y\}+\max\{X,W\}+\\&\qquad+\max\{Z,Y+W\}-(Y+W)\le r_f,\end{aligned}$$

with $r_f\triangleq 2r_1+r_2$. The last constraint can be rewritten as

$$\begin{aligned}2X-W-Y+Z&\le r_f\\2X&\le r_f\\X-Y+Z&\le r_f\\X+W&\le r_f\\X-W+Z&\le r_f\\X+Y&\le r_f\\Z&\le r_f\\W+Y&\le r_f.\end{aligned}$$

This implies that, after substituting the optimal value for X, the optimization problem is now equivalent to maximizing $J(v)$, for $v = (Y, W, Z)$, defined as:

$$\begin{aligned} J(v) &= \min\{f_1(v), f_2(v)\}, \\ f_0(v) &= \min\{\min\{r_f/2, \beta_{11}\} + (Y+W) + Z, \\ &\qquad \frac{1}{2}r_f + \frac{3}{2}(Y+W) + \frac{1}{2}Z\}, \\ f_1(v) &= \min\{f_0(v), r_f + 2Y + W, r_f + Y + Z\}, \\ f_2(v) &= \min\{f_0(v), r_f + Y + 2W, r_f + W + Z\}, \end{aligned}$$

subject to $v = (Y, W, Z)$ being inside the polymatroid:

$$W \le \min\{r_f, \beta_{12}\},\ Y \le \min\{r_f, \beta_{21}\}, \tag{17a}$$
$$Y + W \le \min\{r_f, \beta_{12} + \beta_{21}\}, \tag{17b}$$
$$Z \le \min\{r_f, \beta_{22}\}. \tag{17c}$$

Since all the functions in the min-expression for J are linear, their maximum value is attained at a vertex of the polymatroid in (17). In particular $v_1 = \arg\max_v f_1(v)$ is:

$$\begin{aligned} v_1 : W &= \min\{\beta_{12}, [r_f - \beta_{21}]^+\}, \\ Y &= \min\{\beta_{21}, r_f\}, \\ Z &= \min\{\beta_{22}, r_f\}, \end{aligned}$$

(because all the functions in the min-expression that defines $f_1(v)$ are simultaneously maximized by the vertex of the polymatroid defined in (17) with the largest Y-coordiante) and $v_2 = \arg\max_v f_2(v)$ is:

$$\begin{aligned} v_2 : W &= \min\{\beta_{12}, r_f\}, \\ Y &= \min\{\beta_{21}, [r_f - \beta_{12}]^+\}, \\ Z &= \min\{\beta_{22}, r_f\}. \end{aligned}$$

(because all the functions in the min-expression that defines $f_2(v)$ are simultaneously maximized by the vertex of the polymatroid defined in (17) with the largest W-coordiante) Let v_0 be defined as

$$\begin{aligned} v_0 : W = Y &= \min\{\beta_{12}, \beta_{21}, r_f/2\}, \\ Z &= \min\{\beta_{22}, r_f\}. \end{aligned}$$

Note that whenever $W = Y$ we have that $f_1(v) = f_2(v)$. Since J is a linear program, and because of the symmetry of the objective function in W and Y, one can easily show that

$$J = \max\{J(v_0), J(v_1), J(v_2)\}.$$

Finally, by subtructing the above expression for J from $\beta_{11} + \beta_{12} + \beta_{21} + \beta_{22}$, we obtain

$$d_{(16f)} = \beta_{11} + \beta_{12} + \beta_{21} + \beta_{22} - \max\{J(v_0), J(v_1), J(v_2)\}. \tag{18}$$

The optimal value $d_{(16f)}$ is attained by v_1 or v_2 or v_0 (with $X = J - (Y + W + Z)$).

The diversity $d_{(16g)}$ (corresponding to the constraint in (16g)) is as $d_{(16f)}$ but with the role of the users is swapped.

Appendix B
Achievable DMT with TDMA

For a fixed $\tau \in [0, 1]$, the instantaneous capacity region with TDMA is:

$$\begin{aligned} \mathcal{R}_{\text{tdma}} = \Big\{ & R_1 \le \tau \log\left(1 + |H_{11}|^2\right), \\ & R_2 \le (1-\tau)\log\left(1 + |H_{22}|^2\right) \Big\}. \end{aligned}$$

The outage probability with independent Rayleigh fading is

$$\begin{aligned} \mathbb{P}_{\text{out,tdma}} &= 1 - \exp\left(-\frac{e^{R_1/\tau} - 1}{x^{\beta_{11}}} - \frac{e^{R_2/(1-\tau)} - 1}{x^{\beta_{22}}}\right) \\ &\le \min\left\{1, \frac{e^{R_1/\tau} - 1}{x^{\beta_{11}}} + \frac{e^{R_2/(1-\tau)} - 1}{x^{\beta_{22}}}\right\}. \end{aligned}$$

Since the above upper bound on $\mathbb{P}_{\text{out,tdma}}$ is tight in high-SNR, we have

$$d_{\text{tdma}} = \max_{\tau \in [0,1]} \min\{[\beta_{11} - r_1/\tau]^+, [\beta_{11} - r_2/(1-\tau)]^+\}.$$

Appendix C
Proof of Theorem 2

Without rate splitting in the HK region, a user either sends all private information or all common information. These two modes of operation correspond to either treating the interference as noise at the receiver, or performing joint decoding of the intended message and of the interference as in a MAC channel. We first consider these two cases separately and then we derive the achievable diversity with the HK scheme without rate splitting.

A. Treating interference as noise

Consider the case where the interference is treated as noise at destination 1. User 1 can be successfully decoded at receiver 1 by treating user 2 as noise if the fading gains are in $\mathcal{R}_{\text{NI1}}$, where:

$$\begin{aligned} \mathcal{R}_{\text{NI1}} = \Big\{ & (\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}) \in \mathbb{R}^4 : \\ & \log(1 + x^{r_1}) \le \log\left(1 + \frac{x^{\beta_{11} - \gamma_{11}}}{1 + x^{\beta_{12} - \gamma_{12}}}\right) \Big\}. \end{aligned}$$

By following the same approach used in the derivation of the diversity upper bound, we have that the exponent of the probability that user 1 cannot be decoded successfully at receiver 1 by treating user 2 as noise is given by:

$$\begin{aligned} d_{\text{NI1}} &= \beta_{11} + \beta_{12} - \max\{X_{11} + X_{12}\} \\ &\quad \text{subj. to } 0 \le X_{11} \le \beta_{11},\ 0 \le X_{12} \le \beta_{12}, \\ &\quad \text{and to } [X_{11} - X_{12}]^+ \le r_1 \\ &= \beta_{11} + \beta_{12} - \max\{X_{11} + X_{12}\} \\ &\quad \text{subj. to } 0 \le X_{11} \le \beta_{11},\ 0 \le X_{12} \le \beta_{12}, \\ &\quad \text{and to } X_{11} \le X_{12} + r_1 \\ &= \beta_{11} + \beta_{12} - \max\{\min\{\beta_{11}, X_{12} + r_1\} + X_{12}\} \\ &\quad \text{subj. to } 0 \le X_{12} \le \beta_{12}, \\ &= \beta_{11} + \beta_{12} - \min\{\beta_{11} + \beta_{12}, \beta_{12} + r_1 + \beta_{12}\} \\ &= [\beta_{11} - r_1 - \beta_{12}]^+. \end{aligned}$$

Similarly, the exponent of the probability that user 2 cannot be successfully decoded at receiver 2 by treating user 1 as noise is

$$d_{\rm NI2} = [\beta_{22} - r_2 - \beta_{21}]^+.$$

B. Interference decoding

Consider now the case where receiver 2 behaves like a MAC receiver. User 1 and user 2 can be successfully jointly decoded at receiver 2 if the fading gains are in $\mathcal{R}_{\rm MAC2}$, where:

$$\begin{aligned}\mathcal{R}_{\rm MAC2} = \Big\{&(\gamma_{21},\gamma_{22}) \in \mathbb{R}^2 : \\ &\log(1+x^{r_2}) \le \log(1+x^{\beta_{22}-\gamma_{22}}) \\ &\log(1+x^{r_1}) + \log(1+x^{r_2}) \le \log(1+x^{\beta_{21}-\gamma_{21}} + x^{\beta_{22}-\gamma_{22}})\Big\}.\end{aligned}$$

Notice that $\mathcal{R}_{\rm MAC2}$ differ from the capacity region of a MAC channel in that the rate constraint

$$\log(1+x^{r_1}) \le \log(1+x^{\beta_{21}-\gamma_{21}})$$

is not present. This is so because receiver 2 is not interested in the message sent by user 1; thus, an error in decoding the message from user 1 does not constitute an error for receiver 2. The exponent of the probability that both users cannot be jointly decoded at receiver 2 is given by:

$$d_{\rm MAC2} = \min\{[\beta_{22} - r_2]^+, [\beta_{22} - r_s]^+ + [\beta_{21} - r_s]^+\},$$

with $r_s \triangleq r_1 + r_2$ and where the last argument of the min-expression in $d_{\rm MAC2}$ can be derived as follows:

$$\begin{aligned}d_{\text{sumrate MAC2}} =& \beta_{22} + \beta_{21} - \max\{X_{21} + X_{22}\} \\ & \text{subj. to } 0 \le X_{22} \le \beta_{22},\ 0 \le X_{21} \le \beta_{21}, \\ & \text{and to } \max\{X_{22}, X_{21}\} \le r_s, \\ =& \beta_{22} + \beta_{21} - \min\{\beta_{21}, r_s\} - \min\{\beta_{22}, r_s\} \\ =& [\beta_{22} - r_s]^+ + [\beta_{21} - r_s]^+.\end{aligned}$$

Similarly, the exponent of the probability that user 1 and user 2 cannot be successfully jointly decoded at receiver 1 is

$$d_{\rm MAC1} = \min\{[\beta_{11} - r_1]^+, [\beta_{11} - r_s]^+ + [\beta_{12} - r_s]^+\}.$$

C. Diversity without rate splitting

Without rate splitting, we have:

$$d_{\rm HK-wors} = \max\{d_{00}, d_{01}, d_{10}, d_{11}\}, \tag{19}$$

where "wors" stands for "without rate splitting" and where:

- d_{11} is the diversity when both sources send only private information (which is sum-rate optimal for the unfaded GIFC with very weak interference [7]) given by:
$$d_{11} = \min\{d_{\rm NI1}, d_{\rm NI2}\}.$$
- d_{10} (and similarly for d_{01} but with the role of the users swapped) is the diversity when user 1 sends only private information and the user 2 sends only common information (which is sum-rate optimal for the unfaded GIFC with mixed interference [26]) given by:
$$d_{10} = \min\{d_{\rm NI1}, d_{\rm MAC2}\}.$$
- d_{00} is the diversity when both sources send common information (which is optimal the unfaded GIFC with strong interference [27], [28], [29]) given by:
$$d_{00} = \min\{d_{\rm MAC1}, d_{\rm MAC2}\}.$$

Since

$$\max\{\min\{a,b_1\},\min\{a,b_2\}\} = \min\{a, \max\{b_1,b_2\}\},$$

we further rewrite $d_{\rm HK-wors}$ in (19) as

$$\begin{aligned}&d_{\rm HK-wors} \\ &= \max\Big\{\min\{d_{\rm NI1}, d_{\rm NI2}\}, \min\{d_{\rm NI1}, d_{\rm MAC2}\}, \\ &\qquad \min\{d_{\rm MAC1}, d_{\rm NI2}\}, \min\{d_{\rm MAC1}, d_{\rm MAC2}\}\Big\} \\ &= \max\Big\{\min\{d_{\rm NI1}, \max\{d_{\rm NI2}, d_{\rm MAC2}\}\}, \\ &\qquad \min\{d_{\rm MAC1}, \max\{d_{\rm NI2}, d_{\rm MAC2}\}\}\Big\} \\ &= \min\Big\{\max\{d_{\rm NI1}, d_{\rm MAC1}\},\ \max\{d_{\rm NI2}, d_{\rm MAC2}\}\Big\},\end{aligned}$$

QED.

APPENDIX D
PROOF OF THEOREM 3

The HK region with Gaussian inputs, without time sharing and with a fixed rate split (b_1, b_2) is as in (10) with

$$\begin{aligned}A_1 &= \log\left(1 + \frac{\frac{x^{\beta_{11}-\gamma_{11}}}{1+x^{+b_1}}}{1 + \frac{x^{\beta_{12}-\gamma_{12}}}{1+x^{+b_2}}}\right), \\ D_1 &= \log\left(1 + \frac{x^{\beta_{11}-\gamma_{11}}}{1 + \frac{x^{\beta_{12}-\gamma_{12}}}{1+x^{+b_2}}}\right), \\ E_1 &= \log\left(1 + \frac{\frac{x^{\beta_{11}-\gamma_{11}}}{1+x^{+b_1}} + \frac{x^{\beta_{12}-\gamma_{12}}}{1+x^{-b_2}}}{1 + \frac{x^{\beta_{12}-\gamma_{12}}}{1+x^{+b_2}}}\right), \\ G_1 &= \log\left(1 + \frac{x^{\beta_{11}-\gamma_{11}} + \frac{x^{\beta_{12}-\gamma_{12}}}{1+x^{-b_2}}}{1 + \frac{x^{\beta_{12}-\gamma_{12}}}{1+x^{+b_2}}}\right),\end{aligned}$$

and the quantities A_2, D_2, E_2 and G_2 defined similarly but with the role of the users swapped.

Without loss of generality, we can take $b_1 \ge 0$ and $b_2 \ge 0$; with this, the large-x approximation of the achievable region is:

$$\widetilde{\mathcal{R}}_{\rm HK}(b_1,b_2) = \Big\{(\gamma_{11},\gamma_{12},\gamma_{21},\gamma_{22}) \in \mathbb{R}_+^4 :$$

$$r_1 \le d_1, \tag{20a}$$

$$r_2 \le d_2, \tag{20b}$$

$$r_s \triangleq r_1 + r_2 \le a_1 + g_2, \tag{20c}$$

$$r_s \triangleq r_1 + r_2 \le g_1 + a_2, \tag{20d}$$

$$r_s \triangleq r_1 + r_2 \le e_1 + e_2, \tag{20e}$$

$$r_f \triangleq 2r_1 + r_2 \le a_1 + g_1 + e_2 \tag{20f}$$

$$r_g \triangleq r_1 + 2r_2 \le a_2 + g_2 + e_1 \tag{20g}$$

$$r_1 \le a_1 + e_2, \tag{20h}$$

$$r_2 \le a_2 + e_1, \Big\}, \tag{20i}$$

with

$$
\begin{aligned}
a_1 &= [T_{11} - T_{12}]^+,\\
d_1 &= [X_{11} - T_{12}]^+,\\
e_1 &= [\max\{T_{11}, X_{12}\} - T_{12}]^+,\\
g_1 &= [\max\{X_{11}, X_{12}\} - T_{12}]^+,\\
&\quad X_{ij} \triangleq [\beta_{ij} - \gamma_{ij}]^+, \quad T_{ij} \triangleq [X_{ij} - b_j]^+,
\end{aligned}
$$

and the quantities a_2, d_2, e_2 and g_2 defined similarly but with the role of the users swapped.

We consider here the rate split $b_1 \geq \beta_{21}$ and $b_2 \geq \beta_{12}$ inspired by [6]; with this choice $T_{21} = T_{12} = 0$; we denote the resulting region as $\mathcal{R}_{\mathrm{HK-etw}}$. The evaluation of the diversity lower bound $d_{\mathrm{HK-etw}}$ can be carried out similarly to the evaluation of the diversity upper bound d_{ETW} in the previous section, that is:

$$
d_{\mathrm{HK-etw}}(r_1, r_2) = \min_{\gamma \in (\widetilde{\mathcal{R}}_{\mathrm{HK-etw}})^c} \{\gamma_{11} + \gamma_{12} + \gamma_{21} + \gamma_{22}\}, \tag{21}
$$

where $\widetilde{\mathcal{R}}_{\mathrm{HK-etw}}$ is the large-x approximation of the region $\mathcal{R}_{\mathrm{HK-etw}}$ given by (20) with

$$
\begin{aligned}
a_1 &= [X_{11} - b_1]^+,\\
d_1 &= [X_{11}]^+ = [\max\{X_{11} - b_1, X_{11}\}]^+,\\
e_1 &= [\max\{X_{11} - b_1, X_{12}\}]^+,\\
g_1 &= [\max\{X_{11}, X_{12}\}]^+,
\end{aligned}
$$

and the quantities a_2, d_2, e_2 and g_2 defined similarly but with the role of the users swapped. Notice that:

$$
a_1 \leq \min\{d_1, e_1\} \leq \max\{d_1, e_1\} \leq g_1.
$$

We finally have:

$$
\begin{aligned}
d_{\mathrm{HK-etw}} &= \max_{b_1 \geq \beta_{21}, b_2 \geq \beta_{12}} \min_{\ell \in \{(20\mathrm{a}), \ldots, (20\mathrm{i})\}} \{d_\ell\},\\
d_\ell &\triangleq \beta_{11} + \beta_{12} + \beta_{21} + \beta_{22}\\
&\quad - \max\{X_{11} + X_{12} + X_{21} + X_{22}\},
\end{aligned}
$$

where the maximization of d_ℓ is over $\{X_{cu}\}$, $(c, u) \in \{1, 2\} \times \{1, 2\}$, subject to "$\ell$-the subequation of (20) is NOT satisfied". The optimization problems for d_ℓ, $\ell \in \{(20\mathrm{a}), \ldots, (20\mathrm{i})\}$ are as follows.

A. Solution for $d_{(20\mathrm{a})}$ and $d_{(20\mathrm{b})}$

The diversity $d_{(20\mathrm{a})}$ and $d_{(20\mathrm{b})}$ (corresponding to the constraint in (20a) and (20b), respectively) are:

$$
d_{(20\mathrm{a})} = [\beta_{11} - r_1]^+,
$$

and

$$
d_{(20\mathrm{b})} = [\beta_{22} - r_2]^+,
$$

as for the upper bound.

B. Solution for $d_{(20\mathrm{c})}$ and $d_{(20\mathrm{d})}$

The diversity $d_{(20\mathrm{c})}$ (corresponding to the constraint in (20c)) involves the following linear program:

$$
\begin{aligned}
J = &\max\{X_{22} + X_{21} + X_{11}\}\\
&\text{subj. to } 0 \leq X_{22} \leq \beta_{22},\\
&\text{and to } 0 \leq X_{21} \leq \beta_{21},\ 0 \leq X_{11} \leq \beta_{11},\\
&\text{and to } \max\{X_{22}, X_{21}\} + [X_{11} - b_1]^+ \leq r_s,
\end{aligned}
$$

with $r_s \triangleq r_1 + r_2$. The last constraint can be rewritten as:

$$
\begin{aligned}
X_{11} + X_{22} &\leq r_s + b_1\\
X_{11} + X_{21} &\leq r_s + b_1\\
\max\{X_{22}, X_{21}\} &\leq r_s.
\end{aligned}
$$

This implies that the objective function (after maximization over X_{11}) can be expressed as:

$$
\begin{aligned}
J = \max\min\{&\beta_{11} + X_{21} + X_{22},\\
&r_s + b_1 + X_{22}, r_s + b_1 + X_{21}\},
\end{aligned}
$$

where the maximization is over:

$$
X_{22} \leq \min\{r_s, \beta_{22}\},\ X_{21} \leq \min\{r_s, \beta_{21}\}.
$$

Since all the functions in the min-expression for J are simultaneously maximized by $X_{22} = \min\{r_s, \beta_{22}\}$ and $X_{21} = \min\{r_s, \beta_{21}\}$, we have that $\max\min = \min\max$ and thus

$$
\begin{aligned}
d_{(20\mathrm{c})} &= \beta_{22} + \beta_{21} + \beta_{11} +\\
&\quad - \min\big\{\beta_{11} + \min\{r_s, \beta_{22}\} + \min\{r_s, \beta_{21}\},\\
&\qquad r_s + b_1 + \min\{r_s, \beta_{22}\},\\
&\qquad r_s + b_1 + \min\{r_s, \beta_{21}\}\big\}.\\
&= \max\{[\beta_{22} - r_s]^+ + [\beta_{21} - r_s]^+,\\
&\qquad \beta_{11} - r_s + [\beta_{22} - r_s]^+ + (\beta_{21} - b_1),\\
&\qquad \beta_{11} - r_s + (\beta_{22} - b_1) + [\beta_{21} - r_s]^+\}\big\}.
\end{aligned}
$$

The optimal values are:

$$
\begin{aligned}
\gamma_{11}^{(\mathrm{opt.}(20\mathrm{c}))} &= \max\{0, \beta_{11} - b_1 - [r_s - \beta_{22}]^+,\\
&\quad \beta_{11} - b_1 - [r_s - \beta_{21}]^+\},\\
\gamma_{22}^{(\mathrm{opt.}(20\mathrm{c}))} &= [\beta_{22} - r_s]^+,\\
\gamma_{21}^{(\mathrm{opt.}(20\mathrm{c}))} &= [\beta_{21} - r_s]^+.
\end{aligned}
$$

The diversity $d_{(20\mathrm{d})}$ (corresponding to the constraint in (20d)) is as $d_{(20\mathrm{c})}$ but with the role of the users reversed.

<u>Remark 6:</u> The function $d_{(20\mathrm{c})}$ is non-increasing in b_1, hence it attains its maximum over $\{b_1 \geq \beta_{21}\}$ at $b_1 = \beta_{21}$. For $b_1 = \beta_{21}$, the upper bound $d_{(16\mathrm{c})}$ and lower bound $d_{(20\mathrm{c})}$ coincide in the following cases: (a) when $\beta_{21} \leq r_s$, since $\gamma_{21}^{(\mathrm{opt.}(20\mathrm{c}))} = \gamma_{21}^{(\mathrm{opt.}(16\mathrm{c}))} = 0$, which implies

$$
\begin{aligned}
d_{(20\mathrm{c})}|_{\beta_{21} \leq r_s} &= \max\{[\beta_{22} - r_s]^+, \beta_{11} - r_s + [\beta_{22} - r_s]^+,\\
&\quad \beta_{11} - r_s + \beta_{22} - \beta_{21}\}\\
&= \max\{[\beta_{11} - r_s]^+ + [\beta_{22} - r_s]^+,\\
&\quad \beta_{11} + \beta_{22} - 2r_s + |\beta_{21} - r_s|\}\\
&= d_{(16\mathrm{c})}|_{\beta_{21} \leq r_s};
\end{aligned}
$$

(b) when $\beta_{21} > r_s$ and

$$\begin{aligned}&[\beta_{11} - r_s]^+ + [\beta_{22} - r_s]^+ + \beta_{21} - r_s\\ &= \max\{\beta_{11} - r_s, \beta_{21} - r_s\} + [\beta_{22} - r_s]^+,\end{aligned}$$

that is, when

$$[\beta_{11} - r_s]^+ = [\beta_{11} - \beta_{21}]^+ \iff \beta_{11} \le \min\{\beta_{21}, r_s\} = r_s.$$

By combining these two conditions together, we have that $d_{(16c)} = d_{(20c)}$ when $b_1 = \beta_{21}$ if $\min\{\beta_{21}, \beta_{11}\} \le r_s$. This can be seen for example in Fig. 4, line segment with slope -2, where lower and upper bound coincide for $r_s = 2r \ge \min\{2/3, 1\} \iff r \ge 1/3$.

Similarly, $d_{(20d)} = d_{(16d)}$ for $b_2 = \beta_{12}$ if $\min\{\beta_{12}, \beta_{22}\} \le r_s$.

C. Solution for $d_{(20e)}$

The diversity $d_{(20e)}$ (corresponding to the constraint in (20e)) involves the following linear program:

$$\begin{aligned}J = &\max\{X_{22} + X_{21} + X_{11} + X_{12}\}\\ &\text{subj. to } 0 \le X_{22} \le \beta_{22},\ 0 \le X_{21} \le \beta_{21},\\ &\text{and to } 0 \le X_{11} \le \beta_{11},\ 0 \le X_{12} \le \beta_{12},\\ &\text{and to } \max\{X_{11} - b_1, X_{12}\} +\\ &\quad + \max\{X_{22} - b_2, X_{21}\} \le r_s.\end{aligned}$$

If we rewrite the optimization domain as:

$$\begin{aligned}&X_{22} \le \beta_{22},\ X_{21} \le \beta_{21},\ \max\{X_{22} - b_2, X_{21}\} \le \alpha r_s,\\ &X_{11} \le \beta_{11},\ X_{12} \le \beta_{12},\ \max\{X_{11} - b_1, X_{12}\} \le (1-\alpha) r_s,\\ &\alpha \in [0,1],\end{aligned}$$

we immediately obtain:

$$\begin{aligned}J = \max_{\alpha \in [0,1]} \Big\{ &\min\{\beta_{22}, b_2 + \alpha r_s\} + \min\{\beta_{21}, \alpha r_s\} +\\ &+ \min\{\beta_{11}, b_1 + (1-\alpha) r_s\} + \min\{\beta_{12}, (1-\alpha) r_s\}\Big\},\end{aligned}$$

that is:

$$\begin{aligned}d_{(20e)} = \min_{\alpha \in [0,1]} \Big\{ &[\beta_{22} - b_2 - \alpha r_s]^+ + [\beta_{21} - \alpha r_s]^+\\ &+ [\beta_{11} - b_1 - (1-\alpha) r_s]^+ + [\beta_{12} - (1-\alpha) r_s]^+\Big\}.\end{aligned}$$

Again, $d_{(20e)}$ is a non-increasing function in (b_1, b_2), hence it attains its maximum over $\{b_1 \ge \beta_{21}, b_2 \ge \beta_{12}\}$ at $b_1 = \beta_{21}$ and $b_2 = \beta_{12}$. For $b_1 = \beta_{21}$ and $b_2 = \beta_{12}$, one can easily show that the optimal α is among the elements of the following set:

$$\begin{aligned}\alpha^{(\text{opt.(20e)})} r_s \in \{&0, r_s, \beta_{22} - \beta_{12}, \beta_{21},\\ &- \beta_{11} + \beta_{21} + r_s, -\beta_{12} + r_s\}.\end{aligned}$$

<u>Remark 7:</u> Because we do not have a closed expression for $d_{(20e)}$, it is difficult to compare $d_{(20e)}$ with $d_{(16e)}$.

D. Solution for $d_{(20f)}$ and $d_{(20g)}$

The diversity $d_{(20f)}$ (corresponding to the constraint in (20f)) involves the following linear program:

$$\begin{aligned}J = &\max\{X_{22} + X_{21} + X_{11} + X_{12}\}\\ &\text{subj. to } 0 \le X_{22} \le \beta_{22},\ 0 \le X_{21} \le \beta_{21},\\ &\text{and to } 0 \le X_{11} \le \beta_{11},\ 0 \le X_{12} \le \beta_{12},\\ &\text{and to } \max\{X_{11}, X_{12}\} + \max\{X_{11} - b_1, 0\} +\\ &\quad + \max\{X_{22} - b_2, X_{21}\} \le r_f \triangleq 2r_1 + r_2.\end{aligned}$$

If we rewrite the last constraints as:

$$\begin{aligned}&\max\{X_{22} - b_2, X_{21}\} \le \alpha r_f,\\ &\max\{X_{11}, X_{12}\} + [X_{11} - b_1]^+ \le (1-\alpha) r_f,\\ &\alpha \in [0,1],\end{aligned}$$

we obtain that the optimization domain is:

$$\begin{aligned}&X_{22} \le \min\{\beta_{22}, b_2 + \alpha r_f\},\\ &X_{21} \le \min\{\beta_{21}, \alpha r_f\},\\ &X_{11} \le \min\{\beta_{11}, (1-\alpha) r_f, \frac{b_1 + (1-\alpha) r_f}{2}\},\\ &X_{12} \le \min\{\beta_{12}, (1-\alpha) r_f\},\\ &X_{11} + X_{12} \le b_1 + (1-\alpha) r_f,\\ &\alpha \in [0,1],\end{aligned}$$

and we immediately obtain:

$$\begin{aligned}J = \max_{\alpha \in [0,1]} \Big\{ &\min\{\beta_{22}, b_2 + \alpha r_f\} + \min\{\beta_{21}, \alpha r_f\} +\\ &+ \min\Big\{b_1 + (1-\alpha) r_f,\\ &\min\{\beta_{11}, (1-\alpha) r_f, \frac{b_1 + (1-\alpha) r_f}{2}\} +\\ &+ \min\{\beta_{12}, (1-\alpha) r_f\}\Big\}\Big\},\end{aligned}$$

that is,

$$\begin{aligned}d_{(20f)} = \min_{\alpha \in [0,1]} \Big\{ &[\beta_{22} - b_2 - \alpha r_f]^+ + [\beta_{21} - \alpha r_f]^+ +\\ &+ \max\Big\{\beta_{11} + \beta_{12} - \beta_{21} - (1-\alpha) r_f,\\ &[\beta_{11} - \min\{(1-\alpha) r_f, \frac{b_1 + (1-\alpha) r_f}{2}\}]^+ +\\ &+ [\beta_{12} - (1-\alpha) r_f]^+\Big\}\Big\}.\end{aligned}$$

Again, $d_{(20f)}$ is a non-increasing function in (b_1, b_2), hence it attains its maximum at $b_1 = \beta_{21}$ over $\{b_1 \ge \beta_{21}, b_2 \ge \beta_{12}\}$ at $b_1 = \beta_{21}$ and $b_2 = \beta_{12}$. For $b_1 = \beta_{21}$ and $b_2 = \beta_{12}$, one can easily show that the optimal α is among the elements of the following set:

$$\begin{aligned}\alpha^{(\text{opt.(20f)})} r_f \in \{&0, r_f, \beta_{22} - \beta_{12}, \beta_{21}, r_f + \beta_{21} - \beta_{12} - \beta_{11},\\ &r_f - \beta_{11}, r_f + \beta_{21} - 2\beta_{11}, r_f - \beta_{12}\}.\end{aligned}$$

The diversity $d_{(20g)}$ (corresponding to the constraint in (20g)) is as $d_{(20f)}$ but with the role of the users reversed.

<u>Remark 8:</u> Because we do not have a closed expression for $d_{(20f)}$, it is difficult to compare $d_{(20f)}$ with $d_{(16f)}$.

E. Solution for $d_{(20h)}$ and $d_{(20i)}$

The diversity corresponding to (20h) involves the linear program:

$$
\begin{aligned}
J = \max\{X_{22} + X_{21} + X_{11}\} \\
\text{subj. to } 0 \le X_{22} \le \beta_{22},\ 0 \le X_{21} \le \beta_{21}, \\
\text{and to } 0 \le X_{11} \le \beta_{11}, \\
\text{and to } [X_{11} - b_1]^+ + [\max\{X_{22} - b_2, X_{21}\}]^+ \le r_1.
\end{aligned}
$$

If we rewrite the last constraints as:

$$
\begin{aligned}
X_{11} + X_{22} &\le r_1 + b_1 + b_2 \\
X_{11} + X_{21} &\le r_1 + b_1 \\
X_{11} &\le r_1 + b_1 \\
X_{22} &\le r_1 + b_2 \\
X_{21} &\le r_1,
\end{aligned}
$$

the optimization domain is equivalent to:

$$
\begin{aligned}
&X_{11} \le \min\{\beta_{11}, r_1 + b_1 - X_{21}, r_1 + b_1 + b_2 - X_{22}\} \\
&X_{22} \le \min\{\beta_{22}, r_1 + b_2\} \\
&X_{21} \le \min\{\beta_{21}, r_1\},
\end{aligned}
$$

Hence, the optimization problem is:

$$
\begin{aligned}
J = \max\{\min\{X_{22} + X_{21} + \beta_{11}, \\
X_{22} + r_1 + b_1, X_{21} + r_1 + b_1 + b_2\}\} \\
\text{subj. to } 0 \le X_{22} \le \min\{\beta_{22}, r_1 + b_2\}, \\
\text{and to } 0 \le X_{21} \le \min\{\beta_{21}, r_1\}.
\end{aligned}
$$

Because all functions in the min-espression for J achieve their maximum value at $X_{21}^{(\text{opt.(20h)})} = \min\{\beta_{21}, r_1\}$ and $X_{22}^{(\text{opt.(20h)})} = \min\{\beta_{22}, r_1 + b_2\}$ we have that $\max\min = \min\max$. Finally, by subtructing the optimal expression for J from $\beta_{11} + \beta_{21} + \beta_{22}$ we obtain the desired result. Again, $d_{(20h)}$ is a non-increasing function in (b_1, b_2), hence it attains its maximum at $b_1 = \beta_{21}$ and $b_1 = \beta_{12}$; for these values we get:

$$
\begin{aligned}
&d_{(20h)} = \beta_{22} + \beta_{12} + \beta_{11} - J = \max\big\{ \\
&[\beta_{22} - r_1 - \beta_{12}]^+ + [\beta_{21} - r_1]^+, \\
&[\beta_{22} - r_1 - \beta_{12}]^+ + (\beta_{21} - r_1) + (\beta_{11} - \beta_{21}), \\
&(\beta_{22} - r_1 - \beta_{12}) + [\beta_{21} - r_1]^+ + (\beta_{11} - \beta_{21})\big\}.
\end{aligned}
$$

The diversity $d_{(20i)}$ (corresponding to the constraint in (20i)) is as $d_{(20h)}$ but with the role of the users reversed.

Remark 9: Notice that one of the terms in max-expression that defines $d_{(20h)}$ is $[\beta_{11} - r_1]^+$, which equals $d_{(20a)}$. The same, but with the role of the users swapped, applies to $d_{(20i)}$. This implies that the diversity terms $d_{(20h)}$ and $d_{(20i)}$ are never the minimum in the expression that defines $d_{\mathrm{HK-etw}}$ in Theorem 3 and can thus be dropped.

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