Training, Feedback and Delay: Tradeoffs in Wireless Communication Systems

BY

XINGHAO GU
B.Eng. Central South University, 2012
M.S. University of Illinois at Chicago, 2014

THESIS

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Defense Committee:
Besma Smida, Chair and Advisor
Hulya Seferoglu,
Natasha Devroye,
Rashid Ansari,
Hamza Soury, Caterpillar Inc.

To my parents.

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CONTRIBUTIONS OF AUTHORS

A version of Chapter 2 and Chapter 3 have been published in (Gu et al., 2021). I was the primary author and major driver of the research. Dr. Hamza Soury and Prof. Besma Smida helped me with the development of main ideas and improvement of equation derivations.

A version of Chapter 4 represents a series of my own unpublished work as an extension of Chapter 3 to finite block-length packet. I anticipate the work will be continued after I graduate and will ultimately be published.

A version of Chapter 5 has been published in (Zhu et al., 2017). Dr. Hongyi Zhu was the primary investigator of this part of the work. I was the second author and responsible and developing parts of the ideas including packet structure, number of extra acknowledgements, deriving equations of number of acknowledgements and outage probability, and composing the introduction, preliminary, and simulation results of the manuscript.

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LIST OF ABBREVIATIONS

ARQ	Automatic Repeat Request
AWGN	Additive White Gaussian Noise
BS	Base-station
CI	Truncated Channel Inversion
CP	Constant Transmit Power
CSI	Channel State Information
CC-HARQ	Chase Combining Hybrid Automatic Repeat Request
DL	Downlink
FDD	Frequency Division Duplex
HARQ	Hybrid Automatic Repeat Request
IR-HARQ	Incremental Redundancy Hybrid Automatic Repeat Request
MMSE	Minimum Mean Square Error
NC	No CSI at the transmitter
NCed	Network Coded
SINR	Signal To Interference Plus Noise Ratio

LIST OF ABBREVIATIONS (Continued)

TDD	Time Division Duplex
UL	Uplink

SUMMARY

Most of the recent wireless communication applications require high speed and high rate service. However, less than 70% of the packet size is the actual data. In other words, the data efficiency of a wireless packet is relatively low. The objective of the research is to increase the packet efficiency by investigating the roles of various overhead. In two-way wireless networks, several types of overhead should be considered. For example, the overhead bits used for channel state estimation and its feedback, re-transmission overhead, and control bits are the main research topic this thesis wants to cover.

The first part of this thesis focuses on the analysis and optimization of the throughput in two-way wireless communication networks with infinite block-length. We consider a practical wireless packet capturing the tradeoff between channel estimation pilot bits, channel estimation feedback bits, and re-transmission request bits. We consider three transmitter protocols and receiver schemes, under the assumption of time-correlated fading channels. We evaluate the throughput performance of various protocols while optimizing the packet structure.

The second part of this thesis focuses on the analysis and optimization of the throughput in two-way wireless communication networks with finite block-length. We extend our framework to finite block-length scenarios. We consider two receiver schemes and derive closed form results of the throughput. We compare the difference between infinite block-length and finite block-length in terms of the tradeoffs of channel estimation bits and data bits.

The third part focuses on the network coded (NCed) hybrid-ARQ (HARQ) with packet efficiency considered. We investigate the various types of overhead needed in NCed HARQ systems, which includes the control and extra feedback information between the base station and users. We derive expressions of the outage probabilities and throughput by taking re-transmissions and extra acknowledgements into account. We also obtain the optimal number of users numerically to maximize the downlink and uplink throughput.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

In order to support low-latency, high data rates applications like remote surgery, smart factory automation, and virtual reality video streaming in mobile wireless networks, channel state information (CSI) and its feedback have played a critical role in improving spectrum efficiency. In addition, pushing the demand of high reliability, re-transmission is inevitable for reliable transmission in wireless communication networks due to a failed reception. In practical, a wireless contains bits that used for learning a channel, feedback of the channel state information, bits for requesting re-transmission and actual data. These overhead besides actual data limit the information rate in wireless networks. Transitionally, channel estimation, CSI feedback, and re-transmission requests are usually studied separately, therefore optimized separately. A framework that captures the tradeoff beween all the overhead and data is necessary to improve the throughput performance in practical two-way wireless system. Furthermore, in the fifth generation (5G) mobile wireless networks, the finite block-length coding technique has been developed for reliable and delay restricted applications. These low-latency applications usually require low latency in order of milliseconds and the coding block length is therefore shorter than the traditional applications' packet block length.

1.2 Thesis Contribution

The goal of this research is to provide a fundamental and practical understanding of the value, in terms of system throughput, of feedback in wireless communication. Estimating/learning the channel state and requesting the re-transmission are considered at different levels of the protocol stack and hence studied separately, – one in the physical-layer and the other in the network-layer. In this thesis, several topics are proposed. 1) Development of a framework that combines CSI estimation, feedback, and re-transmission request feedback. 2) Performance evaluation of wireless communication networks while taking into account the channel correlation and channel variation, which add more computation complexity. 3) An analysis of tradeoffs between data, training and feedback that provides insight of future wireless systems design. 4) Moreover, application of the framework in a network coded (NCed) hybrid-ARQ (HARQ) scenario.

1.3 Thesis Organization

The rest of the dissertation is organized as follows. In Chapter 2, we introduce the preliminary knowledge of channel estimation using our practical model. In Chapter 3, we present our analysis of the tradeoff in a point-to-point two-way wireless communication network. A comprehensive study of channel estimation error is presented. In Chapter 4, we investigate the tradeoffs under the assumption of finite block-length and compare the difference of optimal training between infinite block-length and finite block-length packets. In Chapter 5, we investigate how channel estimation bits and extra acknowledgements impacts the performance of network coded (NCed) hybrid automatic repeat request (HARQ). In Chapter 6, we conclude the thesis.

CHAPTER 2

CHANNEL ESTIMATION IN TWO-WAY WIRELESS COMMUNICATION

The contents of this chapters are based on our work that is published in IEEE Open Journal of the Communications Society (Gu et al., 2021)

2.1 Introduction

Channel state information (CSI) is one of the most crucial concepts in current generation wireless communication systems. Path loss, scattering, fading, and movement of objects can cause change of CSI. An accurate CSI at the base station and end user is a prerequisite to embrace all new techniques in current generation wireless systems. In the 4G and 5G wireless communication systems, variety of channel adaptive techniques are widely used to increase the channel capacity. With Channel state information at the transmitter (CSIT) , these new techniques allow the transmitter to adapt the transmit power and rates to increase the throughput of wireless networks.

Two-way wireless communications can be accomplished by frequency division duplex (FDD) or time division duplex (TDD). The first step towards understanding the utility of feedback in two-way networks is to develop a meaningful and general, yet tractable, framework, which can be used to analyze both FDD and TDD systems. We begin by proposing a transmission protocol that captures the tradeoff between feedback and data in two-way scenarios. Then, we derive an expression for the channel state information estimation which takes several practical tradeoffs in two-way communciation networks. The analysis derived here is valid for uplink and downlink by substituting the uplink parameters with downlink parameters (i.e, exchange the subscript u with d).

2.2 Channel model and Estimation

We consider a time-selective fading channel model, which is able to capture static, quasi-static, and block fading. We assume that the channel remains constant within one transmission period but varies between transmissions. The pilot symbols are inserted periodically during each transmission. A simplified model of the channel estimation and its feedback are shown by Figure Figure 1.

At transmission s, the received signal of the i-th channel use, $y_s(i)$, is given by

$$y_s(i) = h_s x_s(i) + w_s(i), \quad i = 1, \cdots, T, \tag{2.1}$$

where $x_s(i)$ is the transmitted signal with maximal power P_l. Note that the subscript $l \in \{d, u\}$ represents downlink or uplink. $w_s(i) \sim \mathcal{CN}(0, N_0)$ is a complex additive white Gaussian noise (AWGN) with zero mean and variance N_0, and h_s is the channel gain that follows a zero mean circular symmetric complex Gaussian distribution with unit variance, i.e., $h_s \sim \mathcal{CN}(0, 1)$, and independent of $w_s(i)$ for all i and s. We define the downlink/uplink transmit power over the noise power as $\gamma_l = P_l/N_0$, with $l \in \{d, u\}$.

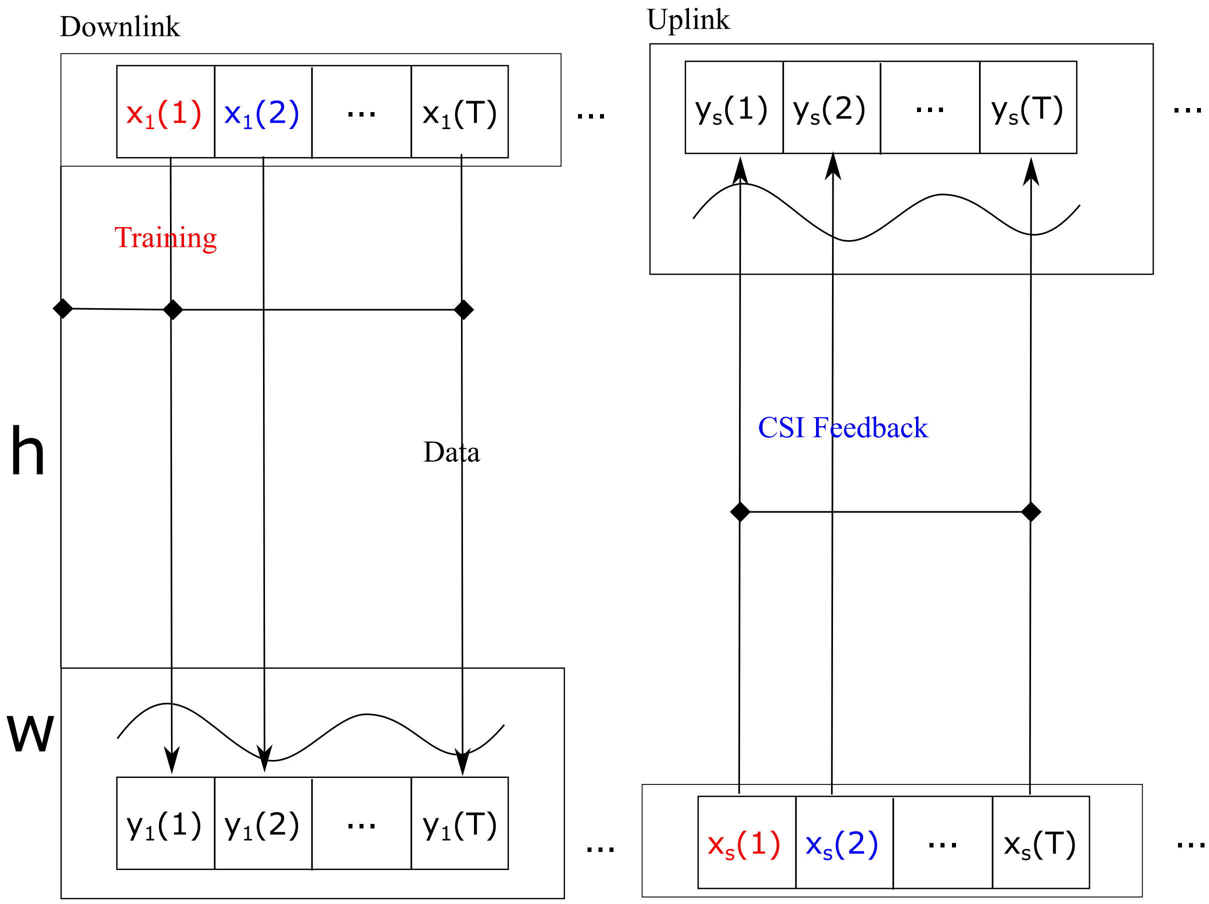


Figure 1: Channel estimation model in fading and noisy channel with CSI feedback. h represents the channel fading coefficient and w represents channel noise. The packet consists of training bits, CSI feedback bits and data bits.

We assume that the channel gains are correlated over several transmissions. Note that due to processing time and ARQ feedback delay, the time between first transmission of a packet and re-transmission of the same packet is long enough so the channel entries can be considered independent (Jin et al., 2011).

2.2.1 Channel estimation at the receiver

During the training phase, the received signal vector at the base station can be written as

$$\mathbf{y}_s = \sqrt{P_u}\ \mathbf{1}_{T_{t,s}} h_s + \mathbf{w}_s, \tag{2.2}$$

where $\mathbf{y}_s = [y_s(1), \cdots, y_s(T_{t,s})]^T$, $\mathbf{1}_{T_{t,s}} = [1, \cdots, 1]^t$ with length $T_{t,s}$, and $\mathbf{w}_s$ contains the noise components that are independent identically distributed (i.i.d.) with covariance matrix $N_0\ \mathbf{I}$, where $\mathbf{I}$ is the identity matrix.

The minimum mean square error (MMSE) estimator of the channel gain h_s at the receiver is given by (Kay, 1993, Eq. (10.31)) as

$$\widehat{h}_s^R = \frac{\sqrt{P_u}}{N_0 + T_{t,s} P_u} \sum_{i=1}^{T_{t,s}} y_s(i). \tag{2.3}$$

It follows from (Equation 2.2) and (Equation 2.3) that $\widehat{h}_s^R$ is a complex Gaussian random variables with zero mean and variance $\widehat{\sigma}_{R,s}^2$ that is given by

$$\widehat{\sigma}_{R,s}^2 = \frac{T_{t,s} P_u}{N_0 + T_{t,s} P_u} = \frac{T_{t,s}\gamma_u}{1 + T_{t,s}\gamma_u}. \tag{2.4}$$

The channel estimation error at the receiver is equal to $\widetilde{h}_s^R = h_s - \widehat{h}_s^R$, which is a circularly symmetric complex Gaussian random variable with zero mean and variance $\widetilde{\sigma}_{R,s}^2$. Using the orthogonality between the MMSE estimator and the error (Kay, 1993), $\widehat{h}_s^R$ and $\widetilde{h}_s^R$ are independent and the mean square error (MSE) of h_s, $\widetilde{\sigma}_{R,s}^2$, is equal to

$$\widetilde{\sigma}_{R,s}^2 = 1 - \widehat{\sigma}_{R,s}^2 = \frac{1}{1 + T_{t,s}\gamma_u}. \tag{2.5}$$

2.2.2 Channel estimation at the transmitter with FDD

In FDD, the transmitter does not have direct access to its own CSI, it relies on the feedback information from the receiver. Because of the transmission time, the feedback CSI describes a delayed channel state $h_{s-\tau}$, where τ is the feedback delay in frames. Therefore, at frame $s - \tau$, the receiver simultaneously estimates $h_{s-\tau}$ to decode the received message and predicts h_s using an innovation process defined later. Once predicted, the receiver will transmit the predicted channel gain, $\widehat{h}_{s,pre}$, to transmitter in order to estimate h_s. Using the classical Jakes model, the correlation coefficient between h_s and $h_{s-\tau}$ is defined as (Tse and Viswanath, 2005a, Eq. (2.58))

$$\rho_\tau = \mathbb{E}[h_s^* h_{s-\tau}] = J_0(2\pi f_\tau T_\tau), \tag{2.6}$$

where f_τ is the Doppler frequency, T_τ is the time difference between time frame s and time frame $s - \tau$, $J_0(\cdot)$ is the zero-th order Bessel function of the first kind (Abramowitz and Stegun, 1964, Eq. (9.1.18)), and $\mathbb{E}[\cdot]$ is the expected value operator. Since $h_{s-\tau}$ and h_s are circularly symmetric jointly Gaussian with zero mean, and using the Gauss-Markov model, the delayed and actual channel gains can be expressed using the following innovation process (Shi et al., 2018)

$$h_{s-\tau} = \rho_\tau h_s + \sqrt{1-\rho_\tau^2}\, e_s, \tag{2.7}$$

where the innovation process $\mathbf{e}_s$ is unit variance complex Gaussian i.i.d. in time and independent of h_s. Under this model, the received signal at the base station during training phase at frame $s-\tau$, can be re-written from (Equation 2.2) as

$$\mathbf{y}_{s-\tau} = \rho_\tau \sqrt{P_u}\ \mathbf{1}_{T_{t,s-\tau}} h_s + \sqrt{(1-\rho_\tau^2)P_u}\ \mathbf{e}_s + \mathbf{w}_{s-\tau},$$

where $\mathbf{e}_s$ is a zero mean complex Gaussian vector with identity covariance matrix and independent of $\mathbf{w}_{s-\tau}$ and h_s. Considering $\sqrt{(1-\rho_\tau^2)P_d}\ \mathbf{e}_s + \mathbf{w}_{s-\tau}$ as a noise vector, h_s can be predicted using MMSE estimator as

$$\widehat{h}^R_{s,pre} = \frac{\rho_\tau \sqrt{P_u}}{N_0 + (1-\rho_\tau^2)P_u + T_{t,s-\tau}\, \rho_\tau^2\, P_u} \sum_{i=1}^{T_{t,s-\tau}} y_{s-\tau}(i). \tag{2.8}$$

Like the non-delayed case, $\widehat{h}^R_{s,pre}$ has complex Gaussian distribution with zero mean and variance $\mathbb{E}[|\widehat{h}^R_{s,pre}|^2]$ that is equal to

$$\widehat{\sigma}^2_{R,s,pre} = \frac{\rho_\tau^2\, T_{t,s-\tau}\, P_u}{N_0 + (1-\rho_\tau^2)P_u + \rho_\tau^2\, T_{t,s-\tau}\, P_u}. \tag{2.9}$$

Applying again the orthogonality principle of the MMSE estimator, the predicted channel error, $\widetilde{h}^{R}_{s,pre} = h_s - \widehat{h}^{R}_{s,pre}$, is a zero mean complex Gaussian random variable with variance equals to the MSE

$$\widetilde{\sigma}^2_{R,s,pre} = 1 - \widehat{\sigma}^2_{R,s,pre} = \frac{1 + (1-\rho^2_{\tau})\gamma_u}{1 + (1-\rho^2_{\tau})\gamma_u + \rho^2_{\tau}\, T_{t,s-\tau}\,\gamma_u}.$$

Once h_s is predicted, the receiver sends back the predicted channel gain $\widehat{h}^{R}_{s,pre}$, to the transmitter during CSI period using $T_{CSI,s}$ CSI symbols. We assume that the feedback channel is modeled as an AWGN channel with downlink transmit power P_d, as described in (Kobayashi et al., 2011). Afterwards, if $\mathbf{z}_s$ denotes the received feedback signal vector at the transmitter, then $\mathbf{z}_s$ can be expressed as

$$\mathbf{z}_s = \sqrt{P_d}\ \mathbf{1}_{T_{CSI,s}} \widehat{h}^{R}_{s,pre}\ \widehat{h}_{s,d} + \sqrt{P_d}\ \mathbf{1}_{T_{CSI,s}} \widehat{h}^{R}_{s,pre}\ \widetilde{h}_{s,d} + \mathbf{n}_s, \tag{2.10}$$

where $\mathbf{n}_s$ is the zero mean complex Gaussian noise with covariance matrix equals to $N_0\,\mathbf{I}$ and independent of $\widehat{h}^{R}_{s,pre}$. $\widehat{h}_{s,d}$ is the MMSE estimator of the downlink channel coefficient using training pilots with variance $\widehat{\sigma}^2_{s,d}$, while $\widetilde{h}_{s,d}$ is the estimation error that is independent of $\widehat{h}_{s,d}$. Both $\widehat{h}_{s,d}$ and $\widetilde{h}_{s,d}$ are independent of $\mathbf{n}_s$. It is worth mentioning that the components of $\mathbf{n}_s$ and $\mathbf{w}_s$ are independent.

At the transmitter side, using the estimator in (Kay, 1993, Eq. (10.31)) and the system model in (Equation 2.10), the MMSE channel estimator at the transmitter is equal to

$$\widehat{h}_s^T = \frac{\widehat{\sigma}^2_{R,s,pre}\widehat{\sigma}_{s,d}\sqrt{P_d}}{N_0 + T_{CSI,s}\,\widehat{\sigma}^2_{R,s,pre}\,P_d}\sum_{i=1}^{T_{CSI,s}} z_s(i). \tag{2.11}$$

Subsequently, $\widehat{h}_s^T$ follows a zero mean complex Gaussian distribution with variance $\widehat{\sigma}^2_{T,s}$ that is expressed as

$$\widehat{\sigma}^2_{T,s} = \frac{T_{CSI,s}\,\widehat{\sigma}^4_{R,s,pre}\,\widehat{\sigma}^2_{s,d}\,\gamma_d}{1 + T_{CSI,s}\,\widehat{\sigma}^2_{R,s,pre}\,\gamma_d}, \tag{2.12}$$

where $\gamma_d = \frac{P_d}{N_0}$ is the downlink SNR, $\widehat{\sigma}^2_{s,d} = \frac{T_{t,s,d}P_d}{N_0+T_{t,s,d}P_d}$, and $T_{t,s,d}$ is the number of downlink training symbols. On the other hand, using again the orthogonality principal, the MSE can be obtained by

$$\widetilde{\sigma}^2_{T,s} = 1 - \widehat{\sigma}^2_{T,s} = \frac{1 + T_{CSI,s}\,\widehat{\sigma}^2_{R,s,pre}\,\gamma_d(1 - \widehat{\sigma}^2_{R,s,pre}\widehat{\sigma}^2_{s,d})}{1 + T_{CSI,s}\,\widehat{\sigma}^2_{R,s,pre}\,\gamma_d}. \tag{2.13}$$

As mentioned above, $\widehat{h}_s^R$ and $\widehat{h}_s^T$ have zero mean complex Gaussian distribution with variances $\widehat{\sigma}^2_{R,s}$ and $\widehat{\sigma}^2_{T,s}$, respectively, but they are not independent. Actually, without that dependency, it is not possible to obtain an accurate estimate of the channel coefficient at the transmitter, $\widehat{h}_s^T$, as the transmitter would have an outdated channel estimate that is independent from the actual channel realization. Indeed, the correlation between $\widehat{h}_s^R$ and $\widehat{h}_s^T$ will improve the total system throughput, even though it presents a challenging computation complexity on the throughput expression. The covariance between $\widehat{h}_s^R$ and $\widehat{h}_s^T$ can be computed using their definitions, (Equation 2.3) and (Equation 2.11), the expression of $\mathbf{y}_s$ in (Equation 2.2), and the expression of $\mathbf{z}_s$ in (Equation 2.10), to get

$$\mathbb{E}[\widehat{h}_s^R \, \widehat{h}_s^{T*}] = \widehat{\sigma}_{R,s}^2 \, \widehat{\sigma}_{T,s}^2 = \rho_s \widehat{\sigma}_{R,s} \, \widehat{\sigma}_{T,s}, \tag{2.14}$$

where $\rho_s = \widehat{\sigma}_{R,s} \, \widehat{\sigma}_{T,s}$ is the correlation coefficient between $\widehat{h}_s^R$ and $\widehat{h}_s^T$.

It is worth noticing that $\widehat{h}_s^T$ is an estimation of $\widehat{h}_s^R$ from a previous realization, and since $\widehat{h}_s^R$ and $\widetilde{h}_s^R$ are orthogonal, the correlation between $\widehat{h}_s^T$ and $\widetilde{h}_s^R$ is negligible, which has been proved numerically. Thereafter, it can be neglected and we consider that $\widetilde{h}_s^R$ and $\widehat{h}_s^T$ are independent complex Gaussian random variables.

2.2.3 Channel estimation at the transmitter with TDD

Unlike FDD, uplink and downlink are reciprocal in TDD, which means that transmitter can rely on the received training to get a prediction of the channel without an extra CSI feedback phase. As in FDD, due to transmission time, the training pilots describe a delayed channel state that is $h_{s-\tau}$. Using $T_{t,s}$ pilots, the transmitter is able to predict $\widehat{h}_s^T$ with an estimate variance similar to $\widehat{\sigma}_{R,s,pre}^2$ in (Equation 2.9), and equals to

$$\widehat{\sigma}_{T,s}^2 = \frac{\rho_\tau^2 \, T_{t,s} \, \gamma_d}{1 + (1 - \rho_\tau^2)\gamma_d + \rho_\tau^2 \, T_{t,s} \, \gamma_d}. \tag{2.15}$$

The MSE is equal to

$$\widetilde{\sigma}_{T,s}^2 = \frac{1 + (1 - \rho_\tau^2)\gamma_d}{1 + (1 - \rho_\tau^2)\gamma_d + \rho_\tau^2 \, T_{t,s} \, \gamma_d}. \tag{2.16}$$

The correlation coefficient between $\widehat{h}_s^R$ and $\widehat{h}_s^T$ in TDD case is similar to that in FDD case i.e., $\rho_s = \widehat{\sigma}_{T,s}\widehat{\sigma}_{R,s}$. Hence, the channel estimation in TDD can be treated as a special case of FDD with no feedback error.

2.3 Conclusion

To conclude this chapter, for both cases FDD and TDD, $\widehat{h}_s^R$, $\widetilde{h}_s^R$, and $\widehat{h}_s^T$ are zero mean circularly symmetric complex Gaussian random variables with variances $\widehat{\sigma}_{R,s}^2$, $\widetilde{\sigma}_{R,s}^2$, and $\widehat{\sigma}_{T,s}^2$, respectively, which means that $|\widehat{h}_s^R|^2$, $|\widetilde{h}_s^R|^2$, and $|\widehat{h}_s^T|^2$ have an exponential distribution with parameters $\widehat{\sigma}_{R,s}^2$, $\widetilde{\sigma}_{R,s}^2$, and $\widehat{\sigma}_{T,s}^2$, respectively. Moreover, we have seen that $\widetilde{h}_s^R$ is independent of $\widehat{h}_s^R$ and $\widehat{h}_s^T$ for all s, though $\widehat{h}_s^R$ and $\widehat{h}_s^T$ are correlated with correlation coefficient ρ_s.

CHAPTER 3

THROUGHPUT PERFORMANCE OF TWO-WAY WIRELESS COMMUNICATION WITH COMBINED CSI AND ARQ FEEDBACK WITH INFINITE PACKET LENGTH

The contents of this chapters are based on our work that is published in IEEE Open Journal of the Communications Society (Gu et al., 2021).

3.1 Introduction

Feedback in wireless systems are designed to increase the throughput performance. Feedback can be used to estimate and indicate the channel state, request re-transmissions after a packet can not be decoded by the receiver, or help improve adaptive transmission strategy. In general, feedback has been studied from a one-way perspective, meaning data travels in one direction, and feedback – often assumed to be perfect – in the other. Two forms of feedback have been played very crucial roles to address these demands. CSI, the signal-to-noise ratio of a certain transmission, is essential for the wireless receiver to decode the packet. Also, CSI can be fed back to the transmitter side and exploited to maximize the data throughput. Re-transmission is inevitable in reliable wireless systems due to a failed reception. Automatic repeat request (ARQ) and HARQ are highly adopted in communication systems.

The goal of this research is to provide a fundamental and practical understanding of the value, in terms of system throughput, of feedback in wireless communication. Feedback has traditionally been used to either learn the channel state, or to request the re-transmission of a failed reception. Despite the established link between the accuracy of the channel state and the performance of the re-transmission protocols, these two feedback usages have been commonly studied separately – one in the physical-layer and the other in the network-layer.

3.2 Related Work

Traditionally, estimating/learning the channel state and requesting the re-transmission are considered at different levels of the protocol stack and hence studied separately. Some research studies have, nonetheless, established a link between re-transmission performances and physical-layer parameters. They can be classified in two groups: Studying re-transmission protocols under the assumption of 1) inaccurate CSI due to transmission noise, or 2) delayed CSI due to re-transmission delay.

Re-transmission protocols under inaccurate/noisy CSI: In (Cao and Kam, 2011), Cao and *al.* studied the impact of the accuracy of CSI at the receiver on the performance of ARQ and HARQ protocols. They derived bounds on the accepted packet error rate and good-put that take into account both the channel estimation error and the re-transmission protocol. Closer to our work is (Ghanavati and Lee, 2018), where the authors took into account the training phase length and minimum mean square error (MMSE) channel estimation when optimizing the total transmission power in point-to-point communication with ARQ re-transmission. However, these papers did not consider the temporal channel variations and the CSI delay. In (Shi et al., 2019), the authors derived outage probabilities for non-orthogonal multiple access (NOMA) with CC-HARQ and IR-HARQ, taking into the channel estimation error at the receiver side.

Delayed CSI and re-transmission delay: In (Kim et al., 2011), the authors proposed an optimal rate adaptation for chase combining HARQ protocol (CC-HARQ) and time-varying channel model, based on the outage probability with delayed CSI that is known at transmitter and receiver. In (Shi et al., 2015), Shi and *al.* adopted an information-theoretic approach to investigate the impact of channel time variation on the incremental redundancy HARQ protocol (IR-HARQ) performance. They have extended their work by analyzing the optimization problem in an energy efficient perspective (Shi et al., 2018), where they considered re-transmission with *a priori* fixed number of re-transmissions. More recently, outage probabilities for NOMA with delayed CSI are derived in (Cai et al., 2019) and statistical CSI are investigated in (Xu et al., 2018). However, in (Kim et al., 2011; Shi et al., 2015; Shi et al., 2018), the CSI estimation error was not considered. In addition, the throughput expression did not take into account the resources dedicated to training and feeding back the CSI. It was assumed also that after sending each packet, the transmitter does not send any further frame until it receives an acknowledgement, which is not throughput-efficient. Therefore, most current re-transmission protocols allow the transmitter to send a number of packets specified by a window size without the need to wait for individual acknowledgement from the receiver.

3.3 Main Results

In this chapter, we develop of a generalized framework for feedback in two-way networks which combines limited CSI feedback and ARQ, captures time-variation of the channel, accounts for training, feedback, and data bits. We derive the outage probability expressions and achievable throughput for both Time-Division Duplex and Frequency-Division Duplex expressions by taking into account the channel correlation, feedback error of CSI from the transmitter to the receiver, and channel variations over time. We evaluate the tradeoffs between training, CSI, ARQ and data bits as function time-variation of the channel. To to specific, we summarize the main contributions as follows:

3.3.1 Accurate channel and re-transmission model

Compared to previous works (Ghanavati and Lee, 2018; Einstein, 1905; Kim et al., 2011; Shi et al., 2015; Shi et al., 2018), we consider both noisy and time-varying channel model, to accurately represent wireless propagation. We extend our model to capture the time-variation of the channel in both frequency division duplex (FDD) and time division duplex (TDD) scenarios. Compared to (Kim et al., 2011; Shi et al., 2015), a more efficient re-transmission protocol is considered, where the transmitter sends a number of packets without waiting for individual acknowledgement from the receiver. In case of error, the receiver can selectively reject a single packet, which will be re-transmitted alone. In such case, the receiver may accept out-of-order packets and buffer them. This protocol design creates a significant channel correlation between consecutive packet transmissions, while the channel correlation between the re-transmission of the same packet is negligible due to the long duration between these transmissions as proved in (Jin et al., 2011).

3.3.2 Efficient resources usage

Most of the existing literature assume an uplink channel to provide information which helps the transmission of the downlink channel. However, the cost of uplink channel usage is not taken into account and the overhead bits can reduce the space for actual information bits. In this chapter, we develop a two-way framework for feedback, which combines limited CSI feedback and ARQ, and accounts for training, feedback, and data bits. This results in a tradeoff between improving rates in one direction and feeding back information to improve rates in the other. The optimization of tradeoffs will increase throughput as they are affected by the rate of channel time-variation.

3.3.3 Accurate performance evaluation

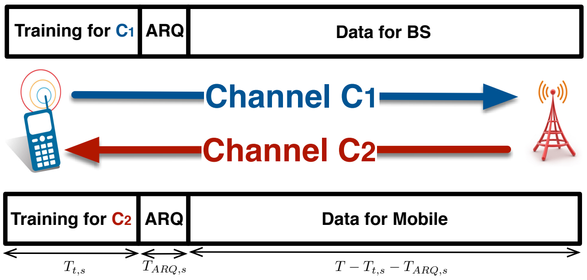
Given the practical channel model we discussed abobe, this chapter is the first to mathematical expressions of the outage probability and throughput of different combinations of transmitter and (H)ARQ protocols. Our study shows the effect of the channel varying rate (slow to fast varying channel) on the throughput of each scheme, and the impact of training, feedback and data phase lengths on system performance. From the mathematical perspective, this chapter offers an explicit form of the MMSE of channel estimation at receiver and transmitter, and closed/integral form expressions of the outage probability and throughput of the above systems using three transmitter protocols based on power and rate adaption such that, no adaptation, power adaptation, and rate adaptation; and three ARQ receiver protocols with basic ARQ and two hybrid ARQ, namely CC-HARQ and IR-HARQ. As opposite to previous works (Cao and Kam, 2011; Ghanavati and Lee, 2018; Einstein, 1905; Shi et al., 2015; Kim et al., 2011), this chapter considers channel estimation error at transmitter and receiver with delayed CSI. Delayed imperfect CSI will create a correlation between the channel estimator at transmitter and receiver. Knowing that in power and rate adaptation protocols the outage event depends on channel estimator at transmitter and receiver, the derivation of the probability of that event becomes complicated using the joint correlated portability density function (PDF) of Rayleigh random variables in terms of the zero-th order Bessel function. For instance, IR-HARQ considers the previously decoded messages where the derivation of the throughput requires the computation of the PDF of the sum of transformation of Rayleigh random variables that was not done before. To summarize, this chapter provides a realistic throughput analysis of the above mentioned protocols that shows the impact of the channel varying rate on tradeoff between training, feedback and data phase lengths through explicit outage probability derivation.

3.4 System Model

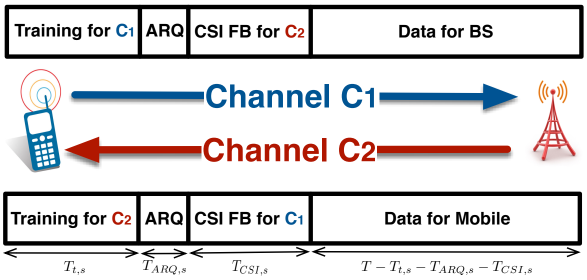
In order to develop a general framework for two-way wireless networks, we start by defining the system model. We propose a transmission protocols that captures the trade-offs between CSI estimation, feedback, re-transmissions, and actual data. Then, we derive an expression for the throughput which demonstrates many of the involved tradeoffs.

3.4.1 Transmission Protocols

We consider both FDD and TDD two-way systems with limited feedback, where two users exchange packets designed as shown in Figure 2. Each packet is divided into three or four phases depending on the used model: CSI training, HARQ feedback, CSI feedback (only for FDD case), and data. Assume that a packet with T symbols (channel uses) that includes $\mathsf{T}_{\mathsf{t,s}}$ training symbols, is sent in transmission s. The training symbols are transmitted by the mobile to the base-station (BS) to estimate the channel gain. In FDD, the system does not have reciprocity between opposite streams (uplink and downlink); this is unlike TDD, where the



(a) TDD



(b) FDD

Figure 2: The two-way packet-structure and inter-dependence of streams for (a) TDD, and (b) FDD.

reciprocity between uplink and downlink allows the mobile to get access to CSI, C_1, through training. Therefore, the CSI feedback phase is only needed in FDD so the transmitter can rely on the feedback information from the receiver. The receiver estimates the CSI and then feeds back the estimates to the transmitter using $\mathrm{T}_{\mathrm{CSI,s}}$ symbols.

For this purpose, two practical adaptive protocols, with very simple encoding and decoding (Goldsmith and Varaiya, 1997a), are considered: *i*) constant transmit power (CP) protocol and *ii*) truncated channel inversion (CI) protocol. In CP protocol, the transmitter adapts the transmission rate to the channel variation by keeping the transmit power constant. While in CI protocol, the transmitter adapts the transmit power by keeping the transmission rate constant when the channel response is above a cut-off level. In addition, we consider a no CSI (NC) protocol to study the impact of the absence of CSI feedback phase on system performance. The last phase is the HARQ feedback phase that is used for re-transmission protocols; it contains $T_{ARQ,s}$ symbols.

To summarize, we consider three transmitter protocols (NC, CP, and CI), combined with three HARQ receiver schemes (basic ARQ, CC-HARQ, and IR-HARQ) to explore the effect of these various protocols on system performance. Let T_x denote the transmitter protocol and R_x denote the receiver scheme, then $T_x \in \{NC, CP, CI\}$ and $R_x \in \{AR, CC, IR\}$.

3.5 Throughput Analysis

The throughput is used to study the system performance of each selected scheme. In order to obtain its expression, we begin by defining the channel achievable rate and the target rate, and deriving the outage probability expressions for each scenario.

3.5.1 Channel Achievable Rate

As described above, the transmitter protocols include NC, CP, and CI, while the receiver ARQ schemes include AR, CC, and IR. Let $I_{T_x,R_x}(m)$ be the channel achievable rate after the m-th transmission at the receiver, using transmitter protocol T_x and receiver scheme R_x. For transmission protocol T_x, we denote $\beta_{T_x,s}$ the received signal to interference plus noise ratio (SINR) at transmission s. It is worth mentioning that $\beta_{T_x,s}$ varies with transmitter protocol only because the SINR does not depend on the receiver scheme. Using the fact that the channel estimation error will add a Gaussian error to the received signal, the SINR at receiver, without CSI or using constant transmit power protocol at transmitter, can be expressed as

$$\beta_{NC,s} = \beta_{CP,s} = \frac{\gamma_u \, |\widehat{h}_s^R|^2}{1 + \gamma_u \, |\widetilde{h}_s^R|^2}. \tag{3.1}$$

Knowing the predicted channel gain $|\widehat{h}_s^T|^2$, the transmitter uses CI protocol to maintain a constant received power by inverting the channel fading, which, in some cases, may lead to infinite transmit power. Therefore, we consider a truncated inversion policy that only compensates for fading above a certain cutoff fade γ_0 (Goldsmith and Varaiya, 1997a), i.e. $|\widehat{h}_s^T|^2 > \gamma_0$. Hence, the SINR using CI protocol is given by

$$\beta_{CI,s} = \frac{\frac{|\widehat{h}_s^R|^2 P_u}{|\widehat{h}_s^T|^2}}{N_0 + \frac{|\widetilde{h}_s^R|^2}{|\widehat{h}_s^T|^2} P_u} = \frac{\gamma_u \, |\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2 + \gamma_u \, |\widetilde{h}_s^R|^2}, \quad \text{if } |\widehat{h}_s^T|^2 > \gamma_0. \tag{3.2}$$

The achievable rate of transmitter protocol T_x with basic ARQ, which takes into account only the most recently received signal burst, can be evaluated as

$$I_{T_x,AR}(m) = \alpha_m \log\left(1 + \beta_{T_x,m}\right), \tag{3.3}$$

where α_m denotes proportion of data symbols in a packet that is equal to $\alpha_m = \frac{T - T_{t,m} - T_{CSI,m} - T_{ARQ,m}}{T}$ for FDD case, and $\alpha_m = \frac{T - T_{t,m} - T_{ARQ,m}}{T}$ for TDD case. When deploying the CC scheme and after combining m received bursts, the total received SINR is equal to $\sum_{s=1}^{m} \beta_{T_x,s}$ (Larsson et al., 2013; Caire and Tuninetti, 2001), so the achievable rate of CC with any transmitter protocol T_x, is given by

$$I_{T_x,CC}(m) = \alpha_m \log\left(1 + \sum_{s=1}^{m} \beta_{T_x,s}\right). \tag{3.4}$$

In addition, using IR scheme, the achievable rate is equal to the sum of all previous received rate terms corresponding to a given message (Larsson et al., 2013). Therefore, the achievable rate of IR and transmitter protocol T_x is equal to

$$I_{T_x,IR}(m) = \sum_{s=1}^{m} \alpha_s \log\left(1 + \beta_{T_x,s}\right). \tag{3.5}$$

3.5.2 Transmitter Target Rate

Another important metric is needed to define the throughput, that is the transmitter target rate at transmission m, $R_{T_x}(m)$. Without CSI, the transmitter is sending at fixed rate R because the channel state is not available. Thus, the transmitter target rate for NC is equal to $R_{NC}(m) = R$. However, in the CP protocol, the transmitter adapts the transmission rate to the channel variation while keeping the transmit power constant, in more details the target rate is adapted as $R_{CP}(m) = \alpha_m \log\left(1 + \gamma_u |\widehat{h}_m^T|^2\right)$. Finally for truncated CI, the transmitter adapts the transmit power according to the channel gain to maintain a constant received power P_u, the target rate is equal to Gaussian channel capacity with SNR γ_u and α_m data symbols, i.e.

$R_{CI}(m) = \alpha_m \log(1+\gamma_u)$, when $|\widehat{h}_m^T|^2 > \gamma_0$, and zero otherwise. To summarize, the transmitter target rate, $R_{T_x}(m)$, varies with the transmitter protocol as follows

$$R_{NC}(m) = R, \tag{3.6}$$

$$R_{CP}(m) = \alpha_m \log(1+\gamma_u |\widehat{h}_m^T|^2), \tag{3.7}$$

$$R_{CI}(m) = \begin{cases} \alpha_m \log(1+\gamma_u) & \text{if } |\widehat{h}_m^T|^2 > \gamma_0 \\ 0 & \text{otherwise.} \end{cases} \tag{3.8}$$

The expected rate at the m-th transmission, $\overline{R}_{T_x}(m)$, is equal to the expected value of the transmitter target rate $R_{T_x}(m)$ over the distribution of $|\widehat{h}_m^T|^2$ that has an exponential distribution with mean $\widehat{\sigma}_{T,m}^2$, i.e. $\overline{R}_{T_x}(m) = \mathbb{E}[R_{T_x}(m)]$. Without CSI, the target is constant, so the expected rate is equal to $\overline{R}_{NC}(m) = R$. Using CP protocol, $\overline{R}_{CP}(m)$ is obtained by averaging (Equation 3.7) over the probability density function (PDF) of $|\widehat{h}_m^T|^2$ as

$$\overline{R}_{CP}(m) = \frac{\alpha_m}{\widehat{\sigma}_{T,m}^2} \int_0^\infty e^{-x/\widehat{\sigma}_{T,m}^2} \log(1+\gamma_d x)\, dx = \alpha_m\, e^{\frac{1}{\gamma_d \widehat{\sigma}_{T,m}^2}} E_1\left(\frac{1}{\gamma_d \widehat{\sigma}_{T,m}^2}\right), \tag{3.9}$$

where $E_1(\cdot)$ is the exponential integral function (Abramowitz and Stegun, 1964, Eq. (5.1.1)). The integral in (Equation 3.9) is solved using the identity (Abramowitz and Stegun, 1964, Eq. (5.1.28)) following an integration by part. In the case of CI protocol, it was proved in

(Goldsmith and Varaiya, 1997a) that the expected rate is the maximum over the threshold, γ_0, of the average transmit rate using $P_{tr,m}$ as transmit power, i.e.

$$\overline{R}_{CI}(m) = \max_{\gamma_0} \ \alpha_m \log\left(1 + P_{tr,m}/N_o\right) \Pr\left[|\widehat{h}_m^T|^2 > \gamma_0\right],$$

where $P_{tr,m}$ is the transmitter power that is determined by solving

$$P_u = \int_{\gamma_0}^{\infty} \frac{P_{tr,m}}{x\,\widehat{\sigma}_{T,m}^2} e^{-x/\widehat{\sigma}_{T,m}^2} dx = \frac{P_{tr,m}}{\widehat{\sigma}_{T,m}^2} E_1\left(\frac{\gamma_0}{\widehat{\sigma}_{T,m}^2}\right).$$

Hence, the expected rate using CI can be expressed as

$$\overline{R}_{CI}(m) = \max_{x \geq 0} \alpha_m \log\left(1 + \frac{\gamma_u\,\widehat{\sigma}_{T,m}^2}{E_1(x)}\right) e^{-x}. \tag{3.10}$$

3.5.3 Outage Probability Definition

In case of AR and CC scenarios, the transmitter keeps sending the same packet, with different coding rates $R_{T_x}(s)$, until an ACK is received or the maximum number of transmission, M, is reached. A successful decoding at transmission s happens when the packet is encoded with rate $R_{T_x}(s)$ below the channel achievable rate $I_{T_x,R_x}(s)$. Therefore, the outage event, at transmission m, occurs when the system transmits with target rate above the channel achievable rate in all previous transmissions $s \leq m$. Let's define $A_{T_x,R_x}(s)$ as the outage event at transmission s $(1 \leq s \leq m)$, then $A_{T_x,R_x}(s) := \{I_{T_x,R_x}(s) < R_{T_x}(s)\}$. Then, for any transmitter protocol T_x, the probability of outage for AR and CC, denoted by $\eta_{T_x,R_x}(m)$, is defined as the probability that $A_{T_x,R_x}(s)$ occurs for $s = 1, \ldots, m$, i.e.

$$\eta_{T_x,R_x}(m) = \Pr\left[A_{T_x,R_x}(1), A_{T_x,R_x}(2), \cdots, A_{T_x,R_x}(m)\right], \text{for } R_x \in \{\text{AR,CC}\}, \tag{3.11}$$

where Pr[A] denotes the probability of event A.

Unlike AR and CC, in IR the system sends the first sub-codeword of length T_1 with coding rate rate $R_{T_x}(1) = \frac{T_1}{\alpha_1 T}$. If a NACK message is returned back from the receiver, the transmitter knows that the first sub-codeword is erroneously decoded and therefore sends the next sub-codeword of length T_2 with code rate $R_{T_x}(2) = \frac{T_2}{\alpha_2 T}$. This process continues until an ACK is received or the maximum number of transmissions, M, is reached. Assuming a perfect coding and decoding, the packet is lost, i.e. outage event, after m transmissions only if the average achievable rate accumulated throughout the transmissions is less than the average transmission rate (Szczecinski et al., 2010). In more details, the packet is decoded successfully only if all the previous sent sub-codewords can be decoded successfully on average. The outage event can be defined by

$$\frac{1}{\sum_{s=1}^{m} T_s} \sum_{s=1}^{m} I'_{T_x,IR}(s) \frac{T_s}{\alpha_s} < \frac{T}{\sum_{s=1}^{m} T_s}, \tag{3.12}$$

where T_s is the size of the s-th sub-codeword and $I'_{T_x,IR}(s) = \alpha_s \log(1 + \beta_{T_x,s})$ is the achievable rate when receiving the s-th codeword only. Knowing that $R_{T_x} = \frac{T_s}{\alpha_s T}$, the outage probability of IR is expressed using (Equation 3.12) as

$$\eta_{T_x,IR}(m) \triangleq \Pr\left[\sum_{s=1}^{m} \frac{I'_{T_x,IR}(s)}{R_{T_x}(s)} < 1\right]. \tag{3.13}$$

On the other hand, the probability of successful transmission at the m-th transmission, $\kappa_{T_x,R_x}(m)$, is defined as the probability that the system decodes successfully the packet after exactly m transmissions and fails to decode it in the previous $m-1$ transmissions, which leads to the expression of $\kappa_{T_x,R_x}(m)$ as

$$\kappa_{T_x,R_x}(m) = \eta_{T_x,R_x}(m-1) - \eta_{T_x,R_x}(m), \tag{3.14}$$

where $\eta_{T_x,R_x}(0) = 1$.

3.5.4 Throughput Expression

Assuming that all nodes have always packets to send (Larsson et al., 2010), the throughput, ν, is defined using the renewal-reward theorem (Caire and Tuninetti, 2001) as the ratio between the expected rate, $\mathcal{R}$, and the expected number of transmissions, $\mathcal{T}$, which are defined through the outage probability and expected rate. Assuming that M is the maximum number of transmissions, i.e. stop transmitting the same packet after M attempts, the expected number of transmissions per packet, $\mathcal{T}_{T_x,R_x}$, is equal to the sum of the outage probabilities during the M transmissions (Caire and Tuninetti, 2001)

$$\mathcal{T}_{T_x,R_x} = \sum_{m=0}^{M-1} \eta_{T_x,R_x}(m). \tag{3.15}$$

The expected rate, $\mathcal{R}_{T_x,R_x}$, in nats/Hz/s is defined as the sum over m of the expected rate at transmission m, $\overline{R}_{T_x,R_x}(m)$, multiplied by the probability of successful transmission at transmission m, $\kappa_{T_x,R_x}(m)$, yield

$$\mathcal{R}_{T_x,R_x} = \sum_{m=1}^{M} \overline{R}_{T_x,R_x}(m)\ \kappa_{T_x,R_x}(m). \tag{3.16}$$

Therefore, the total system throughput can be expressed, using the renewal-reward theorem (Goldsmith and Varaiya, 1997b), as

$$\nu_{T_x,R_x} = \frac{\mathcal{R}_{T_x,R_x}}{\mathcal{T}_{T_x,R_x}} = \frac{\sum_{m=1}^{M} \overline{R}_{T_x,R_x}(m)\ \kappa_{T_x,R_x}(m)}{\sum_{m=0}^{M-1} \eta_{T_x,R_x}(m)}. \tag{3.17}$$

3.6 Outage probability for basic ARQ

In this section, we investigate the outage probability expressions for basic ARQ schemes by deploying the above defined three transmitter protocols, NC, CP, and CI, i.e. $R_x = AR$ and $T_x \in \{NC, CP, CI\}$. Because of its basic concept, the achievable rate of AR at transmission s is independent from the achievable rate of other transmissions, which can be noticed from

(Equation 3.3). Therefore, the events $A_{T_x,AR}(s)$ are independent for $s = 1, \cdots, m$, and the outage probability can be written as the product of outage probabilities of each transmission, yield

$$\begin{aligned}\eta_{T_x,AR}(m) &\triangleq \Pr\left[A_{T_x,AR}(1), A_{T_x,AR}(2), \cdots, A_{T_x,AR}(m)\right] \\ &= \prod_{s=1}^{m} \Pr\left[A_{T_x,AR}(s)\right] = \prod_{s=1}^{m} \Pr\left[I_{T_x,AR}(s) < R_{T_x}(s)\right] \\ &= \prod_{s=1}^{m} \Pr\left[\alpha_s \log\left(1 + \beta_{T_x,s}\right) < R_{T_x}(s)\right], \qquad \text{for } T_x = \mathsf{NC, CP, CI}, \end{aligned} \tag{3.18}$$

where the expression of $I_{T_x,AR}(s)$ is obtained from (Equation 3.3). A compact expression of $\eta_{T_x,AR}(m)$ can be driven as

$$\eta_{T_x,AR}(m) = \prod_{s=1}^{m} \Pr\left[\beta_{T_x,s} < e^{\frac{R_{T_x}(s)}{\alpha_s}} - 1\right], \qquad \text{for } T_x = \mathsf{NC, CP, CI}. \tag{3.19}$$

According to the transmitter protocol T_x, $\eta_{T_x,AR}(m)$ is evaluated in the following subsections.

3.6.1 NO CSI Transmitter Protocol

Without CSI, the transmitter set a fixed rate R and only the imperfect channel estimation at receiver will affect the SINR. Using the expression of $\beta_{NC,s}$ in (Equation 3.1), the outage probability (Equation 3.19) can be obtained as follows

$$\eta_{NC,AR}(m) = \prod_{s=1}^{m} \Pr\left(\frac{\gamma_u |\widehat{h}_s^R|^2}{1 + \gamma_u |\widetilde{h}_s^R|^2} < \theta_s\right) = \prod_{s=1}^{m} \left(1 - \frac{\widehat{\sigma}_{R,s}^2}{\widehat{\sigma}_{R,s}^2 + \widetilde{\sigma}_{R,s}^2 \theta_s} e^{-\frac{\theta_s}{\gamma_u \widehat{\sigma}_{R,s}^2}}\right), \tag{3.20}$$

which is evaluated using the cumulative distribution function (CDF) of $\frac{\gamma_u |\widehat{h}_s^R|^2}{1+\gamma_u |\widetilde{h}_s^R|^2}$ that is derived in Appendix A in (Equation A.1) for independent exponential distributions $|\widehat{h}_s^R|^2$ and $|\widetilde{h}_s^R|^2$. Note that $\theta_s = e^{\frac{R}{\alpha_s}} - 1$.

3.6.2 Constant Transmit Power Transmitter Protocol

By keeping the transmit power constant, the transmitter target rate depends on the transmitter predicted channel coefficient (Equation 3.7), i.e. $R_{CP}(s) = \alpha_s \log(1+\gamma_u |\widehat{h}_s^T|^2)$. Thereafter, by replacing $\beta_{CP,s}$ by its expression from (Equation 3.1), the outage probability (Equation 3.19) can be expressed as

$$\begin{aligned}\eta_{CP,AR}(m) &= \prod_{s=1}^{m} \Pr\left[\frac{|\widehat{h}_s^R|^2}{1+\gamma_u|\widetilde{h}_s^R|^2} < |\widehat{h}_s^T|^2\right] = \prod_{s=1}^{m} \mathbb{E}_{|\widetilde{h}_s^R|^2}\left[\Pr\left[\frac{|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2} < 1+\gamma_u x \,\middle|\, |\widetilde{h}_s^R|^2 = x\right]\right] \\ &= \prod_{s=1}^{m} \frac{1}{\widetilde{\sigma}_{R,s}^2} \int_0^{\infty} F_{\frac{|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2}}(1+\gamma_u x) e^{-\frac{x}{\widetilde{\sigma}_{R,s}^2}} dx, \end{aligned} \quad (3.21)$$

where $F_{\frac{|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2}}(\cdot)$ is the CDF of the ratio $\frac{|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2}$. Note that (Equation 3.21) is obtained by conditioning over the distribution of $|\widetilde{h}_s^R|^2$. The CDF of the ratio of the correlated exponential random variables $|\widehat{h}_s^R|^2$ and $|\widehat{h}_s^T|^2$ is obtained using the distribution of their difference $|\widehat{h}_s^R|^2 - |\widehat{h}_s^T|^2$, as derived in appendix B. Actually, this difference can be seen as an expression that contains four Gaussian random variables, because an exponential random variable is the sum of the squares of two real independent Gaussian random variables. Using a moment generating function (MGF) approach, the CDF of that difference is obtained in (Equation B.9), and subsequently the CDF of the ratio $\frac{|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2}$ is derived in (Equation B.11), the whole proof is more detailed in appendix B. The final outage probability expression is given by

$$\eta_{CP,AR}(m) = \prod_{s=1}^{m} \left(\frac{1}{2} - \int_0^{\infty} \frac{e^{-\frac{x}{\widetilde{\sigma}_{R,s}^2}}}{2\widetilde{\sigma}_{R,s}^2} \frac{\widehat{\sigma}_{R,s}^2 - (1+\gamma_u x)\widehat{\sigma}_{T,s}^2}{\lambda_s(1+\gamma_u x)} dx \right) dx, \tag{3.22}$$

$$\begin{aligned} \lambda_s(x) &\triangleq \left(2\,\mathrm{var}\left(|\widehat{h}_s^R|^2 - x|\widehat{h}_s^T|^2 \right) - \mathbb{E}\left[|\widehat{h}_s^R|^2 - x|\widehat{h}_s^T|^2 \right]^2 \right)^{\frac{1}{2}} \\ &= \sqrt{x^2\,\widehat{\sigma}_{T,s}^4 + \widehat{\sigma}_{R,s}^4 + 2x\,\widehat{\sigma}_{T,s}^2\,\widehat{\sigma}_{R,s}^2(1 - 2\,\rho_s^2)}\ . \end{aligned} \tag{3.23}$$

3.6.3 Channel Inversion Transmitter Protocol

In this protocol, the transmitter rate is constant, $R_{CI}(s) = \alpha_s \log(1+\gamma_u)$, while the SINR is defined in (Equation 3.2) as $\beta_{CI,s} = \frac{\gamma_u\,|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2+\gamma_u\,|\widetilde{h}_s^R|^2}$. From (Equation 3.19), the outage probability of CI protocol and basic ARQ scheme can be written as

$$\eta_{CI,AR}(m) = \prod_{s=1}^{m} \Pr\left[\frac{|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2 + \gamma_u\,|\widetilde{h}_s^R|^2} < 1 \right] = \prod_{s=1}^{m} F_{\frac{|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2+\gamma_u\,|\widetilde{h}_s^R|^2}}(1), \tag{3.24}$$

where $F_{\frac{|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2+\gamma_u\,|\widetilde{h}_s^R|^2}}(\cdot)$ is the CDF of $\frac{|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2+\gamma_u\,|\widetilde{h}_s^R|^2}$. Using the CDF of $|\widehat{h}_s^R|^2 - |\widehat{h}_s^T|^2$ and by averaging over $|\widetilde{h}_s^R|^2$ that is independent of both of them, $F_{\frac{|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2+\gamma_u\,|\widetilde{h}_s^R|^2}}(\cdot)$ is derived as closed form in (Equation B.13). A closed form expression of the outage probability can be derived as

$$\eta_{CI,AR}(m) = \prod_{s=1}^{m} \left(2\frac{\widehat{\sigma}_{T,s}^2\widehat{\sigma}_{R,s}^2(1-\rho_s^2) + \gamma_u\widetilde{\sigma}_{R,s}^2\lambda_s(1)}{\left(\widehat{\sigma}_{R,s}^2 - \widehat{\sigma}_{T,s}^2 + 2\gamma_u\widetilde{\sigma}_{R,s}^2 + \lambda_s(1)\right)\lambda_s(1)} \right), \tag{3.25}$$

where $\lambda_s(1)$ is given by (Equation 3.23).

3.7 Outage Probability for Hybrid ARQ Schemes

In this section, we study the outage probability of HARQ schemes, which includes the CC and IR, $R_x \in \{CC, IR\}$. The outage probability of CC is defined using (Equation 3.11) as the probability that the channel achievable rate $I_{T_x,R_x}(s)$ is less than the target rate $R_{T_x}(s)$ for all transmissions $1 \leq s \leq m$, while the outage probability of IR is given by (Equation 3.12).

3.7.1 HARQ Chase Combining Scheme

In the CC protocol, the user keeps sending the same packet until it receives an ACK or reaches the maximum number of transmission, M. Therefore, the rate is not varying from transmission to another because the same packet is sent. Hence, for CC protocol we consider only the no CSI and channel inversion schemes, where the transmitter target rate is fixed as R and $\log(1+\gamma_u)$, respectively. Using the fact that the achievable rate of CC is an increasing function with respect to s (Equation 3.4), the outage probability depends only on the last event $A_{T_x,R_x}(m)$ as follows

$$\begin{aligned}\eta_{T_x,CC}(m) &\triangleq \Pr\left[I_{T_x,CC}(1) < R_{T_x}(1), \cdots, I_{T_x,CC}(m) < R_{T_x}(m)\right] = \Pr\left[I_{T_x,CC}(m) < R_{T_x}(m)\right] \\ &= \Pr\left[\alpha_m \log\left(1 + \sum_{s=1}^{m} \beta_{T_x,s}\right) < R_{T_x}(m)\right], \text{for } T_x = NC,\ CI, \qquad (3.26)\end{aligned}$$

which is obtained using the channel achievable rate expression of CC, (Equation 3.4). A simplified expression of $\eta_{T_x,CC}(m)$ is given by

$$\eta_{T_x,CC}(m) = \Pr\left[\sum_{s=1}^{m} \beta_{T_x,s} < e^{\frac{R_{T_x}(m)}{\alpha m}} - 1\right], \quad \text{for } T_x = \text{NC, CI}. \tag{3.27}$$

In the following analysis, (Equation 3.27) will be studied to derive the expression of the outage probability using the two different transmitter protocols, NC and CI.

3.7.1.1 No CSI Protocol

The target rate is set to be R, and the received SINR, $\beta_{NC,s}$, is expressed in (Equation 3.1). The outage probability in case of deployment of CC and NC is obtained from (Equation 3.27) as

$$\eta_{NC,CC}(m) = \Pr\left[\sum_{s=1}^{m} \frac{|\widehat{h}_s^R|^2}{1+\gamma_u\,|\widetilde{h}_s^R|^2} < \frac{\theta_m}{\gamma_u}\right] = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \frac{1}{t}\,\mathrm{Im}\left(e^{-it\frac{\theta_m}{\gamma_u}}\varphi_{NC,CC}(t)\right) dt, \tag{3.28}$$

where $\varphi_{NC,CC}(t)$ is the characteristic function (CHF) of $\sum_{s=1}^{m} \frac{|\widehat{h}_s^R|^2}{1+\gamma_u\,|\widetilde{h}_s^R|^2}$, $\mathrm{Im}(\cdot)$ designates the imaginary part of a complex number, and i is the imaginary number, $i^2 = -1$. Actually, (Equation 3.28) is obtained using the CDF inversion formula (Wendel, 1961, Eq. (2)), because $\eta_{NC,CC}(m)$ is equal to the CDF of $\sum_{s=1}^{m} \frac{|\widehat{h}_s^R|^2}{1+\gamma_u\,|\widetilde{h}_s^R|^2}$ applied to $\frac{\theta_m}{\gamma_u}$, where $\theta_m = e^{\frac{R}{\alpha m}} - 1$. The channel coefficients, $\widehat{h}_s^R$ and $\widetilde{h}_s^R$, are independent for $1 \leq s \leq m$, which means that $\varphi_{NC,CC}(t)$

can be expressed as the product over s of the CHFs of each random variable $\frac{|\widehat{h}_s^R|^2}{1+\gamma_u|\widetilde{h}_s^R|^2}$, a closed form expression of that CHF is derived in Appendix A, and from (Equation A.8) we have

$$\varphi_{NC,CC}(t) = \prod_{s=1}^{m}\left(1 + it\frac{\widehat{\sigma}_{R,s}^2}{\gamma_u\,\widetilde{\sigma}_{R,s}^2}e^{\frac{1-it\widehat{\sigma}_{R,s}^2}{\gamma_u\,\widetilde{\sigma}_{R,s}^2}}E_1\left(\frac{1-it\,\widehat{\sigma}_{R,s}^2}{\gamma_u\,\widetilde{\sigma}_{R,s}^2}\right)\right). \tag{3.29}$$

By combining (Equation 3.28) and (Equation 3.29), the outage probability of the couple (NC, CC) has the following expression

$$\eta_{NC,CC}(m) = \frac{1}{2} - \int_0^{\infty}\frac{1}{\pi t}\,\mathrm{Im}\left(e^{-it\frac{\theta_m}{\gamma_u}}\prod_{s=1}^{m}\left(1 + it\frac{\widehat{\sigma}_{R,s}^2}{\gamma_u\widetilde{\sigma}_{R,s}^2}e^{\frac{1-it\widehat{\sigma}_{R,s}^2}{\gamma_u\widetilde{\sigma}_{R,s}^2}}E_1\left(\frac{1-it\widehat{\sigma}_{R,s}^2}{\gamma_u\widetilde{\sigma}_{R,s}^2}\right)\right)\right)dt. \tag{3.30}$$

3.7.1.2 Channel Inversion Protocol

CI scheme consists of sending the packet with the same rate by dividing the transmit power, P_u, by the predicted channel gain $|\widehat{h}_s^T|^2$, under the condition that $|\widehat{h}_s^T|^2 > \gamma_0$. Using these settings, the target rate is equal to $R_{CI}(s) = \alpha_s \log(1+\gamma_u)$ and the received SINR that is expressed in (Equation 3.2), is equal to $\beta_{CI,s} = \frac{\gamma_u|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2+\gamma_u|\widetilde{h}_s^R|^2}$. Thereby, the outage probability expression (Equation 3.27) is evaluated as

$$\eta_{CI,CC}(m) = \Pr\left[\sum_{s=1}^{m}\frac{\gamma_u|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2+\gamma_u|\widetilde{h}_s^R|^2} < \gamma_u\right], \tag{3.31}$$

which is equal to the CDF of $\sum_{s=1}^{m}\frac{\gamma_u|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2+\gamma_u|\widetilde{h}_s^R|^2}$ applied to γ_u.

We may notice that the random variable $\beta_{CI,s} = \frac{\gamma_u|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2+\gamma_u|\widetilde{h}_s^R|^2}$ is composed of two correlated random variables, such that $|\widehat{h}_s^R|^2$ and $|\widehat{h}_s^T|^2$, and an independent random variable from both of them, which is $|\widetilde{h}_s^R|^2$. Using the CDF expression of $\frac{h_1}{h_2+h_3}$ in Appendix B defined in (Equation B.13) for correlated h_1 and h_2, the CDF of $\beta_{CI,s}$ has the following expression

$$F_{\beta_{CI,s}}(x) = \frac{2x\left(\widehat{\sigma}_{T,s}^2 \widehat{\sigma}_{R,s}^2 (1-\rho_s^2) + \gamma_u \widetilde{\sigma}_{R,s}^2 \lambda_s(x/\gamma_u)\right)}{\lambda_s(x/\gamma_u)(\gamma_u \lambda_s(x/\gamma_u) + \gamma_u \widehat{\sigma}_{R,s}^2 - x\,\widehat{\sigma}_{T,s}^2 + 2x\gamma_u \widetilde{\sigma}_{R,s}^2)}, \text{ if } x \geq 0, \tag{3.32}$$

where $\lambda_s(x/\gamma_u)$ is defined in (Equation 3.23).

The PDF of $\beta_{CI,s}$ can be expressed as the first derivative of $F_{\beta_{CI,s}}(x)$ with respect to x

$$f_{\beta_{CI,s}}(x) = \frac{dF_{\beta_{CI,s}}}{dx}(x). \tag{3.33}$$

Thereafter, using the fact that $(\beta_{CI,s})_{1\leq s\leq m}$ are independent, the outage probability when coupling CI and CC is given by the following convolution product

$$\eta_{CI,CC}(m) = f_{\beta_{CI,m}} * \cdots * f_{\beta_{CI,2}} * F_{\beta_{CI,1}}(\gamma_u), \tag{3.34}$$

where " $*$ " denotes the convolution operator.

3.7.2 HARQ Incremental Redundancy Scheme

In IR, the receiver combines the previously received sub-codewords to attempt to decode the whole packet, a different expression of the outage probability is defined for IR rather than the expression of the previous two schemes, AR and CC, which is expressed in (Equation 3.12) as

$$\eta_{T_x,IR}(m) = \Pr\left[\sum_{s=1}^{m} \frac{\alpha_s \log(1+\beta_{T_x,s})}{R_{T_x}(s)} < 1\right], \qquad \text{for } T_x = \text{NC, CP, CI}. \tag{3.35}$$

In the following analysis, we substitute $\beta_{T_x,s}$ and $R_{T_x}(s)$ by their expression according to the transmitter protocol $T_x = NC$, CP, CI.

3.7.2.1 No CSI Protocol

In this case, the rate is fixed to R and the received SINR is evaluated by (Equation 3.1), which gives the expression of the outage probability as follows

$$\eta_{NC,IR}(m) = \Pr\left[\sum_{s=1}^{m} \alpha_s \log\left(1 + \frac{\gamma_u|\widehat{h}_s^R|^2}{1+\gamma_u|\widetilde{h}_s^R|^2}\right) < R\right]. \tag{3.36}$$

It appears that the outage probability is equal to the CDF of $\sum_{s=1}^{m} \alpha_s \log(1 + \beta_{NC,s})$ evaluated at R. Since all the channel coefficients are independent for $s = 1, \cdots, m$, the CDF of that sum applied to x, $F_{SL}(x, \cdot, \cdot, \cdot)$, can be expressed as a convolution product, as described in Appendix A, and is given in (Equation A.12). Thus, the outage probability using IR and NC protocols is equal to

$$\eta_{NC,IR}(m) = F_{SL}(R, \widehat{\Sigma}_m^R, \widetilde{\Sigma}_m^R, \mathbf{A}_m), \tag{3.37}$$

where $\widehat{\Sigma}_m^R = \gamma_u \left[\widehat{\sigma}_{R,1}^2, \cdots, \widehat{\sigma}_{R,m}^2\right]$, $\widetilde{\Sigma}_m^R = \gamma_u \left[\widetilde{\sigma}_{R,1}^2, \cdots, \widetilde{\sigma}_{R,m}^2\right]$, and $\mathbf{A}_m = [\alpha_1, \cdots, \alpha_m]$.

3.7.2.2 Constant Transmit Power Protocol

When deploying CP protocol, the received SINR is equal to the SINR of NC protocol, i.e. $\beta_{CP,s} = \frac{\gamma_u|\widehat{h}_s^R|^2}{1+\gamma_u|\widetilde{h}_s^R|^2}$, while the target rate varies with the channel, as derived in (Equation 3.7), and equals to $R_{CP}(s) = \alpha_s \log(1+\gamma_u |\widehat{h}_s^T|^2)$. Therefore, using its expression in (Equation 3.35), the outage probability of CP and IR is given by

$$\eta_{CP,IR}(m) = \Pr\left[\sum_{s=1}^{m} \frac{\log(1+\beta_{CP,s})}{\log(1+\gamma_u |\widehat{h}_s^T|^2)} < 1\right]. \tag{3.38}$$

From that definition, $\eta_{CP,IR}(m)$ is equal to the CDF of $\sum_{s=1}^{m} \frac{\log(1+\beta_{CP,s})}{\log(1+\gamma_u |\widehat{h}_s^T|^2)}$ evaluated at 1. Noticing that the random variables $LH_s = \frac{\log(1+\beta_{CP,s})}{\log(1+\gamma_u |\widehat{h}_s^T|^2)}$ are independent for $s = 1, \cdots, m$, the outage probability can be written as a convolution product of the PDF of LH_s, $f_{LH_s}(\cdot)$, and its CDF, $F_{LH_s}(\cdot)$, as follows

$$\eta_{CP,IR}(m) = f_{LH_m} * \cdots * f_{LH_2} * F_{LH_1}(1), \tag{3.39}$$

where $f_{LH_s}(\cdot)$ and $F_{LH_s}(\cdot)$ are derived in Appendix B, in (Equation B.15) and (Equation B.16), respectively. In fact, the distribution of LH_s is obtained using the joint distribution of $|\widehat{h}_s^R|^2$ and $|\widehat{h}_s^T|^2$ and some change of variables as described in (Equation B.14).

3.7.2.3 Channel Inversion Protocol

In this case, the target rate is fixed to be $R_{CI}(s) = \alpha_s \log(1+\gamma_u)$, while the received SINR is defined in (Equation 3.2) as $\beta_{CI,s} = \frac{\gamma_u |\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2 + \gamma_u |\widetilde{h}_s^R|^2}$, subsequently the outage probability expression (Equation 3.35) can be expressed as

$$\eta_{CI,IR}(m) = \Pr\left[\sum_{s=1}^{m} \log\left(1 + \frac{\gamma_u |\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2 + \gamma_u |\widetilde{h}_s^R|^2}\right) < \log(1+\gamma_u)\right]. \tag{3.40}$$

Like the previous IR cases, the outage probability is equal to the CDF of $\sum_{s=1}^{m} L_{CI,s}$ evaluated at $\log(1+\gamma_u)$, where $L_{CI,s} = \log\left(1 + \frac{\gamma_u|\widehat{h}_s^R|^2}{|\widehat{h}_s^T|^2+\gamma_u|\widehat{h}_s^R|^2}\right)$. Actually, $(L_{CI,s})_{1\leq s\leq m}$ are independent over s, so the outage probability is expressed as a convolution product of the PDF and CDF of $L_{CI,s}$. Using the CDF and PDF of $\beta_{CI,s}$ derived in (Equation 3.32) and (Equation 3.33), respectively, and a change of variable, the CDF and PDF of $L_{CI,s}$ can be written as

$$F_{L_{CI,s}}(x) = F_{\beta_{CI,s}}(e^x - 1) \tag{3.41}$$

$$f_{L_{CI,s}}(x) = e^x f_{\beta_{CI,s}}(e^x - 1). \tag{3.42}$$

Therefore, the outage probability of using the couple protocol (CI,IR) can be deduced as

$$\eta_{CI,IR}(m) = f_{L_{CI,m}} * f_{L_{CI,m-1}} * \cdots * f_{L_{CI,2}} * F_{L_{CI,1}}\left(\log(1+\gamma_u)\right). \tag{3.43}$$

3.8 Simulation Results

In this section, we first evaluate the tradeoff of training and data in transmissions. We study the impact of training and CSI feedback in different protocols. Then, we optimize the throughput over the training phase duration and CSI phase duration, $T_{t,s}$ and $T_{CSI,s}$, respectively for all re-transmissions. We also validate our derived results with Monte Carlo simulation results. Finally, we investigate how throughput changes with regard to channel correlations, i.e., different mobility, in both FDD and TDD scenarios. We set the maximum number of transmissions to be $M = 3$ and packet size $T = 50$.

3.8.1 Impact of Training effect on throughput

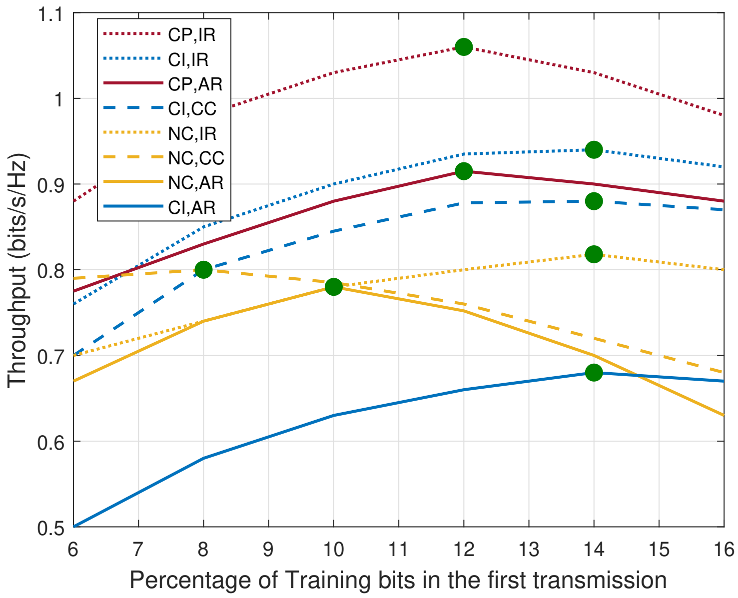


Figure 3: Impact of first transmission training on throughput for different protocols, with SNR= 10 dB, $\rho_d = 0.9$, and $T = 50$. Green circles mark the optimal value of throughput.

When the packet size is fixed, adding more training in the packet reduces the estimation error but reduces the CSI feedback and data length. The throughput could decrease due to redundant training phase and insufficient data phase. Figure 3 shows the impact of the first transmission training on the system throughput, while the training of second and third re-transmission are optimized. We plot the throughput versus the percentage of training symbols

	$\rho_d = 0.9$ fast fading				$\rho_d = 0.1$ slow fading			
	1st	2nd	3rd	Average	1st	2nd	3rd	Average
NC, AR	10	10	10	10	10	10	10	10
NC, CC	8	8	6	7	8	8	6	7
NC, IR	10	8	8	9	10	8	8	9
CP, AR	12	12	12	12	32	32	32	32
CP, IR	12	12	10	11	38	32	32	33
CI, AR	14	14	14	14	32	32	32	32
CI, CC	14	14	14	14	32	30	30	31
CI, IR	14	14	14	14	32	32	26	30

TABLE I: PERCENTAGE OF TRAINING IN EACH TRANSMISSION FOR TWO CHANNEL CORRELATION COEFFICIENT VALUES $\rho_d = 0.9, 0.1$ AND SNR = 10DB.

in the first transmission for the different transmitter and receiver protocols for SNR equals to 10 dB. The impact of training phase length is clearly shown in the figure. The throughput value increases in the beginning and decreases after the optimal point.

To further study the impact of training length at different re-transmissions and protocols, we provide the optimal training phase length at each transmission in Table I for different channel varying conditions, i.e. different values of ρ_d. A larger value of ρ_d indicates a faster varying channel. It is clear from Table I that NC protocol (i.e. fixed rate and power transmission) needs less training than CP and CI. CP and CI need CSI at the transmitter to update the transmitting power or rate. The transmitter relies on the CSI feedback from the receiver through a noisy channel, which is an estimation of the CSI at the receiver. Since, the channel estimation error at the transmitter side is closely related to the channel estimation error at the receiver, the two adaptive transmission protocols are more sensitive to the channel estimation error. Therefore, adding more training for CP and CI to improve the CSI estimation, is worth the loss of data phase length.

HARQ protocols are also compared in Table I, we notice that CC and IR demand more training in the first transmission. However in the second and third transmissions they require less. This can be explained by the fact that AR scheme discards the whole packet if it is not successfully decoded, while CC and IR merge the previous packets to improve the decoding process. More training in the first transmission for CC and IR helps the following re-transmissions, which is one of the tradeoffs that is specific to two-way communication with re-transmission protocols that should be taken into account when optimizing the throughput.

To study the impact of the channel time-variation (how fast the channel varies by (Equation 2.6)) we compare the required training length for different transmitter protocols with different values of ρ_d in Table I. Since NC does not need CSI at the transmitter and CSI estimation is not affected by the channel varying condition, the number of CSI training symbols remains the same for all NC results. As expected, adaptive transmission protocols, CP and CI require less training for slow varying channels and more training for fast varying channels. For fast varying channels, increasing the training will reduces the CSI estimation error at the transmitter (Equation 2.12) and, is worth the loss of the data.

3.8.2 Impact of CSI feedback

The optimal CSI feedback symbols allocation is shown in Table II for several protocols, two values of SNR, and two channel time-varying conditions. Since NC does not need CSI at the transmitter, the results focus only on the two adaptive transmission protocols CP and CI.

	0 dB, Fast fading			0 dB, Slow fading			5 dB, Fast fading			5 dB, Slow fading		
CP, AR	18	18	18	6	6	6	6	6	6	2	2	2
CP, IR	18	18	18	10	10	10	6	6	10	2	2	2
CI, AR	18	18	18	6	6	6	6	6	6	2	2	2
CI, CC	14	14	18	6	10	10	6	6	6	2	2	2
CI, IR	14	14	18	6	10	10	6	6	6	2	2	2

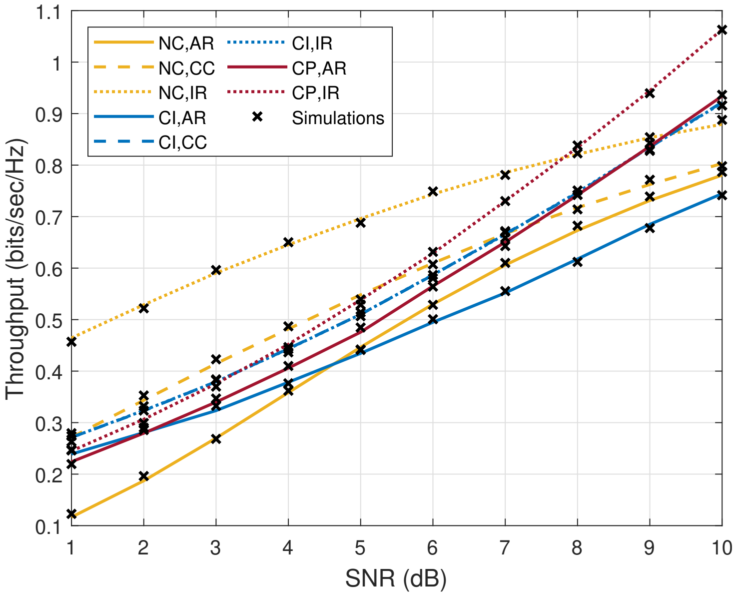
TABLE II: OPTIMAL PERCENTAGE OF CSI FEEDBACK SYMBOLS IN THREE TRANSMISSIONS FOR DIFFERENT VALUES OF SNR AND TWO CHANNEL VARYING CONDITIONS, SLOW FADING CHANNEL $\rho_d = 0.9$ AND FAST FADING CHANNEL $\rho_d = 0.1$.

During the simulation, we noticed that for high values of SNR, only one CSI feedback symbol is needed for SNR greater than 5 dB.

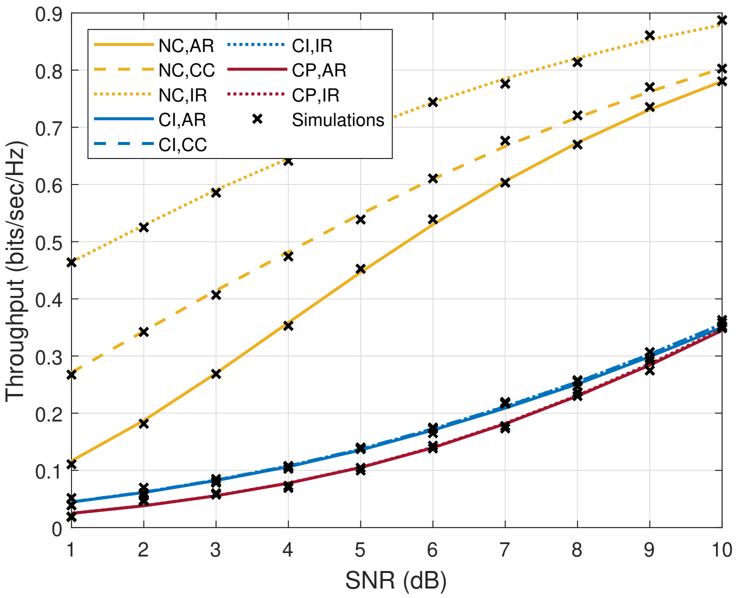
As expected, when the SNR increases, we need less CSI symbols. A similar CSI reduction is observed also when the channel is varying slowly, which means high channel correlation. Comparing Table I and Table II, the CP and CI require less CSI symbols than training symbols because the CSI feedback is used to transmit the estimated channel gain to the transmitter, therefore, an accurate channel gain estimation is prerequisite. Unlike the observation in the previous subsection Sec.3.8.1, where more training is needed in the first transmission for CC and IR, less CSI symbols are needed in the first transmission for most CC and IR protocols. Actually, more packet space has been allocated for training symbols, so it is not worth allocating more CSI symbols in the packet.

3.8.3 Throughput optimization

In this chapter, we considered eight transmitter and receiver protocol combinations in total. We will show how different combinations perform under different SNRs and channel varying conditions. Figure 4 depicts the optimal throughput as a function of the SNR for two channel



(a) $\rho_d = 0.6$



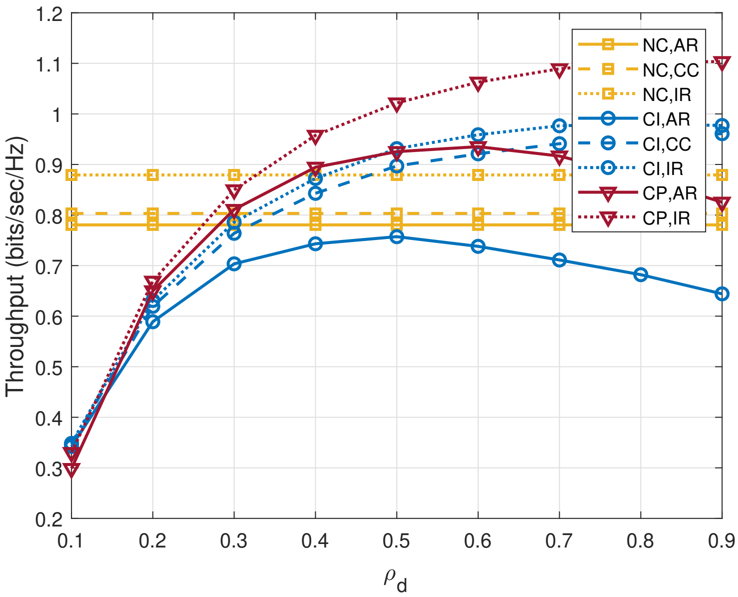
(b) $\rho_d = 0.1$

Figure 4: Optimal throughput for the studied protocols with two values of channel correlation coefficient, (a) $\rho_d = 0.6$, and (b) $\rho_d = 0.1$. Cross markers show the monte carlo simulations that match analytical results.

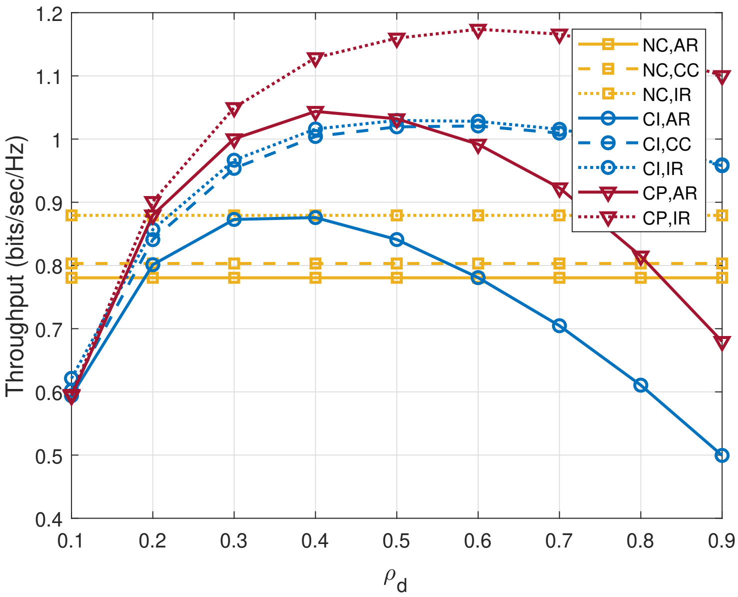
varying conditions, moderate varying channel corresponds to $\rho_d = 0.6$ and fast varying channel corresponds to $\rho_d = 0.1$. Figure 4 shows that NC protocol performs better than CI and CP for fast varying channels and in low to mid SNR for moderate varying channels, and conversely beyond that. In fact, in low SNR and moderate varying channel or moderate SNR and fast varying channel, the CSI at receiver and transmitter is not accurate, which reduces the throughput of transmitter protocols that requires an accurate CSI feedback, such that CP and CI. Therefore, in those regimes, we should not use the adaptive rate or power protocols (CP and CI) and adopt the simple NC protocol.

At the receiver side, the HARQ schemes have better performance than basic ARQ. The IR scheme outperforms the CC scheme for all values of SNR and in both channel varying conditions, which is similar to the results described in (Larsson et al., 2013). It is worth mentioning that for good channel conditions (high SNR and high channel correlation), the combination CP-IR outperforms all other protocol combination by 0.15 bits/s/Hz for $\rho_d = 0.6$. The close performance of receiver protocols (AR, CC, and IR) in Figure 4(b) is due to the fact that for $\rho_d = 0.1$ the channel is nearly uncorrelated, so the system requires more training to cover up this miss. It is also explained in Table I and Table II, which makes the performance of CC and IR converge toward the performance of AR.

We illustrate the effect of channel varying conditions in Figure 5, on the behavior of the optimal throughput with fixed value of SNR = 10 dB for both FDD and TDD cases. The throughput is constant for various ρ_d for NC because NC does not require any CSI feedback to the transmitter. As the result described in the previous figure, the combination CP-IR gives the best throughput over all other combinations for good channel conditions, especially for $\rho_d \geq 0.35$ in FDD and $\rho_d \geq 0.2$ in TDD. Note that the throughput of CP and CI with AR decreases when ρ_d is increasing. The target rate $R_{Tx}(m)$ of CP and CI increases with channel correlation ρ_d. Therefore, the outage probability increases for high ρ_d for CP and CI. We also notice that the overall throughput, which is proportional to the expected rate and inversely proportional to the outage probability, decreases when ρ_d gets closer to 1. Indeed, an asymptotic analysis of the expected rate of CP (Equation 3.9) and CI (Equation 3.10) shows that the expected rate has a logarithmic behavior versus $\widehat{\sigma}^2_{T,s}$. However, the outage probability expressions of AR couples with CP and CI, defined in (Equation 3.25) and (Equation 3.22) respectively, have a linear relation with respect to $\widehat{\sigma}^2_{T,s}$. Since the channel estimator power $\widehat{\sigma}^2_{T,s}$ is increasing with respect to ρ_d (Equation 2.15) and the throughput is expressed as the fraction between the expected rate and the sum of the outage probabilities (Equation 3.17), it follows that the throughput expression has a nominator with logarithmic behavior versus ρ_d and denominator with linear attitude versus ρ_d, i.e. $\nu \sim C_1 \frac{\log(1+C_2\rho_d)}{\rho_d}$, where C_1 and C_2 are constants. Therefore, at some point, the throughput changes behavior with respect to ρ_d from increasing to decreasing for basic ARQ protocols. This throughput decreasing behavior becomes more severe for TDD. For TDD, in (Equation 2.14), the correlation coefficient ρ_s between $\widehat{h}_s^R$ and $\widehat{h}_s^T$ is larger than in the FDD case. Therefore, in (Equation 3.22) and (Equation 3.24), when $\widehat{h}_s^R$ and $\widehat{h}_s^T$, outage probability will increase more, thus additional re-transmissions are required to successfully deliver a message.



(a) FDD



(b) TDD

Figure 5: Impact of the channel variation on the optimal throughput for SNR = 10 dB, with different protocols, for (a) FDD , and (b) TDD.

CHAPTER 4

THROUGHPUT PERFORMANCE OF TWO-WAY WIRELESS COMMUNICATION WITH COMBINED CSI AND ARQ FEEDBACK WITH FINITE PACKET LENGTH

4.1 Introduction

In order to support low-latency applications like remote surgery and smart factory automation in the fifth generation (5G) mobile wireless networks, the finite block-length coding technique has been developed for reliable and delay restricted applications. These low-latency applications usually require low latency in order of milliseconds and the packet size is required to be short.(Devassy et al., 2019).

Shannon's capacity provides the achievable rate with zero error probability under the assumption of unbounded block-length(Tse and Viswanath, 2005b). However, to achieve milliseconds order latency, a packet can only contain finite number of symbols which results in a rate loss. The exact channel capacity of finite block-length packet is unknown but tightly approximated in (Polyanskiy et al., 2010).

Re-transmission is inevitable in reliable wireless systems. ARQ and HARQ are highly adopted in current generation wireless communication systems. HARQ has been proven to be an efficient way to increase spectrum efficiency.

4.1.1 Related Work

Under the assumption of perfect CSI, the performance of wireless communications in finite block-length regime has been studied extensively. The throughput performance of IR-HARQ has been studied in (Makki et al., 2014). The authors derived bounds and an approximated closed form result of throughput. Also, in the imperfect regime, an achievable rate bound of MIMO Systems with Imperfect CSI in the Finite Length Regime is studied in(Potter et al., 2013). More recently, the authors of (Schiessl et al., 2018) defined and derived a random blocklength-equivalent capacity with imperfect CSI at the transmitter and finite block-length. However, to the best of our knowledge, the throughput of HARQ under the imperfect CSI and finite block-length settings has not been studied yet.

4.1.2 Channel Model and Estimation

We consider a single antenna system with classic frequency-flat Rayleigh fading channel. At the m-th transmission, the total of T symbols are transmitted to the receiver through the fading channel. T symbols are split to two phases: the pilot symbols of length $T_{t,s}$, which are used to estimate the channel, and the data symbols with length $T_{d,s}$ which contain the actual data to be transmitted to the receiver. The transmission process can be modeled as

$$y_s(i) = h_s x(i) + w_s(i), \quad i = 1, 2, ..., T,$$

where h_s, denotes the channel fading coefficient at the s-th transmission, following a circularly symmetric complex Gaussian random distribution $\mathcal{CN}(0, 1)$. $x(l)$ is the input symbol and the noise $w_s(i) \sim \mathcal{CN}(0, N_0)$. The channel input is subject to the average power constraint $\mathbb{E}[x^2(i)] \leq P_u$. As discussed in the previous chapters, the MMSE estimator of the channel gain h_s at the receiver is given by (Kay, 1993, Eq. (10.31)) as

$$\widehat{h}_s^R = \frac{\sqrt{P_u}}{N_0 + T_{t,s} P_u} \sum_{i=1}^{T_{t,s}} y_s(i). \tag{4.1}$$

It follows from (Equation 2.2) and (Equation 2.3) that $\widehat{h}_s^R$ is a complex Gaussian random variables with zero mean and variance $\widehat{\sigma}_{R,s}^2$ that is given by

$$\widehat{\sigma}_{R,s}^2 = \frac{T_{t,s} P_u}{N_0 + T_{t,s} P_u} = \frac{T_{t,s} \gamma_u}{1 + T_{t,s} \gamma_u}, \tag{4.2}$$

where $\gamma_u = \frac{P_u}{N_0}$. The channel estimation error at the receiver is equal to $\widetilde{h}_s^R = h_s - \widehat{h}_s^R$, which is a circularly symmetric complex Gaussian random variable with zero mean and variance $\widetilde{\sigma}_{R,s}^2$. Using the orthogonality between the MMSE estimator and the error (Kay, 1993), $\widehat{h}_s^R$ and $\widetilde{h}_s^R$ are independent and the mean square error (MSE) of h_s, $\widetilde{\sigma}_{R,s}^2$, is equal to

$$\widetilde{\sigma}_{R,s}^2 = 1 - \widehat{\sigma}_{R,s}^2 = \frac{1}{1 + T_{t,s} \gamma_u}. \tag{4.3}$$

4.1.3 Information outage probability on finite block-length

The information outage probability is an important performance metric in wireless communication protocol evaluation. Traditionally, we define an outage event that the mutual information between the channel output and the channel input does not cross a target rate R.

When optimal inputs distribution is assumed, the mutual information is equal to the maximal achievable rate for a given error ϵ, which is also known as $\epsilon-$capacity(Koga and others, 2013). An excellent approximation of the maximal achievable rate of a single packet over finite block-length block fading channels is given by (Yang et al., 2013)

$$I \approx \log(1 + \beta_{NC,s}) - \sqrt{\frac{\mathcal{V}(\beta_{NC,s})}{T}} Q^{-1}(\epsilon), \tag{4.4}$$

where $\beta_{NC,s} = \frac{\gamma_u |\widehat{h}_s^R|^2}{1+\gamma_u |\widetilde{h}_s^R|^2}$ and $\mathcal{V}(x) = \sqrt{1 - \frac{1}{(1+x)^2}}$. More formally, we follow the definition of the event $\mathcal{A}_m := \{I_m > R\}$, where I_m is the mutual information after m-th transmission. Under the assumption of Gaussian inputs and noise, using (Equation 4.4), the approximations of mutual information of three (H)ARQ schemes are

$$\begin{aligned} \text{Basic ARQ}: \ I_{AR} &\approx \log(1 + \beta_{NC,s}) - \sqrt{\frac{\mathcal{V}(\beta_{NC,s})}{T}} Q^{-1}(\epsilon). \\ \text{IR-HARQ}: \ I_{IR} &\approx \sum_{s=1}^{m} \log(1 + \beta_{NC,s}) - \sqrt{\frac{\mathcal{V}(\beta_{NC,s})}{T}} Q^{-1}(\epsilon). \end{aligned}$$

4.2 Joint Information Outage analysis of imperfect CSI and (H)ARQ

Traditionally, the outage probability is defined as the the probability that the target rate is above the channel achievable rate. This definition actually assumes that when the block-length is sufficiently large, the error probability is arbitrarily small given the target rate is lower than the achievable rate. However, as discussed in (Yang et al., 2014), the traditional definition of outage probability does not coincide with the real packet error probability when the block-length is small. Indeed, the lowest packet outage probability we can achieve is the block error rate ϵ. Therefore, using (Equation 4.4), we may rewrite the approximated outage probability of one transmission as (Yang et al., 2014, Eq. (59))

$$\eta_{\text{out}}(m) = \mathbb{E}\left[\mathbf{Q}\left(\frac{\sqrt{T}(\log(1+\beta_{NC,s}) - \frac{R}{\alpha_s})}{\sqrt{1 - \frac{1}{(1+\beta_{NC,s})^2}}}\right)\right]. \tag{4.5}$$

Re-transmission of a packet is necessary when the receiver fails to decode the received packet. Two re-transmission patterns are considered in this chapter. 1) Basic ARQ discards previous packet if the received packet fails to be decoded and send a re-transmission request to the transmitter. 2) Incremental redundancy ARQ re-transmit different information in the new packet packet than the previous one. A parent codeword is generated and divided to sets of sub-codewords, and the sub-codewords are transmitted in different IR-HARQ rounds.

4.2.1 Outage probability of basic ARQ

Assume that a packet is transmitted m times before being successfully received, the outage probability can be expressed as

$$\eta_{\text{out,AR}}(m) = \prod_{s=1}^{m}\mathbb{E}\left[\mathbf{Q}\left(\frac{\sqrt{T}(\log(1+\beta_{NC,s}) - \frac{R}{\alpha_s})}{\sqrt{1 - \frac{1}{(1+\beta_{NC,s})^2}}}\right)\right]. \tag{4.6}$$

For Rayleigh fading channels, to find the expectation over the random variable $\beta_{NC,s}$, we need to first derive the statistics of $\beta_{NC,s}$.

$$F_{\beta_{NC,s}}(x) = \mathbf{Pr}\left[\frac{\gamma_u|\widehat{h}_s^R|^2}{1+\gamma_u|\widetilde{h}_s^R|^2} < x\right] = 1 - \frac{1}{1+\frac{\sigma^2_{\widetilde{h}_s^R}}{\sigma^2_{\widehat{h}_s^R}}x} e^{-\frac{x}{\gamma_u \sigma^2_{\widehat{h}_s^R}}}, \tag{4.7}$$

$$f_{\beta_{NC,s}}(x) = \left(\frac{\sigma^2_{\widehat{h}_s^R}\sigma^2_{\widetilde{h}_s^R}}{(\sigma^2_{\widehat{h}_s^R}+\sigma^2_{\widetilde{h}_s^R}x)^2} + \frac{1}{(\sigma^2_{\widehat{h}_s^R}+\sigma^2_{\widetilde{h}_s^R}x)}\right) e^{-\frac{x}{\gamma_u \sigma^2_{\widehat{h}_s^R}}}. \tag{4.8}$$

Using (Equation 4.7), we can derive the outage probability as

$$\eta_{\mathrm{out,AR}}(m) = \prod_{s=1}^{m}\int_0^\infty \mathbf{Q}\left(\frac{\sqrt{T}(\log(1+x)-\frac{R}{\alpha_s})}{\sqrt{1-\frac{1}{(1+x)^2}}}\right)\left(\frac{\sigma^2_{\widehat{h}_s^R}\sigma^2_{\widetilde{h}_s^R}}{(\sigma^2_{\widehat{h}_s^R}+\sigma^2_{\widetilde{h}_s^R}x)^2} + \frac{1}{(\sigma^2_{\widehat{h}_s^R}+\sigma^2_{\widetilde{h}_s^R}x)}\right) e^{-\frac{x}{\gamma_u \sigma^2_{\widehat{h}_s^R}}}dx \tag{4.9}$$

To the best of our knowledge, the integration cannot be solved in closed form. Therefore, the Q-function can be approximated by using the Taylor expansion

$$\mathbf{Q}(\mathbf{x}) \approx \begin{cases} 1, & x \le \theta_m - \sqrt{\frac{\pi}{2b_m^2}} \\ \frac{1}{2} - \frac{b_m}{\sqrt{2\pi}}(x-\theta_m), & \theta_m - \sqrt{\frac{\pi}{2b_m^2}} < x \le \theta_m + \sqrt{\frac{\pi}{2b_m^2}} \\ 0, & x \ge \theta_m + \sqrt{\frac{\pi}{2b_m^2}}, \end{cases}$$

where $\theta_m = e^{\frac{R}{\alpha_s}} - 1$ and $b_m = \sqrt{\frac{T}{e^{\frac{2R}{\alpha}} - 1}}$. Therefore the outage probability of basic ARQ can be approximated by

$$\begin{aligned}\eta_{\text{out,AR}}(m) \approx \prod_{s=1}^{m} \left(\frac{b_m}{\sqrt{2\pi}} + \frac{2}{3} \right) & \left(1 - \frac{\sigma^2_{\widehat{h}^R_s} e^{\frac{\frac{\sqrt{2\pi}}{b_m} - 2\theta_m}{2\sigma^2_{\widehat{h}^R_s}}}}{\sigma^2_{\widetilde{h}^R_s} \left(\theta_m - \frac{\sqrt{2\pi}}{b_m} \right) + \sigma^2_{\widehat{h}^R_s}} \right) - \frac{\sigma^2_{\widehat{h}^R_s} b_m}{\sigma^2_{\widetilde{h}^R_s} \sqrt{2\pi}} \left(\left(e^{\frac{\sqrt{2\pi}}{b_m \sigma^2_{\widehat{h}^R_s}}} - 1 \right) e^{\frac{\frac{\sqrt{2\pi}}{b_m} + 2\theta_m}{2\sigma^2_{\widehat{h}^R_s}}} \right. \\ & - e^{\frac{1}{\sigma^2_{\widetilde{h}^R_s}}} \operatorname{Ei} \left(\frac{\frac{\sqrt{2\pi}}{b_m} - \theta_m}{\sigma^2_{\widehat{h}^R_s}} - \frac{1}{\sigma^2_{\widetilde{h}^R_s}} \right) + e^{\frac{1}{\sigma^2_{\widetilde{h}^R_s}}} \operatorname{Ei} \left. \left(\frac{\frac{\sqrt{2\pi}}{b_m} + \theta_m}{\sigma^2_{\widehat{h}^R_s}} - \frac{1}{\sigma^2_{\widetilde{h}^R_s}} \right) \right) \\ & - \frac{b_m \sigma^4_{\widehat{h}^R_s}}{\sigma^2_{\widetilde{h}^R_s}} e^{\frac{\frac{\sqrt{2\pi}}{b_m} + 2\theta_m}{2\sigma^2_{\widehat{h}^R_s}}} \left(- \frac{e^{\frac{\sqrt{2\pi}}{b_m \sigma^2_{\widehat{h}^R_s}}}}{\sigma^2_{\widetilde{h}^R_s} \left(\sqrt{\frac{\pi}{2}} \frac{1}{b_m} - \theta \right) + \sigma^2_{\widehat{h}^R_s}} + \frac{1}{\sigma^2_{\widetilde{h}^R_s} \left(\sqrt{\frac{\pi}{2}} \frac{1}{b_m} + \theta \right) + \sigma^2_{\widehat{h}^R_s}} \right).\end{aligned} \tag{4.10}$$

4.2.2 Outage probability of IR-HARQ

Unlike AR, during each transmission, IR sends a sub-codeword of length T_1 with the coding rate $R_{T_x}(1) = \frac{T_1}{\alpha_1 T}$. When a NACK is received by the transmitter, the transmitter sends the next part of sub-codeword of length T_2 with code rate $R_{T_x}(2) = \frac{T_2}{\alpha_2 T}$. This process continues until an ACK is received or the maximum number of transmissions, M, is reached. Therefore, the packet is decoded successfully only if all the sub-codewords can be decoded successfully on average after the receiver combines all received packets. The outage probability can be expressed as

$$\eta_{\text{out,IR}}(m) = \mathbb{E}\left[\mathbf{Q} \left(\frac{\sqrt{mT} \left(\sum_{i=1}^{m} \log(1 + \beta_{NC,s}) - \frac{R}{\alpha_m} \right)}{\sum_{i=1}^{m} \sqrt{1 - \frac{1}{(1+\beta_{NC,s})^2}}} \right) \right].$$

The outage probability cannot be solved in closed form. However, based on the simulation results in Chapter 3, we find that the optimal number of training bits in each round is almost the same. In order to solve the outage probability in closed form, we assume that the number training bits are the same in each round. In this case, the outage probability can be solved by adopting the same approximation in (Equation 4.10) as

$$
\begin{aligned}
\eta_{\mathrm{out,IR}}(m) \approx \left(\frac{b_m}{\sqrt{2\pi}} + \frac{2}{3}\right) & \left(1 - \frac{\sigma^2_{\widehat{h}_s^R} e^{\frac{\frac{\sqrt{2\pi}}{b_m} - 2\theta_m}{2\sigma^2_{\widehat{h}_s^R}}}}{\sigma^2_{\widetilde{h}_s^R}\left(\theta_m - \frac{\sqrt{2\pi}}{b_m}\right) + \sigma^2_{\widehat{h}_s^R}}\right) - \frac{\sigma^2_{\widehat{h}_s^R} b_m}{\sigma^2_{\widetilde{h}_s^R}\sqrt{2\pi}} \left(\left(e^{\frac{\sqrt{2\pi}}{b_m \sigma^2_{\widehat{h}_s^R}}} - 1\right) e^{\frac{\frac{\sqrt{2\pi}}{b_m} + 2\theta_m}{2\sigma^2_{\widehat{h}_s^R}}}\right. \\
& \left. - e^{\frac{1}{\sigma^2_{\widetilde{h}_s^R}}} \mathrm{Ei}\left(\frac{\frac{\sqrt{2\pi}}{b_m} - \theta_m}{\sigma^2_{\widehat{h}_s^R}} - \frac{1}{\sigma^2_{\widetilde{h}_s^R}}\right) + e^{\frac{1}{\sigma^2_{\widetilde{h}_s^R}}} \mathrm{Ei}\left(\frac{\frac{\sqrt{2\pi}}{b_m} + \theta_m}{\sigma^2_{\widehat{h}_s^R}} - \frac{1}{\sigma^2_{\widetilde{h}_s^R}}\right)\right) \\
& - \frac{b_m \sigma^4_{\widehat{h}_s^R}}{\sigma^2_{\widetilde{h}_s^R}} e^{\frac{\frac{\sqrt{2\pi}}{b_m} + 2\theta_m}{2\sigma^2_{\widehat{h}_s^R}}} \left(-\frac{e^{\frac{\sqrt{2\pi}}{b_m \sigma^2_{\widehat{h}_s^R}}}}{\sigma^2_{\widetilde{h}_s^R}\left(\sqrt{\frac{\pi}{2}}\frac{1}{b_m} - \theta\right) + \sigma^2_{\widehat{h}_s^R}} + \frac{1}{\sigma^2_{\widetilde{h}_s^R}\left(\sqrt{\frac{\pi}{2}}\frac{1}{b_m} + \theta\right) + \sigma^2_{\widehat{h}_s^R}}\right),
\end{aligned}
\tag{4.11}
$$

where $\theta_m = e^{\frac{R/m}{\alpha_m}} - 1$ and $b_m = \sqrt{\frac{mT}{e^{\frac{2R/m}{\alpha_m}} - 1}}$.

4.3 Throughput Analysis

In order to derive the throughput, we need an expression for average rate per transmission, which depends on the the probability of outage. Follow the definition of (Equation 3.17), the total system throughput can be expressed as

$$
\nu_{\mathrm{sh},R_x} = \frac{\mathcal{R}_{\mathrm{sh},R_x}}{\mathcal{T}_{\mathrm{sh},R_x}}
$$

where $R_x \in \{AR, IR\}$, assuming maximum M transmissions, the expected rate is

$$\mathcal{R}_{sh,R_x} = \sum_{s=1}^{M} R\left(\eta_{out,R_x}(m-1) - \eta_{out,R_x}(m)\right),$$

and the expected number of transmission per packet is

$$\mathcal{T}_{sh,R_x} = \sum_{m=0}^{M-1} \eta_{out,R_x}(m)$$

4.4 Simulation Results

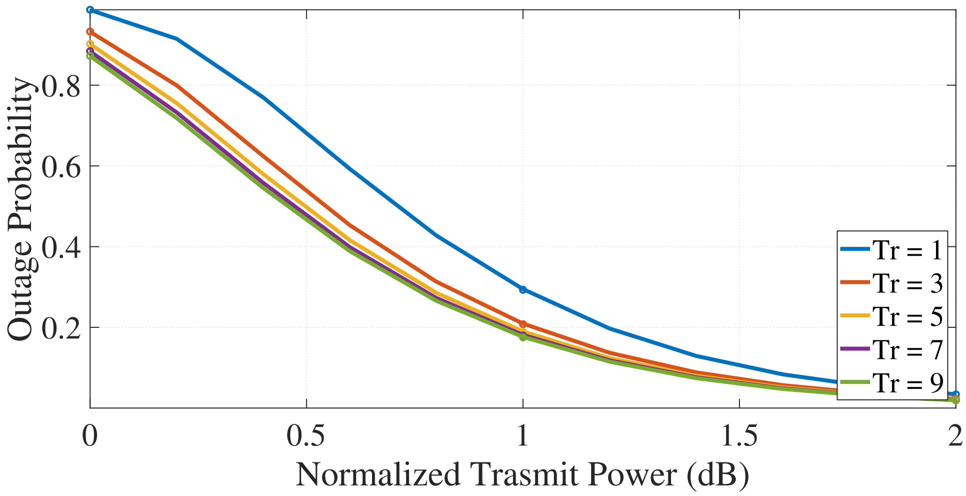
In this section, we simulate the throughput performance results derived previously over a block Rayleigh fading channel. We set the maximum number of transmissions to be $M = 3$.

First we plot and validate the approximation of the outage probability under different transmit power (i,e., γ_u) and various target rates in Figure 6. As shown in the figure, we mark the actual outage probabilities using circles and the approximated values are plotted using curves. The approximation is tight under low to mid SNRs. As reported in (Devassy et al., 2019), the LTE standards employ codes with block-length as short as 100 symbols. We select the packet size T to be 100 to show that our approximation is tight enough even the packet is short. Actually, our approximation becomes tighter when the size of the packet increases. Also, we validate our approximations under different target rates in Figure 6(a) and Figure 6(b). We notice that the outage probability decreases dramatically when the training sizes increase from 1 to 3. The results also coincide with our conclusion in Chapter 3.

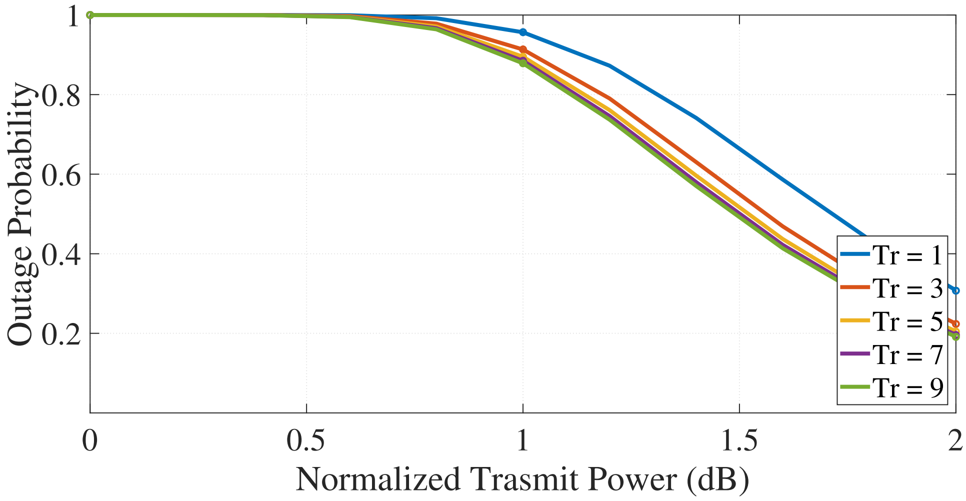
We also plot the throughput of basic ARQ and IR-HARQ systems in Figure 7. Note that the selected training sizes are near the optimal training size. Different from the results in Chapter 3, we find that under curtain packet size, the optimal number of training increases with SNR in mid to high SNR range. Actually, the term $\frac{\sqrt{T}(\log(1+\beta_{NC,s})-\frac{R}{\alpha_s})}{\sqrt{1-\frac{1}{(1+\beta_{NC,s})^2}}}$ inside the Q-function of (Equation 4.6) is a convex function which decreases in the beginning and increases later. Therefore, at curtain point, the behavior of optimal number of training changes with respect to the transmit power. Coincides with the results in Chapter 4, we also find that IR-HARQ shows better throughput performance than basic ARQ.

4.5 Conclusion

In this chapter, we have further expanded the generalized framework in two-way networks to adapt the effect of finite block-length. We derived closed form expressions for basic ARQ and IR-HARQ using a tight approximation. We investigated how finite block-length affects the optimal number of training compared to infinite block-length.

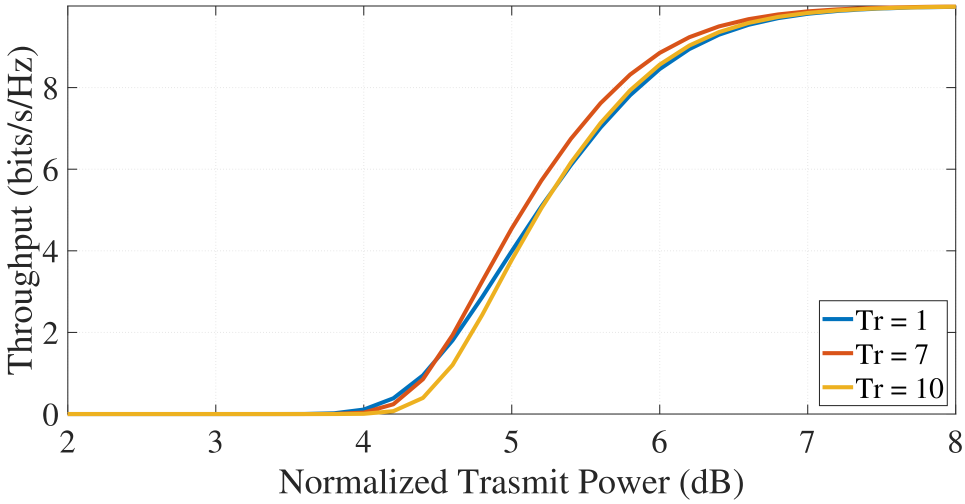


(a) Target rate $\mathsf{R} = 1$.

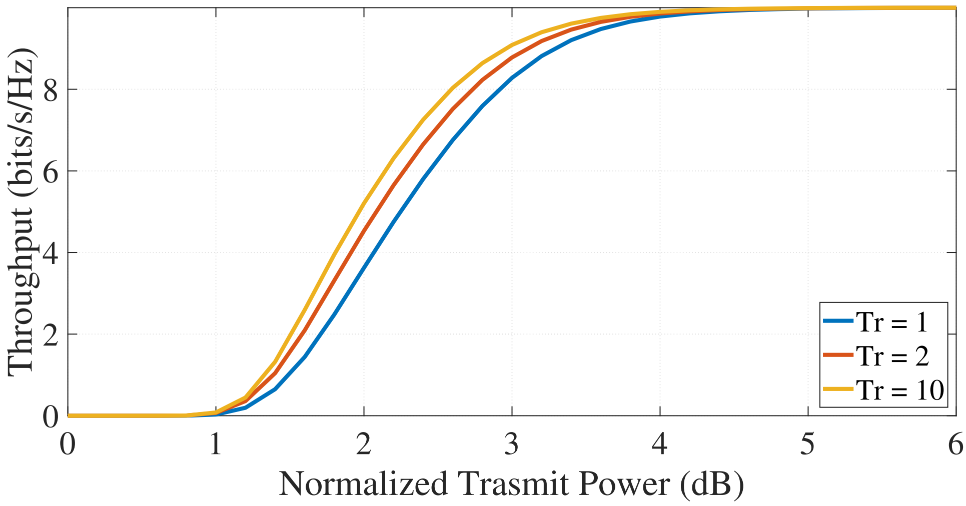


(b) Target rate $\mathsf{R} = 4$.

Figure 6: Outage probability versus normalized transmit power γ_{u} under two different target rates. Tr represents the training bits size.



(a) Basic ARQ



(b) IR-HARQ

Figure 7: Throughput versus normalized transmit power γ_u with packet size equal 100.

CHAPTER 5

ON PRACTICAL NETWORK CODED ARQ FOR TWO-WAY WIRELESS COMMUNICATION

The contents of this chapters are based on our work that is published in the IEEE International Conference on Communications (ICC) (Zhu et al., 2017)

5.1 Introduction

Network coding (NC) has attracted an increasing interest due to its improvement on throughput. The throughput benefit is derived by the efficient use of packet transmissions (Ho and Lun, 2008). For NC, more information can be communicated with fewer packet transmissions, which leads to an improved spectral efficiency (Fragouli, 2011; Keshavarz-Haddad and Riedi, 2014; Zeng et al., 2014). In a series of studies, NC has been proven useful for both fixed reliable wired systems (Lun et al., 2008) and noisy wireless systems (Larsson et al., 2006; Ahmad et al., 2018; Bouteggui et al., 2020; Li et al., 2018).

It has been proven that NC can improve the throughput of ARQ for both multicast channels (Nguyen et al., 2009; Ghaderi et al., 2007; Larsson, 2008) and broadcast channels (Osseiran et al., 2011; Tajbakhsh et al., 2013; Chiti et al., 2013). During re-transmissions, receivers may decode unintended packets as side-information. Then the transmitter can send a network coded packet (NCP) consisting of the packets retransmitted to several receivers who can extract their intended packets from the NCP with side-information. Some network coded (NCed) ARQ schemes have been proposed and proved to be highly effective in improving throughput (Lang et al., 2012; Antonopoulos and Verikoukis, 2012; Larsson et al., 2013). A random linear network coding (RLNC) scheme with ARQ was proposed in (Lucani et al., 2009; Ghanem, 2013) to strengthen the throughput by reducing the transmission delay. A new NC scheme with reverse-link-assistance (RLA) for two-way wireless systems was proposed by (Zhu et al., 2016).

In existing literature, NCed-ARQ throughput was obtained by assuming no overhead during transmissions. However, in real wireless communication systems, the transmitter needs to know exactly what unintended packet has been overheard by several receivers. Thus more feedback between transmitters and receivers are needed(Larsson and Johansson, 2006). The different types of needed feedback have been introduced in (Sundararajan et al., 2009). Similarly, the control information needed to implement NC in multicast channels was also studied in (Sagduyu and Ephremides, 2007), but the resources dedicated to feedback were not taken into account when deriving the throughput. Therefore, the effect of extra overhead to the overall system throughput should be carefully studied. In addition, in a multiple-access-broadcast channel (MABC) with M users, when the users are exchanging packets through a base station , NCed-ARQ improves the performance of the downlink (DL) but the extra-feedback are sent over through the uplink (UL). By allocating more resources to feedback in the uplink packet, we decrease the resources dedicated to the uplink data. Thus, there is a tradeoff between the UL and DL throughput, and this tradeoff is considerably affected by the number of end-users, signal-to-noise ratio (SNR), and size of ARQ feedback.

5.2 Preliminary

We consider an M-user MABC as depicted for $M = 2$ in Figure 9. This model system, in which several end-users wish to exchange messages with a central node, or base-station, is a model that captures the behavior of current and future cellular networks. We assume that the system uses time division duplexing (TDD). During the broadcast period the base-station transmits packets sequentially to the terminal nodes. Then the end-users transmit packets to the base-station sequentially.

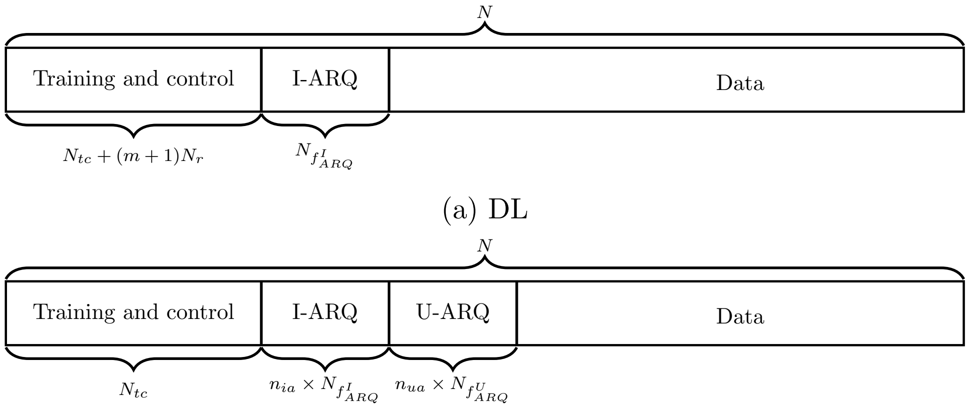
5.2.1 Channel Model

In this chapter, we consider symmetric DL and UL block fading channels with complex valued Gaussian distributed signals and additive white Gaussian Noise. We also assume the channels are completely symmetric with respect to each user i.e., all the involved DL and UL channels undergo identical independent distributed (i.i.d) Rayleigh fading and Gaussian noise. Therefore for both DL and UL, the output of i^{th} symbol of k^{th} transmission of a packet is given by:

$$r_k(i) = h_k x_k(i) + w_k(i), \tag{5.1}$$

where $r_k(i)$ is the received signal, h_k is the channel gain distributed as $\mathcal{CN}(0, 1)$, $x_k(i)$ is the transmitted signal, and $w_k(i)$ is additive white Gaussian noise with mean zero and variance N_0. The capacity for each DL and UL channels is given by

$$C_k = \ln(1 + |h_k|^2 P_l / N_0), l \in \{d, u\} \tag{5.2}$$



(a) DL

(b) UL

Figure 8: Packet structure

where $\mathsf{P_d}$ and $\mathsf{P_u}$ are the DL and UL transmit power, respectively. Hence, $\mathsf{P_d/N_0}$ and $\mathsf{P_u/N_0}$ are their transmit SNR.

5.2.1.1 Scheduling

The NCed-ARQ system consists of one base-station and M end-users. We define a scheduling scheme that the transmission starts from M DL transmissions and followed by one UL transmission per end-user to base-station. We define this procedure as a round of transmission. After a full round of transmission finishes, another round starts.

5.2.1.2 Protocol

With NCed-ARQ, the DL transmission is divided into two phases(Zhu et al., 2016). In the first phase, only DL packets containing RPs are sent from the base-station to end-users. Non-targeted end-users overhear and save some unintended RPs from the base-station. During the first phase, after a packet is sent to its target user, the user sends an intended ACK/NACK back during UL. Also, a packet may be overheard by other users, according to the scheme, the overheard user needs to send an ACK in the UL packet to base-station during its UL time-slot. After each DL packet with RP has been decoded by at least one user, phase II begins. In the second phase, base-station only sends DL packets with NCPs through DL and end-users respond UL packets. When a user successfully decodes its own packet, the user sends an intended ACK to the base-station. Notice UL transmission does not apply NC therefore only RPs are sent during UL.

5.2.1.3 Example ($M = 2$ case)

Figure 9 illustrates NCed-ARQ in the case of $M = 2$ end-users, where a solid line with a blue arrow indicates a real transmission of one packet.We denote overhearing of a packet as a dashed line with a red arrow. We define $W_{B,u}^{\#k}$ as the k-th DL packet from the base-station to end-user u. $\mathrm{IN}_u^{\#k}/\mathrm{IA}_u^{\#k}$ are the ACK/NACK sent by intended user u for packet $W_{B,u}^{\#k}$ and $\mathrm{UA}_{u'}^{\#k,u}$ is the ACK sent by unintended user u' who overhears the packet $W_{B,u}^{\#k}$. TC stands for training and control bits in a packet. In the example, four DL packets are successfully transmitted to the intended users in 4 rounds of transmission. In the first round, $W_{B,1}^{\#1}$ and $W_{B,2}^{\#1}$ are sent and both the packets are failed to be decoded by intended user but decoded by unintended user. During UL transmission, each user sends an intended NACK (IN) and a unintended ACK (UA) along with a regular packet to the base-station. In this example, the UL transmissions are successfully decoded by base-station hence base-station includes an ACK in its DL packets. Similarly in the second round, $W_{B,1}^{\#1}$ and $W_{B,2}^{\#2}$ are sent and each user decodes the other user's packet but its own. Then phase II starts and base-station begins to send NCPs. In the first DL transmission

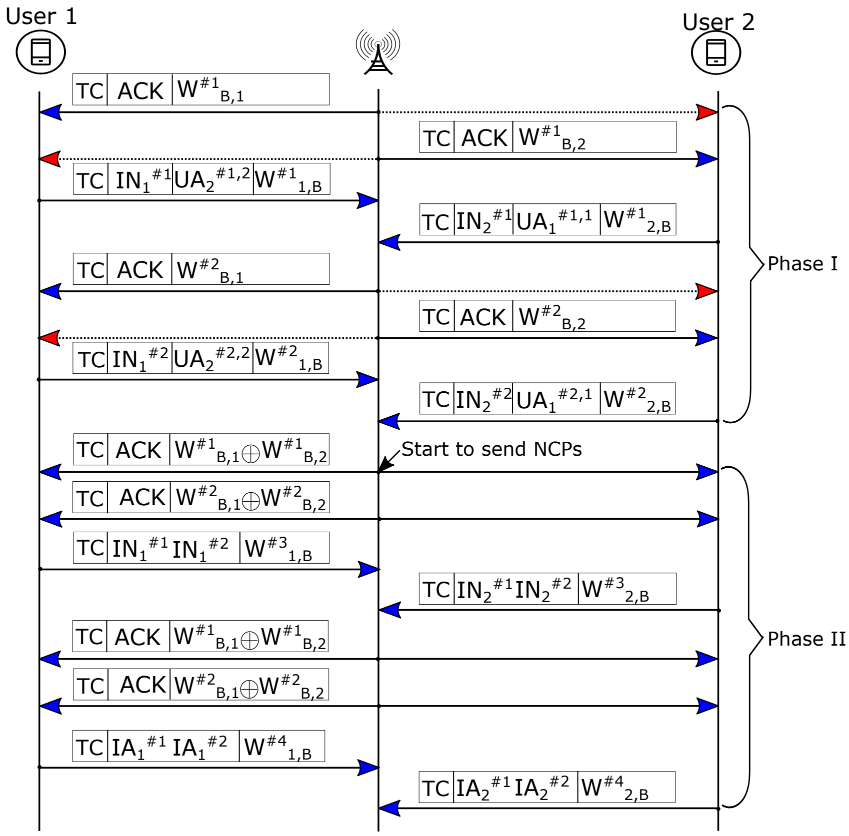


Figure 9: NCed-ARQ for a 2-user case

of round three, the base-station sends $W^{\#1}_{\mathrm{B},1} \oplus W^{\#1}_{\mathrm{B},2}$ as a NCP where $\oplus$ indicates a NC operation of packets. In the second DL transmission, $W^{\#2}_{\mathrm{B},1} \oplus W^{\#2}_{\mathrm{B},2}$ are sent. In this round, each user fails to decode the two NCPs therefore they replies INs in its UL packet. For example, user 1 fails to decode $W^{\#1}_{\mathrm{B},1}$ and $W^{\#2}_{\mathrm{B},1}$, then user 1 sends $\mathrm{IN}^{\#1}_{1}$ and $\mathrm{IN}^{\#2}_{1}$ in UL packet which notifies the base-station to re-transmit the NCP. Then in the fourth round, same NCPs are sent and two users successfully decode the packet and reply IAs.

5.3 Throughput Analysis

In this section, based on the outage probabilities derived previously, we derive the equations for the DL and UL throughput. The target rates per user of DL and UL are defined as R_d and R_u. Assuming one base-station and M users in the two-way wireless system, we define $S_{l,M}$ as the average number of re-transmissions required for the receiver to decode the packet. Then their throughputs are

$$T_{l,M} = \frac{R_l}{S_{l,M}} \qquad \text{for } l \in \{d, u\}. \tag{5.3}$$

To take the two-way channel into consideration, we also consider the sum of DL and UL throughput, which can be expressed as

$$T_{sum,M} = T_{d,M} + T_{u,M}. \tag{5.4}$$

We define $I_{d,k}$ and $I_{u,k}$ as the mutual information at the receiver's decoder after k^{th} re-transmission of a packet in DL and UL, respectively. As assumed, we consider a symmetric system with respect to any user. Then mutual information $I_{d,k}$ and $I_{u,k}$ are identical for each user. We also consider events $A_{d,k} = \{I_{d,k} > R_d\}$ and $A_{u,k} = \{I_{u,k} > R_u\}$ which indicate the mutual information of one packet at the receiver's decoder does not achieve the rate (i.e., decoding is unsuccessful) after k^{th} re-transmission in DL and UL, respectively. Then the successive outage probabilities after k^{th} DL or UL re-transmission are given by

$$p_l(k) = \Pr\{\bar{A}_{l,1}, \bar{A}_{l,2}, ..., \bar{A}_{l,k}\} \qquad \text{for } l \in \{d, u\}. \tag{5.5}$$

5.3.1 Number of Re-transmissions

As introduced in the previous section, we adopt a two-phase NCed-ARQ protocol. The transmission process in the downlink is divided into two phases. In the first phase, only RPs are sent from the base station the users. If a RP has been correctly decoded by one or more receivers, the first phase ends. If the RP is decoded by its intended user, the user will not participate in the phase II of the transmission, which means the RP will not be included in the NCP. If the RP is not decoded by its intended user but received successfully by one or more unintended users, the RP will be included in a NCP which will be sent in the second phase. In the second phase, NCPs will be sent to all users that did not receive its own packet in phase I but received all other unintended packets. We define $\Omega^{\mathrm{I}}(k)$ as the event that a RP is decoded by its intended user after k DL transmissions in the first phase and it does not require a second phase. For the second case that requires Phase II, we define $\Omega^{\mathrm{I+II}}(m,k,t)$ as the event that m unintended users decode the packet after k DL re-transmissions in Phase I but the intended user fails to decode it, and it takes an extra t re-transmissions of an NCP to achieve a successful decoding at the intended user. By these definitions, the probabilities of these two events are

$$\Pr\{\Omega^{\mathrm{I}}(k)\} = (p_d(k-1) - p_d(k))p_d(k-1)^{M-1}. \tag{5.6}$$

$$\begin{aligned}\Pr\{\Omega^{\mathrm{I+II}}(m,k,t)\} &= p_d(k-1)^{M-1}\binom{M-1}{m}\left(\frac{p_d(k)}{p_d(k-1)}\right)^{M-1-m} \\ &\times \left(1-\frac{p_d(k)}{p_d(k-1)}\right)^{m}(p_d(t+k-1) - p_d(t+k)).\end{aligned} \tag{5.7}$$

The average number of re-transmission per DL packet is

$$S_{d,M} = \sum_{k=1}^{\infty} \left(k\Pr\{\Omega^{I}(k)\} + \sum_{m=1}^{M-1} \sum_{t=1}^{\infty} (k + \frac{t}{m+1}) \Pr\{\Omega^{I+II}(m,k,t)\} \right). \tag{5.8}$$

For each UL packet, base-station is the only receiver and there is no NC scheme adopted in UL transmission. Hence, the average number of re-transmissions is given by

$$S_{u,M} = \sum_{k=1}^{\infty} (p_u(k-1) - p_u(k)). \tag{5.9}$$

5.3.2 Outage Probability

The mutual information (in nats/Hz/s) at the k^{th} re-transmission in DL and UL are respectively given by

$$I_{l,k} = \alpha_l \ln(1 + \rho_{l,k}) \qquad \text{for } l \in \{d, u\}, \tag{5.10}$$

where α_l is the average data bit ratio of each packet, SNR random variable $\rho_{l,k} \sim \mathsf{Exp}(\lambda_l)$ and $\lambda_l \triangleq \frac{N_0}{P_l}$. Since the receiver adopts the basic ARQ scheme and discards erroneously received packets, we obtain $\Pr\{\bar{A}_{l,k} | I_{l,k-1}\} = \Pr\{\bar{A}_{l,k}\}$. Then the outage probability for each user in each re-transmission is identically given by

$$\begin{aligned} q_l &= \Pr\{\bar{A}_{l,k}\} = \Pr\{I_{l,k} \le R_l\} = \Pr\{\rho_{l,k} \le e^{R_l/\alpha_l - 1}\} \\ &= 1 - e^{-\lambda_l e^{R_l/\alpha_l - 1}}. \end{aligned} \tag{5.11}$$

We derive the data bit ratios for both DL and UL transmissions. We define N as the packet size and N_{tc} as the training and control bits. In the NCed-ARQ protocol adopted in this paper, each user needs to reply ACK/NACKs for their intended packets and extra ACKs for their unintended packets. The length of ARQ feedback bits for intended and unintended packets are defined as $N_{f^{I}_{ARQ}}$ and $N_{f^{u}_{ARQ}}$, respectively. In DL transmissions, the receiver will send an ACK or NACK to the transmitter. Additionally, when transmitting a NCP, the base-station needs to specify which RPs are included in the NCP. We define N_r as the number of bits to indicate the RP of one user is part of the NCP. Hence, with $m+1$ users, $(m+1)N_r$ bits are included in the NCP. The DL data bit ratio is expressed as

$$\alpha_d = \begin{cases} \alpha_d^{'} & = \frac{N - N_{tc} - N_{f^{I}_{ARQ}}}{N} \quad \text{in Phase I,} \\ \alpha_{d,m}^{''} & = \frac{N - N_{tc} - N_{f^{I}_{ARQ}} - (m+1)N_r}{N} \quad \text{in Phase II.} \end{cases}$$

We define n_{ia} and n_{ua} as the average number of ACK/NACKs for intended packet and ACKs for unintended packets replied in each UL transmission. Then the average UL data bit ratio is

$$\alpha_u = \frac{N - N_{tc} - n_{ia}N_{f^{I}_{ARQ}} - n_{ua}N_{f^{u}_{ARQ}}}{N}. \tag{5.12}$$

The DL outage probability is derived as

$$q_d = \begin{cases} q_d^{'} & = 1 - e^{-\lambda_d e^{R_d/\alpha_d^{'}} - 1} \quad \text{in Phase I,} \\ q_{d,m}^{''} & = 1 - e^{-\lambda_d e^{R_d/\alpha_{d,m}^{''}} - 1} \quad \text{in Phase II.} \end{cases} \tag{5.13}$$

Then, we can derive the outage probabities as

$$
\begin{aligned}
p_d(k) &= q_d^{'k} && \text{in Phase I,} \\
p_d(k+t) &= q_d^{'k} q_{d,m}^{''t} && \text{in Phase II.}
\end{aligned}
\tag{5.14}
$$

Hence, (Equation 5.6) and (Equation 5.7) can be simplified to

$$
\Pr\{\Omega^{\mathrm{I}}(k)\} = (1 - q_d^{'}) q_d^{'(k-1)M}. \tag{5.15}
$$

$$
\Pr\{\Omega^{\mathrm{I+II}}(m,k,t)\} = \binom{M-1}{m} q_d^{'kM} \left(\frac{1-q_d^{'}}{q_d^{'}} \right)^m (q_{d,m}^{''(t-1)} - q_{d,m}^{''t}). \tag{5.16}
$$

Similarly, the UL outage probability is

$$
p_u(k) = q_u^k. \tag{5.17}
$$

We plug these outage probabilities into (Equation 5.8) and (Equation 5.9) and simplify them to

$$
\begin{aligned}
S_{d,M} &= \frac{1}{1-q_d^{'M}} + \frac{q_d^{'M}}{M(1-q_d^{'M})} \sum_{m=1}^{M-1} \binom{M}{m+1} \left(\frac{1-q_d^{'}}{q_d^{'}} \right)^m \frac{1}{1-q_{d,m}^{''}}, \\
S_{u,M} &= \frac{1}{1-q_u}.
\end{aligned}
\tag{5.18}
$$

5.3.3 Number of Extra Acknowledgments

Then we derive the values of n_{ia} and n_{ua} that also depend on events $\Omega^{I}(k)$ and $\Omega^{I+II}(m,k,t)$. If we only have the first phase and no NCP is sent, each user simply replies k IA/INs for its intended packet over k re-transmissions. However, other users may overhear this packet and reply UAs even though the packet is decoded by its intended user. Such UAs are useless for NCed ACK scheme and hurt UL throughput with a lower data bit ratio. Therefore, we assume if a RP is successfully decoded by its intended user, the user will reply IA in advance. Then the base-station sends the control information that notifies the next packet is a new RP to suppress UAs replied by other users. In this way, extra UAs will not be sent, which means each user replies 0 UAs for unintended packets if we only have Phase I. If event $\Omega^{I+II}(m,k,t)$ occurs and $m+1$ users are included with t extra re-transmissions of NCPs, $m+1$ users need to send $(k+t)(m+1)$ IA/INs over $k(m+1)+t$ time-slots. Therefore, $\frac{(k+t)(m+1)}{k(m+1)+t} = \frac{(k+t)}{k+t/(m+1)}$ IA/INs are sent in each packet on average. Similarly, each user sends $\frac{m}{k+t/(m+1)}$ UAs on average.

Considering all possible cases, the average number of ACK/NACKs for intended packet and ACKs for unintended packets are respectively given by

$$n_{ia} = 1 + \sum_{m=1}^{M-1}\sum_{k=1}^{\infty}\sum_{t=1}^{\infty} \frac{tm/(m+1)}{k+t/(m+1)} \Pr\{\Omega^{I+II}(m,k,t)\}, \tag{5.19}$$

$$n_{ua} = \sum_{m=1}^{M-1}\sum_{k=1}^{\infty}\sum_{t=1}^{\infty} \frac{m}{k+t/(m+1)} \Pr\{\Omega^{I+II}(m,k,t)\}. \tag{5.20}$$

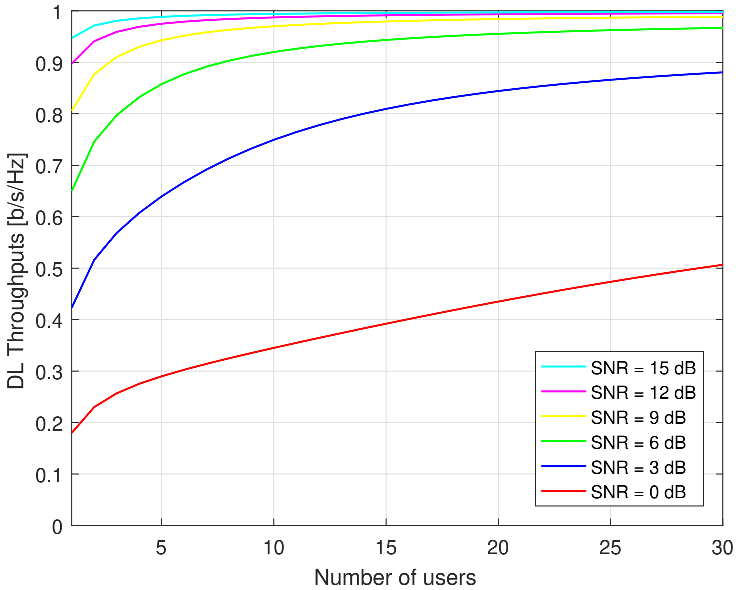
5.4 Simulation Results

In this section, we numerically evaluate the DL, UL and sum throughput derived in the previous section. We assume that each channel is quasi-static and all nodes have the same transmit SNR. We set the size of packet N to be 1000 bits. We set N_{tc} and N_{r} to be 300 bits and 8 bits respectively. More bits for unintended ACK is required as unintended ACK needs to contain source address for base-station to recognize the sender of ARQ. We use 3-bit intended ACK and 10-bit unintended ACK and vary $\mathsf{N}_{\mathsf{f}^{\mathsf{I}}_{\mathsf{ARQ}}}$ and $\mathsf{N}_{\mathsf{f}^{\mathsf{U}}_{\mathsf{ARQ}}}$ by a spread factor. Also we examine how different DL and UL transmission rates affect throughput of the system.

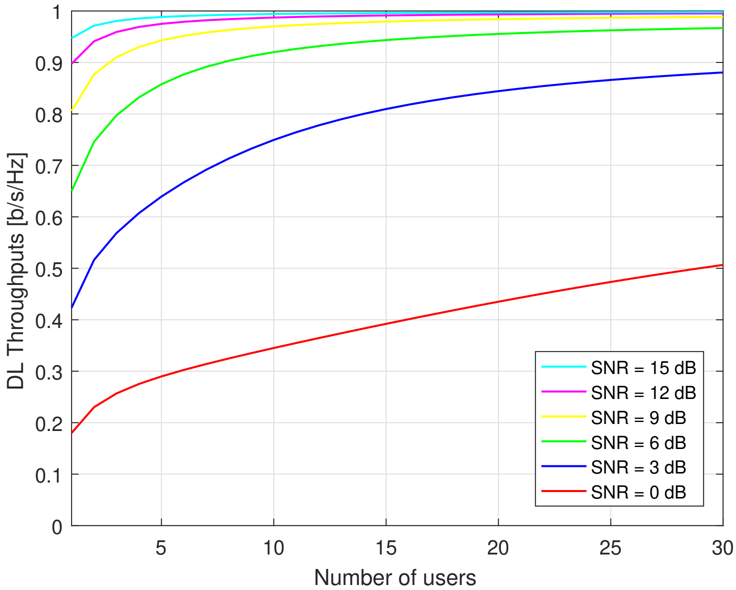
Figure 10 shows the DL and UL throughput versus number of users. Notice that the DL throughput always increases as the number of users become greater in all SNR regimes while UL throughput always decrease. As the number of users increase, more data bits in a UL packet are used for transmitting acknowledgment. Clearly there is a tradeoff between DL and UL throughput so we want to obtain optimal sum throughput when number of user varies. We omit the plots for other DL/UL rate and acknowledgment size settings as for all cases it shows similar results.

Figure 11 and Figure 12 shows the sum throughput versus number of users with different UL rate. In both plots we set 6-bit intended ACK and 20-bit unintended ACK. We also show the optimal number of end-users $\mathsf{M}_{\mathsf{opt}}$ in captions of two plots. Notice when SNR = 0 dB, the actual $\mathsf{M}_{\mathsf{opt}}$ exceeds 30, there is still an optimal value of M. As M increases, a packet may contains only overhead information but no real data, in this case the transmission will be stuck.

Figure 12 and Figure 13 shows the sum throughput versus number of users with different UL rate. The sizes of intended ACK and unintended ACK are doubled in Figure 13. By comparing Figure 12 and Figure 13, we see that when number of users M is quite small, the sum of throughput is almost the same as there is not many overhead acknowledgments sent through UL channels. However as number of users increase, more intended and unintended ACKs are required which impairs the DL performs. The system has larger size of ARQs in Figure 13, therefore we see from Figure 13, sum of throughput decreases more significantly than in Figure 12.



(a) DL Throughput



(b) UL Throughput

Figure 10: $R_d = 1$ b/s/Hz, $R_u = 1$ b/s/Hz and ARQ feedback is spread by 2.

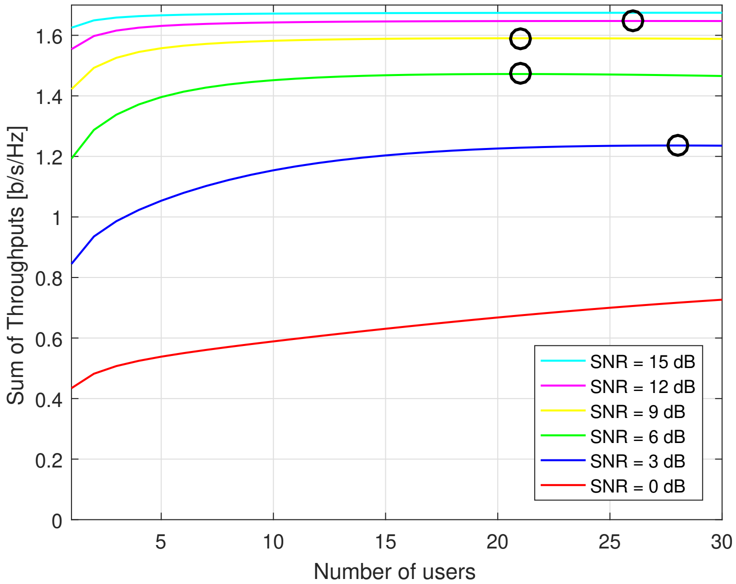


Figure 11: $R_d = 1$ b/s/Hz, $R_u = 0.7$ b/s/Hz and ARQ feedback is spread by 2. $M_{opt} = 30, 28, 21, 21, 26, 30$ for SNR $= 0, 3, 6, 9, 12, 15$ dB (30 indicates the optimal M exceeds 30).

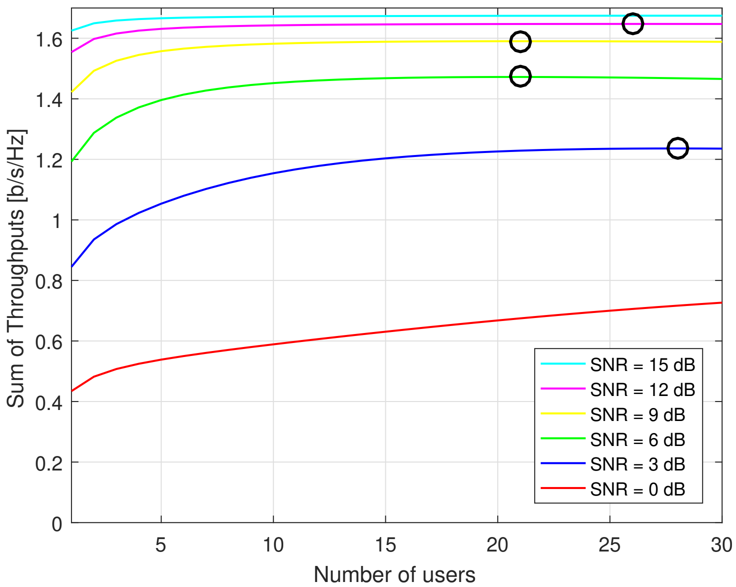


Figure 12: $R_d = 1$ b/s/Hz, $R_u = 1$ b/s/Hz and ARQ feedback is spread by 2. $M_{opt} = 30, 22, 14, 14, 16, 21$ for SNR $= 0, 3, 6, 9, 12, 15$ dB.

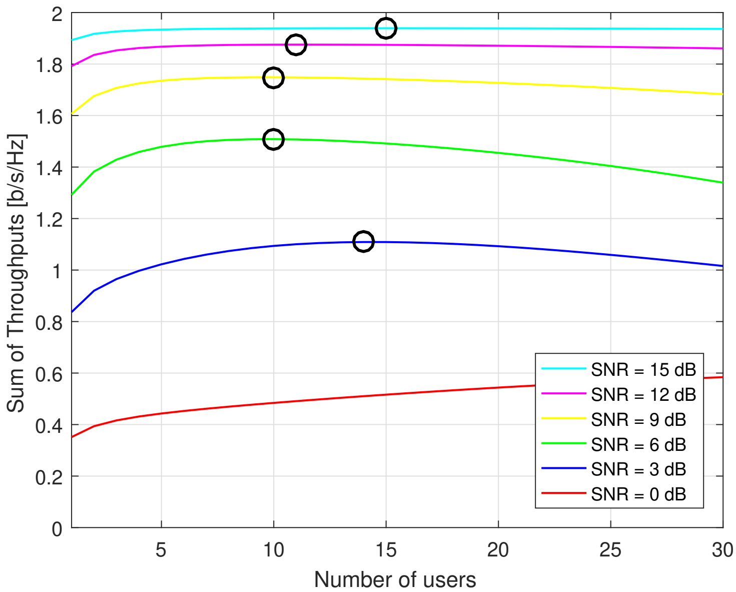


Figure 13: $R_d = 1$ b/s/Hz, $R_u = 1$ b/s/Hz and ARQ feedback is spread by 4. $M_{opt} = 30, 14, 10, 10, 11, 15$ for SNR $= 0, 3, 6, 9, 12, 15$ dB.

CHAPTER 6

CONCLUSION

This dissertation focuses on establishing a practical framework for analyzing two-way wireless networks performance with re-transmissions. In this thesis, we analyzed the tradeoffs between several between bits spent on learning the channel, re-transmission request bits, and data bits in two-way wireless networks. By exploiting the tradeoffs in wireless packets, we are able to shed some light on the design of next generation wireless systems. In particular,

6.1 Contributions

1. In our work *Throughput Performance of Two-way Wireless Communication with Combined CSI and ARQ Feedback with Infinite Packet Length*, we have developed a generalized framework for feedback in two-way networks that combines limited CSI feedback and ARQ. Indeed, throughput expressions, which characterize the tradeoff between resources allocated to training (learn channel), feedback (ARQ, CSI), and data transmission, were derived and numerically evaluated. We investigated the combination of different transmitter and receiver protocols under various channel varying conditions. It has been shown that using imperfect CSI at the transmitter and receiver can enhance the performance in several cases (good conditions) while it will deteriorate in others. In slow fading channels with high SNR regimes, CP with IR has the best throughput result. When dealing with

fast time-varying channels and low SNR regimes, using constant power and rate gives the best throughput performance.

2. In our work *Throughput Performance of Two-way Wireless Communication with Combined CSI and ARQ Feedback with Finite Packet Length*, we explore deeper regarding the short packet effect on the tradeoffs in a wireless packet. As current generation communication systems require ultra-reliable and low-latency communications, the wireless packets should be designed for small packets with only few hundreds of bits. The achievable rates are impacted by the size of the packet and some of our previous results do not hold in the case of short packets. Using the new approximation of finite block-length channel capacity, we derive the outage pro abilities and throughput for basic ARQ and IR-HARQ. We find that using less training bits in the low SNR range and more training bits in the high SNR range could help increase the throughput of wireless systems. This results are slightly different from the results we obtain under the assumption of infinite block-length packet due to the short packet effect.

3. In our work *On Practical Network Coded ARQ for Two-way Wireless Communication*, we study the NCed-ARQ performance in a more practical setting. We take into account the resources(control information, CSI, and extra acknowledgments) dedicated to the exchange of information between base-station and end-users. We derive the expressions for the number of re-transmissions and extra overhead. Based on the analysis, we obtain the expressions for DL and UL throughput which captured the tradeoff between overhead and re-transmission. In moderate SNR, the sum of DL and UL throughput is concave

with respect to the number of end-users. Finally, we maximize the sum of the throughput and derived the optimal number of users numerically.

APPENDICES

Appendix A

TRANSFORMATION OF INDEPENDENT EXPONENTIAL RANDOM VARIABLES

A.1 Statistics of $h_1/(1+h_2)$

Let h_1 and h_2 be two independent random variables following an exponential distribution (i.e. $\sqrt{h_1}$ and $\sqrt{h_2}$ follow a Rayleigh distribution) with means σ_1 and σ_2, respectively, and let $X = \frac{h_1}{1+h_2}$. The CDF of the random variable X, $F_X(x)$, is defined by

$$F_X(x) = \Pr\left[X < x\right] \underset{1+h_2 \geq 0}{=} \Pr\left[h_1 < x + x h_2\right] = \mathbb{E}_{h_2}\left[F_{h_1}(x + x h_2)\right],$$

where $F_{h_1}(h) = 1 - e^{-\frac{h}{\sigma_1}}$ is the CDF of h_1.

Thus, by averaging over the PDF of h_2, the CDF and PDF of $\frac{h_1}{1+h_2}$ can be obtained as

$$F_{\frac{h_1}{1+h_2}}(x) = 1 - \frac{1}{1 + \frac{\sigma_2}{\sigma_1} x} e^{-\frac{x}{\sigma_1}}, \quad \text{if } x \geq 0. \tag{A.1}$$

$$f_{\frac{h_1}{1+h_2}}(x) = \frac{\sigma_1^2 \sigma_2 + \sigma_1 + \sigma_2 x}{(\sigma_1 + \sigma_2 x)^2} e^{-\frac{x}{\sigma_1}}, \quad \text{if } x \geq 0. \tag{A.2}$$

The MGF of X_s, $M_X(t)$, used later to get the distribution of the sum, is defined as

$$M_X(t) = \mathbb{E}\left[e^{tX}\right] = \mathbb{E}\left[e^{t \frac{h_1}{1+h_2}}\right]. \tag{A.3}$$

Appendix A (Continued)

Knowing that the MGF of h_1 is equal to $M_{h_1}(t) = \frac{1}{1-\sigma_1 t}$ (Simon and Alouini, 2005, Eq. (2.8)), the MGF of X can be obtained as the average over h_2 of the following expression

$$M_X(t) = \mathbb{E}\left[\left(1 - t\frac{\sigma_1}{1+h_2}\right)^{-1}\right] = 1 + t\frac{\sigma_1}{\sigma_2}\int_0^\infty \frac{e^{-\frac{x}{\sigma_2}}}{1+x-t\sigma_1}\,dx. \tag{A.4}$$

The last integral (Equation A.4) is solved using the identity (Abramowitz and Stegun, 1964, Eq. (5.1.5)) to get

$$M_X(t) = 1 + t\frac{\sigma_1}{\sigma_2}e^{\frac{1-t\sigma_1}{\sigma_2}}E_1\left(1/\sigma_2 - t\sigma_1/\sigma_2\right). \tag{A.5}$$

The CHF of X is obtained straightforward as

$$\varphi_X(t) = M_X(i\,t) = 1 + it\frac{\sigma_1}{\sigma_2}e^{\frac{1-it\sigma_1}{\sigma_2}}E_1\left(1/\sigma_2 - it\sigma_1/\sigma_2\right). \tag{A.6}$$

A.2 Distribution of the Sum of $h_{1,k}/(1+h_{2,k})$

Let $h_{1,k}$ and $h_{2,k}$ be a set of independent exponential random variables for $k = 1, 2, \cdots, m$ with means $\sigma_{1,k}$ and $\sigma_{2,k}$, respectively. Let $X_k = \frac{h_{1,k}}{1+h_{2,k}}$ for $1 \le k \le m$, and $S = \sum_{k=1}^m X_k$. Actually, S is the sum of independent random variables X_k, which means that its MGF (respectively CHF) is the product of the MGFs (respectively CHFs) of these random variables.

Appendix A (Continued)

Therefore using the results in (Equation A.5) and (Equation A.6), the MGF and CHF of S are given, respectively, by

$$M_S(t) = \prod_{k=1}^{m} \left(1 + t\frac{\sigma_{1,k}}{\sigma_{2,k}} e^{\frac{1-t\sigma_{1,k}}{\sigma_{2,k}}} E_1 \left(\frac{1-t\sigma_{1,k}}{\sigma_{2,k}} \right) \right), \tag{A.7}$$

$$\varphi_S(t) = \prod_{k=1}^{m} \left(1 + it\frac{\sigma_{1,k}}{\sigma_{2,k}} e^{\frac{1-it\sigma_{1,k}}{\sigma_{2,k}}} E_1 \left(\frac{1-it\sigma_{1,k}}{\sigma_{2,k}} \right) \right). \tag{A.8}$$

A.3 Distribution of $\log(1 + h_1/(1 + h_2))$ and its sum

Another transformation of random variable is used during the outage probability derivation in this paper that is a scaled logarithm of $1 + \frac{h_1}{1+h_2}$, i.e. $LH = a \log\left(1 + \frac{h_1}{1+h_2}\right)$. Actually, using the CDF of X from (Equation A.1), the CDF of LH, $F_{LH}(\cdot,\cdot,\cdot,\cdot)$, is derived as

$$F_{LH}(x, \sigma_1, \sigma_2, a) = F_{\frac{h_1}{1+h_2}}(e^{\frac{x}{a}} - 1) = 1 - \frac{\sigma_1 \exp\left(-\frac{e^{x/a}-1}{\sigma_1}\right)}{\sigma_1 - \sigma_2 + \sigma_2 e^{x/a}}, \quad \text{if } x \geq 0. \tag{A.9}$$

By deriving the CDF with respect to x, the PDF of LH, $f_{LH}(\cdot,\cdot,\cdot,\cdot)$, can be written as follows

$$f_{LH}(x, \sigma_1, \sigma_2, a) = \frac{\sigma_1^2\sigma_2 + \sigma_1 - \sigma_2 + \sigma_2 e^{x/a}}{a(\sigma_1 - \sigma_2 + \sigma_2 e^{x/a})^2} \exp\left(-\frac{e^{x/a} - 1}{\sigma_1}\right), \quad \text{if } x \geq 0. \tag{A.10}$$

Furthermore, let $h_{1,k}$ and $h_{2,k}$ be independent exponential random variables for $k = 1, 2, \cdots, m$, with means $\sigma_{1,k}$ and $\sigma_{2,k}$, respectively, the PDF of the sum of $a_k \log 1 + \frac{h_{1,k}}{1+h_{2,k}}$ cannot be expressed in closed form. However, an integral expression can be derived using $f_{LH}(x, \sigma_1, \sigma_2, a)$.

Appendix A (Continued)

To do so, we define the vectors $\Sigma_1 = [\sigma_{1,1}, \sigma_{1,2}, \cdots, \sigma_{1,m}]$, $\Sigma_2 = [\sigma_{2,1}, \sigma_{2,2}, \cdots, \sigma_{2,m}]$, and $\mathbf{A} = [a_1, a_2, \cdots, a_m]$, then the PDF of $SL = \sum_{k=1}^{m} a_k \log 1 + \frac{h_{1,k}}{1+h_{2,k}}$ is given by

$$f_{SL}(x, \mathbf{\Sigma_1}, \mathbf{\Sigma_2}, \mathbf{A}) = f_{LH}(x, \sigma_{1,1}, \sigma_{2,1}, a_1) * f_{LH}(x, \sigma_{1,2}, \sigma_{2,2}, a_2) * \cdots * f_{LH}(x, \sigma_{1,m}, \sigma_{2,m}, a_m), \tag{A.11}$$

where " $*$ " denotes the convolution operator. The CDF of SL can be obtained as

$$F_{SL}(x, \mathbf{\Sigma_1}, \mathbf{\Sigma_2}, \mathbf{A}) = f_{LH}(x, \sigma_{1,m}, \sigma_{2,m}, a_m) * \cdots * f_{LH}(x, \sigma_{1,2}, \sigma_{2,2}, a_2) * F_{LH}(x, \sigma_{1,1}, \sigma_{2,1}, a_1). \tag{A.12}$$

Appendix B

STATISTICS OF CORRELATED EXPONENTIAL DISTRIBUTIONS

Let h_1 and h_2 be two correlated exponential random variables with means σ_1 and σ_2, respectively. The joint PDF of h_1 and h_2 can be written using (Mallik, 2003) as

$$f_{h_1,h_2}(x,y) = \frac{1}{\sigma_1\sigma_2(1-\rho^2)} \exp\left(-\frac{1}{1-\rho^2}\left(\frac{x}{\sigma_1}+\frac{y}{\sigma_2}\right)\right) I_0\left(\frac{2\rho\sqrt{xy}}{(1-\rho^2)\sqrt{\sigma_1\sigma_2}}\right), \tag{B.1}$$

where ρ is related to the correlation coefficient that is defined by $\mathbb{E}[h_1h_2] = (1+\rho^2)\sigma_1\sigma_2$ and $I_0(\cdot)$ is the 0-th order modified Bessel function of the first kind (Abramowitz and Stegun, 1964, Eq. (9.6.16)). In what follows, we aim to obtain the distribution of the random variables $h_1 - h_2$, $\frac{h_1}{h_2}$, and $\frac{h_1}{h_2+h_3}$ for independent h_3, which are needed to get the outage probability of different studied scenarios.

B.1 Distribution of $h_1 - h_2$

Let $X = h_1 - h_2$, as h_1 and h_2 follow an exponential distribution, each one is the sum of the square of two independent Gaussian random variables. More specifically $h_1 = V_1^2 + V_3^2$ and $h_2 = V_2^2 + V_4^2$, where $V = [V_1, V_2, V_3, V_4]$ is a real Gaussian random vector with zero mean and covariance matrix $R_V = \mathbb{E}[V^TV]$. The origin of the correlation between h_1 and h_2 is the correlation between V_1 and V_2, and between V_3 and V_4 (Mallik, 2003). Therefore, the covariance matrix of V is given by

$$R_V = \begin{pmatrix} R_{V,12} & \mathbf{0_2} \\ \mathbf{0_2} & R_{V,12} \end{pmatrix}, \quad \text{where } R_{V,12} = \frac{1}{2}\begin{pmatrix} \sigma_1 & \rho\sqrt{\sigma_1\sigma_2} \\ \rho\sqrt{\sigma_1\sigma_2} & \sigma_2 \end{pmatrix}. \tag{B.2}$$

Note that X can be written in term of the elements of V as

$$X = h_1 - h_2 = (V_1^2 - V_2^2) + (V_3^2 - V_4^2) = U_1 + U_2, \tag{B.3}$$

where $U_1 = V_1^2 - V_2^2$ and $U_2 = V_3^2 - V_4^2$ are i.i.d. random variables, which means that the distribution of X is totally defined by determining the distribution of U_1. Obviously, U_1 is equal to the product of two random variables $P = V_1 - V_2$ and $Q = V_1 + V_2$ (i.e. $U_1 = P\ Q$), where P and Q have joint Gaussian distribution with zero mean, variances $\mathrm{var}(P) = \frac{1}{2}(\sigma_1+\sigma_2-2\rho\sqrt{\sigma_1\sigma_2})$ and $\mathrm{var}(Q) = \frac{1}{2}(\sigma_1 + \sigma_2 + 2\rho\sqrt{\sigma_1\sigma_2})$ respectively, and covariance $\mathbb{E}[PQ] = \frac{\sigma_1-\sigma_2}{2}$. From the PDF of joint Gaussian, the PDF of U_1 can be derived as

$$f_{U_1}(u) = \int_{-\infty}^{\infty} \frac{1}{|t|} f_{P,Q}\left(t, \frac{u}{t}\right) dt = \frac{1}{\pi\sqrt{\sigma_1\sigma_2(1-\rho^2)}} e^{\frac{\sigma_1-\sigma_2}{2\sigma_1\sigma_2(1-\rho^2)}u} K_0\left(\frac{\sigma|u|}{\sigma_1\sigma_2(1-\rho^2)}\right), \tag{B.4}$$

where $K_0(\cdot)$ is the 0-th order modified Bessel function of the second kind (Abramowitz and Stegun, 1964, Eq. (9.6.24)) and σ is defined by

$$\sigma = \sqrt{\mathrm{var}(P)\mathrm{var}(Q)} = \frac{1}{2}\sqrt{(\sigma_1 + \sigma_2)^2 - 4\rho^2\sigma_1\sigma_2}. \tag{B.5}$$

Appendix B (Continued)

Thereby, the MGF of U_1 can be obtained using the identity (Gradshteyn and Ryzhik, 2007, Eq. (6.661.2)) as

$$M_{U_1}(t) = \int_{-\infty}^{\infty} e^{tu} f_{U_1}(u) du = \frac{1}{\sqrt{1 - t^2\sigma_1\sigma_2(1-\rho^2) - t(\sigma_1 - \sigma_2)}}. \tag{B.6}$$

Since U_1 and U_2 are i.i.d., the MGF of X is the square of the MGF of U_1

$$M_X(t) = \frac{1}{1 - t^2\sigma_1\sigma_2(1-\rho^2) - t(\sigma_1 - \sigma_2)}. \tag{B.7}$$

The PDF of $h_1 - h_2$ is defined as the inverse Laplace transform of $M_X(t)$, which can be obtained using the Laplace transform table (Abramowitz and Stegun, 1964, Eq. (29.3.12)) as follows

$$f_{h_1-h_2}(x) = \frac{1}{2\sigma} e^{\frac{-2|x|\sigma + x(\sigma_1-\sigma_2)}{2\sigma_1\sigma_2(1-\rho^2)}} \quad , \ \forall x \in \mathbb{R} \tag{B.8}$$

By a simple integration, the CDF of $h_1 - h_2$ is expressed as

$$F_{h_1-h_2}(x) = \begin{cases} \frac{2\sigma-\sigma_1+\sigma_2}{4\sigma} e^{\frac{2x}{2\sigma-\sigma_1+\sigma_2}}, & \text{if } x \leq 0 \\ 1 - \frac{2\sigma+\sigma_1-\sigma_2}{4\sigma} e^{-\frac{2x}{2\sigma+\sigma_1-\sigma_2}}, & \text{otherwise.} \end{cases} \tag{B.9}$$

B.2 CDF of h_1/h_2

The CDF of $\frac{h_1}{h_2}$ is defined as

$$F_{\frac{h_1}{h_2}}(x) = \Pr\left(\frac{h_1}{h_2} < x\right) \underset{h_2 \geq 0}{=} \Pr(h_1 - xh_2 < 0) = F_{h_1 - xh_2}(0), \tag{B.10}$$

where $F_{h_1-xh_2}(x)$ can be obtained from (Equation B.9) by substituting σ_2 by $x\sigma_2$. Thus the CDF of $\frac{h_1}{h_2}$ can be expressed as

$$F_{\frac{h_1}{h_2}}(x) = \frac{1}{2} - \frac{\sigma_1 - \sigma_2 x}{2\sqrt{\sigma_1^2 + 2\sigma_1\sigma_2(1-2\rho^2)x + \sigma_2^2 x^2}}, \quad \text{if } x \geq 0. \tag{B.11}$$

B.3 CDF of $h_1/(h_2+h_3)$

Assuming that h_3 follows an exponential distribution with mean σ_3 and that is independent of h_1 and h_2, the CDF if $\frac{h_1}{h_2+h_3}$ can be written as

$$F_{\frac{h_1}{h_2+h_3}}(x) = \Pr\left(\frac{h_1}{h_2+h_3} < x\right) \underset{h_2+h_3\geq 0}{=} \Pr\left(h_1 - xh_2 < xh_3\right). \tag{B.12}$$

Conditioning over h_3, the CDF of $\frac{h_1}{h_2+h_3}$ is expressed in terms of the CDF of $h_1 - xh_2$. Therefore, the average CDF is given by

$$\begin{aligned} F_{\frac{h_1}{h_2+h_3}}(x) &= \mathbb{E}\left[F_{h_1-xh_2}(xh_3)\right] = 1 - \frac{2\sigma_x + \sigma_1 - x\sigma_2}{4\sigma_x}\mathbb{E}\left[e^{-\frac{2xh_3}{2\sigma_x+\sigma_1-x\sigma_2}}\right] \\ &= \frac{x\sigma_1\sigma_2(1-\rho^2) + 2x\sigma_x\sigma_3}{\sigma_x(2\sigma_x + \sigma_1 - x\sigma_2 + 2x\sigma_3)}, \quad \text{if } x \geq 0, \end{aligned} \tag{B.13}$$

where $\sigma_x = \frac{1}{2}\sqrt{(\sigma_1 + x\sigma_2)^2 - 4x\rho^2\sigma_1\sigma_2}$.

Appendix B (Continued)

B.4 CDF of $\log(1+h_1/(1+h_3))/\log(1+h_2)$

Like section B.3, assuming that h_3 has an exponential distribution with mean σ_3 and independent of h_1 and h_2, the CDF of the ratio $\frac{\log\left(1+\frac{h_1}{1+h_3}\right)}{\log(1+h_2)}$ is defined as

$$
\begin{aligned}
F_{FL_s}(x) &= \Pr\left[\frac{\log\left(1+\frac{h_1}{1+h_3}\right)}{\log(1+h_2)} < x\right] = \mathbb{E}_{h_3}\left[\Pr\left[\frac{\log\left(1+th_1\right)}{\log(1+h_2)} < x\right]\middle| h_3 = 1/t-1\right] \\
&= \int_0^1 \frac{1}{t^2\sigma_3} e^{-\frac{1}{\sigma_3}\left(\frac{1}{t}-1\right)} \Pr\left[\frac{\log\left(1+th_1\right)}{\log(1+h_2)} < x\right] dt.
\end{aligned}
$$

Therefore, we should focus on the distribution of $X = \frac{\log(1+th_1)}{\log(1+h_2)}$. Using the joint PDF of h_1 and h_2 in (Equation B.1) and a change of variables from (h_1, h_2) to (X, Y), where $Y = \log(1+h_2)$, we can obtain the PDF of X, $f_X(x,t)$, as follows

$$
\begin{aligned}
f_X(x,t) = \int_0^\infty \frac{y}{t\sigma_1\sigma_2(1-\rho^2)} \exp\left(y(x+1) - \frac{1}{1-\rho^2}\left(\frac{e^{xy}-1}{t\sigma_1} + \frac{e^y-1}{\sigma_2}\right)\right) \\
\times I_0\left(\frac{2\rho\sqrt{(e^{xy}-1)(e^y-1)}}{(1-\rho^2)\sqrt{\sigma_1\sigma_2}}\right) dy. \qquad \text{(B.14)}
\end{aligned}
$$

Hence the PDF and CDF of $\frac{\log\left(1+\frac{h_1}{1+h_3}\right)}{\log(1+h_2)}$ can be obtained, respectively, in integral forms as

$$
f_{FL_s}(x) = \int_0^1 \frac{1}{t^2\sigma_3} e^{-\frac{1}{\sigma_3}\left(\frac{1}{t}-1\right)} f_X(x,t)\,dt \qquad \text{(B.15)}
$$

$$
F_{FL_s}(x) = \int_0^x f_{FL_s}(t)\,dt. \qquad \text{(B.16)}
$$

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VITA

NAME	Xinghao Gu
EDUCATION	Ph.D. Electrical and Computer Engineering, University of Illinois at Chicago, 2021 M.Sc. Electrical and Computer Engineering, University of Illinois at Chicago, 2014 B.Eng. Biomedical Engineering, Central South University, 2012
EXPERIENCE	Firmware Engineer at Motorola Mobility (Jan. 2020 - May. 2021) Teaching Assistant at University of Illinois at Chicago (Aug. 2015 - Dec. 2019)
PUBLICATIONS	X. Gu, H. Soury and B. Smida, "On the Throughput of Wireless Communication With Combined CSI and H-ARQ Feedback," in IEEE Open Journal of the Communications Society. H. Zhu, X. Gu, B. Smida and D. J. Love, "On practical network coded ARQ for two-way wireless communication," 2017 IEEE International Conference on Communications (ICC), Paris, 2017.
PRESENTATIONS	"On practical network coded ARQ for two-way wireless communication," 2017 IEEE International Conference on Communications (ICC), Paris, 2017.