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JUSTIFICATIONS FOR USING ANCF FINITE ELEMENTS

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ABSTRACT

This report provides the justifications for introducing and using the finite elements (FE) of the *absolute nodal coordinate formulation* (ANCF). These elements have specific features that distinguish them from conventional finite elements and make them more suited for the large displacement analysis of multibody system (MBS) applications. Furthermore, the continuum kinematic description of fully parameterized ANCF elements cannot be ignored when interpreting the ANCF numerical results and comparing these results with results obtained using semi-discrete models often used with conventional beams and analytical solutions that are based on more simplifying assumptions. For example, torsion is associated with space-curve geometry and is not one of the basic continuum-mechanics shear-strain modes. ANCF displacement fields allow increasing the order of interpolation without increasing the number of nodes or using the noncommutative finite rotations. Such displacement fields also allow developing lower-dimension infinitesimal rotation ANCF/CRBF finite elements without lowering the order of the interpolation.

1. INTRODUCTION

ANCF finite elements were introduced to alleviate known limitations of conventional finite elements and limitations of using the noncommutative finite rotations as nodal coordinates [1 – 57]. Furthermore, the concerns regarding the use of isogeometric analysis (IGA) in multibody system (MBS) applications have been discussed in the literature. For this reason, the use and development of ANCF finite elements are expected to continue since such elements represent the only available option for accurate geometric representation of a wide range of large displacement applications. The implementation of both ANCF and *floating frame of reference* (FFR) formulations [58] in computational MBS algorithms is necessary for the systematic and efficient solution of large and small deformation problems.

This report provides justifications for introducing and using ANCF finite elements which have specific features that distinguish them from conventional finite elements and make them more suited for the large displacement analysis in MBS applications. Nonetheless, the continuum kinematic description of fully parameterized ANCF elements cannot be ignored when interpreting the ANCF numerical results and comparing these results with results obtained using semi-discrete models often used with conventional beams. For example, torsion is associated with space-curve geometry and is not one of the basic continuum-mechanics shear-strain modes. The torsion mode of the chassis of a vehicle represents the twist of a chassis nominal space curve and it is not one of the continuum-mechanics shear-strain modes defined by the dot products of position-gradient vectors. When using a fully continuum model, cross sections defined in a reference configuration cannot be defined in a current configuration because gradient vectors do not maintain constant relative orientations, and consequently, the semi-discrete beam models in which cross sections of a beam rotate with respect to each other are not applicable to fully continuum ANCF models.

Higher-order elements do not always imply FE mesh with higher dimensions or larger number of coordinates. Convergence to correct and smoother solutions may require the use of large number of low-order elements as compared to higher-order elements. For example, large number of linear or bilinear elements is required to describe bending deformations and such large number of elements does not achieve the desired rotation and stress continuity at the nodal points. On the other hand, higher-order elements based on cubic interpolations can describe bending deformations with much smaller number of elements. The use of higher-order elements is common in the FE literature as evident by using the 20-node solid element in commercial FE software. If the 4-node solid element were sufficient and performed well in all applications, there is no justification for developing the 20-node element and implementing it in commercial FE software. ANCF displacement fields allow increasing the order of interpolation without increasing the number of nodes or using the noncommutative finite rotations. Such displacement fields also allow developing lower-dimension infinitesimal rotation ANCF/CRBF finite elements without lowering the order of the interpolation.

2. ANCF DISPLACEMENT FIELD

For ANCF finite elements, the global position of an arbitrary point on an element can be written as $\mathbf{r}(\mathbf{x},t)=\mathbf{S}(\mathbf{x})\mathbf{e}(t)$, where $\mathbf{S}(\mathbf{x})$ is the FE shape-function matrix that depends on the FE spatial coordinates $\mathbf{x}=\begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}^T$, and $\mathbf{e}(t)$ is the vector of element nodal coordinates that depend on time t. At a given node k, absolute position and position-vector gradients define the vector of nodal coordinates as $\mathbf{e}^{ik}=[\mathbf{r}^{ik^T}\ \mathbf{r}_{x_1}^{ik^T}\ \mathbf{r}_{x_2}^{ik^T}\ \mathbf{r}_{x_3}^{ik^T}]^T$, where $\mathbf{r}_{x_k}=\partial\mathbf{r}/\partial x_k$, $k=1,2,3$ [35]. For an arbitrary point, the position vector $\mathbf{r}(\mathbf{x},t)=\mathbf{S}(\mathbf{x})\mathbf{e}(t)$, the velocity vector $\dot{\mathbf{r}}(\mathbf{x},t)=\mathbf{S}(\mathbf{x})\dot{\mathbf{e}}(t)$, and the acceleration vector $\ddot{\mathbf{r}}(\mathbf{x},t)=\mathbf{S}(\mathbf{x})\ddot{\mathbf{e}}(t)$ can be defined and used to formulate the dynamic equations of motion. The use of twelve coordinates per node in the case of three-dimensional fully parameterized ANCF elements have clear geometric and computational advantages that allow for conveniently describing the reference-configuration geometry, capturing deformation modes that cannot be captured by lower-order elements, and obtaining lower-dimension FE mesh in many applications as has been demonstrated in the literature. Therefore, use of higher-order elements, as previously mentioned, does not always imply an FE mesh with larger number of degrees of freedom.

3. JUSTIFICATION FOR USING ANCF ELEMENTS

ANCF elements are fundamentally different from other finite elements. Some important differences which can provide clear explanation for developing and using ANCF finite elements are the following:

1. The coordinates used for ANCF elements are consistent with the kinematic description used in the general continuum-mechanics theory. ANCF elements employ position gradients as nodal coordinates. Position gradients, which are different from displacement gradients, have clear geometric meaning as tangents to coordinate lines. This geometric meaning can be exploited to solve fundamental problems that have not been solved before including motion and shape control of soft robots [59]. ANCF displacement fields allow increasing the order of interpolation without increasing the number of nodes or using the noncommutative finite rotations.
2. Use of position gradients as nodal coordinates ensures the continuity of the rotations, strain, and stress fields at the nodal points of the ANCF elements. Such degree of continuity cannot

be achieved using conventional elements since continuity of the rotations does not imply strain and stress continuity at the nodal points. Higher degree of continuity is important in many problems including bending problems.

3. Because ANCF elements do not employ infinitesimal or finite rotations as nodal coordinates, such elements do not require the use of incremental rotation procedures commonly used in the FE literature and commercial FE software. The resulting ANCF equations can be solved non-incrementally, and therefore, linearization of the kinematic equations is avoided.
4. Three-dimensional ANCF structural elements such as beams and plates lead to a constant mass matrix. Such a constant mass matrix cannot be obtained when using conventional beam, plate, and shell elements that employ infinitesimal or finite rotations as nodal coordinates.
5. Planar and spatial ANCF structural finite elements, such as beams and plates, capture deformation modes that cannot be captured by conventional beam and plate elements. For example, ANCF structural elements captures the deformation of the beam cross section and do not require the use of ad hoc approaches to describe cross section deformations.
6. Geometrically accurate infinitesimal-rotation finite elements can be developed using the ANCF displacement field leading to ANCF/CRBF finite elements that preserve the reference-configuration geometry [60, 61]. Such ANCF/CRBF elements, which can be used with the FFR formulation for the efficient small-deformation analysis, cannot be developed using the displacement fields of conventional finite elements. ANCF displacement fields also allow developing the lower-dimension infinitesimal rotation ANCF/CRBF finite elements without lowering the order of the interpolation.
7. ANCF structural finite elements allows for using both general continuum-mechanics approach and classical beam and plate theories. They can also be used to define more general and more

accurate shear-deformable elements based on the general definition of shear strains used in continuum-mechanics. Therefore, they are more general than elements that are limited to simplified beam and plate theories such as Timoshenko beam and Mindlin plate theories [62].

8. The fact that the displacement field can be written as $\mathbf{r}(\mathbf{x},t)=\mathbf{S}(\mathbf{x})\mathbf{e}(t)=\mathbf{S}(\mathbf{x})(\mathbf{e}_o+\mathbf{e}_d(t))$, where $\mathbf{e}_o$ and $\mathbf{e}_d$ are, respectively, the nodal coordinates in the reference configuration and vector of nodal displacements, allows describing complex curved geometry by proper choice of $\mathbf{e}_o$. Having the position gradients as nodal coordinates allows accomplishing this geometry description. Conventional rotation-based elements often assume rigid cross sections and cannot accurately describe complex reference configuration geometry, requiring use of large number of elements without achieving the desired smoothness.
9. In bending problems, cubic polynomials are required to achieve consistency with the partial differential equation of bending vibration and consistency with the fact that the curvature vector is defined by the second-order derivative. Full conformity of cubic surfaces of solid elements requires the use of forty eight nodal coordinates of the ANCF surface four nodes. This important property is automatically achieved by the ANCF solid elements and is not achieved by conventional brick elements including the 20-node brick element.
10. The fact that the ANCF position and gradients coordinates are independent nodal coordinates allows defining more general boundary conditions. For example, in conventional brick elements, one can fix the position coordinates, but this does not imply zero strain at this point. In the case of ANCF elements different boundary conditions can be conveniently applied including fixing the position and achieving zero strains by using the nodal coordinates directly.

11. In the control of soft robots, the use of the position gradients allows developing a more general inverse dynamics problem using MBS algorithms to simultaneously control the motion and shape, and this can be very difficult to achieve using conventional elements [59].

4. SUMMARY

The development and use of ANCF finite elements is expected to continue due to the lack of a viable alternative for the analysis of large deformation in MBS applications. Large rotation vector formulations (wrongly referred to as geometrically exact beam formulations) and isogeometric analysis (IGA) have serious limitations when used in general MBS algorithms as discussed in the literature. When using fully continuum elements such as ANCF elements, definition of torsion should be associated with space curves or fiber deformation since torsion is not one of the main shear-strain modes used in continuum mechanics. Torsional modes used in the vibration analysis of complex structures are associated with nominal curves whose geometry can accurately be described using higher-order interpolation as offered by ANCF elements. Therefore, it is not clear how torsion can in general be defined as a shear mode for ANCF finite elements since cross sections do not preserve their reference-configuration geometry in a fully continuum model.

Higher-order elements are also commonly used in the FE literature. If the 4-node solid element were adequate for all applications, there is no need for introducing and using the 20-node solid element implemented in commercial FE software. The 20-node solid element implemented in the commercial FE software does not offer the degree of continuity and conformity at the interface surfaces offered by the ANCF solid element. ANCF displacement fields allow increasing the order of interpolation without increasing the number of nodes or using the noncommutative finite rotations [63]. Such displacement fields also allow developing lower-dimension infinitesimal rotation ANCF/CRBF finite elements without lowering the order of the interpolation.

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