Evaluating the Expected Total Cost for Imperfect Production Processes Using a Markovian Approach

BY

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THESIS

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CONTRIBUTION OF AUTHORS

Chapter 1 is an introduction and provides the research objective, contribution, and outline. Chapter 2 is a literature review of my research in four different areas. This chapter is partially published (Al Hajailan, W. I. and David He, "Expected Maintenance Actions for Imperfect Production Processes Using a Markovian Approach." *2020 Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling (APARM)*. IEEE, 2020.)). Chapter 3 provides the first developed Markov Chain model one without time factor. Chapter 4 presents the second developed Markov Chain model two with time factor. Chapter 5 provides scheduling hard time windows between two production processes. Chapter 6 provides the maintenance model for preventive, inspection, and minimal repair. This chapter is published (Al Hajailan, W. I. and David He, "Expected Maintenance Actions for Imperfect Production Processes Using a Markovian Approach." *2020 Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling (APARM)*. IEEE, 2020.)). My advisor Dr. David He administered the research and reviewed the manuscript. Chapter 7 presents sensitivity analysis in two different categories. Finally, Chapter 7 provides the conclusions of my thesis.

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SUMMARY

In this thesis, an irreducible discrete time Markov Chain (MC) model is developed and integrated with the imperfect production and inventory control model. The Markovian approach adapted and uses finite states and two models with/without the time factor are devised. The two MC models developed are designed as MC model one and MC model two, without and with time factor, respectively. In both models, the expected total cost function is obtained and then the optimal production plan is identified. The model two has higher accuracy than the model one in the expected total cost.

Moreover, scheduling hard time window between two production processes are implemented. Two cases are developed, the first case has the same production and demand rates and the second case has different production and demand rates. The developed models demonstrate the efficiency and applicability of the previously developed MC models one and two.

Two well-known heuristic methods are used for the scheduling model utilizing MC model two. Genetic algorithm (GA) is used for two cases. Particle Swarm Optimization (PSO) algorithm is used for only the second case. The optimal solutions for both algorithms as well as Mathematica are close to each other. However, they are different in the model constraints violation values.

The optimal maintenance actions for imperfect production processes are developed. The optimal preventive maintenance, inspection, and minimal repair are obtained for each state. The expected total cost function is developed includes the cost of the maintenance and production planning. By that the expected maintenance actions for each type are obtained for each month in

each state.

Moreover, sensitivity analysis is conducted in two different categories in order to show its effects on the developed MC model two for the fourth scenario. It has the same effects on MC model one and using any scenario. The first uses the production and demand rates and the second category uses the production and inventory control model's parameters. The developed figures for each category are presented to show more details about them carefully. Finally, conclusions are presented of my thesis research.

1. INTRODUCTION

1.1. Background and Motivation

Imperfect production and inventory (P&I) control model are enriched with various studies in order to incorporate the effects of the production process deterioration on determining either the optimal production run time, optimal economic production quantity, or optimal total cost. Rosenblatt and Lee (1986) were the first researchers who studied this problem and derived the rework cost function for the defective products. However, these studies only considered the production cycles for one single state. There is a need to incorporate several states and to use their associated defects rates. MC models are used by several researchers in order to develop the expected total cost function for similar problems. The defect rates are random & stochastic, and the MC structure can be used to model and incorporate their effects. The steady-states operating characteristics are only considered in the developed models and defined by the stationery probabilities. The developed integrated model uses both the imperfect P&I control model along and the MC model in order to evaluate the expected total cost function. The detailed production plan, scheduling hard time windows, and maintenance actions are obtained.

1.2. Defective Rates Data Analysis

Defect rates real data were collected and analysed from a big manufacturing firm in the United States of America. The data included four years of defect rates. When plotting the graphs for any year, we found out that all of them have the same behaviour. The daily defect rates graphs behave like a time series, moreover, the weekly and monthly have same behaviour. However, the yearly defect rates are behaving in a linear relationship with time, see Figure 1, Figure 2, Figure 3, and Figure 4 for the daily, weekly, monthly, and yearly defect rates. This analysis helps us in incorporating the effects of the defect rates by using a MC model. The developed model will use the defect rates for the states of the production process. Moreover, the expected total cost function will be derived by using the integrated model using the MC and the imperfect production and inventory (P&I) control model.

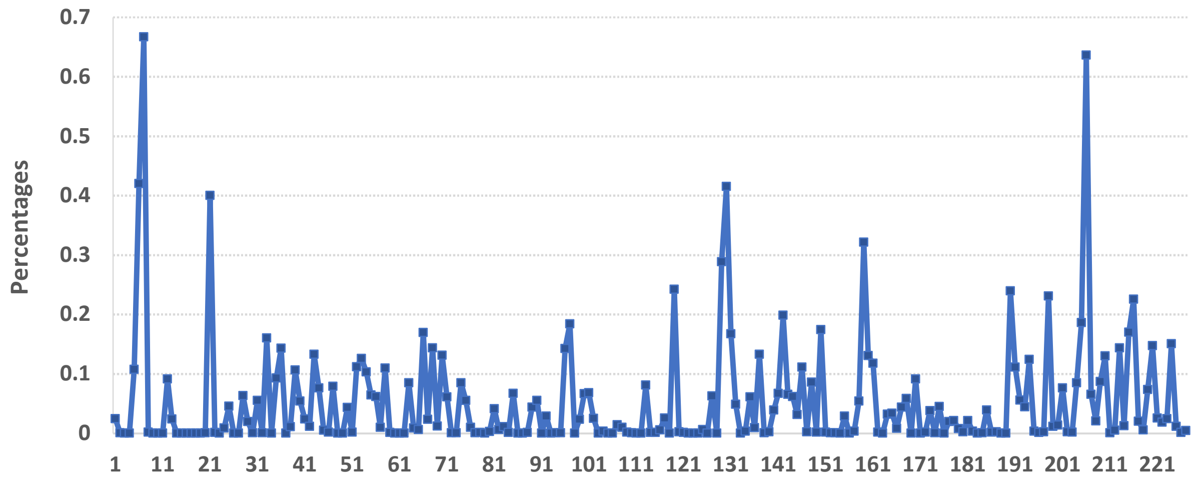


Figure 1. Daily Average Defect Rates

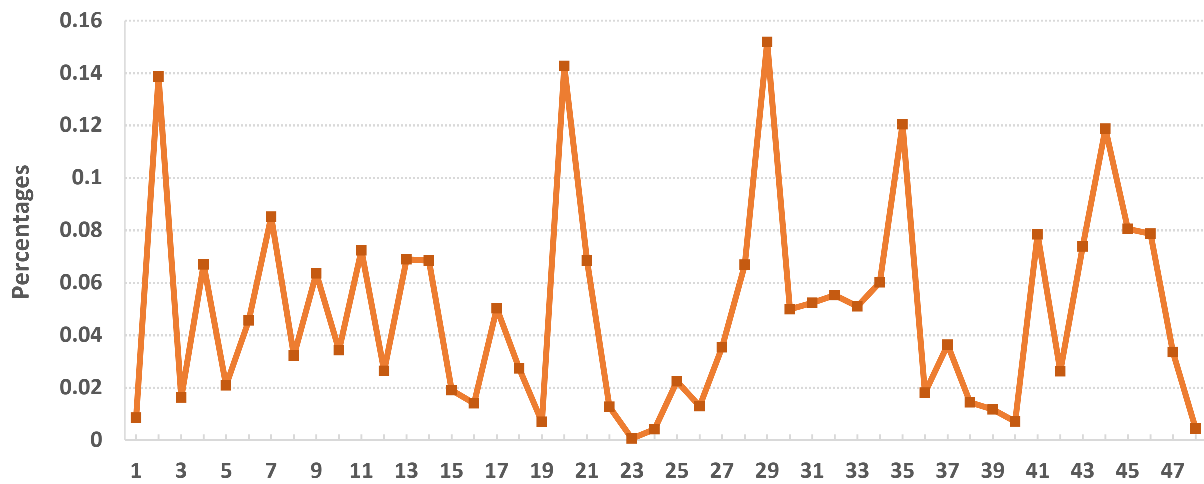


Figure 2. Weekly Average Defect Rates

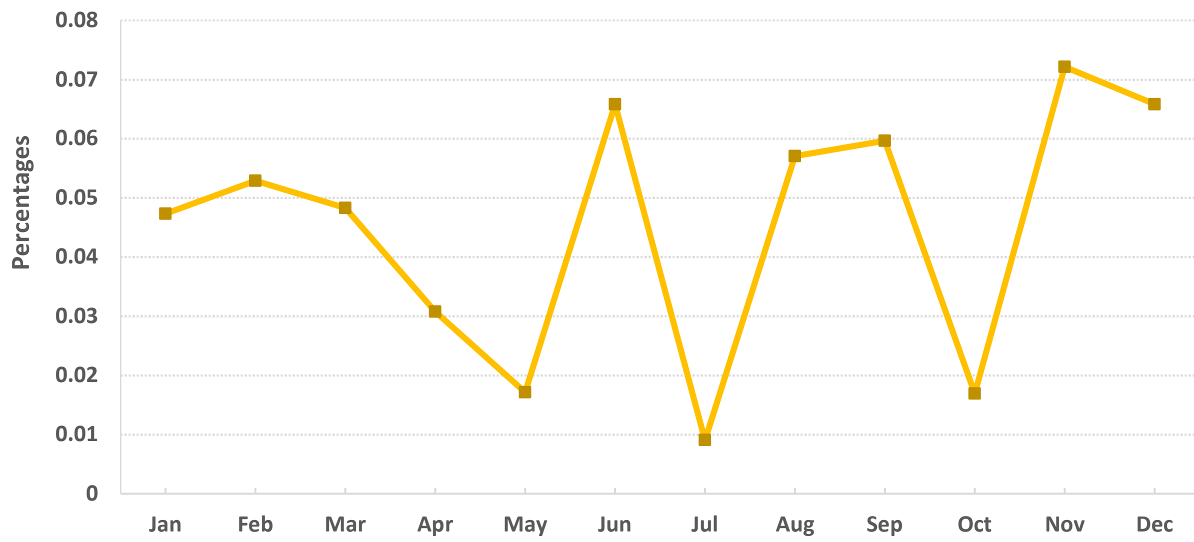


Figure 3. Monthly Average Defect Rates

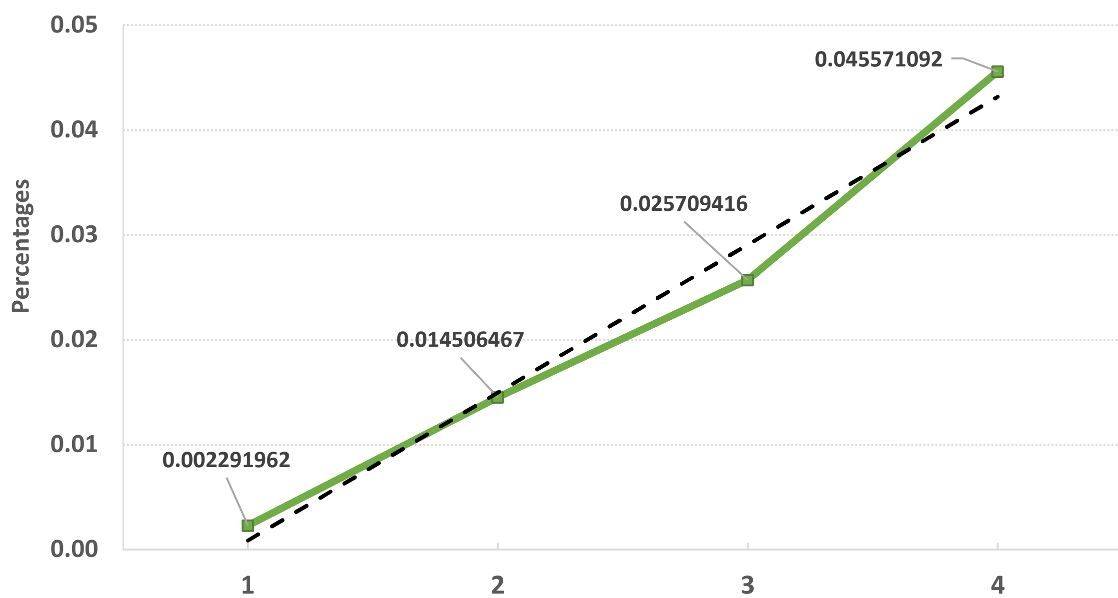


Figure 4. Yearly Average Defect Rates

1.3. Research Objective

This research deals with developing the expected total cost for the imperfect production and inventory control model using MC by different methods and purposes. The developed model enables the evaluation of the objective function which is the expected total cost for any defined number of states. Moreover, the production rate, demand rate, defect rate, optimal cycle time, and the optimal production run time for each state are used in the integrated models. The optimal solution parameters which are the optimal total cost, optimal time when the shortages are met, optimal time when inventory is built, optimal production run times, optimal for the inventory to be used and shortages to be built, probability of being in each state, and the expected state times all are calculated for each state.

This research developed two MC models to show and describe the applicability of the developed models for real life problem. MC model one without time factor and MC model two with time factor are developed with four scenarios and numerical examples are provided. Moreover, scheduling hard time windows and maintenance actions models are developed. The *P*-Value and the 95% confidence interval in each example on the evaluated expected total cost is provided to show the high accuracy achieved by using each model.

1.4. Research Contribution

MC model is used widely to address the stochastic nature for the imperfect production processes by determining the expected total cost by different methods and for different purposes. However, previous researches have not addressed solving the problem of the P&I control model by having several states in order to adapt the stochastic nature for the defect rates among states and developing the desired objective function.

In order to model and use the defect rates data, MC models are developed and used to have that pattern which is shown by the graph for the yearly defect rates. MC model methodology is a very effective tool to express and model that pattern by having the state which will be the average yearly defect rate for the production process. The defect rates are changing and increasing among the states in a yearly linear relationship.

The potential research contribution of this thesis is the development of the MC models that enable us in evaluating the objective function which is the expected total cost for any defined number of states. The state in this problem is defined by the yearly average defect rate. Two models are developed for that purpose, the MC model one without time factor and the MC model two with time factors. The latter model has a better and a higher accuracy in evaluating the expected total cost than the former model.

Moreover, two additional problems are addressed which are associated with the P&I control model by the developed MC models in this research. The first problem is the jobs scheduling hard time windows that the optimal jobs scheduling between two production processes are obtained. The second problem is the optimal maintenance actions which are the required maintenance actions (preventive maintenance, inspection, and minimal repair) are found and optimized for each month in each state.

The first MC model without the time factor are developed in order to be integrated with the P&I control model with the following approaches:

- each state has a unique yearly defect rate
- the required demand rates with the reply production rates are specified for each state
- the balance equations are used to obtain the states' probabilities

- the expected total cost function is developed by using the state's probabilities which is the sum of the multiplication for the state probabilities with the associated state costs which is the sum of the (setup, holding, shortages, and defectives)
- the optimal solution parameters which are the optimal total cost, the optimal time when the shortages are met, the optimal time when inventory is built, the optimal production run times, the optimal for the inventory to be used and shortages to be built, the probability of being in each state, and the expected state times all are calculated and provided for each state.
- four different scenarios of having different combinations of the production and demand rates data for each state are included in the MC model:
    - First scenario is having fixed production and demand rates
    - Second scenario is having variable production and fixed demand rates
    - Third scenario is having fixed production and variable demand rates
    - Fourth scenario is having variable production and demand rates
- each scenario will their related possible optimizing models in order to reduce the expected total cost by the following approaches:
    - optimal pair assignment for the production and demand rates for each state
    - finding the optimal selected production rates for each state
    - finding the optimal production rates for each state
- the accuracy of the expected total cost is evaluated by:
    - finding the expected production, demand, and the defect rates in order to find the optimal total cost when considering all states as one cycle and then finding

the difference between the expected total cost and the optimal total cost, the lower the difference the better indication

    - using the P-Value as a test hypothesis on the expected total cost and the confidence intervals are provided

The second MC model are developed by adding and using the time factors which are the optimal cycle time and the optimal production run time for each state which is implemented in the MC structure:

- each state has a unique yearly defect rate
- the required demand rates with the reply production rates are specified for each state
- the balance equations are used to obtain the states' probabilities
- the expected total cost function is developed by using the state's probabilities which is the sum of the multiplication for the state probabilities with the associated state costs which is the sum of the (setup, holding, shortages, and defectives)
- the optimal solution parameters which are the optimal total cost, the optimal time when the shortages are met, the optimal time when inventory is built, the optimal production run times, the optimal for the inventory to be used and shortages to be built, the probability of being in each state, and the expected state times all are calculated for each state.
- four different scenarios of having different combinations of the production and demand rates data for each state are included in the MC model
    - First scenario is having fixed production and demand rates
    - Second scenario is having variable production and fixed demand rates
    - Third scenario is having fixed production and variable demand rates

- Fourth scenario is having variable production and demand rates

- each scenario will their related possible optimizing models in order to reduce the expected total cost by the following approaches:
  - optimal pair assignment for the production and demand rates for each state
  - finding the optimal selected production rates for each state
  - finding the optimal production rates for each state
- the accuracy of the expected total cost is evaluated by:
  - finding the expected production, demand, and the defect rates in order to find the optimal total cost when considering all states as one cycle and then finding the difference between the expected total cost and the optimal total cost, the lower the difference the better indication

  - using the *P*-Value as a test hypothesis on the expected total cost and the confidence intervals Two MC models are developed. MC model one without time factor and MC model two with time factor. The second model has more accuracy than the first model in the expected total cost.

- the accuracy of the evaluated expected total cost in this model is higher than MC model one which is shown by the obtained higher evaluated *P*-Value for all scenarios and the optimization examples

Scheduling hard time window model is developed using MC model one and MC model two. In this model two production processes will be integrated by using MC model. The scheduling time plans are generated in order to specify the feasible hard time windows between them in order to pass the jobs. Moreover, two cases will be developed for each one in order to specify and to determine the optimal load that is sent to the receiver MC compared by the load of its current state:

- the first case for having same production and demand rates between the sender production process and the receiver production process:
    - the same four scenarios are developed and used
    - different optimization examples are used in order to lower the expected total cost
    - feasible optimal hard time windows are obtained between the concerned states for both the production processes
    - the optimal solution parameters which are the optimal total cost, the optimal time when the shortages are met, the optimal time when inventory is built, the optimal production run times, the optimal for the inventory to be used and shortages to be built, the probability of being in each state, and the expected state times all are calculated for each state
- the second case for having different production and demand rates between the sender production process and the sender production process with specified load
    - the fourth scenario are used
    - feasible optimal hard time windows are obtained
    - the optimal solution parameters which are the optimal total cost, the optimal time when the shortages are met, the optimal time when inventory is built, the optimal production run times, the optimal for the inventory to be used and shortages to be built, the probability of being in each state, and the expected state times all are calculated for each state
    - two famous heuristic search methods are developed which are the genetic algorithm (GA) and the particle swarm optimization (PSO) algorithm:

- the GA is developed by using Evolver in MS Excel for the scheduling case 1 and 2 with MC model two
- the PSO is used by developing a programming code in Python for the scheduling case 2 with MC model two

Moreover, maintenance model is developed. Four different MCs are developed and integrated with the P&I model. They are the preventive maintenance, the inspection, the minimal repair, and the main P&I MC model. The optimal number of the preventive maintenance, the inspection, and the minimal repair are determined by optimizing the integrated expected total cost. The optimal maintenance actions are the number of the actions in each month for each state. Consequently, they can be used which are specified for each month is each state or the expected values for each maintenance actions for the whole production planning period can be used. The detailed optimal production plan is created along with the optimal maintenance actions.

The detailed optimal production plans are obtained for all the developed models in this thesis. The production cycle starts by having shortages. Then the first segment is the optimal time in order for the shortages to be met (T_1). The second segment is the optimal time in order for the inventory to be built (T_2). The sum of these two optimal times for these two segments is called the optimal production run time ($t= T_1 + T_2$). After the second segment the production activity stops. Then in the third segment is the optimal time in order for the inventory to be used and the fourth segment is the optimal time for the shortages to be built ($T_3 + T_4$). Having said that, in each production cycle, the four segments which are specified and added to each generated table for the purpose of finding the optimal expected total cost function. The optimal cycle time is the sum of these four segments ($T = T_1 + T_2 + T_3 + T_4$).

Finally, detailed sensitivity analysis is conducted in two different categories. In the first category is to study the effects of varying the production and demand rate data on the expected values for the total cost, the time for the shortages to be met, the time for the inventory to be built, the production run time, and the defect rates. In the second category, is the effects of varying the P&I control model parameters which are the setup cost, the holding cost, the shortages cost, and the failure rate for the exponential probability density function.

1.5. Research Outline

This thesis is structured as follows. In chapter 2, detailed literature review for the previous and current research topic will be provided. Chapter 3 will present the first developed MC model one without time factor. Moreover, the second developed MC model two with time factor will be provided in chapter 4. The scheduling hard time windows model will be introduced in chapter 5. In section 5.4 two heuristic search method the genetic algorithm (GA) and Particle Swarm Optimization (PSO) are used. In chapter 6, optimal maintenance actions model will be presented. Moreover, the sensitivity analysis will be in conducted in chapter 7. Finally, the conclusion will be presented in chapter 8.

2. LITERATURE REVIEW

(Partially previously published as Al Hajailan, W. I. and He, D., (2020) Expected Maintenance Actions for Imperfect Production Processes Using a Markovian Approach, 2020 Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling (APARM), Vancouver, BC, Canada, 2020, pp. 1-6, doi: 10.1109/APARM49247.2020.9209372.) See Appendix K.

2.1. Imperfect Production and Inventory Control Model

The classical P&I control model assumes that the products are having perfect quality and the production processes don't deteriorate. The optimal production quantity is easily obtained based on those assumptions. Many researchers in the literature have done several studies to show that the production process state is not always perfect, and because of that it affects the products quality. They are named by a deteriorating or imperfect production processes. Under this assumption, both states of in-control and out-of-control are used. The process shift from the former state to the latter state. For the in-control state it is assumed that there are no defective products. During the out-of-control state the defective products start to be produced. The shift happens because the production process deteriorates during each production cycle. Rosenblatt and Lee (1986) were the first researchers who studied this problem and then be able to derive the defective cost function. They considered three different types of defective rates which are constant, linear, or exponential. Porteus (1986) developed a model that has a connection between the product quality and the lot size. The production process goes to out-of-control with a given probability. In this case it will have a rework cost and will have other related operation costs. In this case, it is better to produce in smaller lot to have a small fraction of defectives. He introduced three options for the quality improvement: smaller probability that the process goes to out-of-control, setup cost reduction, or both of them. For each option there is a well-defined form for the cost function and then having the optimal production plan. After that maintenance and inspection policies are introduced by (Lee and Rosenblatt, 1987). They solved that problem by deriving the optimal production run times or the number of produced products and the level of maintenance for inspections. The number of defective products in the in-control state is assumed to be negligible. Lee and Rosenblatt (1989) introduced process restoration that depends on the detection delay and the shortages in the system. They used two different restoration cost functions which are linear and exponential. An optimal maintenance schedule will have an equal interval and the production run time with the level for the shortages are attained. They showed the difference for the optimal cost for both the classical EMQ model and with their developed model. Cheng (1991) developed a model that the unit cost of production depends on the demand and the reliability of the production process which affect the products quality. The economic ordering quantity based on both of them is derived where the problem expressed by geometric program. Tripathy *et al.* (2003) developed a similar EOQ model. They assumed the production cost is related to the reliability; however, it is related to the demand in an inverse way. They claimed that product quality depends on the reliability of the production process and on the used monitoring programs. Liou *et al.* (1994) and (Boone *et al.*, 2000) introduced type-I and type-II errors inspection errors which affects the optimal production run time and help in minimizing the total cost function. Huang and Chiu (1995) extended the works of (Rosenblatt and Lee, 1986) and (Lee and Rosenblatt, 1987 and 1989) by integrating three important production problems. They integrated the production planning, preventive maintenance, and inspection. They claimed that production process deteriorates by worker fatigue and/or machine corrosion. Hariga and Ben-Daya (1998) expanded the model of

(Rosenblatt and Lee, 1986) by having a general distribution for the time shift. They obtained bounds on the optimal cost function with a distribution based and a distribution free. Kim and Hong (1999) extended the work of (Rosenblatt and Lee, 1986) by having an arbitrary distribution function for the shift time instead of the exponential distribution function. They derived the optimal production run times under those different types for the deteriorated production processes which are constant, linear, and exponential increasing. Ben-Daya and Hariga (2000) studied the effects of the imperfect production process and the restoration process on the optimal economical lot scheduling model. Salameh and Jaber (2000) studied the products with imperfect quality to be sold with reduced price. This will decrease the in-hand inventory cost. They found that the economic lot size quantity will increase as the defective rate increases. This contradicts the finding of (Rosenblatt and Lee, 1986) because the defective products are reworked and then kept in the inventory which will increase the inventory cost. Yeh *et al*. (2000) studied the effects of imperfect production process on the optimal production run time for products that are sold with free minimal repair warranty. They used two-state continuous time MC in-control and out-of-control for characterizing the deteriorating production processes which may come from several sources from usage like mentioned before age, corrosion, fatigue, and cumulative wear. Hayek and Salameh (2001) studied similar problem that the effects of the imperfect quality units on the production process with shortages. The total inventory costs are minimized by having the optimal production lot size which considers the rework cost. They mentioned that having too much items in inventory will incur high storage cost. However, having shortages will have low cost but will have a high number of products to be ordered. There is a need to minimize all related costs and then have some portion for shortages which will help to have the optimal quantity for production. Wang and Sheu (2001) generalizes the work of (Porteus, 1986) by studying the effects of the warranty cost on the imperfect economical manufacturing quantity. They used a general discrete distribution function for the production process shifts and then starts having defective products. The defective products will have both the rework and warranty costs. Chiu (2003) developed an imperfect P&I control model to show that the reworked products affect the economic production quantity where the shortages are allowed. Not all defective products are reworked but some percentage of them will be scrapped and then discarded. Chung and Hou (2003) generalizes the work of (Rosenblatt and Lee, 1986) and (Kim and Hong, 1999). They studied the effect of having shortages in the imperfect production process. Having stockout can happen with different situation, so inventory could have shortages which are normal. Wang (2004) extended the work of (Yeh *et al.*, 2000) by assuming general shift distribution from in-control state to the out-of-control state and the work of (Salameh and Jaber, 2000) related to the products that have warranty and then found the optimal run time. They noticed an improvement in the expected total cost by having a lower number of nonconforming products when the failure function improves. Hou (2005) extended the work of (Rosenblatt and Lee, 1986) by adding shortages. He used a continuous-time Markovian production system in order to incorporate the two-state's restoration cost. The defective products are produced during both the in-control and out-of-control states. Moreover, in their model they have noticed a cost savings when considering shortages. Chen and Lo (2006) developed an imperfect P&I control model with shortages for products that are sold with free minimal repair warranty. Their model provides the optimal production run time and the optimal time for the shortages to be replenished. Rahim and Al-Hajailan (2006) extended the work of (Chung and Hou, 2003) by assuming the defect rate is in a linear relationship with time. The rate should be increased with the duration of the out-of-control state. They derived the optimal production run time in order to optimize the developed model cost. Yu and Mao (2010) extended the work of (Rahim and Al-Hajailan, 2006) by adding a free minimal repair warranty model. They assumed that the percentage of defective products in the in-control period is constant and during the out-of-control period is dynamic with respect to the out-of-control time. Their developed model helps in the market competition and for customer satisfaction by providing warranty for their products.

2.2. Markov Chain and Imperfect Production and Inventory Control Model

In the literature MC methodology is used very widely integrated with the imperfect P&I control model. It helps the researchers to model the stochastic nature for the variables that have uncertain or random values. Moreover, current and previous studies dealt with developing the long run expected cost or the long run average cost, the expected total cost or the expected average total cost, and the expected discounted cost for different kind of models and for many different optimization purposes that will be introduced here.

The following studies are about developing the long run expected cost or the long run average cost when the time goes to infinity. Sung and Oh (1987) studied single product on single machine for the P&I control model with shortages and compound poisson demand process to find the optimal (r, Q) policy. The derived long run average cost function that includes production, holding, and shortages costs in order to find the steady-state probability distribution. Kalpakam and Sapna (1997) considered (s, S) inventory system with renewal demand and lost sales. An irreducible MC with steady state variables is used to derive the long run expected cost. The objective function includes setup, purchasing, holding, and shortages costs. Kalpakam and Shanthi (2001) used an irreducible MC stationery distribution to study the lost sales for (S-1, S) policy. He used the steady state variables in order to optimize and define the inventory model long run expected cost that includes the purchase, inventory, shortages, and failure costs. Yin *et al.* (2004) studied stochastic production planning and scheduling model with discrete time and/or continuous time using MC with finite state. They optimized the optimal production rate in order to minimize the long run expected cost by using Hamilton-Jacobi-Bellman equations. The objective function includes production and shortages costs. Isotupa (2006) studied (s, Q) continuous review inventory model for priority and ordinary customers to determine the optimal reorder level and quantity. He developed a MC using the steady state variables to obtain the balance equations and then to determine the average inventory. The derived long run expected cost function includes setup, purchase, shortages, and holding costs. Tai and Ching (2014) studied an inventory and return system by using a queueing model. They developed the expected long run operating cost by using an irreducible MC with steady state probabilities. The objective function includes holding, shipment, transhipment, and lost sales costs. Yao (2017) studied a continuous review stochastic inventory system in order to determine the optimal prices and the inventory levels. He derived the long run average profit function that includes revenue, holding cost, shortages cost, and ordering cost. Elhafsi and Hamouda (2018) studied a P&I control model for two types of demand which are walk-in and long-term. They used finite state irreducible MC model in order to develop the long run expected profit function by using the steady state probabilities. The production scheduling, inventory levels, and the optimal prices are obtained. They considered the production, holding, and shortages costs.

The following studies are about developing the expected total cost or the expected average total cost. Cohen *et al*. (1988) studied (s, S) inventory system for two type of prioritized customers for single product and single location. They used an irreducible MC transition matrix model to approximate the expected total cost subject to a service-level constraint by using the renewal-based approach and steady-state variables. The cost function includes ordering, demand, emergency shortages shipment, and holding costs. The accuracy and the performance of their approach for the true optimal solution is concluded. Schaefer (1989) studied (s, S) inventory model to minimize the expected total cost by using a discrete MC and the steady-state probabilities. The objective function includes ordering, holding, repairing, and shortages costs. Van der Laan *et al.* (1996) studied single product in P&I control model. They used three different types of products which are the returned, remanufactured, or disposed. Several strategies are considered to derive the expected total cost function by using finite state MC and then to minimize the quantity of wastes for these products. The cost function includes production, procurement, remanufacturing, disposal, holding, and shortages costs. Nakashima *et al.* (2002) developed a recovery system for single product and single process within a P&I facility in order to minimize the quantity of wastes. They considered three stochastic variables in their model which are customer demand rate, recovery rate, and disposal rate. They used a discrete time MC with steady state distribution to obtain the expected average total cost per cycle that includes products that could be reused, recycled, or remanufactured. The objective function includes the manufacturing, remanufacturing, holding, backlog, and out-of-date costs. Lian *et al.* (2005) studied stochastic perishable (s, S) inventory model using a multidimensional MC that is finite with discrete time. They used a continuous review model and the expected average total cost that includes holding, replacement, shortages, and ordering costs. Noblesse *et al.* (2014) studied stochastic continuous review single product (s, S) P&I control model to determine the optimal production lead times by using queueing analysis. They used MC to develop the expected total cost that includes ordering, inventory holding, shortages costs. Ozener *et al.* (2014) studied periodic review P&I control model (s, S) for single product. They used discrete time MC with infinite horizon to develop the balance equations and then used the dynamic programming method. The developed expected total cost function includes production, holding, shortages, and ordering costs. Li *et al.* (2016) studied a production facility that satisfies the demand of customers with uncertain times. They used discrete time MC in order to develop the expected total cost by using dynamic programming method. The cost function includes holding and delay penalty costs. Zadeh *et al.* (2016) studied P&I control model for single product in single facility in order to determine the optimal safety stock and the reorder quantity to overcome the inaccuracy in the inventory level. They developed a MC using the steady state variables to obtain the balance equations and then to determine the average inventory. The developed supply chain expected total cost function that includes holding, production, ordering, and lost sales costs. Habibi *et al.* (2018) studied a supply chain that includes optimizing the location and the inventory model. They used MC with steady-state probabilities for the inventory model in continuous time. The expected total cost function includes opening facility location, transportation, holding, shortages, ordering, and purchase costs.

The following studies are about developing the expected discounted cost that is using a positive variable which is less than one in order to have the discounted factor. Lou *et al.* (1994) studied stochastic production planning problem for two machines capacities and demand processes by using finite state MC in an infinite horizon. They optimized the production rate by using dynamic programming in order to minimize the expected discounted cost that includes production, inventory, and shortages costs. Presman *et al.* (1995) studied stochastic production planning problem for N-machines and demand processes by using finite state MC in an infinite horizon. They optimized the production rate by using dynamic programming in order to minimize the expected discounted cost that includes production, inventory, and shortages costs. Presman *et al.* (1997) studied stochastic production system with finite buffers for N-machines to satisfy the demand by using finite state MC in an infinite horizon. They obtain the optimal production rates by using dynamic programming in order to minimize the expected discounted cost that includes production, inventory, and shortages costs. Parlar (2000) used a MC to model a stochastic inventory model with random interruptions. A formula for the distribution of the cycle time and cost are developed and then enabled him to compute the expected discounted cost that is the expected cycle cost divided by the expected cycle time. The cost function includes ordering, holding, and shortages costs. Kenne and Gharbi (2001) developed a new method to optimize the production rates by using Hamilton-Jacobi-Bellman equations. The inventory model is described by MC for multiple machines and products. Their method optimized the expected discounted cost for both inventory and shortages costs. Kenne and Gharbi (2004) studied stochastic machine capacity for one machine and two products. They used finite state continuous time MC in order to obtain the optimal production rate by using Hamilton-Jacobi-Bellman equations. They derived the expected discounted cost objective function that includes the inventory and shortages costs. Hajji *et al.* (2009) studied stochastic production supply chain model to determine the optimal information sharing policy. They used MC with dynamic programming technique for the derived Hamilton-Jacobi-Bellman equations. They obtained the expected discounted cost function that includes ordering, production, holding, and shortages costs. Cadenillas *et al.* (2013) studied P&I control model to determine the optimal inventory level and production rate with full or limited information. They considered the allowed returned or non-returned products. Continuous-time MC is used to derive the expected discounted cost that includes the running cost for the difference between the inventory and the production levels with their targets. Barron (2016) studied stochastic fluid inventory model for single product with infinite horizon. He used finite state continuous time MC to develop the expected discounted cost function that includes setup, delivery, holding, shortages costs. Eruguz *et al.* (2018) studied an inventory model with maintenance. They used continuous time discrete states MC in order to minimize the derived expected discounted costs that include preventive replacement, preventive delivery, transportation, and holding costs. Feng *et al.* (2019) developed MC to study a periodic review stochastic inventory model for two products. They developed the expected discounted costs that includes ordering, shortages, and holding costs.

The following studies have a mix between the three different types of cost functions that are mentioned previously. Song (2009) studied stochastic supply chain to determine the optimal ordering and production policies. He used MC with dynamic programming for the derived Hamilton-Jacobi-Bellman equations. He derived the expected discounted and the long run average costs that includes the material inventory, product holding, and demand shortages costs. ElHafsi *et al.* (2010) developed MC model to study the stochastic supply chain for single product in order to determine the optimal P&I policy by using the dynamic programming method. The derived the expected average total cost or the expected discounted cost that includes holding and shortages costs. Barron *et al.* (2016) studied stochastic P&I control model to select the proper production rates. They used continuous time steady state irreducible MC with finite state for the demand arrival. They derived the expected discounted and the long run average cost functions which consists of lost production, holding, shortage, and unsatisfied demand costs. Barron (2019) studied P&I (s, S) model in order to develop both the expected discounted costs and the expected average total costs by using continuous time MC. He extended the work of (Barron, 2016) by having the lost demand cost to be included with the ordering, purchase, and holding costs.

2.3. Scheduling Time Windows

In the literature the scheduling time windows are used along with the P&I control models for many different purposes. There are several types of time windows that help in constructing constraints to maintain certain level of service or to comply and meet the producer and/or the retailer limitations. There is delivery, soft, hard, and other types of time windows which will be introduced. The soft time window means that the constraints may not to be met and a penalty will be added to the objective function, however, the hard time window must be met, (Ibaraki *et al.*, 2005).

The following studies are about the delivery time window. Christiansen (1999) studied the inventory management with ship routing & scheduling with pickup and delivery time windows. The cost function includes the costs of transportation for harbor, channel, fuel, and diesel oil. Jung *et al.* (2008) studied the safety stock for the production planning and scheduling of the multi-stage supply chain to provide the required service level for the customers with allowable delivery time window. The expected cost function includes the costs of holding, backordering, and the penalty of having a service level below the target. Yeung *et al.* (2011) studied the supply chain scheduling for a manufacturer and a retailer that is having the production time windows and the delivery time windows. The delivery time windows include both normal and express deliveries. The cost function includes the costs of transportation and storage. Amorim *et al.* (2013) studied the batching and lot sizing for the integrated model of production scheduling and distribution planning with customer time windows and perishable products. The cost function includes the costs of production, setup, transportation, and vehicles fixed cost. Zhang *et al.* (2015) studied the order scheduling and inventory matching for the policies of mark-to-order and make-to-stock within the time window of delivery. The cost function includes the costs of delivery penalty, order cancelation penalty, inventory matching, and production. Li *et al.* (2017) studied the integrated model for the production scheduling, inventory, and delivery time windows for splittable or nonsplittable orders. The cost function includes the costs of inventory and transportation. Vahdani *et al.* (2017) studied the supply chain for the integrated problem of the production scheduling and the vehicle routing using the perishable products time delivery windows. The cost function includes the costs of production, holding and transportation. Miranda *et al.* (2018) studied the supply chain for the integrated model of production, inventory, distribution, and routing for multiple delivery time windows. The cost function includes the costs of setup, holding, and routing to determine the optimal lot sizes and the scheduling. Miranda *et al.* (2019) studied the production planning horizon for the production, inventory, distribution, and routing that are having multiple delivery time windows. The cost function includes the costs of setup, holding, and routing to determine the optimal lot (sizes and the scheduling. Tang *et al.* (2019) studied the model of production, inventory, distribution, and routing with multiple delivery time windows. The total cost function includes the costs of setup, holding, and routing in order to determine the optimal lot sizes and scheduling.

The following studies are about the soft and hard time window. Ibaraki *et al.* (2005) studied the vehicle routing problem with soft time window and then solved the production and scheduling problem with cost function that includes the costs of setup and inventory. Jia *et al.* (2014) studied the supply chain of the production scheduling and distribution routing for one supplier and multiple retailers. The obtained the soft time windows for the transportation and hard time windows for the sales to control the quality of the product. The cost function includes the costs of supplier, transportation, retailer, and the penalty for the vehicle returning time interval. Henning *et al.* (2015) studied the routing and scheduling for the split pickup and split delivery with hard time windows that are not permitted to go beyond their constraints. The cost function includes the costs of waiting, route, sailing, port, and handling.

The following studies are about different types of TW. Lin and Krajewski (1992) studied the master production scheduling (MPS) for a rolling schedule which includes the frozen, replanning, and forecast window intervals. The total average cost includes the costs of forecast error, MPS change, setup, and inventory. Chang (1994) studied the capacity time window for the synchronous production. The cost function includes the costs of capacity, holding, and operating. Hishamuddin *et al.* (2014) studied the supply chain for one supplier and one retailer. They obtained the production recovery time windows for the production schedules that are disruptive. The cost function includes the costs of setup, ordering, inventory, backordering, and lost sales. de Moura and Botter (2016) studied the supply chain of auto parts with collecting schedule time windows. The cost function includes the costs of holding and transportation. Kopp *et al.* (2020) studied planning of jobs for the qualification time windows by using scheduling. They used mixed-integer linear programming method and the objective function that includes the costs of backlogging, holding, and penalties of running qualification that is associated for each tool and for each lot family.

The MC model is used with scheduling in order to develop the expected cost function with a discount factor. Moreover, the time windows have not been considered in these studies and they are very limited. Oosterom *et al.* (2017) studied the scheduling replacement for the deteriorating system. They model the deterioration by using a MC model. They derived the total expected discounting function that includes the replacement and the operating costs. Yan and Yang (1987) used a continuous-time MC and a heuristic dynamic programming algorithm to prevent the deadlock scheduling and to control the processing time for multiple machines and products. The derived the expected cost function by using a discount factor that includes the costs of processing, inventory, backlog, and scheduling.

2.4. Maintenance Actions

In practice, most of the production systems are not perfect in that defectives are often produced. Even though the maintenance of such imperfect production systems is studied, the maintenance strategies obtained by a Markovian approach are limited. In this paper, expected maintenance actions for imperfect production processes are developed using a Markovian approach. An integrated maintenance action model is developed to determine the required preventive maintenance (PM), inspection, and minimal repair (MR) for each state. The expected total cost function is obtained that covers the cost of the production process with the cost of maintenance. Imperfect production and inventory (P&I) control model are enriched with many and various studies in order to incorporate the effects of the production process deterioration on determining either the optimal production run time, the optimal economic production quantity, or the optimal total cost.

Maintenance is very helpful in enhancing and lowering the production process failures (Boukas *et al.*, 1996; Meller and Kim, 1996). PM increases the useful life of the production process which is less expensive than replacement (Iakovou *et al.*, 1999). Moreover, PM is the activities that are performed in order to remove or reduce the deterioration and the repair is used to move the current production process failure to a non-failure state (Chen and Trivedi, 2005). However, inspection is used to check the current state of the production process (Berenguer *et al.*, 1997). PM has two types of actions either by using certain fixed time or by checking its current condition. For the time-based PM, the required action is performed each time. However, the condition-based PM depends on the current state of the production process that it may need an action after inspection, see (Chen and Trivedi, 2005) for more details.

Rosenblatt and Lee (1986) were the first researchers who studied the problem of the imperfect or deteriorating production processes and then be able to derive the formula to estimate the expected number of defectives. After that maintenance and inspection policies are introduced by (Lee and Rosenblatt, 1987). They solved that problem by deriving the optimal production run times or the number of produced products and the level of maintenance by inspections. There are numerous papers in the literatures in developing an integrated model for the optimal production quantity, inspection, and the quality control, see (Rahim, 1994).

Moreover, MC model is used and integrated with the P&I control model but without maintenance. There are three different types for the objective function considered which are: the expected long run cost in reference (Tai and Ching, 2014), the expected total cost in reference (Habibi *et al.*, 2018), and the expected discounted cost in reference (Feng *et al.*, 2019).

Furthermore, MC is used with developing the expected discounted cost or the average cost to determine the optimal rates for the production and maintenance in (Boukas *et al.*, 1996; Boukas *et al.*, 1997). Wang and Sheu (2003) developed the expected average cost to find he optimal production, inspection, and maintenance with the two type of errors. Xiang *et al.* (2014) developed the total expected discounted costs to consider the production, yield, and maintenance. However, in this thesis the expected total cost function is developed by using a discrete irreducible MC and with unique MC structure. The optimal production plan along with the optimal maintenance actions are optimized for a defined number of states which has not been addressed before up-to our knowledge.

3. MARKOV CHAIN MODEL ONE WITHOUT TIME FACTOR

The yearly production process defect rates are increasing linearly with respect to time for the data that was collected. It has several states that each state represents the average yearly defective rates and for each state a defect rate is used and associated with it. It is a stochastic nature for the process when the defect rate is changing with respect to time. MC model one is developed and used to evaluate the optimal solution for the objective function which is the expected total cost. The model is using the demand rates, the production rates, and the defective rates that are assigned for each state, see Figure 5 for more detail. Moreover, the optimal time for the shortage to be met T_1, the optimal time for the inventory to be built T_2, the optimal production run time $t = T_1 + T_2$, the optimal for the inventory to be used and shortages to be built $T_3 + T_4$, the optimal total cost, and the expected state time will be calculated for each individual state along with he expected total cost. The accuracy of the model in determining the expected total cost will be compared with both the calculated optimal total cost and by the P-Values.

Four different scenarios are developed in order to model and to have all possible combinations for the production and demand rates data among the states. The first scenarios will have constant production rates and demand rates for each state, the second scenario will have variable production rates and constant demand rates, the third scenario will have constant production rates and variable demand rates, and the fourth scenario will have variable production rates and the demand rates. Each scenario will have several examples to evaluate the expected total cost and different ways to optimize and reducing the cost.

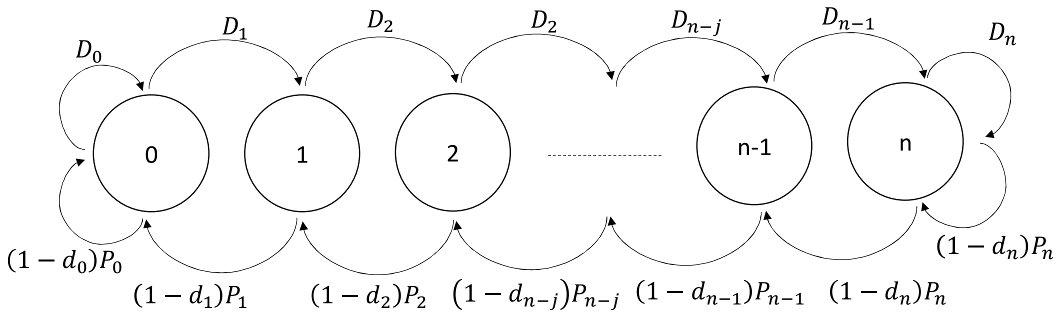


Figure 5. MC Model One with (*n*+1) States

3.1. Notation and P&I Objective Function

The objective function for the imperfect P&I control model from reference (Chung and Hou, 2003) that will be used is

$$TC(t,T_1) = \frac{KD}{Pt} + (h+\pi)\frac{(P-D)}{2t}T_1^2 + h\left[\frac{(P-D)t}{2} - (P-D)T_1\right] + \frac{sD}{t}\int_0^t d(t-x)\lambda e^{-\lambda x}dx \tag{3.1}$$

After doing simplification, the objective function can be stated as

$$TC(t) = \frac{KD}{Pt} + \frac{h(P-D)t}{2}\left(\frac{\pi}{h+\pi}\right) + \frac{sD}{t}\int_0^t d(t-x)\lambda e^{-\lambda x}dx \tag{3.2}$$

The following notation is to be used for the imperfect P&I control model

Parameter	Description
D	demand rate (required product units per time unit)
P	production rate (product units manufactured per time unit)

h	inventory cost for each product unit per unit time
K	production cycle setup cost
s	rework cost for each defective product unit
x	the production process time before shift starts and then defective product units are produced
λ	failure rate per unit time for the exponential distribution function
$f(x)$	the probability density function of the exponential distribution with parameter λ, $f(x) = \lambda\, e^{-\lambda x}$
d	defect rate
π	shortage cost for each unmet product unit
T	production process cycle time
T_1	time for the production process when the shortages are recovered
T_2	time for the production process when the inventory is at peak
T_3	time of no production and inventory is used
T_4	time of no production and shortages start
t	time for the production process when shortages are recovered, and inventory is built ($t=T_1+T_2$)

The following notation is to be used for the developed MC model one:

Parameter	Description
$n+1$	number of states

i	state number
P_i	production rate for state (i)
D_i	demand rate for state (i)
$\hat{P}$	expected production rate
$\widehat{D}$	expected demand rate
d_i	defect rate for state (i) where $(0 \leq d_i \leq 1)$ and $\lim_{i \to \infty} d_i = 1$
$\bar{d}$	expected defect rate
$\bar{P}_i$	probability of being in state (i)
T_{1i}	time for the production process when the shortages are recovered for state (i)
T_{2i}	time for the production process when the inventory is built for state (i)
$T_{3i} + T_{4i}$	time for the inventory used and the shortages to be built
t_i	time for the production process when shortages are met and inventory is built for state (i) ($t_i = T_{1i} + T_{2i}$)
TC_i	total cost for state (i)
$\bar{T}_1$	expected unit of time when shortages are met

$\overline{T}_2$ expected unit of time when inventory is built

$\overline{T}_3 + \overline{T}_4$ expected unit of time for inventory and then shortages are built

$\bar{t}$ expected production run unit of time

$\overline{ETC}$ the objective function which is the expected total cost that covers all related costs

ST production process service time

$\overline{ST}_i$ expected service time for state (i)

A_i variable that is used to calculate $\overline{P}_i$ (A_0=1)

3.2. First Scenario: Fixed Production and Demand Rates

In this scenario, the production and demand rates are fixed among the states. Four states will be used and the balanced equation in order to determine the states' probabilities which will help in finding the expected total cost. See Figure 6 for more detail.

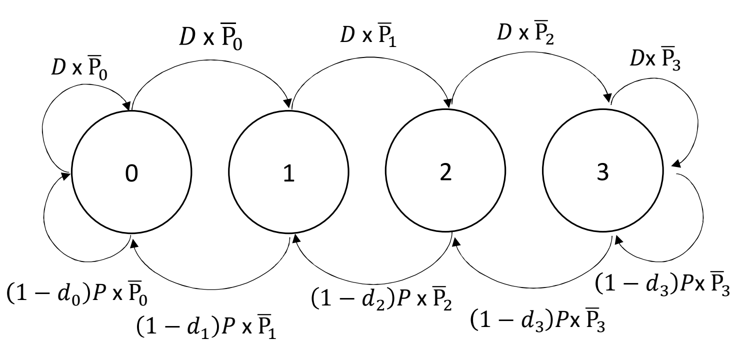


Figure 6. MC Model One for First Scenario with Four States

The state one probability is $\bar{P}_1 = \frac{D}{(1-d_1)P}\bar{P}_0$, state two is $\bar{P}_2 = \frac{D^2}{(1-d_1)(1-d_2)P^2}\bar{P}_0$, state three is $\bar{P}_3 = \frac{D^3}{(1-d_1)(1-d_2)(1-d_3)P^3}\bar{P}_0$, and then for state zero is $\bar{P}_0 = \left[\frac{1}{1+\frac{D}{(1-d_1)P}+\frac{D^2}{(1-d_1)(1-d_2)P^2}+\frac{D^3}{(1-d_1)(1-d_2)(1-d_3)P^3}}\right]$. Moreover, the expected total cost is

$$ETC(n+1) = \sum_{i=0}^{n}(\bar{P}_i \times TC_i) = \sum_{i=0}^{n}\left\{\left\{\left\{\sum_{h=0}^{n}\left[\left(\frac{D}{P}\right)^h \times \frac{1}{\prod_{j=1}^{h}(1-d_j)}\right]^{k^{*a}}\right\}^{-1} \times \left[\left(\frac{D}{P}\right)^i \times \frac{1}{\prod_{j=1}^{i}(1-d_j)}\right]^{k^{*b}}\right\} \times \left[\frac{KD}{Pt_i} + \frac{h(P-D)t_i}{2}\left(\frac{\pi}{h+\pi}\right) + \alpha_i \frac{sD}{t_i}\int_0^{t_i}(t_i - x)f(x)dx\right]\right\} \tag{3.3}$$

There is a condition in deriving the formula for the expected total cost function. It is applied when finding the probability of state zero; (k^{*a}=0 if h=0, else k^{*a}=1) and (k^{*b}=0 if i=0, else k^{*b}=1). The defective products produced is highlighted in Figure 7.

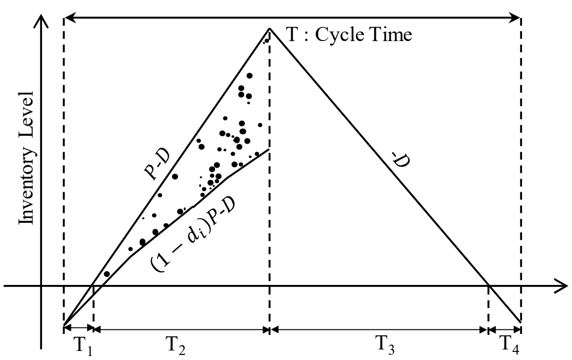


Figure 7. P&I Model for MC model one with First Scenario

The detailed optimal solution and the expected total cost is expressed in Table I which is the base model when the defective products produced in the out-of-control state with duration (t-x). Moreover, both periods are considered that the defective products are produced during in-control and out-of-control states. Two different variations are developed and the defect rates that are assigned to both of them, see Figure 8 for more detail.

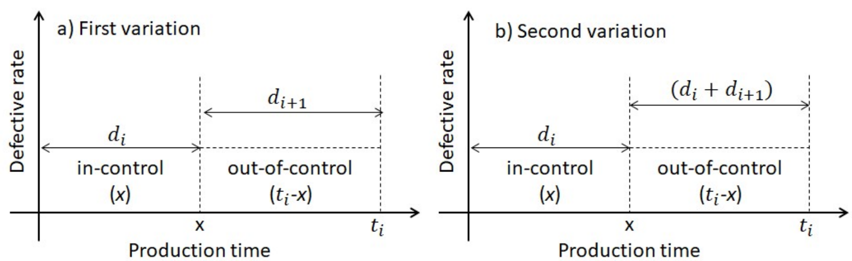


Figure 8. MC Model One for First Scenario and Two Variations

The expected total cost is (95.973) and the expected defect rate is (0.0157306). The expected total cost is calculated by the sum product of the optimal state probabilities with its optimal total cost. The expected defect rate is calculated by the sum product of the optimal state probability with the states defect rate. We will use these both figure to calculate the optimal total cost which is found to be (95.973) when it is considered like one cycle. It is noticed that it is very close to the expected total cost. This help to prove the performance of the accuracy in evaluating the expected total cost that both are close to each other with a zero difference even with the other optimal values. The expected values for T_{1i} , T_{2i}, and $T_{3i} + T_{4i}$ are calculated using same way by the sum product of the state probabilities with their state optimal times values. The P-value for the expected total cost equal (0.5428) which is not significant and the (95%) confidence interval (CI) is (95.856, 96.1529). The developed MC model has a high accuracy in the evaluated expected total cost which is shown by the achieved high P-value.

The solution presents that for state zero, and the optimal total cost is (95.904), the optimal time when the shortages are met is (0.1918), the optimal time when inventory is built is (1.19856), the optimal production run time is which is the sum of (1.3903), the optimal for the inventory to be used and shortages to be built is the sum of (0.6951), the probability being in state zero is (0.4061), and the expected state time is (1.6242) years. The same thing will be for states one, two, and three. See Table I for more detail. During the whole four years period the expected total cost is an accepted hypothesis by the calculated *P*-value, see Table II for more detail. For more details about the first and the second variations, see appendix A.

Table I. MC Model One First Scenario Solution (Base)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
0	600	400	1.00	0.4061	95.904	0.1918	1.1985	0.6951	1.6242
1	600	400	0.6765	0.2747	95.966	0.1917	1.1978	0.6947	1.0988
2	600	400	0.4629	0.1880	96.023	0.1915	1.1972	0.6944	0.7518
3	600	400	0.3233	0.1313	96.124	0.1914	1.1960	0.6937	0.5252
Expected Values					95.973	0.1916	1.1978	0.6947	
Production and Inventory Model					95.973	0.1916	1.1978	0.6947	
Difference					0.0000	0.0000	0.0000	0.0000	

Table II. MC Model One First Scenario Solution and *P*-Values

No.	Example	ETC	Optimal Cost	*P*-Value	CI
1	Base	95.973	95.973	0.5428	(95.856, 96.153)
2	First Variation	96.11	95.966	0.5226	(95.887, 96.465)
3	Second Variation	96.19	96.044	0.5283	(95.851, 96.723)

3.3. Second Scenario: Variable Production and Fixed Demand Rates

In this scenario, the production rates are variable and demand rates are fixed among the states. Four states will be used and the balanced equation in order to determine the states' probabilities which will help in finding the expected total cost. See Figure 9 for more detail.

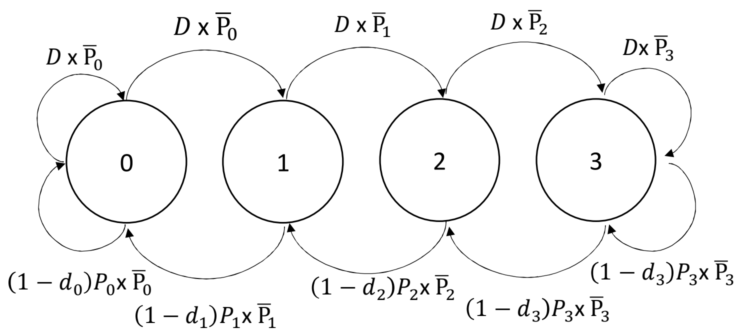


Figure 9. MC Model One for Second Scenario with Four States

The states probability are: $\bar{P}_1 = \frac{D}{(1-d_1)P_1}\bar{P}_0$, $\bar{P}_2 = \frac{D^2}{(1-d_1)(1-d_2)P_1P_2}\bar{P}_0$, $\bar{P}_3 = \frac{D^3}{(1-d_1)(1-d_2)(1-d_3)P_1P_2P_3}\bar{P}_0$, and $\bar{P}_0 = \left[\frac{1}{1+\frac{D}{(1-d_1)P_1}+\frac{D^2}{(1-d_1)(1-d_2)P_1P_2}+\frac{D^3}{(1-d_1)(1-d_2)(1-d_3)P_1P_2P_3}}\right]$. The expected total cost:

$$ETC(n+1) = \sum_{i=0}^{n}\left\{\left(\left\{\sum_{h=0}^{n}\left\{\left[\frac{D^h}{\prod_{j=1}^{h}(1-d_j)P_j}\right]^{k^*a}\right\}\right\}^{-1} \times \left[\frac{D^i}{\prod_{j=1}^{i}(1-d_j)P_j}\right]^{k^*b}\right\} \times \left[\frac{KD}{Pt_i} + \frac{h(P_i-D)t_i}{2}\left(\frac{\pi}{h+\pi}\right) + \alpha_i\frac{sD}{t_i}\int_0^{t_i}(t_i-x)f(x)dx\right]\right\} \quad (3.4)$$

The defective products produced are highlighted in Figure 10. For more detail about the optimal solution, see Table III.

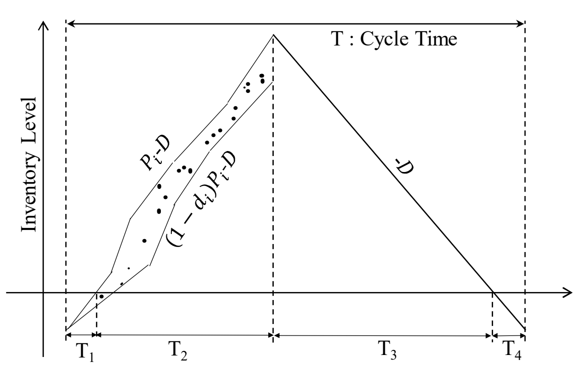


Figure 10. P&I Model for MC Model One with Second Scenario,

Table III. MC Model One Second Scenario Solution (Base)

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	1200	400	1.00	0.5481	135.617	0.0678	0.4238	0.9832	2.1924
1	950	400	0.4273	0.2342	126.413	0.0919	0.5743	0.9160	0.9367
2	750	400	0.2339	0.1282	113.553	0.1296	0.8098	0.8220	0.5128
3	600	400	0.1634	0.0895	96.124	0.1914	1.1960	0.6937	0.3582
Expected Values					127.097	0.0924	0.5777	0.9208	
Production and inventory Model					129.926	0.0825	0.5153	0.6311	
Difference					2.8290	0.0100	0.0623	0.2898	

In this scenario, two optimization problems are provided in order to minimize the expected total cost by using the production rates. Examples 3&5 are determining the optimal selected production rates among the states. Moreover, examples 4&6 are determining the optimal production rates which should be greater than the demand rates in each state. The expected total cost is decreased, see the summary Table IV, and Table V. For more detail about each example, see Appendix B.

Table IV. MC Model One Second Scenario Solution and P-Values

No.	Example Description	ETC	Optimal Cost	P-Value	CI
1	Base - (t-x)	127.097	129.926	0.3626	(90.682, 145.171)
2	Duration (t)	127.138	129.953	0.3624	(90.908, 145.113)
3	Duration (t-x) - Optimal Selected Production Rates	114.966	117.338	0.7541	(90.532, 145.306)
4	Duration - (t-x) - Optimal Production Rates	93.4447	94.884	0.6257	(70.61, 109.636)
5	Duration - (t) - Optimal Selected Production Rates	115.003	117.338	0.7493	(90.542, 145.515)
6	Duration - (t) - Optimal Production Rates	93.5404	94.964	0.6255	(70.932, 109.569)

Table V. MC Model One Second Two Optimal Production Rates Solution

State (*i*)	Example No.	
	3 & 5	4 & 6
0	750	650
1	950	600
2	600	550
3	1,200	500

4.4. Third Scenario: Fixed Production and Variable Demand Rates

In this scenario, the production rates are constant and demand rates are variable among the states. Four states will be used and the balanced equation in order to determine the states' probabilities which will help in finding the expected total cost. See Figure 11 for more detail.

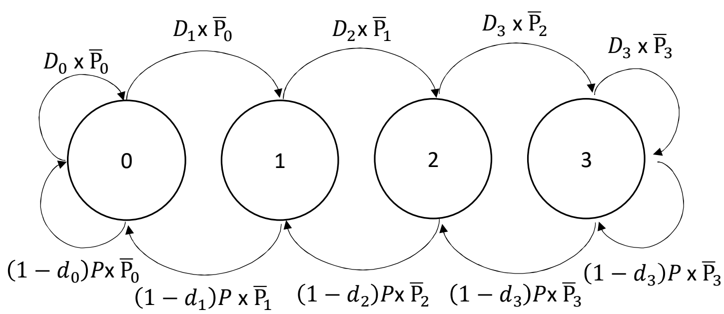


Figure 11. MC Model One for Third Scenario with Four States

The states probabilities are: $\bar{P}_1 = \frac{D_1}{(1-d_1)P}\bar{P}_0$, $\bar{P}_2 = \frac{D_1 D_2}{(1-d_1)(1-d_2)P^2}\bar{P}_0$, $\bar{P}_3 = \frac{D_1 D_2 D_3}{(1-d_1)(1-d_2)(1-d_3)P^3}\bar{P}_0$, and $\bar{P}_0 = \left[\frac{1}{1+\frac{D_1}{(1-d_1)P}+\frac{D_1 D_2}{(1-d_1)(1-d_2)P^2}+\frac{D_1 D_2 D_3}{(1-d_1)(1-d_2)(1-d_3)P^3}}\right]$. The expected total cost

$$ETC(n+1) = \sum_{i=0}^{n}(\bar{P}_i \times TC_i) = \sum_{i=0}^{n}\left\{\left\{\left\{\sum_{h=0}^{n}\left[\prod_{j=1}^{h}\left(\frac{D_j}{(1-d_j)}\right)\times P^h\right]^{k^*a}\right\}^{-1} \times \left[\frac{\prod_{j=1}^{i}(D_j)}{\prod_{j=1}^{i}(1-d_j)\times P^i}\right]^{k^*b}\right\} \times \left[\frac{KD}{Pt_i}+\frac{h(P-D)t_i}{2}\left(\frac{\pi}{h+\pi}\right)+\alpha_i\frac{sD}{t_i}\int_0^{t_i}(t_i-x)f(x)dx\right]\right\} \quad (3.5)$$

The defective products produced are highlighted in Figure 12. For more detail about the optimal solution, see Table VI.

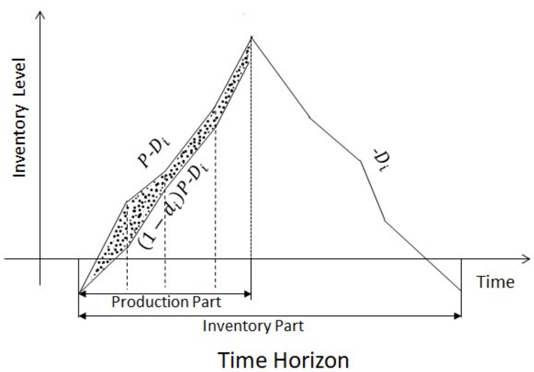


Figure 12. P&I Model for MC Model One with Third Scenario

Table VI. MC Model One Third Scenario Solution (Base)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	1200	850	1.00	0.4361	130.778	0.1494	0.9339	0.4461	1.7446
1	1200	650	0.7188	0.3135	143.406	0.1042	0.6512	0.6392	1.2539
2	1200	500	0.3996	0.1743	141.900	0.0810	0.5063	0.8222	0.6971
3	1200	400	0.1745	0.0761	135.699	0.0678	0.4235	0.9826	0.3043
Expected Values					137.050	0.1171	0.7319	0.6130	
Production and inventory Model					142.207	0.1119	0.6992	0.2764	
Difference					5.1570	0.0052	0.0327	0.3366	

In this scenario, the optimal pair of production and demand rates provided in the Table VII will be found for each state in order to decrease the expected total cost. For more detail about each example, see Appendix C.

Table VII. MC Model One Third Scenario Solution and *P*-Values

No.	Example Description	ETC	Optimal Cost	*P*-Value	CI
1	Base - (*t*-*x*)	137.05	142.207	0.7786	(128.673, 147.218)
2	Duration (*t*)	137.096	142.275	0.7761	(128.704, 147.306)
3	Duration - (*t*-*x*) - Optimal Pair	133.527	141.458	0.2282	(128.609, 147.325)
4	Duration - (*t*) - Optimal Pair	133.561	141.458	0.2287	(128.579, 147.514)

4.5. Fourth Scenario: Variable Production and Demand Rates

In this scenario, the production and demand rates are variable among the states. Four states will be used and the balanced equation in order to determine the states' probabilities which will help in finding the expected total cost. See Figure 13 for more detail.

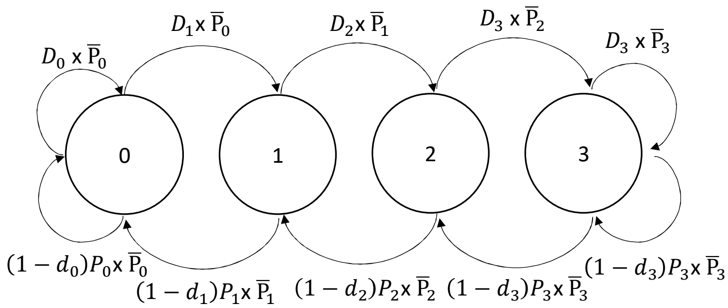


Figure 13. MC Model One for Fourth Scenario with Four States

The states probability are $\bar{P}_1 = \frac{D_1}{(1-d_1)P_1}\bar{P}_0$, $\bar{P}_2 = \frac{D_1 D_2}{(1-d_1)(1-d_2)P_1 P_2}\bar{P}_0$, $\bar{P}_3 = \frac{D_1 D_2 D_3}{(1-d_1)(1-d_2)(1-d_3)P_1 P_2 P_3}\bar{P}_0$, and $\bar{P}_0 = \left[\frac{1}{1+\frac{D_1}{(1-d_1)P_1}+\frac{D_1 D_2}{(1-d_1)(1-d_2)P_1 P_2}+\frac{D_1 D_2 D_3}{(1-d_1)(1-d_2)(1-d_3)P_1 P_2 P_3}}\right]$. The expected total cost

$$ETC(n+1) = \sum_{i=0}^{n}(\bar{P}_i \times TC_i) = \sum_{i=0}^{n}\left\{\left\{\left\{\sum_{h=0}^{n}\left\{\prod_{j=1}^{h}\left[\frac{D_j}{(1-d_j)P_j}\right]\right\}^{k^{*a}}\right\}^{-1} \times \left\{\prod_{j=1}^{i}\left[\frac{D_j}{(1-d_j)P_j}\right]\right\}^{k^{*b}}\right\} \times \left[\frac{KD_i}{P_i t_i} + \frac{h(P_i - D_i)t_i}{2}\left(\frac{\pi}{h+\pi}\right) + \alpha_i \frac{sD_i}{t_i}\int_0^{t_i}(t_i - x)f(x)dx\right]\right\} \tag{3.6}$$

The defective products produced are highlighted in Figure 14. For more detail about the optimal solution, see Table VIII.

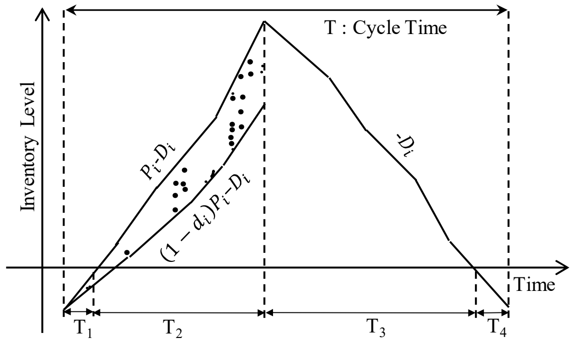


Figure 14. P&I Model for MC Model One with Fourth Scenario

Table VIII. MC Model One Fourth Scenario Solution (Base)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	1200	850	1.00	0.3998	130.778	0.1494	0.9339	0.4461	1.5992
1	950	650	0.6943	0.2776	119.080	0.1585	0.9907	0.5304	1.1103
2	750	500	0.4751	0.1899	107.359	0.1713	1.0708	0.6210	0.7597
3	600	400	0.3318	0.1327	96.124	0.1914	1.1960	0.6937	0.5307
Expected Values					118.485	0.1617	1.0104	0.5356	
Production and Inventory Model					119.228	0.1602	1.0010	0.5164	
Difference					0.743	0.0015	0.0094	0.0191	

In this scenario, three optimization problems are provided in order to minimize the expected total cost. Examples 3&6 are determining the optimal selected production rates. Moreover, examples 4, 5, 7, and 8 are determining the optimal selected and optimized production rates. The expected total cost is decreased, see the summary Table IX and Table X. For more detail about each example, see Appendix D.

Table IX. MC Model One Fourth Scenario Solution and P-Values

No.	Example Description	ETC	Optimal Cost	P-Value	CI
1	Base - (t-x)	118.485	119.228	0.54	(89.569, 137.1)
2	Duration - (t)	118.572	119.228	0.54	(89.811, 137.088)
3	Duration - (t-x) - Optimal Pair	108.355	109.119	0.6669	(89.569, 137.1)
4	Duration - (t-x) - Optimal Selected Production Rates	82.782	95.194	0.62908	(45.531, 138.389)
5	Duration - (t-x) - Optimal Production Rates	76.943	77.37	0.7886	(73.941, 79.439)
6	Duration - (t) - Optimal Pair	108.462	109.119	0.5558	(89.135, 137.9214)
7	Duration - (t) - Optimal Selected Production rates	82.904	95.194	0.5736	(45.696, 138.466)
8	Duration - (t) - Optimal Production Rates	77.1013	77.37	0.7888	(74.286, 79.443)

Table X. MC Model One Scenario Four Optimal Production and Demand Rates Solution

State (i)	Example No.		
	3 & 6	4 & 7	5 & 8
0	(600, 400)	(950, 850)	(950, 850)
1	(750, 500)	(750, 650)	(750, 650)
2	(950, 650)	(600, 500)	(600, 500)
3	(1200, 850)	(1200, 400)	(500, 400)

Regarding the detailed production plan between states. The shortages at the end of the state should equal the shortages in the next state. There are two situations, either both are equal or are not equal. In the second situation, there could be two cases, see Figure 15 for more detail.

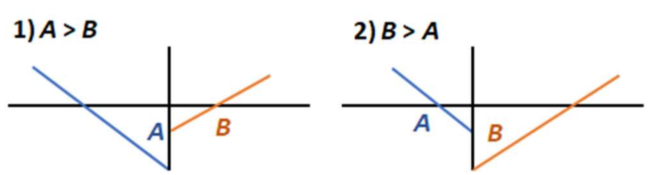


Figure 15. Production Plan for the Shortages Between States

For the first case, the following equations will be used to determine the additional time for production in order to decrease the shortages to match the next state's shortages. The starting states will have its optimal time T_4 and then $T_{4\alpha}$ for production.

$$A = D_A \times T_{4A} \tag{3.5}$$

$$B = (P_B - D_B) \times T_{1B} \tag{3.6}$$

$$A - B = D_A \times T_{4A} - (P_B - D_B) \times T_{1B} \tag{3.7}$$

$$T_{4\alpha} = \frac{A - B}{P_A - D_A} = \frac{D_A \times T_{4A} - (P_B - D_B) \times T_{1B}}{P_A - D_A} \tag{3.8}$$

For the second case, the following equations will be used to determine the additional time in order to increase the shortages to match the next state's shortages. The starting states will have its optimal time T_4 and then $T_{4\alpha}$ that both times have no production activity.

$$B - A = (P_B - D_B) \times T_{1B} - D_A \times T_{4A} \tag{3.9}$$

$$T_{4\beta} = \frac{B - A}{D_A} = \frac{(P_B - D_B) \times T_{1B} - D_A \times T_{4A}}{D_A} \tag{3.10}$$

$$T_{4A^*} = T_4 + T_{4\beta} \tag{3.11}$$

4. MARKOV CHAIN MODEL TWO WITH TIME FACTOR

In this MC model two, a time factor is added and integrated to the model which are the optimal cycle time and the optimal production run time for each state. Also, the number of defective products in each state is included in each state, see Figure 16 for more detail. Moreover, the accuracy of the evaluated expected total cost in this model is higher than the accuracy of MC model one. This finding is proved by the calculated P-Value for all the developed scenarios and their examples.

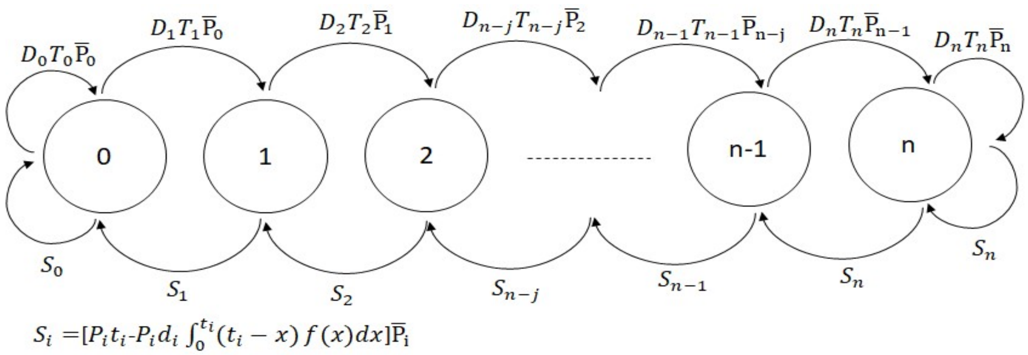


Figure 16. MC Model Two with (n+1) States

4.1. Notation

The following notation will be used in the MC model two along with the previously mentioned notation in section 3.1.

Parameter	Description
S_i	the produced quantity without defective products multiplied by the probability of state (i)
$\bar{S}_i$	the average produced quantity without defective products
R_i	abbreviation symbols used in calculating the state's probabilities

4.2. First Scenario: Fixed Production and Demand Rates

In this scenario, the production and demand rates are fixed among the states. Four states will be used and the balanced equation in order to determine the states' probabilities which will help in finding the expected total cost. See Figure 17 for more detail.

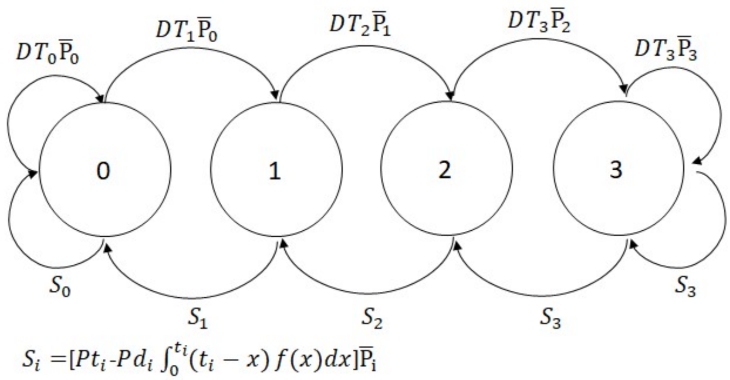


Figure 17. MC model two First Scenario with Four States

We will use the following abbreviation to find the states' probabilities. $R_i = DT_i$ and $S_i = Pt_i - Pd_i \int_o^{t_i}(t_i - x)f(x)dx$. The state one probability is $\bar{P}_1 = \left(\frac{R_1}{S_1}\right)\bar{P}_0$, state two is $\bar{P}_2 = \left(\frac{R_2 R_1}{S_2 S_1}\right)\bar{P}_0$, state three is $\bar{P}_3 = \left(\frac{R_3 R_2 R_1}{S_3 S_2 S_1}\right)\bar{P}_0$, and then for state zero is $\bar{P}_0 = \frac{1}{\left(1+\frac{R_1}{S_1}+\frac{R_2 R_1}{S_2 S_1}+\frac{R_3 R_2 R_1}{S_3 S_2 S_1}\right)}$.

Moreover, the expected total cost for (n+1) states is

$$ETC(n+1) = \sum_{i=0}^{n}(\bar{P}_i \times TC_i) = \sum_{i=0}^{n}\left\{\left\{\sum_{h=0}^{n}\left\{\prod_{j=1}^{h}\left\{\frac{DT_j}{\left[Pt_j - Pd_j\int_o^{t_j}(t_j-x)f(x)dx\right]}\right\}\right\}^{k^{*a}}\right\}^{-1} \times \left\{\prod_{j=1}^{i}\left\{\frac{DT_j}{\left[Pt_j - Pd_j\int_o^{t_j}(t_j-x)f(x)dx\right]}\right\}\right\}^{k^{*b}} \times \left[\frac{KD}{Pt_i} + \frac{h(P-D)t_i}{2}\left(\frac{\pi}{h+\pi}\right) + \alpha_i\frac{sD}{t_i}\int_0^{t_i}(t_i - x)f(x)dx\right]\right\} \quad (4.1)$$

The defective products produced are highlighted in Figure 18. For more detail about the optimal solution, see Table XI. For more detail about each example, see Table XII and Appendix E.

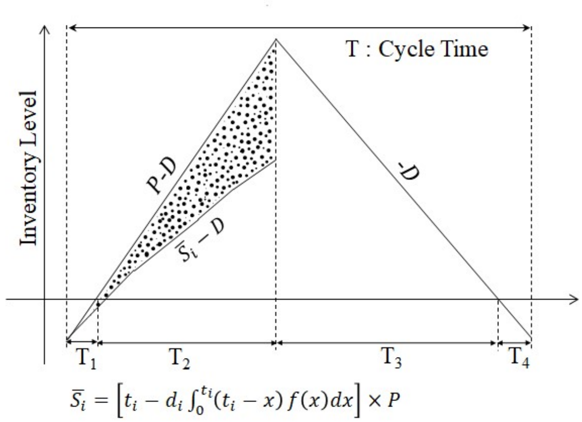


Figure 18. P&I Model for MC model two with First Scenario

Table XI. MC Model Two First Scenario Solution (Base)

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
0	600	400	1.00	0.2489	95.904	0.1918	1.1985	0.6951	0.9955
1	600	400	1.0018	0.2493	95.966	0.1917	1.1978	0.6947	0.9974
2	600	400	1.0051	0.2502	96.023	0.1915	1.1972	0.6944	1.0006
3	600	400	1.0110	0.2516	96.124	0.1914	1.1960	0.6937	1.0065
Expected Values					96.005	0.1916	1.1974	0.6945	
Production and Inventory Model					96.005	0.1916	1.1974	0.6945	
Difference					0.000	0.0000	0.0000	0.0000	

Table XII. MC Model Two First Scenario Solution and *P*-Values

Example		Optimal Cost	*P*-Value	CI
Base	96.005	96.005	0.9941	(95.856, 96.153)
First Variation	96.178	95.988	0.987	(95.887, 96.465)
Second Variation	96.291	96.096	0.9819	(95.851, 96.723)

4.3. Second Scenario: Variable Production and Fixed Demand Rates

In this scenario, the production rates are variable and demand rates are constant among the states. Four states will be used and the balanced equation in order to determine the states' probabilities which will help in finding the expected total cost. See Figure 19 for more detail.

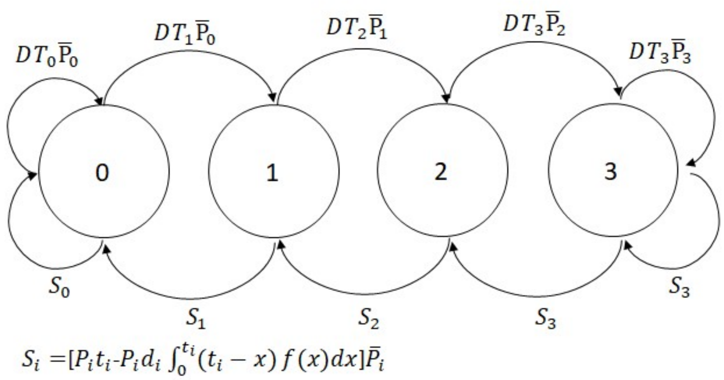


Figure 19. MC model two Second Scenario with Four States

We will use the following abbreviation to find the states' probabilities. $R_i = DT_i$ and $S_i = P_i t_i - P_i d_i \int_o^{t_i} (t_i - x) f(x) dx$. $\bar{P}_1 = \left(\frac{R_1}{S_1}\right)\bar{P}_0$, $\bar{P}_2 = \left(\frac{R_2R_1}{S_2S_1}\right)\bar{P}_0$, $\bar{P}_3 = \left(\frac{R_3R_2R_1}{S_3S_2S_1}\right)\bar{P}_0$, and $\bar{P}_0 = \frac{1}{\left(1+\frac{R_1}{S_1}+\frac{R_2R_1}{S_2S_1}+\frac{R_3R_2R_1}{S_3S_2S_1}\right)}$. The expected total cost

$$ETC(n+1) = \sum_{i=0}^{n} (\bar{P}_i \times TC_i) = \sum_{i=0}^{n} \left\{ \left\{ \sum_{h=0}^{n} \left\{ (D)^h \times \prod_{j=1}^{h} \left\{ \frac{T_j}{\left[P_j t_j - P_j d_j \int_o^{t_j} (t_j - x) f(x) dx \right]} \right\} \right\}^{k^*a} \right\}^{-1} \times \left\{ (D)^i \times \prod_{j=1}^{i} \left\{ \frac{T_j}{\left[P_j t_j - P_j d_j \int_o^{t_j} (t_j - x) f(x) dx \right]} \right\} \right\}^{k^*b} \times \left[\frac{KD}{P_i t_i} + \frac{h(P_i - D) t_i}{2} \left(\frac{\pi}{h+\pi} \right) + \alpha_i \frac{sD}{t_i} \int_0^{t_i} (t_i - x) f(x) dx \right] \right\} \tag{4.2}$$

The defective products produced are highlighted in Figure 20. For more detail about the optimal solution, see Table XIII. For more detail about each example, see Table XIV, Table XV, and Appendix F.

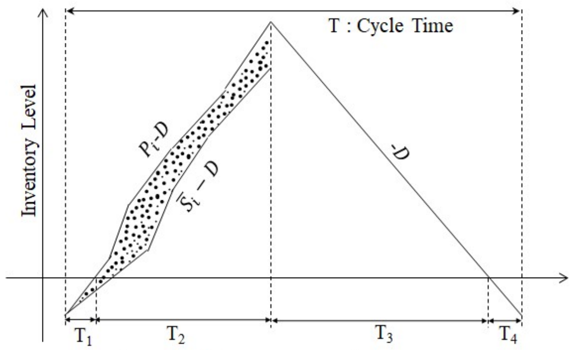


Figure 20. P&I Model for MC model two with Second Scenario

Table XIII. MC Model Two Second Scenario Solution (Base)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
0	1200	850	1.00	0.2495	130.778	0.1494	0.9339	0.4461	0.9979
1	1200	650	1.0010	0.2497	143.406	0.1042	0.6512	0.6392	0.9990
2	1200	500	1.0025	0.2501	141.900	0.0810	0.5063	0.8222	1.0004
3	1200	400	1.0047	0.2507	135.699	0.0678	0.4235	0.9826	1.0026
Expected Values					137.947	0.1005	0.6284	0.7229	
Production and Inventory Model					143.927	0.0958	0.5987	0.4591	
Difference					5.9800	0.0048	0.0297	0.2637	

Table XIV. MC Model Two Second Scenario Solution and P-Values

No.	**Example Description**	**ETC**	**Optimal Cost**	***P*-Value**	**CI**
1	Base - (*t-x*)	117.876	122.395	0.9956	(90.682, 145.171)
2	Duration (*t*)	117.914	122.417	0.9917	(90.909, 145.113)
3	Duration (*t-x*) - Optimal Selected Production Rates	117.876	122.399	0.9959	(90.676, 145.174)
4	Duration - (*t-x*) - Optimal Production Rates	90.0661	91.697	0.9931	(70.61, 109.637)
5	Duration - (*t*) - Optimal Selected Production Rates	117.915	122.424	0.9921	(90.118, 145.118)
6	Duration - (*t*) - Optimal Production Rates	90.144	91.761	0.987	(70.933, 109.569)

Table XV. MC Model Two Scenario Two Optimal Production Rates Solution

State (i)	Example No.	
	3 & 5	4 & 6
0	1,200	650
1	950	600
2	750	550
3	600	500

4.4. Third Scenario: Fixed Production and Variable Demand Rates

In this scenario, the production rates are constant and demand rates are variable among the states. Four states will be used and the balanced equation in order to determine the states' probabilities which will help in finding the expected total cost. See Figure 21 for more detail.

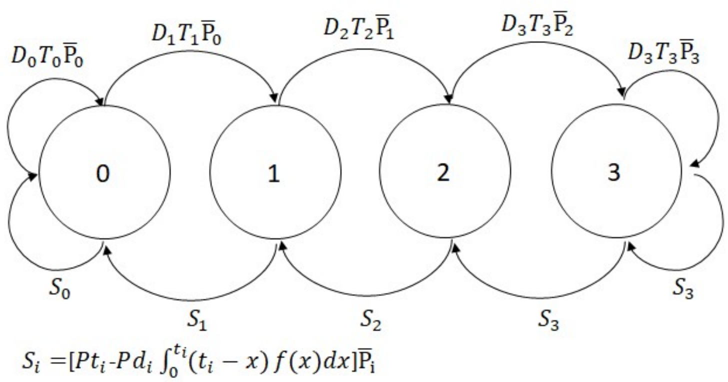


Figure 21. MC model two Third Scenario with Four States

We will use the following abbreviation to find the states' probabilities. $R_i = D_i T_i$ and $S_i = Pt_i - Pd_i \int_o^{t_i} (t_i - x) f(x) dx$. $\bar{P}_1 = \left(\frac{R_1}{S_1}\right) \bar{P}_0$, $\bar{P}_2 = \left(\frac{R_2 R_1}{S_2 S_1}\right) \bar{P}_0$, $\bar{P}_3 = \left(\frac{R_3 R_2 R_1}{S_3 S_2 S_1}\right) \bar{P}_0$, and $\bar{P}_0 = \frac{1}{\left(1 + \frac{R_1}{S_1} + \frac{R_2 R_1}{S_2 S_1} + \frac{R_3 R_2 R_1}{S_3 S_2 S_1}\right)}$. The expected total cost

$$TC(n+1) = \sum_{i=0}^{n}(\bar{P}_i \times TC_i) =$$

$$\sum_{i=0}^{n}\left\{\left\{\sum_{h=0}^{n}\left\{\prod_{j=1}^{h}\left\{\frac{D_j \times T_j}{\left[Pt_j - Pd_j\int_o^{t_j}(t_j - x)f(x)dx\right]}\right\}\right\}^{k^*a}\right\}^{-1} \times$$

$$\left\{\prod_{j=1}^{i}\left\{\frac{D_j \times T_j}{\left[Pt_j - Pd_j\int_o^{t_j}(t_j - x)f(x)dx\right]}\right\}\right\}^{k^*b} \times \left[\frac{KD_i}{Pt_i} + \frac{h(P-D_i)t_i}{2}\left(\frac{\pi}{h+\pi}\right) + \alpha_i\frac{sD_i}{t_i}\int_0^{t_i}(t_i - \tag{4.3}$$

$$x)f(x)dx\right]\Bigg\}$$

The defective products produced are highlighted in Figure 22. For more detail about the optimal solution, see Table XVI. For more detail about second example, see Table XVII and Appendix G.

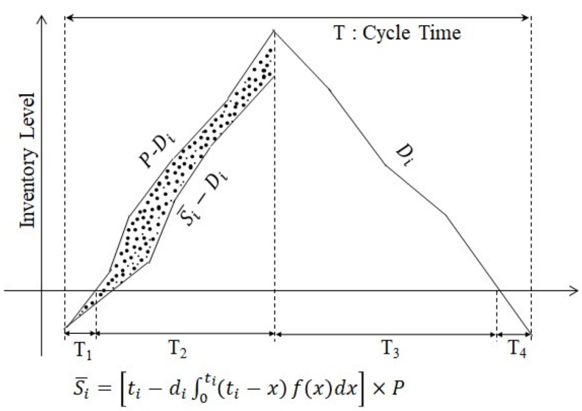


Figure 22. P&I Model for MC model two with Third Scenario

Table XVI. MC Model Two Third Scenario Solution (Base)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	1200	850	1.00	0.2495	130.778	0.1494	0.9339	0.4461	0.9979
1	1200	650	1.0010	0.2497	143.406	0.1042	0.6512	0.6392	0.9990
2	1200	500	1.0025	0.2501	141.900	0.0810	0.5063	0.8222	1.0004
3	1200	400	1.0047	0.2507	135.699	0.0678	0.4235	0.9826	1.0026
Expected Values					137.947	0.1005	0.6284	0.7229	
Production and Inventory Model					143.927	0.0958	0.5987	0.4591	
Difference					5.9800	0.0048	0.0297	0.2637	

Table XVII. MC Model Two Third Scenario Solution and P-Values

No.	Example Description		Optimal Cost	P-Value	CI
1	Base - (t-x)	137.947	143.927	0.9997	(128.673, 147.218)
2	Duration (t)	138.008	144.01	0.9992	(128.704, 2147.306)

4.5. Fourth Scenario: Variable Production and Demand Rates

In this scenario, the production and demand rates are variable among the states. Four states will be used and the balanced equation in order to determine the states' probabilities which will help in finding the expected total cost. See Figure 23 for more detail.

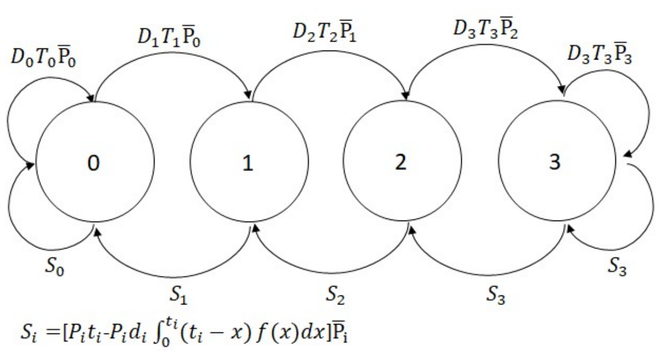


Figure 23. MC model two Fourth Scenario with Four States

We will use the following abbreviation to find the states' probabilities. $R_i = D_i T_i$ and $S_i = P_i t_i - P_i d_i \int_o^{t_i}(t_i - x)f(x)dx$. $\bar{P}_1 = \left(\frac{R_1}{S_1}\right)\bar{P}_0$, $\bar{P}_2 = \left(\frac{R_2 R_1}{S_2 S_1}\right)\bar{P}_0 \bar{P}_3 = \left(\frac{R_3 R_2 R_1}{S_3 S_2 S_1}\right)\bar{P}_0$, and $\bar{P}_0 = \frac{1}{\left(1+\frac{R_1}{S_1}+\frac{R_2 R_1}{S_2 S_1}+\frac{R_3 R_2 R_1}{S_3 S_2 S_1}\right)}$. The expected total cost

$$ETC(n+1) = \sum_{i=0}^{n}(\bar{P}_i \times TC_i) =$$

$$\sum_{i=0}^{n}\left\{\left\{\sum_{h=0}^{n}\left\{\prod_{j=1}^{h}\left\{\frac{D_j \times T_j}{\left[P_j t_j - P_j d_j \int_o^{t_j}(t_j - x)f(x)dx\right]}\right\}\right\}^{k*a}\right\}^{-1} \times \left\{\prod_{j=1}^{i}\left\{\frac{D_j \times T_j}{\left[P_j t_j - P_j d_j \int_o^{t_j}(t_j - x)f(x)dx\right]}\right\}\right\}^{k*b} \times \left[\frac{KD_i}{P_i t_i} + \frac{h(P_i - D_i)t_i}{2}\left(\frac{\pi}{h+\pi}\right) + \alpha_i \frac{sD_i}{t_i}\int_0^{t_i}(t_i - x)f(x)dx\right]\right\} \quad (4.4)$$

The defective products produced are highlighted in Figure 24. For more detail about the optimal solution see Table XVIII. For more detail about other examples, see Table XIX, Table XX, and Appendix H.

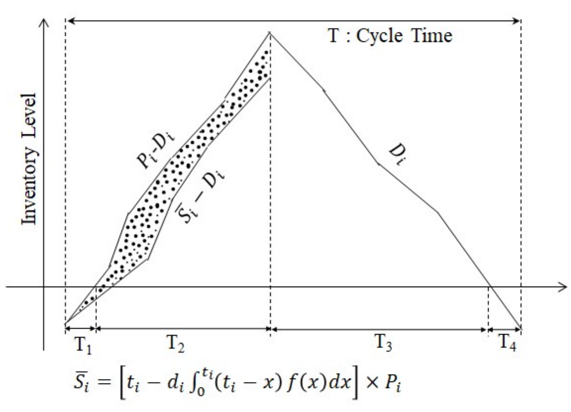


Figure 24. P&I Model for MC model two with Fourth Scenario

Table XVIII. MC Model Two Fourth Scenario Solution (Base)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	1200	850	1.00	0.2490	130.778	0.1494	0.9339	0.4461	0.9959
1	950	650	1.0016	0.2494	119.080	0.1585	0.9909	0.5305	0.9975
2	750	500	1.0045	0.2501	107.359	0.1714	1.0714	0.6214	1.0004
3	600	400	1.0104	0.2516	96.124	0.1915	1.1972	0.6944	1.0062
Expected Values					113.286	0.1678	1.0487	0.5734	
Production and Inventory Model					114.138	0.1657	1.0358	0.5509	
Difference					0.8520	0.0021	0.0129	0.0226	

Table XIX. MC Model Two Fourth Scenario Solution and P-Values

No.	Example Description	ETC	Optimal Cost	P-Value	CI
1	Base - (*t-x*)	113.286	114.138	0.9952	(89.569, 137.101)
2	Duration - (*t*)	113.356	114.094	0.9908	(89.811, 137.088)
3	Duration - (*t-x*) - Optimal Selected Production Rates	92.027	114.285	0.9966	(45.531, 138.388)
4	Duration - (*t-x*) - Optimal Production Rates	76.6808	77.132	0.9922	(73.941, 79.439)
5	Duration - (*t*) - Optimal Selected Production Rates	92.21	114.375	0.9935	(45.696, 138.466)
6	Duration - (*t*) - Optimal Production Rates	76.8482	77.345	0.9856	(74.286, 79.443)

Table XX. MC Model Two Fourth Scenario Optimal Production and Demand Rates Solution

State (i)	Example No.		
	3	5	4 & 6
0	(950, 850)	(950, 850)	(950, 850)
1	(750, 650)	(750, 650)	(750, 650)
2	(600, 500)	(1200, 500)	(600, 500)
3	(1200, 400)	(600, 400)	(500, 400)

Two MC models are developed which are the MC model one without time factor and MC model two with time factor. Detailed production plans for each state in each example are provided. The used collected data for the defect rates are random and stochastic. Each state is assigned with a defect rate. They are used and implemented their roles by using a Markovian approach and integrated with the imperfect production and inventory control model.

5. SCHEDULING HARD TIME WINDOWS

Two different production processes with single product can interact with each other. The jobs requests can be passed from a production process to another different production process. As a result, there will be two different processes which are the production process one (PP1) and the production process two (PP2). One of them will be a sender and the other will be a receiver production process. By using the developed MC model one and MC model two, the scheduling between theses two production processes can be constructed. The defect dates that are collected will be used for that purpose. The name of MCA will be assigned to the receiver production process (PP1) in order to fulfil the required demand from the sender production presses (PP2) which will be MCB. In this chapter, a scheduling model is developed in order to determine the optimal time when to pass jobs from production process two (PP2) to the other production process one (PP1). The developed MC model will be used in order to find the optimal scheduling hard time windows using the expected total cost function. Both the developed MC model one and MC model two will be considered. Two cases will be used in order to address either using the same or different production and demand rates for both production processes. The first case will be with same production and demand rates and the second case will have different production and demand rates for the concerned states. In the second case, the cost functions for the receiver MC will be developed in order to address it in the developed expected total cost function. Consequently, each state (state 2 and 3) in the receiver MCA will have two cost functions and with the required load percentages.

The scheduling hard time window using MC models will be introduced in order to determine the optimal feasible hard time windows for the jobs that should be passed between the two different production processes. As a result, the current setup of both of them is that the jobs will be passed from the sender PP2 (MCB) to the receiver PP1 (MCA).

5.1. Notation

The following notation is to be used for the P&I control model with shortages and the MC models.

Parameter	Description
P_{A_i}, P_{B_i}	production rate for MCA and MCB for state (i)
D_{A_i}, D_{B_i}	demand rate for MCA and MCB for state (i)
d_{A_i}, d_{B_i}	defect rate for MCA and MCB for state (i)
$\bar{P}_{A_i}, \bar{P}_{B_i}$	probability of being in state (i) for MCA and MCB
T_{A_i}, T_{B_i}	optimal cycle time ($T_1 + T_2 + T_3 + T_4$) for MCA and MCB for state (i)
t_{A_i}, t_{B_i}	optimal production run time for MCA and MCB for state (i)
S_{A_i}, S_{AiX}	flow rate for MCA from state (i) to state (i-1) multiplied by the probability of state (i)
S_{B_i}	flow for MCB from state (i) to state (i-1) multiplied by the probability of state (i)
S_{AiY}	flow from MCA to MCB from state (i) to state (i-1) multiplied by the probability of state (i)

5.2. Scheduling Between Two Production Processes Having Same Production and Demand Rates (Case 1)

Case 1 will use the four scenarios, however, the production and demand rates for the concerned states will be similar. Four states will be used and the balanced equation in order to determine the states' probabilities which will help in finding the expected total cost.

5.2.1. Scheduling Model by Using MC Model (One)

In this section, the scheduling model will be introduced by using the MC model one. In this section, the scheduling model will be introduced by using the MC model Two. The scheduled jobs from PP2 (state one and two) to PP1 (state two and three), the production plan will be taken from PP2 (state two and two) because we are directing the jobs using the current optimal cycle time (and) as shown on . However, the scheduling cost will be taken from PP1 (state two and three) since the jobs is processed on it. See Figure 25 for more detail.

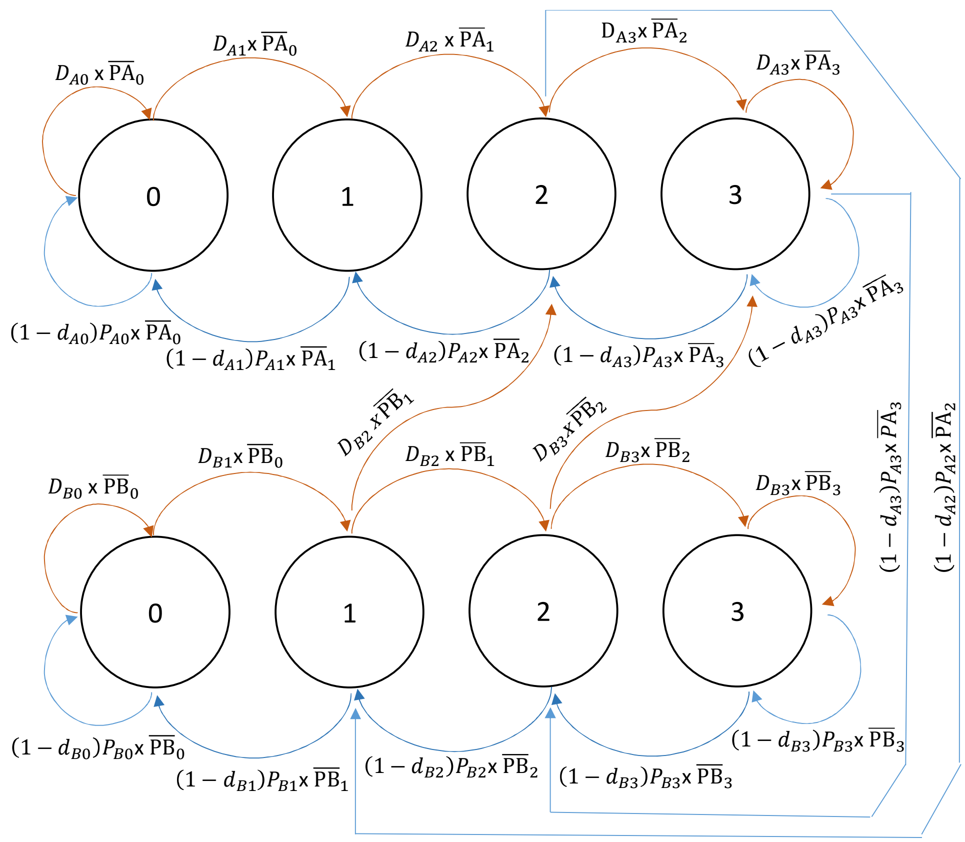


Figure 25. Scheduling Model Using MC Model One, Case One, and the Fourth Scenario

The balance equations for the developed scheduling MCA are

State zero $$D_{A1}\overline{\mathrm{PA}}_0 = (1-d_{A1})P_{A1}\overline{PA}_1 \tag{5.1}$$

State one $$[D_{A2} + (1-d_{A1})P_{A1}]\overline{\mathrm{PA}}_1 = D_{A1}\,\overline{PA}_0 + (1-d_{A2})P_{A2}\,\overline{PA}_2 \tag{5.2}$$

State two $$[\mathrm{D}_{A3} + 2(1-d_{A2})P_{A2}]\,\overline{\mathrm{PA}}_2 = D_{A2}\overline{PA}_1 + (1-d_{A3})\mathrm{P}_{A3}\,\overline{PA}_3 + D_{B2}\overline{PB}_1 \tag{5.3}$$

State three $$2(1-d_{A3})P_{A3}\overline{\mathrm{PA}}_3 = \mathrm{D}_{A3}\overline{\mathrm{PA}}_2 + D_{B3}\overline{PB}_2 \tag{5.4}$$

The balance equations for the developed scheduling MCB are

State zero $$D_{B1}\mathrm{x}\,\overline{\mathrm{PB}}_0 = (1-d_{B1})P_{B1}x\overline{PB}_1 \tag{5.5}$$

State one $$[2D_{B2} + (1-d_{B1})P_{B1}]x\overline{PB}_1 = D_{B1}\overline{PB}_0 + (1-d_{B2})P_{B2}\,\overline{PB}_2 + (1-d_{A2})P_{A2}\overline{PA}_2 \tag{5.6}$$

State two $$[2D_{B3} + (1 - d_{B2})P_{B2}]\overline{PB}_2 = D_{B2}\overline{PB}_1 + (1 - d_{B3})P_{B3}\overline{PB}_3 + (1 - d_{A3})P_{A3}\overline{PA}_3 \tag{5.7}$$

State three $$(1 - d_{B3})P_{B3}\overline{\mathrm{PB}}_3 = D_{B3}\overline{PB}_2 \tag{5.8}$$

The last constraint is the sum of all probabilities should be equal to on: $\overline{\mathrm{PA}}_0 + \overline{\mathrm{PA}}_1 + \overline{\mathrm{PA}}_2 + \overline{\mathrm{PA}}_3 + \overline{\mathrm{PB}}_0 + \overline{\mathrm{PB}}_1 + \overline{\mathrm{PB}}_2 + \overline{\mathrm{PB}}_3 = 1$.

The expected total cost is (119.962), the expected production rate is (1037.13), and the expected demand rate is (740.701), and the expected defect rate is (0.0160767). We will use the last three figures to calculate the optimal total cost which is found to be (120.962) and is close to the expected total cost with a difference equal to (1.139) even with the other optimal values. The *P*-Value for the expected total cost equal (0.3499) which is not significant and the (95%) confidence interval is (101.9187, 127.0735). The developed MC model one has a high accuracy in the evaluated expected total cost which is shown by the achieved *P*-Value.

The production plan for PP1 (state zero) will have the optimal total cost is (135.346), the optimal time when the shortages are met is (0.1546), the optimal time when inventory is built (0.9664), the optimal production run time is (1.121), the optimal time for the inventory to be used and shortages to be built is (0.3567), the probability of being in state zero is (0.1878), and the expected state time is (1.5027) years. The same practice will be stated for states one, two, three and also for the PP2. See Table XXI for more detail.

Table XXI.Scheduling Solution MC Model One Fourth Scenario Case 1 (Base)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1450	1100	1.00	0.1878	135.346	0.1546	0.9664	0.3567	1.5027
A1	1150	850	0.7500	0.1409	123.799	0.1647	1.0295	0.4215	1.1271
A2	750	500	0.5132	0.0964	107.359	0.1713	1.0708	0.6210	0.7712
A3	600	400	0.3585	0.0673	96.124	0.1914	1.1960	0.6937	0.5387
B0	1200	850	1.00	0.2029	130.778	0.1494	0.9339	0.4461	1.6234
B1	950	650	0.6943	0.1409	119.080	0.1585	0.9907	0.5304	1.1271
B2	750	500	0.4751	0.0964	107.359	0.1713	1.0708	0.6210	0.7712
B3	600	400	0.3318	0.0673	96.124	0.1914	1.1960	0.6937	0.5387
Expected values					119.823	0.1637	1.0232	0.5048	
Production and Inventory Model					120.962	0.1629	1.0180	0.4726	
Difference					1.139	0.0008	0.0051	0.0322	

The optimal solution, for the production process 2 (state 1) which is MCB, it can pass jobs to production process 1 (state 2) which is MCA from 2.7505 years until 3.401 years. For Production process 2 (state 2) which is MCB, it can pass jobs to production process 1 (state 3) which MCA from 3.5217 years until 3.9397 years. See Figure 26 for more detail.

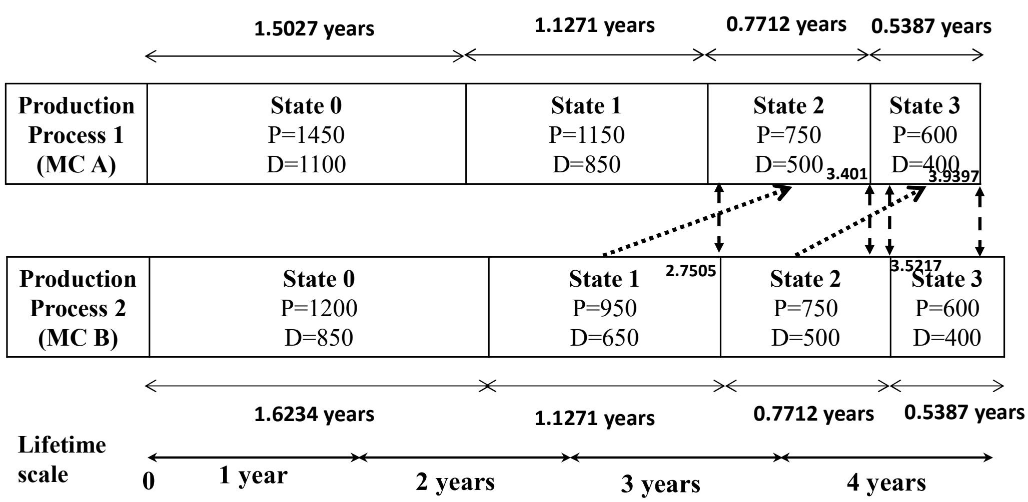


Figure 26. Scheduling Plan Solution Using MC Model One Fourth Scenario Case 1

In the following Table XXII, the summary for all four scenarios optimal solutions are presented. The associated *P*-Values for each run is an accepted hypothesis. For more detail about each run, see appendix I.

Table XXII. Scheduling Summary Solution for MC model one, Case 1, Four Scenarios, and the related *P*-Values with Confidence Intervals

Scenario No.	Description	E(*TC*)	Optimal Cost	*P*-Value	CI
1st	Base - (*t*-*x*)	107.321	108.486	0.546	(102.331, 115.97)
	First Variation - (*t*-*x*)	107.421	108.535	0.5408	(102.444, 116.115)
	Second Variation - (*t*-*x*)	107.471	108.496	0.5354	(102.505, 116.216)
2nd	Base Model - (*t*-*x*)	130.215	133.836	0.1317	(104.915, 134.301)
	Duration – (*t*-*x*) – Optimal Production Rates	123.377	131.623	0.2103	(86.663, 133.041)
	Duration – (*t*)	130.252	133.858	0.1316	(105.066, 134.32)
	Duration – (*t*) – Optimal Production Rates	123.437	131.651	0.2102	(86.872, 133.056)
3rd	Base – (*t*-*x*)	125.233	135.339	0.4235	(116.387, 144.004)
	Duration – (*t*)	125.295	135.419	0.4226	(116.459, 144.078)
4th	Base – (*t*-*x*)	119.823	120.962	0.3499	(101.919, 127.074)
	Duration – (*t*-*x*) – Optimal Production Rates	107.196	111.045	0.5719	(77.756, 124.831)
	Duration – (*t*)	119.916	120.962	0.3501	(102.095, 127.135)
	Duration – (*t*) – Optimal Production Rates	107.331	111.045	0.572	(78.003, 124.904)

5.2.2. Scheduling Model by Using Markov Chain Model (Two)

In this section, the scheduling model will be introduced by using the MC model Two. The scheduled jobs from PP2 (state one and two) to PP1 (state two and three), the production plan will be taken from PP2 (state two and two) because we are directing the jobs using the current optimal cycle time (and) as shown on . However, the scheduling cost will be taken from PP1 (state two and three) since the jobs is processed on it. For more detail, see Figure 27.

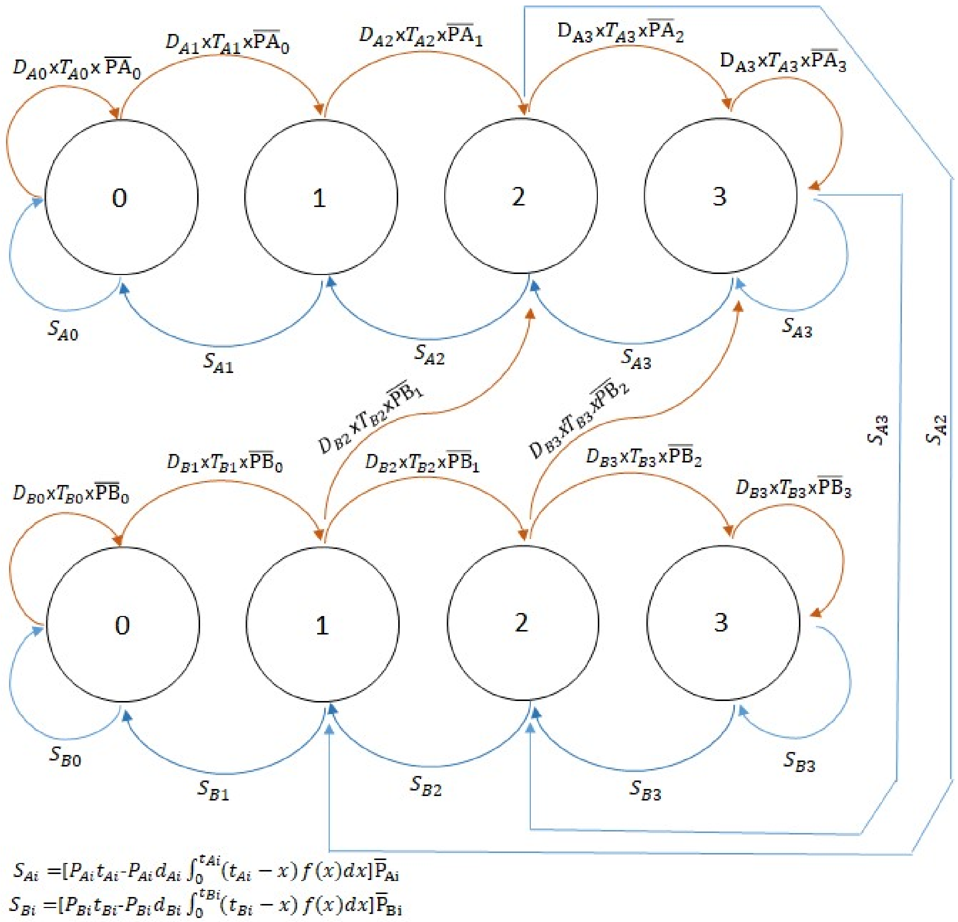


Figure 27. Scheduling Model Using MC Model Two Case 2

The balance equations for the developed scheduling MCA are

Abbreviation $$S_{Ai} = \left[P_{Ai}t_{Ai} - P_{Ai}d_{Ai}\int_0^{t_{Ai}}(t_{Ai}-x)\,f(x)dx\right]\bar{P}_{Ai} \qquad (5.9)$$

State zero $$DT_{A1}\overline{\text{PA}}_0 = S_{A1} \qquad (5.10)$$

State one $$DT_{A2}\overline{\text{PA}}_1 + S_{A1} = DT_{A1}\overline{PA}_0 + S_{A2} \qquad (5.11)$$

State two $$DT_{A3}\overline{\text{PA}}_2 + 2S_{A2} = DT_{A2}\overline{PA}_1 + S_{A3} + DT_{B2}\overline{PB}_1 \qquad (5.12)$$

State three $$2S_{A3} = DT_{A3}\overline{PA}_2 + DT_{B3}\overline{PB}_2 \qquad (5.13)$$

The balance equations for the developed scheduling MCB are

Abbreviation $S_{Bi} = \left[P_{Bi}t_{Bi} - P_{Bi}d_{Bi}\int_0^{t_{Bi}}(t_{Bi} - x)\,f(x)dx\right]\bar{P}_{Bi}$ (5.14)

State zero $DT_{B1}\overline{\mathrm{PB}}_0 = S_{B1}$ (5.15)

State one $2DT_{B2}\overline{\mathrm{PB}}_1 + S_{B1} = DT_{B1}\overline{PB}_0 + S_{B2} + S_{A2}$ (5.16)

State two $2DT_{B3}\overline{\mathrm{PB}}_2 + S_{B2} = DT_{B2}\overline{PB}_1 + S_{B3} + S_{A3}$ (5.17)

State three $S_{B3} = DT_{B3}\overline{PB}_2,$ (5.18)

The last constraint is the sum of all probabilities should be equal to on: $\overline{\mathrm{PA}}_0 + \overline{\mathrm{PA}}_1 + \overline{\mathrm{PA}}_2 + \overline{\mathrm{PA}}_3 + \overline{\mathrm{PB}}_0 + \overline{\mathrm{PB}}_1 + \overline{\mathrm{PB}}_2 + \overline{\mathrm{PB}}_3 = 1.$

To calculate the expected total cost is by the sum product of the optimal state's probability with its optimal total cost. The expected production rate is calculated by the sum product of the optimal state's probability with the states' production rate. The expected demand rate is calculated by the sum product of the optimal state's probability with the state's demand rate. Consequently, by using these methods, the expected total cost is (114.43), the expected production rate is (928.964), the expected demand rate is (654.184), and the expected defect rate is (0.0220824). We will use the last three figures to calculate the optimal total cost which is found to be (115.683) and is close to the expected total cost with a difference equal to (1.253) even with the other optimal values. The *P*-Value for the expected total cost equal (0.9884) which is not significant and the (95%) confidence interval is (101.948, 127.082). The developed MC model has a high accuracy in the evaluated expected total cost which is shown by the achieved high *P*-Value.

The production plan for PP1 (state zero) will have the optimal total cost is (135.346), the optimal time when the shortages are met is (0.1546), the optimal time when inventory is built (0.9664), the optimal production run time is (1.121), the optimal time for the inventory to be used and shortages to be built is (0.3567), the probability of being in state zero is (0.1218), and the expected state time is (0.9742) years. The same practice will be stated for states one, two, three and for the PP2's states. See Table XXIII for more detail.

Table XXIII. Scheduling MC model two, Case 1, Fourth Scenario Optimal Solution Base

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1450	1100	1.00	0.1218	135.346	0.1546	0.9664	0.3567	0.9742
A1	1150	850	1.0016	0.1220	123.799	0.1648	1.0297	0.4216	0.9758
A2	750	500	1.0046	0.1223	107.382	0.1749	1.0932	0.6341	0.9787
A3	600	400	1.0071	0.1226	96.163	0.1969	1.2306	0.7138	0.9811
B0	1200	850	1.00	0.1271	130.778	0.1494	0.9339	0.4461	1.0168
B1	950	650	1.0016	0.1273	119.080	0.1585	0.9909	0.5305	1.0184
B2	750	500	1.0076	0.1281	107.379	0.1680	1.0503	0.6092	1.0246
B3	600	400	1.0133	0.1288	96.153	0.1868	1.1672	0.6770	1.0304
Expected values					114.430	0.1692	1.0576	0.5494	
Production and Inventory Model					115.683	0.1679	1.0497	0.5114	
Difference					1.253	0.0013	0.0080	0.0380	

For Production process 2 (state 1) which is MCB, it can pass jobs to production process 1 (state 2) which is MCA from 2.0352 years until 2.9287 years. For Production process 2 (state 2) which is MCB, it can pass jobs to production process 1 (state 3) which MCA from 3.0598 years until 3.9098 years. See Figure 28 for more detail.

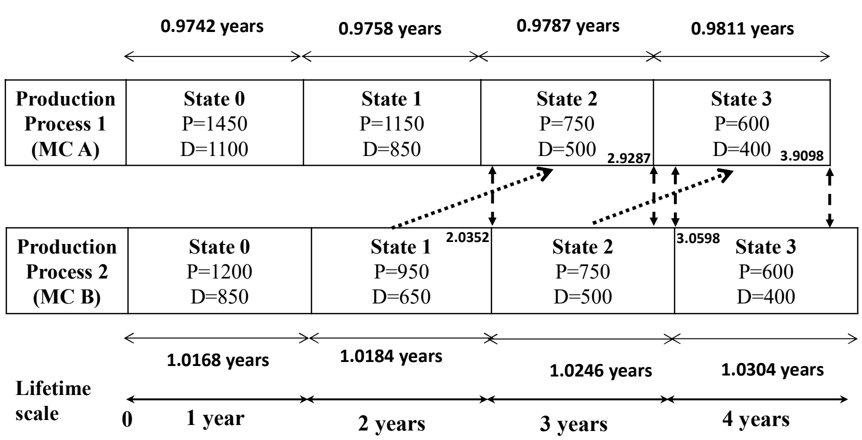


Figure 28. Scheduling Plan Solution for MC Model Two Fourth Scenario Case 1

In the following Table XXIV, the summary for all four scenarios optimal solutions are presented. The associated *P*-Values for each run is an accepted hypothesis. For more detail about each run, see appendix J.

Table XXIV. Scheduling Summary Solution for MC model two, Case 1, Four Scenarios, and the related *P*-Values with Confidence Intervals on the Expected Total Cost

Scenario No.	Description	E(*TC*)	Optimal Cost	*P*-Value	CI
1st	Base - (*t-x*)	108.908	109.637	0.8621	(102.459, 116.421)
	First Variation - (*t-x*)	109.058	109.613	0.8689	(102.569, 116.559)
	Second Variation - (*t-x*)	109.409	109.624	0.8545	(102.835, 117.134)
2nd	Base Model - (*t-x*)	119.521	125.353	0.9841	(104.997, 134.3)
	Duration – (*t-x*) – Optimal Production Rates	109.581	121.657	0.9667	(86.991, 133.013)
	Duration – (*t*)	119.554	125.367	0.9777	(105.152, 134.314)
	Duration – (*t*) – Optimal Production Rates	109.706	121.839	0.9734	(87.042, 133.042)
3rd	Base – (*t-x*)	129.659	142.932	0.8654	(116.573, 144.848)
	Duration – (*t*)	129.749	143.048	0.8682	(116.643, 144.914)
4th	Base – (*t-x*)	114.43	115.683	0.9884	(101.948, 127.082)
	Duration – (*t-x*) – Optimal Production Rates	101.095	105.143	0.9788	(77.898, 124.839)
	Duration – (*t*)	114.497	115.632	0.9808	(102.121, 127.137)
	Duration – (*t*) – Optimal Production Rates	101.142	105.004	0.971	(78.12, 124.909)

5.3. Scheduling Between Two Production Processes Having Different Production and Demand Rates (Case 2)

In this section, case number two will be described. There will be two cost functions associated with the selected state in the receiver MCA. The first one will be for the jobs coming from the existing receiver MCA and the other one for the jobs coming from the sender MCB. The sum of these two cost functions for each state in the receiver MC will be multiplied by the appropriate load percentages. The load for the receiver MCA for state (*i*) will be higher than the load from the sender MCB. The sum of these two factors will equal to one.

The receiver PP1 which will accept the scheduled jobs from the sender PP2 with a workload 40% of the time and the remaining workload 60% will be devoted to the jobs coming from the same PP1. In this case the production and demand rates for all state in PP1 and PP2 are not the same. There will be two cost functions associated with state two and with state three for the PP1. The first one will be for the jobs coming from the same PP1 (state one) and the other one for the scheduled jobs coming from the sender PP2 (state one).

5.3.1. Scheduling Model by Using Markov Chain Model (One)

In this section the scheduling case 2 will be introduced using MC model one. For more detail, see Figure 29.

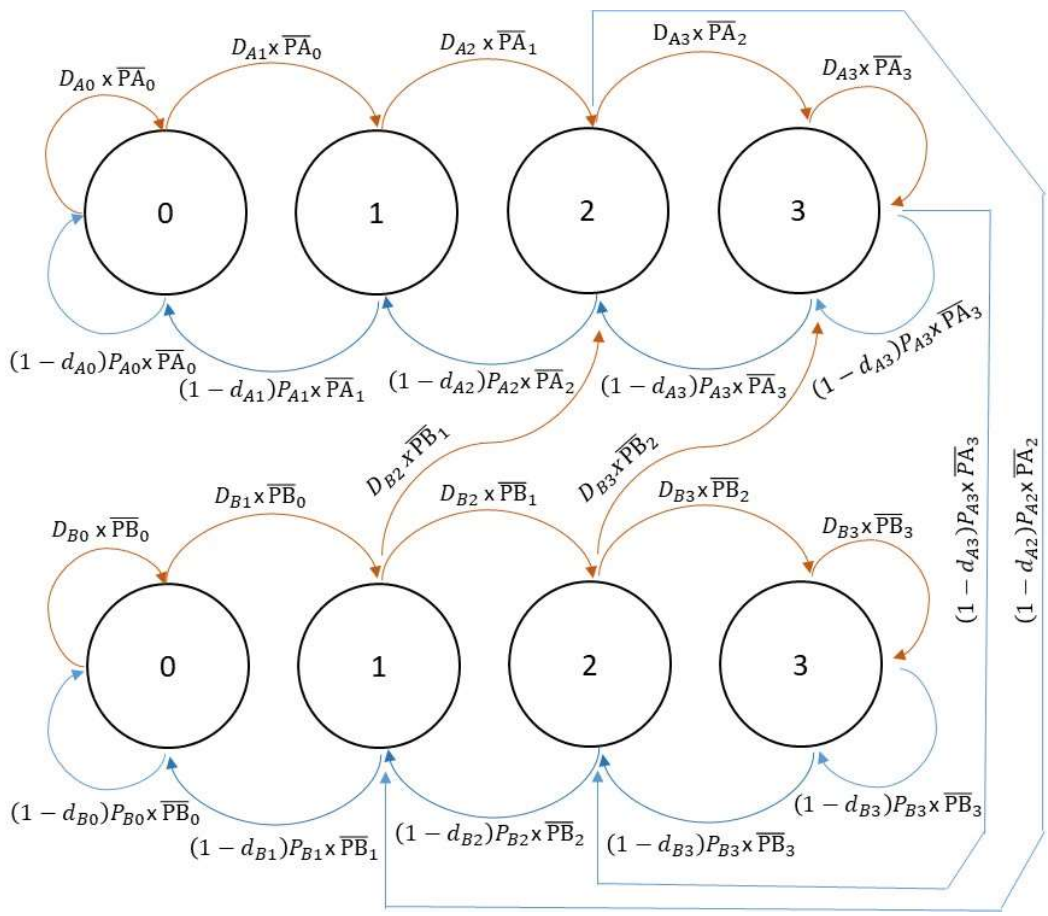


Figure 29. Scheduling Model Using MC Model One, Case 2, and the Fourth Scenario

The balance equations for the developed scheduling MCA are

State zero $\quad D_{A1}\overline{\mathrm{PA}}_0 = (1-d_{A1})P_{A1}\overline{PA}_1$ (5.19)

State one $\quad [D_{A2} + (1-d_{A1})P_{A1}]\overline{\mathrm{PA}}_1 = D_{A1}\,\overline{PA}_0 + (1-d_{A2})P_{A2}\,\overline{PA}_2$ (5.20)

State two $\quad [\mathrm{D}_{A3} + 2(1-d_{A2})P_{A2}]\,\overline{\mathrm{PA}}_2 = D_{A2}\overline{PA}_1 + (1-d_{A3})\mathrm{P}_{A3}\,\overline{PA}_3 + D_{B2}\overline{PB}_1$ (5.21)

State three $\quad 2(1-d_{A3})P_{A3}\overline{\mathrm{PA}}_3 = \mathrm{D}_{A3}\overline{\mathrm{PA}}_2 + D_{B3}\overline{PB}_2$ (5.22)

The balance equations for the developed scheduling MCB are

State zero $\quad D_{B1}\mathrm{x}\,\overline{\mathrm{PB}}_0 = (1-d_{B1})P_{B1}x\overline{PB}_1$ (5.23)

State one $[2D_{B2} + (1 - d_{B1})P_{B1}]x\overline{PB}_1 = D_{B1}\overline{PB}_0 + (1 - d_{B2})P_{B2}\,\overline{PB}_2 + (1 - d_{A2})P_{A2}\overline{PA}_2$ (5.24)

State two $[2D_{B3} + (1 - d_{B2})P_{B2}]\overline{PB}_2 = D_{B2}\overline{PB}_1 + (1 - d_{B3})P_{B3}\overline{PB}_3 + (1 - d_{A3})P_{A3}\overline{PA}_3$ (5.25)

State three $(1 - d_{B3})P_{B3}\overline{\mathrm{PB}}_3 = D_{B3}\overline{PB}_2$ (5.26)

The last constraint is the sum of all probabilities should be equal to on: $\overline{\mathrm{PA}}_0 + \overline{\mathrm{PA}}_1 + \overline{\mathrm{PA}}_2 + \overline{\mathrm{PA}}_3 + \overline{\mathrm{PB}}_0 + \overline{\mathrm{PB}}_1 + \overline{\mathrm{PB}}_2 + \overline{\mathrm{PB}}_3 = 1$.

The expected total cost is (121.098), the expected production rate is (1052.4), the expected demand rate is (757.876), the expected defect rate is (0.0166030). We will use the last three figures to calculate the optimal total cost which is found to be (121.082) and is close to the expected total cost with a difference equal to (0.016) even with the other optimal values. The *P*-Value for the expected total cost equal (0.5118) which is not significant and the (95%) confidence interval is (108.9617, 127.6051). The developed MC model has a high accuracy in the evaluated expected total cost which is shown by the achieved *P*-Value.

The production plan for PP1 (state zero) will have the optimal total cost is (135.346), the optimal time when the shortages are met is (0.1546), the optimal time when inventory is built (0.9664), the optimal production run time is (1.121), the probability of being in state zero is (0.1406), and the expected state time is (1.1249) years. The same practice will be for states one, two, three and for the PP2. However, for the scheduled jobs coming from the sender PP2 (state one and two), the production plan for the receiver PP1 (state two and state three) will be taken from the rows (A2y and A3y), respectively. However, the production plan in rows (A2x and A3x) will be for the jobs coming from same PP1 (state one and state two). See Table XXV for more detail.

Table XXV. Scheduling MC Model One Case 2 Fourth Scenario Solution base (t-x)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1450	1100	1.00	0.1406	135.346	0.1546	0.9664	0.3567	1.1249
A1	1200	900	0.7610	0.1070	124.713	0.1659	1.0370	0.4010	0.8561
A2x	1000	750	0.5858	0.0494	113.948	0.1816	1.1350	0.4389	0.3954
A2y	1000	500		0.0329	131.400	0.1050	0.6561	0.7611	0.2636
A3x	850	650	0.4422	0.0373	103.102	0.2047	1.2793	0.4566	0.2984
A3y	850	400		0.0249	120.984	0.1073	0.6707	0.8752	0.1989
B0	1200	850	1.0000	0.2401	130.7780	0.1494	0.9339	0.4461	1.9211
B1	950	650	0.6943	0.1667	119.0800	0.1585	0.9907	0.5304	1.3338
B2	750	500	0.4928	0.1183	107.3590	0.1713	1.0708	0.6210	0.9466
B3	600	400	0.3442	0.0827	96.124	0.1914	1.1960	0.6937	0.6612
Expected values					121.098	0.1606	1.0039	0.5050	
Production and Inventory Model					121.082	0.1641	1.0255	0.4623	
Difference					0.016	0.0035	0.0216	0.0427	

After careful examination about the optimal scheduling times, case 2 does not have a feasible solution using MC model one. The current used problem data and MC model one structure, the optimal scheduling hard time windows between them does not exist. These two production processes work only without scheduling with each other. The optimization model prefers that both to work independently. As a result, each production process should have a developed optimal individual detailed production plan. For more information, see Figure 30.

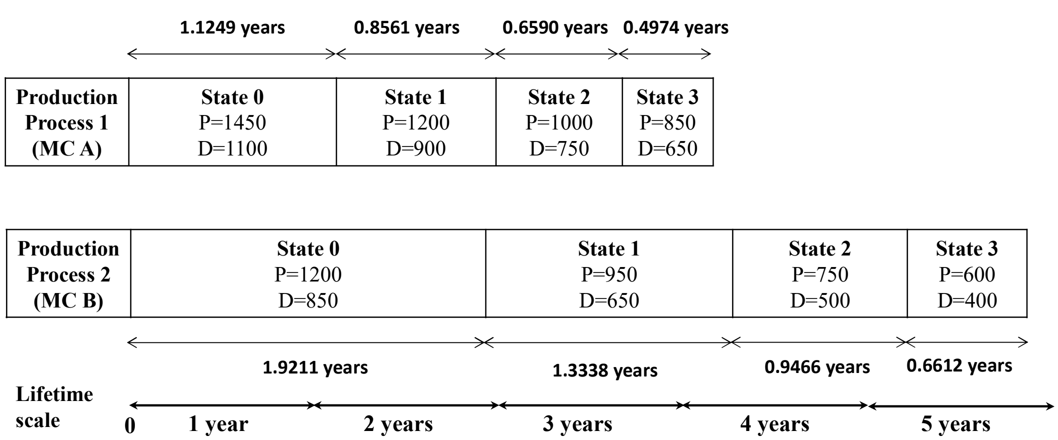


Figure 30. Scheduling Plan Solution for MC Model One Fourth Scenario Case 2

5.3.2. Scheduling Model by Using Markov Chain Model (Two)

In this section the scheduling case 2 will be introduced using MC model two. For more detail, see Figure 31.

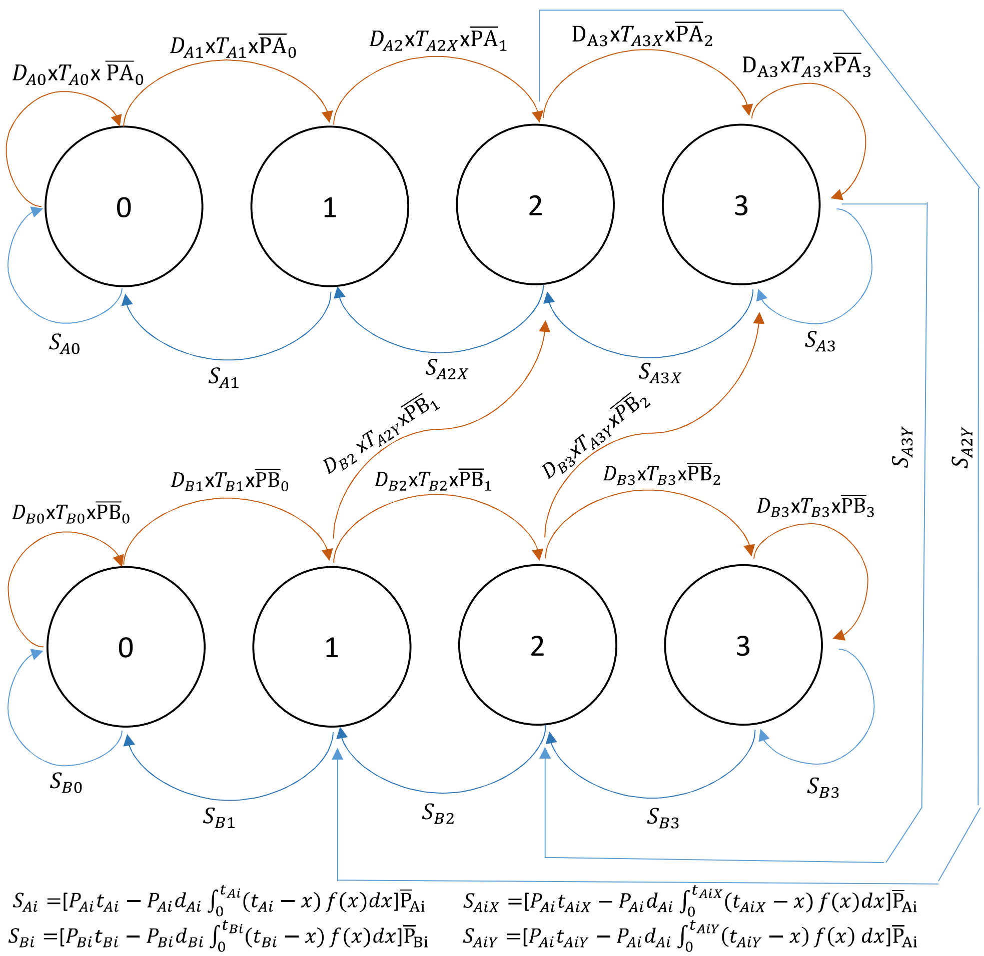


Figure 31. Scheduling Model Using MC Model Two, Case 2, and the Fourth Scenario

The following abbreviation will be used in presenting the balance equations

Abbreviation

$$S_{A2X} = [P_{A2}t_{A2X}\text{-}P_{A2}d_{A2}\int_0^{t_{A2X}}(t_{A2X}-x)\,f(x)dx]\overline{P}_{A2} \tag{5.27}$$

$$S_{A3X} = [P_{A3}t_{A3X}\text{-}P_{A3}d_{A3}\int_0^{t_{A3X}}(t_{A3X}-x)\,f(x)dx]\overline{P}_{A3} \tag{5.28}$$

$$S_{A2Y} = [P_{A2}t_{A2Y}\text{-}P_{A2}d_{A2}\int_0^{t_{A2Y}}(t_{A2Y}-x)\,f(x)dx]\overline{P}_{A2} \tag{5.29}$$

$$S_{A3Y} = [P_{A3}t_{A3Y}\text{-}P_{A3}d_{A3}\int_0^{t_{A3Y}}(t_{A3Y}-x)\,f(x)dx]\overline{P}_{A3} \tag{5.30}$$

The following formulas will be used to show a different representation which will be used in the balance equations

$$D_{A2}T_{A2X}\overline{PA}_1 = P_{A2}(T_{A12x} + T_{A22x})\overline{PA}_1 \tag{5.31}$$

$$D_{A3}T_{A3X}\overline{\mathrm{PA}}_2 = P_{A3}(T_{A13x} + T_{A23x})\overline{PA}_2 \tag{5.32}$$

$$D_{B2}T_{A2Y}\overline{PB}_1 = P_{A2}(T_{A12y} + T_{A22y})\overline{PB}_1 \tag{5.33}$$

$$D_{B3}T_{A3Y}\overline{\mathrm{PB}}_2 = P_{A3}(T_{A13y} + T_{A23y})\overline{PB}_2 \tag{5.34}$$

The balance equations for the developed scheduling MCA are

State zero $$D_{A1}T_{A1}\overline{\mathrm{PA}}_0 = S_{A1} \tag{5.35}$$

State one $$D_{A2}T_{A2X}\overline{\mathrm{PA}}_1 + S_{A1} = D_{A1}T_{A1}\overline{PA}_0 + S_{A2X} \tag{5.36}$$

State two $$\mathrm{D}_{A3}T_{A3X}\overline{\mathrm{PA}}_2 + S_{A2X} + S_{A2Y} = D_{A2}T_{A2X}\overline{PA}_1 + S_{A3X} + D_{B2}T_{A2Y}\overline{PB}_1 \tag{5.37}$$

State three $$S_{A3X} + S_{A3Y} = D_{A3}T_{A3X}\overline{PA}_2 + D_{B3}T_{A3Y}\overline{PB}_2 \tag{5.38}$$

The balance equations for the developed scheduling MCB are

State zero $$D_{B1}T_{B1}\overline{\mathrm{PB}}_0 = S_{B1} \tag{5.39}$$

State one $$D_{B2}T_{B2}\overline{\mathrm{PB}}_1 + D_{B2}T_{A2Y}\overline{\mathrm{PB}}_1 + S_{B1} = D_{B1}T_{B1}\overline{PB}_0 + S_{B2} + S_{A2Y} \tag{5.40}$$

State two $$D_{B3}T_{B3}\overline{\mathrm{PB}}_2 + D_{B3}T_{A3Y}\overline{\mathrm{PB}}_2 + S_{B2} = D_{B2}T_{B2}\overline{PB}_1 + S_{B3} + S_{A3Y} \tag{5.41}$$

State three $$S_{B3} = D_{B3}T_{B3}\overline{PB}_2 \tag{5.42}$$

The last constraint is the sum of all probabilities should be equal to on: $\overline{PA}_0 + \overline{PA}_1 + \overline{PA}_2 + \overline{PA}_3 + \overline{PB}_0 + \overline{PB}_1 + \overline{PB}_2 + \overline{PB}_3 = 1$.

The expected total cost is (118.026), the expected production rate is (999.032), the expected demand rate is (724.252), the expected defect rate is (0.0220837). We will use the last three figures to calculate the optimal total cost which is found to be (117.392) and is close to the expected total cost with a difference equal to (0.0.634) even with the other optimal values. The *P*-Value for the expected total cost equal (0.9516) which is not significant and the (95%) confidence interval is (108.9617, 127.6051). The developed MC model has a high accuracy in the evaluated expected total cost which is shown by the achieved *P*-Value.

The production plan for PP1 (state zero) will have the optimal total cost is (135.346), the optimal time when the shortages are met is (0.1546), the optimal time when inventory is built (0.9664), the optimal production run time is (1.121), the probability of being in state zero is (0.1244), and the expected state time is (0.9948) years. The same practice will be for states one, two, three and for the PP2. However, for the scheduled jobs coming from the sender PP2 (state one and two), the production plan for the receiver PP1 (state two and state three) will be taken from the rows (A2y and A3y), respectively. However, the production plan in rows (A2x and A3x) will be for the jobs coming from same PP1 (state one and state two). See Table XXVI for more detail.

Table XXVI. Scheduling MC Model Two Case 2 Fourth Scenario Solution base (t-x)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1450	1100	1.00	0.1244	135.346	0.1546	0.9664	0.3567	0.9948
A1	1200	900	1.0016	0.1246	124.713	0.1659	1.0372	0.4010	0.9965
A2x	1000	750	1.0047	0.0750	113.948	0.1818	1.1362	0.4393	0.5997
A2y	1000	500		0.0500	131.400	0.1049	0.6559	0.7608	0.3998
A3x	850	650	1.0107	0.0754	103.102	0.2051	1.2816	0.4574	0.6033
A3y	850	400		0.0503	120.984	0.1072	0.6703	0.8747	0.4022
B0	1200	850	1.0000	0.1246	130.7780	0.1494	0.9339	0.4461	0.9966
B1	950	650	1.0016	0.1248	119.0800	0.1585	0.9909	0.5305	0.9982
B2	750	500	1.0049	0.1252	107.3590	0.1714	1.0713	0.6213	1.0015
B3	600	400	1.0108	0.1259	96.124	0.1916	1.1975	0.6945	1.0073
Expected values					118.026	0.1636	1.0225	0.5307	
Production and Inventory Model					117.392	0.1704	1.0649	0.4687	
Difference					0.634	0.0068	0.0423	0.0621	

For Production process 2 (state 1) which is MCB, it can pass jobs to production process 1 (state 2) which is MCA from 1.9948 years until 2.9909 years. For Production process 2 (state 2) which is MCB, it can pass jobs to production process 1 (state 3) which MCA from 2.9963 years until 3.9964 years. See Figure 32 for more detail.

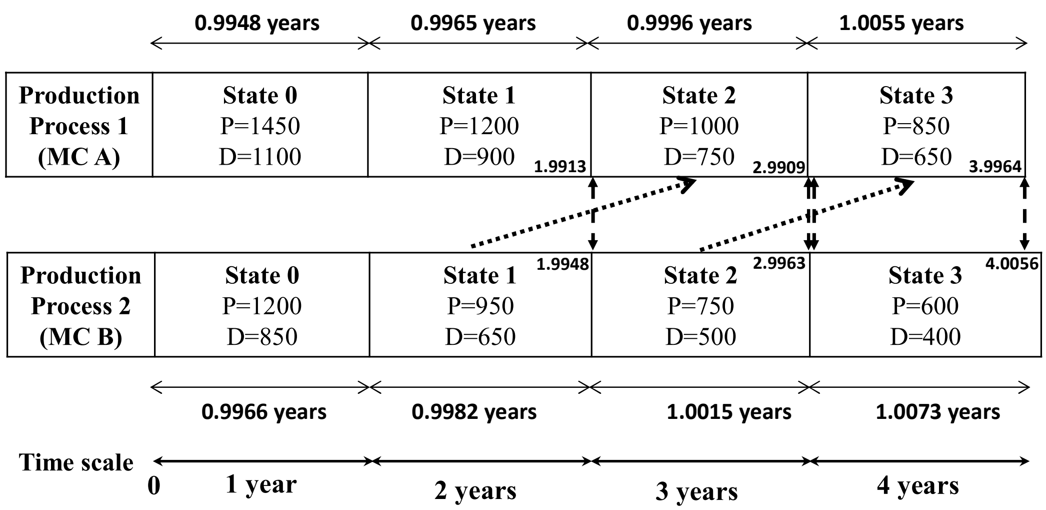


Figure 32. Scheduling Plan Solution for MC Model two Fourth Scenario Case 2

5.4. Heuristic Search Methods

Heuristic search methods are mostly used to deal with complex nonlinear objective functions. Moreover, local optimal solution can be avoided in order to have the desired global optimal solution. Each method allows to have a custom search settings in order to deal with the type of the model's objective function and their constraints. In this section, two well-known heuristic search methods will be used in order to obtain the optimal solution and compare them with the results of the previous sections 5.2 and 5.3. The first method is the Genetic algorithm that will be used for MC2 and both case 1 and case 2. The second method is the Particle swarm optimization that will be used only for MC 2 with case 2. The fourth scenario model will only be considered.

5.4.1. Problem Formulation

In this section, the developed model formulas for the scheduling model of different production & demand rates (Case 2), MC model two, and the fourth scenario will be provided.

The nonlinear programming representation by using the listed equations in this section will be as follows:

Minimize	*ETC*(*n*+1)
Subject to	The balance equations for MC A and MC B The sum of probabilities equal 1
Decision parameters	The production times and probabilities for MC A and MC B
Bounds	All decision parameters are greater than zero

The following formulas will be used for MC (A) in order to provide the problem formulations. The optimal production run times

State 0
$$t_{A_0} = T_{A_{10}} + T_{A_{20}} \tag{5.43}$$

State 1
$$t_{A_1} = T_{A_{11}} + T_{A_{21}} \tag{5.44}$$

State 2*x*
$$t_{A_{2x}} = T_{A_{12x}} + T_{A_{22x}} \tag{5.45}$$

State 2*y*
$$t_{A_{2y}} = T_{A_{12y}} + T_{A_{22y}} \tag{5.46}$$

State 3*x*
$$t_{A_{3x}} = T_{A_{13x}} + T_{A_{23x}} \tag{5.47}$$

State 3*y*
$$t_{A_{3y}} = T_{A_{13y}} + T_{A_{23y}} \tag{5.48}$$

The expected number of defectives for each state

State 0 (i=0) and

State 1 (i=1)

$$E(MCA_i) = \int_0^{t_{A_i}} d_{Ai} P_{A_i}\left(t_{A_i} - x\right) f(x)dx \tag{5.49}$$

State 2x

$$E(MCA_{2x}) = \int_0^{t_{A_{2x}}} d_{A2} P_{A_2}\left(t_{A_{2x}} - x\right) f(x)dx \tag{5.50}$$

State 2y

$$E\left(MCA_{2y}\right) = \int_0^{t_{A_{2y}}} d_{A2} P_{A_2}\left(t_{A_{2y}} - x\right) f(x)dx \tag{5.51}$$

State 3x

$$E(MCA_{3x}) = \int_0^{t_{A_{3x}}} d_{A3} P_{A_3}\left(t_{A_{3x}} - x\right) f(x)dx \tag{5.52}$$

State 3y

$$E\left(MCA_{3y}\right) = \int_0^{t_{A_{3y}}} d_{A3} P_{A_3}\left(t_{A_{3y}} - x\right) f(x)dx \tag{5.53}$$

The total cost for each state

State 0 (i=0)

and State 1

(i=1)

$$\begin{aligned} TC(MCA_i) &= \frac{KD_{A_i}}{P_{A_i}(T_{A_{1i}} + T_{A_{2i}})} + (h+\pi)\frac{\left(P_{A_i} - D_{A_i}\right)}{2\left(T_{A_{1i}} + T_{A_{2i}}\right)} T_{A_{1i}}^2 \\ &+ h\left[\frac{(P_{A_i} - D_{A_i})(T_{A_{1i}} + T_{A_{2i}})}{2} - (P_{A_i} - D_{A_i})T_{A_{1i}}\right] \\ &+ \frac{sD_{A_i}}{P_{A_i}(T_{A_{1i}} + T_{A_{2i}})} E(MCA_i) \end{aligned} \tag{5.54}$$

$$TC(MCA_{2x}) = \frac{KD_{A_2}}{P_{A_2}(T_{A_{12}} \quad + T_{A_{22x}})} + (h+\pi)\frac{\left(P_{A_2} - D_{A_2}\right)}{2\left(T_{A_{12}} \quad + T_{A_{22}} \quad\right)} T_{A_{12}}^2 \tag{5.55}$$

State 2x

$$+ h\left[\frac{(P_{A_2} - D_{A_2})(T_{A_{12x}} + T_{A_{22}} \quad)}{2} - (P_{A_2}\right.$$

$$\left. - D_{A_2})T_{A_{12}} \quad\right] + \frac{sD_{A_2}}{P_{A_2}(T_{A_{12x}} + T_{A_{22x}})} E(MCA_{2x})$$

$$TC(MCA_{2y}) = \frac{KD_{B_2}}{P_{A_2}(T_{A_{12}} \quad + T_{A_{22}} \quad)} \tag{5.56}$$

$$+ (h+\pi)\frac{\left(P_{A_2} - D_{B_2}\right)}{2\left(T_{A_{12}} \quad + T_{A_{22}} \quad\right)} T_{A_{12}}^2$$

State 2y

$$+ h\left[\frac{(P_{A_2} - D_{B_2})(T_{A_{12}} \quad + T_{A_{22y}})}{2} - (P_{A_2} - D_{B_2})T_{12} \quad\right]$$

$$+ \frac{sD_{B_2}}{P_{A_2}(T_{A_{12y}} + T_{A_{22y}})} E\left(MCA_{2y}\right)$$

$$TC(MCA_{3x}) = \frac{KD_{A_3}}{P_{A_3}(T_{A_{13}} \quad + T_{A_{23}} \quad)} + (h+\pi)\frac{\left(P_{A_3} - D_{A_3}\right)}{2\left(T_{A_{13}} \quad + T_{A_{23}} \quad\right)} T_{A_{13}}^2 \tag{5.57}$$

State 3x

$$+ h\left[\frac{(P_{A_3} - D_{A_3})(T_{A_{13x}} + T_{A_{23x}})}{2} - (P_{A_3}\right.$$

$$\left. - D_{A_3})T_{A_{13}} \quad\right] + \frac{sD_{A_3}}{P_{A_3}(T_{A_{13x}} + T_{A_{23x}})} E(MCA_{3x})$$

$$TC(MCA_{3y}) = \frac{KD_{B_3}}{P_{A_3}(T_{A_{13y}} + T_{A_{23y}})} \tag{5.58}$$

State $3y$

$$+ (h+\pi)\frac{\left(P_{A_3} - D_{B_3}\right)}{2\left(T_{A_{13y}} + T_{A_{23}}\ \right)} T_{A_{13y}}^2$$

$$+ h\left[\frac{(P_{A_3} - D_{B_3})(T_{A_{13y}} + T_{A_{23}}\ )}{2} - (P_{A_3} - D_{B_3})T_{13y}\right]$$

$$+ \frac{sD_{B_2}}{P_{A_3}(T_{A_{13y}} + T_{A_{23y}})} E\left(MCA_{3y}\right)$$

The following formulas are abbreviations used in the balance equations

$$S_{A1} = \left[P_{A_1} t_{A1} - E(MCA_0)\right]\bar{P}_{A1} \tag{5.59}$$

$$S_{Aix} = \left[P_{A_i} t_{Aix} - E(MCA_{ix})\right]\bar{P}_{Ai} \tag{5.60}$$

$$S_{Aiy} = \left[P_{A_i} t_{Aiy} - E\left(MCA_{iy}\right)\right]\bar{P}_{Ai} \tag{5.61}$$

The following formulas are different representations used in the balance equations

$$D_{A1}T_{A1}\overline{\mathrm{PA}}_0 = P_{A1}t_{A_1}\overline{PA}_0 \tag{5.62}$$

$$D_{A2}T_{A2X}\overline{\mathrm{PA}}_1 = P_{A2}t_{A_{2x}}\overline{PA}_1 \tag{5.63}$$

$$\mathrm{D}_{\mathrm{A3}}T_{A3X}\overline{\mathrm{PA}}_2 = P_{A3}t_{A_{3x}}\overline{PA}_2 \tag{5.64}$$

$$D_{B1}T_{B1}\overline{\mathrm{PB}}_0 = P_{B1}t_{B_1}\overline{PB}_0 \tag{5.65}$$

$$D_{B2}T_{B2}\overline{\mathrm{PB}}_1 = P_{B2}t_{B_2}\overline{PB}_1 \tag{5.66}$$

$$D_{B3}T_{B3}\overline{\mathrm{PB}}_2 = P_{B3}t_{B_3}\overline{PB}_2 \tag{5.67}$$

$$D_{B2}T_{A2Y}\overline{\mathrm{PB}}_1 = P_{A2}t_{A_{2y}}\overline{PB}_1 \tag{5.68}$$

$$D_{B3}T_{A3Y}\overline{\mathrm{PB}}_2 = P_{A3}t_{A_{3y}}\overline{PB}_2 \tag{5.69}$$

The following formulas are the balance equations (MC A Constraints)

State 0 $$D_{A1}T_{A1}\overline{\mathrm{PA}}_0 = S_{A1} \tag{5.70}$$

State 1 $$D_{A2}T_{A2X}\overline{\mathrm{PA}}_1 + S_{A1} = D_{A1}T_{A1}\overline{PA}_0 + S_{A2X} \tag{5.71}$$

State 2 $$\mathrm{D}_{A3}T_{A3X}\overline{\mathrm{PA}}_2 + S_{A2X} + S_{A2Y} = D_{A2}T_{A2X}\overline{PA}_1 + S_{A3X} + D_{B2}T_{A2Y}\overline{PB}_1 \tag{5.72}$$

State 3 $$S_{A3X} + S_{A3Y} = D_{A3}T_{A3X}\overline{PA}_2 + D_{B3}T_{A3Y}\overline{PB}_2 \tag{5.73}$$

The following formulas will be used for MCB in order to provide the problem formulations.

The optimal production run times

State 0 $$t_{B_0} = T_{B_{10}} + T_{B_{20}} \tag{5.74}$$

State 1 $$t_{B_1} = T_{B_{11}} + T_{B_{21}} \tag{5.75}$$

State 2 $$t_{B_2} = T_{B_{12}} + T_{B_{22}} \tag{5.76}$$

State 3 $$t_{B_3} = T_{B_{13}} + T_{B_{23}} \tag{5.77}$$

The expected number of defectives for each state

State *i* $$E(MCB_i) = \int_0^{t_{B_i}} d_{Bi}P_{B_i}(t_{B_i} - x)f(x)dx \tag{5.78}$$

The total cost for each state

State i

$$TC(MCB_i) = \frac{KD_{B_i}}{P_{B_i}(T_{B_{1i}} + T_{B_{2i}})} + (h+\pi)\frac{(P_{B_i} - D_{B_i})}{2(T_{B_{1i}} + T_{B_{2i}})}T_{B_{1i}}^2 + h\left[\frac{(P_{B_i} - D_{B_i})(T_{B_{1i}} + T_{B_{2i}})}{2} - (P_{B_i} - D_{B_i})T_{B_{1i}}\right] + \frac{sD_{B_i}}{P_{B_i}(T_{B_{1i}} + T_{B_{2i}})}E(MCB_i) \quad (5.79)$$

The following formulas are abbreviations used in the balance equations

State i

$$S_{Bi} = \left[P_{B_i}t_{Bi} - E(MCB_i)\right]\bar{P}_{Bi} \quad (5.80)$$

The following formulas are the balance equations (MC B Constraints)

State 0

$$D_{B1}T_{B1}\overline{PB}_0 = S_{B1} \quad (5.81)$$

State 1

$$D_{B2}T_{B2}\overline{PB}_1 + D_{B2}T_{A2Y}\overline{PB}_1 + S_{B1} = D_{B1}T_{B1}\overline{PB}_0 + S_{B2} + S_{A2Y} \quad (5.82)$$

State 2

$$D_{B3}T_{B3}\overline{PB}_2 + D_{B3}T_{A3Y}\overline{PB}_2 + S_{B2} = D_{B2}T_{B2}\overline{PB}_1 + S_{B3} + S_{A3Y} \quad (5.83)$$

State 3

$$S_{B3} = D_{B3}T_{B3}\overline{PB}_2 \quad (5.84)$$

The sum of all state’s probabilities should be equal to 1 (Constraint #9)

$$\overline{PA}_0 + \overline{PA}_1 + \overline{PA}_2 + \overline{PA}_3 + \overline{PB}_0 + \overline{PB}_1 + \overline{PB}_2 + \overline{PB}_3 = 1 \quad (5.85)$$

The following conditions on the load

$$x_2 + y_2 = 1 \quad (5.86)$$

$$x_3 + y_3 = 1 \quad (5.87)$$

The expected total cost function for (*n*+1) states is

The objective function

$$
\begin{aligned}
ETC(n+1) = {} & \overline{PA}_0 \times TC(MCA_0) + \overline{PA}_1 \times TC(MCA_1) \\
& + (x_2) \times \overline{PA}_2 \times TC(MCA_{2x}) \\
& + (y_2) \times \overline{PA}_2 \times TC\left(MCA_{2y}\right) \\
& + (x_3) \times \overline{PA}_3 \times TC(MCA_{3x}) \\
& + (y_3) \times \overline{PA}_3 \times TC\left(MCA_{3y}\right) + \overline{PB}_0 \times TC(MCB_0) \\
& + \overline{PB}_1 \times TC(MCB_1) + \overline{PB}_2 \times TC(MCB_2) \\
& + \overline{PB}_3 \times TC(MCB_3)
\end{aligned} \tag{5.88}
$$

The decision parameters for both MC A (16 variables) and MC B (12 variables)

MC A

$$
\begin{gathered}
T_{A_{10}}, T_{A_{20}}, T_{A_{11}}, T_{A_{11}}, T_{A_{12x}}, T_{A_{22x}}, T_{A_{12y}}, T_{A_{22y}}, T_{A_{13x}}, T_{A_{23x}}, \\
T_{A_{13y}}, T_{A_{23y}}, \overline{PA}_0, \ \overline{PA}_1, \ \overline{PA}_2, \ \overline{PA}_3
\end{gathered} \tag{5.89}
$$

MC B

$$
\begin{gathered}
T_{B_{10}}, T_{B_{20}}, T_{B_{11}}, T_{B_{21}}, T_{B_{12}}, T_{B_{22}}, T_{B_{13}}, T_{B_{23}}, \overline{PB}_0, \overline{PB}_1, \ \overline{PB}_2, \\
\overline{PB}_3
\end{gathered} \tag{5.90}
$$

5.4.2. Genetic Algorithm Method

The Evolver optimization add-in for Microsoft Excel is used to perform the optimization task. It is an industrial edition with version 8.0.1. Moreover, the genetic algorithm method was first introduced by (Holland, 1975).

The genetic algorithm is a well know approach that is used to enhance the optimal solution among the common evolutionary algorithms. It is a population-based approach that it seeks to select the better individuals that will be used to survive and then to replicate. Moreover, it needs to preserve a balance between the exploitation and the exploration. The first one is used to evaluate the solutions and to select only the optimal one. However, the second one is to find a better new search space. The Genetic algorithm depends on the selection, the crossover, and the mutation in order to have a better solution. The selection is done by having the chromosome for the mating process, the crossover is to have new generation, and the mutation is determining how many chromosomes should be changed. For more detail, see (Hussain and Muhammad, 2019).

The used settings for GA method, the population size is (100), crossover rate is (0.8), and mutation rate is (0.05). The following mutations are used, the default, Cauchy, Boundary, and non-uniform. For the crossovers, heuristic, arithmetic, and default are used. The other operators that are used too, default parent selection, default backtrack, linear, and local research.

The expected total cost by using Mathematica for the scheduling of MC2 and case 1 equal (114.42972789) with *P*-Value (0.9884) and confidence interval (101.948, 127.082). However, by using GA, the expected total cost is (114.41000953) with *P*-Value (0.9857) and the confidence interval is (101.9422, 127.0748). The saving is equal to (0.01971835) which is around (%0.01723184). The total number of run time hours for the GA method is around (46 hours). The GA allows to have a custom search setting in order to look for the desired optimal results. I have started by having the optimal solution from Mathematica and then let the GA to look and enhance it and then having a lower expected total cost. For more detail about the optimal solution, see Table XXVII.

Table XXVII. Genetic Solution for MC Model Two Case 1 Fourth Scenario Solution

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1450	1100	1.00	0.1217	135.346	0.1546	0.9665	0.3567	0.9736
A1	1150	850	1.0019	0.1219	123.799	0.1647	1.0295	0.4215	0.9754
A2	750	500	1.0038	0.1224	107.379	0.1744	1.0924	0.6334	0.9792
A3	600	400	1.0034	0.1228	96.159	0.1962	1.2292	0.7127	0.9825
B0	1200	850	1.00	0.1270	130.778	0.1494	0.9339	0.4461	1.0161
B1	950	650	1.0016	0.1272	119.080	0.1585	0.9908	0.5305	1.0177
B2	750	500	1.0063	0.1280	107.377	0.1687	1.0506	0.6097	1.0241
B3	600	400	1.0062	0.1288	96.150	0.1874	1.1681	0.6778	1.0304
Expected values					114.410	0.1692	1.0575	0.5494	
Production and Inventory Model					115.671	0.1676	1.0475	0.5104	
Difference					1.261	0.0016	0.0099	0.0389	

The expected total cost by using Mathematica for the scheduling of MC2 and case 2 equal (118.02558327) with P-Value (0.9516) and confidence interval (108.9617, 127.6051). However, by using GA, the expected total cost is (118.008132843) with P-Value (0.9842) and confidence interval (108.962, 127.605). The saving is equal to (0.0174504363) which is around (%0.0147852997). The total number of run time hours for the GA is around (42 hours). For more detail about the optimal solution, see Table XXVIII.

Table XXVIII. Genetic Solution for MC Model Two Case 2 Fourth Scenario Solution (t-x)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1450	1100	1.00	0.1242	135.346	0.1546	0.9664	0.3567	0.9940
A1	1200	900	1.0019	0.1245	124.713	0.1659	1.0370	0.4010	0.9959
A2x	1000	750	1.0037	0.0750	113.948	0.1816	1.1350	0.4389	0.5997
A2y	1000	500		0.0500	131.400	0.1050	0.6561	0.7611	0.3998
A3x	850	650	1.0062	0.0754	103.102	0.2047	1.2793	0.4566	0.6035
A3y	850	400		0.0503	120.984	0.1073	0.6707	0.8752	0.4023
B0	1200	850	1.0000	0.1245	130.7778	0.1494	0.9339	0.4461	0.9961
B1	950	650	1.0019	0.1248	119.0799	0.1585	0.9907	0.5304	0.9981
B2	750	500	1.0039	0.1252	107.3586	0.1713	1.0708	0.6210	1.0019
B3	600	400	1.0060	0.1260	96.124	0.1914	1.1960	0.6937	1.0079
Expected values					118.008	0.1635	1.0219	0.5305	
Production and Inventory Model					117.381	0.1711	1.0697	0.4708	
Difference					0.627	0.0076	0.0477	0.0597	

5.4.3. Particle Swarm Optimization Method

The PSO was first developed by (Kennedy and Eberhart, 1995) in order to solve and optimize the nonlinear functions. It has the general idea of the movements of birds or fish in a group and the collision is avoided. The interactions between them while moving and maintaining the same distance between each other. This happens because the information about the position is shared between them. Moreover, it is similar to the genetic algorithm and the evolutionary programming in general. The tuning of the best position and the best global in PSO is like the crossover operator in GA that is called the fitness.

The constraint problems can be solved by using PSO. There is a famous way to incorporate those constraints into the objective function. The penalty function can be used so the constraints can be incorporated to the objective function. The penalty function value should be selected properly in order to find a feasible solution. There are two types of penalties, either it may have a fixed value or a dynamic value. The latter method is better than the former method. There are three parameters that are used in the PSO algorithm. The inertia weight that is used for updating the current velocity by using the history of the velocities. Two constants that are positive and called the cognitive and the social parameters. For more detail about setting these parameters, refer to (Parsopoulos and Vrahatis, 2002). These two constants are used for the acceleration for both the positions that are the personal and the global. Small or large values will affects reaching the desired area either far or close (He *et al.*, 2016).

The settings that are used for the developed PSO code in Python for the expected total cost function; the particle size is (100), number of iterations is (3500), inertia constant is (0.75), cognative constant is (2), and the social constant is (2). For the finding the optimal cost considered as one cycle; the particle size is (65), number of iterations is (1000), inertia constant is (0.75), cognative constant is (2), and the social constant is (2). The Python code is using an imported code for the PSO algorithm that is from the reference (Dao, 2020).

The expected total cost by using Mathematica is (118.02558327) with *P*-Value (0.9516) and GA is (118.008132843) with *P*-Value (0.9842). However, by using PSO, the expected total cost is (118.475807725) with *P*-Value (0.8947) and the confidence interval is (109.772, 128.295). For more detail about the optimal solution using Python, see Table XXIX.

Table XXIX. PSO Method Solution for MC Model Two Case 2 Fourth Scenario Solution (*t*-*x*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
A0	1450	1100	1.00	0.1145	135.846	0.1318	1.0377	0.3721	0.9164
A1	1200	900	1.1293	0.1294	125.082	0.1320	1.0416	0.3912	1.0348
A2x	1000	750	1.2886	0.0886	114.581	0.1605	1.0246	0.3950	0.7085
A2y	1000	500		0.0590	132.318	0.1322	0.7119	0.8441	0.4723
A3x	850	650	1.1192	0.0769	104.601	0.1647	1.0879	0.3854	0.6153
A3y	850	400		0.0513	121.565	0.1304	0.7168	0.9531	0.4102
B0	1200	850	1.0000	0.1135	131.892	0.1012	0.8968	0.4110	0.9077
B1	950	650	1.0377	0.1177	119.736	0.1277	1.0756	0.5553	0.9419
B2	750	500	1.0976	0.1245	107.653	0.1688	0.9886	0.5787	0.9962
B3	600	400	1.0924	0.1239	97.060	0.2085	1.0314	0.6200	0.9915
Expected values					118.476	0.1469	0.9861	0.5175	
Production and Inventory Model					116.959	0.1712	1.0702	0.4688	
Difference					1.517	0.0243	0.0840	0.0487	

5.4.4. Constraints Violation Values

In this section, a comparison for the constraint's violation value at the optimal solution for the scheduling model case 2, MC model two, and the fourth scenario. The three method that are Mathematica, GA, and PSO will be compared. The constraint violation value that is close to zero means that it is better satisfied. In this model we have 9 constraints. Each MC (A and B) has four constraints that each should have a value of zero. The 9th constraint is the sum of all probabilities that should be equal to one. The methods are ranked according to the lowest violation values. For more detail, see the Table XXX.

Table XXX. The Constraints' Violation Values for Mathematica, GA-Evolver, and PSO-Python

No.	Mathematica	GA - Evolver	PSO - Python
Constraint A0	7.7017E-05	0.05	0.3552828
Constraint A1	7.0863E-04	0.0465490	0.1794144
Constraint A2	1.1235E-03	0.05	0.0839731
Constraint A3	3.5900E-05	0.05	1.3167086
Constraint B0	3.0065E-05	0.0499999	0.2950729
Constraint B1	1.0796E-03	0.0209038	0.3653206
Constraint B2	4.9625E-04	0.050	0.6579693
Constraint B3	9.7375E-05	0.0174527	0.6203244
Constraint 9	1.00E-06	1.00E-04	6.4349E-04
Expected Total Cost	**118.0255833**	**118.0081328**	**118.4758077**
***P*-Value**	**0.9516**	**0.9482**	**0.8947**

The scheduling hard time windows are developed by using MC model one and MC model two. Two cases are considered in dealing with the production and demand rates among the states for both the production processes. The developed model could be used in any related production facility that is having a single product machine. The scheduling times could be achieved by entering and using their specification data and figures.

6. MAINTENANCE MODEL

(Previously published as Al Hajailan, W. I. and He, D., (2020) Expected Maintenance Actions for Imperfect Production Processes Using a Markovian Approach, 2020 Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling (APARM), Vancouver, BC, Canada, 2020, pp. 1-6, doi: 10.1109/APARM49247.2020.9209372.) See appendix K.

In this chapter, an integrated model will be presented in order to find the optimal production plan and the required maintenance action in each month and for each state. Maintenance actions will be developed by using four different MC models. The first model will be for the main P&I control model, the second model for the PM (time-based), the third model for the inspection (time-based), and the fourth model will be for the minimal repair (MR) which is a (condition-based).

In the developed integrated MC model, the PM actions will affect both the inspection and the MR. Moreover, the MR actions will affect both the PM and the inspection. Finally, all the maintenance actions will interact and affect the P&I control MC model. As a result, by using these interactions and the P&I MC model, the optimized desired decision variables will be obtained.

6.1. Notation

In this section, the notations that are used in the developed integrated models will be defined.

Parameter	Description
C_{PM},C_{Ins},C_{MR}	preventive maintenance, inspection, minimal repair costs
$\bar{P}_{P\&I_i}$	probability of being in state (i) for the P&I MC
$\bar{P}_{PM_i}$	probability of being in state (i) for the PM MC

$\bar{P}_{Ins_i}$	probability of being in state (i) for the inspection MC
$\bar{P}_{MR_i}$	probability of being in state (i) for the MR MC
$n^*_{PM_i}, n^*_{Ins_i}, n^*_{MR_i}$	optimal number of PM, inspection, and MR in each month for state (i)
$MTTF_i$	mean time to failure for state (i)
a_i , b_i	decision variable for PM and inspection at the end of state (i)
R_i, Q_i, X_i	flow rate from state (i) to state (i +1)
W_i, S_i, U_i, Y_i	flow rate from state (i) to state (i -1)

6.2. Assumptions

In this section, the assumptions that are used for the developed integrated models will be introduced:

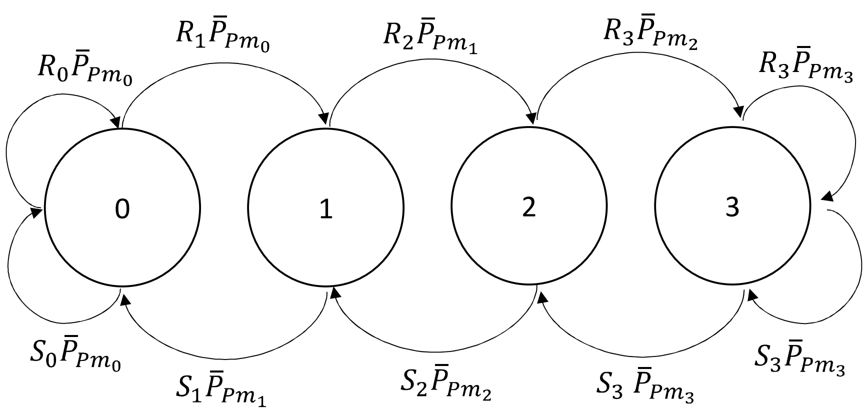
- The setup cost is used before each production cycle
- At the end of each production cycle, there is a rework cost for each defective product
- Maintenance action times are considered negligible
- Equal maintenance time intervals are used
- Backordering is allowed

6.3. Maintenance Markov Chain Models

In this section, detailed description for each developed maintenance MC model will be provided.

6.3.1. Preventive Maintenance

The required PM actions will be determined by the following MC model which will be affected by the MR actions. It will used to find the optimal number of actions required in each month and for each state. The flow from state (*i*) to state (*i*+1) is the required PM actions and then it will reply with the effects which is the flow from state (*i*) to state (*i*-1), they are (R_i) and (S_i), respectively. The parameter (a_i) is used to determine if PM is required at the end of state (i). See Figure 33 for more detail.



$R_i = \left(\bar{P}_{P\&I_i} \times 4 \times 12\right) \times n_{pm_i} + 1 \times a_i$

$S_i = \left[\left(\bar{P}_{P\&I_i} \times 4 \times 12\right) \times n_{pm_i} + 1 \times a_i\right] \times \left(1 - \bar{P}_{mm_i}\right)$

Figure 33. Preventive Maintenance MC Model

Preventive Maintenance MC model balance equations are

Abbreviations

$$R_i = \left(\bar{P}_{P\&I_i} \times 4 \times 12\right) \times n_{pm_i} + 1 \times a_i \tag{6.1}$$

$$S_i = \left[\left(\bar{P}_{P\&I_i} \times 4 \times 12\right) \times n_{pm_i} + 1 \times a_i\right] \times \left(1 - \bar{P}_{mm_i}\right) \tag{6.2}$$

State zero

$$R_1\bar{P}_{PM_0} = S_1\bar{P}_{PM_1} \tag{6.3}$$

State one

$$(R_2 + S_1)\bar{P}_{PM_1} = R_1\bar{P}_{PM_0} + S_2\bar{P}_{PM_2} \tag{6.4}$$

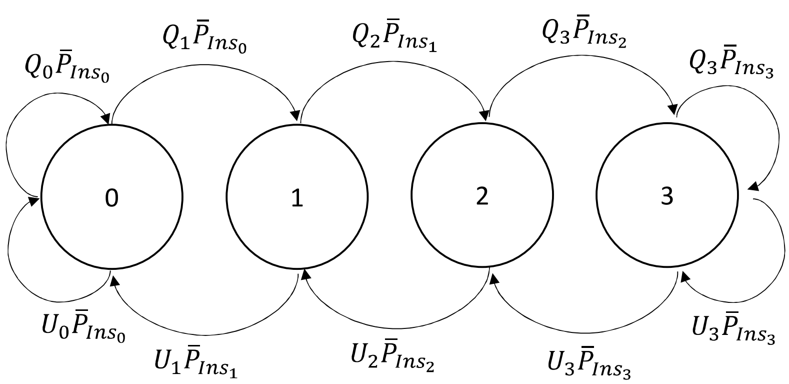
State two $$(R_3 + S_2)\bar{P}_{PM_2} = R_2\bar{P}_{PM_1} + S_3\bar{P}_{PM_3} \quad (6.5)$$

State three $$R_3\bar{P}_{PM_2} = S_3\bar{P}_{PM_3} \quad (6.6)$$

The sum of the probabilities is: $\bar{P}_{PM_0} + \bar{P}_{PM_1} + \bar{P}_{PM_2} + \bar{P}_{PM_3} = 1$.

6.3.2. Inspection

The required inspection actions will be determined by the following MC model and will be affected by the PM, the MR, and the MTTF. It will used to find the optimal number of actions required in each month and for each state. The flow from state (i) to state (i+1) is the required inspection actions and then it will reply with the effects which is the flow from state (i) to state (i-1), they are (Q_i) and (U_i), respectively. The parameter (b_i) is used to determine if inspection is required at the end of state (i). See Figure 34 for more detail.



$Q_i = (\bar{P}_{P\&I_i} \times 4 \times 12) \times n_{Ins_i} + 1 \times b_i$

$U_i = [(\bar{P}_{P\&I_i} \times 4 \times 12) \times n_{Ins_i} + 1 \times b_i] \times (1 - \bar{P}_{Pm_i} \times \bar{P}_{mm_i}) \times MTTF_i$

Figure 34. Inspection Action MC Model

Inspection MC model balance equations are

Abbreviations $$Q_i = (\bar{P}_{P\&I_i} \times 4 \times 12) \times n_{Ins_i} + 1 \times b_i \quad (6.7)$$

$$U_i = \left[\left(\bar{P}_{P\&I_i} \times 4 \times 12\right) \times n_{Ins_i} + 1 \times b_i\right] \times \left(1 - \bar{P}_{Pm_i} \times \bar{P}_{mm_i}\right) \times MTTF_i \tag{6.8}$$

State zero
$$Q_1 \bar{P}_{Insp_0} = U_1 \bar{P}_{Insp_1} \tag{6.9}$$

State one
$$(Q_2 + U_1)\bar{P}_{Ins_1} = Q_1 \bar{P}_{Ins_0} + U_2 \bar{P}_{Ins_2} \tag{6.10}$$

State two
$$(Q_3 + U_2)\bar{P}_{Ins_2} = Q_2 \bar{P}_{Ins_1} + U_3 \bar{P}_{Ins_3} \tag{6.11}$$

State three
$$Q_3 \bar{P}_{Ins_2} = U_3 \bar{P}_{Ins_3} \tag{6.12}$$

The sum of the probabilities is: $\bar{P}_{Ins_0} + \bar{P}_{Ins_1} + \bar{P}_{Ins_2} + \bar{P}_{Ins_3} = 1$.

6.3.3. Minimal Repair

The required MR actions will be determined by the following MC model and will be affected by the PM and the MTTF. It will used to find the optimal number of actions required in each month and for each state. The flow from state (*i*) to state (*i*+1) is the required MR actions and then it will reply with the effects which is the flow from state (*i*) to state (*i*-1), they are (X_i) and (Y_i), respectively. The parameter (b_i) is used to determine if inspection is required at the end of state (*i*) and then it may require MR. See Figure 35 for more detail. The MR actions required will be determined after and an inspection is performed and then a decision will be taken to determine if MR is required. Normal distribution function will be used to approximate the Binomial distribution probabilities in order to evaluate the associated probabilities for each inspection and then to estimate the number of MR actions.

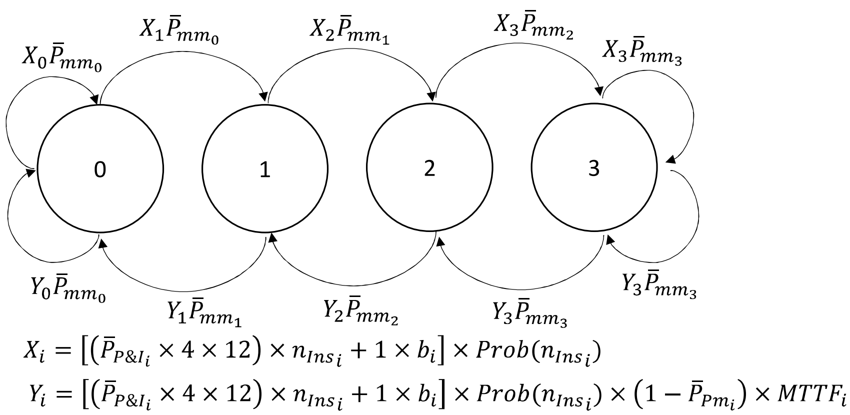


Figure 35. Minimal Repair Actions MC Model

Minimal repair MC model balance equations are

$$X_i = \left[\left(\bar{P}_{P\&I_i} \times 4 \times 12\right) \times n_{Ins_i} + 1 \times b_i\right] \times Prob(n_{Ins_i}) \tag{6.13}$$

Abbreviations

$$Y_i = \left[\left(\bar{P}_{P\&I_i} \times 4 \times 12\right) \times n_{Ins_i} + 1 \times b_i\right] \times Prob(n_{Ins_i}) \times \left(1 - \bar{P}_{Pm_i}\right) \times MTTF_i \tag{6.14}$$

State zero

$$X_1\bar{P}_{MR_0} = Y_1\bar{P}_{MR_1} \tag{6.15}$$

State one

$$(X_2 + Y_1)\bar{P}_{mMR_1} = X_1\bar{P}_{MR_0} + Y_2\bar{P}_{MR_2} \tag{6.16}$$

State two

$$(X_3 + Y_2)\bar{P}_{MR_2} = X_2\bar{P}_{MR_1} + Y_3\bar{P}_{MR_3} \tag{6.17}$$

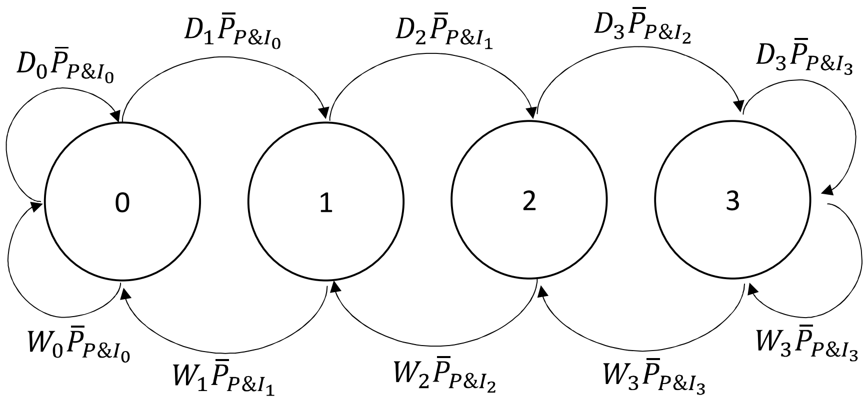
State three

$$X_3\bar{P}_{MR_2} = Y_3\bar{P}_{MR_3} \tag{6.18}$$

The sum of the probabilities is: $\bar{P}_{MR_0} + \bar{P}_{MR_1} + \bar{P}_{MR_2} + \bar{P}_{MR_3} = 1$

6.3.4. P&I Control Model

In this MC model, the optimal production plan and the optimal maintenance actions will be obtained. The required demand is passed from state (i) to the state (i+1) and then it replies to the required demand by having a production cycle, they are (D_i) and (W_i), respectively. However, the reply will have the effects of having PM, inspection, and MR actions. The effects used are their optimal state probabilities and the number of actions performed in each month and for each state. See Figure 36 for more detail.



$$W_i = \left[1 - d_i \frac{\left(1 - \bar{P}_{Pm_i} \times \bar{P}_{Ins_i} \times \bar{P}_{mm_i}\right)}{\left(\bar{P}_{Pm_i} n^*_{Pm_i} + \bar{P}_{Ins_i} n^*_{Ins_i} + \bar{P}_{mm_i} n^*_{mm_i}\right)}\right] P_i$$

Figure 36. P&I Control MC Model

P&I control MC model balance equations are

Abbreviation $$W_i = \left[1 - d_i \frac{\left(1 - \bar{P}_{Pm_i} \times \bar{P}_{Ins_i} \times \bar{P}_{mm_i}\right)}{\left(\bar{P}_{Pm_i} n^*_{Pm_i} + \bar{P}_{Ins_i} n^*_{Ins_i} + \bar{P}_{mm_i} n^*_{mm_i}\right)}\right] P_i \quad (6.19)$$

State zero $$D_1\bar{P}_{P\&I_0} = W_1\bar{P}_{P\&I_1} \quad (6.20)$$

State one $$(D_2 + W_1)\bar{P}_{P\&I_1} = D_1\bar{P}_{P\&I_0} + W_2\bar{P}_{P\&I_2} \quad (6.21)$$

State two $$(D_3 + W_2)\bar{P}_{P\&I_2} = D_2\bar{P}_{P\&I_1} + W_3\bar{P}_{P\&I_3} \quad (6.22)$$

State three $$D_3\bar{P}_{P\&I_2} = W_3\bar{P}_{P\&I_3} \quad (6.23)$$

The sum of the probabilities is: $\bar{P}_{P\&I_0} + \bar{P}_{P\&I_1} + \bar{P}_{P\&I_2} + \bar{P}_{P\&I_3} = 1$.

6.4. Expected Maintenance Actions Cost Function

In this section, the developed expected maintenance cost (EMC_i) for state (i) for all maintenance actions are

$$
\begin{aligned}
EMC_i &= EC(PM_i) + EC(Ins_i) + EC(MR_i) \\
&= \frac{D_i C_{PM_i}}{(P_i t_i)} \bar{P}_{PM_i} + \frac{D_i C_{Ins_i}}{(P_i t_i)} \bar{P}_{Ins_i} + \frac{D_i C_{MR_i}}{(P_i t_i)} \bar{P}_{MR_i}
\end{aligned} \tag{6.24}
$$

6.5. Expected Maintenance Total Cost Function

In this section, the expected total cost (ETC) for the developed integrated model that includes the P&I control model cost, and the maintenance cost will be presented. The total cost (TC_i) for

$$
\begin{aligned}
TC_i = {} & \frac{KD_i}{P_i t_i} + \frac{h(P_i - D_i)t_i}{2}\left(\frac{\pi}{h+\pi}\right) + \frac{sD_i}{t_{t_i}} \int_0^{t_{t_i}} d_{t_i}(t_{t_i} - x)\lambda e^{-\lambda x} dx + \frac{D_i C_{PM_i}}{(P_i t_i)} \bar{P}_{PM_i} \\
& + \frac{D_i C_{Ins_i}}{(P_i t_i)} \bar{P}_{Ins_i} + \frac{D_i C_{MR_i}}{(P_i t_i)} \bar{P}_{MR_i}
\end{aligned} \tag{6.25}
$$

Moreover, the objective function which is the expected total cost will be

$$
\begin{aligned}
ETC &= \sum_{i=0}^{4} \left(\bar{P}_{P\&I_i} \times TC_i\right) \\
&= \bar{P}_{P\&I_0} \times TC_0 + \bar{P}_{P\&I_1} \times TC_1 + \bar{P}_{P\&I_2} \times TC_2 + \bar{P}_{P\&I_3} \times TC_3
\end{aligned} \tag{4.26}
$$

6.6. Numerical Example

In this section, we will present an example to illustrate the developed integrated MC model. The data from (Hou, 2005) will be used: K=100, π=2.5, h=0.4, s=0.1, and λ=0.2. The maintenance costs will be: C_{PM}=200, C_{Ins}=50, and C_{MR}=100. The defect rates are: d_0= 0.002292, d_1=0.014506, d_2=0.025709, and d_3=0.045571. This data covers four years in sequence, and we will use four states that the optimal production plan and the optimal maintenance actions will be established for them. The production and demand rate for state zero is (1200, 850), state one is (950, 650), state two (750, 500), and state three is (600, 400). The Weibull probability density function from reference [13] will be used to evaluate the MTTF for each state, $f(x) = \lambda\, \nu\, x^{(\nu-1)}\, e^{-\lambda\, x^{\nu}}$ with scale parameter $\lambda = 0.05$ and shape parameter $\nu = 1$. We will consider only the steady-state operating characteristics in our model. The probability that will be used to determine the required MR action after each inspection is using the Normal distribution which will approximate the Binomial distribution with parameter n=4 and p=0.6.

The solution presents that for state zero the optimal cost is (155.775), the optimal time for the shortages to be satisfied is (0.1785), the optimal time for the inventory to be built is (1.1157), the optimal production run time is (1.2943), the optimal time for the inventory to be used and shortages to be built is (0.5329) and the expected state time is (1.4399) years. The same practice will be stated for state one, two, and three. The expected total cost is (172.821) with P-Value equal (0.989), however, without maintenance the expected total cost is (118.485) with P-Value equal (0.54). Moreover, the expected production rate is (940.648), the expected demand rate is (649.666), the expected defect rate is (0.01767), the expected number of maintenance actions that is required for PM is (2.1157) with P-Value equal (0.9099), for inspection is (2.6592) with P-Value equal (0.6579), and for MR is (1.7603) with P-Value equal (0.363). By using all the expected numbers mentioned in order to evaluate the optimal cost for the developed integrated MC model that is considered as one cycle gives an estimated cost equal to (193.694). For more detail, see Table XXXI.

Table XXXI. Maintenance Model Optimal Solution

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	**1200**	**850**	**1.00**	**0.3600**	**155.775**	**0.1785**	**1.1157**	**0.5329**	**1.4399**
1	**950**	**650**	**0.7613**	**0.2741**	**203.847**	**0.0789**	**0.4931**	**0.2640**	**1.0962**
2	**750**	**500**	**0.6991**	**0.1916**	**170.447**	**0.2508**	**1.5674**	**0.9091**	**0.7664**
3	**600**	**400**	**0.9101**	**0.1744**	**161.856**	**0.1603**	**1.0018**	**0.5810**	**0.6975**
Expected values					**172.821**	**0.1619**	**1.0118**	**0.5397**	
Production and Inventory Model					**193.694**	**0.2658**	**1.6610**	**0.8630**	
Difference					**20.873**	**0.1039**	**0.6492**	**0.3233**	

The optimal solution for the states' probability for the PM, inspection, MR, and for the P&I MC are presented in Figure 37. Moreover, the optimal number of maintenance actions for each month and for state are presented in Figure 38. The optimal solution suggests that the PM is required at the end of state one, state two, and state three. However, an inspection is only required at the end of state zero then it may require an MR action. The expected maintenance cost is (44.9844) with *P*-Value equal (0.6706) and the expected P&I model is (127.8363) with *P*-Value equal (0.738).

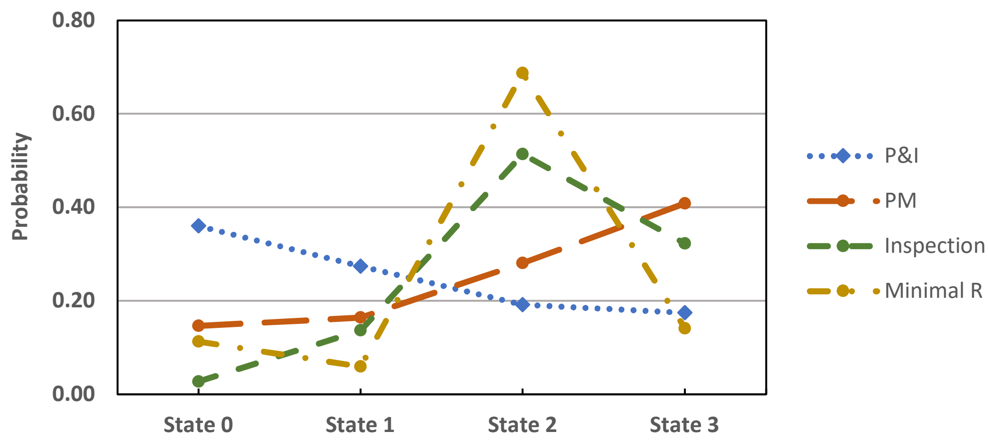


Figure 37.Optimal P&I and maintenance actions' probabilities

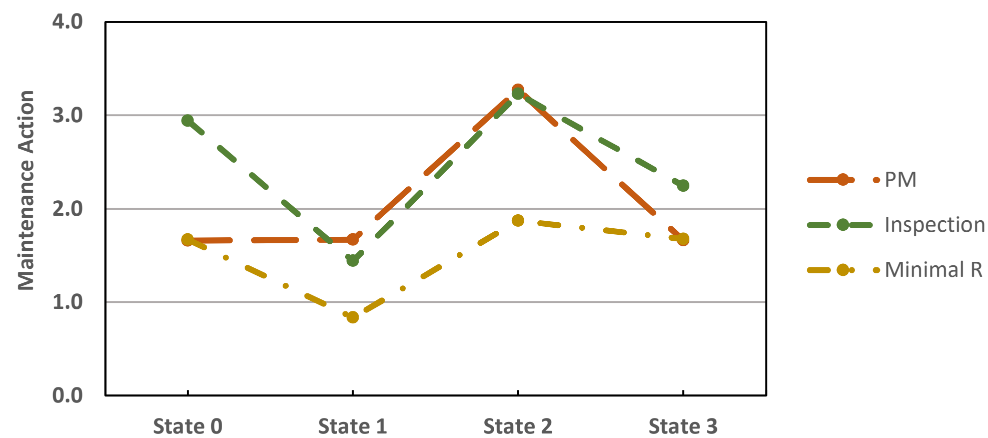


Figure 38.Optimal maintenance actions required in each month and for each state

The probability of doing MR for each inspection for state zero is (0.33922), for state one is (0.25168), for state two is (0.28127), and for state three is (0.38574). Moreover, the mean time to failure for state zero is (0.04011), state one is (0.00802), state two is (0.0778), and for state three is (0.0.03248). Since the optimal numbers of PM, inspection, and MR are found. The maintenance action policy for our model will be using an equal space interval for each state. For state zero the PM is performed every (18.0793) days, inspection is performed every (10.1991) days, and MR is performed every (17.9589) days. The same practice will be stated for state one, two, and three.

However, the expected number of days for the PM is every (15.5394) days, for the inspection is every (12.2032) days, and for the MR is every (17.6837) days that could be used for all whole four states during the whole four years. Or the following expected number of days using the P&I probabilities will be for PM is every (16.332) days, for the inspection is every (13.484), and for the MR is every (22.49) days. The associated *P*-Values for them are high which are an accepted hypothesis. For more detail see Table XXXII and Table XXXIII.

Table XXXII. Detailed Maintenance actions and Probabilities for Each State

State (i)	PM	Inspection	MR	N(PM)	N(Insp.)	N(MR)
0	0.14625	0.02707	0.11278	1.6594	2.9414	1.6705
1	0.16436	0.13664	0.05923	1.6703	1.4418	0.8362
2	0.28083	0.51375	0.68699	3.2719	3.2279	1.8723
3	0.40856	0.32254	0.14100	1.6634	2.2454	1.6751
Expected Values				2.1156	2.6592	1.7603
P-Value				0.9099	0.6579	0.363
P&I Prob				1.9720	2.4639	1.4813
P-Value				0.8297	0.9997	0.8978

Table XXXIII. Optimal Maintenance Interval Solution for Each State

State (*i*)	Days/PM	Days/Insp.	Days/MR
0	18.079	10.199	17.959
1	17.961	20.807	35.878
2	9.169	9.294	16.024
3	18.035	13.361	17.909
Expected Days	15.539	12.203	17.684
P-Value	0.9101	0.674	0.4288
Using P&I Prop.	16.332	13.484	22.49
P-Value	0.8291	0.9806	0.9140

The developed maintenance actions model provides optimal solution in order to carry out the maintenance activities on the production process. It will enhance the health and will reduce the process failures among the duration of the four states. Moreover, detailed production plan is provided. Any related production facility could adapt this model and utilized it in order to enhance their production processes. By using the developed MC models, it will overcome the effects of the failures and deterioration and then enhance the health for the process.

7. SENSITIVTY ANALYSIS

In this chapter sensitivity analysis will be performed in order to show the behaviour of the developed MC model one and two. We will provide the figures for MC model two using the first scenario. However, MC model one has same behaviour and with all other scenarios.

7.1. Production and Demand Rates

In this section we will explore the effects of varying the production and demand rates for each state on the optimal solution using the developed MC model two for scenario one. The same behaviour is observed for the other scenarios. There are five parameters that will be examined which are the expected total cost ($\overline{TC}$), the expected time when shortages are met ($\bar{T}_1$), the expected time when inventory is built ($\bar{T}_2$), the expected production run time ($\bar{t}$), and the expected defect rate ($\bar{d}$). The production and demand rate data are manipulated by having five percent increase or decrease from the base variable, the data is presented in Table XXXIV. Four states will be considered to perform the sensitivity analysis.

Table XXXIV. Sensitivity Analysis Data for the Production and Demand Rates

State (i)	10% ↓	5% ↓	Base	5% ↑	10% ↑
0	(660, 440)	(630, 420)	(600, 400)	(570, 380)	(540, 360)
1	(660, 440)	(630, 420)	(600, 400)	(570, 380)	(540, 360)
2	(660, 440)	(630, 420)	(600, 400)	(570, 380)	(540, 360)
3	(660, 440)	(630, 420)	(600, 400)	(570, 380)	(540, 360)

It is noticed that the expected total cost ($\overline{TC}$) is getting lower by having lower production rates. However, by having lower demand rates, it is getting higher. See Figure 39 for more detail. Moreover, for the expected ($\bar{T}_1$), it is getting lower by having lower demand rates and by having lower production rates, it is getting higher. Also, same behaviour is noticed for the other expected values of ($\bar{T}_2$), ($\bar{t}$), and ($\bar{d}$). See Figure 40, Figure 41, Figure 42, and Figure 43 for more detail.

The sensitivity analysis solution figures are summarized by having the average, the maximum, the minimum, the range, and the standard deviation for each variable for the reader convenience, see Table XXXV for more detail.

Table XXXV. Sensitivity Analysis Summary Statistics for Production and Demand Rates Effects

Variables	Average	Max	Min	Range	Standard Deviation
$\overline{TC}$	94.694	106.308	75.146	31.162	8.223
$\overline{T}_1$	0.1984	0.2992	0.1415	0.1577	0.0389
$\overline{T}_2$	1.2401	1.8701	0.8847	0.9854	0.2434
$\bar{t}$	1.4385	2.1693	1.0262	1.1431	0.2824
$\bar{d}$	0.02209	0.02212	0.02207	4.800E-05	1.194E-05

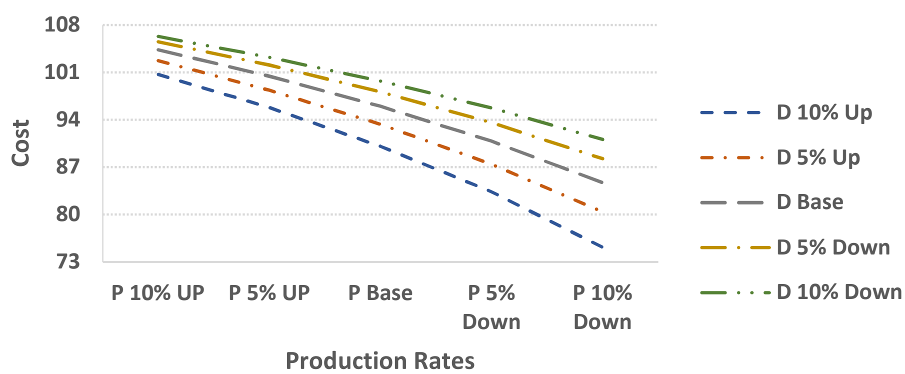


Figure 39. Expected ($\overline{TC}$) w.r.t. Production and Demand Rates

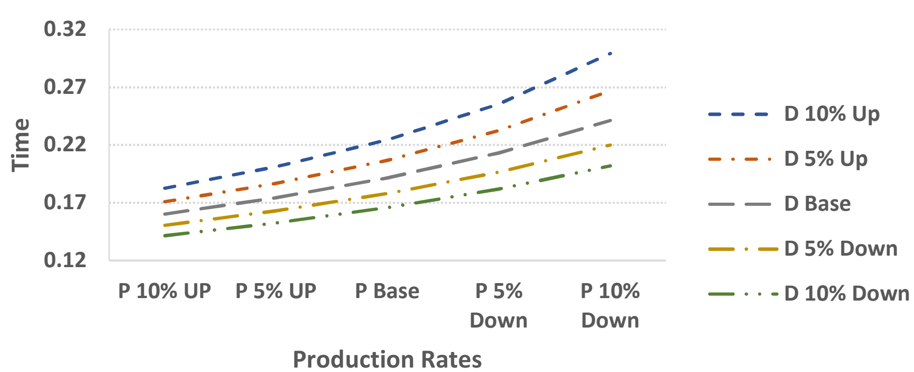


Figure 40. Expected ($\overline{T}_1$) Met w.r.t. Production and Demand Rates

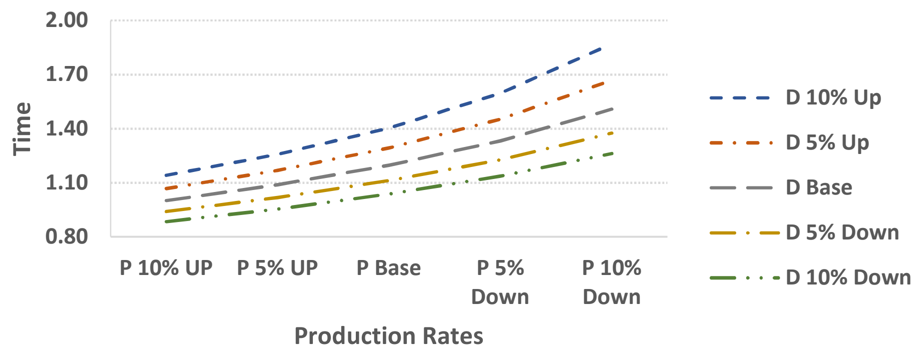


Figure 41. Expected ($\bar{T}_2$) w.r.t. Production and Demand Rates

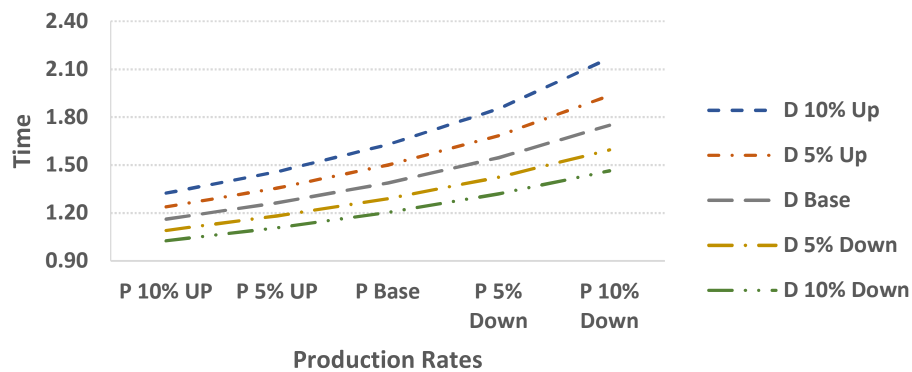


Figure 42. Expected ($\bar{t}$) w.r.t. Production and Demand Rates

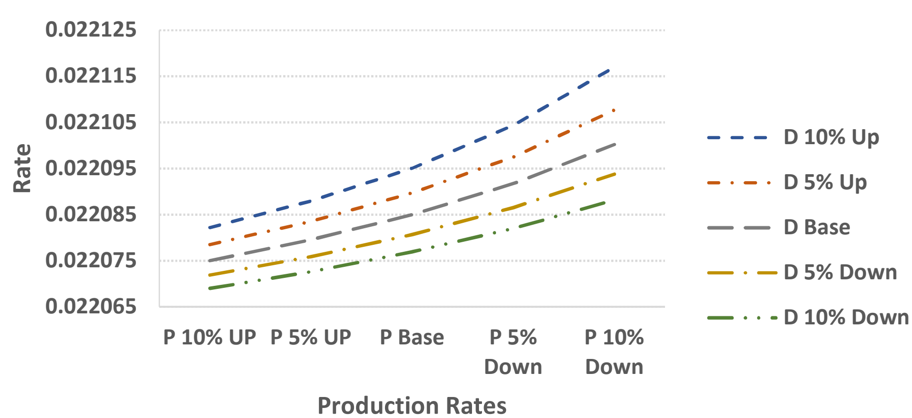


Figure 43. Expected ($\bar{d}$) w.r.t. Production and Demand Rates

7.2. Production and Inventory Control Model's Parameters

In this category, we will test the effects of the setup cost (K), the holding cost (h), the shortages cost ($\boldsymbol{\pi}$), rework cost (s), and the failure rate (λ)by having an increasing 20%. For more detail, see Table XXXVI.

Table XXXVI. Sensitivity Analysis Data for the P&I Control Model Parameters

Parameter	Base	20% ↑	40% ↑	60% ↑	80% ↑	100% ↑
K	**100**	120	140	160	180	200
h	0.4	0.48	0.56	0.64	0.72	0.8
π	2.5	3	3.5	4	4.5	5
s	0.1	0.12	0.14	0.16	0.18	0.2
λ	0.2	0.24	0.28	0.32	0.36	0.4

It is noticed that the expected total cost ($\overline{TC}$) is getting higher by having higher values for (K, h, π, s, and λ), see Figure 44 for more detail. However, the expected ($\bar{T}_1$) is getting higher by having higher values for (K and h) and getting lower by having higher values for (π, s, and λ), see Figure 45 for more detail. Moreover, for the expected ($\bar{T}_2$) is getting higher by having higher values for (K and h) and getting lower by having higher values for (π, s, and λ), see Figure 46 for more detail. Also, the expected ($\bar{t}$) is getting higher by having higher values for only the setup cost and is getting lower by having higher values for (h, π, s, and λ), see Figure 47 for more detail. Finally, the expected ($\bar{d}$) is getting higher by having higher values for (K and λ) and is getting lower by having higher values for (h, π, and s), see Figure 48 for more detail.

The sensitivity analysis behaviour is summarized in the following Table XXXVII. Moreover, the solution figures are summarized too by having the statistics like the average, the maximum, the minimum, the range, and the standard deviation for each variable, see Table XXXVIII for more detail.

Table XXXVII. Sensitivity Analysis Variables Behavior Summary for the P&I Control Model Parameters

Parameter	$\overline{TC}$	$\overline{T}_1$	$\overline{T}_2$	$\bar{t}$	$\bar{d}$
K	↑	↑	↑	↑	↑
h	↑	↑	↓	↓	↓
π	↑	↓	↑	↓	↓
s	↑	↓	↓	↓	↓
λ	↑	↓	↓	↓	↑

Table XXXVIII. Sensitivity Analysis Summary Statistics for P&I Control Model Parameters Effects

Variable	Average	Max	Min	Range	Standard Deviation
$\overline{TC}$	103.934	103.934	96.005	7.930	12.036
$\overline{T}_1$	0.1957	0.1957	0.0993	0.0964	0.0403
$\overline{T}_2$	1.2084	1.2084	0.7940	0.4144	0.1861
$\bar{t}$	1.4041	1.4041	1.0481	0.3560	0.1933
$\bar{d}$	0.02209	0.02209	0.02207	2.116E-05	1.626E-05

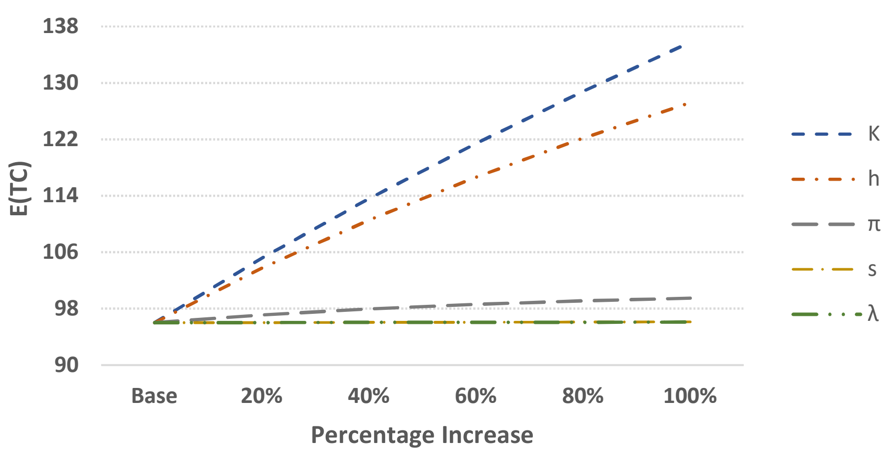


Figure 44. Expected ($\overline{TC}$) w.r.t. P&I Control Model Parameters

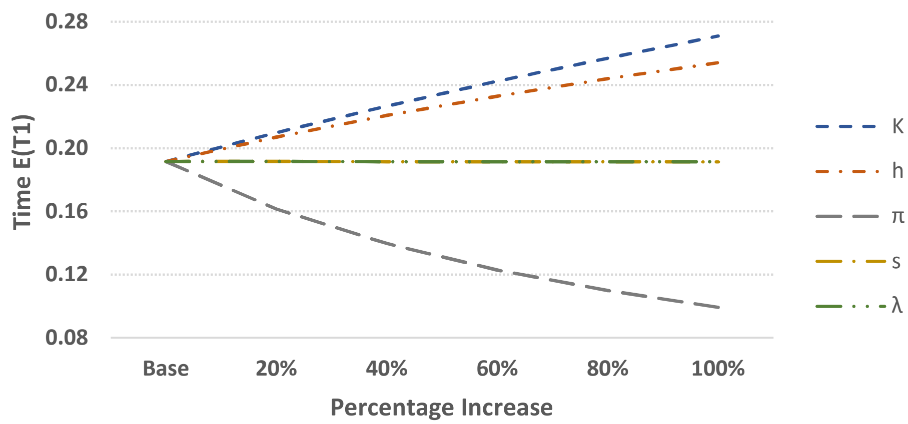


Figure 45. Expected ($\bar{T}_1$) w.r.t. P&I Control Model Parameters

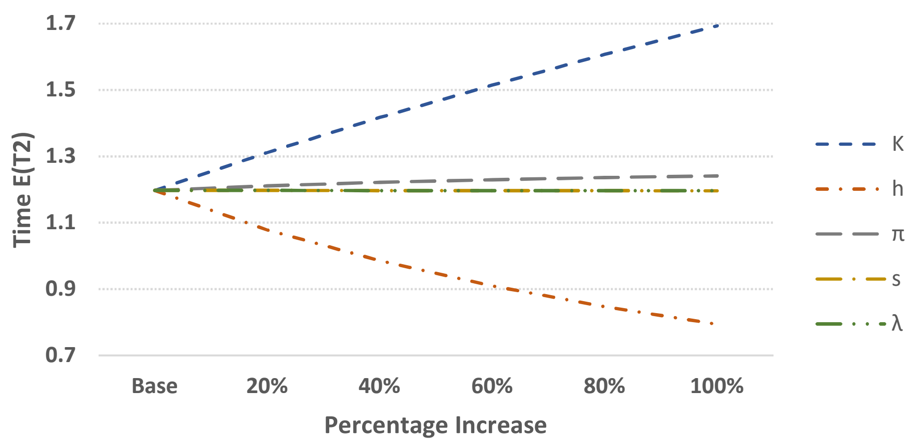


Figure 46. Expected ($\bar{T}_2$) w.r.t. P&I Control Model Parameters

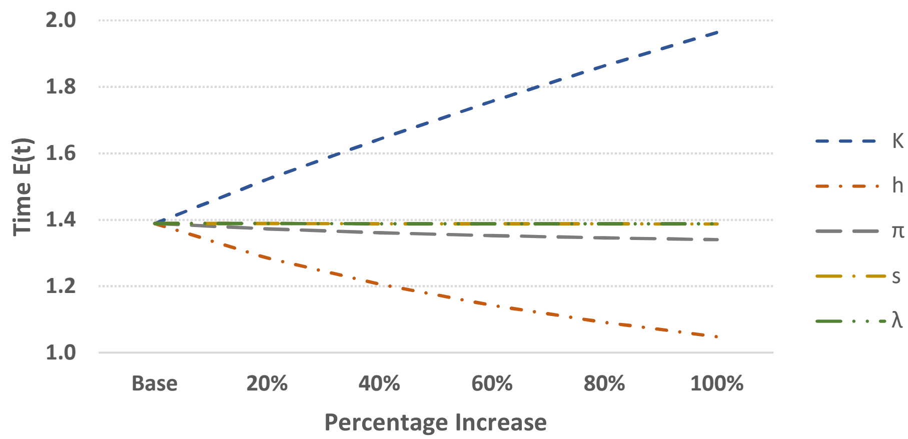


Figure 47. Expected ($\bar{t}$) w.r.t. P&I Control Model Parameters

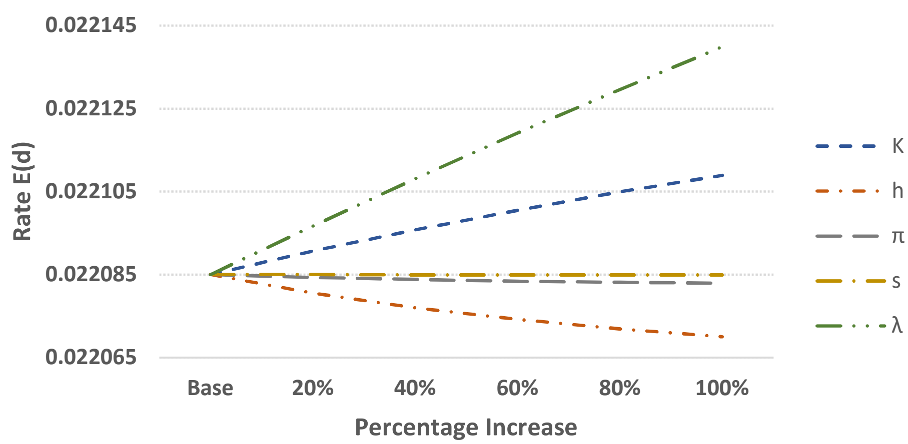


Figure 48. Expected ($\bar{d}$) w.r.t. P&I Control Model Parameters

8. CONCLUSIONS

The developed MC model in this thesis uses an irreducible discrete time and with finite states. Two different MC models are developed, MC model one without time factor and MC model two with time factor. Four different scenarios are created for each MC model one and two. The first scenario has fixed production and demand rates among the states. The second scenario has variable production rates and fixed demand rates. The third scenario has fixed production rates and variable demand rates. The fourth scenario has variable production and demand rates. Different optimization examples are conducted for each scenario in order to reduce the expected total cost. Detailed production plan is established that has the required optimal time for $T_1, T_2,$ and $T_3 + T_4$ for each cycle in each state. The accuracy in the expected total cost for MC model two is higher than MC model one. Moreover, both of them have high P-Values which is an accepted hypothesis.

Scheduling models are developed in order to pass jobs from sender production process to production processes which is the receiver. This problem is solved by establishing the hard time windows. Two cases are considered when having same production and demand that is case one. The second case has different production and demand rates. Optimal feasible scheduling hard time window is obtained in order to specify the times when the sender production process can pass jobs to the receiver production process. The related calculated P-Values are high and are accepted hypothesis.

Moreover, an integrated maintenance MC model is developed in order to determine the optimal maintenance actions for the preventive maintenance, inspection, and minimal repair in each month and for each state. The optimal production plan for each state is obtained. The objective function which is the expected total cost function is developed which covers the cost of the production process and the cost of the maintenance. Equal maintenance action interval plan is used for each state. Moreover, the related calculated expected values for the maintenance actions can be used. The developed MC models are interacting and affecting each other. The calculated associated P-Values for the expected values are high and are accepted hypothesis.

Sensitivity analysis is performed with two different approaches. In the first approach, the effects of increasing and decreasing the production and demand rates on the five different parameters for the MC model with fixed production rates which is scenario one. Similar behaviours are noticed for scenario two, three, and four. However, in the second approach, the effects of increasing the setup cost, holding cost, shortage cost, rework cost, and failure rate are performed and illustrated.

The developed models in this thesis can be used in any production facility with similar problem and having a single product. The provided Chapter 7, sensitivity analysis, can answer that question. Since the optimal production times values are needed, the production managers are looking for ways that the developed model can be used for their production process. The minimization of the objective function is the main criteria in making the decision. The values of the developed model's parameters are tested by having percentage increases to show the effects on the desired decision parameters. The two categories cover the effects of production & demand rates and the P&I model's parameters on the expected total cost, the expected production run times for shortages to be recovered and inventory at peak, and the expected defect rates.

The developed and used MC models are irreducible and with discrete time. Moreover, as the problem dimension increased among the provided scenarios for MC model one and MC model two, the processing time that are needed to solve and evaluate the expected total cost are increased too. Moreover, for the scheduling MC model, the time is increased substantially in order to solve the problem by having two production processes interacting with each other. Also, the time is increased and noticed from using MC model one to using MC model two. The maintenance model takes longer time. As the dimension of the MC models increases, the required times to find the optimal expected total cost increases. When using GA and PSO code to solve the scheduling problem, it takes long time which depends on the desired time to run and the number of iterations. The processing time is increased among all the developed models and scenarios, as a result, this problem is NP-Complete. For more detail, see Figure 49.

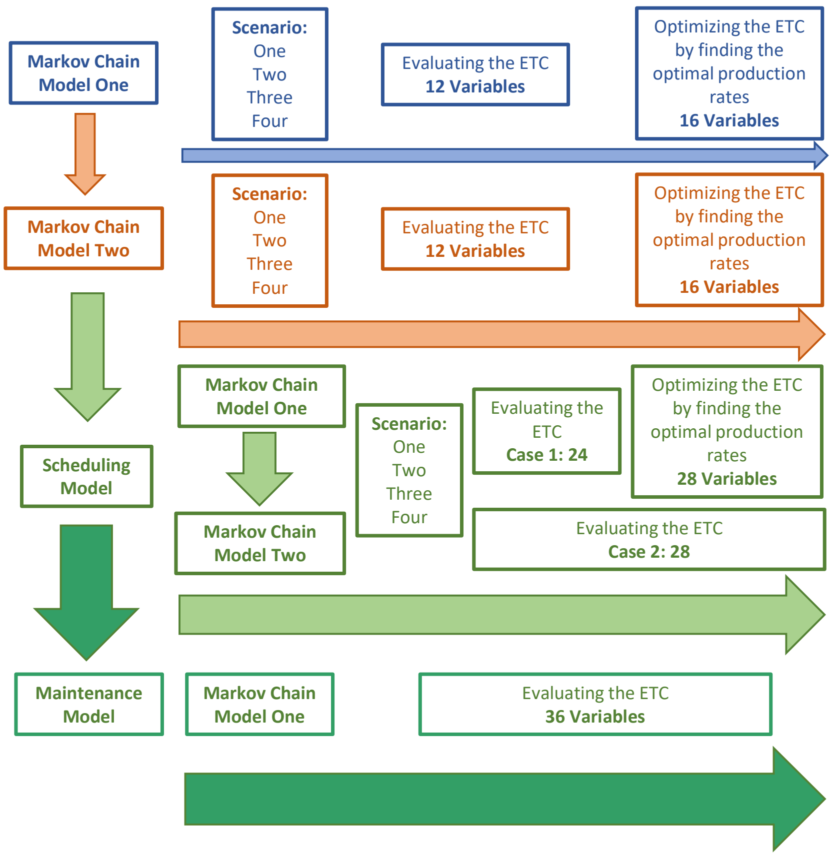


Figure 49. The Development of the MC Models (NP-Complete) Problem

REFERENCES

© 2020 IEEE. Reprinted, with permission, from [Al Hajailan, W. I. and David He, "Expected Maintenance Actions for Imperfect Production Processes Using a Markovian Approach," 2020 Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling (APARM), Vancouver, BC, Canada, 2020, pp. 1-6, doi: 10.1109/APARM49247.2020.9209372.].

Amorim, P., et al. "Lot Sizing Versus Batching in the Production and Distribution Planning of Perishable Goods." *International Journal of Production Econxomics* 146.1 (2013): 208-18.

Barron, Yonit. "Critical Level Policy for a Production-Inventory Model with Lost Sales." *International Journal of Production Research* 57.6 (2019): 1685-705.

Barron, Yonit, David Perry, and Wolfgang Stadje. "A make-to-Stock production/inventory Model with MAP Arrivals and Phase-Type Demands." *Annals of Operations Research* 241.1-2 (2016): 373-409.

Barron, Y. "An (s, k, S) Fluid Inventory Model with Exponential Leadtimes and Order Cancellations." *Stochastic Models* 32.2 (2016): 301-32.

Ben-Daya, M., and M. Hariga. "Economic Lot Scheduling Problem with Imperfect Production Processes." *Journal of the Operational Research Society* 51.7 (2000): 875-81.

Berenguer, Christophe, Chengbin Chu, and Antoine Grall. "Inspection and Maintenance Planning: An Application of Semi-Markov Decision Processes." *Journal of Intelligent Manufacturing* 8.5 (1997): 467-76.

Boone, Tonya, et al. "The Impact of Imperfect Processes on Production Run Times." *Decision Sciences* 31.4 (2000): 773-87.

Boukas, EK, Q. Zhang, and G. Yin. "On Robust Design for a Class of Failure Prone Manufacturing Systems." *Recent Advances in Control and Optimization of Manufacturing Systems* 214 (1996): 197-222.

Boukas, EK, et al. "Manufacturing Systems: Wear Modeling and Numerical Procedures." *Stochastic Analysis and Applications* 15.3 (1997): 269-93.

Cadenillas, Abel, Peter Lakner, and Michael Pinedo. "Optimal Production Management when Demand Depends on the Business Cycle." *Operations research* 61.4 (2013): 1046-62.

Chang, Yl. "Time Window Capacity Analysis for Synchronous Production." *Production Planning & Control* 5.1 (1994): 66-81.

Chen, Cheng-Kang, and Chih-Chung Lo. "Optimal Production Run Length for Products Sold with Warranty in an Imperfect Production System with Allowable Shortages." *Mathematical and Computer Modelling* 44.3-4 (2006): 319-31.

Chen, Dongyan, and Kishor S. Trivedi. "Optimization for Condition-Based Maintenance with Semi-Markov Decision Process." *Reliability Engineering & System Safety* 90.1 (2005): 25-9.

Cheng, TCE. "An Economic Order Quantity Model with Demand-Dependent Unit Production Cost and Imperfect Production Processes." *IIE transactions* 23.1 (1991): 23-8.

Chiu, Yuanshyi Peter. "Determining the Optimal Lot Size for the Finite Production Model with Random Defective Rate, the Rework Process, and Backlogging." *Engineering optimization* 35.4 (2003): 427-37.

Christiansen, M. "Decomposition of a Combined Inventory and Time Constrained Ship Routing Problem." *Transportation Science* 33.1 (1999): 3-16.

Chung, Kun-Jen, and Kuo-Lung Hou. "An Optimal Production Run Time with Imperfect Production Processes and Allowable Shortages." *Computers & Operations Research* 30.4 (2003): 483-90.

Cohen, M. A., P. R. Kleindorfer, and H. L. Lee. "Service Constrained (S, S) Inventory Systems with Priority Demand Classes and Lost Sales." *Management Science* 34.4 (1988): 482-99.

Dao, S.D. 2020 “Particle Swarm Optimization (PSO) for Constrained Optimization Problems”, *Research Note*, online at: https://learnwithpanda.com/2020/05/19/particle-swarm-optimization-pso-for-constrained-optimization-problems

de Moura, Delmo Alves, and Rui Carlos Botter. "Delivery and Pick-Up Problem Transportation - Milk Run Or Conventional Systems." *Independent Journal of Management & Production* 7.3 (2016): 746-70.

Elhafsi, Mohsen, and Essia Hamouda. "Managing an Integrated Production and Inventory System Selling to a Dual Market: Long-Term and Walk-in." *European Journal of Operational Research* 268.1 (2018): 215-30.

ElHafsi, Mohsen, Herve Camus, and Etienne Craye. "Managing an Integrated Production Inventory System with Information on the Production and Demand Status and Multiple Non-Unitary Demand Classes." *European Journal of Operational Research* 207.2 (2010): 986-1001.

Eruguz, Ayse Sena, Tarkan Tan, and Geert-Jan van Houtum. "Integrated Maintenance and Spare Part Optimization for Moving Assets." *IISE Transactions* 50.3 (2018): 230-45.

Feng, Youyi, Jianjun Xu, and Shaohui Zheng. "Dynamic Optimal Policy for an Inventory System of Two Substitutable Products with Positive Replenishment Lead Times." *Operations research* (2019).

Habibi, Farhad, Ehsan Asadi, and Seyed Jafar Sadjadi. "A Location-Inventory-Routing Optimization Model for Cost Effective Design of Microalgae Biofuel Distribution System: A Case Study in Iran." *Energy Strategy Reviews* 22 (2018): 82-93.

Hajji, A., A. Gharbi, and J. P. Kenne. "Joint Replenishment and Manufacturing Activities Control in a Two Stage Unreliable Supply Chain." *International Journal of Production Research* 47.12 (2009): 3231-51.

Hariga, M., and M. Ben-Daya. "Note: The Economic Manufacturing lot-sizing Problem with Imperfect Production Processes: Bounds and Optimal Solutions." *Naval Research Logistics (NRL)* 45.4 (1998): 423-33.

Hayek, Pascale A., and Moueen K. Salameh. "Production Lot Sizing with the Reworking of Imperfect Quality Items Produced." *Production planning & control* 12.6 (2001): 584-90.

He, Yan, Wei Jin Ma, and Ji Ping Zhang. "The parameters selection of pso algorithm influencing on performance of fault diagnosis." *MATEC Web of conferences*. Vol. 63. EDP Sciences, 2016.

Hennig, F., et al. "Alternative Approaches to the Crude Oil Tanker Routing and Scheduling Problem with Split Pickup and Split Delivery." *European Journal of Operational Research* 243.1 (2015): 41-51.

Hishamuddin, Hawa, Ruhul A. Sarker, and Daryl Essam. "A Recovery Mechanism for a Two Echelon Supply Chain System Under Supply Disruption." *Economic Modelling* 38 (2014): 555-63.

Holland, John. "Adaptation in Natural and Artificial Systems: An Introductory Analysis with Application to Biology." *Control and artificial intelligence* (1975)Web.

Hou, KL. "Optimal Production Run Length for Deteriorating Production System with a Two-State Continuous-Time Markovian Processes Under Allowable Shortages." *Journal of the Operational research Society* 56.3 (2005): 346-50.

Huang, Bo-Shi, and Huan-Neng Chiu. "The Quality Management of the Imperfect Production Process Under Two Monitoring Policies." *International Journal of Quality & Reliability Management* 12.3 (1995): 19-31.

Hussain, Abid, and Yousaf Shad Muhammad. "Trade-off between exploration and exploitation with genetic algorithm using a novel selection operator." *Complex & Intelligent Systems* (2019): 1-14.

Iakovou, E., CM Ip, and C. Koulamas. "Throughput-Dependent Periodic Maintenance Policies for General Production Units." *Annals of Operations Research* 91 (1999): 41-7.

Ibaraki, T., et al. "Effective Local Search Algorithms for Routing and Scheduling Problems with General Time-Window Constraints." *Transportation Science* 39.2 (2005): 206-32.

Isotupa, K. P. S. "An (s,Q) Markovian Inventory System with Lost Sales and Two Demand Classes." *Mathematical and Computer Modelling* 43.7-8 (2006): 687-94.

Jia, Tao, et al. "Integrated Inventory Routing Problem with Quality Time Windows and Loading Cost for Deteriorating Items Under Discrete Time." *Mathematical Problems in Engineering* 2014 (2014): 537409.

Jung, June Young, et al. "Integrated Safety Stock Management for Multi-Stage Supply Chains Under Production Capacity Constraints." *Computers & Chemical Engineering* 32.11 (2008): 2570-81.

Kalpakam, S., and K. P. Sapna. "A Lost Sales Inventory System with Supply Uncertainty." *Computers & Mathematics with Applications* 33.3 (1997): 81-93.

Kalpakam, S., and S. Shanthi. "A Perishable Inventory System with Modified (S-1,S) Policy and Arbitrary Processing Times." *Computers & Operations Research* 28.5 (2001): 453-71.

Kenne, J. P., and A. Gharbi. "A Simulation Optimization Based Control Policy for Failure Prone One-Machine, Two-Product Manufacturing Systems." *Computers & Industrial Engineering* 46.2 (2004): 285-92.

Kenne, J. P., and A. Gharbi. "A Simulation Optimization Approach in Production Planning of Failure Prone Manufacturing Systems." *Journal of Intelligent Manufacturing* 12.5-6 (2001): 421-31.

Kennedy, James, and Russell Eberhart. "Particle swarm optimization." *Proceedings of ICNN'95-International Conference on Neural Networks*. Vol. 4. IEEE, 1995.

Kim, Chang Hyun, and Yushin Hong. "An Optimal Production Run Length in Deteriorating Production Processes." *International Journal of Production Economics* 58.2 (1999): 183-9.

Kopp, Denny, et al. "Qualification Management in Wafer Fabs: Optimization Approach and Simulation-Based Performance Assessment." *Ieee Transactions on Automation Science and Engineering* 17.1 (2020): 475-89.

Lee, Hau L., and Meir J. Rosenblatt. "A Production and Maintenance Planning Model with Restoration Cost Dependent on Detection Delay." *IIE transactions* 21.4 (1989): 368-75.

Lee, Hau L., and Meir J. Rosenblatt. "Simultaneous Determination of Production Cycle and Inspection Schedules in a Production System." *Management science* 33.9 (1987): 1125-36.

Li, Feng, Zhi-Long Chen, and Lixin Tang. "Integrated Production, Inventory and Delivery Problems: Complexity and Algorithms." *Informs Journal on Computing* 29.2 (2017): 232-50.

Li, Qing, Qi-Ming He, and Xiaoli Wu. "Timing Order Fulfillment of Capital Goods Under a Constrained Capacity." *Annals of Operations Research* 241.1-2 (2016): 431-56.

Lian, Z. T., L. M. Liu, and M. F. Neuts. "A Discrete-Time Model for Common Lifetime Inventory Systems." *Mathematics of Operations Research* 30.3 (2005): 718-32.

LIN, NP, and L. KRAJEWSKI. "A Model for Master Production Scheduling in Uncertain Environments." *Decision Sciences* 23.4 (1992): 839-61.

Liou, Ming-Jong, Sheng-Tsaing Tseng, and Tsong-Ming Lin. "The Effects of Inspection Errors to the Imperfect EMQ Model." *IIE transactions* 26.2 (1994): 42-51.

Lou, S., S. P. Sethi, and Q. Zhang. "Optimal Feedback Production Planning in a Stochastic 2-Machine Flowshop." *European Journal of Operational Research* 73.2 (1994): 331-45.

Meller, RD, and DS Kim. "The Impact of Preventive Maintenance on System Cost and Buffer Size." European Journal of Operational Research 95.3 (1996): 577-91.

Miranda, Pedro L., Reinaldo Morabito, and Deisemara Ferreira. "Mixed Integer Formulations for a Coupled Lot-Scheduling and Vehicle Routing Problem in Furniture Settings." *Infor* 57.4 (2019): 563-96.

Miranda, Pedro L., Reinaldo Morabito, and Deisemara Ferreira "Optimization Model for a Production, Inventory, Distribution and Routing Problem in Small Furniture Companies." *Top* 26.1 (2018): 30-67.

Nakashima, K., et al. "Analysis of a Product Recovery System." *International Journal of Production Research* 40.15 (2002): 3849-56.

Noblesse, Ann M., et al. "Lot Sizing and Lead Time Decisions in production/inventory Systems." *International Journal of Production Economics* 155 (2014): 351-60.

Ozener, Okan Orsan, Refik Gullu, and Nesim Erkip. "Near-Optimal Modified Base Stock Policies for the Capacitated Inventory Problem with Stochastic Demand." *Asia-Pacific Journal of Operational Research* 31.3 (2014): 1450019.

Parlar, M. "Probabilistic Analysis of Renewal Cycles: An Application to a Non-Markovian Inventory Problem with Multiple Objectives." *Operations research* 48.2 (2000): 243-55.

Parsopoulos, Konstantinos E., and Michael N. Vrahatis. "Particle swarm optimization method for constrained optimization problems." *Intelligent Technologies–Theory and Application: New Trends in Intelligent Technologies* 76.1 (2002): 214-220.

Porteus, Evan L. "Optimal Lot Sizing, Process Quality Improvement and Setup Cost Reduction." *Operations research* 34.1 (1986): 137-44.

Presman, E., S. Sethi, and Q. Zhang. "Optimal Feedback Production Planning in a Stochastic N-Machine Flowshop." *Automatica* 31.9 (1995): 1325-32.

Presman, E., S. P. Sethi, and W. Suo. "Existence of Optimal Feedback Production Plans in Stochastic Flowshops with Limited Buffers." *Automatica* 33.10 (1997): 1899-903.

Rahim, M. A. "Joint determination of production quantity, inspection schedule, and control chart design." *IIE transactions* 26.6 (1994): 2-11.

Rahim, MA, and WI Al-Hajailan. "An Optimal Production Run for an Imperfect Production Process with Allowable Shortages and Time-Varying Fraction Defective Rate." *The International Journal of Advanced Manufacturing Technology* 27.11-12 (2006): 1170-7.

Rosenblatt, Meir J., and Hau L. Lee. "Economic Production Cycles with Imperfect Production Processes." *IIE transactions* 18.1 (1986): 48-55.

Salameh, MK, and MY Jaber. "Economic Production Quantity Model for Items with Imperfect Quality." *International Journal of Production Economics* 64.1-3 (2000): 59-64.

Schaefer, M. K. "Replenishment Policies for Inventories of Recoverable Items with Attrition." *Omega-International Journal of Management Science* 17.3 (1989): 281-7.

Song, Dong-Ping. "Optimal Integrated Ordering and Production Policy in a Supply Chain with Stochastic Lead-Time, Processing-Time, and Demand." *Ieee Transactions on Automatic Control* 54.9 (2009): 2027-41.

Sung, C. S., and G. T. Oh. "(R,q) Policy for a Single-Product Production Inventory Problem with a Compound Poisson Demand Process." *Journal of the Operations Research Society of Japan* 30.2 (1987): 132-49.

Tai, Allen H., and Wai-Ki Ching. "Optimal Inventory Policy for a Markovian Two-Echelon System with Returns and Lateral Transshipment." *International Journal of Production Economics* 151 (2014): 48-55.

Tang, Liang, et al. "Supply Chain Scheduling in a Collaborative Manufacturing Mode: Model Construction and Algorithm Design." *Annals of Operations Research* 275.2 (2019): 685-714.

Tripathy, PK, WM Wee, and PR Majhi. "An EOQ Model with Process Reliability Considerations." *Journal of the Operational Research Society* 54.5 (2003): 549-54.

Vahdani, Behnam, S. T. A. Niaki, and S. Aslanzade. "Production-Inventory-Routing Coordination with Capacity and Time Window Constraints for Perishable Products: Heuristic and Meta Heuristic Algorithms." *Journal of Cleaner Production* 161 (2017): 598-618.

van der Laan, E., R. Dekker, and M. Salomon. "Product Remanufacturing and Disposal: A Numerical Comparison of Alternative Control Strategies." *International Journal of Production Economics* 45.1-3 (1996): 489-98.

van Oosterom, Chiel, Hao Peng, and Geert-Jan van Houtum. "Maintenance Optimization for a Markovian Deteriorating System with Population Heterogeneity." *Iise Transactions* 49.1 (2017): 96-109.

Wang, Chih-Hsiung. "The Impact of a Free-Repair Warranty Policy on EMQ Model for Imperfect Production Systems." *Computers & Operations Research* 31.12 (2004): 2021-35.

Wang, Chih-Hsiung, and Shey-Huei Sheu. "The Effects of the Warranty Cost on the Imperfect EMQ Model with General Discrete Shift Distribution." *Production Planning & Control* 12.6 (2001): 621-8.

Wang, CH, and SH Sheu. "Determining the Optimal Production-Maintenance Policy with Inspection Errors: Using a Markov Chain." *Computers & Operations Research* 30.1 (2003): 1-17.

Xiang, Yisha, et al. "Joint Production and Maintenance Planning with Machine Deterioration and Random Yield." *International Journal of Production Research* 52.6 (2014): 1644-57.

Yan, Hong-Sen, and Hong-Bing Yang. "Deadlock-Free Scheduling of Knowledgeable Manufacturing Cell with Multiple Machines and Products." *Proceedings of the Institution of Mechanical Engineers Part B-Journal of Engineering Manufacture* 229.10 (2015): 1834-47.

Yao, Dacheng. "Joint Pricing and Inventory Control for a Stochastic Inventory System with Brownian Motion Demand." *IISE Transactions* 49.12 (2017): 1101-11.

Yeh, Ruey Huei, Wen-Tsung Ho, and Sheng-Tsaing Tseng. "Optimal Production Run Length for Products Sold with Warranty." *European Journal of Operational Research* 120.3 (2000): 575-82.

Yeung, Wing-Kwan, Tsan-Ming Choi, and T. C. E. Cheng. "Supply Chain Scheduling and Coordination with Dual Delivery Modes and Inventory Storage Cost." *International Journal of Production Economics* 132.2 (2011): 223-9.

Yin, K. K., G. G. Yin, and H. Liu. "Stochastic Modeling for Inventory and Production Planning in the Paper Industry." *AIChE Journal* 50.11 (2004): 2877-90.

Yu, Jian, and Chunmiao Mao. "The Model of Production Run Time for Products Sold with FMR Warranty in the Imperfect Production System". *2010 International Conference on Management and Service Science.*

Zadeh, Amir Hassan, Ramesh Sharda, and Narges Kasiri. "Inventory Record Inaccuracy due to Theft in Production-Inventory Systems." *International Journal of Advanced Manufacturing Technology* 83.1-4 (2016): 623-31.

Zhang, Tao, et al. "Multi-Level Inventory Matching and Order Planning Under the Hybrid make-to-Order/Make-to-Stock Production Environment for Steel Plants Via Particle Swarm Optimization." *Computers & Industrial Engineering* 87 (2015): 238-49.

APPENDIX

Appendix A

Table XXXIX. MC Model One First Scenario and First Variation

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
0	600	400	1.00	0.4113	95.977	0.1916	1.1977	0.6947	1.6450
1	600	400	0.6713	0.2761	96.090	0.1914	1.1965	0.6939	1.1042
2	600	400	0.4530	0.1863	96.243	0.1912	1.1947	0.6929	0.7452
3	600	400	0.3073	0.1264	96.394	0.1909	1.1931	0.6920	0.5055
Expected values					96.110	0.1914	1.1962	0.6938	
Production and Inventory Model					95.966	0.1917	1.1979	0.6948	
Difference					0.145	0.0003	0.0016	0.0010	

Table XL. MC Model One First Scenario and Second Variation

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
0	600	400	1.00	0.4096	95.989	0.1916	1.1976	0.6946	1.6385
1	600	400	0.6730	0.2757	96.164	0.1913	1.1956	0.6934	1.1027
2	600	400	0.4562	0.1869	96.373	0.1909	1.1932	0.6920	0.7475
3	600	400	0.3120	0.1278	96.624	0.1905	1.1904	0.6904	0.5112
Expected values					96.190	0.1912	1.1953	0.6933	
Production and Inventory Model					96.044	0.1915	1.1970	0.6943	
Difference					0.145	0.0003	0.0017	0.0010	

Appendix B

Table XLI. MC Model One Second Scenario and Duration (t)

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
0	1200	400	1.00	0.5481	135.621	0.0678	0.4238	0.9831	2.1924
1	950	400	0.4273	0.2342	126.449	0.0919	0.5741	0.9157	0.9367
2	750	400	0.2339	0.1282	113.638	0.1295	0.8093	0.8214	0.5128
3	600	400	0.1634	0.0895	96.334	0.1910	1.1939	0.6925	0.3582
Expected values					127.138	0.0924	0.5774	0.9206	
Production and Inventory Model					129.953	0.0824	0.5152	0.6305	
Difference					2.8150	0.0099	0.0621	0.2901	

Table XLII. MC Model One Second Scenario and Optimal Selected Production Rates (*t*-*x*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	750	400	1.00	0.5489	113.470	0.1299	0.8118	0.8240	2.1957
1	950	400	0.4273	0.2345	126.470	0.0892	0.5574	0.8890	0.9381
2	600	400	0.2924	0.1605	96.027	0.1932	1.2076	0.7004	0.6419
3	1200	400	0.1021	0.0560	135.709	0.0686	0.4286	0.9943	0.2242
Expected values					114.966	0.1271	0.7942	0.8290	
Production and Inventory Model					117.338	0.1178	0.7365	0.8502	
Difference					2.3720	0.0092	0.0577	0.0212	

Table XLIII. MC Model One Second Scenario and Optimal Production Rates (*t*-*x*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	650	400	1.00	0.3839	103.015	0.1648	1.0300	0.7467	1.5357
1	600	400	0.6765	0.2597	95.966	0.1917	1.1978	0.6947	1.0389
2	550	400	0.5050	0.1939	86.893	0.2309	1.4433	0.6279	0.7755
3	500	400	0.4233	0.1625	74.619	0.2959	1.8496	0.5364	0.6500
Expected values					93.445	0.2059	1.2869	0.6760	
Production and Invenotry Model					94.884	0.1960	1.2253	0.4996	
Difference					1.4394	0.0099	0.0616	0.1764	

Table XLIV. MC Model One Second Scenario and Optimal Selected Production Rates (*t*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	750	400	1.00	0.5489	113.478	0.1301	0.8129	0.8251	2.1957
1	950	400	0.4273	0.2345	126.449	0.0917	0.5731	0.9140	0.9381
2	600	400	0.2924	0.1605	96.143	0.1907	1.1920	0.6914	0.6419
3	1200	400	0.1021	0.0560	136.044	0.0637	0.3978	0.9230	0.2242
Expected values					115.003	0.1271	0.7942	0.8300	
Production and Inventory Model					117.338	0.1178	0.7365	0.8502	
Difference					2.3350	0.0092	0.0577	0.0202	

Table XLV. MC Model One Second Scenario and Optimal Production Rates (*t*)

State (*i*)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	650	400	1.00	0.3839	103.025	0.1648	1.0299	0.7467	1.5357
1	600	400	0.6765	0.2597	96.033	0.1915	1.1971	0.6943	1.0389
2	550	400	0.5050	0.1939	87.031	0.2306	1.4415	0.6271	0.7755
3	500	400	0.4233	0.1625	74.914	0.2951	1.8443	0.5348	0.6500
Expected values					93.540	0.2057	1.2855	0.6755	
Production and Inventory Model					94.964	0.1959	1.2244	0.4988	
Difference					1.4240	0.0098	0.0610	0.1766	

Appendix C

Table XLVI. MC Model One Third Scenario and Duration (*t*)

State (*i*)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	1200	850	1.00	0.4361	130.796	0.1494	0.9337	0.4460	1.7446
1	1200	650	0.7188	0.3135	143.471	0.1042	0.6510	0.6390	1.2539
2	1200	500	0.3996	0.1743	141.970	0.0810	0.5060	0.8218	0.6971
3	1200	400	0.1745	0.0761	135.783	0.0677	0.4233	0.9820	0.3043
Expected values					137.096	0.1171	0.7317	0.6128	
Production and Inventory Model					142.275	0.1118	0.6989	0.2762	
Difference					5.1790	0.0052	0.0328	0.3365	

Table XLVII. MC Model One Third Scenario and Optimal Pair (*t*-*x*)

State (*i*)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	1200	850	1.00	0.6390	130.778	0.1494	0.9339	0.4461	2.5559
1	1200	400	0.3382	0.2161	135.640	0.0678	0.4237	0.9830	0.8645
2	1200	500	0.1447	0.0924	141.900	0.0810	0.5063	0.8222	0.3697
3	1200	650	0.0821	0.0525	143.552	0.1041	0.6506	0.6386	0.2098
Expected values					133.527	0.1231	0.7692	0.6070	
Production and Inventory Model					141.458	0.1154	0.7211	0.1381	
Difference					7.9310	0.0077	0.0482	0.4688	

Table XLVIII. MC Model One Third Scenario and Optimal Pair (*t*)

State (*i*)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	1200	850	1.00	0.6390	130.796	0.1494	0.9337	0.4460	2.5559
1	1200	400	0.3382	0.2161	135.667	0.0678	0.4236	0.9828	0.8645
2	1200	500	0.1447	0.0924	141.970	0.0810	0.5060	0.8218	0.3697
3	1200	650	0.0821	0.0525	143.754	0.1040	0.6498	0.6378	0.2098
Expected values					133.561	0.1231	0.7691	0.6068	
Production and Inventory Model					141.458	0.1154	0.7211	0.1381	
Difference					7.8970	0.0077	0.0480	0.4687	

Appendix D

Table XLIX. MC Model One Fourth Scenario Duration (*t*)

State (*i*)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	1200	850	1.00	0.3998	130.796	0.1494	0.9337	0.4460	1.5992
1	950	650	0.6943	0.2776	119.173	0.1584	0.9901	0.5301	1.1103
2	750	500	0.4751	0.1899	107.494	0.1711	1.0696	0.6204	0.7597
3	600	400	0.3318	0.1327	96.334	0.1910	1.1939	0.6925	0.5307
Expected values					118.572	0.1616	1.0097	0.5352	
Production and Inventory Model					119.228	0.1602	1.0010	0.5164	
Difference					0.656	0.0014	0.0087	0.0187	

Table L. MC Model One Fourth Scenario Optimal Pair (*t-x*)

State (*i*)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	600	400	1.00	0.3993	95.904	0.1918	1.1985	0.6951	1.5974
1	750	500	0.6765	0.2701	107.294	0.1714	1.0714	0.6214	1.0806
2	950	650	0.4751	0.1897	119.157	0.1584	0.9901	0.5301	0.7589
3	1200	850	0.3526	0.1408	131.149	0.1490	0.9314	0.4449	0.5632
Expected values					108.355	0.1739	1.0870	0.6087	
Production and Inventory Model					109.119	0.1718	1.0738	0.5874	
Difference					0.7636	0.0021	0.0132	0.0213	

Table LI. MC Model One Fourth Scenario Optimal Selected Production Rates (*t*-*x*)

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	950	850	1.00	0.3455	78.592	0.3141	1.9630	0.2679	1.3820
1	750	650	0.8794	0.3038	77.494	0.3086	1.9289	0.3442	1.2154
2	600	500	0.7522	0.2599	76.055	0.3024	1.8900	0.4385	1.0395
3	1200	400	0.2627	0.0908	135.699	0.0678	0.4235	0.9826	0.3631
Expected values					82.782	0.2870	1.7939	0.4003	
Production and Inventory Model					95.194	0.2321	1.4507	0.4186	
Difference					12.412	0.0549	0.3433	0.0183	

Table LII. MC Model One Fourth Scenario Optimal Production Rates (*t*-*x*)

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	950	850	1.00	0.3066	78.592	0.3141	1.9630	0.2679	1.2262
1	750	650	0.8794	0.2696	77.494	0.3086	1.9289	0.3442	1.0784
2	600	500	0.7522	0.2306	76.055	0.3024	1.8900	0.4385	0.9223
3	500	400	0.6305	0.1933	74.619	0.2959	1.8496	0.5364	0.7731
Expected values					76.943	0.3064	1.9150	0.3797	
Production and Inventory Model					77.370	0.3077	1.9233	0.3550	
Difference					0.427	-0.001	-0.008	0.0247	

Table LIII. MC Model One Fourth Scenario Optimal Pair (*t*)

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	600	400	1.00	0.3993	95.915	0.1917	1.1984	0.6951	1.5974
1	750	500	0.6765	0.2701	107.371	0.1713	1.0707	0.6210	1.0806
2	950	650	0.4751	0.1897	119.322	0.1582	0.9890	0.5295	0.7589
3	1200	850	0.3526	0.1408	131.511	0.1487	0.9292	0.4438	0.5632
Expected values					108.462	0.1738	1.0863	0.6083	
Production and Inventory Model					109.119	0.1718	1.0738	0.5874	
Difference					0.7636	0.0020	0.0125	0.0209	

Table LIV. MC Model One Fourth Scenario Optimal Selected Production Rates (t)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
0	**950**	**850**	**1.00**	**0.3455**	**78.624**	**0.3140**	**1.9624**	**0.2678**	**1.3820**
1	**750**	**650**	**0.8794**	**0.3038**	**77.651**	**0.3082**	**1.9261**	**0.3437**	**1.2154**
2	**600**	**500**	**0.7522**	**0.2599**	**76.266**	**0.3018**	**1.8862**	**0.4376**	**1.0395**
3	**1200**	**400**	**0.2627**	**0.0908**	**135.783**	**0.0677**	**0.4233**	**0.9820**	**0.3631**
Expected values					**82.904**	**0.2867**	**1.7919**	**0.3998**	
Production and Inventory Model					**95.194**	**0.2321**	**1.4507**	**0.4186**	
Difference					**12.290**	**0.0546**	**0.3412**	**0.0188**	

Table LV. MC Model One Fourth Scenario Optimal Production Rates (t)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
0	**950**	**850**	**1.00**	**0.3066**	**78.624**	**0.3140**	**1.9624**	**0.2678**	**1.2262**
1	**750**	**650**	**0.8794**	**0.2696**	**77.651**	**0.3082**	**1.9261**	**0.3437**	**1.0784**
2	**600**	**500**	**0.7522**	**0.2306**	**76.266**	**0.3018**	**1.8862**	**0.4376**	**0.9223**
3	**500**	**400**	**0.6305**	**0.1933**	**74.914**	**0.2951**	**1.8443**	**0.5348**	**0.7731**
Expected values					**77.101**	**0.3060**	**1.9122**	**0.3790**	
Production and Inventory Model					**77.370**	**0.3077**	**1.9233**	**0.3550**	
Difference					**0.269**	**-0.002**	**-0.011**	**0.0240**	

Appendix E

Table LVI. MC Model Two First Scenario and First Variation

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
0	**600**	**400**	**1.00**	**0.2472**	**95.977**	**0.1916**	**1.1977**	**0.6947**	**0.9888**
1	**600**	**400**	**1.0050**	**0.2484**	**96.090**	**0.1914**	**1.1965**	**0.6940**	**0.9937**
2	**600**	**400**	**1.0138**	**0.2506**	**96.243**	**0.1912**	**1.1948**	**0.6930**	**1.0024**
3	**600**	**400**	**1.0267**	**0.2538**	**96.394**	**0.1909**	**1.1932**	**0.6920**	**1.0151**
Expected values					**96.178**	**0.1913**	**1.1955**	**0.6934**	
Production and Inventory Model					**95.988**	**0.1916**	**1.1976**	**0.6946**	
Difference					**0.189**	**0.0003**	**0.0021**	**0.0012**	

Table LVII. MC Model Two First Scenario and Second Variation

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	600	400	1.00	0.2461	95.989	0.1916	1.1976	0.6946	0.9843
1	600	400	1.0068	0.2478	96.164	0.1913	1.1956	0.6935	0.9910
2	600	400	1.0190	0.2508	96.373	0.1909	1.1933	0.6921	1.0030
3	600	400	1.0380	0.2554	96.624	0.1905	1.1906	0.6905	1.0217
Expected values					96.291	0.1911	1.1942	0.6927	
Production and Inventory Model					96.096	0.1914	1.1964	0.6939	
Difference					0.1951	0.0004	0.0022	0.0013	

Appendix F

Table LVIII. MC Model Two Second Scenario and Duration (t)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	1200	400	1.00	0.2484	135.621	0.0678	0.4238	0.9831	0.9937
1	950	400	1.0018	0.2489	126.449	0.0919	0.5743	0.9160	0.9955
2	750	400	1.0062	0.2500	113.638	0.1296	0.8100	0.8222	0.9999
3	600	400	1.0175	0.2528	96.335	0.1914	1.1965	0.6940	1.0110
Expected values					117.914	0.1205	0.7531	0.8531	
Production and Inventory Model					122.417	0.1032	0.6449	0.8898	
Difference					4.5030	0.0173	0.1082	0.0367	

Table LIX. MC Model Two Second Scenario Optimal Selected Production Rates (t-x)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	950	400	1.00	0.2492	126.382	0.0919	0.5744	0.9162	0.9969
1	1200	400	1.0007	0.2494	135.640	0.0678	0.4237	0.9830	0.9976
2	750	400	1.0030	0.2500	113.553	0.1296	0.8102	0.8224	0.9999
3	600	400	1.0088	0.2514	96.124	0.1916	1.1975	0.6945	1.0057
Expected values					117.876	0.1204	0.7524	0.8537	
Production and Inventory Model					122.399	0.1031	0.6444	0.8865	
Difference					4.5230	0.0173	0.1080	0.0328	

Table LX. MC Model Two Second Scenario Optimal Production Rates (*t*-*x*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	650	400	1.00	0.2486	103.015	0.1648	1.0300	0.7467	0.9945
1	600	400	1.0018	0.2491	95.966	0.1917	1.1981	0.6949	0.9964
2	550	400	1.0057	0.2501	86.893	0.2311	1.4444	0.6283	1.0002
3	500	400	1.0144	0.2522	74.620	0.2964	1.8525	0.5372	1.0088
Expected values					90.066	0.2213	1.3829	0.6514	
Production and Inventory Model					91.697	0.2094	1.3088	0.6631	
Difference					1.6309	0.0119	0.0741	0.0118	

Table LXI. MC Model Two Second Scenario Optimal Selected Production Rates (*t*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	1200	400	1.00	0.248498	126.388	0.0919	0.5744	1.3326	0.9940
1	950	400	1.0014	0.248836	135.667	0.0678	0.4237	0.6757	0.9953
2	750	400	1.0058	0.249937	113.638	0.1296	0.8100	0.8222	0.9997
3	600	400	1.0170	0.252729	96.335	0.1914	1.1965	0.6940	1.0109
Expected values					117.915	0.1205	0.7530	0.8802	
Production and Inventory Model					122.424	0.1032	0.6448	0.8385	
Difference					4.5090	0.0173	0.1082	0.0417	

Table LXII. MC Model Two Second Scenario Optimal Production Rates (*t*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	650	400	1.00	0.24742	103.025	0.1648	1.0299	0.7467	0.9897
1	600	400	1.0035	0.248293	96.034	0.1916	1.1976	0.6946	0.9932
2	550	400	1.0109	0.250123	87.031	0.2310	1.4435	0.6279	1.0005
3	500	400	1.0273	0.254164	74.915	0.2959	1.8492	0.5363	1.0167
Expected values					90.144	0.2213	1.3832	0.6506	
Production and Inventory Model					91.761	0.2094	1.3088	0.5709	
Difference					1.6167	0.0119	0.0744	0.0797	

Appendix G

Table LXIII. MC Model Two Third Scenario and Duration (*t*)

State (*i*)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	**1200**	**850**	**1.00**	**0.2490**	**130.796**	**0.1494**	**0.9337**	**0.4460**	**0.9960**
1	**1200**	**650**	**1.0020**	**0.2495**	**143.471**	**0.1041**	**0.6509**	**0.6389**	**0.9980**
2	**1200**	**500**	**1.0049**	**0.2502**	**141.970**	**0.0810**	**0.5060**	**0.8218**	**1.0009**
3	**1200**	**400**	**1.0092**	**0.2513**	**135.783**	**0.0677**	**0.4233**	**0.9821**	**1.0051**
Expected values					**138.008**	**0.1005**	**0.6279**	**0.7229**	
Production and Inventory Model					**144.010**	**0.0957**	**0.5981**	**0.9173**	
Difference					**6.0020**	**0.0048**	**0.0298**	**0.1944**	

Appendix H

Table LXIV. MC Model Two Fourth Scenario and Duration (*t*)

State (*i*)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	1200	850	1.00	0.2480	130.796	0.1494	0.9337	0.4460	0.9922
1	950	650	1.0030	0.2488	119.173	0.1585	0.9904	0.5303	0.9951
2	750	500	1.0087	0.2502	107.494	0.1713	1.0708	0.6211	1.0008
3	600	400	1.0200	0.2530	96.335	0.1913	1.1959	0.6936	1.0120
Expected values					113.356	0.1678	1.0485	0.5734	
Production and Inventory Model					114.094	0.1658	1.0362	0.5512	
Difference					0.7380	0.0020	0.0123	0.0223	

Table LXV. MC Model Two Fourth Scenario Optimal Selected Production Rates (*t-x*)

State (*i*)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
0	950	850	1.00	0.2487	78.59	0.3141	1.9630	0.2679	0.9949
1	750	650	1.0028	0.2494	77.49	0.3085	1.9280	0.3441	0.9977
2	600	500	1.0077	0.2507	76.06	0.3019	1.8870	0.4378	1.0026
3	1200	400	1.0099	0.2512	135.70	0.0677	0.4232	0.9819	1.0048
Expected values					92.027	0.2478	1.5485	0.5088	
Production and Inventory Model					114.285	0.1653	1.0333	0.5516	
Difference					22.258	0.0824	0.5152	0.0428	

Table LXVI. MC Model Two Fourth Scenario Optimal Production Rates (*t*-*x*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
0	950	850	1.00	0.2483	78.59	0.3141	1.9630	0.2679	0.9933
1	750	650	1.0028	0.2490	77.49	0.3086	1.9290	0.3442	0.9961
2	600	500	1.0078	0.2503	76.05	0.3024	1.8903	0.4385	1.0010
3	500	400	1.0164	0.2524	74.62	0.2960	1.8500	0.5365	1.0096
Expected values					76.681	0.3052	1.9078	0.3974	
Production and Inventory Model					77.132	0.3066	1.9164	0.3711	
Difference					0.451	0.0014	0.0086	0.0263	

Table LXVII. MC Model Two Fourth Scenario Optimal Selected Production Rates (*t*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
0	950	850	1.00	0.2476	78.624	0.3140	1.9624	0.2678	0.9905
1	750	650	1.0053	0.2489	77.651	0.3079	1.9247	0.3435	0.9957
2	1200	500	1.0145	0.2512	76.267	0.3010	1.8811	3.0550	1.0048
3	600	400	1.0188	0.2523	135.783	0.0676	0.4227	0.2452	1.0091
Expected values					92.210	0.2471	1.5442	0.9811	
Production and Inventory Model					114.375	0.1650	1.0312	0.5526	
Difference					22.165	0.0821	0.5129	0.4285	

Table LXVIII. MC Model Two Fourth Scenario Optimal Production Rates (*t*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
0	950	850	1.00	0.2469	78.624	0.3140	1.9624	0.2678	0.9875
1	750	650	1.0053	0.2482	77.651	0.3082	1.9262	0.3438	0.9927
2	600	500	1.0145	0.2505	76.266	0.3019	1.8867	0.4377	1.0018
3	500	400	1.0309	0.2545	74.914	0.2952	1.8449	0.5350	1.0180
Expected values					76.848	0.3047	1.9046	0.3972	
Production and Inventory Model					77.345	0.3060	1.9123	0.3708	
Difference					0.497	-0.001	-0.008	0.0264	

Appendix I

Table LXIX. Scheduling with MC Model One First Scenario Base (*t*-*x*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	750	400	1.00	0.2627	113.470	0.1297	0.8104	0.8225	2.1020
A1	750	400	0.5412	0.1422	113.513	0.1296	0.8101	0.8222	1.1376
A2	750	400	0.2962	0.0778	113.553	0.1296	0.8098	0.8220	0.6227
A3	750	400	0.1655	0.0435	113.623	0.1295	0.8094	0.8215	0.3480
B0	600	400	1.00	0.2102	95.904	0.1918	1.1985	0.6951	1.6816
B1	600	400	0.6765	0.1422	95.966	0.1917	1.1978	0.6947	1.1376
B2	750	400	0.3703	0.0778	113.553	0.1296	0.8098	0.8220	0.6227
B3	750	400	0.2069	0.0435	113.623	0.1295	0.8094	0.8215	0.3480
Expected values					107.321	0.1515	0.9468	0.7774	
Production and Inventory Model					108.486	0.1459	0.9119	0.7858	
Difference					1.165	0.0056	0.0349	0.0084	

Table LXX. Scheduling with MC Model One First Scenario (*t*-*x*) First Variation

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	750	400	1.00	0.2627	113.521	0.1296	0.8100	0.8222	2.1020
A1	750	400	0.5412	0.1422	113.601	0.1295	0.8095	0.8217	1.1376
A2	750	400	0.2962	0.0778	113.708	0.1294	0.8088	0.8210	0.6227
A3	750	400	0.1655	0.0435	113.815	0.1293	0.8081	0.8203	0.3480
B0	600	400	1.00	0.2102	95.977	0.1916	1.1977	0.6947	1.6816
B1	600	400	0.6765	0.1422	96.090	0.1914	1.1965	0.6940	1.1376
B2	750	400	0.3703	0.0778	113.708	0.1294	0.8088	0.8210	0.6227
B3	750	400	0.2069	0.0435	113.815	0.1293	0.8081	0.8203	0.3480
Expected values					107.421	0.1514	0.9461	0.7767	
Production and Inventory Model					108.535	0.1458	0.9116	0.7855	
Difference					1.114	0.0055	0.0345	0.0088	

Table LXXI. Scheduling with MC Model One First Scenario (*t*-*x*) Second Variation

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	750	400	1.00	0.2627	113.529	0.1296	0.8100	0.8221	2.1020
A1	750	400	0.5412	0.1422	113.652	0.1295	0.8092	0.8213	1.1376
A2	750	400	0.2962	0.0778	113.799	0.1293	0.8082	0.8203	0.6227
A3	750	400	0.1655	0.0435	113.976	0.1291	0.8071	0.8192	0.3480
B0	600	400	1.00	0.2102	95.989	0.1916	1.1976	0.6946	1.6816
B1	600	400	0.6765	0.1422	96.164	0.1913	1.1957	0.6935	1.1376
B2	750	400	0.3703	0.0778	113.799	0.1293	0.8082	0.8203	0.6227
B3	750	400	0.2069	0.0435	113.976	0.1291	0.8071	0.8192	0.3480
Expected values					107.471	0.1513	0.9457	0.7764	
Production and Inventory Model					108.496	0.1459	0.9118	0.7857	
Difference					1.025	0.0054	0.0338	0.0094	

Table LXXII. Scheduling with MC Model One Second Scenario Base (*t*-*x*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1450	400	1.00	0.3136	141.341	0.0538	0.3365	1.0247	2.5092
A1	1150	400	0.3529	0.1107	134.160	0.0715	0.4470	0.9722	0.8856
A2	750	400	0.1932	0.0606	113.553	0.1296	0.8098	0.8220	0.4848
A3	600	400	0.1350	0.0423	96.124	0.1914	1.1960	0.6937	0.3386
B0	1200	400	1.00	0.2591	135.617	0.0678	0.4238	0.9832	2.0728
B1	950	400	0.4273	0.1107	126.413	0.0919	0.5743	0.9160	0.8856
B2	750	400	0.2339	0.0606	113.553	0.1296	0.8098	0.8220	0.4848
B3	600	400	0.1634	0.0423	96.124	0.1914	1.1960	0.6937	0.3386
Expected values					130.215	0.0844	0.5278	0.9435	
Production and Inventory Model					133.836	0.0723	0.4521	0.9700	
Difference					3.6210	0.0121	0.0757	0.0265	

Table LXXIII. Scheduling with MC Model One Second Scenario Optimal Production Rates (*t*-*x*)

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1450	400	1.00	0.2924	141.341	0.0538	0.3365	1.0247	2.3393
A1	1150	400	0.3529	0.1032	134.160	0.0715	0.4470	0.9722	0.8256
A2	600	400	0.2415	0.0706	96.023	0.1915	1.1972	0.6944	0.5650
A3	500	400	0.2024	0.0592	74.619	0.2959	1.8496	0.5364	0.4735
B0	1200	400	1.00	0.2416	135.617	0.0678	0.4238	0.9832	1.9324
B1	950	400	0.4273	0.1032	126.413	0.0919	0.5743	0.9160	0.8256
B2	600	400	0.2924	0.0706	96.023	0.1915	1.1972	0.6944	0.5650
B3	500	400	0.2450	0.0592	74.619	0.2959	1.8496	0.5364	0.4735
Expected values					123.377	0.1111	0.6942	0.8936	
Production and Inventory Model					131.623	0.0780	0.4876	0.9539	
Difference					8.246	0.0331	0.2066	0.0603	

Table LXXIV. Scheduling with MC Model One Second Scenario (*t*)

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1450	400	1.00	0.3136	141.344	0.0538	0.3365	1.0246	2.5092
A1	1150	400	0.3529	0.1107	134.188	0.0715	0.4469	0.9721	0.8856
A2	750	400	0.1932	0.0606	113.638	0.1295	0.8093	0.8214	0.4848
A3	600	400	0.1350	0.0423	96.334	0.1910	1.1939	0.6925	0.3386
B0	1200	400	1.00	0.2591	135.621	0.0678	0.4238	0.9831	2.0728
B1	950	400	0.4273	0.1107	126.449	0.0919	0.5741	0.9157	0.8856
B2	750	400	0.2339	0.0606	113.638	0.1295	0.8093	0.8214	0.4848
B3	600	400	0.1634	0.0423	96.334	0.1910	1.1939	0.6925	0.3386
Expected values					130.252	0.0844	0.5275	0.9433	
Production and Inventory Model					133.858	0.0723	0.4520	0.9698	
Difference					3.6060	0.0121	0.0755	0.0266	

Table LXXV. Scheduling with MC Model One Second Scenario Optimal Production Rates (*t*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
A0	1450	400	1.00	0.2924	141.344	0.0538	0.3365	1.0246	2.3393
A1	1150	400	0.3529	0.1032	134.188	0.0715	0.4469	0.9721	0.8256
A2	600	400	0.2415	0.0706	96.142	0.1914	1.1960	0.6937	0.5650
A3	500	400	0.2024	0.0592	74.914	0.2951	1.8443	0.5348	0.4735
B0	1200	400	1.00	0.2416	135.621	0.0678	0.4238	0.9831	1.9324
B1	950	400	0.4273	0.1032	126.449	0.0919	0.5741	0.9157	0.8256
B2	600	400	0.2924	0.0706	96.142	0.1914	1.1960	0.6937	0.5650
B3	500	400	0.2450	0.0592	74.914	0.2951	1.8443	0.5348	0.4735
Expected values					123.437	0.1109	0.6934	0.8932	
Production and Inventory Model					131.651	0.0780	0.4875	0.9537	
Difference					8.214	0.0329	0.2059	0.0605	

Table LXXVI. Scheduling with MC Model One Third Scenario (*t*-*x*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
A0	1200	1050	1.00	0.2002	95.180	0.2536	1.5851	0.2627	1.6015
A1	1200	950	0.8033	0.1608	117.002	0.1867	1.1668	0.3562	1.2865
A2	1200	500	0.3435	0.0688	141.900	0.0810	0.5063	0.8222	0.5502
A3	1200	400	0.1200	0.0240	135.699	0.0678	0.4235	0.9826	0.1922
B0	1200	850	1.00	0.2926	130.778	0.1494	0.9339	0.4461	2.3407
B1	1200	650	0.5496	0.1608	143.406	0.1042	0.6512	0.6392	1.2865
B2	1200	500	0.2351	0.0688	141.900	0.0810	0.5063	0.8222	0.5502
B3	1200	400	0.0821	0.0240	135.699	0.0678	0.4235	0.9826	0.1922
Expected values					125.233	0.1557	0.9729	0.5035	
Production and Inventory Model					135.339	0.1366	0.8538	0.4874	
Difference					10.106	0.0191	0.1191	0.0160	

Table LXXVII. Scheduling with MC Model One Third Scenario (t)

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
A0	1200	1050	1.00	0.2002	95.215	0.2535	1.5847	0.2626	1.6015
A1	1200	950	0.8033	0.1608	117.158	0.1865	1.1655	0.3558	1.2865
A2	1200	500	0.3435	0.0688	141.970	0.0810	0.5060	0.8218	0.5502
A3	1200	400	0.1200	0.0240	135.783	0.0677	0.4233	0.9820	0.1922
B0	1200	850	1.00	0.2926	130.796	0.1494	0.9337	0.4460	2.3407
B1	1200	650	0.5496	0.1608	143.471	0.1042	0.6510	0.6390	1.2865
B2	1200	500	0.2351	0.0688	141.970	0.0810	0.5060	0.8218	0.5502
B3	1200	400	0.0821	0.0240	135.783	0.0677	0.4233	0.9820	0.1922
Expected values					125.295	0.1556	0.9725	0.5033	
Production and Inventory Model					135.419	0.1365	0.8533	0.4872	
Difference					10.124	0.0191	0.1191	0.0161	

Table LXXVIII. Scheduling with MC Model One Fourth Scenario Optimal Production Rates (t-x)

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
A0	1450	1100	1.00	0.1684	135.346	0.1546	0.9664	0.3567	1.3471
A1	1150	850	0.7500	0.1263	123.799	0.1647	1.0295	0.4215	1.0103
A2	600	500	0.6415	0.1080	76.055	0.3024	1.8900	0.4385	0.8642
A3	500	400	0.5377	0.0905	74.619	0.2959	1.8496	0.5364	0.7243
B0	1200	850	1.00	0.1819	130.778	0.1494	0.9339	0.4461	1.4552
B1	950	650	0.6943	0.1263	119.080	0.1585	0.9907	0.5304	1.0103
B2	600	500	0.5938	0.1080	76.055	0.3024	1.8900	0.4385	0.8642
B3	500	400	0.4978	0.0905	74.619	0.2959	1.8496	0.5364	0.7243
Expected values					107.196	0.2130	1.3310	0.4533	
Production and Inventory Model					111.045	0.1860	1.1628	0.4525	
Difference					3.849	0.0269	0.1682	0.0008	

Table LXXIX. Scheduling with MC Model One Fourth Scenario (t)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1450	1100	1.00	0.1878	135.371	0.1546	0.9663	0.3566	1.5027
A1	1150	850	0.7500	0.1409	123.924	0.1646	1.0286	0.4211	1.1271
A2	750	500	0.5132	0.0964	107.494	0.1711	1.0696	0.6204	0.7712
A3	600	400	0.3585	0.0673	96.334	0.1910	1.1939	0.6925	0.5387
B0	1200	850	1.00	0.2029	130.796	0.1494	0.9337	0.4460	1.6234
B1	950	650	0.6943	0.1409	119.173	0.1584	0.9901	0.5301	1.1271
B2	750	500	0.4751	0.0964	107.494	0.1711	1.0696	0.6204	0.7712
B3	600	400	0.3318	0.0673	96.334	0.1910	1.1939	0.6925	0.5387
Expected values					119.916	0.1636	1.0224	0.5044	
Production and Inventory Model					120.962	0.1629	1.0180	0.4726	
Difference					1.046	0.0007	0.0043	0.0318	

Table LXXX. Scheduling with MC Model One Fourth Scenario Optimal Production Rates (t)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1450	1100	1.00	0.1684	135.371	0.1546	0.9663	0.3566	1.3471
A1	1150	850	0.7500	0.1263	123.924	0.1646	1.0286	0.4211	1.0103
A2	600	500	0.6415	0.1080	76.266	0.3018	1.8862	0.4376	0.8642
A3	500	400	0.5377	0.0905	74.914	0.2951	1.8443	0.5348	0.7243
B0	1200	850	1.00	0.1819	130.796	0.1494	0.9337	0.4460	1.4552
B1	950	650	0.6943	0.1263	119.173	0.1584	0.9901	0.5301	1.0103
B2	600	500	0.5938	0.1080	76.266	0.3018	1.8862	0.4376	0.8642
B3	500	400	0.4978	0.0905	74.914	0.2951	1.8443	0.5348	0.7243
Expected values					107.331	0.2126	1.3290	0.4527	
Production and Inventory Model					111.045	0.1860	1.1628	0.4525	
Difference					3.714	0.0266	0.1662	0.0002	

Appendix J

Table LXXXI. Scheduling with MC Model Two First Scenario (*t-x*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
A0	750	400	1.00	0.1118	113.470	0.1297	0.8104	0.8225	0.8941
A1	750	400	1.0013	0.1119	113.513	0.1296	0.8101	0.8223	0.8952
A2	750	400	1.0039	0.1122	114.861	0.1508	0.9425	0.9566	0.8976
A3	750	400	1.0713	0.1197	113.654	0.1326	0.8285	0.8409	0.9578
B0	600	400	1.00	0.1397	95.904	0.1918	1.1985	0.6951	1.1179
B1	600	400	1.0018	0.1400	95.966	0.1916	1.1976	0.6946	1.1200
B2	750	400	0.9451	0.1321	114.502	0.1139	0.7117	0.7224	1.0566
B3	750	400	0.9488	0.1326	113.649	0.1267	0.7921	0.8040	1.0608
Expected values					108.908	0.1473	0.9203	0.7883	
Production and Inventory Model					109.637	0.1422	0.8885	0.7937	
Difference					0.729	0.0051	0.0319	0.0053	

Table LXXXII. Scheduling with MC Model Two First Scenario First Variation (*t-x*)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
A0	750	400	1.00	0.1113	113.521	0.1296	0.8100	0.8222	0.8907
A1	750	400	1.0035	0.1117	113.601	0.1295	0.8096	0.8218	0.8938
A2	750	400	1.0106	0.1125	114.990	0.1504	0.9398	0.9539	0.9001
A3	750	400	1.0830	0.1206	113.845	0.1323	0.8268	0.8392	0.9646
B0	600	400	1.00	0.1388	95.977	0.1916	1.1977	0.6947	1.1108
B1	600	400	1.0050	0.1395	96.090	0.1913	1.1958	0.6936	1.1163
B2	750	400	0.9518	0.1322	114.651	0.1138	0.7111	0.7218	1.0573
B3	750	400	0.9601	0.1333	113.841	0.1266	0.7911	0.8029	1.0665
Expected values					109.058	0.1470	0.9187	0.7875	
Production and Inventory Model					109.613	0.1421	0.8884	0.7941	
Difference					0.555	0.0048	0.0303	0.0066	

Table LXXXIII. Scheduling with MC Model Two First Scenario Second Variation (t-x)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	750	400	1.00	0.1095	115.562	0.1328	0.9904	0.9828	0.8759
A1	750	400	1.0048	0.1100	113.652	0.1295	0.8068	0.8192	0.8801
A2	750	400	1.0147	0.1111	115.321	0.1520	0.9520	0.9660	0.8888
A3	750	400	1.0881	0.1191	114.120	0.1327	0.8511	0.8608	0.9531
B0	600	400	1.00	0.1398	96.061	0.1919	1.1483	0.6701	1.1187
B1	600	400	1.0066	0.1408	96.276	0.1916	1.1347	0.6631	1.1260
B2	750	400	0.9583	0.1340	114.861	0.1133	0.7046	0.7156	1.0720
B3	750	400	0.9704	0.1357	114.021	0.1261	0.7841	0.7964	1.0856
Expected values					109.409	0.1476	0.9254	0.7986	
Production and Inventory Model					109.624	0.1422	0.8887	0.7936	
Difference					0.215	0.0054	0.0367	0.0050	

Table LXXXIV. Scheduling with MC Model Two Second Scenario (t-x)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1450	400	1.00	0.1208	141.341	0.0538	0.3365	1.0247	0.9668
A1	1150	400	1.0007	0.1209	134.160	0.0715	0.4471	0.9724	0.9675
A2	750	400	1.0030	0.1212	113.568	0.1317	0.8235	0.8358	0.9697
A3	600	400	0.9933	0.1200	96.288	0.2032	1.2677	0.7354	0.9603
B0	1200	400	1.00	0.1276	135.617	0.0678	0.4238	0.9832	1.0209
B1	950	400	1.0009	0.1277	126.413	0.0919	0.5743	0.9161	1.0218
B2	750	400	1.0223	0.1305	113.573	0.1272	0.7948	0.8067	1.0436
B3	600	400	1.0280	0.1312	96.230	0.1825	1.1412	0.6618	1.0494
Expected values					119.521	0.1164	0.7275	0.8655	
Production and Inventory Model					125.353	0.0948	0.5927	0.9080	
Difference					5.832	0.0216	0.1348	0.0425	

Table LXXXV. Scheduling with MC Model Two Second Scenario Optimal Production Rates (*t*-*x*)

State (*i*)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
A0	1450	400	1.00	0.1193	141.341	0.0538	0.3365	1.0247	0.9547
A1	1150	400	1.0007	0.1194	134.160	0.0715	0.4471	0.9724	0.9554
A2	600	400	1.0039	0.1198	96.052	0.1868	1.1683	0.6775	0.9584
A3	500	400	0.9587	0.1144	75.410	0.3516	2.1291	0.6202	0.9153
B0	1200	400	1.00	0.1263	135.617	0.0678	0.4238	0.9832	1.0107
B1	950	400	1.0009	0.1265	126.413	0.0919	0.5744	0.9161	1.0116
B2	600	400	1.0812	0.1366	96.023	0.1911	1.1948	0.6929	1.0927
B3	500	400	1.0897	0.1377	74.999	0.2654	1.6744	0.4849	1.1013
Expected values					109.581	0.1604	0.9970	0.7920	
Production and Inventory Model					121.657	0.1052	0.6577	0.8811	
Difference					12.076	0.0552	0.3393	0.0891	

Table LXXXVI. Scheduling with MC Model Two Second Scenario (*t*)

State (*i*)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
A0	1450	400	1.00	0.1205	141.344	0.0538	0.3365	1.0246	0.9637
A1	1150	400	1.0014	0.1206	134.188	0.0715	0.4470	0.9723	0.9650
A2	750	400	1.0059	0.1212	113.654	0.1317	0.8232	0.8356	0.9694
A3	600	400	1.0014	0.1206	96.513	0.2030	1.2690	0.7360	0.9650
B0	1200	400	1.00	0.1272	135.621	0.0678	0.4238	0.9831	1.0178
B1	950	400	1.0018	0.1275	126.449	0.0919	0.5743	0.9159	1.0197
B2	750	400	1.0258	0.1305	113.657	0.1271	0.7946	0.8066	1.0441
B3	600	400	1.0368	0.1319	96.436	0.1824	1.1402	0.6613	1.0553
Expected values					119.554	0.1166	0.7285	0.8651	
Production and Inventory Model					125.367	0.0949	0.5932	0.9073	
					5.813	0.0216	0.1353	0.0422	

Table LXXXVII. Scheduling with MC Model Two Second Scenario Optimal Production Rates (t)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1450	400	1.00	0.1219	141.344	0.0538	0.3365	1.0246	0.9755
A1	1150	400	1.0014	0.1221	134.188	0.0715	0.4471	0.9724	0.9769
A2	600	400	1.0076	0.1229	96.157	0.1896	1.1742	0.6819	0.9829
A3	500	400	0.9948	0.1213	75.024	0.3359	1.8075	0.5359	0.9704
B0	1200	400	1.00	0.1246	135.621	0.0678	0.4238	0.9831	0.9971
B1	950	400	1.0018	0.1249	126.449	0.0919	0.5743	0.9161	0.9989
B2	600	400	1.0445	0.1302	96.143	0.1914	1.1922	0.6918	1.0415
B3	500	400	1.0598	0.1321	75.408	0.2711	1.6369	0.4770	1.0567
Expected values					109.706	0.1600	0.9551	0.7825	
Production and Inventory Model					121.839	0.1048	0.6552	0.8816	
Difference					12.133	0.0552	0.2999	0.0991	

Table LXXXVIII. Scheduling with MC Model Two Third Scenario (t-x)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i}+T_{4i}$	$\overline{ST}_i$
A0	1200	1050	1.00	0.1421	95.180	0.2536	1.5851	0.2627	1.1370
A1	1200	950	1.0007	0.1422	117.002	0.1865	1.1657	0.3558	1.1378
A2	1200	500	0.9966	0.1416	143.162	0.0709	0.4431	0.7196	1.1331
A3	1200	400	0.9531	0.1355	136.025	0.0632	0.3952	0.9169	1.0837
B0	1200	850	1.00	0.1075	130.778	0.1494	0.9340	0.4461	0.8596
B1	1200	650	1.0019	0.1077	143.406	0.1042	0.6513	0.6392	0.8612
B2	1200	500	1.0395	0.1117	144.064	0.0964	0.6028	0.9789	0.8936
B3	1200	400	1.0399	0.1117	136.067	0.0729	0.4559	1.0576	0.8940
Expected values					129.659	0.1274	0.7961	0.6583	
Production and Inventory Model					142.932	0.1080	0.6747	0.6166	
Difference					13.273	0.0194	0.1214	0.0417	

Table LXXXIX. Scheduling with MC Model Two Third Scenario (t)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
A0	1200	1050	1.00	0.1417	95.215	0.2535	1.5843	0.2625	1.1338
A1	1200	950	1.0023	0.1420	117.158	0.1862	1.1640	0.3553	1.1364
A2	1200	500	0.9992	0.1416	143.228	0.0709	0.4430	0.7194	1.1329
A3	1200	400	0.9577	0.1357	136.106	0.0632	0.3951	0.9166	1.0858
B0	1200	850	1.00	0.1073	130.796	0.1494	0.9338	0.4460	0.8585
B1	1200	650	1.0029	0.1076	143.471	0.1042	0.6510	0.6390	0.8610
B2	1200	500	1.0420	0.1118	144.105	0.0963	0.6018	0.9773	0.8946
B3	1200	400	1.0447	0.1121	136.146	0.0729	0.4554	1.0565	0.8969
Expected values					129.749	0.1272	0.7949	0.6583	
Production and Inventory Model					143.048	0.1078	0.6738	0.6167	
Difference					13.299	0.0194	0.1210	0.0417	

Table XC. Scheduling with MC Model Two Fourth Scenario Optimal Production Rates (t-x)

State (i)	P_i	D_i	A_i	$\overline{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
A0	1450	1100	1.00	0.1219	135.346	0.1546	0.9664	0.3567	0.9756
A1	1150	850	1.0016	0.1221	123.799	0.1648	1.0299	0.4216	0.9772
A2	600	500	1.0064	0.1227	76.071	0.2971	1.8501	0.4294	0.9819
A3	500	400	0.9657	0.1178	74.989	0.3317	2.0378	0.5924	0.9421
B0	1200	850	1.00	0.1254	130.778	0.1494	0.9339	0.4461	1.0030
B1	950	650	1.0016	0.1256	119.080	0.1586	0.9911	0.5306	1.0046
B2	600	500	1.0504	0.1317	76.056	0.3018	1.8820	0.4368	1.0536
B3	500	400	1.0588	0.1328	74.830	0.2749	1.7153	0.4975	1.0621
Expected values					101.095	0.2294	1.4278	0.4636	
Production and Inventory Model					105.143	0.1983	1.2393	0.4649	
Difference					4.048	0.0311	0.1885	0.0013	

Table XCI. Scheduling with MC Model Two Fourth Scenario (t)

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
A0	1450	1100	1.00	0.1212	135.371	0.1546	0.9668	0.3568	0.9697
A1	1150	850	1.0032	0.1216	123.924	0.1646	1.0286	0.4211	0.9729
A2	750	500	1.0094	0.1224	107.519	0.1749	1.0929	0.6339	0.9789
A3	600	400	1.0181	0.1234	96.372	0.1963	1.2279	0.7121	0.9873
B0	1200	850	1.00	0.1267	130.796	0.1493	0.9335	0.4459	1.0133
B1	950	650	1.0030	0.1270	119.173	0.1586	0.9899	0.5301	1.0164
B2	750	500	1.0116	0.1281	107.513	0.1679	1.0495	0.6087	1.0251
B3	600	400	1.0229	0.1296	96.362	0.1866	1.1657	0.6761	1.0365
Expected values					114.497	0.1691	1.0571	0.5493	
Production and Inventory Model					115.632	0.1680	1.0500	0.5118	
Difference					1.135	0.0011	0.0071	0.0375	

Table XCII. Scheduling with MC Model Two Fourth Scenario Optimal Production Rates (t)

State (i)	P_i	D_i	A_i	$\bar{P}_i$	TC_i	T_{1i}	T_{2i}	$T_{3i} + T_{4i}$	$\overline{ST}_i$
A0	1450	1100	1.00	0.1214	135.371	0.1546	0.9663	0.3566	0.9712
A1	1150	850	1.0031	0.1218	123.924	0.1647	1.0294	0.4214	0.9742
A2	600	500	1.0122	0.1229	76.276	0.2975	1.8564	0.4308	0.9830
A3	500	400	0.9836	0.1194	75.206	0.3292	2.0068	0.5840	0.9553
B0	1200	850	1.00	0.1248	130.796	0.1494	0.9337	0.4460	0.9985
B1	950	650	1.0030	0.1252	119.173	0.1585	0.9907	0.5304	1.0015
B2	600	500	1.0519	0.1313	76.267	0.3023	1.8785	0.4361	1.0503
B3	500	400	1.0679	0.1333	75.108	0.2751	1.7156	0.4977	1.0662
Expected values					101.142	0.2295	1.4262	0.4629	
Production and Inventory Model					105.004	0.1986	1.2412	0.4653	
Difference					3.862	0.0309	0.1850	0.0024	

Appendix K

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JOURNAL SUBMISSION:

Wael I. Al-Hajailan and David He, "Scheduling Hard Time Windows Between Two Production Processes Using a Markov Chain Model with Time Factor", *Production Planning and Control*.

Wael I. Al-Hajailan and David He, "Evaluating the Expected Total Cost for Imperfect Production Processes Using a Markovian Approach with the Time Factor", *Production and Manufacturing Research*.

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